Default risk in corporate yield spreads

Georges Dionne, Geneviève Gauthier, Khemais Hammami,
Mathieu Maurice and Jean-Guy Simonato*

May 5, 2008

Abstract

An important research question examined in the credit risk literature focuses on the proportion of corporate yield spreads attributed to default risk. This topic is reexamined in the light of the different issues associated with the computation of transition and default probabilities obtained from historical default data. We find that the out of sample estimated default-risk proportion in corporate yield spreads is highly sensitive to the ex-ante estimated term structure of default probabilities used as inputs. This proportion can become a large fraction of the yield spread when sensitivity analysis are made with respect to the period over which the probabilities are estimated and the recovery rates. The computation of approximate confidence sets evaluates the precision of the estimated proportions which are also shown to be sensitive to the different filtering procedures required to treat the historical default data base.

Keywords: Corporate yield spread, default risk, estimation period, generator, recovery rate, data filtration, confidence intervals.

JEL classification: G24, G32, G33.

*Dionne, Gauthier and Simonato are at HEC Montréal. Hammami and Maurice are at Caisse de Dépôt et Placement du Québec. Corresponding author: Jean-Guy Simonato, HEC Montréal, 3000, Chemin Côte-Sainte-Catherine, Montreal (Qc) Canada, H3T 2A7. E-mail: jean-guy.simonato@hec.ca. The authors acknowledge the financial support of the National Science and Engineering Research Council of Canada (NSERC), of the Fonds québécois de recherche sur la nature et les technologies (FQRNT), of the Social Sciences and Humanities Research Council of Canada (SSHRC), of the Institut de Finance Mathématique de Montréal (IFM2), and of the Centre for Research on E-finance (CREF). Previous versions of the paper were presented at the Second International Conference on Credit Risk in Montreal, the 2004 SCSE annual conference in Québec, the 2004 C.R.E.D.I.T. Risk Conference in Venesia, and at the finance department of the University of Toronto. J. Christensen, P. François, E. Hansen, D. Lando, and T. Schuermann made very useful comments on previous versions of this manuscript.

1 Introduction

An important research question studied in the credit risk literature looks at the proportion of corporate yield spreads explained by default risk i.e. the part of the spread rewarding the investor for the actuarial expected default loss. This question is not only important for the pricing of bonds and credit derivatives but also for computing banks' optimal economic capital for credit risk (Crouhy, Galai, and Mark, 2000; Gordy, 2000). Elton, Gruber, Agrawal and Mann (Elton *et al.*, 2001) have verified that only a small fraction of corporate yield spreads can be attributed to default risk or expected default loss. They got their result from a reduced form model and have shown that the expected default loss explains no more than 25% of corporate spot spreads. The remainder is attributed to a tax premium and a risk premium for systematic risk. Huang and Huang (2003) reached a similar conclusion with a structural model. They verified that, for investment-grade bonds (Baa and higher ratings), only 20% of the spread is explained by default risk.

One of the key inputs needed for such assessments is an estimate of the term structure of default probability, that is, the probability of defaulting for different time horizons. These quantities may be inferred from databases on historical default frequencies from Moody's and Standard and Poor's. For example, one can first come up with an estimate of transition probabilities between rating classes and then use them to compute the term structure of the default probability. This is the approach used in Elton *et al.* (2001). Although this method appears straightforward, obtaining probability estimates with such a procedure is not a trivial exercise. Many important issues arise in the process and the different choices might lead to different results.

A first issue concerns the period over which the estimation is to be performed. As shown in Bangia *et al.* (2002), transition-matrix estimates are sensitive to the period in which they are computed. Business and credit cycles might have a serious impact on the estimated transition matrices and recovery rates and might lead to highly different estimates for the default-risk proportion.

A second issue calling for close attention is the statistical approach. Because defaults and rating transitions are rare events, the typical cohort approach used by Moody's and Standard and Poor's will produce transition probabilities matrices with many cells equal to zero. This does not mean that the probability of the cell is nil but that its estimate is nil. Such a characteristic could lead to underestimate the default-risk fraction in corporate yield spreads. Lando and Skodeberg (2002) have shown that a continuous-time analysis of rating transitions using generator matrices improves the estimates of rare transitions even when they are not observed in the data, a result that cannot be obtained with the discrete-time cohort approach of Carty and Fons (1993) and Carty (1997).

A third issue arising in computing default and transition probabilities is the data filtering process which determines the information considered about issuers' movements in the database. For example, one must decide whether to consider issuers that are present at the beginning of the estimation period but leave for reasons other than default (withdrawn rating or right censoring). Another choice is whether to consider issuers entering the database after the starting date of estimation. Again, these choices might have non-negligible impacts on the final estimates.

Finally, a fourth important consideration associated with the computations of the default and transition probabilities is the statistical precision with which these quantities are calculated. The statistical uncertainty associated with these estimates should be accounted for and reflected on the default proportions estimates on the form of confidence intervals.

In this article, we revisit the estimation of default spreads in light of the above considerations. For this purpose, we introduce a simple continuous-time model of corporate zero-coupon bond where default time and default probabilities are characterized by a generator matrix describing the credit rating migrations of the firm. The modeling approach is interesting in our context because it allows adressing data filtering issues when estimating the generator. Such a model can also be conveniently simulated. This enables us to address the inference issue and get approximate confidence intervals for the default spread proportions. To use historical databases for assessing the various alternatives associated with the estimation of physical default probabilities, our model is build under an assumption of risk neutrality. Therefore, our estimates and analysis only account for expected default loss and do not include any of the various risk premia potentially present in corporate spreads[1].

Our empirical analysis proceeds as follows. We first look at the issues associated with the choice of the estimation periods and statistical approach for estimating transition matrices and default probability computations. More specifically, the sensitivity to the estimation period is illustrated with a rolling window approach estimating ex-ante time-varying transition and default probabilities that are then used as inputs to get default spread proportions. This approach considers that the

[1]Credit spreads are usually thought of being formed of various parts: 1) the expected default loss; 2) a risk premium on changes in default intensity; 3) a jump risk premium on the default event; 4) a risk premium on recovery risk; 5) a tax effect, and 6) a liquidity premium. The estimated credit spread obtained with the model used here will only include the first part without any risk premia.

recent history of credit migration and default data is the most relevant one to assess the probabilities of defaulting. Comparisons are then made between the estimated proportions calculated with the cohort and the continuous-time generator approach. The results show that the average default spread proportion for 10 years to maturity Baa bonds can jump from 35% (Table 4) for the case obtained with a fixed cohort transition matrix to 54% (Table 7, 1987-1996 period) with an ex-ante time-varying cohort transition matrix and recovery approach. These estimates are also variable through time. For example, for the first half of our sample (1987-1991 period), the estimated proportion jumps from 31% (Table 4) to 74% (Table 7). These results are confirmed with the more robust generator estimation approach.

We then address the data filtering issues. Three data filtering procedures considering different types of information are considered: the first excludes issuers entering after the starting date of estimation (entry firms hereafter) and withdrawn-rating observations; the second one excludes only entry firm observations; and the third considers entry firms observations and withdrawn-rating observations. Our results show that the estimated proportions are sensitive to the choices relative to withdrawn-ratings and entry firms. Indeed, for a Baa rated firm, the estimated proportion varies from 42% to 53% (Table 10, 1987-1996 period) according to the chosen filtration approach. Finally, we study the statistical inference issues. For this purpose, we use a simulation approach to compute approximate confidence intervals in the spirit of Christensen, Hansen, and Lando (2004). In many cases, the 95% confidence sets are wide, illustrating the precision of the point estimates.

The rest of the paper is organized as follows. In Section 2, we describe how the empirical bond-spread curves are estimated. Section 3 presents the default spread model used for estimating the default proportion of the corporate yield spread for different rating categories and maturities. Section 4 explains the estimation methodologies. The numerical findings are then presented in Section 5. More precisely, Subsections 5.1 and 5.2 present the results about the default-risk proportions obtained with this model and examine their sensitivity to the sample period and estimation methodology of probabilities. The results about the information considered in the default database and inference are then presented in Subsections 5.3 and 5.4. Section 6 concludes.

2 Empirical bond-spread curves

Our bond price data come from the Lehman Brothers Fixed Income Database (Warga, 1998). We choose this data to enable comparisons with other articles in this literature using the same information. The data contains information on monthly prices (quote and matrix), accrued interest, coupons, ratings, callability and returns on all investment-grade corporate and government bonds for the period from January 1987 to December 1996. All bonds with matrix prices and options were eliminated; bonds not included in Lehman Brothers' bond indexes and bonds with an odd frequency of coupon payments were also dropped. A detailed description of the bond filtering procedure and treatment of accrued interest is available upon request.

Month-end estimates of the yield-spread curves on zero-coupon bonds for each rating class are needed to implement the models. These yield-spread curves are computed from zero-coupon yield curves obtained with the Nelson and Siegel (1987) approach on government and corporate bonds grouped in three categories: Aa, A, and Baa. When estimating the zero-coupon yield curves of corporate bonds, in a first pass, we remove all bonds with a pricing error higher than \$5. We then repeat the Nelson and Siegel (1987) calibration procedure and data removal procedure until all bonds with a pricing error larger than \$5 have been eliminated. Using this procedure, 776 bonds were eliminated (one Aa, 90 A and 695 Baa) out of a total of 33,401 bonds found in the industrial sector, which is the focus of this study. Our results are coherent, in that all of our estimated empirical bond-spread curves, defined as the difference in yield to maturity of corporate and government zero-coupon bonds, are positive. Moreover, the bond-spread curves between a high rating class and a lower rating class are also positive.

Table 1 reports the average corporate yield spreads for two to ten years of maturity. The results are very close to those presented in Table 1 of Elton *et al.* (2001) for the industrial sector. The small discrepancies might be explained by differences in data filtration and estimation algorithms. In the first panel, the results cover the entire 10-year period, while the second and third panels refer to two sub-periods of five years. Finally, Table 2 compares the average root mean squared errors of the difference between theoretical bond prices computed using the Nelson-Siegel model and the actual bond prices for treasuries and industrial corporate bonds. Again, our results are similar to those reported in the literature.

3 Default spread model

We define here the corporate yield spread as the difference between the yield curves of the risky zero-coupon bond and the risk-free, zero-coupon bond. Therefore, to characterize corporate yield spreads, one need only to model the values of a risk-free and a corporate zero-coupon bond. The model developed here, unlike that of Elton *et al.* (2001), avoids specifying a coupon rate that might absorb effects unrelated to default risk. The model we propose thus focuses on zero-coupon bonds and assumes that a corporate yield spread might be totally explained by the recovery rate and the possibility of default. The model will be used to measure how much of the observed corporate yield spread is explained by these two components.

Our model relies on a constant recovery rate ρ and the intensity $\{\lambda_t : t \geq 0\}$ associated with the distribution of τ, the default time. The risk-free discount factor for the time interval $(t, T]$ is $\beta(t,T) = \exp\left(-\int_t^T r(s)\, ds\right)$ where $r(s)$ denotes the instantaneous continuously compounded risk-free rate. In the following, it is assumed that:

(i) There exists a martingale measure Q under which the discounted value of any risk-free, zero-coupon bond is a martingale.

(ii) In case of default, a constant fraction ρ of the market value of an equivalent risky bond is recovered at the default time.

(iii) Under the martingale measure Q, the default time intensity is driven by a time-homogeneous Markov process X describing the credit rating migrations of the firms. This Markov process X is characterized by the generator matrix $\mathbf{\Lambda}$ and we assume that $\mathbf{\Lambda}$ is diagonable.

In this context, Appendix A shows that the intensity can be written as:

$$\lambda_t = \frac{\sum_{k=1}^m a_k d_k \exp(d_k t)}{1 - \sum_{k=1}^m a_k \exp(d_k t)} \tag{1}$$

where the constants $d_1, ..., d_m$ are the eigenvalues associated with the generator matrix $\mathbf{\Lambda}$ and the constants $a_1, ..., a_m$ are functions of the components of the eigenvectors of $\mathbf{\Lambda}$ and are described explicitly in Appendix A.

(iv) Investors are risk neutral with respect to default risk.

Assumption (i) is needed to price a bond as its expected discounted payoff. Assumption (ii) is as in Duffie and Singleton (1999). Assumption (iii) links the default intensity to the credit rating migration's generator. Therefore, the default time of high-rated bonds reflects the downgrade risk which is the main source of risk for this type of bonds. Finally, assumption (iv), which implies that the distribution of the default time τ will remain the same under the empirical probability measure P and the martingale measure Q, is required to allow the use of databases containing information about default probabilities in our empirical analysis. Under these assumptions, the time t value of a corporate zero-coupon bond paying one dollar at time T is

$$\widetilde{P}\left(t,T\right)=P\left(t,T\right)\exp\left(-\left(1-\rho\right)\int_{t}^{T}\lambda_{s}ds\right) \tag{2}$$

where $P(t,T)$ is the price of a risk-free zero-coupon bond. This result is a particular case of the Duffie and Singleton (1999) approach. A derivation of the bond price equation is in Appendix B. Given this pricing equation, the corporate yield spread curve at time t is given by

$$S\left(t,T\right)=\frac{\ln P\left(t,T\right)}{T-t}-\frac{\ln\widetilde{P}\left(t,T\right)}{T-t}=\frac{1-\rho}{T-t}\int_{t}^{T}\lambda_{s}ds. \tag{3}$$

The spreads can then be computed using the following discrete approximation of equation (3):

$$\frac{1-\rho}{T-t}\int_{t}^{T}\lambda_{s}ds\cong\frac{1-\rho}{n}\sum_{j=1}^{n}\hat{\lambda}_{j\Delta t} \tag{4}$$

where $\Delta t=(T-t)/n=10^{-6}$ and $\widehat{\lambda}_{j}$ is the estimated default intensity process.

4 Generator estimation

The corporate yield spread's model proposed in the previous section requires estimating a generator since such a quantity appears in the construction of the intensity (1). This section describes the different methodologies that may be used to obtain such estimates.

To obtain default probabilities estimates, a first approach imposing little structure on the data, requires forming a cohort at a point in time and counting the defaults after one period, two periods, and so on. The drawback of such an approach stems from the large standard errors associated with the estimates. Generating accurate estimates needs the observation of many defaults, an unlikely possibility when working with investment grade bonds. For such a case, many estimated probabilities would simply be zero. This approach would also make it difficult to include the information provided by new firms entering the database and would not capture the downgrade risk.

Another approach found in the literature uses estimates of periodic transition matrices available from Moody's or Standard and Poor's via the cohort method of Carty and Fons (1993) and Carty (1997). The transitions from one credit rating class to another are counted and estimates of transition probabilities are calculated using the number of bonds in the cohort at the beginning of the period. Probabilities of defaulting for more than one period can then be conveniently computed from this transition matrix using simple matrix multiplications. This convenience comes at the cost of imposing a Markovian structure on the data and it is not clear if such a structure holds. As with the preceding approach, there are also several drawbacks associated with such estimates of default probabilities. Defaults and rating transitions are rare events and these transition matrices contain many cells with estimated probabilities equal to zero. This might lead to an underestimation of the default-spread. Again, as with the preceding approach, if one builds confidence intervals around these estimates, the results turn out to be unsatisfactory. With a small sample size, the default-spread could be misestimated because of large sampling errors.

Lando and Skodeberg (2002) have suggested estimating a Markov-process generator rather than a one-year transition matrix. Such a generator can then be used to compute transition matrices for any desired horizon. As with the cohort approach, this method also imposes a Markovian structure. Lando and Skodeberg (2002) have shown that this continuous-time analysis of rating transitions using generator matrices improves the estimates of rare transitions even when they are not observed in the data, a result that cannot be obtained with the discrete-time analysis of Carty and Fons (1993) and Carty (1997). A continuous-time analysis of defaults permits estimates of default probabilities even for cells that have no defaults. This is possible because the approach draws on the information in the transition from one class to another to infer better estimates of the default probabilities. Finally, as shown in Christensen, Hansen, and Lando (2004), inference in such a framework is informative and can be conveniently computed.[2]

As just discussed, the generator may be estimated using raw data about credit migration's timing. We use this approach herein under the label of "continuous-time generator". However, for sake of comparison with the widely used cohort approach, we must construct a generator

[2]Other recent references about estimating transition matrices and the resulting inferences issues are Jafry and Schuermann (2004) and Hanson and Schuermann (2006).

estimate from transition probability matrices obtained with the cohort estimation approach. As shown in Israel *et al.* (2001), the existence of such a generator for a given transition probability matrix is not guaranteed. However, as proposed by these authors, a solution to this problem is to obtain a generator that will produce a transition matrix close to the original transition matrix. We therefore use the procedure suggested in Israel *et al.* (2001) to verify the existence and obtain the underlying generators for the transition matrices that will be used in our empirical analysis. Using these generators, we will then compute the intensities with equation (1). We label this approach "cohort".

5 Empirical findings

5.1 Sample period

A first key issue associated with estimating transition and default probabilities is the choice of the estimation period. Although we do not observe default probabilities, we can observe substantial variations in spreads over time. These variations can be caused by changes in expected recovery rates, liquidity or risk premia, but also to changes in default probabilities. Figure 1 plots the times series of empirical yield spreads for Aa, A and Baa industrial bonds with ten years to maturity. Given the wide variations in the spread level over time, it is not clear that using a long history of past data to assess the probabilities of defaulting is the best approach for our purposes. With the model described in Section 3, a constant term structure of default probabilities will get a constant credit spread. A long history of default data updated regularly will most likely produce term structures of default probabilities and credit spreads that will be fairly constant through time. This would be at odd with the substantial time variations observed in spreads. Here, we adopt the view that the most recent ex-ante credit-migration and default history is perhaps a more valid indicator of the subjective probability of defaulting used by investors to determine the proper yield for bonds in the various credit classes. We will thus assume that, at a given year, economic agents use the most recent rating transition data to form their anticipations about survival and default probabilities for various horizons. The default probabilities will be estimated using a rolling window approach that will get new transition and default probabilities each year. For example, with a 1-year window, the default proportions for each month in 1987 would be assessed with default data from January 1986 to December 1986.

With such an approach, the length of the window is an important consideration. To provide some rough guidance about what should be a proper length, Table 3 shows the sample correlations between yield spreads and estimated default spreads obtained with our model and various window lengths. In this table, the time series of estimated default spreads are computed with transition matrices estimated with the cohort approach. For the whole sample, we see that short window lengths are associated with positive but modest correlations. A detailed look of the data shows that these low correlations are mostly caused by the high negative correlation in the first year of the sample. Removing these first 12 observations obtains correlations of 0.32, 0.69 and 0.50 for Aa, A and Baa bonds with a one-year window. These correlations are then decreasing as the window length is increased and become negative with longer window lengths. Figures 2 and 3 plots the time series of yield spreads and estimated credit spreads on a two scale graph for the one and ten years window length cases. As shown in these graphs, a short window length seems in better agreement with the yield spreads than a long length. Although the one-year window length obtains higher correlations, it is still important to look at how different window lengths affect the estimated default proportions. We will therefore analyze the results with window lengths of one, two and three years.

To assess how different treatments of default data impact on the estimated proportions of credit spreads, a benchmark case is required. Table 4 shows the estimated proportions for such a case which are computed with a constant transition probability matrix and recovery rates as in Elton *et al.* (2001). This transition matrix is the one used in their analysis and was estimated using Moody's default data over the 1970-1993 period with the cohort approach. Although their model is different from ours because it deals with coupon bonds and a different recovery assumption, the results are almost identical. The estimated proportions with our model are 5%, 12% and 35% for 10 years to maturity Aa, A and Baa bonds while the Elton *et al.* (2001) model gets 5%, 12% and 37%. This suggests that the results presented next cannot be attributed to differences in our modeling approach or recovery assumptions.

Table 5 presents the results obtained with the time-varying probabilities' term structure computed with the window approach described above in this section. As with the previous table, the transition matrices are estimated using a cohort approach. Window lengths of 1, 2 and 3 years are considered. For 2 years to maturity bonds, the proportions are roughly doubled for the 1987-1991 period for all credit classes and windows lengths. For the ten years to maturity case, the proportions are also roughly doubled except for the Baa case that goes from 35% in Table 4 to numbers around 47% for this case. Results are also presented for the first and second halves of our sample, that is the 1987-1991 and 1992-1996 sub-periods. As seen in the table, the proportions vary substantially across sub-samples and lengths of the window. For the first part of the sample, a shorter window length produces higher estimated default proportions while the reverse situation occurs for the second part of the sample. This can be explained by looking at the estimated term structure of default probabilities shown in Table 6. Comparing the estimates for different window lengths shows that, for the second half of the sample, a longer window length tends to include years with many defaults, which in turns gets high estimates of default probabilities. If investors give high weights to the information provided by the more recent default history when forming their expectations, it is not clear that the results obtained with a longer window length such as three years are relevant. They are nevertheless indicative of the sensitivity of the estimated proportions about different sample periods for the default data.

Another input of our model varying much across time and assumed constant is the recovery rate. Figure 4 plots the average recovery rates obtained from Moody's (2005). These rates are defined as the ratio of the defaulted bond's market price observed 30 days after the default date to the face value, for all bonds irrespective of their rating. The average recovery rates vary significantly trough time. They range from a high of 62% to a low of 28%. The average recovery rate during the 1987-1991 sub-period is equal to 40.8% while that of the 1992-1996 sub-period is equal to 45.5%. It is also documented in Moody's (2005) that the recovery rates are even lower for industrial bonds. Because these recovery rates are for all bond ratings, they can be interpreted as the recovery rates of bonds with an average risk. They should thus approximate well the expected recovery rates of Baa ratings, a category falling between the high quality investment grade bonds (like Aaa, Aa and A) and the speculative grades (like Ba, B and Caa-C). Table 7 shows the average proportions obtained for Baa bonds using these time varying recovery rates. Again, these rates are used ex-ante. Thus, to get the 1987 average default proportion, the average recovery rate estimated for 1986 was used. Using these time varying recovery rates does affect the results. For example, for the one-year window case with 10 years to maturity, the proportions that were of 47%, 64% and 29% (Table 5) for the whole sample and the two sub samples go up to 53%, 74% and 33% (Table 7). The effect is similar but less pronounced for the two years to maturity case.

5.2 Generator's estimation

As mentioned in the introduction, estimating transition matrices, generators and default probabilities to measure the proportion of the spread from default data requires a choice about the statistical approach. It is not clear that the results about default proportions are invariant to these different approaches. We have already used the cohort approach in the previous subsection. The goal here is to verify if the continuous-time estimation of the generator produces similar results.

Table 8 presents the results with the time-varying probabilities calculated with the window approach but now using generators estimated with the continuous-time approach. From a comparison with Table 5, we find that the impacts are small in all cases. Our earlier results, which were obtained with the cohort method applied to small sample sizes, might have inherited of the large sampling errors associated with this approach. We find here that the generator approach, which has been found to have better statistical properties, brings similar results and confirms our preceding findings.

5.3 Data filtering

We discuss here the impact of the data filtering process and the information considered when estimating transition matrices and generators. Such an analysis is important for financial institutions that are building their own internal rating system for Basel II and for the regulators who will have to monitor these systems.

When working with default databases, one must deal with issuers' movements in the database. For example, a decision must be made about whether to consider issuers that are present at the beginning of the estimation period but leave for reasons other than default. These cases will be referred to here as withdrawn rating (or right censoring). Another decision is whether or not to consider issuers that entering the database after the starting date of estimation. These cases will be referred to as entry firms. Excluding withdrawn-rating and entry firms are more in the spirit of the standard cohort analysis of Moody's, which also produces statistics including withdrawals (right censoring).

To show the impact of these decisions on the resulting data set with which a generator is estimated, Table 9 examines the data composition with respect to the three filtering alternatives. First, we exclude entry firm and withdrawn-rating data. Second, we include entry firm and right censored data. Finally, we exclude entry firm data but include withdrawals. The analysis was done for the 1987-1996 period and for the 1987-1991 and 1992-1996 sub-periods. We observe, from Table 9, that the proportions of default issuers (Defaults/Issuers) vary substantially when the filtering approach is varied. For example, when compared with the case of entry and withdrawal exclusion, this proportion decreases when including withdrawals and entry firm data. These differences in proportions might affect the estimated generators and default probabilities. A sensitivity analysis about these issues on the corporate default proportions is thus assessed here.

Tables 10 presents a sensitivity analysis about the data filtering procedure. As the results show, important differences are observed. For a Baa rated firm, the 10 years to maturity default spread proportion goes from 42% to 53%. The case of excluded withdrawn-ratings and entry data report higher default proportions. A detailed examination of the results also shows that these are essentially caused by higher estimates of default probabilities. We observe, from Table 9 that the number of defaults is the same in the first and third cases while the numbers of issuers and rating observations are higher in the third case. Inclusion of the withdrawals reduces default probabilities and default risk proportions in yield spreads. The same conclusion is obtained when entry firms are added. Default-risk proportions and implied default probabilities are even lower.

5.4 Inference

As argued in the introduction, inference is another important issue associated with computing default proportions. Because defaults are rare events, default probabilities are typically estimated with much uncertainty. This uncertainty should be reflected in the estimated default proportions based on the latter. To evaluate this uncertainty, we propose here a procedure to compute approximate confidence intervals for the estimated default proportions. Our inference procedure is based on simulation and proceeds as follows.

In a first step, a generator matrix is estimated with default data for a given period using the maximum likelihood estimator given in Lando and Skodeberg (2002). Let the length of this period be T. The estimated generator is considered the *true* generator governing the data generating process.

The second step uses the estimated generator obtained in the first step and the sample of issuers at the beginning of the period to simulate one rating history for each issuer. For each issuer with initial rating i, we simulate the waiting time for leaving this state with an exponential distribution with mean $\frac{1}{|\lambda_{ii}|}$, where λ_{ii} are the elements of the generator matrix for $j = i$. If the waiting time is longer than period T, the issuer stays in its current rating for all the period. If the waiting time is shorter than T, we simulate a uniform distributed random variable between 0 and 1 to determine the issuer's next rating, using the migration intensities $\frac{\lambda_{ij}}{|\lambda_{ii}|}$ for all j different from i so that the migration intensity is different from zero. We then repeat the same task with the new rating until the cumulative waiting time is greater than T or the issuer gets default as a new rating. This procedure is carried out for each issuer with a rating at the beginning of the period. Using these rating histories for all issuers, a generator is estimated to obtain a term structure of default probabilities and an estimate of the average default-risk proportion in yield spreads for each of the maturities.

The second step is repeated 10,000 times to generate 10,000 estimates of average default risk proportion in yield spreads. We then compute different statistics (mean, median, percentiles 2.5 and 97.5 used as our approximate confidence intervals) of average default proportion for each rating and maturity. Table 11 reports the distribution of issuers by rating at the starting date of the simulation period. Tables 12 and 13 report the results for the approximate confidence intervals obtained with the simulation approach for the two sub-periods of our sample. It should be noticed that to simplify the simulation procedure, we use the same generator for all years within the period examined. In each sub-period, the generator is estimated with the default data covering the period over which the proportions are assessed. Such a procedure might amplify (or shrink) the proportions in a high (low) default sub-period[3].

Despite this different procedure, the estimated average proportions reported in Tables 12 and 13 are of similar magnitude than those reported in Table 10 for the one and two year window lengths. The results are showing wide variation across sub-samples and credit ratings. We see that for the 1987-1991 period, the 10 years to maturity Aa case that was originally estimated to be around 4% for the benchmark case in Table 4 is now estimated to be around 14% with 95% approximate confidence intervals of 8.37% and 21.63%. For the 1992-1996 period, the reverse effect is obtained with lower estimated proportions than the original benchmark case and a tight 95% approximate interval of 1.20% and 4.0%. Similar finding, but with higher magnitudes, are also found for the A and Baa classes. For Baa, the 31% benchmark case of Table 4 is now around 71% with confidence

[3]For example, in a high default sub-period, we assume that a ten-year bond is priced with probabilities from the high default period even if this period is not expected to last for ten years. The reverse effect might also be obtained for a low default period. Low default probabilities are used to price a ten-year bond even if the low default cycle is not expected to last for ten years.

intervals of 56.45% and 85.66% for the ten year to maturity case in the 1986-1991 sub period.

6 Conclusion

We have revisited the estimation of default-risk proportions in corporate yield spreads. Past studies have found that only a small proportion of the spreads can be attributed to default risk. Such results do not hold for all periods of our 1987-1996 sample when sensitivity analysis are made with respect to the sample period used to estimate ex-ante default and transition probabilities. We find here that the 1987-1991 period corresponds to a high default cycle, while the 1992-1996 period corresponds to a low default cycle. The estimated proportions can reach 74% of the estimated spread for maturities of ten years for Baa bonds during the 1987-1991 period. We also find that the estimated proportion of default in credit spread is sensitive to changes in recovery rates or data filtration approach for estimating default probabilities. Finally, the sampling variability is estimated to be large in many cases. These conclusions are important for financial institutions planning to use internal rating systems and for the regulators that will have to monitor these systems.

Our study could be extended in several directions by relaxing some of the restrictive assumptions used here. First, the assumption of risk neutrality could be relaxed. The computation of risk-neutral probabilities different from the default probabilities under the objective measure could then be obtained. Building confidence intervals around such estimates might produce results that leave a small place for taxes once liquidity premia are taken into account. This would produce results consistent with the vast and successful literature on derivative securities in which the inclusion of taxes has been found to be of little help.

Finally, it should be noticed that we have observed substantial increases in the estimated proportion in the first half of our sample only. The results in the 1991-1996 low default period maintain that a small proportion of the spread is attributable to the default risk.

References

[1] Altman, E.I., Resti, A., Sironi, A., 2003. Default recovery rates in credit risk modeling: a review of the literature and empirical evidence. Working paper, Stern School of Business, New York University.

[2] Bangia, A. Diebold, F.X., Kronimus, A., Schagen, C., Schuermann, T., 2002. Ratings migration and the business cycle, with application to credit portfolio stress testing. *Journal of Banking and Finance* 26, 445- 474.

[3] Carty, L., Fons, J, 1993. Measuring changes in credit quality. *Journal of Fixed Incomes* 4, 27-41.

[4] Carty, L., 1997. Moody's rating migration and credit quality correlation, 1920-1996. Special Comment, Moody's Investors Service, New York.

[5] Christensen, J., Hansen, E., Lando, D., 2004. Confidence sets for continuous-time rating transition probabilities. *Journal of Banking and Finance* 28, 2575-2602.

[6] Crouhy, M., Galai, D., Mark, R., 2000. A comparative analysis of current credit risk models. *Journal of Banking and Finance* 24, 59-117.

[7] Duffie, D,. Singleton, K.J., 1999. Modeling term structures of defaultable bonds. *Review of Financial Studies* 12, 687-720.

[8] Elton E. J., Gruber, M.J., Agrawal, D., Mann, C., 2001. Explaining the rate spread on corporate bonds. *The Journal of Finance* 56, 247-277.

[9] Gordy, M., 2000. A comparative anatomy of credit risk models. *Journal of Banking and Finance* 24, 119-149.

[10] Hanson, S., Schuermann, T., 2006. Confidence intervals for probabilities of default. *Journal of Banking and Finance* 30, 2281-2301.

[11] Huang, J-Z., Huang, M., 2003. How much of the corporate-treasury yield spread is due to credit risk?. Working paper, Graduate School of Business, Stanford University.

[12] Israel, R.B., Rosenthal, J.S., Wei, J.Z., 2001. Finding generators for Markov chains via empirical transition matrices, with applications to credit ratings. *Mathematical Finance* 2, 245-265.

[13] Jafry, Y., Schuermann, T., 2004. Measurement, estimation and comparison of credit migration matrices. *Journal of Banking and Finance* 28, 2603-2639.

[14] Lando, D., Skodeberg, T.M., 2002. Analyzing rating transitions and rating drift with continuous observations. *Journal of Banking and Finance* 26, 423-444.

[15] Moody's, 2005, Default and recovery rates of corporate bond issuers, 1920-2004.

[16] Nelson, R., Siegel, F., 1987. Parsimonious modeling of yield curves. *Journal of Business* 60, 473-489.

[17] Warga, A., 1998. Fixed income database. University of Houston, Houston, Texas.

.

A Intensity under assumption (iii)

If the generator matrix $\mathbf{\Lambda}$ is diagonable, then one can write $\mathbf{\Lambda} = \mathbf{PDP}^{-1}$ where the columns of the matrix $\mathbf{P}$ contain the eigen vectors of $\mathbf{\Lambda}$ and $\mathbf{D} = (d_i)$ is a diagonal matrix filled with the eigen values of $\mathbf{\Lambda}$. Let $\mathbf{Q}_t = (Q[X_t = j\,|X_0 = i])_{i,j=1,\ldots,m}$ denotes the transition matrix of the Markov process X. Then

$$\begin{aligned}\mathbf{Q}_t = \exp(\mathbf{\Lambda}t) &= \sum_{k=1}^{\infty} \frac{(\mathbf{\Lambda}t)^k}{k!} = \sum_{k=1}^{\infty} \frac{\mathbf{PD}^k\mathbf{P}^{-1}t^k}{k!} = \mathbf{P}\exp(\mathbf{D}t)\,\mathbf{P}^{-1} \\ &= \left(\sum_{k=1}^{m} p_{ik}\exp(d_k t)\,p_{kj}^{-1}\right)_{i,j=1,\ldots,m}\end{aligned}$$

where p_{ij} are the components of $\mathbf{P}$, p_{ij}^{-1} are the components of $\mathbf{P}^{-1}$, and the first equality is justified by the definition of the generator of a time-homogenous Markov process. Let τ_i be the default time of a firm initially rated i and note that the default state corresponds to state m. The cumulative distribution of τ_i is $Q[\tau_i \le t] = Q[X_t = m\,|X_0 = i]$. Therefore, the intensity associated with τ_i is

$$\lambda_{i,t} = \frac{\frac{\partial}{\partial t}Q[X_t = \text{ default}\,|X_0 = i]}{1 - Q[X_t = \text{default}\,|X_0 = i]} = \frac{\sum_{k=1}^{m} p_{ik}p_{km}^{-1}d_k\exp(d_k t)}{1 - \sum_{k=1}^{m} p_{ik}p_{km}^{-1}\exp(d_k t)} = \frac{\sum_{k=1}^{m} a_k d_k\exp(d_k t)}{1 - \sum_{k=1}^{m} a_k\exp(d_k t)}.$$

Q.E.D.

B Default spread model derivation

In case of default, the bondholder recovers, at time τ, a fraction of the market value of an equivalent bond. The value of the corporate zero-coupon bond is expressed as the expectation, under the martingale measure Q, of its discounted payoff:

$$\begin{aligned}\widetilde{P}(t,T) &= \mathrm{E}_t^Q\left[\beta(t,T)\,\mathbf{1}_{\tau>T} + \beta(t,\tau)\,\rho\widetilde{P}(\tau,T)\,\mathbf{1}_{\tau\le T}\right] \\ &= \mathrm{E}_t^Q\left[\exp\left(-\int_t^T [r(s) + (1-\rho)\,\lambda_s]\,ds\right)\right] \\ &= \mathrm{E}_t^Q\left[\exp\left(-\int_t^T r(s)ds\right)\right]\exp\left(-(1-\rho)\int_t^T \lambda_s ds\right) \\ &= P(t,T)\exp\left(-(1-\rho)\int_t^T \lambda_s ds\right)\end{aligned}$$

where the second line is obtain using results from Duffie and Singleton(1999). Q.E.D.

C Data description for transition matrix estimation

The rating transition histories used to estimate the generator are taken from Moody's Corporate Bond Default Database (January, 09, 2002). We consider only issuers domiciled in the United States and having at least one senior unsecured estimated rating. We started with 5,719 issuers (in all industry groups) with 46,305 registered debt issues and 23,666 ratings observations. For each issuer we checked the number of default dates in the Master Default Table (Moody's, January, 09, 2002). We obtained 1,041 default dates for 943 issuers in the period 1970-2001. Some issuers (91) had more than one default date. In the rating transition histories, there are 728 withdrawn ratings that are not the last observation of the issuer. Theses irrelevant withdrawals were eliminated and so we obtained 22,938 ratings observations.

The most important and difficult task is to get a proper definition of default. In order to compare our results with recent studies, we treat default dates as do Christensen *et al.* (2004). First, all the non withdrawn-rating observations up to the date of default have typically been unchanged. However, the ratings that occur within a week before the default date were eliminated. Rating changes observed after the date of default were eliminated unless the new rating reached the B3 level (which is a subcategory of the B rating) or higher and the new ratings were related to debt issued after the date of default. In theses cases we treated theses ratings as related to a new issuer. It is important to emphasize that the first rating date of the new issuer is the latest date between the date of the first issue after default and the first date we observe an issuer rating higher than or equal to B3. The same treatment is applied for the case of two and three default dates. Finally, few issuers have a registered default date before the first rating observation in the Senior Unsecured Estimated Rating Table (Moody's, January, 09, 2002). In theses cases, we considered that there was no default. With this procedure we got 5821 issuers with 965 default dates. We aggregated all rating notches and so we got the nine usual ratings Aaa, Aa, A, Baa, Ba, B, Caa-C, Default and NR (Not Rated) with 15,564 rating observations.

Table 1: Measured corporate yield spreads

Years to maturity	2	3	4	5	6	7	8	9	10
					1987-1996				
Average Treasury yields (%)	6.454	6.709	6.920	7.090	7.226	7.337	7.426	7.500	7.562
Average Aa yield spread (%)	0.413	0.416	0.447	0.477	0.502	0.526	0.548	0.569	0.590
Average A yield spread (%)	0.612	0.672	0.722	0.752	0.769	0.776	0.779	0.778	0.776
Average Baa yield spread (%)	1.180	1.206	1.229	1.237	1.234	1.224	1.210	1.193	1.174
					1987-1991				
Average Treasury yields (%)	7.601	7.775	7.928	8.054	8.157	8.241	8.309	8.364	8.410
Average Aa yield spread (%)	0.512	0.495	0.515	0.545	0.579	0.617	0.656	0.697	0.738
Average A yield spread (%)	0.737	0.802	0.845	0.869	0.882	0.889	0.893	0.895	0.896
Average Baa yield spread (%)	1.421	1.400	1.402	1.400	1.391	1.379	1.363	1.346	1.328
					1992-1996				
Average Treasury yields (%)	5.306	5.643	5.912	6.126	6.296	6.433	6.544	6.636	6.713
Average Aa yield spread (%)	0.315	0.336	0.379	0.409	0.425	0.434	0.439	0.441	0.442
Average A yield spread (%)	0.487	0.543	0.599	0.635	0.655	0.664	0.665	0.661	0.655
Average Baa yield spread (%)	0.939	1.012	1.056	1.074	1.077	1.070	1.057	1.040	1.019

This table reports the average corporate spot yield spreads for industrial Aa, A and Baa corporate bonds for maturities from two to ten years. Corporate bond spreads are calculated as the difference between the corporate spot rates and treasury spot rates for a given maturity. Spot rates were computed using the Nelson-and-Siegel (1987) model. The first panel contains the average treasury spot rates and corporate yield spreads over the entire 10-year period of our sample. The second panel contains the averages for the first five years of our sample and the third panel contains the averages for the second five years.

Table 2: Average root mean squared errors

	Treasuries	Aa	A	Baa
1987-1996	0.220	0.525	0.812	1.458
1987-1991	0.304	0.555	0.876	1.387
1992-1996	0.136	0.496	0.748	1.529

This table presents the average root mean squared errors obtained from the difference between theoretical bond prices computed using the Nelson-and-Siegel model and the actual bond prices for treasuries and industrial Aa, A and Baa corporate bonds. The estimation procedure is described in Section 2. Root mean squared error is measured in cents per dollar. For a given class of bonds, the root mean squared error is calculated once per period (month). The number reported is the average of all root mean squared errors within a given class over the months of the corresponding period.

Table 3: Correlations between the yield spreads and estimated default spreads for varying sampling window lengths

Window	1 year	2 years	3 years	5 years	7 years	10 years
			Industrial Aa bonds			
1987-1996	0.12	0.18	0.08	-0.23	-0.67	-0.75
1987	-0.94	-0.94	-0.94	-0.94	-0.94	-0.94
1988-1996	0.32	0.32	0.12	-0.20	-0.72	-0.82
			Industrial A bonds			
1987-1996	0.36	0.39	0.34	0.27	-0.45	-0.70
1987	-0.99	-0.99	-0.99	-0.99	-0.99	-0.99
1988-1996	0.69	0.48	0.36	0.30	-0.50	-0.78
			Industrial Baa bonds			
1987-1996	0.34	0.14	-0.04	-0.30	-0.62	-0.59
1987	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97
1988-1996	0.50	0.18	-0.02	-0.28	-0.69	-0.64

This table reports the correlations between the 10 years to maturity estimated default spread, Equation (3), and the corresponding Nelson-Siegel yield spread. The generator have been estimated using the cohort method and several sampling window lengths. Recovery rates are 59.59% for Aa, 60.63% for A, and 49.42% for Baa.

Table 4: Average default spread proportions: constant Moody's cohort transition matrix

	2 years	10 years
Proportions (%) Aa		
1987-1996	0.55	4.97
1987-1991	0.43	3.90
1992-1996	0.66	6.03
Proportions (%) A		
1987-1996	2.27	11.79
1987-1991	1.82	10.04
1992-1996	2.72	13.54
Proportions (%) Baa		
1987-1996	9.68	34.78
1987-1991	7.37	31.03
1992-1996	12.00	38.53

This table reports the average of two and ten years to maturity default spread proportions obtained with the Moody's matrix reported in Elton *et al.* (2001) and used over the entire 10-year period. This matrix is estimated with Moody's data over the 1970-1993 period. Recovery rates are 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Averages are computed for the entire 10-year period and the two 5-year sub-periods of our sample. As the transition matrix is the same for the two sub-periods while larger spreads are observed in the first sub-period, the default spread proportions are smaller in this first sub-period because of a constant numerator and larger denominators.

Table 5: Average default spread proportions: time-varying Moody's cohort transition matrices

	2 years to maturity			10 years to maturity		
Window	1 year	2 years	3 years	1 year	2 years	3 years
	Proportions in %: Industrial Aa bonds					
1987-1996	1.45	1.44	1.48	10.88	11.14	11.49
1987-1991	2.61	2.41	2.16	17.76	16.47	14.60
1992-1996	0.29	0.46	0.81	3.99	5.80	8.38
	Proportions in %: Industrial A bonds					
1987-1996	5.42	4.93	4.77	20.02	20.40	20.64
1987-1991	9.43	7.86	6.72	31.83	29.43	26.00
1992-1996	1.42	1.99	2.81	8.21	11.37	15.29
	Proportions in %: Industrial Baa bonds					
1987-1996	16.89	16.39	18.09	46.56	47.68	48.93
1987-1991	23.53	21.83	21.79	64.23	62.22	55.70
1992-1996	10.26	10.95	14.39	28.90	33.14	42.16

This table reports the average of two and ten years to maturity default spread proportions for generators estimated with the cohort approach and a rolling window of 1, 2, and 3 years lengths of ex-ante default data. Recovery rates are 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Averages are computed for the entire 10-year period and the two 5-year sub-periods of our sample.

Table 6: Estimated term structure of default probabilities: time-varying Moody's cohort transition matrices

	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
	Default probabilities in %: 1 year window									
1	1.55	0.15	0.07	0.59	0.23	0.59	0.27	0.02	0.01	0.03
2	3.38	0.56	0.29	1.65	0.85	1.53	0.63	0.09	0.05	0.10
3	5.46	1.20	0.66	3.13	1.81	2.80	1.08	0.21	0.13	0.22
4	7.76	2.03	1.17	4.96	3.07	4.39	1.61	0.39	0.23	0.38
5	10.23	3.02	1.80	7.08	4.60	6.24	2.21	0.62	0.37	0.57
6	12.84	4.14	2.54	9.43	6.37	8.32	2.88	0.91	0.54	0.80
7	15.53	5.37	3.38	11.96	8.33	10.57	3.61	1.24	0.74	1.05
8	18.27	6.68	4.29	14.61	10.45	12.95	4.38	1.60	0.97	1.32
9	21.04	8.06	5.28	17.34	12.69	15.42	5.18	2.00	1.22	1.62
10	23.81	9.50	6.32	20.12	15.02	17.94	6.02	2.42	1.50	1.94
	Default probabilities in %: 2 years window									
1	0.81	0.77	0.10	0.30	0.29	0.30	0.41	0.13	0.02	0.02
2	1.87	1.82	0.41	0.90	1.07	0.93	1.00	0.33	0.07	0.07
3	3.16	3.11	0.89	1.77	2.30	1.90	1.77	0.60	0.17	0.15
4	4.64	4.60	1.54	2.88	3.91	3.18	2.71	0.94	0.31	0.27
5	6.29	6.26	2.33	4.20	5.85	4.75	3.81	1.34	0.49	0.42
6	8.07	8.04	3.24	5.69	8.07	6.56	5.03	1.80	0.72	0.60
7	9.95	9.93	4.25	7.31	10.49	8.57	6.37	2.31	0.98	0.81
8	11.91	11.89	5.34	9.05	13.07	10.74	7.80	2.85	1.28	1.06
9	13.93	13.91	6.51	10.87	15.77	13.03	9.29	3.43	1.61	1.32
10	15.99	15.96	7.73	12.76	18.53	15.41	10.84	4.04	1.97	1.62
	Default probabilities in %: 3 years window									
1	0.66	0.53	0.51	0.21	0.20	0.38	0.28	0.27	0.09	0.02
2	1.49	1.32	1.25	0.70	0.75	1.18	0.80	0.66	0.22	0.08
3	2.50	2.34	2.18	1.46	1.64	2.38	1.55	1.18	0.41	0.18
4	3.66	3.55	3.30	2.44	2.83	3.93	2.52	1.83	0.66	0.31
5	4.95	4.92	4.57	3.61	4.27	5.79	3.70	2.59	0.96	0.49
6	6.36	6.42	5.97	4.95	5.93	7.90	5.05	3.45	1.31	0.71
7	7.87	8.03	7.47	6.42	7.77	10.22	6.55	4.39	1.70	0.96
8	9.45	9.71	9.06	8.00	9.74	12.69	8.18	5.40	2.13	1.24
9	11.09	11.46	10.71	9.66	11.82	15.27	9.90	6.46	2.60	1.55
10	12.76	13.24	12.40	11.38	13.97	17.92	11.69	7.57	3.09	1.88

Estimated term structure of default probabilities for horizons of one to ten years and window lengths of 1, 2 and 3 years of ex-ante default data.

Table 7: Average default spread proportions: time-varying Moody's cohort transition matrices and recovery rates

	2 years to maturity			10 years to maturity		
Window	1 year	2 years	3 years	1 year	2 years	3 years
			Proportions in %: Industrial Baa bonds			
1987-1996	19.65	17.84	19.83	53.44	52.88	54.16
1987-1991	27.58	23.58	23.82	73.79	68.56	61.29
1992-1996	11.72	12.09	15.84	33.09	37.20	47.04

This table reports the average of two and ten years to maturity default spread proportions for generators estimated with the cohort approach and a rolling window of 1, 2, and 3 years lengths of ex-ante default data. Time varying recovery rates, obtained from Moody's 2005 database, have been used here and are defined as the ratio of the defaulted bond's market price, observed 30-days after its default date, to its face value (par) for all bonds.

Table 8: Average default spread proportions: time-varying Moody's generators

Window	2 years to maturity 1 year	2 years	3 years	10 years to maturity 1 year	2 years	3 years
	Proportions in %: Industrial Aa bonds					
1987-1996	1.61	1.94	2.22	8.94	10.37	11.18
1987-1991	3.06	3.70	3.40	14.87	16.61	15.13
1992-1996	0.16	0.18	1.03	3.01	4.13	7.24
	Proportions in %: Industrial A bonds					
1987-1996	3.11	3.20	3.37	15.78	17.80	19.12
1987-1991	5.12	5.02	4.67	25.03	26.48	24.30
1992-1996	1.10	1.38	2.07	6.53	9.11	13.95
	Proportions in %: Industrial Baa bonds					
1987-1996	12.46	14.19	16.64	42.76	48.80	52.84
1987-1991	15.20	16.28	16.49	59.50	63.05	58.83
1992-1996	9.73	12.09	16.80	26.02	34.55	46.84

This table reports the average of two and ten years to maturity default spread proportions for estimated generators computed using the continuous-time approach and a rolling window of 1, 2, and 3 years lengths of ex-ante default data. Recovery rates are 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Averages are computed for the entire 10-year period and the two 5-year sub-periods of our sample.

Table 9: Sensitivity to data filtering

	Excluding withdrawals and entry			Including withdrawals and entry			Including withdrawals and excluding entry		
	87-96	87-91	92-96	87-96	87-91	92-96	87-96	87-91	92-96
Issuers	1,239	1,539	1,432	3,879	2,656	3,090	1,977	1,977	1,867
Rating observations	2,731	2,672	2,236	7,652	4,690	4,829	4,590	3,667	3,213
Defaults	250	196	92	399	267	132	250	196	92
Defaults/Issuers	20.18%	12.74%	6.42%	10.29%	10.05%	4.27%	12.65%	9.91%	4.93%

This table reports the number of firms, transitions, and defaults used to estimate the continuous-time transition matrices. There are three cases: 1) exclusion of withdrawals and exclusion of entry firms; 2) inclusion of withdrawals and inclusion of entry firms; 3) inclusion of withdrawals and exclusion of entry firms.

Table 10: Average default spread proportions: sensitivity to data filtering

	2 years to maturity			10 years to maturity		
	wre & efe	wri & efe	wri & efi	wre & efe	wri & efe	wri & efi
			window length: 1 year			
			Proportions (in %): industrial Aa bonds			
1987-1996	1.61	1.55	1.37	8.94	8.94	8.01
1987-1991	3.06	2.94	2.59	14.87	14.93	13.34
1992-1996	0.16	0.16	0.15	3.01	2.95	2.68
			Proportions (in %): industrial A bonds			
1987-1996	3.11	3.09	2.71	15.78	15.80	14.25
1987-1991	5.12	5.11	4.45	25.03	25.37	22.75
1992-1996	1.10	1.06	0.97	6.53	6.22	5.75
			Proportions (in %): industrial Baa bonds			
1987-1996	12.46	12.42	11.51	42.76	42.35	38.81
1987-1991	15.20	15.44	14.11	59.50	59.89	54.24
1992-1996	9.73	9.39	8.91	26.02	24.81	23.38
			window length: 2 years			
			Proportions (in %): industrial Aa bonds			
1987-1996	1.94	1.81	1.52	10.37	10.17	8.76
1987-1991	3.70	3.45	2.88	16.61	16.40	13.92
1992-1996	0.18	0.18	0.16	4.13	3.95	3.59
			Proportions (in %): industrial A bonds			
1987-1996	3.20	3.12	2.64	17.80	17.45	15.09
1987-1991	5.02	4.95	4.15	26.48	26.37	22.42
1992-1996	1.38	1.29	1.13	9.11	8.52	7.75
			Proportions (in %): industrial Baa bonds			
1987-1996	14.19	13.65	12.06	48.80	46.74	41.17
1987-1991	16.28	15.83	13.48	63.05	61.06	52.10
1992-1996	12.09	11.47	10.63	34.55	32.43	30.24
			window length: 3 years			
			Proportions (in %): industrial Aa bonds			
1987-1996	2.22	1.97	1.72	11.18	10.65	9.37
1987-1991	3.40	3.07	2.68	15.13	14.77	12.89
1992-1996	0.87	0.87	0.77	7.24	6.54	5.84
			Proportions (in %): industrial A bonds			
1987-1996	3.37	3.17	2.62	19.12	18.08	15.46
1987-1991	4.67	4.49	3.65	24.30	23.62	19.74
1992-1996	2.07	1.85	1.60	13.95	12.54	11.17
			Proportions (in %): industrial Baa bonds			
1987-1996	16.64	15.43	13.30	52.84	48.74	42.48
1987-1991	16.49	15.45	12.61	58.83	55.13	46.26
1992-1996	16.80	15.40	13.98	46.84	42.36	38.69

This table reports the average of two and ten years to maturity to maturity default spread proportions for generators estimated with the continuous-time approach and a rolling window. Three cases are examined for each maturity; wre & efe : withdrawn ratings exclusion and entry firm exclusion; wri & efe : withdrawn rating inclusion and entry firm exclusion; wri & efi : withdrawn rating inclusion and entry firm inclusion. Recovery rates are 59.59% for Aa, 60.63% for A, and 49.42% for Baa.

Table 11: Rating distributions

	Aaa	Aa	A	Baa	Ba	B	CCC-C
1987-1991	58	210	456	287	343	179	6
1992-1996	51	160	416	321	275	197	12

This table reports the distribution of issuers, by rating, at the starting date of the estimation period used to construct the confidence sets of average default-spread proportions. There are two periods of simulations: 1987-1991 and 1992-1996.

Table 12: Approximate confidence intervals and sensitivity to data filtering: January 1987 to December 1991

	2 years to maturity			10 years to maturity		
	wre & efe	wri & efe	wri & efi	wre & efe	wri & efe	wri & efi
			Proportions (%) Aa bonds			
Mean	3.18	2.42	2.24	14.08	12.53	11.95
Standard Error	1.89	1.37	1.28	3.43	2.88	2.80
Percentile 2.5	0.43	0.43	0.42	8.37	7.68	7.25
Percentile 9.5	7.50	5.57	5.15	21.63	18.79	18.03
			Proportions (%) A bonds			
Mean	4.66	4.17	3.58	25.71	23.14	20.91
Standard Error	1.00	0.86	0.79	3.18	2.86	2.71
Percentile 2.5	2.90	2.63	2.15	19.76	17.91	15.87
Percentile 9.5	6.77	6.01	5.26	32.16	29.15	26.52
			Proportions (%) Baa bonds			
Mean	21.49	18.22	16.18	70.56	59.81	56.22
Standard Error	4.72	4.27	4.09	7.44	6.50	6.31
Percentile 2.5	13.17	10.72	9.27	56.45	47.48	44.50
Percentile 9.5	31.65	27.42	25.24	85.66	73.15	68.86

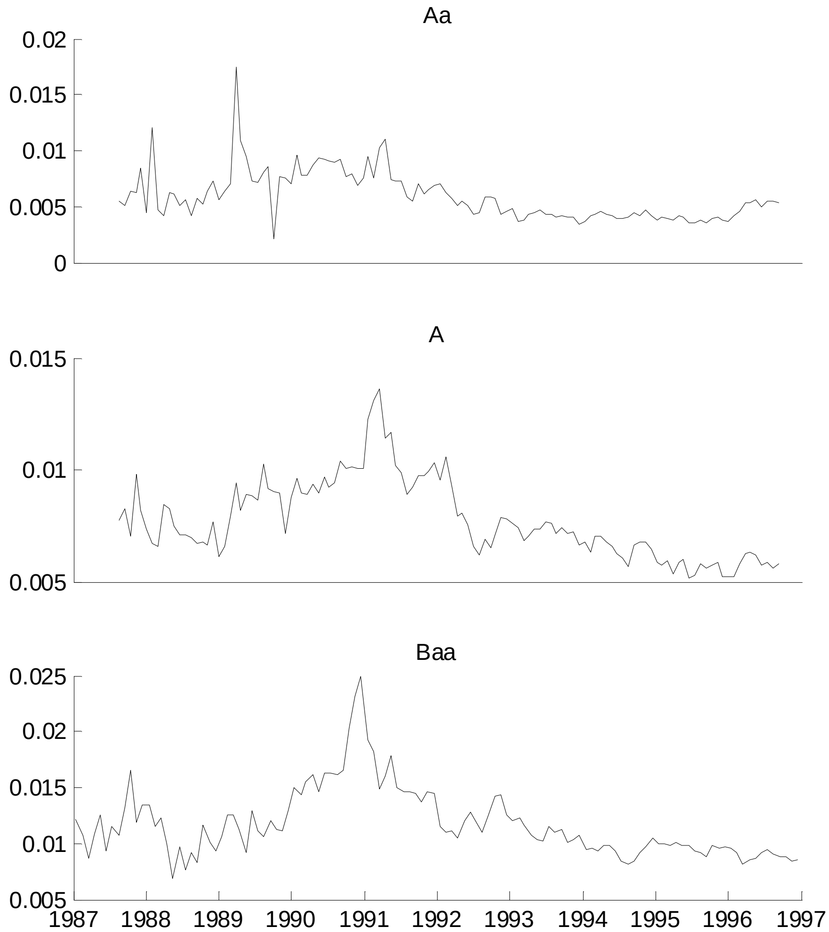
This table reports the mean, standard error, median, and percentiles 2.5 and 97.5 of average default-spread proportions for 2 and 10 years to maturity zero-coupon bonds obtained from Monte Carlo simulations using the continuous-time transition matrix estimated for the 5-year period starting from January 1986 to December 1991 with Moody's default data. wre & efe : withdrawn ratings exclusion and entry firm exclusion; wri & efe : withdrawn rating inclusion and entry firm exclusion; wri & efi : withdrawn rating inclusion and entry firm inclusion. The recovery rates are 59.59% for Aa, 60.63% for A, and 49.42% for Baa.

Table 13: Approximate confidence intervals and sensitivity to data filtering: January 1992 to December 1996

	2 years to maturity			10 years to maturity		
	wre & efe	wri & efe	wri & efi	wre & efe	wri & efe	wri & efi
			Proportions (%) Aa bonds			
Mean	0.16	0.14	0.09	2.41	2.08	1.42
Standard Error	0.06	0.06	0.04	0.72	0.64	0.49
Percentile 2.5	0.06	0.05	0.02	1.20	1.00	0.60
Percentile 9.5	0.30	0.26	0.19	4.00	3.47	2.51
			Proportions (%) A bonds			
Mean	1.72	1.38	0.94	6.24	5.03	3.50
Standard Error	0.62	0.51	0.39	1.62	1.35	1.08
Percentile 2.5	0.66	0.50	0.27	3.43	2.65	1.62
Percentile 9.5	3.04	2.49	1.79	9.65	7.90	5.85
			Proportions (%) Baa bonds			
Mean	15.40	13.05	10.22	20.80	16.91	12.52
Standard Error	6.37	5.84	5.45	4.59	4.02	3.62
Percentile 2.5	4.73	3.02	1.51	12.65	9.80	6.36
Percentile 9.5	29.40	25.99	22.53	30.63	25.44	20.49

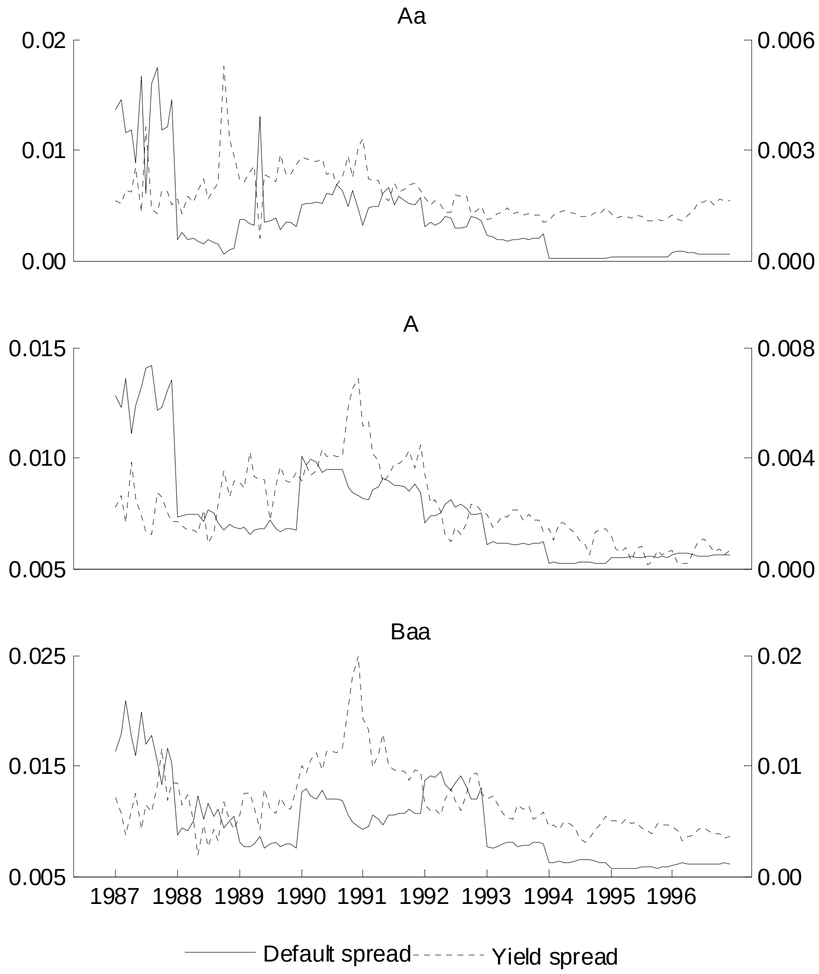
This table reports the mean, standard error, median, and percentiles 2.5 and 97.5 of average default-spread proportions for 2 and 10 years to maturity zero-coupon bonds obtained from Monte Carlo simulations using the continuous-time transition matrix estimated for the 5-year period starting from January 1991 to December 1996 with Moody's default data. wre & efe : withdrawn ratings exclusion and entry firm exclusion; wri & efe : withdrawn rating inclusion and entry firm exclusion; wri & efi : withdrawn rating inclusion and entry firm inclusion. The recovery rates are 59.59% for Aa, 60.63% for A, and 49.42% for Baa.

Figure 1: Ten years to maturity corporate yield spread levels



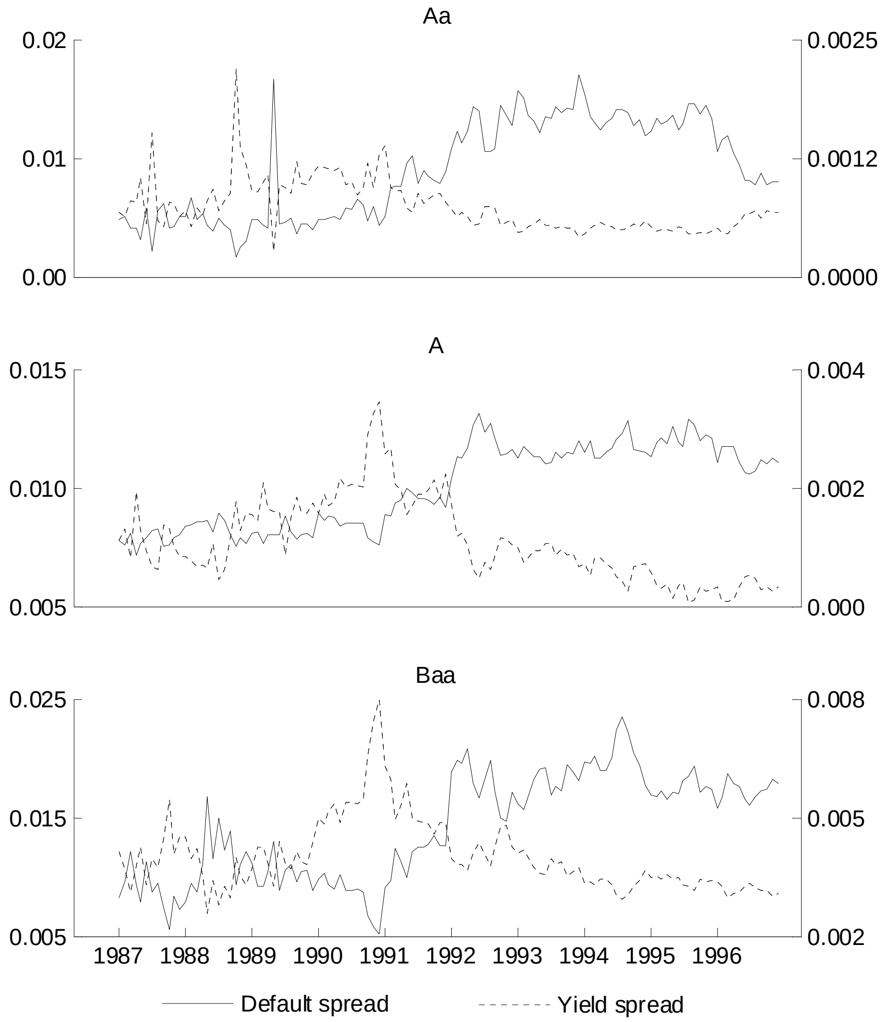
This figure shows the ten year to maturity zero-coupon yield spread levels estimated with the Nelson-Siegel approach on samples of corporate coupon bonds from January 1987 to December 1996.

Figure 2: One year window length



This figure shows on two scale graphs the yield spreads and the estimated default spreads for ten years to maturity Baa zero-coupon bonds from January 1987 to December 1996. For each year, the estimated spreads are estimated with default probabilities got with a one-year window of out of sample default data and the cohort approach.

Figure 3: Ten years window length



This figure shows on two scale graphs the yield spreads and the estimated default spreads for ten years to maturity Baa zero-coupon bonds from January 1987 to December 1996. For each year, the estimated spreads are estimated with default probabilities got with a ten year window of out of sample default data and the cohort approach.

Figure 4: Annual defaulted bonds recovery rates

This figure shows the time series of average recovery rates obtained from Moody's (2005) and defined as the ratio of the defaulted bond's market price, observed 30-days after its default date, to its face value (par) for all bonds.