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Estimation of GARCH process in the presence of structural change

Jean-Guy Simonato *
McGill University, Montreal, Quebec, Canada

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This paper blends the switching regression model of Goldfeld and Quandt (1973) with the GARCH model of Bollerslev (1987) in order to estimate and test for the significance of GARCH parameters in the presence of structural breaks. The GARCH parameter values, the switch date at which one regime supersedes another and the gradualness of each regime switch are all determined endogenously.

1. Introduction

The descriptive validity of univariate ARCH and GARCH models in characterizing the conditional heteroscedasticity of many financial assets has already been well documented in the literature [see Bollerslev et al. (1992)]. Under GARCH, shock to variance persists according to an autoregressive moving average structure of the squared residuals. Empirical evidence from financial-market data seems to suggest that persistence in variance measured by GARCH models is quite substantial. Lamoureux and Lastrapes (1990) argue that the high degree of persistence found by GARCH models might be due to a misspecification of the variance. They find that failure to account for structural change will cause the persistence in variance to be overstated. As they point out the primary difficulty in an empirical investigation of this question is the determination of the timing of structural shifts. For this purpose, the switching regression technique of Goldfeld and Quandt (1973) is blended with the univariate model of Bollerslev (1987). The switching regression technique allows parameter values in the regime, the switch date, and the gradualness of each regime switch to be determined endogenously. Using this general framework, structural changes and GARCH models can be simultaneously examined.

2. Econometric methodology

Assume that T observations on a dependent variable y are available and are generated by r distinct regimes, i.e.

$$y_t = \beta_j x_{jt} + \epsilon_{jt}, \qquad j = 1, \dots, r. \tag{1}$$

Correspondence to: Jean-Guy Simonato, Faculty of Management, McGill University, 1001 Sherbrooke Street West, Montreal, Quebec, H3A 1G5, Canada.
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In eq. (1), j is the regime index, x_{jt} is an exogenous or predetermined variable vector and ϵ_{jt} is the regression residual. These residuals are normally distributed and are assumed to follow a GARCH process, i.e.

$$(\epsilon_{jt} \mid \epsilon_{j,t-1}, \epsilon_{j,t-2}, \ldots) \sim \mathrm{N}(0, h_{jt}), \tag{2}$$

$$h_{jt} = a_j + \sum_{i=1}^{q} b_{i,j} \epsilon_{j,t-i}^2 + \sum_{i=1}^{p} c_{i,j} h_{j,t-i}. \tag{3}$$

The Goldfeld and Quandt switching-regression method (GQSRM) provides a flexible way to identify the changes in regime. To do this GQSRM introduces $r+1$ transition variables, D_{jt} defined as

$$D_{jt} = \int_{-\infty}^{z_t} \left[2\pi^{1/2}\sigma_j^*\right]^{-1} \mathrm{e}^{-\frac{1}{2}((\xi - z_j^*)/\sigma_j^*)} \,\mathrm{d}\xi, \tag{4}$$

where j now runs from 1 to $r-1$ and the endpoint values are $D_{rt} = 0$ and $D_{0t} = 1$ by definition. Eq. (4) is simply the cumulative normal density function. The switch date is represented by the mean z^* and the gradualness of the regime change by the standard deviation, σ^*. Dummy variables can then be formed using these transition variables defined in eq. (4):

$$\gamma_{ik} = \prod_{j=0}^{k-1} D_{jt} \prod_{j=k}^{r} (1 - D_{jt}). \tag{5}$$

Using these dummy variables to separate the regimes, we obtain a composite equation given by

$$y_t = \sum_{j=1}^{r} (\beta_j x_{jt} + \epsilon_{jt})(\gamma_{jt}). \tag{6}$$

This composite equation can be estimated using maximum likelihood, the variance of the error term being given by

$$h_t = \sum_{j=1}^{r} h_{jt} \gamma_{jt}^2. \tag{7}$$

The likelihood function, apart from some initial conditions and constants is

$$L(\theta) = -\tfrac{1}{2} \sum_{t=1}^{T} \left(\log h_t + \epsilon_t h_t^{-1} \epsilon_t\right), \tag{8}$$

where θ represents the vector of all unknown parameters. The model can easily be extended in a multivariate setting such as Bollerslev (1990).

3. Application: Spurious GARCH residuals

As discussed in Lamoureux and Lastrapes (1990), large persistence in variance (I-GARCH) can be caused by failure to identify structural changes. Consequently, one should find less persistence in the variance of a small data set than in a larger one since the probability of multiple structural change is greater in a data set covering a longer period. An interesting question, not examined by Lamoureux and Lastrapes (1990), is to seek if GARCH features are still present in a small sample where all the structural breaks have been identified.

A potential application of the proposed model is thus to discriminate between true GARCH residuals and spurious GARCH residuals induced by an unidentified structural change. An example of a financial time series in which structural breaks have been known to occur and in which ARCH and GARCH models have been used extensively is short-run nominal exchange rates. A structural break can be found in European exchange rates where an increase in policy coordination took place among several European countries around March 1979. This known regime shift in European exchange rates provides a good opportunity to test if GARCH residuals can be attributed to an unidentified structural change that can be detected by the model. In order to minimize the probability of other structural shifts, a small sample size is chosen around the regime change date. The data set consists of weekly exchange rates between the U.S. dollar and the German mark (DM), the French franc (FF), and the Swiss franc (SW) from the DRI database. The data, in this application, covers the period from the first week of 1978 to week 52 of 1979 for a total of 104 observations.

As in Bollerslev (1990) the following model is fitted to the data:

$$y_{it} = 100 \log(s_{it}/s_{it-1}) = \mu_i + \epsilon_{it}, \tag{9}$$

Table 1
Swiss frank, 1978:1 to 1979:52 (p-values in parentheses). [a]

	SRG	UG	SR
μ_1	0.1954 (0.2781)	0.0591 (0.3596)	0.3946 (0.1191)
μ_2	0.1099 (0.2543)	–	0.0753 (0.3209)
a_1	2.160 (0.2101)	0.6564 (0.0038)	5.553 (0.000)
a_2	0.5561 (0.2350)	–	1.280 (0.0000)
b_1	0.0229 (0.4370)	0.2665 (0.0190)	–
b_2	0.0907 (0.2608)	–	–
c_1	0.5951 (0.1009)	0.5770 (0.0000)	–
c_2	0.4629 (0.2477)	–	–
z^*	54.59 (0.0000)	–	50.38 (0.0000)
σ^*	4.96	–	1.53
−2 log. lik.	199.28	217.59	199.99
Like. ratio	–	18.30	0.70
p-value	–	0.005	0.95

[a] SRG is the univariate switching regression model with GARCH residuals; UG is the univariate GARCH model without regime change; SR is the univariate switching regression model with constant variances; z^* is the estimated break point, and σ^* is the standard error associated with the break point.

where s_{it} refers to the ith spot rate, i = DM, SF, FF. The Ljung–Box (LB) and Lagrange multiplier (LM) test for up to 10th order serial correlation in squared residuals reveal that GARCH residuals are present only for Swiss frank with p-values of 0.018 and 0.002, respectively. For the other two series, the hypothesis that no serial correlation is present in the squared residuals cannot be rejected at the 5 percent level over the sample period. A visual inspection of the residual plot for the Swiss franc suggests the presence of a one time shift in variance. The GARCH residuals for the Swiss franc might thus be spurious. In order to test for this hypothesis the GQSRM with one switch point and GARCH(1,1) residuals is estimated. Results for this model specification can be found in table 1. This table also reports nested constraint versions of the model among which we find the univariate GARCH, and the univariate switching regression with constant variance in each regime. Using a likelihood ratio test, the switching regression with constant variances cannot be rejected while the univariate GARCH is rejected at the 5 percent level. GARCH parameters that are significant when there is no break in the model, become statistically insignificant or marginally significant after the introduction of a stochastic break. The estimated switch point is very close to the date where the actual change in policy took place.

4. Conclusion

In this paper, a model capturing regime changes and GARCH effect is developed. This model assumes that the observed data series contains, in addition to the GARCH effect, one or more regime changes affecting the regression coefficients and the GARCH residuals. The model is tested on short-run nominal exchange rates where regime changes have been known to occur. The results demonstrate that over a small sample period, spurious GARCH residuals caused by an unidentified structural change disappear when the possibility of a stochastic regime change is introduced.

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