Chapter 3 The Value of a Forward Contract and Its Implications

Quiz Questions

Q1. Which of the following statements are false? Why?

(a) The forward rate $F_{t,T}$ is the certainty equivalent of the future spot rate. Therefore, the expected spot rate is equal to $F_{t,T}$.
(b) Market makers set the forward rate $F_{t,T}$ so that it is equal to the future spot rate.
(c) If you expect the spot rate to increase, it is more accurate to use the forward rate $F_{t,T}$ when recording an accounts receivable on the balance sheet a time *t*. Otherwise, use the spot rate. This rule ensures that your profits are maximized because your sales figures are maximized.
(d) The forward rate $F_{t,T}$ is the risk-adjusted expected value of the future spot exchange rate.
(e) It is expensive to record an accounts receivable on the balance sheet using the forward rate when it is lower than the spot rate.
(f) In perfect markets, the currency of invoicing is irrelevant, because both parties to a contract can immediately hedge in the forward market.
(g) Bidders to an international tender should be asked to submit prices in their home currency.

A1. (a) False. The forward rate is the certainty equivalent of the future spot rate, that is, the *risk-adjusted* expected value.
(b) False. The future spot rate is unknown.
(c) False. It is always best to use the forward rate, because it is the certainty equivalent of the future spot rate. The actual cash flows are unaffected by the way you account for the sale.
(d) True.
(e) False. The actual cash flows are unaffected by the way you account for the sale.
(f) True, but only when there is no time difference between the moments when the prices are set and when an order is made.
(g) True.

Q2. Given the following data, compute the value today of an outstanding forward purchase contract initiated at t_0 for 1,000,000 units of foreign currency (where the exchange rate is HC/FC). Does the new value represent a gain or loss to the holder of the old contract? (Hint: First compute the new forward rate.)

		Spot rate S_t	Old forward rate, $F_{t,T}$	$r_{t,T}$	$r^*_{t,T}$
(a)	BEF/DEM	20.5	22.0	3.5%	2.5%
(b)	JPY/NLG	57.5	54.2	1.25%	3.0%
(c)	ITL/FRF	283.0	289.4	4.5%	3.5%
(d)	CHF/GBP	2.2	1.8	2.0%	3.0%

A2. (a) The new forward rate is:

$$F_{t,T} = 20.5 \times \frac{1.035}{1.025} = 20.7.$$

Therefore, the value of the outstanding contract equals $[(20.7 - 22.0)/1.035] \times 1{,}000{,}000$ = BEF -1,256,039. To the holder of the old forward contract, this means a loss because the forward rate for DEM decreased since time t_0. In other words, to replace the old contract with a new one, the holder of the old contract must pay the counterpart BEF 1,256,039.

(b) The new forward rate is:

$$F_{t,T} = 57.5 \times \frac{1.0125}{1.03} = 56.52.$$

Therefore, the value of the outstanding contract equals [(56.52 - 54.2)/1.0125] × 1,000,000 = JPY 2,291,358. This means a gain to the holder of the contract.

(c) The new forward rate is:

$$F_{t,T} = 283 \times \frac{1.045}{1.035} = 285.73.$$

Therefore, the value of the outstanding contract equals [(285.73 - 289.4)/1.045] × 1,000,000 = ITL -3,511,962. This means a loss to the holder of the contract.

(d) The new forward rate is:

$$F_{t,T} = 2.2 \times \frac{1.02}{1.03} = 2.18.$$

Therefore, the value of the outstanding contract equals [(2.18 - 1.8)/1.02] × 1,000,000 = CHF 372,549. This means a gain to the holder of the contract.

Q3. Repeat the previous question using the same data, but with an outstanding forward sale contract.

A3. (a) The new forward rate is:

$$F_{t,T} = 20.5 \times \frac{1.035}{1.025} = 20.7.$$

Therefore, the value of the outstanding contract equals [(22.0 - 20.7)/1.035] × 1,000,000 = BEF 1,256,039. To the holder of the old forward sale contract, this means a gain, because this holder can sell DEM at a higher rate at time T than can someone who initiates a forward sale contract now. In other words, to replace the old contract by a new one, the holder will demand BEF 1,256,039.

(b) The new forward rate is $F_{t,T}$ = 56.52. Therefore, the value of the outstanding contract equals [(54.2 - 56.52)/1.0125] × 1,000,000 = JPY -2,291,358. This means a loss to the holder of a forward sale contract.

(c) The new forward rate is $F_{t,T}$ = 285.73. Therefore, the value of the outstanding contract equals [(289.4 - 285.73)/1.045] × 1,000,000 = ITL 3,511,962. This means a gain to the holder of the contract.

(d) The new forward rate is $F_{t,T}$ = 2.18. Therefore, the value of the outstanding contract equals [(1.8 - 2.18)/1.02] × 1,000,000 = CHF -372,549. This means a loss to the holder of the contract.

Note: The gains (losses) to the holder of the foward purchase contract in exercise 2 are the same but opposite in sign to the losses (gains) to the holder of the forward sales contract in exercise 3.

Q4. Lucky Lucas, a French chain of western-style restaurants, has received a shipment of Argentinian beef worth ARP 100,000 to be paid in 90 days. The invoice must be translated into FRF. The spot rate is FRF/ARP 4.2, and the forward rate is FRF/ARP 4.1.

(a) Using the spot rate, how would you record the invoice at time *t*? What is the accounting entry at time *T* if the spot rate at *T* equals 4.25?

(b) Using the forward rate, how would you record the invoice at time *t*? What is the accounting entry at time *T* if the spot rate equals 4.25?

(c) Are the profits and cash flows affected by the way in which the recording is made?

(d) Which method is more economically correct, *ex ante*? Why?

A4. (a) Recording an accounts payable at the spot rate at time *t*:

Sales	4,200,000		
A/P		4,200,000	

At time T:

A/P	4,200,000		
Bank account		4,250,000	
Capital gains or losses	50,000		(gain)

(b) Recording an accounts payable at the forward rate at time t:

Inventory	4,100,000		
A/P		4,100,000	

At time T:

A/P	4,100,000		
Bank account		4,250,000	
Capital gains or losses	150,000		(loss)

(c) The payment made from the bank account (FRF 4,250,000) is clearly not affected. Under the second method, sales are lower by FRF 100,000, but the capital gain is higher by FRF 100,000. Thus, in terms of overall profits, it also makes no difference.

(d) The method in which the forward rate is used is more accurate because the forward rate is the certainty equivalent of the future spot rate.

Exercises

E1. How do you evaluate the following claim: "The forward rate, if computed from IRP, entirely ignores expectations. In reality, the market evaluates the currency’s prospects, and takes into account not just the expected value but also the risks. Any theory which would have us mechanically compute the forward rate from the current spot rate and the interest rates is totally crazy."

Before formulating your comments, think about the direction of causality (if any) implied by IRP.

A1. The two claims—(1) the forward rate reflects the risk-adjusted expectations and (2) the forward rate can be computed from the spot and interest rates—are perfectly compatible because the spot rate and the interest rates are not determined exogenously. Rather, the spot rate takes into account (1) the risk-adjusted expected future value of the currency, (2) the foreign interest rate that is earned between t and T when foreign exchange is bought spot rather than forward, and (3) the domestic interest foregone when one buys spot rather than forward. Stated differently, both the spot and the forward rate are based on the risk-adjusted expectations, and the difference between these exchange rates merely reflects the net (dis)advantage that arises from postponed payment and delivery.

E2. Suppose that you sold forward (360 days) GBP 1m at the forward rate CAD/GBP 1.82, to hedge a payment from a customer. Eleven months later the GBP trades at CAD/GBP 2.1, and (annualized) interest rates for 30 days are 12 percent for the CAD, 18 percent for the GBP. Unexpectedly, the customer pays one month early. Consider the following alternatives. You may:

(a) Invest the GBP 1m for one month, and deliver them to the bank you signed the forward contract with.
(b) Sell the GBP 1m spot, and negotiate an early termination of the outstanding forward sale.
(d) Sell the GBP 1m spot, and buy them forward 30 days so that you can deliver the required amount to your bank.

Analyze each alternative. If the cash flows differ, trace the basis of the difference.

A2.

(a) At time $t_2 = T$ - 30 days:

- Invest GBP 1,000,000 for 1 month at 18 percent interest *p.a.* At time *T*, you receive GBP 15,000 in interest: GBP 1,000,000 $\times$ 0.15 = GBP 15,000.
- Sell the interest forward at the forward price $F_{t,T}$ = 2.1 $\times$ (1.01/1.015) = 2.08965517.

At time *T*:
Deliver GBP 1,000,000 to bank at CAD/GBP 1.82. Receive CAD 1,820,000.00
Deliver GBP 15,000 to bank at CAD/GBP 2.0895517. Receive CAD 31,344.83
TOTAL CAD 1,851,344.83

Another solution that is equal in value is to convert the t_2 value of the interest payment into CAD and to invest this at the CAD rate until *T*. Check this.

(b) At time $t_2 = T$ - 30 days:

- Sell GBP 1,000,000 spot at CAD/GBP 2.1 = CAD 2,100,000. To negotiate the early settlement of the forward contract the bank will charge you PV($F_{t2,T}$-$F_{t,T}$) or CAD 266,985.32.
- Invest the CAD (2,100,000 - 266,985.32) = 1,833,014.70 received at the CAD rate (12 percent) for 30 days.

At time *T*:
CAD deposit expires TOTAL CAD 1,851,344.83

(c) At time $t_2 = T$ - 30 days:

- Sell GBP 1,000,000 spot at CAD/GBP 2.1 = CAD 2,100,000. Invest the CAD 2,100,000 at the CAD rate (12 percent) for 30 days. At *T*, you receive CAD 2,121,000.
- Contract to purchase forward GBP 1,000,000 in 30 days at the new forward rate $F_{t2,T}$ = 2.08965517.

At time T:
Deliver GBP 1,000,000 to bank at 1.82 CAD/GBP. Receive CAD 1,820,000.00

Receive GBP 1,000,000 at 2.08965517 CAD/GBP.	Pay CAD –2,089,655.17
Expiration value of a CAD deposit.	Receive <u>CAD 2,121,000.00</u>
	TOTAL CAD 1,851,344.83

E3. You, a Belgian importer, have made a large order for Dotty Dolls. You should receive the shipment in six months, just in time for pre-Christmas shopping. The sales contract demands immediate payment upon receipt of the shipment. The unit price for your bulk order of 50,000 dolls is HKD 10. The spot exchange rate BEF/HKD is 6, and the simple p.a. interest rates for a six month investment in Belgium and Hong Kong are, respectively, 8 percent and 12 percent.

(a) How would you hedge your payment for the dolls?
(b) Suppose that three months after hedging the purchase, there is a fire in the doll factory. The dolls will not be delivered, but you still have an outstanding forward purchase contract for HKD 500,000. If the spot rate is now 6.1, and the simple *p.a.* interest rates for a three-month investment are 8 percent and 13 percent, in Belgium and Hong Kong, respectively, what is the value of your forward contract? Have you made a gain or loss?

A3. (a) 50,000 dolls at HKD 10 per doll means that you will owe HKD 500,000 in six months. You can hedge HKD 500,000 at the forward rate BEF/HKD 5.88679 (BEF/HKD 6 $\times$ (1.04/1.06)) for a total cost of BEF_T 2,943,396.
(b) Because your forward purchase contract has three months to go, you first need to know the current value of a three-month forward rate, which is BEF/HKD 6.1 $\times$ (1.02/1.0325) = 6.02615. You can now compute the value of the outstanding contract: (6.0262 - 5.8868)/1.02 = BEF 0.1366251 per HKD. Since you purchased forward HKD 500,000 at BEF/HKD 5.88679 and since you can sell these at 6.02615, you made money: this means a total time-*t* gain of BEF 68,312.58.

E4. Graham Cage, the mayor of Atlantic Beach, in the US, has received bids from three dredging companies for a beach renewal project. The work is carried out in three stages, with partial payment to be made at the completion of each stage. The current FC/USD spot rates are DEM/USD 1.6, FRF/USD 5.5, and CAD/USD 1.3. The effective USD returns that correspond to the completion of each stage are the following: $r_{0,1}$ = 6.00 percent, $r_{0,2}$ = 6.25 percent and $r_{0,3}$ = 6.50 percent. The companies' bids are shown below. Each forward rate corresponds to the expected completion date of each stage.

Company	stage 1	stage 2	stage 3
Hamburg Dredging	DEM 1,700,000	DEM 1,800,00	DEM 1,900,000
Forward rate DEM/USD	$F_{0,1}$ = 1.65	$F_{0,2}$ = 1.70	$F_{0,3}$ = 1.75
Marseille Dredging	FRF 5,200,000	FRF 5,800,000	FRF 6,500,000
Forward rate FRF/USD	$F_{0,1}$ = 5.50	$F_{0,2}$ = 5.45	$F_{0,3}$ = 5.35
Vancouver Dredging	CAD 1,300,000	CAD 1,400,000	CAD 1,500,000
Forward rate CAD/USD	$F_{0,1}$ = 1.35	$F_{0,2}$ = 1.30	$F_{0,3}$ = 1.25

(a) Which offer should Mayor Cage accept?
(b) Was he wise to accept the bids in each company's own currency? Please explain.

A4. (a) Mayor Cage should accept the bid made by Hamburg Dredging.

Company	Stage	USD value of bid at time 0	
Hamburg Dredging	1	1,700,000/1.65 × 1/1.06	= 971,984
	2	1,800,000/1.70 × 1/1.0625	= 996,540
	3	1,900,000/1.75 × 1/1.065	= 1,019,450
Total time-0 value of the bid		2,987,974	
Marseille Dredging	1	5,200,000/5.5 × 1/1.06	= 891,938
	2	5,800,000/5.45 × 1/1.0625	= 1,001,619
	3	6,500,000/5.35 × 1/1.065	= 1,140,801
Total time-0 value of the bid		3,034,358	
Vancouver Dredging	1	1,300,000/1.35 × 1/1.06	= 908,456
	2	1,400,000/1.3 × 1/1.0625	= 1,013,575
	3	1,500,000/1.25 × 1/1.065	= 1,126,761
Total time-0 value of the bid		3,048,791	

(b) Yes. The mayor can hedge using a standard forward contract. If the bids had been offered in CAD, each holder would have to use an expensive hedge *or* bear substantial risk. Both would have increased their bids.

E5. Suppose you have a clause in a forward purchase contract which gives you a partial timing option: you can buy USD against DEM at a forward rate $F_{t_0,T}$, either at time T, or two months earlier. Right now, you are at the intermediate decision date (T – 2 months) . So you have to decide whether to buy now or at T.

(a) Assume that, at the beginning of this two-month period, the term structure of compound interest rates is flat for maturities up to one year, with a *p.a.* DEM interest rate of 10 percent and a USD interest rate of 6 percent. Would you decide to buy the USD now ($t = T$ - 2 months), or would you rather wait? Make this decision in each of the following nine situations regarding the current spot rates S_t and contractual prices $F_{t_0,T}$:

Contract price $F_{t_0,T}$:	1.5	2	2.5 (DEM/USD)
Current spot rate			
1.5	…	…	…
2	…	…	…
2.5	…	…	…

(b) Repeat (a), but reverse the interest rates.

(c) How would your answers change if the clause is modified as follows: if you buy immediately, then the amount of DEM would be discounted (at the prevailing DEM rate), while the amount of USD payable would be discounted (at the prevailing USD interest rate). You should be able to do this without any computations.

A5. (a) Compare the pay off from exercising now, (S_t - $F_{t,T}$), versus the present value of the pay off from exercising at *T*, $\frac{S_t}{1 + r^*_{t,T}} - \frac{F_{t,T}}{1 + r_{t,T}}$.

Pay-off from exercising now, (S_t - $F_{t,T}$):

Contract price $F_{t,T}$:	1.5	2	2.5(DEM/USD)
Current spot rate			
1.5	0.0	-0.5	-1.0
2	0.5	0.0	-0.5
2.5	1.0	0.5	0.0

Pay-off from exercising at T, $\frac{S_t}{1 + r^*_{t,T}} - \frac{F_{t,T}}{1 + r_{t,T}}$:

Contract price $F_{t,T}$:	1.5	2	2.5(DEM/USD)
Current spot rate			
1.5	0.010	-0.482	-0.975
2	0.505	0.013	-0.479
2.5	1.000	0.508	0.016

In all cases, it is preferable to wait to exercise until time *T*. The reason is that the interest that can be earned on DEM exceeds the interest that can be earned on USD. So it is better to stay in DEM for the time being:

$$S_t - F_{t,T} = \frac{S_t}{1 + r^*_{t,T}} - \frac{F_{t,T}}{1 + r_{t,T}} + \left[\frac{\mathrm{r}^*_{t,T}}{1 + r^*_{t,T}} \times S_t - \frac{\mathrm{r}_{t,T}}{1 + r_{t,T}} \times F_{t_0,T} \right]$$

= Present value of the later cash flows + the time value effect.

(b) The pay-off from exercising now, (S_t - $F_{t,T}$), is the same as in (a).

Pay-off from exercising at *T*, $\frac{S_t}{1 + r^*_{t,T}} - \frac{F_{t,T}}{1 + r_{t,T}}$:

Contract price $F_{t,T}$:	1.5	2	2.5(DEM/USD)
Current spot rate			
1.5	0.010	0.505	1.000
2	-0.482	0.013	0.508
2.5	-0.975	-0.479	0.016

In all case it is preferable to exercise at time *T-2.*

(c) The formula for the pay off now, at *t*, would become:

$$\frac{S_t}{1+\mathrm{r}^*_{t,T}} - \frac{F_{t,T}}{1+\mathrm{r}_{t,T}}.$$

This is exactly the same as the present value of the pay off at *T*. In this case, you are indifferent between buying now and within two months.

Mind-Expanding Exercises

ME1. Suppose that you expect to receive FRF 1 at a future date T. We can translate this FRF cash flow into home currency, LUF, and discount; or discount at a FRF cost of capital, and then translate. For simplicity, assume away uncertainty about the FRF cash flows. Which of the following alternatives are correct? Why? Under what assumptions?

(a) Translate FRF cash flows into home currency using the expected future spot rate, and discount at the home currency risk-free rate.
(b) Translate FRF cash flows into home currency using the forward rate for that maturity, and discount at the home currency risk-free rate.
(c) Discount at the FRF risk-free rate, and translate using the expected future spot rate.
(d) Discount at the FRF risk-free rate, and translate using the current spot rate.
(e) Translate FRF cash flows at the current spot rate, and discount at the home currency risk-free rate.

A1. (a) This method discounts a risky expected cash flow $E_t(S_T)$ (an amount of expected LUF) at the risk-free LUF rate. Therefore, (a) assumes risk neutrality in nominal terms. This is generally incorrect. You should either have adjusted $E(S_T)$ for risk, or discounted $E(S_T)$ at a risk-adjusted rate.
(b) This is correct: $F_{t,T}$ is the risk-adjusted expectation, so you need no more risk-adjustment in the discount rate.
(c) This is, in general, a joke: you should have used S_t, not $E_t(S_T)$—see the next formula.
(d) This is correct. Compute the present value in FRF of the cash flow using the risk-free rate, $\frac{1}{1 + r^*_{t,T}}$, and translate the FRF present value into LUF at today's spot rate. Alternatively, use IRP to show that (d) is equivalent to (b).
(e) This is a joke too, in general. You should have used $F_{t,T}$, not S_t.

ME2. By "marking to market a forward contract" we mean "adjusting the forward rate fixed in an old contract to the currently prevailing forward rate" (see also Chapter 4). For instance, an old DEM_{T2} contract at F_{t,T_2} = BEF/DEM 20.5 would be replaced, at time $T_1 < T_2$, by a new forward contract at the then prevailing rate $F_{T_1,T_2} = 20.6$.

Clearly the change in the terms of the contract will require some compensation from the loser to the winner. What payment should be made? Assume it is a purchase for DEM 10m, and that $r_{T_1,T_2} = 0.02$. Check if both parties are indifferent to marking to market.

A2. For the buyer, the old contract has a positive value of:

$$\frac{(20.6 - 20.5)}{1.02} \times 10{,}000{,}000 = \text{DEM } 980{,}392,$$

while the value of the new contract is zero; so the seller is adequately compensated if the buyer pays him DEM 980,392. The buyer can invest this amount at a return of 2 percent, yielding 1,000,000—exactly the difference between the BEF_{T_2} 206,000,000 due under the new contract, and the BEF_{T_2} 205,000,000 due under the old contract.

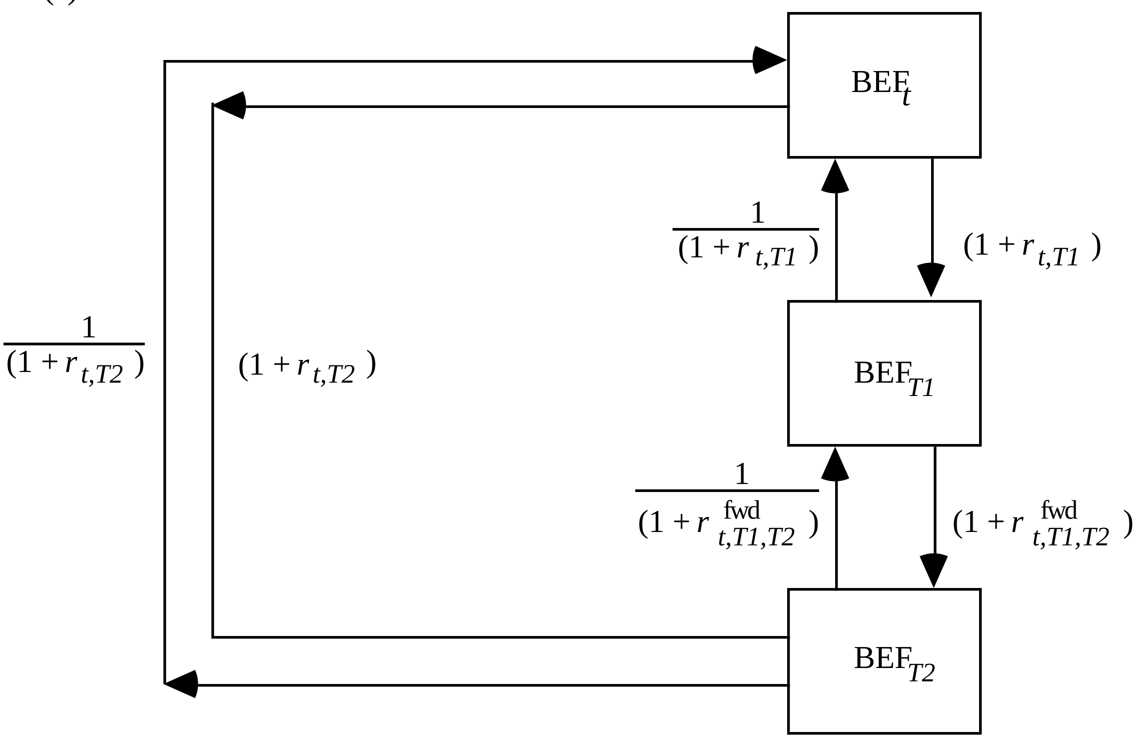
The seller can likewise borrow the DEM 980,392 he needs to settle the old contract; the loan plus interest will exactly wipe out the difference between the old and new BEF_{T_2} inflows.

ME3. In principle, a Future Rate Agreement (FRA) in principle fixes an interest rate for a deposit or loan starting at a future time $T_1 > t$ and expiring at $T_2 > T_1$. For instance, a six-to-nine-month FRA at 10 percent (simple annualized interest) fixes the return on a three-month deposit, to be made six months from now, at 10%/4 = 2.5%. Thus, the input of this transaction is BEF_{T_1}, and the output is $BEF_{T_2} = BEF_{T_1} \times (1 + r^{fwd}_{t,T_1,T_2})$.

(a) Make a diagram showing all possible transactions; derive the no-arbitrage bounds; check that the least-cost dealing computations are unnecessary in the absence of spreads when the no-arbitrage conditions are met.

(b) In practice, the deposit is notional, or theoretical. You do not have to effectively make a deposit; instead, at time T_1 there is a cash settlement of the difference between the contracted forward interest rate (r^{fwd}) and the time T_1 prevailing market rate. How do we compute the amount to be paid or received at T_1?

A3. (a)



A clockwise round-trip, starting and ending in, for example, BEF_{T_2}, yields the no-arbitrage condition:

$$\frac{1}{1+r_{t,T_2}}\ (1+r_{t,T_1})\ (1+r^{fwd}_{t,T_1,T_2}) \le 1.$$

The counter-clockwise trip gives:

$$\frac{1}{1+r^{fwd}_{t,T_1,T_2}}\ \frac{1}{1+r_{t,T_1}}\ (1+r_{t,T_2}) \le 1.$$

Taken together, the round-trip applications lead to the no-arbitrage condition:

$$(1 + r_{t,T_2}) = (1 + r_{t,T_1}) \times (1 + r^{\text{fwd}}_{t,T_1,T_2}),$$

and this satisfies the *Law of One Price* for all least-cost dealing applications.

(b) Because settlement is at T_1, while interest on a genuine deposit normally would have been paid at T_2, you have to discount the gain or the loss. The theoretical payment is:

$$[\text{notional deposit}] \times \frac{r^{\text{fwd}}_{t,T_1,T_2} - r_{T_1,T_2}}{1 + r_{T_1,T_2}}$$

For example, consider a nine-to-twelve-month LUF 50,000,000 notional deposit at a forward interest rate of 12 percent (that is, a forward return of 3 percent). If LIBOR at time T_1 turns out to be 10 percent (or a return of 2.5 percent), the investor will receive the discounted shortfall[1], or:

$$50{,}000{,}000 \times \frac{0.03 - 0.025}{1.025} = 243{,}902 \ (\text{BEF}_{T_1})$$

ME4. A friend, who works in the London City, claims that he has a bright idea for a new financial product: the Forward Operation on a Forward Exchange Liquidation (FOOFEL). A FOOFEL starts with a notional forward purchase contract signed at t and expiring at T_2; and it stipulates that, at a pre-specified time T_1 ($< T_2$), the loser will buy back this contract from the winner.

(a) What payment should be made from the loser to the winner (as a function of S, F, r, r^* observed at T_1)?

(b) Explain the difference and similarity with a forward contract F_{t,T_2} having as an additional clause that it will be marked to market at T_1.

(c) Is the FOOFEL aimed at hedgers, speculators, or both? If speculators use it, is this a bet on time-T_1 spot rates, or on interest rates, or what?

(d) How should the initial contract price be set such that the FOOFEL has a zero initial value?

A4. (a) The market value of the 'old' FOOFEL contract at T_1 is:

$$\frac{S_{T_1}}{1 + r^*_{T_1,T_2}} - \frac{F_{t,T_2}}{1 + r_{T_1,T_2}}$$

(b) The time-*T1* cash flow is the same for both. But under the FOOFEL, the new contract ends up in the hands of the loser (who bought it from the winner), while in the other case, the old contract remains with the original holder.

(c) From equation [9], it is a bet on the spot rate plus the two interest rates, but the *ex ante* variance of the time-*T1* spot rate is probably the biggest source of potential variations in the FOOFEL's time-*T1* value. But you need no FOOFEL for such a bet. In fact, you get exactly the same bet by signing, at t, a contract for delivery at *T2* at a price X. This contract will have exactly the same time-*T1* market value as the FOOFEL.

(d) From the previous question, we can get exactly the same time-*T1* value by signing a forward purchase contract (t,T_2, X). This contract has zero time-t value for $X = F_{t,T_2}$.

1. In practice, the LIBOR of two days before T_2 is used.

Chapter 4 Forward Contracts with Market Imperfections

Quiz Questions

Q1. Which of the following are risks that arise when you hedge by buying a forward contract in financial markets that are imperfect?
(a) Credit risk: the risk that the counterpart to a forward contract defaults.
(b) Hedging risk: the risk that you are not able to find a counterpart for your forward contract if you want to close out early.
(c) Reverse risk: the risk that results from a sudden unhedged position because the counterpart to your forward contract defaults.
(d) Spot rate risk: the risk that the spot rate has changed once you have signed a forward contract.

A1. Surely (a) & (c) if the counterpart is not top notch or has not put up substantial margin.
(b) is not a major risk because you can otherwise close out in the forward market or hedge via the money markets.
(d) is a risk in the sense that, at time T, you may regret your forward purchase. (d) is not a risk on the sense that your cash flow is not affected by S_T, barring reverse risk.

Q2. Which of the following statements are true?
(a) Margin is a payment to the bank to compensate it for taking on credit risk.
(b) If you hold a forward purchase contract for JPY which you wish to reverse, and the JPY has increased in value, you owe the bank the discounted difference between the current forward rate and the historic forward rate, that is, the market value.
(c) If the balance in your margin account is not sufficient to cover the losses in the value of your forward contract and you fail to post additional margin, the bank must speculate in order to recover the losses.
(d) Under the system of daily recontracting, the value of an outstanding forward contract is recomputed every day. If the forward rate for GBP/DEM drops each day for ten days until the forward contract expires, the purchaser of DEM forward must pay the forward seller of DEM the market value of the contract for each of those ten days. If the purchaser cannot pay, the bank seizes his or her margin.

A2. (a) Margin is not a payment; it is a security deposit.
(b) No. The contract has increased in value. That is, you made a gain rather than a loss.
(c) No. The bank will seize the margin and reverse the forward contract.
(d) True.

Q3. Innovative Bicycle Makers must hedge an accounts payable of MAD 100,000 due in 90 days for bike tires purchased in Malaysia. Suppose that the GBP/MAD forward rates and the GBP effective returns are as follows:

Time	$t = 0$	$t = 1$	t = 2	$t = 3$
Forward rate	4	4.2	3.9	4
Effective return	12%	8.5%	4%	0%

(a) What are IBM's cash flows given a variable-collateral margin account?
(b) What are IBM's cash flows given periodic contracting?

A3.

Forward price, $F_{t,3}$, in GBP/MAD	GBP return, $r_{t,3}$	Variable Collateral	Periodic Recontracting
At time 0: $F_{0,3} = 4$	12%	IBM buys forward at $F_{0,3} = 4$	IBM buys forward at $F_{0,3} = 4$
At time 1: $F_{1,3} = 4.5$	8.5%	Market value of the contract is $\frac{4.5 - 4}{1.085} = 0.461$. IBM's margin account is worth 0.461.	Market value of the contract is $\frac{4.5 - 4}{1.085} = 0.461$. IBM receives 0.461 for the old contract, and signs a new contract at $F_{1,3} = 4.5$.
At time 2: $F_{2,3} = 3.7$	4%	Market value of the contract is $\frac{3.7 - 4}{1.04} = -0.288$. IBM deposits at least -0.288 in its margin account as collateral.	Market value of the contract is $\frac{3.7 - 4.5}{1.04} = -0.769$. IBM buys back the old contract for -.769, and signs a new contract at $F_{2,3} = 3.7$.
At time 3: $S_3 = 4$	0%	IBM pays per MAD: 0 4 The collateral in IBM's margin account is returned to IBM, including the interest earned on it.	A's payments adjusted for time value: time 3: (purchase of MAD) = 3.7 time 2: 0.769×1.04 = 0.8 time 1: -0.461×1.085 = -0.5 4.0

Q4. Which of the following statements are correct?

(a) A forward purchase contract can be replicated by borrowing foreign currency, converting it to domestic currency, and investing the domestic currency.

(b) A forward purchase contract can be replicated by borrowing domestic currency, converting it to foreign currency, and investing the foreign currency.

(c) A forward sale contract can be replicated by borrowing foreign currency, converting it to domestic currency, and investing the domestic currency.

(d) A forward sale contract can be replicated by borrowing domestic currency, converting it to foreign currency, and investing the foreign currency.

A4. (b), (c).

Q5. The following spot and forward rates are in units of BEF/FC. The forward spread is quoted in centimes.

	Spot bid-ask	1-month		3-month		6-month		12-month	
1 NLG	18.21–18.30	+0.6	+0.8	+2.1	+2.7	+3.8	+4.9	+6.9	+9.1
1 FFR	5.95–6.01	-0.1	-0.2	-0.3	-0.1	-0.7	-0.3	-0.9	+0.1
1 CHF	24.08–24.24	+3.3	+3.7	+9.9	+10.8	+19.3	+21.1	+36.2	+39.7
100 JPY	33.38–33.52	+9.5	+9.9	+28.9	+30.0	+55.2	+57.5	+99.0	+105.
1 ECU	39.56–39.79	-1.7	-1.0	-3.4	-1.8	-5.8	-2.9	-10.5	-5.2

A5. This question is incomplete! Please see Question 6.

Q6. Choose the correct answer.

i. The one-month forward bid-ask qutoes for CHF are:
a. 27.387–27.942 b. 25.078–24.357 c. 24.113–24.277 d. 24.410–24.610

ii. The three-month forward bid-ask quotes for ECU are:
a. 39.526–39.772 b. 36.167–37.992 c. 39.641–40.158 d. 39.397–39.699

iii. The six-month forward bid-ask quotes for JPY are:
a. 38.902–39.273 b. 88.584–91.025 c. 33.686–33.827 d. 33.932–34.095

iv. The twelve-month forward bid-ask qutoes for NLG are:
a. 18.731–19.352 b. 25.113–27.404 c. 17.305–17.716 d. 18.279–19.391

A5. i. c.; ii. a.; iii. d.; iv. d.

Q6. Suppose you are quoted the following DEM/FC spot and forward rates:

	Spot bid-ask	3-mo. forward bid-ask	*p.a.* 3 month Euro-interest	6-mo. forward bid-ask	*p.a.* 6-month Euro-interest
DEM			5.65–5.90		5.47–5.82
USD	0.5791–0.5835	0.5821–0.5867	3.63–3.88	0.5839–0.5895	3.94–4.19
ECU	0.5120–0.5159	0.5103–0.5142	6.08–6.33	0.5101–0.5146	5.60–6.25
FFR	3.3890–3.4150	3.3350–3.4410	6.05–6.30	3.3720–3.4110	5.93–6.18
JPY*	0.5973–0.6033	0.5987–0.5025	1.71–1.96	0.5023–0.5099	2.47–2.75
GBP	0.3924–0.3954	0.3933–0.3989	5.09–5.34	0.3929–0.3001	5.10–5.35

*The DEM/JPY exchange rate is for 100 JPY.

(a) What are the three-month synthetic-forward DEM/USD bid-ask rates?
(b) What are the six-month synthetic-forward DEM/ECU bid-ask rates?
(c) What are the six-month synthetic-forward DEM/FFR bid-ask rates?
(d) What are the three-month synthetic-forward DEM/JPY bid-ask rates?
(e) In (a)–(d), are there any arbitrage opportunities? Are there opportunities for least-cost dealing at the synthetic rate?

A6.(a) 0.5816–0.5868; (b) 0.5117–0.5148; (c) 3.381–3.409; (d) 0.6028–0.6096.
(e) DEM/USD: no arbitrage opportunity; DEM/ECU: least cost dealing opportunity for sellers of ECU; DEM/FFR: least-cost dealing opportunity for both buyers and sellers of FFR; DEM/JPY: arbitrage opportunity.

Q7. True or False: Occasionally arbitrage bounds are violated using domestic ("on-shore") interest rates because:
(a) Offshore or euromarkets are perfect markets while "on-shore" markets are imperfect.

(b) Offshore or euromarkets are efficient markets while "on-shore" markets are inefficient.

A7. Neither (a) nor (b). Neither market is perfect—although off-shore markets tend to be less imperfect.

Exercises

E1. Michael Milkem, an ambitious MBA student from Anchorage, Alaska, is looking for free lunches on the foreign exchange markets. Keeping his eyes glued to his Reuters screen until the wee hours, he spots the following quotes in Tokyo:

Exchange rates:spot	DEM/USD 1.59–1.60	JPY/USD 100–101
	DEM/GBP 2.25–2.26	JPY/GBP 150–152
180-day Forward	DEM/USD 1.615–1.626	JPY/USD 97.96–98.42
	DEM/GBP 2.265–2.274	JPY/GBP 146.93–149.19
Interest rates (simple, p.a.)		
180 days	USD 5%–5.25%	JPY 3%–3.25%
	DEM 8%–8.25%	GBP 7%–7.25%

Given the above qutoes, has Michael found any arbitrage opportunities?

A1. The synthetic 180-day forward quotes are DEM/USD 1.6113–1.6254, JPY/USD 98.9038–100.1378, DEM/GBP 2.258–2.2736, JPY/GBP 146.924–149.2464. There is an opportunity for least-cost dealing when selling USD against JPY, and when buying GBP against DEM, but Michael is only interested in a free lunch (and not in the cheapest way to take a position in a currency). So, because the arbitrage bounds for the JPY/USD rate are violated, he will buy USD with JPY in the direct market and sell the USD synthetically in order to make a risk-free profit.

E2. Polyglot Industries will send its employee Jack Pundit to study French in an intensive training course at the Sorbonne. Jack will need FRF 10,000 at t = 3 months when classes begins, and FRF 6,000 at t = 6 months, t = 9 months, and t = 12 months to cover his tuition and living expenses. The exchange rates and *p.a.* interest rates are the following:

USD/FRF	Exchange rate	*p.a.* interest rate USD	*p.a.* interest rate FFR
spot	5.820–5.830		
90 days	5.765–5.770	3.82–4.07	8.09–8.35
180 days	5.713–5.720	3.94–4.19	8.00–8.26
270 days	5.660–5.680	4.13–4.38	7.99–8.24
360 days	5.640–5.670	4.50–4.75	7.83–8.09

Polyglot wants to lock in the FRF value of Jack's expenses. Is it indifferent between buying FRF forward and investing in FRF for each time period that he should receive his allowance?

A2. The synthetic USD/FRF forward rates are:

USD/FRF	Exchange Rate
90 days	5.76–5.77
180 days	5.70–5.72
270 days	5.65–5.68
360 days	5.63–5.67

Because the rates on the synthetic market equal or exceed those on the direct forward market, Polyglot will always prefer to buy FRF forward directly.

E3. Check that a money market hedge is equivalent to an outright forward transaction. Analyze, for instance, a forward sale of DKK 1 against DEM.

A3. Six months: borrow DEM $\frac{1}{1.025}$, convert spot, and invest at an effective return of 5.0625 percent; your DEM debt is 1, your DKK inflow will be $\frac{1}{1.025} \times 1.050625 = 1.025$, QED. Selling DKK 1 at a forward rate of 1.025 gives the same result.

Twelve months: borrow DEM $\frac{1}{1.05}$, convert spot, and invest at an effective return of 10.25 percent; your DEM debt is 1, your DKK inflow will be $\frac{1}{1.05} \times 1.1025 = 1.05$, QED. This is equivalent to selling forward at 1.05.

Exercises 4 through 6 use the following time-0 data for the fictitious currency, the Walloon Franc (WAF) and the Flemish Yen (FLY), on Jan. 1, 2000. The spot exchange rate is 1 WAF/FLY.

	Interest rates		Swap rate
	FLY	WAF	WAF/FLY
180 days	5%	10.125%	0.025
360 days	5%	10.250%	0.050

E4. On June 1, 2000, the FLY has depreciated to WAF 0.90, but the six-month interest rates have not changed. In early 2001, the FLY is back at par. Compute the gain or loss (and the cumulative gain or loss) on two consecutive 180-day forward sales (the first one is bought at 2/1/2000), when you start with a FLY 500,000 forward sale. First do the computations without increasing the size of the forward contract. Then verify how the results are affected if you do increase the contract size, at the roll-over date, by a factor $1 + r^*_{T_1,T_2}$—that is, from FLY 500,000 to FLY 525,000.

A4. The first 180d: $(1.025 - 0.90) \times 500{,}000$ = WAF 62,500 profit.

The new forward rate: $\frac{0.9}{1.025} \times 1.050625 = 0.9225$. So if you do not adjust the contract size, your second profit will tbe $(0.9225 - 1) \times 500{,}000 = -38{,}750$. The total, not corrected for time value, is 62,500 - 38,750 = 23,750.

The cumulative profit makes sense only if you bring in interest rates. First, you reinvest the first gain: $62{,}500 \times 1.050625 = 65{,}664$. The second time you increase the contract size to $500{,}000 \times 1.025 = 512{,}500$ so that your ex post result from the second contract is

512,500 $\times$ (0.9225 - 1) = -39,718.7. Thus, your total profit is 65,664 - 39,718.7 = 25,945.3.

E5. Repeat the previous exercise, except that after six months the exchange rate is at WAF/FLY 1, not 0.9.

A5. The first 180d: (1.025 - 1) $\times$ 500,000 = WAF 12,500 profit.

The new forward rate: $\frac{1}{1.025} \times 1.050625 = 1.025$. So if you do not adjust the contract size, your second profit will be (1.025 - 1) $\times$ 500,000 = 12,500. Notice how the total, without correction for time value, now is 25,000.

The cumulative profit makes sense only if you bring in interest rates. First, you reinvest the first gain: 12,500 $\times$ 1.050625 = 13,132.8. The second time you increase the contract size to 500,000 $\times$ 1.025 = 512,500 so that your ex post gain from the second contract is 512,500 $\times$ (1.025-1) = 12,812.5. Thus, your total profit is 13,132.8 + 12,812.5 = 25,945.3, as before.

Conclusion:

- When rolling over short-term contracts, the result is "essentially" independent of the intermediate spot rate: the profit is around 25,000.
- We can entirely eliminate the uncertainty about the intermediate spot rate by slightly increasing the forward contract's size at each roll-over date. Then, the profit is 25,945.30 independent of the intermediate spot rate.
- The final result always depends on the interest rates at the roll-over date.

E6. Compare the analyses in exercises 4 and 5 with a rolled-over money-market hedge. That is, what would have been the result if you had borrowed WAF for six months (with conversion and investment of FLY—the money-market replication of a 6-month forward sale), and then rolled-over (that is, renewed) the WAF loan and the FLY deposit, principal plus interest?

A6. Borrow FLY $\frac{500{,}000}{1.025}$ = 487,804.88, convert into WAF, and invest. The values are:

	WAF deposit	FLY debt	net value
time-0	487,804.88	487,804.88	0.0
time-1	512,500.00	500,000.00	12,500.0
time-2	538,445.30	512,500.00	25,945.3

Rolling over money market hedges is the same as rolling over forward contracts. Clearly, the intermediate spot exchange rates here are irrelevant, and the only risk is interest rate risk.

Chapter 5 Currency Futures Markets

Quiz Questions

Q1. For each pair shown below, which of the two describes a forward contract? Which describes a futures contract?

(a) standardized/made to order
(b) interest rate risk/no interest rate risk
(c) ruin risk/no ruin risk
(d) short maturities/even shorter maturities
(e) no secondary market/liquid secondary market
(f) for hedgers/speculators
(g) more expensive/less expensive
(h) no credit risk/credit risk
(i) organized market/no organized market

A1.

	Forward contract	Futures contract
(a)	made to order	standardized
(b)	no interest rate risk	interest rate risk
(c)	no ruin risk	ruin risk
(d)	short maturities	even shorter maturities
(e)	no secondary market	liquid secondary market
(f)	for hedgers	for speculators
(g)	more expensive	less expensive
(h)	credit risk	no credit risk
(i)	no organized market	organized market

Q2. Match the vocabulary below with the following statements.

(a) organized market
(b) standardized contract
(c) standardized expiration
(d) clearing corporation
(e) daily recontracting
(f) marking to market
(g) convergence
(h) settlement price
(i) default risk of a futures
(j) initial margin
(k) maintenance margin
(l) margin call
(m) variation margin
(n) open interest
(o) interest rate risk
(p) cross hedge
(q) delta hedge
(r) delta-cross hedge
(s) ruin risk

________ 1. Daily payment of the change in a forward or futures price.
________ 2. The collateral deposited as a guarantee when a futures position is opened.
________ 3. Daily payment of the discounted change in a forward or futures price.
________ 4. The minimum level of collateral on deposit as a guarantee for a futures position.
________ 5. A hedge on a currency for which no futures contracts exist and for an expiration other than what the buyer or seller of the contract desires.
________ 6. An additional deposit of collateral for a margin account that has fallen below it maintenance level.
________ 7. A contract for a standardized number of units of a good to be delivered at a specific date.

_______ 8. A hedge on a foreign-currency accounts receivable or accounts payable that is due on a day other than the last Wednesday of March, June, September, or December.
_______ 9. The number of outstanding contracts for a given type of futures.
_______ 10. The one-day futures price change.
_______ 11. A proxy for the closing price which is used to ensure that a futures price is not manipulated.
_______ 12. Generally, the Wednesday of March, June, September, or December.
_______ 13. Organization that acts as a "go-between" for buyers and sellers of futures contracts.
_______ 14. The risk that the interim cash flows must be invested or borrowed at an unfavorable interest rate.
_______ 15. A hedge on a currency for which no futures contract exists.
_______ 16. The risk that the price of a futures contract drops (rises) so far that the purchaser (seller) has a severe short-term cash flow problems due to marking to market.
_______ 17. The property whereby the futures equals the spot price at expiration.
_______ 18. Centralized market (either an exchange or a computer system) where supply and demand are matched.

A2. 1. (f); 2. (j); 3. (e); 4. (k); 5. (r); 6. (m); 7. (b); 8. (q); 9. (n); 10. (f); 11. (h); 12. (c); 13. (d); 14. (o); 15. (p); 16. (s); 17. (g); 18. (a).

The table below is an excerpt of futures prices from *The Wall Street Journal* of Tuesday, March 22, 1994. Use this table to answer questions 3 through 6.

						Lifetime		Open
	Open	High	Low	Settle	Change	High	Low	Interest
JAPAN YEN (CME) — 12.5 million yen; $ per yen (.00)								
June	.9432	.9460	.9427	.9459	+ .0007	.9945	.8540	48,189
Sept	.9482	.9513	.9482	.9510	+ .0007	.9900	.8942	1,782
Dec	.9550	.9610	.9547	.9566	+ .0008	.9810	.9525	384
Est vol 13,640; vol Fri 15,017; open int 50,355, + 414								
DEUTSCHEMARK (CME) — 125,000 marks; $ per marks								
June	.5855	.5893	.5847	.5888	+ .0018	.6162	.5607	87.662
Sep	.5840	.5874	.5830	.5871	+ .0018	.6130	.5600	2,645
Dec	.5830	.5860	.5830	.5864	+ .0018	.5910	.5590	114
Est vol 40,488; vol Fri 43,717; open int 90,421, -1,231								
CANADIAN DOLLAR (CME) — 100,000 dlrs.; $ per Can $								
Jun	.7296	.7329	.7296	.7313	+ .0021	.7805	.7290	43,132
Sep	.7293	.7310	.7290	.7297	+ .0018	.7740	.7276	962
Dec	.7294	.7295	.7285	.7282	+ .0016	.7670	.7270	640
Mar95	.7263	.7263	.7263	.7267	+ .0015	.7605	.7260	152
Est vol 5,389; vol Fri 4,248; open int 44,905, -1,331								

Q3. What is the CME contract size for:
(a) Japanese yen?
(b) German mark?
(c) Canadian dollar?

A3. (a) 12.5 million yen; (b) 125,000 marks; (c) 100,000 dollars.

Q4. What is the open interest for the September contract for:
(a) Japanese yen?
(b) German mark?
(c) Canadian dollar?

A4. (a) 1,782; (b) 2,645; (c) 962 contracts.

Q5. What are the daily high, low, and settlement prices for the December contract for:
(a) Japanese yen?
(b) German mark?
(c) Canadian dollar?

A5. (a) high: 0.9610, low: 0.9547, settle: 0.9566; (b) high: 0.5860, low: 0.5830, settle: 0.5864; (c) high: 0.7295, low: 0.7285, settle: 0.7282.

Q6. What is the day's cash flow from marking to market for the holder of a:
(a) JPY June contract?
(b) DEM June contract?
(b) CAD June contract?

A6.(a) $0.0007/100 \times 12.5$ million = USD 87.50 (inflow).
(b) $0.0018 \times 125{,}000$ = USD 2.25 (inflow).
(c) $0.0021 \times 100{,}000$ = USD 210 (inflow).

Exercises

E1. On the morning of Monday, August 21, you purchase a futures contract for 1 unit of CHF at a rate of USD/CHF 0.7. The subsequent settlement prices are shown in the table below.
(a) What are the daily cash flows from marking to market?
(b) What is the cumulative total cash flow from marking to market (ignoring discounting)?
(c) Is the total cash flow greater than, less than, or equal to the difference between the price of your original futures contract and the price of the same futures contract on August 30?

August	21	22	23	24	25	28	29	30
Futures rate	0.71	0.70	0.72	0.71	0.69	0.68	0.66	0.63

A1. (a)

August	21	22	23	24	25	28	29	30
Cash flow	0.01	-0.01	0.02	-0.01	-0.02	-0.01	-0.02	-0.03

(b) -0.07.
(c) Equal to.

E2. On November 15, you sold ten futures contracts for CAD 100,000 each, at a rate of USD/CAD 0.75. The subsequent settlement prices are shown in the table below.
(a) What are the daily cash flows from marking to market?
(b) What is the total cash flow from marking to market (ignoring discounting)?
(c) If you deposit USD 75,000 into your margin account, and your broker requires USD 50,000 as maintenance margin, when will you receive a margin call and how much will you have to deposit?

November	16	17	18	19	20	23	24	25
futures rate	0.74	0.73	0.74	0.76	0.77	0.78	0.79	0.80

A2. (a)

November	16	17	18	19	20	23	24	25
cash flow	0.01	0.01	-0.01	-0.02	-0.01	-0.01	-0.01	-0.01

(b) 1m $\times$ -.05 = USD -50,000

(c)

November	16	17	18	19	20	23*	24	25
Margin account	85,000	95,000	85,000	65,000	55,000	~~45,000~~ 75,000	65,000	55,000

You will get a margin call from your broker on November 23 for a deposit of variation margin equaling USD 30,000.

E3. On the morning of December 6, you purchased a futures contracts for one USD at a rate of BEF/USD 55. The following table gives the subsequent settlement prices and the *p.a.* bid-ask interest rates on a BEF investment made until December 10th.

December	6	7	8	9	10
futures price	56	57	54	52	55
bid-ask interest rates on BEF in %	12.00–12.25	11.50–11.75	13.00–13.25	13.50–13.75	NA

(a) What are the daily cash flows from marking to market?
(b) What is the total cash flow from marking to market (ignoring discounting)?
(c) If you must finance your losses and invest your gains from marking to market, what is the value of the total cash flows on December 10?

A3. (a)

December	6	7	8	9	10
Cash flow	1	1	-3	-2	3

(b) USD 0.
(c) Using the convention of 360-days per year:

December	6	7	8	9	10
Cash flow	1	1	-3	-2	3
Future value of cash flow invested until Dec. 11th	1.0013333	1.0009583	-3.0022083	-2.0007639	3

The total future value of the cash flows equals -0.00068.

E4. You want to hedge the DEM value of a CAD 1m inflow using futures contracts. On Germany's exchange, there is a futures contract for USD 100,000 at DEM/USD 1.5.
(a) Your assistant runs a bunch of regressions:

(1) $\Delta S[\text{DEM/CAD}] = \alpha_1 + \beta_1\ \Delta f[\text{USD/DEM}]$.

(2) $\Delta S[\text{DEM/CAD}] = \alpha_2 + \beta_2\ \Delta f[\text{DEM/USD}]$.

(3) $\Delta S[\text{CAD/DEM}] = \alpha_3 + \beta_3\ \Delta f[\text{DEM/USD}]$.

(4) $\Delta S[\text{CAD/DEM}] = \alpha_4 + \beta_4\ \Delta f[\text{USD/DEM}]$.

Which regression is relevant to you?

(b) If the relevant β is 0.83, how many contracts do you buy? Sell?

A4. (a) regression (2). Both sides of the regression take the DEM as the home currency (= DEM). The left-hand side is the spot rate that you are exposed to, and the right-hand side is the futures rate you use as a hedge.

(b) You sell USD $\frac{1{,}000{,}000}{100{,}000} \times 0.83 = 8.3$ contracts, or after rounding, 8 USD contracts.

E5. In the previous question, we assumed that there was a USD futures contract in Germany, with a fixed number of USD (100,000 units) and a variable DEM/USD price. What if there is no German futures exchange? Then you would have to go to a US exchange, where the number of DEM per contract is fixed (at, say, 125,000), rather than the number of USD. How many USD/DEM contracts will you buy?

A5. If hedging is done on a U.S. futures exchange, you buy forward eight contracts worth USD 100,000 each for a total of USD 800,000. At the futures rate of DEM/USD 1.5, this corresponds to 800,000 × 1.5 = DEM 1,200,000, or about ten contracts of DEM 125,000 each.

E6. A German exporter wants to hedge an outflow of NZD 1m. She decides to hedge the risk with a DEM/USD contract and a DEM/AUD contract. The regression output is, with the t-statistics shown in parentheses, and $R^2 = 0.59$:

$$\Delta S_{[\text{DEM/AUD}]} = a + \underset{(1.57)}{0.15}\ \Delta f_{[\text{DEM/USD}]} + \underset{(17.2)}{0.7}\ \Delta f_{[\text{DEM/AUD}]}$$

(a) How will you hedge if you use both contracts, and if the face value of a USD contract is for USD 50,000 and AUD contract for AUD 75,000?

(b) Should you use the USD contract, in view of the low t-statistic? Or should you only use the AUD contract?

A6. (a) USD: $0.15 \times \frac{1{,}000{,}000}{50{,}000} = 3$ contracts

AUD: $0.70 \times \frac{1{,}000{,}000}{75{,}000} = 9(.33)$ contracts.

(b) The t-statistic is rather low, so on the basis of this sample there is no way to say, with reasonable confidence, whether or not the USD contract actually reduces the risk.

Mind-Expanding Exercises

ME1. Consider the two possible following sequences of interest rates and futures prices (GBP/IEP) time 2:

	1/1/2000	7/1/2000	1/1/2001
Futures price			
Path A:	1.05	0.92	1
Path B:		1.02	1
P.a. simple Interest rate	360 days	180 days	
GBP (HC)	0.050	0.060	n.a.
IEP (FC)	0.025	0.035	n.a.

Assume that you have a short futures position for IEP 50,000 and that there is marking to market twice a year. Check that by increasing the futures position on July 1, 2000, the hedge becomes path-independent.

The following question is based on rolled-over forward contracts. Suppose that you have a long-term open position that you want to hedge, but there is no corresponding long-term futures contract. Thus, you must roll over short-term contracts. For example, rather than taking out a single five-year contract, you use five consecutive one-year contracts. When hedging a position, you increase the size of the new forward hedge hedge at each roll-over date by a factor of $(1 + r^*_{t,t+1})$ (not by the factor $(1 + r_{t,t+1})$ used in Appendix 5B).

A1. Since you are looking at a short position (a futures sale), the cash flow from marking to market is (old price - new price) $\times$ 50,000. For a unit contract, the cash flows will be:

	Number of contracts	Cash flows path A	Cash flows path B
t = 0	1.05	—	—
t = 1	1.05	(1.05 - 0.92)(1.05) = 0.1365	(1.05 - 1.02)(1.05) = 0.0315
t = 2	1.05 $\times$ 1.06	(0.92 - 1)(1.05)(1.06) = -0.08904	(1.02 - 1)(1.05)(1.06) = 0.02226
Total		0.1365 $\times$ 1.06 - 0.08904 = 0.05565	0.0315 $\times$ 1.06 + 0.02226 = 0.05565

For our contract, we just multiply by 50,000; in both cases, the result is IEP_3 2,782.50.

ME2. Consider the following possible paths of spot rates:

	1/1/2000	7/1/2000	1/1/2001
Spot rate			
Path *A* :		1.00	1.20
	1.00		
Path *B* :		1.20	1.20
p.a. simple Interest rates	360 days	180 days	
HC	0.21	0.21	n.a.
FC	0.10	0.10	n.a.
1 yr fwd *A* :		1.10	n.a.
B :		1.32	n.a.

Show that, when initially selling forward 1,000 units of foreign exchange for six months and adjusting the size of the new hedge on July 1:

(a) The price path does not matter

(b) The results are the same as for a sequence of two one-year money market hedges.

A2. (a) Rolling over the forwards:

	Number of contracts	Cash flows path A	Cash flows path B
t = 0	1,000	—	—
t = 1	1,000	(1.1 - 1.0) × 1,000 = 100	(1.1 - 1.2) × 1,000 = -100
t = 2	1,100	(1.1 - 1.2) × 1,100 = -110	(1.32 - 1.2) × 1,000 = 132
Total		(100 × 1.21) - 110 = 11	(-100 × 1.21) + 132 = 11

(b) Rolling over money market hedges: the sale of FC_1 1,000 corresponds to a FC_0 loan of \f(1,000,1.1) = 909.09, which yields a HC_0 deposit of 909.09 (as $S_0 = 1$). In the table below, we compute the consecutive values of the FC loan and the HC deposit.

	HC deposit	FC debt
t = 0	909.09	909.09
t = 1	909.09 × 1.21 = 1,100	909.09 × 1.1 = 1,000
t = 2	1,100 × 1.21 = 1,331	1,000 × 1.1 = 1,100

To redeem the debt, you need 1,100 × 1.2 = 1,320, which leaves a net cash position of 1,331 - 1,320 = 11 at time t = 2. This is the same outcome as the one obtained with rolled over forward contracts.