

Is the correlation in international equity returns constant: 1960–1990?

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We study the correlation of monthly excess returns for seven major countries over the period 1960–90. We find that the international covariance and correlation matrices are unstable over time. A multivariate GARCH(1,1) model with constant conditional correlation helps to capture some of the evolution in the conditional covariance structure. However tests of specific deviations lead to a rejection of the hypothesis of a constant conditional correlation. An explicit modelling of the conditional correlation indicates an increase of the international correlation between markets over the past thirty years. We also find that the correlation rises in periods of high volatility. There is some preliminary evidence that economic variables such as the dividend yield and interest rates contain information about future volatility and correlation that is not contained in past returns alone. (JEL G15, F3).

The correlation matrix of international asset returns plays a special role in the finance literature. Since the seminal work of Levy and Sarnat (1970), Grubel and Fadner (1971), Lessard (1973) and Solnik (1974), international diversification of equity portfolios has been advocated on the basis of the low correlation between national stock markets. The covariance between national markets could change because the volatility of national markets evolves over time, but also because the interdependence across markets changes. Looking at the market correlation allows one to focus on the interdependence between markets. The covariance/correlation matrix is one of the inputs for the computation of a trading portfolio. Knowledge

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about its behavior, stability and predictability is then crucial. It is also of particular importance for testing international pricing theories as misspecification could lead to false conclusions.¹

It is often stated that the progressive removal of impediments to international investment, as well as the growing political, economic and financial integration, affects international market linkages. This could lead to a progressive increase in the international correlation of financial markets reflecting the 'global finance' phenomenon. Kaplanis (1988) studied the stability of the correlation and covariance matrices of monthly returns of ten markets over a fifteen year period (1967–82). She compared matrices estimated over sub-periods of 46 months using Box (1949) and Jenrich (1970) tests. The null hypothesis that the correlation matrix is constant over two adjacent sub-periods could not be rejected at the 15 percent confidence level. The covariance matrix was much less stable (rejection at the 5 percent confidence level for most sub-periods). This result could be caused by changes in the conditional variances with constant international conditional correlations. Ratner (1992) also claimed that the international correlations remained constant over the period 1973–89. On the other hand, Koch and Koch (1991) looked at the correlation of eight markets using daily data for three separate years (1972, 1980 and 1987) and concluded from simple Chow tests that 'international markets have recently grown more interdependent'. Von Furstenberg and Jeon (1989) reached a similar conclusion using a VAR approach for four markets and a very short time period (1986–88). King, Sentana and Wadhvani (1992) claimed that this is only a transitory increase caused by the 1987 crash. Indeed, a question often raised is whether the international correlation increases in periods of high turbulence. The international correlation increases when global factors dominate domestic ones and affect all financial markets. The dominance of global factors tends to be associated with very volatile markets (the oil crises, the Gulf war, etc.). Using high-frequency data surrounding the crash of 1987, King and Wadhvani (1990) and Bertero and Mayer (1990) found that international correlation tends to increase during the stock market crisis.

Our objective is to test the hypothesis of a constant international conditional correlation, investigating various types of deviations. We explicitly model the conditional multivariate distribution of international asset returns and test for the existence of predictable time-variation in conditional correlation for the period 1960–90. Previous works have considered unconditional correlation computed over different sub-periods; here, we come up with an explicit model for conditional correlation. Previous studies have also looked at a fairly short history of monthly returns or focused on the recent period surrounding the crash of 1987. Here, we attempt to discover longer term phenomena by looking at monthly data for the seven major stock markets over the period 1960–90. This is a fairly original database which includes good-quality price indices and dividend yields over the total period. The data are described in Appendix A. The period covers several business and market cycles, with the steady growth of the sixties, the oil crises and the 1987 market crash.

A preliminary look at the data gives an indication of the stability of the correlation of markets. National stock markets have not been strongly correlated over the past thirty years. Table 1 reports the unconditional correlation of national

TABLE 1. Basic statistics.

	GE	FR	UK	SW	JA	CA	US
Mean	0.38	0.29	0.47	0.37	0.65	0.30	0.25
S.D.	5.16	5.87	6.19	5.15	4.97	4.81	4.36
	GE	FR	UK	SW	JA	CA	US
GE	1.00						
FR	0.45	1.00					
UK	0.34	0.42	1.00				
SW	0.60	0.51	0.45	1.00			
JA	0.24	0.26	0.25	0.29	1.00		
CA	0.30	0.42	0.48	0.46	0.27	1.00	
US	0.38	0.43	0.50	0.55	0.30	0.71	1.00

This table gives the mean and standard deviation of monthly excess returns (expressed in percentage monthly units) and the unconditional correlation matrix of national excess returns. The period is 1/1960–8/1990.

monthly excess returns estimated from 1960 to 1990. The lowest coefficient is 0.24 (Germany and Japan) and the highest is 0.71 (Canada and the USA). The average coefficient is around 0.5. The mean excess returns per month vary from 0.251 percent for the USA to 0.647 percent for Japan. The standard deviations are more similar across countries ranging from 4.36 percent per month for the USA to 6.19 percent per month for the UK.

To get a visual impression of the instability of the correlation across markets, we plotted the mean correlation of the US market with the other six markets in Figure 1. The correlations are estimated over a sliding window of five years. The correlation fluctuates over time. The inclusion of October 1987 in the calculation of the 5-year correlation leads to an increase in the correlation for a period of 5 years.

The global test for a constant unconditional correlation matrix performed by Kaplanis (1988) can be replicated on this longer time period. We estimate the unconditional correlation matrix for the seven countries over six sub-periods of five years and test for the equality of the correlation matrix over adjacent sub-periods as well as over non-adjacent sub-periods. The Jenrich test² of equality of two matrices calculated over different time periods, has an asymptotic chi-square distribution with 21 degrees of freedom for a seven by seven matrix.³ A similar test can be applied to the covariance matrix although the number of degrees of freedom is now 28 since the diagonal elements can vary over time. The results are also reported in Table 2. The null hypothesis of a constant correlation matrix is rejected at the 15 percent confidence level in 10 out of 15 comparisons and at the 5 percent level in 5 out of 15 comparisons. The same test applied to the covariance matrix leads to a rejection of the hypothesis of a constant covariance matrix at the 1 percent level in all comparisons but one. These results confirm the findings by Kaplanis that the covariance matrix is less stable than the correlation matrix. However our *p*-values for the correlation

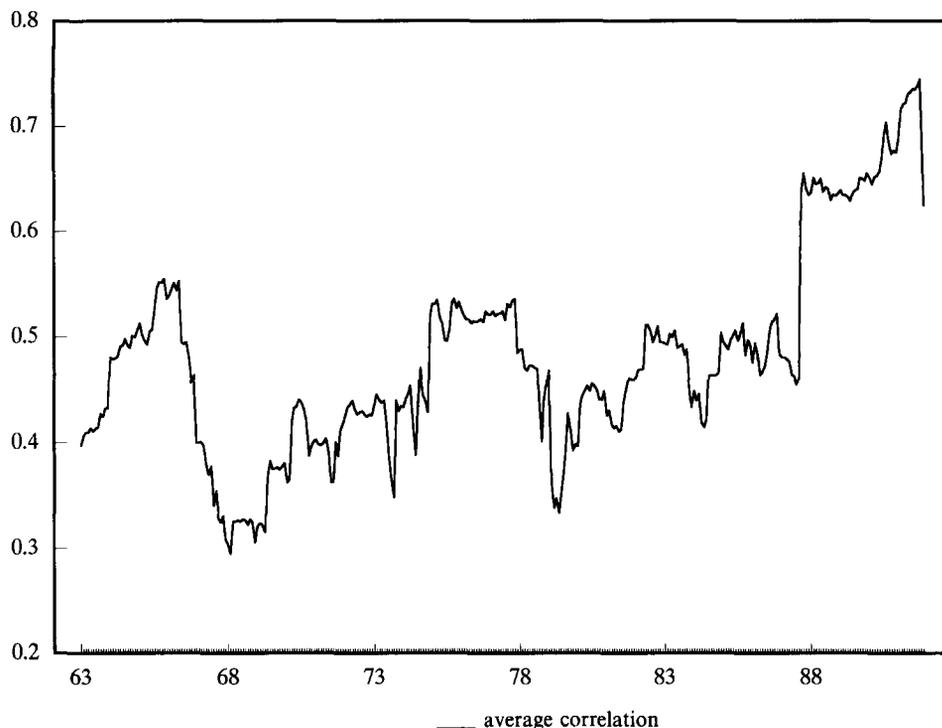


FIGURE 1. Correlation of the US stock market. This figure reports the (unweighted) average correlation of the US stock market with the other seven stock markets. The correlation is computed over sliding windows of four years, using local currency monthly total returns. The period is December 1959–91.

matrix are somewhat lower than hers; this could be explained by an increased instability in the eighties, since her data end in 1982.

What can explain this instability of the unconditional correlation matrix? A first alternative is that the conditional correlation remains constant over time but the market expected returns and variances vary over time. Indeed, we have extensive international evidence of predictable time-variations in the equity return distribution. Expected returns seem to depend worldwide on a set of information variables such as the dividend yield and various interest-rate-related variables.⁴ The variance of returns has been shown to be heteroscedastic. The conditional variance of national equity markets has been modelled with good success using a univariate GARCH approach⁵ for several national markets. A second alternative or, rather, additional explanation is that the interdependence of national equity markets is changing through time. Growing international integration could lead to a progressive increase in market correlation. Markets could be more highly correlated in periods of high volatility. The correlation could be higher when the markets go down, rather than up. The correlation could be higher in some periods of the business cycle, for example periods characterized by high levels of interest rates and dividend yields. These are some of the arguments often heard in financial circles.

TABLE 2. Test of the equality of the correlation and covariance matrices over time.

Periods compared	Correlation matrix		Covariance matrix	
	Test	p-value	Test	p-value
1960/65 to 1965/70	22.89	0.349	34.95	0.171
1965/70 to 1970/75	31.70	0.062	64.09	0.000
1970/75 to 1975/80	44.22	0.002	67.07	0.000
1975/80 to 1980/85	28.69	0.121	49.95	0.006
1980/85 to 1985/90	30.59	0.080	77.23	0.000
1960/65 to 1970/75	31.72	0.062	76.19	0.000
1960/65 to 1975/80	39.67	0.008	102.18	0.000
1960/65 to 1980/85	24.41	0.273	90.02	0.000
1960/65 to 1985/90	44.07	0.002	52.47	0.003
1965/70 to 1975/80	7.96	0.995	70.09	0.000
1965/70 to 1980/85	22.93	0.347	72.81	0.000
1965/70 to 1985/90	45.79	0.001	52.74	0.003
1970/75 to 1980/85	20.52	0.488	74.21	0.000
1970/75 to 1985/90	122.06	0.010	71.89	0.000
1975/80 to 1985/90	49.22	0.000	79.96	0.000

Correlation and covariance matrices of monthly national excess returns for seven countries are computed over periods of five years. We use the Jenrich test to test the equality of the correlation (and covariance) matrix over two periods. The rest is asymptotically distributed as a chi-square with 21 degrees of freedom for the correlation matrix and 28 degrees of freedom for the covariance matrix.

To test the assumption of a constant conditional correlation, we model the asset return dynamics explicitly using a bivariate GARCH model for each pair of markets. We also condition the first two moments of the distribution on a set of information variables observable at the start of the period used to calculate return, namely the dividend yield, the short- and long-term interest rates and a January seasonal. The parsimonious GARCH representation best fitted to our purpose is the GARCH constant-conditional-correlation model put forward in Bollerslev (1990). This specification has been extensively used to model international asset returns; it can be found in Ng (1991), Baillie and Bollerslev (1990) and Giovannini and Jorion (1989) among others. This will be our model of the null hypothesis of constant correlation and we will test different deviations from this model. To investigate whether the conditional correlation of markets becomes higher in turbulent periods, we develop a test on the conditional correlation inspired from the threshold GARCH models of Gouriéroux and Monfort (1992) and Engle and Ng (1993).

Looking at correlation alone one cannot reach conclusions with regard to market integration. In an asset pricing sense, markets can be fully integrated with or without correlation across asset returns. This paper focuses on the interdependence across markets and does not provide an asset pricing test of market integration.

The paper proceeds as follows. In Section I, we introduce our base model for the conditional multivariate asset process. This is a bivariate GARCH(1,1) model with information variables and constant conditional correlation. In Section II, we test various forms of deviations from constant conditional correlation. The findings are summarized in Section III.

I. Modelling the multivariate conditional return process

I.A. A multivariate conditional model

The multivariate process for asset returns can be written as:

$$\begin{aligned} R_t &= m_{t-1} + e_t, \\ \langle 1 \rangle \quad m_{t-1} &= E(R_t | F_{t-1}), \\ e_t | F_{t-1} &\sim N(0, H_t), \end{aligned}$$

where R_t is a vector of asset excess returns (denoted R_t^i for country i), m_{t-1} is the vector of expected returns conditioned on the information set F_{t-1} , e_t is the vector of innovations or unexpected returns assumed to be conditionally normal with a conditional covariance matrix H_t . Elements of H_t are denoted $h_t^{i,j}$ for the off-diagonal terms and h_t^i for the diagonal terms (variances).

As mentioned in the introduction, there is extensive empirical evidence that expected returns depend on a set of information variables such as the dividend yield and various interest-rate related variables. The expected excess return for market i (its national risk premium) is conditioned on a set of information variables Z_{t-1}^i . We assume a linear relation between expected excess returns and the vector of information variables:

$$\langle 2 \rangle \quad R_t^i = b^i Z_{t-1}^i + e_t^i.$$

In line with some previous research (Harvey, 1991 and Solnik, 1993), we include in the information set of country i the national dividend yield, short-term and long-term interest rates as well as a January seasonal.⁶ These variables are commonly used in models of the US market. We adopt the same national risk premium model for each country by using only domestic information variables. A 'better' risk premium model could certainly be found ex post but it would be the result of data mining.

We use a multivariate GARCH specification to model the time-variation in variances and also include the information variables discussed above. Several parsimonious multivariate GARCH specifications have been used in the literature. All these specifications are first-order processes denoted GARCH(1,1). To test the assumption of a constant conditional correlation across markets, we use as the null hypothesis the constant-conditional-correlation model proposed by Bollerslev (1990). We use excess equity returns in local currency. Because of interest rate parity (the forward currency basis is equal to the interest rate differential), this is equal to the currency-hedged excess return from any nationality viewpoint. This invariance to the numeraire used would not be preserved if the

constant-conditional-correlation representation was written on unhedged returns.⁷

The variance term for each market is assumed to be a function of the past innovation and conditional variance of this market, as well as some national information variables. However the conditional correlation between the two markets is assumed to be constant over time:

$$\langle 3 \rangle \quad h_t^i = a^i + b^i e_{t-1}^i e_{t-1}^i + c^i h_{t-1}^i + d^{i'} Z_{t-1}^i \text{ and } h_t^{i,us} = r^{i,us} \sqrt{h_t^i / h_t^{us}}.$$

The combination of equations $\langle 2 \rangle$ and $\langle 3 \rangle$ constitutes our base model with constant conditional correlation. We estimate this base model and then attempt to test for potential deviations from constant conditional correlation.

I.B. Estimation of the base model with constant conditional correlation

The conditional log-likelihood function at time t , $l_t(q)$, can be expressed as:

$$\langle 4 \rangle \quad l_t(q) = -\frac{N}{2} \ln(2\pi) - \frac{N}{2} \ln|H_t| - \frac{1}{2} e_t' H_t e_t,$$

where q is the vector of all the parameters to be estimated and N the dimension of the model (the number of countries). Thus, the log-likelihood for the whole sample from time 1 to T , $L(q)$, is given by:

$$\langle 5 \rangle \quad L(q) = \sum_{t=1}^T l_t(q).$$

This log-likelihood is maximized using the Berndt, Hall, Hall and Hausman (1974) algorithm⁸. A multivariate GARCH estimation with seven countries is not feasible for technical reasons as explained below. The full model would require the estimation of 98 coefficients (35 for the mean equations, 35 for the GARCH terms and the information variables in the conditional variances and 28 for the correlation coefficients). The large number of coefficients to be estimated makes the iteration process very long. Moreover the relatively small size of the database (368 observations) and the low frequency of the data (monthly observations) make the convergence of the iterative method very difficult especially if some of the coefficients are not strongly significant.⁹ Hence we only conduct the estimations for country pairs. To simplify the exposition we focus on the correlation of the US market with foreign markets and hence estimate six bivariate GARCH processes. We had to estimate the GARCH models with a large number of parameters for each country pair: ten in the mean equations (five for each country), seven to model the GARCH process plus one term for each information variable in the conditional variance equation. This is very computer-time-consuming and leads to a convergence problem unless good starting values are found for the parameters. This is particularly true for the parameters in the variance equations. We use only the national dividend yield and short-term interest rate as information variables for each variance equation. This gives a total of 21 parameters to be estimated. A sufficient, but not necessary, condition for positive definitiveness of the covariance matrix is to constrain the coefficients of the information variables

TABLE 3. Estimation of the national risk premium models.

A. Return equations						
	b_0	b_1	b_2	b_3	b_4	Lik
GE	0.015 (0.84)	2.21 (0.60)	-0.89 (-0.43)	-2.74 (-0.76)	0.014 (1.53)	1261.3
FR	0.001 (0.00)	5.46 (1.84)	1.22 (0.74)	-3.62 (-1.23)	0.035 (3.49)	1209.9
UK	-0.050 (-4.30)	13.92 (5.14)	-2.58 (-1.43)	2.14 (0.90)	0.017 (1.92)	1246.7
SW	0.013 (1.38)	6.94 (1.17)	-2.27 (-1.41)	-4.55 (-1.17)	0.023 (3.00)	1298.1
JA	0.012 (0.90)	1.22 (0.51)	-2.64 (-0.92)	0.47 (0.13)	0.020 (1.85)	1255.4
CA	-0.012 (-0.94)	8.75 (2.23)	-1.24 (-0.68)	-0.65 (-0.27)	0.021 (3.46)	1406.6
US	-0.021 (-2.20)	15.16 (3.69)	-3.28 (-2.15)	-0.87 (-0.42)	0.009 (1.49)	

B. Variance equations and correlation						
	$a \cdot 10^3$	b	c	d_1	d_2	r
GE	0.705 (1.91)	0.130 (2.57)	0.740 (8.75)	-0.184 (-1.92)	0.053 (1.66)	0.353 (8.39)
FR	0.558 (1.00)	0.086 (1.38)	0.657 (2.35)	0.009 (0.11)	0.037 (0.68)	0.407 (8.54)
UK	0.199 (0.57)	0.118 (1.70)	0.630 (3.61)	-0.018 (-0.22)	0.079 (1.76)	0.469 (11.4)
SW	0.193 (2.95)	0.010 (0.12)	0.975 (86.6)	-0.091 (-4.46)	0.015 (4.28)	0.508 (15.3)
JA	-0.007 (-0.01)	0.070 (2.62)	0.901 (22.9)	-0.002 (-0.17)	0.016 (1.07)	0.297 (5.43)
CA	-0.55 (-1.18)	0.125 (2.28)	0.435 (1.80)	0.225 (1.20)	0.126 (1.98)	0.723 (28.5)
US	0.271 (1.09)	0.055 (1.58)	0.747 (4.34)	-0.044 (-0.64)	0.035 (1.16)	

Estimation of bivariate projections:

$$R_t^i = b_0^i + b_1^i DIV_{t-1}^i + b_2^i ST_{t-1}^i + b_3^i LT_{t-1}^i + b_4^i JAN_t + e_t^i \quad \langle 2 \rangle$$

$$h_t^i = a^i + b^i e_{t-1}^i e_{t-1}^{i,us} + c^i h_{t-1}^i + d^i Z_{t-1}^i \quad \text{and} \quad h_t^{i,us} = r^{i,us} \sqrt{h_t^i / h_t^{us}} \quad \langle 3 \rangle$$

where R_t^i is the excess return for market i in period t , h_t^i is the conditional variance of market i , $h_t^{i,us}$ is the conditional covariance between market i and the US DIV_{t-1}^i , ST_{t-1}^i , LT_{t-1}^i and JAN_t are respectively the dividend yield, the short-term interest rate, the long-term interest rate of country i observed in $t-1$ and a January dummy that takes the value one if t is the month of January and zero otherwise. The t -statistics adjusted for heteroscedasticity are reported in parentheses. Lik is the value of the log-likelihood of the bivariate models. The period of estimation is 1/1960-8/1990. Estimation results for the USA are those obtained for the UK/US pair.

to be all positive as the variables are always positive. This is not satisfactory from an economic viewpoint as it would rule out the possibility of a negative influence of one of the variables on the variance or covariance. Negative coefficients could still lead to positive definitiveness of the covariance matrix over the sample space. In practice, we never encountered the problem of a negative conditional variance in our sample or for any plausible value of the information variables.

Results of the estimation are given in Table 3. The top panel gives the parameters of the mean equations. The bottom panel gives the parameters of the covariance equations. As mentioned above, these are the results of a bivariate estimation of the USA and a foreign country. The last line gives the results of the estimation of the USA derived from the UK/US estimation. The values for the USA derived from the other bivariate models are very similar.

The results for the conditional expected returns (Panel A) confirm previous findings. The coefficients of the dividend yield tend to be positive and the coefficients of the short-term interest rate tend to be negative; there is a positive January seasonal in all countries.

The coefficients for the variance equations (Panel B) show significant GARCH effects for many countries. The dividend yield tends to have mixed signs¹⁰ in the variance equations. There is one negative coefficient for Switzerland, statistically significant at the 1 percent level. The interest rate has a positive coefficient in all variance equations. The coefficients have the same sign for each country, but their significance level is somewhat marginal. The fact that the conditional variance is predictably lower in periods of high dividend yields could have a plausible explanation: with high dividend yields, more of the current value of a firm is derived from near-term dividends, which can be regarded as more predictable than remote future dividends. The positive relation between volatility and interest rate levels is not surprising either. Inflationary periods lead to higher volatility in financial prices.

We test the significance of the various factors by estimating three nested models. The simplest model is the bivariate homoscedastic model with no information variables; it requires the estimation of only two unconditional mean terms and three covariance terms. Its computed likelihood function is referred to as Lik1. The second model is a bivariate homoscedastic model with information variables in the mean equation. Its computed likelihood function is referred to as Lik2. The base model is the bivariate GARCH(1,1) with information variables in the mean and variance equations and constant conditional correlation. The estimation of this base model is reported in Table 3 and the computed likelihood function is referred to as Lik3. The three specifications are nested, so we can perform likelihood-ratio tests (LR test hereafter).

The results reported in Table 4 indicate that both the modelling of the conditional expected return and variance are significant. The first section of Table 4 gives the value of the likelihood function of the simple bivariate homoscedastic model. The second section gives the value of the likelihood function of the bivariate homoscedastic model with information variables in the mean, as well as the *p*-value of the LR test against the null of the simple model. The hypothesis of a constant expected return is rejected at the 1 percent level in all cases. The third

TABLE 4. Likelihood ratio tests of bivariate GARCH specifications.

1. Log-likelihood of bivariate homoscedastic						
Market	GE/US	FR/US	UK/US	SW/US	JA/US	CA/US
Lik1	1225.5	1186.9	1184.5	1264.4	1228.1	1355.6
2. Bivariate conditional mean/homoscedastic						
Market	GE/US	FR/US	UK/US	SW/US	JA/US	CA/US
Lik2	1237.5	1200.2	1204.2	1282.2	1246.0	1368.9
<i>p</i> -value/model 1	0.002	0.001	0.000	0.000	0.000	0.001
3. Base model						
Market	GE/US	FR/US	UK/US	SW/US	JA/US	CA/US
Lik3	1261.3	1209.9	1246.7	1298.1	1255.4	1406.6
<i>p</i> -value/model 1	0.000	0.000	0.000	0.000	0.000	0.000
<i>p</i> -value/model 2	0.000	0.013	0.000	0.001	0.016	0.000

section gives the value of the likelihood function of the base model as well as LR test against the alternative of the two homoscedastic models. Both simpler models are rejected. We see that the hypothesis of constant variances is rejected at the 5 percent level in all cases. It should be stressed that we estimate bivariate models of the US and foreign equity markets; the predictability in the conditional US expected return and variance affects the significance levels reported for each bivariate estimation.

We can test whether the estimated GARCH parametrization provides an adequate description of the heteroscedasticity by performing Ljung–Box portmanteau misspecification tests on the squared residuals as suggested by Li and McLeod (1983). This test is performed on the standardized squared residuals of country *i*, $e_t^i e_t^i / h_t^i$, obtained for the constant correlation specification. The Ljung–Box statistics with 20 lags ranges from 7 to 15 for the six country-pairs, while the 5 percent critical level is 31.44. We cannot reject the specification of variances at this confidence level for any country. A more direct test has been suggested by Bollerslev (1990). The GARCH parametrization assumes that all relevant past information F_{t-1} is reflected in the GARCH estimate of the variance H_{t-1} :

$$\langle 6 \rangle \quad E(e_t^i \cdot e_t^j | F_{t-1}) = h_t^{i,j}.$$

This can be tested by including in the information set F_{t-1} past residuals and regressing $e_t^i e_t^j / h_t^{i,j} - 1$ on $1/h_t^i, e_{t-1}^i e_{t-1}^i / h_t^i, \dots, e_{t-k}^i e_{t-k}^i \cdot 2/h_t^i$. Stopping at $k = 5$, we test whether the estimated regression coefficients are different from zero with a F-test. The 5 percent critical value for a F(6,361) is 2.13. The F-tests reported in Table 5 range from 0.28 for Canada to 1.61 for France and do not lead to a rejection of the constant correlation specification. A similar test is performed for the cross-product of residuals. We regress $e_t^i \cdot e_t^j / h_t^{i,j} - 1$ on $1/h_t^{i,j}, e_{t-1}^i \cdot e_{t-1}^j / h_t^{i,j}, \dots, e_{t-5}^i \cdot e_{t-5}^j / h_t^{i,j}$ as well as $e_{t-1}^i e_{t-1}^i / h_t^{i,j}$ and $e_{t-1}^j e_{t-1}^j / h_t^{i,j}$. The 5 percent critical value for a F(8,359) is 1.97 and all F-tests fall below that value as seen in Table 5. Altogether, no evidence is found to reject our base model, using

TABLE 5. Heteroscedasticity specification tests.

Squared residuals	GE	FR	UK	SW	JA	CA	US
<i>F</i> -test	0.571	1.611	1.374	0.469	0.896	0.283	0.518
<i>p</i> -value	(0.754)	(0.143)	(0.224)	(0.831)	(0.498)	(0.945)	(0.794)
Cross-product	GE/US	FR/US	UK/US	SW/US	JA/US	CA/US	
<i>F</i> -test	0.592	0.658	0.825	1.195	1.296	0.451	
<i>p</i> -value	(0.785)	(0.728)	(0.581)	(0.301)	(0.244)	(0.890)	

This table gives the results of heteroscedasticity specification tests as suggested in Bollersleu (1990). Standardized squared residuals and cross-product of residuals are regressed on lagged variables. The residuals are from the base model (a bivariate GARCH (1, 1) with constant conditional correlation). *F*-tests are reported with their *p*-values in parentheses.

standard heteroscedasticity tests. However, more powerful tests can be designed, by an explicit modelling of the deviations from our constant correlation base model.

II. Is the conditional correlation constant?

There are many reasons why the international market correlation should not remain constant over time. We examine three potential sources of deviation from our base model of constant conditional correlation: a time trend, the presence of threshold and asymmetry, and the influence of economic variables. Due to the large number of parameters to be estimated we have to study separately each cause of deviation from the base model.

II.A. A time trend

It is often stated that the progressive removal of impediments to international investment as well as the growing political, economic and financial integration affects international market linkages. More integrated economies could mean that national firms are more and more influenced by global factors. The extent of international activities by companies themselves is growing. Most companies can now be regarded as a diversified portfolio of international activities through their exports and through foreign implantation. Their stock price should behave like that of an internationally diversified portfolio and, hence, be more correlated with that of other firms worldwide. This could lead to a progressive increase in the international correlation of equity markets.

We use our long period of time to test this increase in correlation. There are no specific events (or too many of them) that should be used as cutoff points or structural breaks (the monthly frequency of our data would also limit the power of studies over short sub-periods). We use a simple test to detect a progressive increase in correlation over the past 30 years. We augment our base model to include a linear time-trend in the correlation specification and test the null hypothesis that this coefficient is zero. In other words, the covariance term of

equation <3> is now written as:

$$\langle 7 \rangle \quad h_t^{i,us} = (r_0^{i,us} + r_1^{i,us}t)\sqrt{h_t^i/h_t^{us}}$$

To remove potential biases created by a structural increase in conditional variance, we also add a trend term in each variance equation. Needless to say, this specification is a simplistic modelling of the increase in correlation over time, and other specifications of the trend could have been used. In the absence of a formal model, our sole purpose is to test the existence of a trend. Estimation of the augmented model are reported in Table 6, where we only report the coefficients of the time trends. The coefficients for the variances are small and statistically insignificant. There is no secular increase in expected market volatility. The GARCH(1,1) specification seems to adequately capture the time evolution of the market variances and no additional time-trend needs to be added in the variance equation. The conclusion is quite different for the correlation, and hence the covariance, across markets. There is a positive time-trend in conditional correlation for all countries. The trend is statistically significant at the 5 percent level for four countries out of six. A similar conclusion is obtained if we estimate a GARCH model with a trend solely in the correlation but not in the variances. In other words, the international correlation seems to be increasing over time, even after the variance terms have been modelled with a GARCH parametrization. The average increase in correlation over 30 years is 0.36, or slightly more than 0.01 per year. Obviously such a constant linear trend is not consistent with the definition of a correlation coefficient. Other forms of trend could be modelled, but no theory exists of the exact form of this trend. We tried an exponential trend

TABLE 6. Time-trend in correlation.

Market	GE/US	FR/US	UK/US	SW/US	JA/US	CA/US
d^i	0.01 (0.44)	0.21 (0.98)	0.35 (1.75)	-0.00 (-0.03)	0.01 (0.43)	0.07 (1.70)
d^{us}	0.17 (1.03)	0.15 (0.96)	0.20 (1.06)	0.17 (0.96)	0.23 (1.05)	0.10 (1.52)
r_0	0.27 (3.16)	0.13 (1.22)	0.09 (0.95)	0.41 (5.39)	0.10 (0.99)	0.65 (11.61)
r_1	0.20 (1.55)	0.49 (3.15)	0.76 (6.13)	0.25 (2.40)	0.38 (2.33)	0.16 (1.82)
LR test	10.0	17.4	30.4	11.2	17.8	11.2
p -value	(0.019)	(0.001)	(0.000)	(0.011)	(0.001)	(0.011)

This table gives the estimates of the augmented model with time-trend. This model is similar to the base model with an additional time-trend in the variance equations (d^i for country i and d^{us} for the US) and in the correlation. The correlation is now estimated as:

$$h_t^{i,us} = (r_0^{i,us} + r_1^{i,us}t)\sqrt{h_t^i/h_t^{us}} \quad \langle 7 \rangle$$

T-statistics appear below the estimated coefficients. The likelihood ratio (p -value in parentheses) is a test of the augmented model against the base model.

of the form by modelling the correlation as follows:

$$\langle 8 \rangle \quad h_t^{i,us} = (r_0^{i,us} + r_1^{i,us} (1 - e^{-at})) \sqrt{h_t^i} \sqrt{h_t^{us}}$$

If a is positive, correlation varies from r at the beginning of the period to $r_0 + r_1(1 - e^{-aT})$ at the end of the period T ; if the two bounds r_0 and $r_0 + r_1$ are between -1 and $+1$, then the correlation is always defined. To assure the convergence of the algorithm, a constraint can be imposed on r_0 and $r_0 + r_1$ such that the correlation is always defined. Empirically for the pair JA/US, we found a correlation increasing from -0.397 to 0.356 at an exponential rate 11.818 .¹¹ For the other pairs of countries, the constraints were binding out of sample (GE/US and FR/US) or in sample (UK/US, SW/US and CA/US) with a negative value for the rate of convergence ($a < 0$). Such a result could be explained by a sharp increase in correlation at the end of the period (the eighties). The proposed simple econometric methodology (a linear time-trend) however satisfies our objective to test, and reject, the null hypothesis of a constant conditional correlation.

II.B. Threshold and asymmetry in the conditional correlation

International market linkages are due to common factors that affect all economies at the same time. It is often claimed that there are periods when international factors dominate purely domestic factors and vice versa.

The constant correlation GARCH model assumes that the conditional covariance, $h_t^{i,us}$, estimated from information available at time $t - 1$, is equal to a constant correlation times the product of the two conditional standard deviations $\sqrt{h_t^i} \sqrt{h_t^{us}}$ following a first order GARCH process. It is often claimed that the international correlation increases in periods of high market turbulence, when international factors dominate. For example, all markets went down together during the second oil shock of 1974 and in October 1987, and went up at the end of the Gulf War. This hypothesis can be tested by introducing a threshold in the constant correlation GARCH model. Threshold GARCH models have been extensively used in the univariate case but not in the multivariate case (see Appendix B for a discussion and results of the univariate asymmetric threshold GARCH model for the seven countries under study). To test the hypothesis of higher international correlation during turbulent periods, we introduce a threshold effect in our bivariate constant correlation GARCH specification. With this threshold on correlation, the covariance term of the GARCH specification can be written as:

$$\langle 9 \rangle \quad h_t^{i,us} = (r_0^{i,us} + r_1^{i,us} S_{t-1}) \sqrt{h_t^i} \sqrt{h_t^{us}},$$

where S_{t-1} is a dummy variable that takes the value 1 if the estimated conditional variance of the US market is greater than its unconditional value and 0 otherwise. The unconditional variance of innovations from the base model is taken as the exogenous threshold. In other words, the time- t international correlation is conditional on the time- t volatility of the US market; note that both the correlation and the US volatility are estimated using only information observable at time $t - 1$. The coefficient r_1 will be positive if the correlation increases when the conditional US variance is high; it will be zero if there is no threshold effect.

TABLE 7. Threshold in correlation.

	r_0	r_1	r	LR test (p -value)
GE/US	0.327 (5.52)	0.134 (1.75)	0.353	2.4 (0.121)
FR/US	0.331 (5.44)	0.194 (2.43)	0.407	4.8 (0.028)
UK/US	0.468 (9.34)	0.057 (0.74)	0.469	0.6 (0.438)
SW/US	0.458 (8.70)	0.192 (3.12)	0.508	8.0 (0.010)
JA/US	0.265 (3.82)	0.101 (0.99)	0.297	2.2 (0.138)
CA/US	0.729 (26.02)	0.024 (0.65)	0.723	1.0 (0.317)

The table gives the estimation of the bivariate GARCH(1,1) models with threshold correlation. The correlation is conditioned on the size of the US expected variance:

$$h_t^{i,us} = (r_0^{i,us} + r_1^{i,us} S_{t-1}) \sqrt{h_t^i / h_t^{us}} \quad (9)$$

The coefficient r is the estimate from the base model with constant correlation, as given in Table 3. The likelihood ratio test from the base model with constant correlation is indicated in the last column, with p -value in parentheses.

The results of the estimation of the threshold correlation specification are reported in Table 7. The estimated coefficient r_1 is positive for all countries and individually significant at the 5 percent confidence level in two cases. The magnitude of the coefficient is quite large in most cases. We can illustrate the results in the case of France. The constant correlation estimated was equal to 0.407; this compares with a constant term r_0 of 0.331 and a turbulence effect r_1 of 0.194. In other words the correlation is estimated to be equal to 0.331 in periods of low volatility and 0.525 in periods of high volatility. The average correlation in the six country-pairs was equal to 0.460 when using the constant correlation model. The average over all countries of the correlation r_0 is equal to 0.430 while the average turbulence effect r_1 is equal to 0.117. The correlation coefficient seems to increase by about 27 percent in periods of high turbulence.

We only introduced one exogenous threshold arbitrarily set at the unconditional variance. We would have liked to use several thresholds but were limited by the size of our database to get reliable statistical results. We also replicated these tests by conditioning the correlation on the conditional variance of the foreign market rather than that of the US market. The conclusions were quite similar and the results are not reported here.

We can say more about correlation by looking at asymmetry. In the previous threshold GARCH model, a positive or negative shock (e_{t-1}) observed at time $t - 1$ has the same impact on correlation. It is interesting to test whether negative and positive shocks have a different impact on the conditional correlation. Rather than conditioning the correlation on the US market volatility (and hence indirectly

on the absolute magnitude of e_{t-1}^{us} , we condition the correlation on both the sign and magnitude of past¹² shocks e_{t-1}^{us} . This threshold, asymmetric correlation GARCH specification can be written as:

$$\langle 10 \rangle \quad h_t^{i,us} = (r_1^{i,us} S_{1,t-1} + r_2^{i,us} S_{2,t-1} + r_3^{i,us} S_{3,t-1} + r_4^{i,us} S_{4,t-1}) \sqrt{h_t^i / h_t^{us}},$$

where $S_{k,t-1}$ are dummy variables that take the values:

- $S_{1,t-1} = 1$ if e_{t-1}^{us} is less than $-\sigma^{us}$,
 - $S_{2,t-1} = 1$ if e_{t-1}^{us} is less than 0,
 - $S_{3,t-1} = 1$ if e_{t-1}^{us} is greater than 0,
 - $S_{4,t-1} = 1$ if e_{t-1}^{us} is greater than $+\sigma^{us}$,
- and zero otherwise.

σ^{us} (the unconditional standard deviation of the innovations e^{us} from the base model), $-\sigma^{us}$ and zero are the exogenous thresholds used for the augmented model. Note that two dummy variables will be equal to one for large negative shocks ($S_{1,t-1}$ and $S_{2,t-1}$) and for large positive shocks ($S_{3,t-1}$ and $S_{4,t-1}$). This model allows one to capture asymmetry in the impact of shocks ($r_2 \neq r_3$) and different impacts for large and small shocks ($r_1 \neq 0$ and $r_4 \neq 0$). If the impact of small and large shocks are similar, we should find that the two coefficients r_1 and r_4 are equal to zero.

The results are reported in Table 8. The results confirm the previous findings. Several coefficients r_1 and r_4 are significantly positive, indicating that large shocks

TABLE 8. Asymmetry in correlation.

	r_1	r_2	r_3	r_4	LR test (p-value)
GE/US	0.307 (2.95)	0.313 (5.10)	0.327 (3.76)	0.304 (2.33)	10.2 (0.016)
FR/US	0.165 (1.59)	0.365 (5.92)	0.309 (3.73)	0.345 (2.92)	6.2 (0.102)
UK/US	0.005 (0.05)	0.489 (8.34)	0.452 (6.64)	0.117 (0.91)	0.6 (0.896)
SW/US	0.282 (3.54)	0.468 (8.60)	0.544 (8.95)	-0.003 (-0.36)	10.2 (0.016)
JA/US	-0.201 (-1.20)	0.329 (3.82)	0.332 (4.67)	0.084 (0.46)	2.8 (0.423)
CA/US	-0.115 (-1.64)	0.780 (27.06)	0.749 (25.40)	-0.099 (-1.29)	4.8 (0.187)

This table gives the estimation of a bivariate asymmetric correlation GARCH(1,1) where the correlation is modelled as:

$$h_t^{i,us} = (r_1^{i,us} S_{1,t-1} + r_2^{i,us} S_{2,t-1} + r_3^{i,us} S_{3,t-1} + r_4^{i,us} S_{4,t-1}) \sqrt{h_t^i / h_t^{us}} \quad \langle 10 \rangle$$

The correlation is conditioned on the sign and size of past US shocks. No asymmetry corresponds to the case: $r_1 = r_4$ and $r_2 = r_3$. No threshold effect corresponds to the case: $r_1 = r_4 = 0$. The likelihood ratio test comparing the asymmetric model to the base model with constant correlation is indicated in the last column, with p-value in parentheses.

tend to increase the conditional correlation. However we have little evidence of asymmetry as the coefficients r_2 and r_3 are quite similar; furthermore we find no evidence that the coefficient r_1 is systematically greater than r_4 . The correlation seems to increase in periods of high turbulence (following large positive or negative shocks) but is no more sensitive to negative than to positive shocks.¹³

II.C. Influence of information variables on correlation

We have seen that the information variables affect the conditional covariance matrix. Even with a constant correlation, the conditional covariance between two national markets is a function of the information variables, as each national conditional standard deviation is. For example, if all national variances increase in periods of high interest rates, the covariances will increase in parallel. The remaining question is whether the covariances will increase more (or less) than the variances. This can be investigated by testing an augmented model, identical to the base model but with the hypothesis that the conditional correlation itself is predictable based on past values of the information variables. Hence the covariance equation in model <3> is replaced by:

$$\langle 11 \rangle \quad h_t^{i,us} = (r' + g_1^{i,us} DIV_{t-1}^{us} + g_2^{i,us} ST_{t-1}^{us}) \sqrt{h_t^i / h_t^{us}}$$

The information set is supposed to be equal to the US dividend yield and interest rate. Estimates of the correlation parameters are given in Table 9. This table also gives a likelihood-ratio test of the augmented model against the base model. Several of the coefficients g 's are significant at the 5 percent level but not at the 1 percent level. The p -values of the likelihood ratio tests show a rejection of the base model for half of the country-pairs at the 5 percent level. With one

TABLE 9. Information variables in correlation.

Market	GE/US	FR/US	UK/US	SW/US	JA/US	CA/US
r'	0.467 (2.24)	0.248 (1.41)	0.684 (4.35)	0.355 (2.68)	0.375 (1.78)	0.936 (9.27)
g_1	-114.4 (-1.32)	-8.47 (-0.11)	-157.6 (-2.16)	1.37 (0.02)	-147.7 (-1.85)	-96.2 (-2.17)
g_2	43.5 (2.23)	25.3 (1.49)	47.8 (2.91)	27.8 (2.21)	63.3 (3.33)	15.9 (1.36)
LR test	0.0	0.2	7.6	9.6	8.6	5.6
p -value	(0.999)	(0.999)	(0.022)	(0.008)	(0.013)	(0.060)

This table gives the estimates of the augmented model with information variables (US dividend yield and US short-term interest rate) in the conditional correlation. The conditional correlation is now estimated as:

$$h_t^{i,us} = (r' + g_1^{i,us} DIV_{t-1}^{us} + g_2^{i,us} ST_{t-1}^{us}) \sqrt{h_t^i / h_t^{us}} \quad \langle 11 \rangle$$

T-statistics appear below the estimated coefficients. The likelihood ratio (p -value in parentheses) is a test of the augmented model against the base model.

exception, the sign pattern is consistent. The conditional correlation is predicted to increase in periods of low dividend yield and high interest rates.

This result is consistent with the relation between conditional correlation and variance found in the previous section. For example, we found that the conditional variance increases with the level of interest rates and that the correlation increases in periods of high volatility. We confirm here that the conditional correlation is higher in periods of high interest rates.

III. Summary and conclusions

We studied the correlation of monthly excess returns for seven major countries over the period 1960–90. We find that the international covariance and correlation matrices are unstable over time. A multivariate GARCH(1,1) model with constant conditional correlation helps capture some of the evolution in the conditional covariance structure. We include information variables in the mean and variance equations. The volatility of markets changed somewhat over the period 1960–90 and the proposed GARCH model allows one to capture this evolution in variances. However tests of specific deviations lead to a rejection of the hypothesis of a constant conditional correlation. An explicit model of the conditional correlation indicates an increase in the international correlation between markets over the past 30 years. We also find that the correlation rises in periods when the conditional volatility of markets is large. There is some preliminary evidence that economic variables such as the dividend yield and interest rates contain information about future volatility and correlation that is not contained in past returns alone. However, further theoretical work is required to provide a satisfactory model. The effects are not of great magnitude but are often statistically significant.

Our results confirm and complete some previous findings mentioned in the introduction. These studies (e.g. Koch and Koch 1991, and Von Furstenberg and Jeon, 1989) typically computed correlation coefficients over short sub-periods, using high-frequency data, and then looked at their evolution over time. Here we explicitly model the multivariate asset return process and use a much longer time period. In their APT test of global market integration, King *et al.* (1992) briefly looked at changes in conditional correlation. They also explicitly model the conditional correlation, using monthly data over the period 1970–88. In both cases, a very parsimonious parametrization is used, but the models for the covariance and correlation are different. They assume that the national markets are correlated through common factors with constant factor loadings. In their factor model, the correlation can only increase over time if the variances of the common factor increase relative to the national residual variances. King *et al.* (1992) calculated the monthly conditional correlation and found only a modest (negligible) increase over time, without explicit significance test. A secular trend due to increased international market integration, or transitory changes due to business cycles, cannot be modelled¹⁴ and tested. On the other hand, our parsimonious parametrization allows a direct test of the existence of a time trend

and various influences in the conditional correlation. We do find evidence of predictable components in the time-variation of international correlation.

Some words of caution are in order. We are able to reject the hypothesis of constant international correlation but fall short of a full model. We test separately several deviations from a constant correlation model because the econometric methodology used does not allow us to include all of them in a single model. In our GARCH equation, we allowed the coefficients of the (positive) information variables to be negative. This means that the covariance matrix could lose its positive-definiteness for some large values of the dividend yield. Negative coefficients could still lead to positive-definiteness of the covariance matrix over the sample space. In practice, we never encountered the problem of a negative conditional variance in our sample or for any plausible value of the information variables and we felt that there was no economic rationale to rule out negative coefficients. The next step would be to describe the evolution in correlation over time by a richer model than the linear (or exponential) time-trend used in the hypothesis testing. Some form of event modelling would be more satisfactory if structural breaking points could be identified. Indicators such as the number of restrictions to free capital movements could also be used.

The methodology developed in this paper could be a useful basis for a more detailed study of the international integration of financial markets. However such conclusions cannot be reached by looking at the correlation alone and an international asset pricing model must be explicitly used. The methodology could also be applied to other areas of finance in which correlation is involved, such as the pricing of financial instruments like differential swaps or options on multiple assets.

Future research should also focus on the fundamental determinants of international correlation across equity markets. This correlation is likely to be affected by the industry mix of each national market as well as the correlation of the countries' business cycles.

Appendix A: Data description

The data on international financial markets vary greatly in terms of availability, quality and comparability. We have chosen to restrict the time period and the number of countries selected to insure good quality of the data. We get good month-end data for stock prices, as well as dividend yield, long- and short-term market interest rates used as instruments, for the seven major financial markets over the period January 1960 to August 1990. The countries are: France (FR), Germany (GE), Switzerland (SW), UK (UK), Japan (JA), Canada (CA) and the USA (US). These are the seven largest markets in terms of size and their combined stock market capitalization is more than 90 percent of that of the world. In many tables, these countries will be called by the letters indicated in parentheses.

Common stock returns: Month-end stock market indices calculated in local currency come from Morgan Stanley Capital International (MSCI). The MSCI sample covers around 65 percent of each market capitalization, with an attempt to stratify their sampling by industry breakdown so that each industry is represented in the national index in proportion to its national weight; hence the selection of individual companies is not solely based on their market capitalization. The data since 1970 is publicly available and widely used; the price indices from 1960 to 1969 were back-calculated by MSCI.¹⁵ For each market we calculated a time-series

of monthly excess returns in local currency. These are returns in excess of the risk-free rate of the country considered; the short-term interest rates used as risk-free rates are described below. We use returns in local currency to focus on the correlation across markets¹⁶ rather than across currencies.

Dividend yield: The dividend yields are calculated in the usual fashion by averaging the dividend paid over the past twelve months. These data are published by MSCI since 1970. For the period 1960–69, we collected dividend yields from various sources. The major source is the OECD, but we also checked with local statistical publications for each country.

Long-term interest rates: We use the yield-to-maturity on long-term Government bonds. The maturity ranges from 5 to 15 years depending on the country. From 1971 on, we use bond yield indices calculated by Lombard Odier. These bond yields are calculated daily from a small sample of plain-vanilla, actively traded, long-term government bonds in each currency. While the number of bonds in each index is limited, the bond prices and yields are current. These indices have been published daily in the Wall Street Journal (Europe) since the early eighties. Prior to 1971, we use average long-term bond yields published by Morgan Guaranty. Free and active bond markets did not develop in many countries until the 1970s. Hence the quality of the yields prior to 1970 is less reliable for some of the countries under study.

Short-term interest rates: We use one-month Eurocurrency interest rates as risk-free rates. These are the only true market rates¹⁷ for many countries and they are fully comparable. The data come from Morgan Guaranty and Lombard Odier for the period 1970–90 and from the OECD for the period 1960–70. An active Eurocurrency market did not exist prior to 1971 for most currencies. Hence we used domestic interest rates on instruments that were priced on a free money market. This means that these rates are not fully comparable across countries as these instruments differ in terms of characteristics and fiscal treatment. This is not a serious problem for our study as we never engage in the calculation of interest rate differentials or in international comparisons of interest rates.

Appendix B: Threshold and asymmetry in the expected variance

The traditional GARCH models have the property that the impact of the past shock e_{t-1} on the conditional variance (represented by the degree of persistence b) is independent of the sign and the size of the shock. For example a univariate GARCH(1,1) model has a conditional variance equation of the form:

$$\langle B1 \rangle \quad h_t = a + be_{t-1}^2 + ch_{t-1}.$$

This specification imposes two constraints on the impact of past innovations, e_{t-1} , on the conditional variance h_t . First, negative and positive innovations have the same influence, only the absolute value of the shock matters. Second, the persistence of a shock is the same whatever the magnitude of the past shock; larger innovations create more volatility at a rate proportional to the square of the size of the innovation. These properties have been criticized and several extensions have been proposed for univariate GARCH specifications. There are many ways to introduce asymmetry (elasticity function of the sign of the shock) and thresholds (elasticity function of the magnitude of the past variance and of the shock) in univariate GARCH models. One approach is to specify a particular non-linear function of past shocks and variances (e.g. Nelson, 1991, Glosten *et al.*, 1993 and Hentschel, 1991). Another approach is to specify a piece-wise linear function with arbitrary threshold levels (Engle and Ng, 1993, Zakoïan, 1990, Rabemananjara and Zakoïan, 1991 and Gouriéroux and Monfort, 1992). Thresholds are exogenous to assure the efficiency of the estimators (see Friedman and Laibson, 1989 for an endogenous threshold). The basic idea is to allow different impact coefficients b for different signs and values of the past innovation e_{t-1} . An asymmetric, threshold GARCH(1,1) univariate specification that accounts for a differential impact of the past innovation, depending on its

sign and magnitude, is:

$\langle B2 \rangle$

$$h_t = a' + b_1 S_{1,t-1} (e_{t-1}^2 - \sigma^2) + b_2 S_{2,t-1} e_{t-1}^2 + b_3 S_{3,t-1} e_{t-1}^2 + b_4 S_{4,t-1} (e_{t-1}^2 - \sigma^2) + c' h_{t-1}$$

where $S_{k,t-1}$ are dummy variables that take the values:

$S_{1,t-1} = 1$ if e_{t-1} is less than $-\sigma$,

$S_{2,t-1} = 1$ if e_{t-1} is less than 0,

$S_{3,t-1} = 1$ if e_{t-1} is greater than 0,

$S_{4,t-1} = 1$ if e_{t-1} is greater than σ ,

and zero otherwise.

σ (the unconditional standard deviation of the innovations e from the GARCH model), $-\sigma$ and zero are the exogenous thresholds used in this threshold, asymmetric model. Note that two dummy variables will be equal to one for large positive or negative innovations. This model allows us to capture asymmetry in the impact of innovation ($b_2 \neq b_3$) and different impacts for large and small news ($b_1 \neq 0$ and $b_4 \neq 0$). If the impact of small and large innovations are similar, we should find that the two coefficients b_1 and b_4 are equal to zero; on the other side, it is often claimed that large shocks should have a shorter persistence than small shocks (b_1 and b_2 negative). Similarly, it is often stated (see Black, 1976, Nelson, 1990 and Campbell and Hentschel, 1992) that negative news should increase volatility more than positive news ($b_2 > b_3$). A sufficient condition for the positiveness¹⁸ of the conditional variance is that $b_1 + b_2 > 0$ and $b_3 + b_4 > 0$.

The results of the estimation of the univariate threshold GARCH specification are given in Table 10. The mean equation includes the information variables as before, and only the

TABLE 10. Threshold and asymmetry in the conditional variance.

	b_1	b_2	b_3	b_4	b'	Lik	Lik'	LR test (p-value)
GE	-0.055 (-0.43)	0.141 (1.47)	0.281 (2.44)	-0.260 (-1.95)	0.105 (3.19)	588.3	584.5	7.6 (0.055)
FR	-0.348 (-1.97)	0.424 (2.71)	0.145 (0.81)	-0.129 (0.54)	0.139 (2.00)	535.6	530.1	11.0 (0.011)
UK	0.480 (1.75)	-0.051 (-0.37)	0.398 (2.82)	-0.378 (-2.38)	0.232 (2.39)	555.6	549.0	13.0 (0.005)
SW	-0.085 (-1.34)	0.066 (1.21)	0.209 (3.94)	-0.258 (-3.59)	0.058 (2.46)	588.3	581.7	13.2 (0.004)
JA	0.205 (1.93)	-0.128 (-1.34)	0.187 (2.29)	-0.233 (-2.57)	0.067 (2.41)	602.4	591.4	22.0 (0.000)
CA	-0.131 (-1.05)	0.146 (1.45)	0.165 (1.52)	-0.117 (-0.89)	0.080 (2.89)	611.7	610.5	2.4 (0.493)
US	-0.241 (-2.11)	0.250 (2.55)	-0.032 (-0.45)	0.135 (2.11)	0.058 (2.12)	649.3	643.4	11.8 (0.008)

The table gives the estimation of the univariate GARCH(1,1) model with threshold, asymmetric conditional variance. The variance is conditioned on the sign and the size of the past innovation and on the past conditional variance. The coefficients b_k ($k = 1, 2, 3, 4$) capture the asymmetry in the conditional variance. b_1 and b_4 capture the size effect and b_2 and b_3 the sign effect. The t-statistics are reported in parentheses. No asymmetry corresponds to the case: $b_1 = b_4 = 0$ and $b_2 = b_3 = b'$. Lik and Lik' are the values of the log-likelihood function of the unconstrained and constrained GARCH(1,1) models. The likelihood ratio test (LR test) is indicated in the last column with the p-value in parentheses. The statistics of the test follows asymptotically a chi-square with 3 degrees of freedom.

estimated b coefficients are reported in the first four columns of the table. The next column recalls the single coefficient b' of the univariate GARCH without thresholds. The log-likelihood are reported for the threshold GARCH (Lik) and simple GARCH (Lik') specifications, as well as a likelihood ratio test between the two specifications (3 degrees of freedom). Significant threshold effects are found for a majority of the countries. Most authors found that negative news had a stronger impact on the volatility in the case of the US stock market. Our data for the USA confirm that finding ($b_2 > b_3$), but we cannot reach the same conclusion for other countries except France. On the other side, the coefficients for large shocks are negative in 11 out of 14 cases, providing evidence that large shocks have a smaller persistence than small shocks.

Notes

1. The use of a time-varying conditional covariance/correlation matrix can be found in studies based on international asset pricing models (see Engel and Rodrigues, 1989 and Chan *et al.*, 1992 for example).
2. Kaplanis (1988) also used a stability test designed by Box (1949) for the covariance matrix. The results are quite similar. Both tests are asymptotically equivalent but their small-sample properties are not known. We conducted both types of tests on our data with similar conclusions and only reported the Jenrich test in Table 2.
3. Simulations conducted by Kaplanis showed that the length of the time period of 50 months is adequate for the asymptotic results. Tests of covariance stability, such as Jenrich's test, may lead to an incorrect rejection of the null if asset returns follow stable Paretian distributions (see Loretan and Phillips, 1994).
4. Some recent studies for the US market are Breen *et al.* (1989), Campbell (1987), Fama and French (1988), Ferson (1989), Ferson and Harvey (1991). Bekaert and Hodrick (1992) look at the stock markets and exchange rates of Japan, UK and Germany for the period 1/1981–12/1989; Campbell and Hamao (1992) look at Japan for the period 1/1970–3/1990; Cumby (1990) looks at Germany, UK and Japan for the period 1/1975–12/1987 using monthly observations of quarterly returns hence requiring an adjustment for overlapping; Cutler *et al.* (1991) look at 12 foreign stock and bond markets as well as their associated exchange rates with observations starting between 1/1960 and 7/1969, depending on the country, and ending in 1988; Harvey (1991) looks at 16 foreign stock markets for the period 1/1970–5/1989. Solnik (1993) looks at 7 foreign stock markets for the period 1/1971–8/1990.
5. A review of the literature can be found in Bollerslev *et al.* (1992).
6. Conditional volatility is not included in the risk premium equation since most studies have failed to discover a significant influence using monthly data (see for example Baillie and De Gennaro, 1990 and Sentana and Wadhvani, 1991). We got similar indications on our data.
7. For example, a constant-correlation GARCH model written on US dollar returns, i.e. using the US dollar as a base currency, would generally not give a constant-correlation GARCH model translated in another base currency.
8. We are grateful to Ken Kroner for providing us with a copy of his Fortran GARCH program.
9. GARCH effects for example tend to decrease by temporal aggregation as the distribution of returns tends toward normality (see Diebold, 1988).
10. This finding is consistent with previous research. For example, Attanasio (1991), Sentana and Wadhvani (1991), Glosten *et al.* (1989) included the dividend yield in univariate GARCH specifications with mixed results.
11. Parameters estimates are: $r = -0.397$ (-0.73), $r' = 0.753$ (1.41) and $a = 11.818$ (1.01) with Lik = 1251.6, LR test = 8.0 (p -value = 0.018).
12. It is not possible to condition on the future shock e_t and we cannot test whether the correlation is higher when the markets go up or go down.

13. The statistical significance of this phenomenon is rather low, so the conclusions are only tentative.
13. Economic variables do enter 'observable' factors, but again the assumption of constant b 's imply that only changes in relative variance of these 'observable' factors can affect the correlation across markets.
15. We thank Morgan Stanley for making these data available to us for research purposes.
16. By interest rate parity the forward basis is equal to the short-term interest rate differential between the country of the asset and that of the investor. Hence the excess returns in local currency are (approximately) equal to the excess returns hedged against currency risk obtained by an investor of any nationality.
17. Remember that the volume of Eurodollar transactions is enormous and the LIBOR (London Interbank Offer Rate) has become the reference short-term dollar interest rate for borrowing in the USA. This is demonstrated by the fact that the Eurodollar futures contract has the largest transaction volume of all financial contracts in terms of underlying capital. A similar comment applies to other currencies.
18. We did not impose this constraint in our estimation and the sum turned out to be slightly negative in a couple of cases.

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