Real-Time Trading Models and the Statistical Properties of Foreign Exchange Rates

Ramazan Gençay* Giuseppe Ballocchi Michel Dacorogna
Richard Olsen Olivier Pictet

Olsen & Associates
Seefeldstrasse 233, CH-8008, Zurich, Switzerland

December 1998

JEL No: G14, C45, C52, C53

Key Words: Real-time trading models, exponential moving averages, robust kernels, technical trading.

*This paper was written while Gençay visited Olsen & Associates as a research scholar. He is grateful to them for their generosity and for providing an excellent research platform for high frequency finance research. Gençay also thanks the Social Sciences and Humanities Research Council of Canada and the Natural Sciences and Engineering Research Council of Canada for financial support. Address for correspondance: Ramazan Gençay, Department of Economics, Mathematics and Statistics, University of Windsor, 401 Sunset Avenue, Windsor, Ontario, Canada. Email: gencay@uwindsor.ca, Tel: (519) 253 3000 ext. 2382, Fax: (519) 973 7096.

Abstract

Real-time trading models use high frequency live data feeds and their recommendations are transmitted to the traders through data feed lines instantaneously. In this paper, a widely used real-time trading model is used to evaluate the statistical properties of foreign exchange rates. The out-of-sample test period is seven years of five-minute series on three major foreign exchange rates against the US Dollar and one cross rate. Performance of the real-time trading models is measured by the annualized return, two measures of risk corrected annualized return, deal frequency and maximum drawdown. The simulated probability distributions of these performance measures are calculated with the three traditional processes, the random walk, GARCH and AR-GARCH. The null hypothesis that the real-time performances of the foreign exchange series are generated from these traditional processes is tested under the probability distributions of the performance measures.

All four currencies yield postive annualized returns in the studied sampling period. These annualized returns are net of transaction costs. The results indicate that the excess returns of the real-time trading models, after taking the transaction costs and correcting for market risk, are not spurious. The results reject the random walk, GARCH(1,1) and AR-GARCH(1,1) processes as the data generating mechanisms for the high frequency foreign exchange rates. One important reason for the rejection of the GARCH type processes as a data generating mechanishm of foreign exchange returns is the aggregation property of the GARCH processes. The GARCH process behaves more like a homoskedastic process at lower frequencies. Since the real-time trading model's trading frequency is less than two deals per week, the trading model does not pick up the five minute level heteroskedastic structure at the weekly frequency.

The results indicate that the foreign exchange series may possess a multi-frequency conditional mean and conditional heteroskedastic dynamics. The traditional heteroskedastic models fail to capture the entire dynamics by only capturing a slice of this dynamics at a given frequency. Therefore, a more realistic processes for foreign exchange returns should give consideration to the scaling behavior of returns at different frequencies and this scaling behavior should be taken into account in the construction of a representative process.

1. Introduction

The foreign exchange market is the largest financial market worldwide which involves dealers in different geographical locations, time zones, working hours, time horizons, home currencies, information access, transaction costs, and other institutional constraints. The time horizons vary from intra-day dealers, who close their positions every evening, to long-term investors and central banks. In this highly complex and heterogeneous market structure, the market participants are faced with different constraints and use different strategies to reach their financial goals such as by maximizing their profits or maximizing their utility function after adjusting for market risk.

Real-time trading models use high frequency live data feeds and their recommendations are transmitted to the traders through data feed lines instantaneously. In this paper, a popular real-time trading model is used to evaluate the statistical properties of foreign exchange rates. The out-of-sample test period is seven years of five-minute series on three major foreign exchange rates and one cross rate. Performances of the real-time trading models is measured by the annualized return, two measures of risk corrected annualized return, deal frequency and maximum drawdown.

The main essence behind the real time trading models is that they are designed to capture the conditional mean dynamics of the return process for foreign exchange rates under changing market conditions, trading intervals, opening and closing times, market holidays and different market horizons varying from short-term to long-term horizons. Until recently, the common ground was that the return process follows a random walk and therefore there would be nothing in the conditional mean to capture. The objective of this study is to examine whether the excess returns of the real-time trading models of foreign exchange rates are obtained by *pure luck* or whether these models in fact capture subtle persistence in the conditional mean dynamics of the foreign exchange returns. The contributions of this study are twofold. The first contribution involves the examination of the performance of the trading models with traditional statistical processes such as the random walk, generalized autoregressive conditional heteroskedastic process (GARCH) and the autoregressive GARCH (AR-GARCH) process. The second contribution is to test whether a sophisticated trading model has any value added over a model based on a simple technical indicator. The null hypothesis of whether the data generating process for the foreign exchange returns is generated from these processes is tested by comparing the performance of the actual data under the simulated distribution of the performance measures for these processes.

The sensitiveness of the performance measures is also investigated with reference to the different market horizons.

The real-time trading models studied here are based on technical analysis. In the earlier literature, simple technical indicators for the securities market have been tested by Brock et al. (1992). Their study indicates that patterns uncovered by technical rules cannot be explained by simple linear processes or by changing the expected returns caused by changes in volatility. LeBaron (1992,1997) and Levich and Thomas (1993) follow the methodology of Brock et al. (1992) and use bootstrap simulations to demonstrate the statistical significance of the technical trading rules against well-known parametric null models of exchange rates.

In Sullivan et al. (1997), an extensive study of the trading rule performance is examined by extending the Brock et al. (1992) data for the period of 1987-1996. The results of Sullivan et al. (1997) indicate that the trading rule performance remains superior for the time period that Brock et al. (1992) studied, however, these gains disappear in the last ten years of the Dow Jones Industrial Average (DJIA) series. In fact, Sullivan et al. (1997) report that the performance of the out-of-sample results of the technical indicators are completely reversed and the best performing trading rule is not even statistically significant at standard critical levels for the period from 1987 to 1996. One conclusion of their paper is that their findings are not representative as the results can be attributed to an unusually large one-day movement which occured on October 1987. The other conclusion is that the technical trading rules did historically well by producing superior performance, but that, more recently, the markets have become more efficient and hence such opportunities have disappeared.

In Gençay (1998a), the DJIA data set of Brock et al. (1992), is studied with simple moving average indicators within the nonparametric conditional mean models. The results indicate that nonparametric models with buy-sell signals of the moving average models provide more accurate sign and mean squared prediction errors (MSPE) relative to a random walk and GARCH models. Gençay (1998b) shows that past buy-sell signals of simple moving average rules provide statistically significant sign predictions for modelling the conditional mean of the returns for the foreign exchange rates. The results in Gençay (1998b) also indicate that past buy-sell signals of the simple moving average rules are more powerful for modelling the conditional mean dynamics in the nonparametric models.

Overall, the scope of the most recent literature supports the technical analysis but it is limited to simple univariate technical rules. One particular exception is the study by Dacorogna et al. (1995) which examines real-time trading models of foreign exhanges under heterogeneous trading strategies. Dacorogna et al. (1995) point out that there is no particular trading strategy that is systematically better than all the others at all times. Rather, excess return should be evaluated within the context of different trading profiles and this requires various ways of evaluating risk and return in a trading model. They also point out that the most profitable trading models actively trade when many agents are in the market (high liquidity) and do not trade at other times of the day and on weekends. This profitability of the real-time trading models is attributed to the simultaneous presence of heterogeneous agents who utilize different trading strategies based on their trading horizon. Therefore, it is the identification of the heterogeous market microstructure in a trading model which leads to an excess return after adjusting for market risk.

Real-time trading models studied in Pictet et al. (1992) and Dacorogna et al. (1995) utilize technical based information and are also available commercially through Olsen & Associates. Olsen & Associates (O&A) has collected and analyzed large amounts of foreign exchange quotes by market makers around the clock (up to 10000 non-equally spaced prices per day for the German Mark against the US Dollar). Based on this data, O&A has developed a highly sophisticated set of real-time trading models. These models give explicit trading recommendations under realistic constraints and explicitly take into account the heterogeneous structure of the foreign exchange markets by utilizing information on opening hours of a market, different time zones and local holidays. The models[1] have been running real-time for more than eight years with intra-day data, thus providing a long period of time series data for an *ex ante* test.

The period of study covers the first trading day in 1990 until December 31, 1996, a period of seven years at the five minutes frequency. Other studies such as Sullivan et al. (1997) found technical indicators to be ineffective for the DJIA between 1987-1997. Although our study focuses solely on the spot exchange markets, it will provide a benchmark for whether a similar absence of profitability also prevails in spot exchange markets in a seven year period between 1990-1996.

[1]Another aspect of O&A trading models is that trading recommendations transmitted to the customers include a theoretical trading model execution price which is derived from bid/ask quotes using conservative assumptions. The customer who executes the recommendation compares the actual price he got with the trading model execution price. The experience of eight years of trading model operation indicates that the theoretical trading model execution price is rather conservative, since customers often achieve prices that are more favorable to them. This means that the actual performance of the models is typically slightly better than what is calculated on the basis of the theoretical trading model execution price.

The results of this paper indicate that:

- The four currencies pairs, USD-DEM (US Dollar-Deutsche Mark), USD-CHF (US Dollar-Swiss Franc), USD-FRF (US Dollar-French Franc) and DEM-JPY (Deutsche Mark-Japanese Yen) yield 9.63, 3.66, 8.20 and 6.43 percent annualized returns in the studied sampling period. These annualized returns are net of transaction costs.

- The simulated probability distributions of the performance measures are calculated with the random walk, GARCH(1,1) and AR(4)-GARCH(1,1) processes. The null hypothesis that the real-time performances of the foreign exchange series are generated from these traditional processes is rejected under the probability distributions of the performance measures.

- One important reason for the rejection of the GARCH(1,1) process as a data generating mechanishm of foreign exchange returns is the aggregation property of the GARCH(1,1) process. The high frequency GARCH(1,1) process behaves more like a homoskedastic process at lower frequencies. Since the real-time trading model's trading frequency is less than two deals per week, the trading model does not pick up the five minute level heteroskedastic structure at the weekly frequency. Rather, the heteroskedastic structure behaves as if it is measurement noise where the model takes positions and this leads to the rejection of the GARCH(1,1) as a data generation process of the foreign exchange series.

  In a GARCH process, the conditional heteroskedasticity exists in the frequency that the data has been generated. As it is moved away from this frequency to lower frequencies, the heteroskedastic structure slowly dies away leaving itself to a more homogeneous structure in time. A more elaborate processes such as the multiple horizon ARCH models[2]

  possess conditionally heteroskedastic structure at all frequencies in general. The existence of multiple frequency heteroskedastic structure may be more in line with the heterogeneous structure of the foreign exchange markets.

  Similarly, one possible explanation for the rejection of the AR(4)-GARCH(1,1) process is the relationship between the dealing frequency of the model and the

[2]Andersen and Bollerslev (1997) analysed the intraday periodicity and volatility persistence in financial markets. They show that intraday periodicity in the return volatility in foreign exchange markets has a strong impact on the dynamic properties of high frequency returns. Múller et al. (1997) designed a multiple horizon HARCH process to address the conditionally heteroskedastic nature of the foreign exchange returns at different frequencies.

frequency of the simulated data. The AR(4)-GARCH(1,1) process is generated at the 5 minutes frequency but the model's dealing frequency is one or two deals per week. Therefore, the model picks up the high frequency serial correlation as a noise and this serial correlation works against the process. This cannot be treated as a failure of the real-time trading model. Rather, this strong rejection is an evidence of the failure of the aggregation properties of the AR(4)-GARCH(1,1) process over lower frequencies.

- A benchmark model based on a simple exponential moving average (EMA) has also been examined. This simple EMA model yields annualized return of 3.33, 4.40, 6.01 and 7.09 for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY. These annualized returns take the transaction costs into account. The return performance of the EMA model confirms the earlier findings that the simple technical models may be profitable. The examination of the EMA model also leads to the rejection of the random walk, GARCH(1,1) and the AR(4)-GARCH(1,1) processes as the data generating process of foreign exchange dynamics. These rejections are weaker relative to the real-time trading model.

- The annualized return is a performance measure which does not utilize the entire equity curve. Rather, only first and the last points of the equity curve are used in the calculation of the annualized returns. Therefore, a straight line equity curve as well as an equity curve which is subject to extreme variations can have the same annualized returns. A more stringent performance measure is the one which would consider the entire equity path in its calculation. In this paper, two risk corrected return measures, named as X_{eff} and R_{eff}, are used which utilize the entire equity curve in their calculations. The results indicate that these stringent performance measures are more reliable in the statistical testing of the trading models. The paper also addresses the limitations of the Sharpe ratio relative to the X_{eff} and R_{eff}.

- Overall, the results indicate that the real-time trading models yield positive returns after taking the transaction costs into account. The findings point out that the performance of these models are not obtained by *luck* when evaluated under the simulated distributions of the performance measures with the traditional statistical processes.

  The results also indicate that the foreign exchange series may possess a multi-frequency conditional mean and conditional heteroskedastic dynamics. The traditional heteroskedastic models fail to capture the entire dynamics by only capturing a slice of this dynamics at a given frequency. Therefore, a more realistic processes for foreign exchange returns should give consideration to the scaling behavior of foreign exchange returns at different frequencies and this scaling behavior should be taken into account in the construction of a representative process.

The paper is organized as follows. In section two, the performance measures are explained. The simulation models and the simulation methodology are presented in section three. The technical indicators and their robusness properties are explained in section four. The trading models are described in section five. We discuss the empirical results in section six. We conclude afterwards.

2. Performance Measures

In this section we describe the performance measures[3] used to evaluate the trading models in this paper. The *total return*, R_T, is a measure of the overall success of a trading model over a period T, and defined by

$$R_T \equiv \sum_{j=1}^{n} r_j \tag{1}$$

where n is the total number of transactions during the period T and j is the jth transaction and r_j is the return from the jth transaction. The total return expresses the amount of profit made by a trader always investing up to his initial capital or credit limit in his home currency. The annualized return, $\bar{R}_{T,A}$, is calculated by multiplying the total return with the ratio of the number of days in a year to the total number of days in the entire period.

The *maximum drawdown*, D_T, over a certain period $T = t_E - t_0$, is defined by

$$D_T \equiv \max(R_{t_a} - R_{t_b} \mid t_0 \leq t_a \leq t_b \leq t_E) \tag{2}$$

where R_{t_a} and R_{t_b} are the total returns of the periods from t_0 to t_a and t_b respectively.

The trading model performance needs to account for a high total return; a smooth, almost linear increase of the total return over time; a small clustering of losses and no bias towards low frequency trading models. A measure frequently used by practitioners to evaluate portfolio models is the Sharpe ratio. Unfortunately, the Sharpe

[3]The performance measures of this paper are also used in Pictet et al. (1992).

ratio is numerically unstable for small variances of returns and cannot consider the clustering of profit and loss trades. As the basis of a risk-sensitive performance measure, a trading model return variable $\tilde{R}$ is defined to be the sum of the total return R_T and the unrealized current return r_c. The variable $\tilde{R}$ reflects the additional risk due to unrealized returns. Its change over a time interval Δt is

$$X_{\Delta t} \; = \; \tilde{R}_t \; - \; \tilde{R}_{t-\Delta t} \tag{3}$$

where t expresses the time of the measurement. In this paper, Δt is allowed to vary from seven days to 301 days.

A risk-sensitive measure of trading model performance can be derived from the utility function framework (Keeney and Raiffa (1976)). Let us assume that the variable $X_{\Delta t}$ follows a Gaussian random walk with mean $\overline{X}_{\Delta t}$ and the risk aversion parameter α is constant with respect to $X_{\Delta t}$. The resulting utility $u(X_{\Delta t})$ of an observation is $-\exp(-\alpha X_{\Delta t})$, with an expectation value of $\overline{u} = u(\overline{X}_{\Delta t}) \exp(\alpha^2 \sigma^2_{\Delta t}/2)$, where $\sigma^2_{\Delta t}$ is the variance of $X_{\Delta t}$. The expected utility can be transformed back to the *effective return*, $X_{eff} = -\log(-\overline{u})/\alpha$ where

$$X_{eff} \; = \; \overline{X}_{\Delta t} \; - \; \frac{\alpha \sigma^2_{\Delta t}}{2}. \tag{4}$$

The risk term $\alpha\sigma^2_{\Delta t}/2$ can be regarded as a risk premium deducted from the original return where $\sigma^2_{\Delta t}$ is computed by

$$\sigma^2_{\Delta t} \; = \; \frac{n}{n-1} \left(\overline{X^2_{\Delta t}} - \overline{X}^2_{\Delta t} \right). \tag{5}$$

Unlike the Sharpe ratio, this measure is numerically stable and can differentiate between two trading models with a straight line behaviour ($\sigma^2_{\Delta t} = 0$) by choosing the one with the better average return[4].

The measure X_{eff} still depends on the size of the time interval Δt. It is hard to compare X_{eff} values for different intervals. The usual way to enable such a comparison is through the annualization factor, $A_{\Delta t}$, where $A_{\Delta t}$ is the ratio of the number of Δt in a year divided by the number of Δt's in the full sample.

$$X_{eff,ann,\Delta t} \; = \; A_{\Delta t} \, X_{eff} \; = \; \overline{X} - \frac{\alpha}{2} \, A_{\Delta t} \, \sigma^2_{\Delta t} \tag{6}$$

where $\overline{X}$ is the annualized return and it is no longer dependent on Δt. The factor $A_{\Delta t}\sigma^2_{\Delta t}$ has a constant expectation, independent of Δt. This annualized measure still

[4]An example for the limitation of the Sharpe ratio is its inability to distinguish between two straigth line equity curves with different slopes.

has a risk term associated with Δt and is insensitive to changes occurring with much longer or much shorter horizons. To achieve a measure that simultaneously considers a wide range of horizons, a weighted average of several $X_{eff,ann}$ is computed with n different time horizons Δt_i, and thus takes advantage of the fact that annualized $X_{eff,ann}$ can be directly compared

$$X_{eff} \;=\; \frac{\sum_{i=1}^{n} w_i X_{eff,ann,\Delta t_i}}{\sum_{i=1}^{n} w_i} \tag{7}$$

where the weights w are chosen according to the relative importance of the time horizons Δt_i and may differ for trading models with different trading frequencies. In this paper, α is set to $\alpha = 0.10$.

The risk term of X_{eff} is based on the volatility of the total return curve against time, where a steady, linear growth of the total return represents the zero volatility case. This volatility measure of the total return curve treats positive and negative deviations symmetrically, whereas foreign exchange dealers become more risk averse in the loss zone and do hardly care about the clustering of positive profits. A measure which treats the negative and positive zones asymmetrically is defined to be R_{eff} (Müller, Dacorogna and Pictet (1993)) where R_{eff} has a high risk aversion in the zone of negative returns and a low one in the zone of profits whereas X_{eff} assumes constant risk aversion. A high risk aversion in the zone of negative returns means that the performance measure is dominated by the large drawdowns. The R_{eff} has two risk aversion levels: a low one, α_+, for positive $\Delta\tilde{R}$ (profit intervals) and a high one, α_-, for negative $\Delta\tilde{R}$ (drawdowns)

$$\alpha \;=\; \begin{cases} \alpha_+ & for \quad \Delta\tilde{R} \geq 0 \\ \alpha_- & for \quad \Delta\tilde{R} < 0 \end{cases}$$

where $\alpha_+ < \alpha_-$. The high value of α_- reflects the high risk aversion of typical market participants in the loss zone. Trading models may have some losses but, if the loss observations strongly *vary* in size, the risk of very large losses becomes unacceptably high. On the side of the positive profit observations, a certain regularity of profits is also better than a strong variation in size. However, this distribution of positive returns is never *vital* for the future of market participant as the distribution of losses (drawdowns). Therefore, α_+ is much smaller than α_-. In this paper, we assume that $\alpha+ = \alpha_-/4$ and $\alpha_- = .20$.

Amongst annualized return, X_{eff} and R_{eff}, the last two performance measures examine the entire equity curve contrary to the annualized total return. The X_{eff} and R_{eff} are more stringent performance measures by taking into account the entire path in the equity curve. The annualized return, on the other hand, leaves a large degree of freedom to an infinite number of equity curve paths by only considering the beginning and the end points of the equity curve performance.

3. Simulation Methodology

The distributions of the performance measures under various null processes will be calculated by using a simulation methodology. The random walk process is defined by

$$r_t = \alpha + \epsilon_t \tag{8}$$

where $r_t = log(p_t/p_{t-1})$ and $\epsilon_t \sim$ N(0,σ^2). The random walk estimation involves the regression of the actual foreign exchange returns on a constant. A simulation sample for the random walk series with drift is obtained by sampling from the Gaussian random number generator with the mean and the standard deviation of the residual series. The simulated residuals are added to the conditional mean defined by $\hat{\alpha}$, to form a new series of returns. The new series of the returns has the same drift in prices, the same variance and the same unconditional distribution. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

The GARCH(1,1) process is written as

$$r_t = \gamma_0 + \epsilon_t \tag{9}$$

where $\epsilon_t = h_t^{1/2} z_t$, $z_t \sim$ N(0,1) and $h_t = \alpha_0 + \alpha_1 h_{t-1} + \beta_1 \epsilon_{t-1}^2$. GARCH specification (Bollerslev (1986)) allows the conditional second moments of the return process to be serially correlated. This specification implies that periods of high (low) volatility are likely to be followed by periods of high (low) volatility. GARCH specification allows the volatility to change over time and the expected returns are a function of past returns as well as volatility. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the GARCH(1,1) process are generated from the simulated normalized residuals and the estimated parameters.

The AR(p)-GARCH(1,1) process is written as

$$r_t = \gamma + \sum_{i=1}^{p} \gamma_i r_{t-i} + \epsilon_t \tag{10}$$

where $\epsilon_t = h_t^{1/2} z_t$ $z_t \sim$ N(0,1) and $h_t = \alpha_0 + \alpha_1 h_{t-1} + \beta_1 \epsilon_{t-1}^2$. The estimated parameters of the AR(p)-GARCH(1,1) processes together with the simulated residuals are used to generate the simulated returns from these processes. As before, half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid (ask) prices.

4. Exponential Moving Averages with Robust Kernels

The moving average indicators are used to summarize the past behavior of a time series at a given point in time. In many cases, they are used in the form of a *momentum or differential*, the difference between two moving averages. The moving averages can be defined with their *weight* or *kernel* function. The choice of the kernel function has an influence on the behavior of the moving average indicator. A particular type of moving average called *exponential average* plays an important role in the technical analysis literature. Exponential moving average (EMA) operator is a simple average operator with

$$w_{ema}(t;\tau) = \frac{e^{-t/\tau}}{\tau}$$

an exponential decaying kernel. τ determines the range of the operator and t indexes the time. An EMA is written as

$$EMA_p(\tau, t) = \int_{-\infty}^{t} w_{ema}(t - t')p(t')dt'$$

where

$$w_{ema}(t - t';\tau) = \frac{e^{-(t-t')/\tau}}{\tau}$$

Figure 1 demonstrates the kernel function of an exponential moving average with $\tau = 0.5, 20$ and their differential kernel. The sequential computation of exponential moving averages is simple with the help of a recursion formula and it is more efficient than the computation of any differently weighted moving averages.

This basic exponential average kernel can be iterated to provide a family of *iterated exponential moving average* kernels (Müller (1989, 1991), Zumbach (1998))

$$w_{iema}(t;\tau,n) = \frac{1}{(n-1)!}\frac{e^{-t/\tau}}{\tau}(t/\tau)^{n-1}$$

As n gets larger the more weight is allocated towards the middle range of the kernel. In the limit as n goes to infinity, the iterated exponential average behaves like a bell-shaped curve. This implies that the center of the weight is placed in the middle range of the kernel rather than the most recent past. For instance, a second iterative EMA, EMA^2 is written as

$$EMA_p^2(\tau,t) = \int_{-\infty}^{t} \frac{(t-t')}{\tau} w_{ema}(t-t')p(t')dt'$$

where

$$w_{ema}(t-t';\tau) = \frac{e^{-(t-t')/\tau}}{\tau}$$

In general, an n iterative EMA, EMA^n is written as

$$EMA_p^n(\tau,t) = \frac{1}{(n-1)!}\int_{-\infty}^{t} \frac{(t-t')^{n-1}}{\tau^{n-1}} w_{ema}(t-t')p(t')dt'.$$

where

$$w_{ema}(t-t';\tau) = \frac{e^{-(t-t')/\tau}}{\tau}$$

In Figure 2, The iterative moving averages for $n = 1, 2$ and $n = 4$ are plotted which indicate that as n gets larger the center of the weight distribution moves to the middle part of the kernel function. *A simple arithmetic moving average* of length m has a rectengular kernel which makes it very sensitive to the observations leaving the average when the average moves over time. A more robust class of kernels which remedy this sensitivity are the ones which assign an exponentally decaying weights to the observations in the more distant past. These class of robust kernels are obtained from the simple arithmetic average of the iterated exponential average kernels

$$w_{ma}(t;\tau,n) = (1/n)\sum_{j=1}^{n} w_{iema}(t;\tau',j) \tag{11}$$

where $\tau' = 2\tau/(n+1)$ so that the range, r, is independent of n. The robust exponential moving average is written as

$$MA_p^n(\tau,t) = \int_{-\infty}^{t} w_{ma}(t-t',\tau,n)p(t')dt' \tag{12}$$

This is a special case where all weights assigned to each iterative kernel is the same in equation (11). Examples of these robust kernels are presented in Figure 3 where equally weighted iterative exponential moving average kernels are plotted up to $n = 8$. The property of this kernel is that its kernel function has a plateau before it asymptotically declines to zero. This kernel has the property that it is robust against the exteme variations leaving the average by assigning expontially decaying weights. It has also the property that it assigns relatively uniform weights to the most recent history where a simple exponential average would be very sensitive to such a new information. Therefore, a robust kernel has the property that it preserves only the desirable robustness properties of the simple average and exponential average kernels but ignores their highly noisy unrobust properties. In Figure 4, a robust differential kernel is presented which is based on the difference between the exponential moving average with $\tau = 1$ and a robust kernel with $w_{ma}(\tau = 1, n = 8)$. By construction, the area under the kernel sums to zero. The differential kernel assigns positive weights to the recent past and negative weights to the distant past. The real-time trading model of this paper uses a similar robust differential kernel in the construction of the gearing function.

5. Trading Models

A distinction should be made between a price change forecast and an actual trading recommendation. A trading recommendation naturally includes a price change forecast, but it must also account for the specific constraints of the dealer of the respective trading model because a trading model is constrained by its past trading history and the positions to which it is committed. A price forecasting model, on the other hand, is not limited to similar types of constraints. A trading model thus goes beyond predicting a price change such that it must decide if and at what time a certain action has to be taken.

Trading models offer a real-time analysis of foreign exchange movements and generate explicit trading recommendations. These models are based on the continuous collection and treatment of foreign exchange quotes by market makers around the clock at the tick-by-tick frequency level. There are three important reasons to utilize high frequency data in the real-time trading models. The first one is that the model indicators acquire robustness by utilizing the intraday volatility behavior in their build-up. The second reason is that any position taken by the model may need to be reversed quickly although these position reversals may not need to be observed often.

The stop-loss objectives need to satisfied and the high frequency data provides an appropriate platform for this requirement. More importantly, the customer's trading positions and strategies within a trading model can only be replicated with a high statistical degree of accuracy by utilizing high frequency data in a real-time trading model.

The trading models imitate the trading conditions of the real foreign exchange market as closely as possible. They do not deal directly but instead instruct human foreign exchange dealers to make specific trades. In order to imitate real-world trading accurately, they take transaction costs into account in their return computation, they do not trade outside market working hours except for executing stop-loss and they avoid trading too frequently. In short, these models act realistically in a manner which a human dealer can easily follow.

Every trading model is associated with a local market that is identified with a corresponding geographical region. In turn, this is associated with generally accepted office hours and public holidays. The local market is defined to be open at any time during office hours provided it is neither a weekend nor a public holiday. The O&A trading models presently support the Zurich, London, Frankfurt, Vienna and New York markets. Typical opening hours for a model are between 8:00 and 17:30 local time, the exact times depending on the particular local market.

The central part of a trading model is the analysis of the past price movements which are summarized within a trading model in terms of indicators. The indicators are then mapped into actual trading positions by applying various rules. For instance, a model may enter a long position if an indicator exceeds a certain threshold. Other rules determine whether a deal may be made at all. Among various factors, these rules determine the timing of the recommendation. A trading model thus consists of a set of indicator computations combined with a collection of rules. The former are functions of the price history. The latter determine the applicability of the indicator computations to generating trading recommendations. The model gives a recommendation not only for the direction but also for the amount of the exposure. The possible exposures (gearings) are $\pm\frac{1}{2}$, ± 1 or 0 (no exposure).

5.1. The Real Time Trading (RTT) Model

The real-time trading model studied in this paper is classified as a one-horizon, high risk/high return model. The RTT is a trend-following model and takes positions when an indicator crosses a threshold. The indicator is a momentum based on specially weighted moving averages with repeated application of the exponential moving average operator. In case of extreme foreign exchange movements, however, the model adopts an overbought/oversold (contrarian) behaviour and recommends taking a position against the current trend. The contrarian strategy is governed by rules that take the recent trading history of the model into account. The RTT model goes neutral only to save profits or when a stop-loss is reached. Its profit objective is typically at three percent. When this objective is reached, a gliding stop-loss prevents the model from losing a large part of the profit already made by triggering its going neutral when the market reverses. The gearing function for the RTT is

$$g(I_p) = sign(I_p)\ f(|I_p|)\ c(I)$$

where

$$I_p = p - MA_p^4(\tau = 20)$$

where p is the logarithmic price and

$$f(|I_p|) = \begin{cases} if & |I_p| > b & 1 \\ if & a < |I_p| < b & 0.5 \\ if & |I_p| < a & 0 \end{cases}$$

and

$$c(I) = \begin{cases} +1 & if & |I_p| < d \\ -1 & if & |I_p| > d \end{cases}$$

where $a < b < d$. $f(|I_p|)$ measure the size of the signal and $c(|I_p|)$ acts as a contrarian strategy. a and b are functions of *current position, volatility and trading frequency*. d is a function of *position in*, *previous position in*, *sign of the the return of the previous position*. Since X_{eff} and R_{eff} are implicit functions of the gearing function, the optimization of the RTT model is based on the X_{eff} and R_{eff} performance. The model is subject to the open-close and holiday closing hours. The model has maximum stop-loss and maximum gain limits set by the environment.

5.2. A Simple Exponential Average (EMA) Model

The EMA model indicator is momentum based indicator consisting of a difference between two exponential moving averages of range $\tau = 0.5$ and $\tau = 20$ days. The gearing function for the EMA model is

$$g(I_p) = sign(I_p)\ f(|I_p|)$$

where

$$I_p = EMA(\tau = 0.5) - EMA(\tau = 20)$$

and

$$f(|I_p|) \quad = \quad \begin{cases} if \quad |I_p| > a \quad 1 \\ if \quad |I_p| < a \quad 0 \end{cases}$$

where $a > 0$. The model is subject to the open-close and holiday closing hours. The model has maximum stop-loss and profit objective which are set to the same values as in the RTT model.

6. Empirical Results

Imitating the real world requires a system that collects all available price quotes and reacts to each foreign exchange rate movements in real time. For the trading models, O&A have mainly used Reuters data, but other high frequency data suppliers provide similar information in their foreign exchange quotes. Using propriatory data filtering methods, the data is collected, validated and stored for real-time trading model performance evaluation. The current tick frequency is approximately 10000 ticks per business day for Deutsche Mark (USD-DEM); approximately 5000 for the other major rates[5]). Altogether, the O&A database currently contains more than 16 million ticks for USD-DEM.

The data is the five minutes[6] ϑ-time series from January, 1, 1990 to December, 31 1996 for the three major foreign exhange rates, USD-DEM, USD-CHF (Swiss Franc), USD-FRF (French Franc), and the cross-rate DEM-JPY (Deutsche Mark - Japanese Yen). The high frequency data inherits intra-day seasonalities and requires deseasonalization. This paper uses the deseasonalization methodology advocated in Dacorogna et al. (1993) named as the ϑ-time seasonality correction method. The ϑ-time method uses the business time scale and utilizes the average volatility to represents the activity of the market. The ϑ-time method is based on three geographical

[5]The abbreviations used here are defined by the International Organization for Standardization (code 4217).

[6]The real-time system uses tick-by-tick data for its trading recommendations. The historical realizations and the simulations in this paper are carried out with 5 minute data as it is computationally expensive to use the tick-by-tick data for the simulations. Although, the data frequency used in this paper is slighly lower, the historical performance of the currency pairs from the 5 minute series are exactly compatible with performance of the real-time trading model which utilize the tick-by-tick data.

markets namely the East Asia, Europe and the North America. A more detailed exposition of the ϑ methodology is presented in Dacorogna et al. (1993.)

The optimization and the validation of the trading models are done with data prior to January 1, 1990. Therefore, our results here provide a complete *ex-ante* test for the trading performance measures under the studied processes with seven years of 5 minute frequency data. The simulations for each process are done for 1000 replications.

6.1. Random Walk Process

6.1.1 Real Time Trading (RTT) Model

The simulations for the RTT model are reported in Tables 1 to 4. The first and the second columns are the historical realization and the p-value of the corresponding performance measures. The p-value[7] represents the fraction of simulations generating a performance measure larger than the original. The remaining columns report the $5th$ and the $95th$ percentiles, mean, standard deviation, skewness and the kurtosis of the simulations.

The methodology of this paper places a historical realization in the simulated distribution of the performance measure under the assumed process and calculates its p-value. This tells us whether the historical realization is likely to be generated from this particular distribution or not. More importantly, it would tell if the historical performance is likely occur in the future. A small p-value (less than 5 percent) indicates that the historical performance lies in the right tail (or the left tail) and the studied performance distribution is not representative of the data generating process assuming that the trading model is a good one. If the process which generated the performance distribution is close to the data generating process of the foreign exchange returns, the historical performance would lie within two standard deviations

[7]The p-value represents a decreasing index of the reliability of a result. The higher the p-value, the less we can believe that the observed relation between variables in the samples is a reliable indicator of the relation between the respective variables in the population. Specifically, the p-value represents the probability of error that is involved in accepting our observed result as valid, that is, as *representative of the population*. For example, the p-value of 0.05 indicates that there is a 5 percent probability that the relation between the variables found in our sample is a *fluke*. In other words, assuming that in the population there was no relation between those variables whatsoever, and by repeating the experiment, we could expect that approximately every 20 replications of the experiment there would be one in which the relation between the variables in question would be equal or stronger than ours. In many areas of research, the p-value of 5 percent is treated as a *borderline acceptable* level.

of the performance distribution indicating that the studied process may be retained as the representative of the data generating process.

After the transaction costs, actual data with the USD-DEM, USD-CHF, USD-FRF and DEM-JPY yield an annualized total return of 9.63, 3.66, 8.20 and 6.43 percent, respectively. The USD-CHF has the weakest performance relative to the other three currencies. The X_{eff} and R_{eff} performance of the USD-DEM, USD-FRF and DEM-JPY are all positive and range between 3-4 percent. For the USD-CHF, the X_{eff} and R_{eff} are -1.68 and -4.23 percent reflecting the weakness of its performance.

The p-values of the annualized return for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY are 0.3, 8.9, 1.2 and 2.1 percent, respectively. For the USD-DEM and USD-FRF, the p-values are less than 2 percent level and it is about 2 percent for the USD-CHF. In the case of the USD-CHF, the p-value for the annualized return is 8.9 which is well above the 5 percent level. As indicated in section 2, the annualized only utilizes two points of the equity curve leaving a large degrees of freedom to infinitely many equity curves which would be compatible for a given total return. X_{eff} and R_{eff} are more stringent performance measures which utilize the entire equity curve in their calculations. The p-values of X_{eff} and R_{eff} are 0.0, 0.0 percent for USD-DEM; 0.7 and 0.6 percent for USD-CHF; 0.2 and 0.1 percent for USD-FRF and 0.2 and 0.1 percent for DEM-JPY. The p-values for the X_{eff} and R_{eff} are all less than one percent rejecting the null hypothesis that the random walk is the data generating process of exchange rate returns.

The maximum drawdowns for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY are 11.02, 16.08, 11.36 and 12.03 percent. The mean maximum drawdowns from the simulated random walk processes are 53.79, 63.68, 47.68 and 53.49 for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY, respectively. The mean of the simulated maximum drawdowns are three or four times larger than the actual maximum drawdowns. The deal frequencies are 1.68, 1.29, 1.05, 2.14 per week for the four currency pairs from the actual data. The deal frequencies indicate that the RTT model trades on average no more than 2 trades per week although the data feed is at the 5 minute frequency. The mean simulated deal frequencies are 2.46, 1.98, 1.65 and 3.08 which are significantly larger than the actual drawdowns.

The values for the maximum drawdown and the deal frequency indicate that the random walk simulation yield larger maximum drawdown and deal frequency values relative to the values of these statistics from the actual data. In other words, the random walk simulations deal more frequently and result in more volatile equity curves on the average relative to the equity curve from the actual data. Correspondingly, the p-values indicate that the random walk process cannot be the representative of the actual foreign exchange series under these two performance measures. The summary statistics of the simulated performance measures have negligible skewness and statistically insignificant excess kurtosis. This indicates that the distribution of the performance measures are symmetric and do not exhibit tick tails.

The means of the simulations indicate that the distributions are correctly centered at the average transaction costs which is expected under the random walk process. For instance, the mean simulated deal frequency of the USD-DEM is 2.46 deals per week or 127.92 (2.46×52) deals per year. The percentage spread for the USD-DEM is 0.00025 which in turn indicate an average transaction cost of -3.20. Given that the mean of the simulated annualized return is -3.44, we can conclude that the mean of simulated annualized return distribution is centered around the mean transaction cost.

The behavior of the performance measures across 7 day, 29 day, 117 day and 301 day horizons are also investigated with X_{eff} and R_{eff}. The importance of the performance analysis at various horizons is that it permits a more detailed analysis of the equity curve at the predetermined points in time. These horizons correspond approximately to a week, a month, four months and a year's performance. The X_{eff} and R_{eff} values indicate that the RTT model's performance improves over longer time horizons. This is in accordance with the low dealing frequency of the RTT model. In all horizons, the p values for the X_{eff} and R_{eff} are less than a half percent for USD-DEM, USD-FRF and DEM-JPY. For USD-CHF, the p-values are less than 2.4 percent for all horizons. Overall, the multi-horizon analysis indicate the rejection of the random walk process as a candidate for the foreign exchange returns.

6.1.2 Exponential Moving Average (EMA) Model

The simulation results for the EMA model are presented in Tables 5-8. The annualized returns for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY are 3.33, 4.40, 6.01 and 7.09 percent, respectively. Relative to the annualized return performance of the RTT model, the EMA model's return performance is lower for the USD-DEM and the USD-FRF. On the other hand, the annualized returns of the EMA model for USD-CHF and DEM-JPY are slighly higher. One noticeable difference is that the EMA model has higher maximum drawdowns for all four currency pairs relative to the RTT model. The EMA model has also a smaller deal frequency relative to the RTT model. The reason for the difference between the deal frequencies between the two models is that the RTT model has a contrarian strategy whereas the EMA model does not. The second reason is that the long indicator of the RTT model is 16 days wheras the EMA model has a long indicator of 20 days. A relatively longer average lets the EMA model to engage lower number of deals.

The p-values for the annualized returns are 12.6, 8.8, 6.4 and 6.6 percent for the four currencies pairs. All four p-values are greater than the 5 percent level of confidence. Therefore, it is not possible to reject the null hypothesis that the data generating process is random walk with the EMA model under the simulated annualized return distributions. The examination of the p-values for the X_{eff} and R_{eff} indicate that these values are 2.8 and 2.3 percent for the USD-DEM; 0.9 and 0.4 percent for the USD-CHF; 0.6 and 0.4 percent for the USD-FRF and 1.4 and 1.0 percent for the DEM-JPY. Under these more stringent performance measure, the null hypothesis that the random walk process is the data generating process for the foreign exchange returns is rejected.

The p-values for the maximum drawdown and the deal frequency indicate that the EMA model's historical performance stays well below the ones generated under the random walk simulations. In other words, the random walk simulations always generate larger drawdowns relative to the historical drawdowns from the four currency pairs. In fact, the mean simulated drawdowns for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY are 49.70, 60.72, 47.51 and 44.28 percent which are at least three times larger than the historical drawdowns. Similarly, the mean of the simulated deal frequencies for the random walk process stays approximately 20 percent above the historical realizations. The multi-horizon analysis of the EMA model indicate that the model's performance improves in longer horizons. This is mostly due to the low dealing frequency. The p-values of the four currency pairs for X_{eff} and R_{eff} remain less than 3.3 percent for the 301 day horizon.

The overall evaluation of the EMA model is that this simple technical model generate net positive returns for all currencies after taking the transaction costs into account. The simulated probability distributions of the performance measures also indicate that the null hypothesis of whether the foreign exchange returns can be characterized by random walk process is rejected. Both the RTT model as well the EMA model are able to generate positive annualized returns (after taking the transaction costs into account)and the performance of the RTT model is superior to the EMA model with higher returns and smaller drawdowns. The EMA model has smaller deal frequency per week relative to the RTT model.

6.2. GARCH(1,1) Process

A more realistic process for the foreign exchange returns is the GARCH(1,1) process which allows for conditional herterskedasticity. The GARCH(1,1) estimation results are presented in Table 9. The numbers in parantheses are the robust standard errors and the GARCH(1,1) parameters are statistically significant at the 5 percent level for all currency pairs. The Ljung-Box statistic is calculated up to 12 lags for the standardized residuals and it is distributed with χ^2 with 12 degrees of freedom. The Ljung-Box statistic indicate no serial correlation for the USD-DEM and USD-FRF but the USD-FRF and DEM-JPY remains serially correlated. The variance of the normalized residuals are near one. There is no evidence of skewness but the excess kurtosis remains large for the residuals.

6.2.1 Real Time Trading (RTT) Model

In Tables 10-13, the RTT model simulations with the GARCH(1,1) process are presented. Since GARCH(1,1) allows for conditional heteroskedasticity, it is expected that the simulated performance of the RTT model would yield higher p-values and therefore leading to the failure of rejection of the null hypothesis that GARCH(1,1) is the data generating mechanishm for the foreign exchange returns. The results however indicate smaller p-values which is in favor of a stronger rejection of this process relative to the random walk process.

One important reason for the rejection of the GARCH(1,1) process as a data generating mechanishm of foreign exchange returns is the aggregation property of the GARCH(1,1) process[8]. The GARCH(1,1) process behaves more like a homoskedastic process as the frequency is reduced from high to low frequency. Since the RTT model's trading frequency is less than two deals per week, the trading model does not pick up the five minute level heteroskedastic structure at the weekly frequency. Rather, the heteroskedastic structure behaves as if it is measurement noise where the model takes positions and this leads to the stronger rejection of the GARCH(1,1) as a candidate for the foreign exchange data generating mechanism.

In a GARCH process, the conditional heteroskedasticity exists in the frequency that the data has been generated. As it is moved away from this frequency to lower

[8] Guillaume (1995) show that the use of an alternative time scale can eliminate the inefficiencies in the estimation of a GARCH model caused by intra-daily seasonal patterns. However, the temporal aggregation properties of the GARCH models do not hold at the intra-daily frequencies, revealing the presence of several time-horizon components.

frequencies, the heteroskedastic structure slowly dies away leaving itself to a more homogeneous structure in time. A more elaborate processes such as the multiple horizon ARCH models (as in the HARCH process of Müller et al. (1997)) possess conditionally heteroskedastic structure at all frequencies in general. The existence of multiple frequency heteroskedastic structure seem to be more in line with the heterogeneous structure of the foreign exchange markets.

The p-values of the annualized return for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY are 0.4, 8.4, 0.9 and 0.9 percent, respectively. All four currency pairs except USD-CHF yield p-values which are less than one percent. The X_{eff} and R_{eff} are 0.1 and 0.0 percent for USD-DEM; 1.4 and 0.9 percent for USD-CHF; 0.1 and 0.1 percent for USD-FRF and 0.4 and 0.4 percent for DEM-JPY.

The historical maximum drawdown and deal frequency of the RTT model is small relative to the ones generated from the simulated data. The maximum drawdowns for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY are 11.02, 16.08, 11.36 and 12.03 for the four currencies. The mean simulated drawdowns are 53.33, 60.58, 46.00 and 48.77 for the four currencies. The mean simulated maximum drawdowns are three or four times larger than the historical ones. The historical deal frequencies are 1.68, 1.29, 1.05 and 2.14. The mean simulated deal frequencies are 2.39, 1.87, 1.59 and 2.66 for the four currencies. The difference between the historical deal frequencies and the mean simulated deal frequencies remain large. Therefore, the examination of the GARCH(1,1) process with the maximum drawdown and the deal frequency indicate that the historical realizations of these two measures stay outside of the 5 percent level of simulated distributions of these two performance measures.

The mean simulated deal frequency for the USD-DEM is 2.39 trade per week. In annual terms, this is approximately 124.28 deals per year. The half spread for the USD-DEM series is about 0.00025 and this yield 3.10 percent when multiplied with the number of deals per year. The -3.10 percent would be the annual transaction cost of the model. For the model to be profitable, it should yield more than 3.10 per year. Table 10 indicates that the RTT model generates an excess annual return of 9.63 percent whereas the mean of the annualized return from the GARCH(1,1) process stay at the -3.27 percent level.

The multi-horizon examination of the equity curve with the X_{eff} and R_{eff} performance measures indicate that the GARCH(1,1) process as a candidate for the data generation mechanism is strongly rejected at all horizons from seven horizon to a horizon as long as 301 days.

6.2.2 Exponential Moving Average (EMA) Model

The simulation performance of the EMA model under the GARCH(1,1) process is presented in Tables 14-17. The p-values for the annualized returns for the for currency pairs are 14.1, 9.6, 4.6 and 2.8 percent, respectively. Based on the annualized return performance, the null hypothesis that the GARCH(1,1) process is the data generating process of foreign exchange returns cannot be rejected for the USD-DEM and USD-CHF. The other currency pairs stay below the 5 percent level but relatively close to the 5 percent level providing a weak level of confidence. In comparison with the RTT model, the p-values of the EMM model substantially higher indicating the relative weakness of this model as a model of foreign exchange dynamics.

The p-values of the X_{eff} and R_{eff} are 2.3, 1.8 percent for USD-DEM; 0.7, 0.5 percent for USD-CHF; 0.6, 0.4 percent for USD-FRF and 0.5, 0.2 percent for the DEM-JPY. Relative to the annualized return performance, the p-values of the X_{eff} and R_{eff} remain statistically significant as the largest values is not greater than 2.3 percent. As indicated earlier, the annualized return is a performance measure does not utilize the entire equity curve. Rather, only first and the last points of the equity curve are used in the calculation of the annualized returns. Therefore, a straight line equity curve as well as an equity curve which is subject to extreme variations can have the same annualized returns. Based on the X_{eff} and R_{eff} p-values, it can be concluded that the GARCH(1,1) process be rejected as a data generating mechanism of foreign exchange returns with the EMA model. The finding of the EMA model is line with the earlier literature, such as Brock et. al (1989), who also obtained similar findings.

6.3. AR(4)-GARCH(1,1) Process

A further direction is to investigate whether a conditional mean dynamics with GARCH(1,1) innovations would be a more successful characterization of the dynamics of the high frequency foreign exchange returns. The conditional mean of the foreign exchange returns are estimated with four lags of these returns. The additional lags did not lead to substantial increases in the likelihood value. The results of the AR(4)-GARCH(1,1) are presented in Table 18. The numbers in parantheses are the robust standard errors and all four lags are statistically significant at the 5 percent level. The negative autocorrelation is large and higly significant for the first lag of the returns. This is consistent with the high frequency behavior of the foreign exchange returns and is also observed in Dacorogna (1993). The Ljung-Box statistic indicate no serial correlation in the normalized residuals. The variance of the normalized residuals are near one. There is no evidence of skewness but the excess kurtosis remains large for the residuals.

6.3.1 Real Time Trading (RTT) Model

The p-values of the annualized returns are presented in Tables 19-22 which are 0.1, 3.7, 0.3 and 0.5 percent for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY. The results indicate that the AR(4)-GARCH(1,1) process is rejected under the RTT model as a data generating process of foreign exchange returns. One possible explanation of this failure is the relationship between the dealing frequency of the model and the frequency of the simulated data. The AR(4)-GARCH(1,1) process is generated at the 5 minutes frequency but the model's dealing frequency is between one or two deals per week. Therefore, the model picks up the high frequency serial correlation as a noise and this serial correlation works against the process. This can not be treated as a failure of the RTT model. Rather, this strong rejection is an evidence of the failure of the aggregation properties of the AR(4)-GARCH(1,1) process over lower frequencies.

The rejection of the AR(4)-GARCH(1,1) process with the X_{eff} and R_{eff} are even more stronger. The p-values of the X_{eff} and R_{eff} are 0.1, 0.0 percent for USD-DEM; 1.9, 2.3 percent for USD-CHF; 0.2, 0.1 percent for USD-FRF and 0.1, 0.1 percent for DEM-JPY. The p-values remain low at all horizons for the X_{eff} and R_{eff}. The p-values of the maximum drawdown and the deal frequency also indicate almost in all replications the AR(4)-GARCH(1,1) generate higher maximum drawdowns and deal frequencies.

6.3.2 Exponential Moving Average (EMA) Model

The performance of the EMA model with the AR(4)-GARCH(1,1) process are presented in Tables 23-26. The p-values are 10.0, 5.9, 3.2 and 1.8 percent for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY. The p-values for the USD-DEM and USD-CHF are higher than than the 5 percent level. The results with the X_{eff} and R_{eff}, on the other hand, indicate that the p-values remain under the 5 percent levels. In fact, the p-vales for the X_{eff} and R_{eff} are 3.8, 3.7 percent for the USD-DEM; 1.7, 2.4 percent for the USD-CHF; 0.7, 0.6 percent for the USD-FRF and 0.5, 0.4 percent for the DEM-JPY. In comparison with the p-values of the same process for the RTT model, the p-values of the EMA model remains high. This is mostly due to the simplistic nature of the EMA model which cannot capture the dynamics of the foreign exchange returns as successfully as the RTT model. The multi-horizon dynamics of the EMA model is weak with the USD-DEM series as the p-values of the X_{eff} and R_{eff} over the 5 percent borderline for all horizons except the 301 day horizon. The USD-CHF series also behave similarly in the first two horizons. The performance however goes against the AR(4)-GARCH(1,1) process in longer horizons. For the USD-FRF and DEM-JPY, all horizons have p-values less than 5 percent indicating the rejection of the AR(4)-GARCH(1,1) as the data generating process of the foreign exchange returns.

Overall, both the RTT model and the EMA model generate net positive annualized returns for the seven years of high frequency analsis of the four currencies studied here. The performance of the RTT model dominates the EMA model for all currencies. The RTT model also yield smaller drawdowns implying less volatile equity curves. The simulation results indicate that the random walk, the GARCH(1,1) and the AR(4)-GARCH(1,1) cannot sucessfully characterize the dynamics of the foreign exchange returns. In particular, the results indicate that the temporal aggreagation properties of the GARCH(1,1) and AR(4)-GARCH(1,1) processes fail to match the temporal aggregation properties of the actual foreign exchange returns.

7. Conclusions

The results indicate that the excess returns of the real-time trading models, after taking the transaction costs and correcting for market risk, are not spurious. The results also reject the random walk, GARCH(1,1) and AR-GARCH(1,1) processes as the data generating mechanisms for the high frequency foreign exchange dynamics.

The results indicate that USD-DEM, USD-CHF, USD-FRF and DEM-JPY yield 9.63, 3.66, 8.20 and 6.43 percent annualized returns in the studied sampling period. These annualized returns are net of transaction costs. The simulated probability distributions of the performance measures are calculated with the random walk, GARCH(1,1) and AR(4)-GARCH(1,1) processes. The null hypothesis that the real-time performances of the foreign exchange series are generated from these traditional processes is rejected under the probability distributions of the performance measures.

One important reason for the rejection of the GARCH(1,1) process as a data generating mechanishm of foreign exchange returns is the aggregation property of the GARCH(1,1) process. The GARCH(1,1) process behaves more like a homoskedastic process at lower frequencies. Since the real-time trading model's trading frequency is less than two deals per week, the trading model does not pick up the five minute level heteroskedastic structure at the weekly frequency. Rather, the heteroskedastic structure behaves as if it is measurement noise where the model takes positions and this leads to the rejection of the GARCH(1,1) as a data generation process of the foreign exchange series.

In a GARCH process, the conditional heteroskedasticity exists in the frequency that the data has been generated. As it is moved away from this frequency to lower frequencies, the heteroskedastic structure slowly dies away leaving itself to a more homogeneous structure in time. A more elaborate processes such as the multiple horizon ARCH models possess conditionally heteroskedastic structure at all frequencies in general. The existence of multiple frequency heteroskedastic structure may be more in line with the heterogeneous structure of the foreign exchange markets.

Similarly, one possible explanation for the rejection of the AR(4)-GARCH(1,1) process is the relationship between the dealing frequency of the model and the frequency of the simulated data. The AR(4)-GARCH(1,1) process is generated at the 5 minutes frequency but the model's dealing frequency is one or two deals per week. Therefore, the model picks up the high frequency serial correlation as a noise and this serial correlation works against the process. This can not be treated as a failure of the real-time trading model. Rather, this strong rejection is an evidence of the failure of the aggregation properties of the AR(4)-GARCH(1,1) process over lower frequencies.

A benchmark model based on a simple exponential moving average (EMA) has also been examined. This simple EMA model yields annualized return of 3.33, 4.40, 6.01 and 7.09 for the USD-DEM, USD-CHF, USD-FRF and DEM-JPY. These annualized returns take the transaction costs into account. The return performance of the EMA model confirms the earlier findings that the simple technical models may be profitable. The examination of the EMA model also leads to the rejection of the random walk, GARCH(1,1) and the AR(4)-GARCH(1,1) processes as the data generating process of foreign exchange dynamics. These rejections are weaker relative to the real-time trading model.

The annualized return is a performance measure which does not utilize the entire equity curve. Rather, only first and the last points of the equity curve are used in the calculation of the annualized returns. Therefore, a straight line equity curve as well as an equity curve which is subject to extreme variations can have the same annualized returns. A more stringent performance measure is the one which would consider the entire equity path in its calculation. In this paper, two risk corrected return measures, named as X_{eff} and R_{eff}, are used which utilize the entire equity curve in their calculations. The results indicate that these stringent performance measures are more reliable in the statistical testing of the trading models. The paper also addresses the limitations of the Sharpe ratio relative to the X_{eff} and R_{eff}.

Overall, the results indicate that the real-time trading models yield positive returns after taking the transaction costs into account. The findings point out that the performance of these models are not obtained by *luck* when evaluated under the simulated distributions of the performance measures with the traditional statistical processes.

The results also indicate that the foreign exchange series may possess a multi-frequency conditional mean and conditional heteroskedastic dynamics. The traditional heteroskedastic models fail to capture the entire dynamics by only capturing a slice of this dynamics at a given frequency. Therefore, a more realistic processes for foreign exchange returns should give consideration to the scaling behavior of returns at different frequencies and this scaling behavior should be taken into account in the construction of a representative process.

References

Bollerslev, T., 1986, Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics* 31, 307-327.

Andersen, T. G. and T. Bollerslev, 1997, Intraday periodicity and volatility persistence in financial markets, *Journal of Empirical Finance*, 4, 115-158.

Brock, W. A., J. Lakonishok and B. LeBaron, 1992, Simple technical trading rules and the stochastic properties of stock returns, *Journal of Finance* 47, 1731-1764.

Dacorogna, M. M., U. A. Müller, R. J. Nagler, R. B. Olsen, and O. V. Pictet, 1993, A geographical model for the daily and weekly seasonal volatility in the foreign exchange market, *Journal of International Money and Finance*, 12, 413-438.

Dacorogna, M. M., U. A. Müller, C. Jost, O. V. Pictet, R. B. Olsen and J. R. Ward, 1995, Heterogeneous real-time trading strategies in the foreign exchange market, *The European Journal of Finance*, 1, 383-404.

Engle, R. F., 1982, Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U. K. Inflation, *Econometrica* 50, 987-1008.

Engle, R. F., D. M. Lilien and R. P. Robins, 1987, Estimating time varying risk premia in the term structure: The ARCH-M model, *Econometrica* 55, 391-408.

Gençay, R., 1998a, The predictability of security returns with simple technical trading rules, *Journal of Empirical Finance*, 5, 347-359.

Gençay, R., 1998b, Linear, nonlinear and essential foreign exchange prediction with simple technical rules, *Journal of International Economics*, forthcoming.

Guillaume, D. M., O. V. Pictet and M. M. Dacorogna, 1995, On the intra-daily performance of GARCH process, Olsen Research Institute Discussion Paper, Zurich, Switzerland.

Keeney, R. L. and H. Raiffa, 1976, Decision with multiple objectives: Preferences and value tradeoffs, John Wiley & Sons, New York.

Kuan, C.-M and H. White, (1994), Artificial neural networks: An econometric perspective, *Econometric Reviews* 13, 1-91.

LeBaron, B., 1997, Technical trading rules and regime shifts in foreign exchange, forthcoming in *Advanced Trading Rules*, edited by E. Acar and S. Satchell, Butterworth-Heinemann.

LeBaron, B., 1992, Do moving average trading rule results imply nonlinearities in foreign exchange markets?, Social Systems Research Institute, University of Wisconsin-Madison.

Levich, R. M. and L. R. Thomas, 1993, The significance of technical trading-rule profits in the foreign exchange market: A bootstrap approach, *Journal of International Money and Finance* 12, 451-474.

Müller, U. A., 1989, Indicators for trading systems, Olsen Research Institute Discussion Paper, Zurich, Switzerland.

Müller, U. A., 1991, Specially weighted moving averages with repeated application of the EMA operator, Olsen Research Institute Discussion Paper, Zurich, Switzerland.

Müller, U. A., M. M. Dacorogna and O. V. Pictet, 1993, A trading model performance measure with strong risk aversion against drawdowns, Olsen Research Institute Discussion Paper, Zurich, Switzerland.

Müller, U. A., M. M. Dacorogna, R. D. Davé, Richard B. Olsen, O. V. Pictet and J E. von Weizsaˇcker, 1997, Volatilities of Different Time Resolutions - Analyzing the Dynamics of Market Components, *Journal of Empirical Finance*, 4, 213-240.

Pictet, O. V., M. M. Dacorogna, U. A. Müller, R. B. Olsen and J. R. Ward, 1992, Real-time trading models for foreign exchange rates, Neural Network World, 6, 713-744.

Sullivan, R., A. Timmermann and H. White, 1997, Data-Snooping, Technical Trading Rule Performance and the Bootstrap, University of California-San Diego, Economics Department Discussion Paper 97-31.

Zumbach, G., 1998, Operators on inhomogeneous time series, Olsen Research Institute Discussion Paper, Zurich, Switzerland.

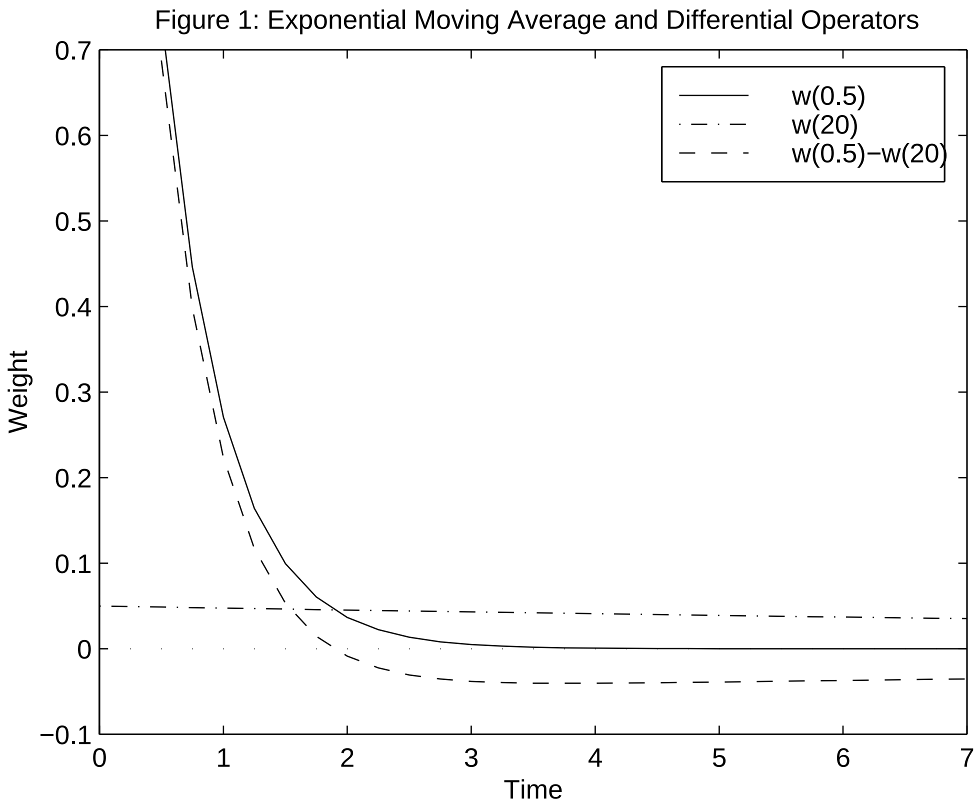

Figure 1: Exponential moving average (EMA) operator is a simple average operator with

$$w_{ema}(t;\tau) = \frac{e^{-t/\tau}}{\tau}$$

an exponential decaying kernel. τ determines the range of the operator and t indexes the time. An EMA is written as

$$EMA_p(\tau,t) = \int_{-\infty}^{t} w_{ema}(t-t')p(t')dt'$$

where $w_{ema}(t-t';\tau) = \frac{e^{-(t-t')/\tau}}{\tau}$.

The figure above demonstrates the kernel function of an exponential moving average with $\tau = 0.5$ and $\tau = 20$ and their differential kernel. The sequential computation of exponential moving averages is simple with the help of a recursion formula and it is more efficient than the computation of any differently weighted moving averages.

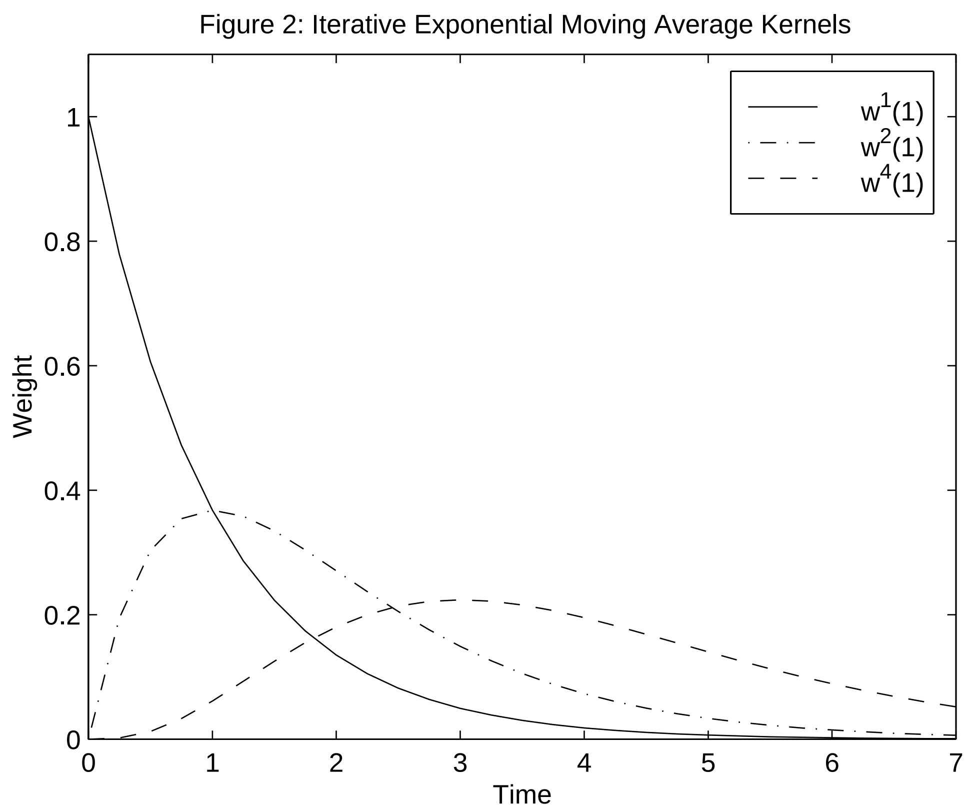

Figure 2: An n iterative EMA, EMA^n is written as

$$EMA_p^n(\tau, t) = \frac{1}{(n-1)!}\int_{-\infty}^{t} \frac{(t-t')^{n-1}}{\tau^{n-1}} w_{ema}(t-t')p(t')dt'$$

where $w_{ema}(t-t';\tau) = \frac{e^{-(t-t')/\tau}}{\tau}$.

In the figure above, the iterative moving averages for $n = 1, 2$ and $n = 4$ are plotted which indicate that as n gets larger the center of the weight distribution moves to the middle part of the kernel function.

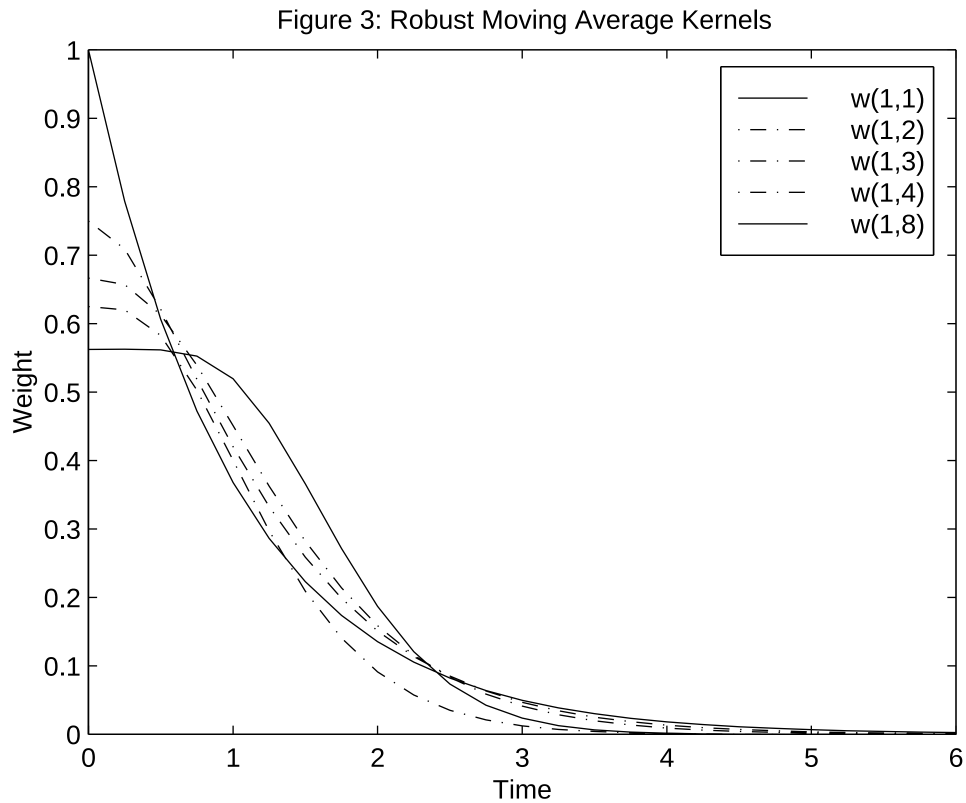

Figure 3: The robust exponential moving average is written as

$$MA_p^n(\tau, t) = \int_{-\infty}^{t} w_{ma}(t - t', \tau, n)p(t')dt'.$$

This is a special case where all weights assigned to each iterative kernel is the same in equation (11). In the figure above, the examples of these robust kernels are plotted with n up 8. The property of this kernel is that its kernel function has a plateau before it asymptotically declines to zero. This kernel has the property that it is robust the exteme variations leaving the average by assigning expontially decaying weights. It has also the property that it is assigns relatively uniform weights to the most recent history wheras a simple exponential average would be very sensitive such a new information. Therefore, a robust kernel has the property that it preserves only the desirable robustness properties of the simple average and exponential average kernels but ignores their highly noisy unrobust properties.

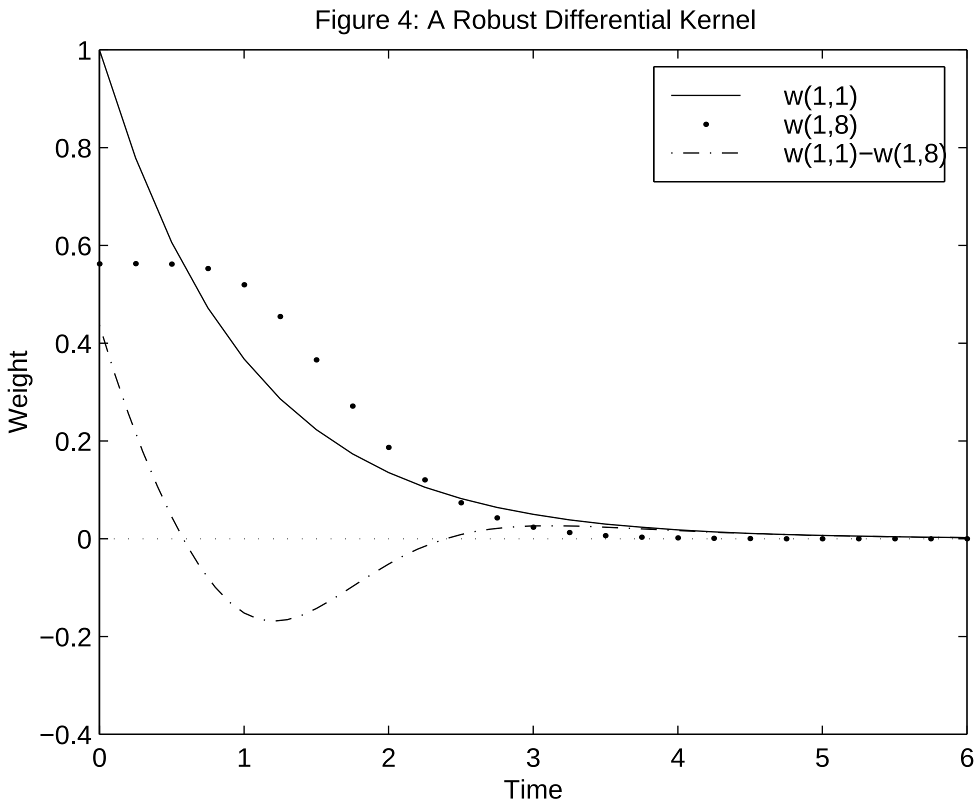

Figure 4: A robust differential kernel is presented which is based on the difference between the exponential moving average with $\tau = 1$ and a robust kernel with $w_{ma}(\tau = 1, n = 8)$. By construction, the area under the kernel sums to zero. The differential kernel assigns positive weights to the recent past and negative weights to the distant past. The real-time trading model of this paper uses a similar robust differential kernel in the construction of the gearing function.

Table 1
Random Walk Simulations with RTT Model
USD-DEM, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	9.63	0.3	-11.38, 4.03	-3.44	4.74	0.09	-0.13
Xeffective	3.78	0.0	-20.25, -4.14	-12.11	5.09	0.13	-0.23
Reffective	4.43	0.0	-26.42, -7.70	-16.80	5.90	0.03	-0.20
Max Drawdown	11.02	100.0	25.26, 94.86	53.79	21.36	-0.71	0.21
Deal frequency	1.68	100.0	2.20, 2.71	2.46	0.16	-0.10	-0.19
Horizon: 7 days							
Xeffective	3.47	0.0	-19.65, -4.24	-11.83	4.76	0.08	-0.15
Reffective	1.80	0.0	-24.14, -7.21	-15.51	5.20	0.05	-0.15
Horizon: 29 days							
Xeffective	3.27	0.0	-20.21, -4.36	-12.10	4.95	0.07	-0.23
Reffective	2.16	0.0	-27.05, -8.07	-17.45	5.91	0.02	-0.28
Horizon: 117 days							
Xeffective	4.07	0.0	-20.85, -3.42	-12.21	5.44	0.10	-0.32
Reffective	5.10	0.0	-31.01, -6.53	-18.10	7.49	0.26	0.25
Horizon: 301 days							
Xeffective	4.62	0.0	-23.37, -2.42	-11.89	6.32	0.39	0.02
Reffective	6.83	0.0	-27.85, -3.25	-14.56	7.49	0.35	0.16

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-DEM series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The random walk estimation involves the regression of the actual USD-DEM returns on a constant. A simulation sample for the random walk series with drift is obtained by sampling from the Gaussian random number generator with the mean and the standard deviation of the residual series. The simulated residuals are added to the conditional mean defined by $\hat{\alpha}$, to form a new series of returns. The new series of the returns has the same drift in prices, the same variance and the same unconditional distribution. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures.*

Table 2

Random Walk Simulations with RTT Model

USD-CHF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	3.66	8.9	-12.95, 5.10	-4.15	5.61	-0.01	0.23
Xeffective	-1.68	0.7	-26.23, -6.00	-16.26	6.19	-0.02	0.03
Reffective	-4.23	0.6	-33.48, -9.80	-22.40	7.10	-0.14	-0.01
Max Drawdown	16.08	100.0	29.28, 108.66	63.68	25.19	-0.82	0.84
Deal frequency	1.29	100.0	1.82, 2.14	1.98	0.10	-0.05	-0.05
Horizon: 7 days							
Xeffective	-2.96	1.2	-24.72, -6.45	-15.79	5.62	-0.04	0.18
Reffective	-4.81	0.4	-31.01, -10.55	-21.09	6.19	-0.06	0.15
Horizon: 29 days							
Xeffective	-4.10	2.2	-25.72, -6.29	-16.08	5.83	-0.01	0.14
Reffective	-8.97	2.3	-34.76, -11.35	-23.61	7.05	-0.11	-0.00
Horizon: 117 days							
Xeffective	-0.67	0.9	-27.49, -4.95	-16.34	6.88	-0.02	-0.13
Reffective	-1.77	0.9	-39.14, -8.91	-23.97	9.31	0.17	0.51
Horizon: 301 days							
Xeffective	1.94	0.5	-32.61, -2.87	-16.48	8.74	0.42	0.34
Reffective	1.71	0.7	-35.24, -4.47	-18.84	9.44	0.26	0.09

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-CHF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The random walk estimation involves the regression of the actual USD-CHF returns on a constant. A simulation sample for the random walk series with drift is obtained by sampling from the Gaussian random number generator with the mean and the standard deviation of the residual series. The simulated residuals are added to the conditional mean defined by $\hat{\alpha}$, to form a new series of returns. The new series of the returns has the same drift in prices, the same variance and the same unconditional distribution. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures.*

Table 3
Random Walk Simulations with RTT Model
USD-FRF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	8.20	1.2	-9.92, 5.25	-2.28	4.61	0.01	0.04
X effective	4.80	0.2	-17.92, -2.24	-10.18	4.85	0.10	0.21
R effective	4.95	0.1	-23.03, -4.36	-14.13	5.63	-0.00	0.25
Max Drawdown	11.36	100.0	21.26, 83.70	47.68	19.63	-0.84	1.02
Deal frequency	1.05	100.0	1.51, 1.79	1.65	0.08	-0.13	-0.22
Horizon: 7 days							
Xeffective	3.18	0.2	-17.32, -2.56	-9.99	4.60	0.02	-0.02
Reffective	1.97	0.2	-21.08, -5.05	-13.09	5.02	-0.00	0.01
Horizon: 29 days							
Xeffective	4.41	0.2	-17.36, -2.40	-10.15	4.64	0.02	0.08
Reffective	3.99	0.1	-23.15, -5.68	-14.54	5.47	-0.02	0.12
Horizon: 117 days							
Xeffective	3.83	0.2	-18.87, -1.95	-10.24	5.09	0.15	0.10
Reffective	3.77	0.1	-26.40, -4.28	-15.24	6.78	0.23	0.24
Horizon: 301 days							
Xeffective	6.35	0.2	-20.77, -0.42	-10.01	6.12	0.32	0.27
Reffective	7.38	0.2	-25.15, -0.45	-12.37	7.43	0.21	0.20

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-FRF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The random walk estimation involves the regression of the actual USD-FRF returns on a constant. A simulation sample for the random walk series with drift is obtained by sampling from the Gaussian random number generator with the mean and the standard deviation of the residual series. The simulated residuals are added to the conditional mean defined by $\hat{\alpha}$, to form a new series of returns. The new series of the returns has the same drift in prices, the same variance and the same unconditional distribution. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures.*

Table 4
Random Walk Simulations with RTT Model
DEM-JPY, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	6.43	2.1	-11.67, 4.66	-3.50	4.96	-0.09	-0.12
Xeffective	3.81	0.2	-20.30, -3.17	-11.69	5.17	-0.02	0.02
Reffective	3.45	0.1	-25.66, -6.19	-15.99	6.02	-0.12	0.19
Max Drawdown	12.03	100.0	23.12, 94.17	53.49	21.87	-0.67	0.16
Deal frequency	2.14	100.0	2.75, 3.28	3.08	0.16	0.89	0.00
Horizon: 7 days							
Xeffective	1.87	0.3	-19.66, -3.53	-11.59	4.96	-0.06	-0.14
Reffective	0.58	0.2	-23.75, -5.93	-14.96	5.47	-0.06	-0.13
Horizon: 29 days							
Xeffective	2.12	0.3	-19.93, -3.39	-11.69	5.09	-0.02	-0.23
Reffective	0.19	0.3	-26.45, -6.52	-16.56	6.10	-0.02	-0.23
Horizon: 117 days							
Xeffective	4.90	0.2	-20.88, -2.74	-11.79	5.48	0.01	0.11
Reffective	5.17	0.2	-29.41, -5.42	-17.08	7.32	0.11	0.59
Horizon: 301 days							
Xeffective	5.39	0.1	-23.06, -1.93	-11.77	6.63	0.40	0.81
Reffective	5.56	0.3	-26.64, -2.37	-13.86	7.52	0.19	0.66

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual DEM-JPY series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The random walk estimation involves the regression of the actual DEM-JPY returns on a constant. A simulation sample for the random walk series with drift is obtained by sampling from the Gaussian random number generator with the mean and the standard deviation of the residual series. The simulated residuals are added to the conditional mean defined by $\hat{\alpha}$, to form a new series of returns. The new series of the returns has the same drift in prices, the same variance and the same unconditional distribution. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures.*

Table 5
Random Walk Simulations with EMA Model
USD-DEM, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	3.33	12.6	-10.52, 5.95	-2.20	5.01	0.07	0.03
Xeffective	-0.67	2.8	-19.88, -2.44	-10.96	5.43	0.11	0.02
Reffective	-2.48	2.3	-26.19, -5.15	-15.80	6.46	0.04	0.31
Max Drawdown	17.73	98.5	21.91, 90.24	49.70	21.34	-0.86	0.67
Deal frequency	0.77	100.0	0.90, 1.06	0.98	0.05	-0.06	-0.17
Horizon: 7 days							
Xeffective	-2.09	4.8	-19.18, -2.24	-10.80	5.08	0.07	0.03
Reffective	-4.29	3.3	-23.97, -5.67	-14.82	5.65	0.05	0.04
Horizon: 29 days							
Xeffective	-2.81	6.1	-19.93, -2.27	-10.94	5.30	0.08	0.05
Reffective	-7.18	7.4	-27.17, -5.71	-16.53	6.52	0.03	0.10
Horizon: 117 days							
Xeffective	-1.03	4.3	-20.72, -1.42	-10.93	5.79	0.14	0.10
Reffective	-3.96	4.8	-30.42, -4.26	-16.86	8.02	0.30	0.59
Horizon: 301 days							
Xeffective	1.65	2.0	-22.95, -0.70	-10.90	6.77	0.41	0.20
Reffective	1.75	3.2	-26.43, -0.74	-13.31	8.02	0.34	0.62

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-DEM series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The random walk estimation involves the regression of the actual USD-DEM returns on a constant. A simulation sample for the random walk series with drift is obtained by sampling from the Gaussian random number generator with the mean and the standard deviation of the residual series. The simulated residuals are added to the conditional mean defined by $\hat{\alpha}$, to form a new series of returns. The new series of the returns has the same drift in prices, the same variance and the same unconditional distribution. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures.*

Table 6

Random Walk Simulations with EMA Model

USD-CHF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	4.40	8.8	-12.87, 6.34	-3.33	5.81	0.06	-0.06
Xeffective	-0.46	0.9	-25.98, -5.64	-15.25	6.34	0.16	-0.05
Reffective	-3.00	0.4	-34.94, -10.12	-21.98	7.60	0.13	0.16
Max Drawdown	17.10	99.9	28.01, 107.44	60.72	25.19	-0.84	0.46
Deal frequency	0.85	100.0	1.10, 1.28	1.19	0.06	-0.25	0.36
Horizon: 7 days							
Xeffective	-2.86	2.2	-25.07, -5.17	-15.05	5.91	0.07	-0.03
Reffective	-5.66	1.1	-32.09, -9.94	-20.99	6.65	0.05	-0.03
Horizon: 29 days							
Xeffective	-3.45	2.7	-25.16, -4.88	-15.24	6.10	0.06	-0.01
Reffective	-7.26	1.8	-36.60, -11.02	-23.61	7.72	0.02	0.03
Horizon: 117 days							
Xeffective	0.39	0.7	-27.30, -4.73	-15.25	6.84	0.26	0.25
Reffective	-2.53	1.1	-40.19, -9.07	-23.34	10.02	0.64	1.23
Horizon: 301 days							
Xeffective	2.09	1.2	-29.98, -3.15	-15.15	8.47	0.63	0.89
Reffective	2.17	1.3	-34.81, -3.86	-17.68	9.49	0.47	0.62

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-CHF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The random walk estimation involves the regression of the actual USD-CHF returns on a constant. A simulation sample for the random walk series with drift is obtained by sampling from the Gaussian random number generator with the mean and the standard deviation of the residual series. The simulated residuals are added to the conditional mean defined by $\hat{\alpha}$, to form a new series of returns. The new series of the returns has the same drift in prices, the same variance and the same unconditional distribution. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures.*

Table 7
Random Walk Simulations with EMA Model
USD-FRF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	6.01	6.4	-9.78, 6.35	-1.84	4.96	-0.02	-0.13
X effective	2.25	0.6	-19.18, -1.88	-10.31	5.36	0.06	-0.20
R effective	2.02	0.4	-25.20, -4.59	-14.91	6.38	-0.00	-0.02
Max Drawdown	14.52	99.6	21.44, 84.77	47.51	20.44	-0.80	0.39
Deal frequency	0.70	100.0	0.88, 1.04	0.96	0.05	-0.11	-0.14
Horizon: 7 days							
Xeffective	1.36	1.1	-18.19, -1.79	-10.19	5.03	-0.04	-0.17
Reffective	-0.07	0.4	-22.91, -4.71	-14.04	5.57	-0.05	-0.21
Horizon: 29 days							
Xeffective	1.15	1.3	-18.34, -1.93	-10.28	5.14	-0.04	-0.19
Reffective	-0.91	1.0	-25.64, -5.18	-15.64	6.28	-0.11	-0.21
Horizon: 117 days							
Xeffective	1.92	0.9	-20.03, -1.19	-10.36	5.65	0.10	-0.12
Reffective	2.57	0.6	-29.43, -3.85	-15.92	7.75	0.28	0.24
Horizon: 301 days							
Xeffective	4.06	0.7	-22.19, 0.17	-10.18	6.75	0.38	0.04
Reffective	5.17	0.6	-26.18, 0.27	-12.59	7.95	0.23	0.10

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-FRF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The random walk estimation involves the regression of the actual USD-FRF returns on a constant. A simulation sample for the random walk series with drift is obtained by sampling from the Gaussian random number generator with the mean and the standard deviation of the residual series. The simulated residuals are added to the conditional mean defined by $\hat{\alpha}$, to form a new series of returns. The new series of the returns has the same drift in prices, the same variance and the same unconditional distribution. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 8

Random Walk Simulations with EMA Model

DEM-JPY, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	7.09	6.6	-8.71, 7.56	-0.75	5.06	-0.07	-0.02
Xeffective	2.63	1.4	-18.35, -0.74	-9.24	5.33	0.15	0.23
Reffective	2.97	1.0	-24.36, -2.42	-13.54	6.65	0.07	0.49
Max Drawdown	13.06	99.6	20.45, 79.17	44.28	18.91	-0.95	1.03
Deal frequency	0.75	99.9	0.82, 1.08	0.95	0.08	-0.12	-0.95
Horizon: 7 days							
Xeffective	1.81	2.0	-17.04, -0.89	-8.90	5.04	0.07	0.11
Reffective	0.57	1.3	-21.54, -3.70	-12.56	5.63	0.11	0.24
Horizon: 29 days							
Xeffective	1.94	1.6	-17.60, -0.56	-9.16	5.17	0.08	0.09
Reffective	1.47	0.6	-25.08, -3.61	-14.27	6.43	0.14	0.20
Horizon: 117 days							
Xeffective	3.43	0.9	-19.19, -0.18	-9.24	5.87	0.27	0.44
Reffective	4.26	0.9	-29.00, -2.07	-14.60	8.45	0.53	1.49
Horizon: 301 days							
Xeffective	3.76	1.1	-22.51, 1.29	-9.60	7.18	0.48	0.88
Reffective	4.31	1.0	-24.87, 2.29	-11.29	8.42	0.22	0.71

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual DEM-JPY series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The random walk estimation involves the regression of the actual DEM-JPY returns on a constant. A simulation sample for the random walk series with drift is obtained by sampling from the Gaussian random number generator with the mean and the standard deviation of the residual series. The simulated residuals are added to the conditional mean defined by $\hat{\alpha}$, to form a new series of returns. The new series of the returns has the same drift in prices, the same variance and the same unconditional distribution. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures.*

Table 9

GARCH(1,1) Parameter Estimates, 1990-1996, 5 minute frequency

	USD-DEM	USD-CHF	USD-FRF	DEM-JPY
α_0	4.95 (4.23)	0.11 (0.12)	9.38 (7.09)	2.97 (4.03)
α_1	0.1111 (0.0005)	0.1032 (0.0007)	0.1572 (0.0007)	0.0910 (0.0005)
β_1	0.8622 (0.0007)	0.8578 (0.0009)	0.8137 (0.0009)	0.8988 (0.0006)
LL	6.45	6.17	6.29	6.34
$Q(12)$	5.08	32.96	4.04	55.94
$\hat{\epsilon}_{\sigma^2}$	1.04	1.03	1.07	1.05
$\hat{\epsilon}_{sk}$	-0.07	-0.03	-0.05	0.16
$\hat{\epsilon}_{ku}$	11.73	7.28	22.93	27.73

Notes: α_0 values are 10^{-9}. The numbers in paratheses are the standard errors. The standard errors of α_0 are 10^{-11}. LL is the average log likelihood value. $Q(12)$ is the Ljung and Box portmanteu test for serial correlation and distributed χ^2 with 12 degrees of freedom. The $\chi^2_{0.05}(12)$ is 21.03. $\hat{\epsilon}_{\sigma^2}$, $\hat{\epsilon}_{sk}$ and $\hat{\epsilon}_{ku}$ are the variance, skewness and the excess kurtosis of the residuals.

Table 10

GARCH(1,1) Simulations with RTT Model

USD-DEM, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	9.63	0.4	-11.14, 5.12	-3.27	4.90	-0.08	-0.01
X effective	3.78	0.1	-20.40, -3.16	-11.88	5.18	-0.07	-0.11
R effective	4.43	0.0	-26.60, -6.37	-16.50	6.10	-0.14	-0.05
Max Drawdown	11.02	100.0	24.17, 93.96	53.33	21.50	-0.73	0.30
Deal frequency	1.68	100.0	2.14, 2.64	2.39	0.15	-0.02	-0.15
Horizon: 7 days							
Xeffective	3.47	0.2	-19.56, -3.49	-11.64	4.90	-0.06	-0.03
Reffective	1.80	0.0	-24.19, -6.58	-15.37	5.38	-0.08	-0.05
Horizon: 29 days							
Xeffective	3.27	0.2	-19.95, -3.29	-11.86	5.00	-0.12	-0.04
Reffective	2.16	0.1	-26.92, -6.75	-17.20	6.04	-0.20	-0.03
Horizon: 117 days							
Xeffective	4.07	0.1	-21.24, -2.77	-11.91	5.56	0.03	-0.28
Reffective	5.10	0.1	-30.17, -5.44	-17.57	7.60	0.29	0.31
Horizon: 301 days							
Xeffective	4.62	0.2	-22.73, -1.48	-11.73	6.42	0.28	0.16
Reffective	6.83	0.3	-27.64, -2.01	-14.28	7.72	0.26	0.38

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-DEM series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 11
GARCH(1,1) Simulations with RTT Model
USD-CHF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	3.66	8.4	-12.56, 4.87	-3.84	5.46	-0.08	-0.04
Xeffective	-1.68	1.4	-24.23, -5.25	-14.89	5.80	-0.02	-0.09
Reffective	-4.23	0.9	-32.01, -9.33	-20.59	6.73	-0.02	0.05
Max Drawdown	16.08	100.0	28.25, 105.09	60.58	24.16	-0.76	0.28
Deal frequency	1.29	100.0	1.72, 2.02	1.87	0.09	-0.12	0.22
Horizon: 7 days							
Xeffective	-2.96	1.6	-23.49, -6.05	-14.72	5.44	-0.07	-0.08
Reffective	-4.81	1.0	-29.35, -9.96	-19.61	6.01	-0.07	-0.06
Horizon: 29 days							
Xeffective	-4.10	2.6	-23.87, -5.76	-14.90	5.58	-0.04	-0.13
Reffective	-8.97	2.8	-32.67, -10.69	-21.80	6.76	-0.03	-0.11
Horizon: 117 days							
Xeffective	-0.67	0.8	-25.50, -4.55	-15.06	6.27	0.01	-0.15
Reffective	-1.77	0.7	-36.29, -8.68	-22.21	8.69	0.25	0.45
Horizon: 301 days							
Xeffective	1.94	0.9	-27.44, -2.59	-14.54	7.68	0.42	0.71
Reffective	1.71	1.5	-31.28, -3.58	-17.01	8.58	0.30	0.94

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-CHF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 12

GARCH(1,1) Simulations with RTT Model

USD-FRF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	8.20	0.9	-9.21, 5.50	-2.01	4.44	-0.11	-0.10
X effective	4.80	0.1	-17.54, -2.17	-9.84	4.67	-0.03	-0.03
R effective	4.95	0.1	-22.39, -4.17	-13.71	5.54	-0.09	0.22
Max Drawdown	11.36	99.9	21.85, 78.99	46.00	18.13	-0.82	0.99
Deal frequency	1.05	100.0	1.46, 1.72	1.59	0.08	-1.05	8.78
Horizon: 7 days							
Xeffective	3.18	0.3	-16.67, -2.28	-9.73	4.42	-0.06	-0.00
Reffective	1.97	0.1	-20.62, -4.78	-12.88	4.86	-0.03	0.11
Horizon: 29 days							
Xeffective	4.41	0.1	-17.02, -2.44	-9.84	4.49	-0.08	-0.12
Reffective	3.99	0.0	-22.63, -5.14	-14.19	5.44	-0.00	0.57
Horizon: 117 days							
Xeffective	3.83	0.4	-18.01, -1.68	-9.87	4.95	-0.00	0.14
Reffective	3.77	0.2	-26.15, -4.02	-14.77	6.77	0.21	1.07
Horizon: 301 days							
Xeffective	6.35	0.1	-20.12, -0.58	-9.72	5.87	0.33	0.35
Reffective	7.38	0.2	-24.06, -1.06	-12.00	7.06	0.33	0.85

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-FRF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 13

GARCH(1,1) Simulations with RTT Model

DEM-JPY, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	6.43	1.2	-10.75, 3.59	-3.51	4.36	-0.12	0.03
Xeffective	3.81	0.4	-17.00, -2.75	-9.86	4.48	-0.12	0.11
Reffective	3.45	0.4	-21.74, -4.82	-13.35	5.21	-0.19	0.35
Max Drawdown	12.03	100.0	22.13, 86.56	48.77	19.70	-0.71	0.26
Deal frequency	2.14	100.0	2.53, 2.78	2.66	0.08	-1.39	15.91
Horizon: 7 days							
Xeffective	1.87	0.6	-17.11, -3.02	-9.90	4.34	-0.12	0.08
Reffective	0.58	0.6	-20.50, -4.92	-12.57	4.83	-0.07	0.22
Horizon: 29 days							
Xeffective	2.12	0.5	-17.31, -2.76	-9.95	4.43	-0.10	-0.03
Reffective	0.19	0.7	-22.38, -5.08	-13.75	5.29	-0.08	0.01
Horizon: 117 days							
Xeffective	4.90	0.2	-17.74, -2.10	-9.90	4.67	-0.12	0.05
Reffective	5.17	0.2	-24.77, -3.69	-14.33	6.25	0.04	0.36
Horizon: 301 days							
Xeffective	5.39	0.4	-19.13, -1.74	-9.76	5.35	0.29	0.68
Reffective	5.56	0.5	-23.34, -2.57	-12.08	6.47	0.34	1.07

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual DEM-JPY series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 14
GARCH(1,1) Simulations with EMA Model
USD-DEM, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	3.33	14.1	-9.83, 5.97	-1.98	4.88	0.01	-0.07
Xeffective	-0.67	2.3	-19.63, -2.05	-10.76	5.30	0.12	-0.07
Reffective	-2.48	1.8	-26.07, -5.04	-15.69	6.38	0.05	0.01
Max Drawdown	17.73	99.2	22.60, 85.88	48.61	19.71	-0.86	0.71
Deal frequency	0.77	100.0	0.88, 1.05	0.96	0.05	-0.06	0.00
Horizon: 7 days							
Xeffective	-2.09	3.8	-18.40, -2.60	-10.59	4.96	0.03	-0.05
Reffective	-4.29	2.8	-23.51, -5.50	-14.63	5.56	0.03	-0.05
Horizon: 29 days							
Xeffective	-2.81	5.8	-19.20, -2.60	-10.75	5.16	0.12	-0.04
Reffective	-7.18	7.2	-27.32, -6.14	-16.46	6.46	0.16	0.09
Horizon: 117 days							
Xeffective	-1.03	3.1	-20.42, -1.90	-10.77	5.69	0.20	-0.00
Reffective	-3.96	3.8	-31.27, -4.67	-16.88	8.16	0.45	0.73
Horizon: 301 days							
Xeffective	1.65	2.1	-22.35, -0.52	-10.73	6.68	0.48	0.43
Reffective	1.75	2.4	-26.80, -1.28	-13.28	7.69	0.23	-0.09

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-DEM series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 15
GARCH(1,1) Simulations with EMA Model
USD-CHF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	4.40	9.6	-12.86, 6.13	-3.16	5.80	0.03	-0.23
X effective	-0.46	0.7	-26.27, -4.95	-15.13	6.35	0.12	-0.18
R effective	-3.00	0.5	-34.67, -9.17	-21.92	7.59	0.06	0.13
Max Drawdown	17.10	99.9	27.98, 109.09	60.09	24.80	-0.80	0.18
Deal frequency	0.85	100.0	1.08, 1.28	1.18	0.06	-0.10	0.38
Horizon: 7 days							
Xeffective	-2.86	1.7	-24.57, -5.53	-14.87	5.92	0.04	-0.22
Reffective	-5.66	1.2	-31.76, -10.13	-20.83	6.71	0.03	-0.25
Horizon: 29 days							
Xeffective	-3.45	2.7	-25.46, -5.05	-15.09	6.15	0.01	-0.32
Reffective	-7.26	1.5	-37.07, -9.83	-23.56	7.93	-0.05	-0.23
Horizon: 117 days							
Xeffective	0.39	0.7	-26.72, -3.87	-15.18	6.88	0.18	0.00
Reffective	-2.53	1.5	-39.88, -7.71	-23.42	9.98	0.41	0.65
Horizon: 301 days							
Xeffective	2.09	1.3	-30.15, -2.48	-14.94	8.45	0.55	0.63
Reffective	2.17	1.5	-34.44, -2.83	-17.51	9.43	0.43	0.64

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-CHF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 16

GARCH(1,1) Simulations with EMA Model

USD-FRF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	6.01	4.6	-8.91, 5.88	-1.60	4.61	-0.12	-0.03
Xeffective	2.25	0.6	-18.14, -1.83	-10.03	5.01	-0.04	-0.18
Reffective	2.02	0.4	-25.25, -4.27	-14.69	6.19	-0.02	0.03
Max Drawdown	14.52	99.7	20.79, 80.38	46.04	18.73	-0.77	0.51
Deal frequency	0.70	100.0	0.85, 1.02	0.93	0.05	-1.44	13.00
Horizon: 7 days							
Xeffective	1.36	0.8	-17.54, -2.43	-9.91	4.70	-0.06	-0.04
Reffective	-0.07	0.6	-22.08, -5.40	-13.76	5.28	-0.03	-0.01
Horizon: 29 days							
Xeffective	1.15	0.9	-18.06, -1.97	-9.98	4.87	-0.03	-0.07
Reffective	-0.91	0.9	-25.40, -5.43	-15.27	6.18	-0.02	-0.00
Horizon: 117 days							
Xeffective	1.92	0.7	-18.90, -1.22	-10.08	5.34	0.07	-0.22
Reffective	2.57	0.5	-29.72, -3.32	-15.78	8.00	0.49	0.78
Horizon: 301 days							
Xeffective	4.06	0.8	-20.53, 0.42	-9.87	6.39	0.35	0.50
Reffective	5.17	0.7	-26.23, 0.42	-12.35	7.91	0.27	0.31

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-FRF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 17

GARCH(1,1) Simulations with EMA Model

DEM-JPY, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	7.09	2.8	-8.77, 5.80	-1.59	4.38	-0.11	-0.19
Xeffective	2.63	0.5	-16.67, -0.91	-8.72	4.73	0.02	-0.27
Reffective	2.97	0.2	-22.03, -3.34	-12.73	5.78	0.02	-0.15
Max Drawdown	13.06	99.8	19.69, 75.94	43.01	17.61	-0.79	0.31
Deal frequency	0.75	97.8	0.77, 0.93	0.84	0.05	-1.85	19.57
Horizon: 7 days							
Xeffective	1.81	1.2	-16.00, -1.11	-8.65	4.50	-0.05	-0.15
Reffective	0.57	0.6	-19.98, -3.38	-11.94	5.10	0.01	0.01
Horizon: 29 days							
Xeffective	1.94	0.6	-16.32, -1.22	-8.76	4.59	0.01	-0.20
Reffective	1.47	0.3	-22.86, -4.23	-13.26	5.73	0.12	0.09
Horizon: 117 days							
Xeffective	3.43	0.4	-17.14, -0.35	-8.68	5.04	0.07	-0.26
Reffective	4.26	0.3	-25.94, -2.47	-13.56	7.20	0.32	0.29
Horizon: 301 days							
Xeffective	0.76	3.6	-19.12, 0.14	-8.73	5.87	0.34	0.10
Reffective	2.31	2.4	-22.65, 0.30	-11.09	7.22	0.35	0.51

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual DEM-JPY series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 18

AR(4)-GARCH(1,1) Parameter Estimates, 1990-1996, 5 minute frequency

	USD-DEM	USD-CHF	USD-FRF	DEM-JPY
α_0	3.90 (3.40)	8.19 (9.03)	7.28 (5.80)	2.92 (3.93)
α_1	0.099 (0.0005)	0.0874 (0.0006)	0.1349 (0.0007)	0.088 (0.0005)
β_1	0.8796 (0.0006)	0.8833 (0.0007)	0.8411 (0.0008)	0.9008 (0.0006)
γ_1	-0.176 (0.001)	-0.208 (0.001)	-0.200 (0.002)	-0.130 (0.002)
γ_2	-0.011 (0.001)	-0.031 (0.002)	-0.025 (0.002)	-0.090 (0.002)
γ_3	0.003 (0.001)	-0.001 (0.002)	-0.005 (0.002)	-0.005 (0.002)
γ_4	-0.004(0.001)	-0.002 (0.001)	-0.008 (0.002)	-0.010 (0.002)
$\bar{LL}$	6.46	6.19	6.30	6.35
$Q(12)$	0.21	0.69	0.10	0.81
$\hat{\epsilon}_{\sigma^2}$	1.04	1.03	1.07	1.05
$\hat{\epsilon}_{sk}$	-0.07	-0.04	-0.05	0.15
$\hat{\epsilon}_{ku}$	12.29	7.86	21.84	27.98

Notes: α_0 values are 10^{-9}. The numbers in paratheses are the standard errors. The standard errors of α_0 are 10^{-11}. LL is the average log likelihood value. $Q(12)$ refer to the Ljung-Box portmanteu test for serial correlation and it is distributed χ^2 with 12 degrees of freedom. The $\chi^2_{0.05}(12)$ is 21.03. $\hat{\epsilon}_{\sigma^2}$, $\hat{\epsilon}_{sk}$ and $\hat{\epsilon}_{ku}$ are the variance, skewness and the excess kurtosis of the residuals.

Table 19

AR(4)-GARCH(1,1) Simulations with RTT Model

USD-DEM, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	9.63	0.1	-10.46, 3.13	-3.68	4.13	-0.01	-0.16
X effective	3.78	0.1	-16.72, -3.16	-9.95	4.27	-0.02	-0.18
R effective	4.43	0.0	-21.37, -5.28	-13.37	4.93	-0.07	-0.15
Max Drawdown	11.02	100.0	21.73, 84.55	49.07	19.16	-0.59	0.03
Deal frequency	1.68	100.0	1.89, 2.35	2.12	0.14	-0.04	-0.26
Horizon: 7 days							
Xeffective	3.47	0.1	-16.53, -2.86	-9.72	4.13	0.01	-0.18
Reffective	1.80	0.2	-19.63, -4.95	-12.33	4.45	-0.01	-0.17
Horizon: 29 days							
Xeffective	3.27	0.1	-16.90, -3.24	-9.94	4.21	0.00	-0.11
Reffective	2.16	0.1	-21.54, -5.87	-13.67	4.87	-0.02	-0.02
Horizon: 117 days							
Xeffective	4.07	0.1	-17.37, -2.83	-9.97	4.50	0.00	-0.26
Reffective	5.10	0.0	-24.10, -4.75	-14.17	5.98	0.23	0.21
Horizon: 301 days							
Xeffective	4.62	0.1	-18.19, -2.01	-9.83	4.95	0.19	0.16
Reffective	6.83	0.1	-22.90, -2.91	-12.15	6.15	0.34	0.53

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-DEM series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the AR(4)-GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 20

AR(4)-GARCH(1,1) Simulations with RTT Model

USD-CHF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	3.66	3.7	-11.57, 3.36	-4.06	4.54	-0.06	-0.05
Xeffective	-1.68	1.9	-19.54, -3.94	-11.72	4.77	-0.06	-0.06
Reffective	-4.23	2.3	-24.29, -6.48	-15.75	5.50	-0.10	0.20
Max Drawdown	16.08	99.7	24.11, 93.75	53.88	21.64	-0.62	0.03
Deal frequency	1.29	99.9	1.40, 1.65	1.52	0.08	-0.02	0.04
Horizon: 7 days							
Xeffective	-2.96	2.8	-19.15, -4.22	-11.47	4.53	-0.05	-0.02
Reffective	-4.81	2.4	-23.05, -6.74	-14.63	4.92	-0.05	0.01
Horizon: 29 days							
Xeffective	-4.10	4.6	-19.36, -4.23	-11.66	4.59	-0.04	-0.02
Reffective	-8.97	9.7	-25.17, -7.02	-16.06	5.39	-0.07	0.02
Horizon: 117 days							
Xeffective	-0.67	1.3	-19.93, -3.51	-11.81	5.00	-0.07	-0.07
Reffective	-1.77	1.4	-27.35, -6.41	-16.98	6.57	0.09	0.42
Horizon: 301 days							
Xeffective	1.94	0.8	-21.30, -2.21	-11.51	6.08	0.30	0.44
Reffective	1.71	1.1	-25.53, -2.90	-13.95	7.27	0.46	1.59

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-CHF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the AR(4)-GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 21
AR(4)-GARCH(1,1) Simulations with RTT Model
USD-FRF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	8.20	0.3	-9.07, 3.99	-2.62	3.95	-0.09	-0.09
X effective	4.80	0.2	-15.12, -1.63	-8.36	4.13	-0.06	-0.10
R effective	4.95	0.1	-19.30, -3.22	-11.25	4.81	-0.11	0.11
Max Drawdown	11.36	99.9	18.51, 76.40	42.67	17.57	-0.73	0.24
Deal frequency	1.05	100.0	1.26, 1.49	1.37	0.07	-0.40	1.86
Horizon: 7 days							
Xeffective	3.18	0.2	-14.72, -1.62	-8.26	3.95	-0.07	-0.10
Reffective	1.97	0.2	-17.58, -3.20	-10.51	4.29	-0.08	-0.12
Horizon: 29 days							
Xeffective	4.41	0.0	-14.92, -1.86	-8.39	4.01	-0.11	-0.12
Reffective	3.99	0.0	-19.12, -3.51	-11.47	4.69	-0.15	-0.07
Horizon: 117 days							
Xeffective	3.83	0.3	-15.58, -1.42	-8.37	4.28	-0.03	0.01
Reffective	3.77	0.3	-21.10, -3.52	-12.05	5.59	0.08	0.41
Horizon: 301 days							
Xeffective	6.35	0.2	-16.81, -0.30	-8.26	4.95	0.18	-0.05
Reffective	7.38	0.2	-20.78, -0.77	-10.21	5.99	0.24	0.31

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-FRF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the AR(4)-GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 22
AR(4)-GARCH(1,1) Simulations with RTT Model
DEM-JPY, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	6.43	0.5	-10.51, 2.15	-4.14	3.88	-0.12	0.05
Xeffective	3.81	0.1	-16.15, -2.91	-9.35	3.94	-0.11	0.09
Reffective	3.45	0.1	-19.55, -4.81	-12.26	4.48	-0.19	0.28
Max Drawdown	12.03	100.0	21.29, 81.86	48.09	18.74	-0.60	0.17
Deal frequency	2.14	100.0	2.35, 2.58	2.46	0.08	-0.54	4.42
Horizon: 7 days							
Xeffective	1.87	0.5	-15.53, -3.23	-9.40	3.85	-0.10	0.08
Reffective	0.58	0.5	-18.59, -4.85	-11.58	4.21	-0.09	0.06
Horizon: 29 days							
Xeffective	2.12	0.4	-15.72, -3.38	-9.43	3.90	-0.08	0.05
Reffective	0.19	0.4	-19.78, -5.42	-12.51	4.53	-0.07	0.08
Horizon: 117 days							
Xeffective	4.90	0.1	-16.09, -2.48	-9.38	4.08	-0.13	0.10
Reffective	5.17	0.1	-21.44, -4.27	-13.07	5.25	-0.04	0.44
Horizon: 301 days							
Xeffective	5.39	0.0	-17.47, -1.95	-9.24	4.58	0.19	0.42
Reffective	5.56	0.2	-21.15, -2.91	-11.36	5.59	0.31	0.79

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual DEM-JPY series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the AR(4)-GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 23
AR(4)-GARCH(1,1) Simulations with EMA Model
USD-DEM, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	3.33	10.0	-8.49, 4.95	-1.90	4.11	-0.02	-0.05
Xeffective	-0.67	3.8	-15.50, -1.31	-8.18	4.36	0.03	-0.02
Reffective	-2.48	3.7	-20.25, -3.20	-11.69	5.21	-0.05	0.09
Max Drawdown	17.73	96.9	19.30, 73.09	41.69	16.77	-0.77	0.38
Deal frequency	0.77	75.1	0.73, 0.87	0.80	0.04	-0.04	-0.18
Horizon: 7 days							
Xeffective	-2.09	6.8	-15.00, -1.32	-8.07	4.16	-0.00	-0.01
Reffective	-4.29	6.8	-18.46, -3.31	-10.83	4.59	-0.00	0.02
Horizon: 29 days							
Xeffective	-2.81	9.4	-15.27, -1.26	-8.17	4.30	0.06	-0.00
Reffective	-7.18	17.4	-20.71, -3.66	-11.96	5.18	0.06	0.15
Horizon: 117 days							
Xeffective	-1.03	5.8	-16.11, -0.69	-8.16	4.58	0.08	-0.11
Reffective	-3.96	8.9	-23.32, -2.40	-12.48	6.33	0.27	0.37
Horizon: 301 days							
Xeffective	1.65	1.8	-17.25, -0.05	-8.16	5.19	0.32	0.23
Reffective	1.75	1.8	-21.94, -0.56	-10.44	6.34	0.28	0.24

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-DEM series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the AR(4)-GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 24
AR(4)-GARCH(1,1) Simulations with EMA Model
USD-CHF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	4.40	5.9	-11.20, 4.82	-2.89	4.86	0.01	-0.20
X effective	-0.46	1.7	-19.99, -2.89	-11.18	5.17	0.07	-0.10
R effective	-3.00	2.4	-25.68, -5.55	-15.97	6.17	-0.02	0.16
Max Drawdown	17.10	99.0	23.18, 92.51	50.85	21.04	-0.76	0.19
Deal frequency	0.85	95.1	0.85, 1.02	0.93	0.05	-0.12	-0.12
Horizon: 7 days							
Xeffective	-2.86	4.8	-19.44, -3.03	-10.97	4.95	0.02	-0.21
Reffective	-5.66	4.9	-23.94, -5.67	-14.85	5.49	0.01	-0.23
Horizon: 29 days							
Xeffective	-3.45	6.8	-19.83, -2.92	-11.14	5.07	0.03	-0.20
Reffective	-7.26	7.0	-26.94, -6.63	-16.62	6.23	-0.02	-0.14
Horizon: 117 days							
Xeffective	0.39	1.5	-20.34, -2.50	-11.24	5.48	0.12	-0.01
Reffective	-2.53	2.9	-30.53, -5.37	-17.29	7.87	0.39	0.79
Horizon: 301 days							
Xeffective	2.09	1.6	-22.61, -1.44	-11.08	6.41	0.33	0.30
Reffective	2.17	2.4	-27.46, -1.31	-13.67	7.76	0.30	0.53

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-CHF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the AR(4)-GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 25
AR(4)-GARCH(1,1) Simulations with EMA Model
USD-FRF, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	6.01	3.2	-8.18, 5.05	-1.63	4.04	-0.11	0.08
Xeffective	2.25	0.7	-14.98, -0.75	-7.83	4.30	-0.02	-0.02
Reffective	2.02	0.6	-19.69, -2.61	-11.23	5.24	0.01	0.32
Max Drawdown	14.52	98.8	17.92, 71.43	40.24	16.74	-0.80	0.41
Deal frequency	0.70	98.0	0.72, 0.87	0.79	0.04	-0.34	0.90
Horizon: 7 days							
Xeffective	1.36	1.6	-14.42, -1.22	-7.74	4.11	-0.07	0.05
Reffective	-0.07	1.2	-18.11, -3.20	-10.47	4.56	-0.04	0.08
Horizon: 29 days							
Xeffective	1.15	1.5	-14.74, -0.79	-7.79	4.22	-0.03	0.07
Reffective	-0.91	1.7	-19.84, -2.69	-11.47	5.14	-0.01	0.26
Horizon: 117 days							
Xeffective	1.92	1.1	-15.16, -0.48	-7.86	4.52	0.07	-0.04
Reffective	2.57	0.7	-22.85, -1.84	-12.02	6.47	0.42	0.78
Horizon: 301 days							
Xeffective	4.06	0.5	-16.02, 0.43	-7.67	5.15	0.31	0.43
Reffective	5.17	0.3	-20.54, 0.74	-9.82	6.44	0.26	0.47

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual USD-FRF series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the AR(4)-GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 26
AR(4)-GARCH(1,1) Simulations with EMA Model
DEM-JPY, 1990-1996, 5 minute frequency

Description	Historical Realization	p-value (in%)	Percentile [5%, 95%]	Mean	St. Dev	Skew.	Kurt.
Annu. Return	7.09	1.8	-8.19, 5.20	-1.58	4.00	-0.09	-0.18
X effective	2.63	0.5	-14.48, -0.47	-7.47	4.26	0.01	-0.24
R effective	2.97	0.4	-19.26, -2.31	-10.72	5.14	-0.03	-0.16
Max Drawdown	13.06	99.8	17.80, 69.48	39.45	16.12	-0.75	0.21
Deal frequency	0.75	61.1	0.70, 0.84	0.77	0.04	-0.75	4.68
Horizon: 7 days							
Xeffective	1.81	1.3	-14.26, -0.67	-7.40	4.09	-0.05	-0.18
Reffective	0.57	0.9	-17.39, -2.46	-10.02	4.53	-0.04	-0.15
Horizon: 29 days							
Xeffective	1.94	0.9	-14.35, -0.77	-7.51	4.17	-0.01	-0.23
Reffective	1.47	0.4	-19.19, -2.84	-11.04	5.04	0.03	-0.11
Horizon: 117 days							
Xeffective	3.43	0.5	-15.00, -0.00	-7.43	4.50	0.06	-0.20
Reffective	4.26	0.5	-21.73, -1.52	-11.39	6.23	0.25	0.31
Horizon: 301 days							
Xeffective	3.76	0.2	-15.71, 0.43	-7.45	5.10	0.32	0.29
Reffective	4.31	0.6	-20.35, 0.84	-9.59	6.41	0.32	0.45

Notes: The column under *Historical Realization* presents the performance of the trading model with the actual DEM-JPY series from January 1, 1990 until December 31, 1996 with 5 minute frequency. The results under columns p-value, *Percentile*, *Mean*, *St.Dev.*, *Skewness* and *Kurtosis* present the values of these statistics from 1000 replications with the random walk process. The parameters and the normalized residuals are estimated from the foreign exchange returns using the maximum likelihood procedure. The simulated returns for the AR(4)-GARCH(1,1) process are generated from the simulated residuals and the estimated parameters. From the new series of returns, the simulated price process is recovered recursively by setting the initial price to the true price at the beginning of the sample. The trading models use the bid and ask prices as inputs. Half of the average spread is subtracted (added) from the simulated price process to obtain the simulated bid and ask prices.

p-values are reported in percentage terms (e.g. 8.4 refers to 8.4 percent). The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures*.

Table 27
p-value Comparisons for RTT Model
with RW, GARCH(1,1) and AR(4)-GARCH(1,1)

Currency	Random Walk	GARCH(1,1)	AR(4)-GARCH(1,1)
Annu. Return			
USD-DEM	0.3	0.4	0.1
USD-CHF	8.9	8.4	3.7
USD-FRF	1.2	0.9	0.8
DEM-JPY	2.1	1.2	0.5
Xeffective			
USD-DEM	0.0	0.1	0.1
USD-CHF	0.7	1.4	1.9
USD-FRF	0.2	0.1	0.2
DEM-JPY	0.2	0.4	0.1
Reffective			
USD-DEM	0.0	0.0	0.0
USD-CHF	0.6	0.9	2.3
USD-FRF	0.1	0.1	0.1
DEM-JPY	0.1	0.4	0.1

Note: p-values are expressed in percentage. The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures.*

Table 28
p-value Comparisons for EMA Model
with RW, GARCH(1,1) and AR(4)-GARCH(1,1)

Currency	Random Walk	GARCH(1,1)	AR(4)-GARCH(1,1)
Annu. Return			
USD-DEM	12.6	14.1	10.0
USD-CHF	8.8	9.6	5.9
USD-FRF	6.4	4.6	3.2
DEM-JPY	6.6	2.8	1.8
Xeffective			
USD-DEM	2.8	2.3	3.8
USD-CHF	0.9	0.7	1.7
USD-FRF	0.6	0.6	0.7
DEM-JPY	1.4	0.5	0.5
Reffective			
USD-DEM	2.3	1.8	3.7
USD-CHF	0.4	0.5	2.4
USD-FRF	0.4	0.4	0.6
DEM-JPY	1.0	0.2	0.4

Note: p-values are expressed in percentage. The definitions of the performance measures for the *Annu. Return*, *Xeffective*, *Reffective*, *Max Drawdown* and *Deal Frequency* are presented in section on *Performance Measures.*