Beta Forecasting at Long Horizons

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Abstract

Systematic (CAPM beta) risk forecasting for long horizons, such as six months and one year, play an important role in financial management. This paper evaluates a variety of beta forecasting procedures for these long forecast horizons. The widely utilized Fama-MacBeth approach based on five years of monthly returns is found to be unreliable in terms of mean absolute (and squared) forecast error and statistical bias. The most accurate forecasts are found to be generated from an autoregressive model of realized beta. In addition to analyzing the statistical properties of these forecasts, the economic significance between the different approaches is demonstrated through evaluating investment projects.

Key Words: NPV Analysis, Realized Beta, Systematic Risk.

1 Introduction

Accurate forecasts at long horizons, such as one year, of the Capital Asset Pricing Model[1] (CAPM) beta play an important role in financial management, including cost of capital estimation and performance measurement. Beta forecasts are usually generated through estimation of the slope coefficient from a linear regression of individual stock returns onto a constant and market index returns, typically with five years of monthly data as in Fama and MacBeth (1973). This method of estimating beta forecasts is the baseline for many empirical studies using the CAPM and for numerous professional advisers on beta such as Bloomberg, Reuters, Standard & Poors and Value Line.

Motivated from advances in the financial econometrics of volatility measurement, namely realized volatility measurement, see Andersen et al. (2001a), Andersen et al. (2001b) and Andersen et al. (2003), CAPM realized betas were developed in the work of Barndorff-Nielsen and Shephard (2004), Andersen et al. (2005) and Andersen et al. (2006). These betas are computed over a period from a sufficiently high number of intra-period returns and are econometrically consistent over a fixed interval. In recent studies, CAPM realized betas have served as a benchmark to evaluate the accuracy of beta forecasting approaches, see for example, Hooper et al. (2008), Chang et al. (2012), Reeves and Wu (2013), Papageorgiou et al. (2016) and Cenesizoglu et al. (2016). In a forecast evaluation study with realized betas computed over 6 months and one year, this paper evaluates the standard Fama and MacBeth (1973) forecasting approach with proposed forecasting procedures based on realized beta estimation. Chang et al. (2012) also study the same long forecast horizons for beta as this paper, however, their forecasting approach is restricted to settings where there is availability of accurate stock option data. Whereas, our study is mainly restricted only on the basis of availability of accurate daily stock return data, thus has far greater general applicability.

[1]see Sharpe (1964) , Lintner (1965) and Mossin (1966).

The primary aim of this paper is to evaluate (both statistically and economically) the performance of a variety of models for forecasting long horizon beta. To accomplish that we implement two major classes of realized beta models (constant and autoregressive) and compare their forecasting accuracy with the industry standard Fama-Macbeth beta. Our study uses daily data from 1^{st} January 1952 to 31^{st} December 2011 for 15 stocks from the DJIA index, and with the DJIA index as the market portfolio. By measuring forecast performance through Mean Absolute Forecast Error (MAE), Mean Squared Forecast Error (MSE) and by testing for bias in the forecasts, we find that the industry standard Fama-MacBeth beta has not only a forecast error much larger than the other time-series model, but it is also downward biased, under-estimating the future value of beta.

We also test the forecast ability of realized betas constructed from varying lengths from 6 months up to 60 months of daily data and also implement five specifications of an autoregressive realized beta model, with lags 1 up to 5. Our general finding is that both constant realized betas and autoregressive models outperform the constant Fama-Macbeth beta in terms of MAE and MSE. The best constant method, realized beta with 18 months of daily data, reduces the mean absolute error by 26.5% (25.7%) for 6 month (one year) forecasts, and the best autoregressive, the AR(1), reduces it by 30.3% (29.8%) for 6 month (one year) forecasts, when compared to the standard Fama-MacBeth beta. In addition, we also perform Mincer-Zarnowitz regressions (see Mincer and Zarnowitz (1969)) to test whether predictions from forecasting models are biased. We find that the constant realized beta models and autoregressive realized beta models not only reduce forecasting error, but are also less biased than Fama-MacBeth betas, which are downward biased. Overall, we find an autoregressive model of realized beta with one lag, to be statistically unbiased, leading this model to be our preferred approach.

Our study focuses on Dow stocks due to their very high liquidity which permits the use of daily historical return data going back to 1952. However, our conclusions are not restricted to Dow stocks, with conclusions extending to other stocks that are sufficiently liquid that allow accurate daily return measurement. This set of stocks has been constantly increasing over time with overall improvements in market liquidity and typically includes sets of stocks such as those currently trading in the S&P 500 index, where for most stocks, daily returns over a number of years can be relied upon.

In addition to the statistical results, this paper demonstrates strong economic significance in an application to cost of capital measurement and evaluating investment projects. In these applications, the Fama-MacBeth betas again result in downward biases and greater variability in cost of capital measurements which distort net present value calculations. Whereas, realized beta and autoregressive models display more favourable performance.

This paper is organized as follows: Section 2 reviews the realized beta estimator and discusses approaches to forecast beta at long horizons. Section 3 describes the data sources and sample of US stocks. Section 4 is on the empirical forecast performance of the various approaches for both the 6 month and one year forecast horizons. Section 5 demonstrates the economic significance between the different forecasting approaches through evaluating investment projects. The final section concludes the paper.

2 Realized Betas and Forecasting Approaches

In this section, we first discuss the estimation and theoretical justification of realized beta estimators. We then discuss the popular beta forecasting approach of Fama and MacBeth (1973) and new approaches to forecasting long horizon betas utilizing realized beta estimates.

2.1 Realized Beta Measurement

We firstly briefly review the realized beta estimator and its theoretical justification that was developed and discussed in Barndorff-Nielsen and Shephard (2004) and Andersen et al.

(2006). Suppose that prices follow a multivariate continuous-time stochastic volatility diffusion, with the N x 1 logarithmic vector price process p_t

$$dp_t = \mu_t dt + \theta_t dW_t \tag{1}$$

where μ_t is the vector of instantaneous drift, $\Omega_t = \theta_t \theta_t'$ is the diffusion (variance-covariance) matrix and W_t represents a vector of standard Brownian motion innovations. The variance-covariance matrix and the drift vectors are not correlated with the Brownian motion process and are strictly stationary. To facilitate the interpretation, we can think of N as the number of stocks plus the market index with the N^{th} element containing information of the index and each i^{th} element with information on stocks. By defining a time interval (for example, a day or month) and denoting it h we define $r_{t+h,h} \equiv p_{t+h} - p_t$ as the continuously compounded return in this period.

The realized beta of a period can be defined as the realized covariance between a security and the market index divided by the realized variance of the market. With Δ being the sampling frequency, or the size by which we divide the h period, the realized covariance during a time interval h, at time $t+h$, of a security i and the market index M is defined as:

$$\hat{\nu}_{iM,t,t+h} = \sum_{j=1,\ldots,[h/\Delta]} r_{i,t+j.\Delta,\Delta} \cdot r_{N,t+j.\Delta,\Delta} \tag{2}$$

and the realized market variance as:

$$\hat{\nu}^2_{M,t,t+h} = \sum_{j=1,\ldots,[h/\Delta]} r^2_{N,t+j.\Delta,\Delta}. \tag{3}$$

The realized beta as the ratio of the realized covariance to the realized market variance is then:

$$\hat{\beta}_{i,t,t+h} = \frac{\hat{\nu}_{iM,t,t+h}}{\hat{\nu}^2_{M,t,t+h}} = \frac{\sum\limits_{j=1,\ldots,[h/\Delta]} r_{i,t+j.\Delta,\Delta} \cdot r_{N,t+j.\Delta,\Delta}}{\sum\limits_{j=1,\ldots,[h/\Delta]} r^2_{N,t+j.\Delta,\Delta}} \xrightarrow{p} \frac{\int_0^h \Omega_{iN,t+\tau} d\tau}{\int_0^h \Omega_{NN,t+\tau} d\tau} = \beta_{i,t,t+h} \tag{4}$$

This equation shows that finely sampled realized beta converges in probability to the true latent beta. This CAPM realized beta is equivalent to a linear regression of stock intra-period returns on market intra period returns with the suppression of the constant term.

In this paper we use equation 4 in the computation of realized betas which serve as benchmarks in the evaluation of forecasts, with h defined as 6 months or one year and Δ defined as daily. This follows the realized beta construction in the forecast evaluation study of Chang et al. (2012) at the 6 months and one year horizons. A number of other beta forecast evaluation studies, at shorter forecast horizons, have also utilized realized betas, see for example, Hooper et al. (2008), Reeves and Wu (2013), Papageorgiou et al. (2016) and Cenesizoglu et al. (2016).

In general, important considerations need to be taken into account in realized beta measurements. Most importantly, the Δ sampling frequency should not be too high relative to the liquidity of the asset, to avoid poor return measurement which leads to bias (typically downward) in the beta measurement.[2] It is also the case that the h period is often chosen with consideration given to capturing the time-varying nature of beta. The length of the h period also must be long enough (relative to the Δ sampling frequency) to ensure a sufficient number of return observations so that the variability of the beta estimates is controlled.

[2]Dimson (1979) betas are sometimes utilized in settings where the return measurement is too high relative to the liquidity of the asset. In these estimations with leads and lags of betas, some of the bias can be corrected however, this typically comes at the cost of greater variability in the beta estimates.

2.2 Forecasting Approaches

The standard approach in long horizon beta forecasting follows that of Fama and MacBeth (1973) where the beta forecast is the regression slope coefficient from 5 years of monthly stock returns regressed onto a constant and the market returns. The popularity of this approach can be attributed to its simplicity and that Fama and MacBeth (1973) is a seminal paper in the financial economics literature.

In addition to this standard approach, this paper also studies forecasting approaches based on realized beta estimates. These realized beta approaches are based on estimations over various prior periods and include autoregressive modeling.[3] The first set of realized beta models that we study are constant realized beta models. In this group of models the forecast of beta for the next period is generated using the prior realized beta. The forecasting equation takes the form:

$$\beta_{i,t+1} = RBeta_{i,t,n} \tag{5}$$

where i indexes each firm, t indexes the time at which the forecast is made and n represents the number of months of prior daily data used to compute the realized beta. We calculate realized betas with n equal to 6, 12, 18, 24, 48 and 60 months, denoted by 6M [4], 12M, 18M, 24M, 48M and 60M.

We also apply several specifications of the AR(p) model, computed on half-yearly and yearly realized betas formed from daily returns, with the general form:

$$\beta_{i,t} = \phi_0 + \sum_{j=1}^{p} \phi_j \beta_{i,t-j} + \epsilon_t, \quad \epsilon_t \sim iid(0, \sigma^2) \;\; t = 1, 2, \ldots, n. \tag{6}$$

where p denotes the number of lags of the realized betas used in the forecast. In this paper we use $p = 1, \ldots, 5$ with three different in-sample estimation sizes. For the 6-month

[3] Andersen et al. (2005) and Andersen et al. (2006) were the first in depth studies of autoregressive models for CAPM realized betas, which was done at the monthly and quarterly frequencies.

[4] 6M is only used to forecast six-month-ahead betas.

forecast we estimate AR(p) models with 40 and 60 observations and for the yearly forecast we estimate AR(p) models using 30 observations. These autoregressive models are then utilized to generate one-step-ahead forecasts.

3 Data

We calculate realized betas from daily returns for 15 US companies with the Dow Jones Industrials Average (DJIA) index as the market portfolio. Equity data is sourced from CRSP through Wharton Research Data Services (WRDS) and the DJIA index from Datastream. We use the return series from CRSP which adjusts for capital structure events as well as for cash dividends at the ex-dividend date. In order to be included in the sample a company has to be listed in the DJIA index as of 31^{st} December 2011 and needs to have complete daily return information going as far back as the 1950's. With this criteria we were able to collect daily data from 15 large US companies from 1^{st} January 1952 to 31^{st} December 2011 which gives us a time-series with 60 years (or 120 half-years) of realized betas.

4 Empirical Forecast Performance

In this section we first discuss the methodology to our evaluation of competing beta forecasting approaches, followed by the presentation and discussion of empirical results.

4.1 Methodology

In our study we calculate realized betas following Barndorff-Nielsen and Shephard (2004) and Andersen et al. (2006) from equation 4 once every half-year (or year for the one-year-ahead forecasting sample) in order to have a sample of realized betas with non-overlapping windows of daily returns. The realized beta is calculated at the last trading day of June and December for the 6-month realized beta series and the last trading day of December for the yearly series. The out-of-sample forecast evaluation period for the half-year forecast is the 55 half-years starting from the second half of 1984. For the yearly forecast, the forecast evaluation period starts in 1987 and contains 25 years of forecasting periods.

The one period ahead forecast of beta, β_{n+1}, generated by each model for each company is compared with the benchmark beta, which we define as the 6-month (or one year) realized beta calculated with daily returns at that given date. As betas are used by both executives of companies as well as external analysts, it is hard to define a preferred direction of forecast error (over or under estimation of the true value).[5] Hence, our baseline measures of forecast ability of the models - mean absolute error (MAE) and mean squared error (MSE) - are based on a symmetric, quadratic and absolute, loss function as follows:

$$MAE_i = \frac{1}{m}\sum_{j=1}^{m}|\widehat{\beta_{i,j}} - \beta_{i,j}| \tag{7}$$

$$MSE_i = \frac{1}{m}\sum_{j=1}^{m}(\widehat{\beta_{i,j}} - \beta_{i,j})^2 \tag{8}$$

where m is the total number of forecasting periods, i indexes each company, $\widehat{\beta_j}$ is the forecast for the j^{th} period beta and β_j is the 6-month (yearly) realized beta calculated from daily returns in the j^{th} period.

In addition, to evaluate the presence of systematic bias in the forecast of each model, we follow the seminal work of Mincer and Zarnowitz (1969) and estimate a regression of the next period's realized beta on the current forecast of beta:

[5] Errors in beta forecasting are not particularly worse if positive or negative because it will, in either case, lead to incorrect inference on cost of capital calculation which can be bad or good depending on the use. For example, if beta is used as an input when defining a hurdle rate for investment projects, on one hand overestimation of beta could lead to higher capital cost and thus a lower (or negative) present value of projects, which would be erroneously discarded. On the other hand, and following the same logic, underestimation could lead to approval of projects with true negative present value.

$$\beta_{t+h} = \alpha + \gamma\beta_{t+h|t} + e_{t+h|h} \tag{9}$$

where t represents the current time period and the forecast is made for a h-step ahead (in this paper h is always 1). With the parameters of this regression estimated, we test the joint hypothesis

$$H_0 : \alpha = 0, \gamma = 1$$

of no systematic bias in forecast. We also test individually the null hypotheses $H_0 : \alpha = 0$ and $H_0 : \gamma = 1$ to investigate the origin of the bias. In the individual test, if $\alpha = 0$ we can conclude that there is no systematic bias, and if $\gamma = 1$ the forecast can be considered efficient.

4.2 Results

In this section we present and discuss the main results of both the 6 month and the one year ahead forecasts of beta. First, we look at the time-series of realized betas in figures 1 and 2 which display features of stationarity. In particular, over the stocks, betas have not risen since the 1950's, indicating that the chose of daily return measurement was still appropriate for these stocks in the early part of the sample as it did not generate a downward bias in the beta measurements. It is also important to consider the dynamic structure of realized betas when modelling its behaviour, and consequently, estimating next period's value. With that in mind the Autocorrelation (ACF) and Partial Autocorrelation (PACF) functions of realized betas were calculated for our entire sample. These are not displayed in this paper due to space limitations, though these are available from the authors upon request. These functions suggest that the structure of realized betas are better modelled by low order autoregressive processes as the PACFs usually cut-off after lag 1 and ACFs slowly decay.

We now move to the results regarding the forecast ability of the models ranked as contenders in this paper. In table 1 we present the values of the mean absolute error for the 6-month-ahead forecast. We present the numbers for each model for each company and then the average of each model. The first column is the standard FM beta used as a forecast for the next period. Columns 2 to 7 present different specifications of the constant realized beta model, columns 8 o 12 present five specifications of the AR(p) model with in-sample parameters estimated over 40 periods (half-years) and in columns 13 to 17 the AR(p) models with 60 in-sample estimation periods. We highlight in each row, the best performing model within each category (constant and autoregressive).

For the half-year forecast we have consistent results across all models by measuring forecast precision by either MAE or MSE, though displayed tables are only for MAE due to space limitations. The FM beta is the worst performer of all models in the overall average as well as in individual companies where it is the model with highest error in 13 (12) out of 15 companies in terms of MAE (MSE). Realized beta with 18 months of daily data is the constant model with smallest error in the overall average of MAE and MSE, although it is not the model with smallest error in the largest number of companies. Using this model to forecast beta for 6 months ahead yields a reduction of 43.55% in MSE (24.87% in root mean squared error[6]) and 26.54% in MAE compared to using FM beta. For both sample sizes of the AR(p) models, 40 and 60 in-sample estimation periods, the models with one lag have the smallest error. The AR(1) model with 60 in-sample estimation periods is the best model of all categories in terms of lower MSE with an overall square error of 0.0537 that represents a reduction of 48% (or 27.89% in root mean squared error) compared to FM beta while AR(1) with 40 in-sample estimation periods has an overall MSE of 0.0540 for a reduction of 47.71% (or 27.69% in root mean squared error) versus FM beta. In terms of MAE, the model with smallest error of all is the AR(1) with 40 in-sample estimation periods with an absolute error of 0.1763, or a reduction of 30.28% to FM beta, while AR(1) with 60 in-sample estimation

[6]Root mean squared error is calculated by taking the square root of average MSE for each model.

periods has an overall MAE of 0.1764, or a reduction of 30.24% compared to FM beta.

Next, in table 2 we present the values of the mean absolute error for the one-year-ahead forecast. The first column is the standard FM beta used as a forecast for the next period. Columns 2 to 6 present different specifications of the constant realized beta model and columns 7 o 11 present five specifications of the AR model with in-sample parameters estimated over 30 periods (years). We highlight in each row, the best performing model within each category (constant and autoregressive).

Firstly though, in terms of MSE, the FM beta performs worse in all situations having the highest MSE in 10 of 15 firms and the largest overall MSE. Amongst the constant models the realized beta calculated with 24 months of daily data has the lowest average MSE and also the lowest MSE in 5 of 15 companies. Compared to the FM beta, using the 24M realized beta provides a reduction of 41.44% in MSE (23.48% in root mean squared error). Moving to the autoregressive specifications, the model with 1 lag is the best amongst the AR(p) models in 10 of 15 companies and has the smallest overall MSE, with a reduction of 46.18% in MSE (26.64% in root mean squared error) when compared to FM betas used for forecasting purposes.

The results from MAE are mostly consistent with those from MSE. The standard FM beta is again the worst performer overall. Amongst the AR(p) models, the model with 1 lag is again the best performer with average MAE of 0.1670 which represents a reduction in absolute forecast error of 29.82% in comparison with the FM beta. The results from the constant models are slightly different in terms of MAE with the model with realized betas calculated over 18 months of daily data (18M beta) now being the model with smallest overall error, a MAE of 0.1768, with a reduction of 25.71% compared with FM beta. However, this model is the best performer of all constant models in only 3 of 15 companies and its overall value is very similar to that of 24M beta.

In tables 3 and 4 we compare the summary statistics of the best forecasting models (in terms of MAE and MSE) of the AR(1) models, with that of the standard constant Fama-MacBeth beta model. For each company we report the number of observations, or out-of-sample forecasting evaluation periods, the mean, standard deviation, minimum and maximum of forecast error. The error in forecast is the difference between the estimated beta at time t and the realized beta at time $t+1$ with time measured either in half-years or years.

For the 6 month forecast (table 3) we find that the dispersion of forecasting error is much larger when using the constant FM beta to forecast next period's stock beta. Here the standard deviation of forecast errors using the FM model is the largest, when compared to those of both AR(1) specifications, in 14 out of 15 stocks. Additionally, FM betas calculated as the standard in the literature again have the largest range of error with values ranging from -0.93 to 1.20, while the two autoregressive specifications have values from -0.87 to 0.95 and -0.84 to 0.91 (with 40 and 60 in-sample estimation periods, respectively).

Similar results are found for the forecasts of betas for one year, see table 4. The FM beta model has a larger standard deviation than the AR(1) for 11 out of the 15 US stocks in our sample. Moreover, it has also the widest range of error with its global minimum and maximum of -0.70 and 1.14 compared with those of -0.74 and 0.66 of the autoregressive model. These results support initial findings that using an autoregressive specification for realized beta improves forecasting ability when estimating stocks betas one year in advance when compared to the standard practice in industry.

Having analysed the error characteristics of several classes of models, leading to a preliminary conclusion that the AR(1) is the best model in terms of smallest errors (with the constant model with 18 months also performing well in terms of MAE and MSE), we now analyse additional statistical properties of constant and autoregressive models, in comparison with the standard FM beta, by implementing the methodology introduced in Mincer and Zarnowitz (1969).

When implementing the Mincer-Zarnowitz (MZ) regression we first analyze potential presence of bias in the forecast. To achieve that we test the joint null hypothesis that α equals zero and γ equals one simultaneously. We do that by conducting an F-test where the unrestricted model is the regression of the actual values on the forecasts and the restricted model is built by applying the restrictions of $\alpha = 0$ and $\gamma = 1$ to the MZ equation.[7] In order to further investigate the characteristics of each model's forecast, we also want to investigate whether the bias, if any, comes from the intercept (α) or from the slope (γ).

First we analyse the bias in forecasts for the 6 months scenario in tables 5, 6 and 7. For each model we have three columns: The first two columns present the regression coefficients α and γ with individual tests of significance where bold values highlight rejection of the null at the 5% level; the third column shows the p-values of the joint hypothesis of systematic bias and again bold values are rejection from the null at the 5% significance level, or bias in the forecast. Here the problems with using the industry standard FM beta to forecast next period's beta is evident, with all companies presenting bias in the forecast with very strong rejection of the joint null hypothesis (in 14 of the 15 companies the rejection is significant at 1% or better). The positive significant values of α are indication that the FM betas understate the value of realized beta one period ahead. The constant realized beta models are also mostly biased with the bias reducing, almost monotonically, with the increase in number of months. The best model in terms of MAE and MSE, the 18M beta, is biased in 10 of the 15 companies, however, due to its performance in MAE and MSE, this is still considered the best constant model.

We next analyse the bias in autoregressive models where we find substantial improvement. Starting with AR(p) models with 40 periods of in-sample estimation, we can see that the models with 2 and 3 lags are the best with only 1 biased result amongst 15 companies.

[7]The errors in the restricted model are essentially the forecasting errors of the MZ equation when restricted: $y_{t+h} = 0 + 1y_{t+h|t} + u_t$ with y being in our study the stock betas.

Although not the best in terms of bias, the AR(1) model still does a good job with only 3 biased companies. This model also presents strong support of the null when it is unbiased with all p-values higher than 20%. The final results of bias are in table 7 with the autoregressive models with 60 in-sample estimation periods. The AR(1) model is the best overall model for the 6 month forecasts with bias in only one company, Caterpillar, and strong support of the null of no bias with p-values as high as 0.9824 in Merck for example. The other specifications of the AR(p) model with 60 in-sample estimation periods also perform well in terms of bias but when we consider the previous results in terms of MAE and MSE, the AR(1) is still the best performing forecasting model.

Next, in tables 8 and 9 we report the results for the main models (FM, constant and autoregressive) for 1 year forecasts. Here again we can see that the FM betas are mostly biased. The forecast from this model is biased in 13 out of the 15 companies in the sample, with the two companies whose null is not rejected at 5% (Chevron and Caterpillar), still having a p-value under 10%. Most of the time the bias comes from both the intercept and the slope of the regression with the positive α values, as in the 6 month scenario, suggesting underestimation of the forecast. Moving to the constant realized beta models, we can see a general improvement in the bias as we increase the length of the measure (from 12 to 60 months of daily returns). Both the 18M and the 24M models, the ones with lowest error measured by MAE and MSE, perform better than the FM betas with only 6 out of 15 companies presenting systematic bias. The constant model which presents less bias is the realized beta with 60 months of daily data with only 5 biased results. However, as this model had relatively weak performance by MAE and MSE, the complete picture still indicates the 18M and 24M as the two most reliable constant models to forecast beta.

Moving to table 9 we can clearly see the improvement in bias reduction by using autoregressive models to forecast next period's (one year ahead) betas. AR(p) models are the least biased of all with the bias increasing with the number of lags used in the specification, again supporting the evidence that betas are at most autoregressive at one period. The AR(1) model outperforms all others with only one incidence of bias in a company's beta forecast, namely Boeing. This again provides strong support for the choice of the AR(1) as the most accurate beta forecasting model.

In summary, the analysis of forecasting bias using the Mincer-Zarnowitz regression provides additional support to the AR(1) as the most suitable forecasting model for both 6 months as well as yearly beta forecasts. The results also show a substantial bias in the standard FM beta.

5 An Economic Example: Evaluating Investments

One of the most important applications of long range beta forecasting in finance is in cost of capital calculations, see Pratt and Grabowski (2014). Graham and Harvey (2001), in a survey regarding the practice of corporate finance, treats in length the estimation of the cost of capital. The study surveys 392 US company executives about their practice on cost of capital, capital budgeting and capital structure and document the widespread use of CAPM as a capital budgeting tool in the US. The results show that 73.5% of their respondents always or almost always use the CAPM and the authors also discuss that it had increased in popularity from previous studies.

To illustrate the impact of our results in economic applications we analyse four investment projects. The first three investment projects have the same initial investment of $8,000, a time frame of 5 years, and a cash flow of $15,000 which differ only in the timing that cash flows happen. In project A, cash is returned in equal amounts during the project of $3,000 per year. Project B has a decreasing cash generation over the five years of $5,000, $4,000, $3,000, $2,000 and $1,000. While project C has an increasing cash generation over the five years of $1,000, $2,000, $3,000, $4,000 and $5,000. The fourth project is a perpetual cash flow (which is more similar to a going concern company) with an initial investment of \$10,000 and a cash flow of \$650 per year with indefinitely long duration.

In order to assess the economic effects of using different beta forecasts we calculate the net present value (NPV) of all four projects for all companies across all time periods of the out-of-sample evaluation period. In the NPV calculations we use a discount rate (r) from CAPM assuming a 2% risk free rate and a 5.5% risk premium of the market over the risk free rate, i.e. $r = 2\% + \beta 5.5\%$ and

$$NPV = -C_0 + \sum_{i=1}^{\infty} \frac{C_i}{(1+r)^i} \tag{10}$$

where C_0 is the initial investment and C_i is the cash flow in year i.

For the one year sample we have 375 NPV values for each project/forecast method combination. For the 6 month beta we have a total of 825 NPVs. The results are presented in figures 3 and 4, which present the histogram of NPVs for each project for 6-month and one year betas respectively, and in table 10 with the summary statistics. From table 10 we can quantify this difference. In panel (a), one year betas sample, we can see that the difference in the maximum values reach more than \$14,000 and for the minimum values about \$700 in the perpetual project which has a mean NPV of -\$961 and -\$358 for AR(1) and FM beta, respectively. We can also see a similar economic difference when looking at the 25^{th} and 75^{th} percentiles with the difference reaching approximately \$500 for the 25^{th} percentile in the perpetuity. We can reach the same conclusion by looking at panel (b) where the dollar value difference in NPVs reach as much as approximately \$13,000 between AR(1) and FM betas for the minimum values of the perpetuity and as much as approximately \$500 for the 25^{th} and 75^{th} percentiles in the perpetual project.

Figures 3 and 4 reveal that for all four projects the NPVs calculated from the AR(1) forecast method have less extreme values than that calculated with FM betas, and are also more concentrated around the mean. This is evidence that there is indeed a significant economic difference in using each forecast method. These results provide us with sufficient evidence that using the more accurate beta forecasting model by our econometric tests (the AR(1) model of realized betas) can significantly change the investment, or project evaluation, decision when compared to what is still the industry standard (the Fama-MacBeth beta forecast).

6 Conclusions

Beta forecasting for long horizons is important to many firms, and particularly when firms are calculating the cost of capital. In this paper we have demonstrated the inaccuracy of the standard forecasting approach which follows Fama and MacBeth (1973), both from statistical and economic perspectives. We also propose a new approach to long range beta forecasting with an AR(1) model of realized beta, for in settings where at least 20 years of daily return data is available, which has a relatively low mean absolute (and squared) forecast error and is statistically unbiased.

In settings where accurate daily return data is available only for a limited historical period, we find that a constant realized beta model based on the prior 18 months of daily returns, produces forecasts with the lowest mean absolute (and squared) forecast error. However, these constant beta forecasts can often contain a significant statistical bias, highlighting that additional historical data is important in long range beta forecasting for horizons of 6 months or one year.

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Table 1: **MAE of 6-Month-Ahead Forecast of Betas**

								40 periods					60 periods				
Company	**FM**	**6M**	**12M**	**18M**	**24M**	**48M**	**60M**	**AR(1)**	**AR(2)**	**AR(3)**	**AR(4)**	**AR(5)**	**AR(1)**	**AR(2)**	**AR(3)**	**AR(4)**	**AR(5)**
Coca Cola	0.3350	0.1812	0.1621	**0.1598**	0.1737	0.1994	0.2084	0.1797	**0.1767**	0.1779	0.1812	0.1808	0.1819	0.1778	**0.1757**	0.1762	0.1760
Du Pont	0.1483	0.1805	0.1484	0.1420	0.1383	**0.1364**	0.1388	0.1449	**0.1336**	0.1373	0.1372	0.1417	0.1459	**0.1348**	0.1360	0.1365	0.1421
Exxon	0.2682	0.2257	0.2205	0.1960	**0.1948**	0.2221	0.2285	**0.1988**	0.2009	0.2009	0.2052	0.2133	0.2001	0.2032	**0.1994**	0.2003	0.2081
GE	0.2222	**0.1449**	0.1465	0.1543	0.1679	0.1955	0.1958	**0.1323**	0.1353	0.1386	0.1378	0.1417	**0.1304**	0.1323	0.1345	0.1327	0.1339
IBM	0.2711	0.2071	0.2079	0.1760	0.1756	**0.1740**	0.1805	0.1810	0.1862	**0.1719**	0.1746	0.1767	0.1755	0.1820	0.1686	**0.1669**	0.1691
Chevron	0.2617	0.2000	0.2116	**0.1969**	0.2015	0.2295	0.2349	0.2031	0.2069	**0.1997**	0.2055	0.2195	0.1981	0.1986	**0.1930**	0.1956	0.1988
United Tech	0.2696	0.1570	0.1635	0.1468	**0.1439**	0.1511	0.1535	0.1405	0.1441	**0.1313**	0.1329	0.1352	0.1401	0.1412	**0.1334**	0.1406	0.1396
P&G	0.2345	**0.1639**	0.1795	0.1905	0.1884	0.1928	0.2001	0.1807	**0.1802**	0.1834	0.1829	0.1792	0.1749	0.1706	0.1707	0.1687	**0.1665**
Caterpillar	0.2495	**0.1955**	0.2030	0.2144	0.2250	0.2483	0.2450	**0.1964**	0.2004	0.2027	0.2049	0.2083	**0.2076**	0.2092	0.2107	0.2117	0.2115
Boeing	0.3280	**0.2289**	0.2546	0.2770	0.2764	0.2879	0.2643	**0.2193**	0.2215	0.2269	0.2383	0.2513	**0.2206**	0.2234	0.2324	0.2407	0.2496
Pfizer	0.3326	0.1925	**0.1812**	0.1904	0.2068	0.2214	0.2119	**0.1891**	0.1910	0.2013	0.2064	0.1987	**0.1750**	0.1768	0.1847	0.1834	0.1918
J&J	0.2211	0.1790	0.1922	0.1946	0.1865	0.1804	**0.1779**	**0.1749**	0.1777	0.1883	0.1867	0.1901	0.1887	0.1900	0.1911	**0.1864**	0.1895
3M	0.1461	0.1471	0.1582	0.1408	**0.1369**	0.1460	0.1382	**0.1313**	0.1368	0.1341	0.1336	0.1361	0.1281	**0.1264**	0.1273	0.1311	0.1326
Merck	0.2280	0.1837	0.1818	0.1857	**0.1814**	0.1977	0.1960	**0.1598**	0.1616	0.1654	0.1719	0.1766	**0.1598**	0.1629	0.1685	0.1684	0.1714
Alcoa	0.2770	**0.2104**	0.2140	0.2212	0.2201	0.2164	0.2173	0.2126	0.2149	**0.2113**	0.2160	0.2312	**0.2193**	0.2224	0.2229	0.2278	0.2306
Average	0.2529	0.1865	0.1883	**0.1858**	0.1878	0.1999	0.1994	**0.1763**	0.1779	0.1781	0.1810	0.1854	**0.1764**	0.1768	0.1766	0.1778	0.1807

This table shows the Mean Absolute Error of 6-month-ahead beta forecasts for each company and the average for the entire sample for each model: Fama-MacBeth; constant models of realized beta with varying estimation sizes (6, 12, 18, 24, 48 and 60 months of daily data); Autoregressive models with 1-5 lags and estimation sizes of 40 and 60 periods (half-years). The out of sample evaluation starts from 1984:2 to 2011:2 with a total of 55 forecasting periods. Values in bold show the model with the smallest MAE within each model class.

Table 2: **MAE of One-Year-Ahead Forecast of Betas**

Company	FM	12M	18M	24M	48M	60M	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
Coca Cola	0.3225	**0.1366**	0.1598	0.1640	0.1911	0.1889	**0.1564**	0.1685	0.1733	0.1717	0.1741
Du Pont	0.1306	0.1276	0.1171	0.1198	**0.1086**	0.1149	0.1074	0.1056	0.1030	0.1034	**0.0992**
Exxon	0.2661	0.2265	**0.1876**	0.1904	0.2257	0.2355	0.2016	**0.2050**	0.2123	0.2146	0.2244
GE	0.2290	**0.1548**	0.1674	0.1804	0.1860	0.1785	0.1191	0.1170	0.1121	**0.1082**	0.1180
IBM	0.2659	0.1399	0.1150	**0.1094**	0.1197	0.1278	**0.1267**	0.1274	0.1329	0.1353	0.1404
Chevron	0.2278	0.1975	**0.1920**	0.1971	0.2119	0.2189	**0.1966**	0.2055	0.2115	0.1974	0.2024
United Tech	0.2520	0.1334	0.1362	**0.1315**	0.1323	0.1400	0.1151	0.1142	**0.1108**	0.1277	0.1276
P&G	0.2248	0.1732	0.1819	0.1782	**0.1588**	0.1705	0.1686	**0.1628**	0.1832	0.1888	0.1957
Caterpillar	0.2417	**0.2207**	0.2438	0.2368	0.2466	0.2339	**0.2177**	0.2198	0.2243	0.2276	0.2342
Boeing	0.2895	0.2638	0.2754	**0.2575**	0.2989	0.2622	**0.2532**	0.2798	0.2883	0.2687	0.2640
Pfizer	0.2983	**0.1732**	0.1819	0.1922	0.1942	0.1811	**0.1553**	0.1584	0.1558	0.1568	0.1573
J&J	0.1917	0.1674	0.1708	0.1745	0.1663	**0.1569**	0.1824	0.1941	**0.1701**	0.1590	0.1690
3M	0.1485	0.1579	0.1362	0.1464	0.1475	**0.1310**	**0.1188**	0.1219	0.1221	0.1286	0.1296
Merck	0.2142	0.1741	0.1874	**0.1731**	0.1853	0.1799	**0.1437**	0.1484	0.1571	0.1678	0.1759
Alcoa	0.2663	0.2306	**0.1992**	0.2149	0.2122	0.2165	**0.2422**	0.2547	0.2612	0.2713	0.2781
Average	0.2379	0.1785	**0.1768**	0.1778	0.1857	0.1824	**0.1670**	0.1722	0.1745	0.1751	0.1793

This table shows the Mean Absolute Error of one-year-ahead beta forecasts for each company and the average for the entire sample for each model: Fama-MacBeth; constant models of realized beta with varying estimation sizes (12, 18, 24, 48 and 60 months of daily data); Autoregressive models with 1-5 lags and an estimation size of 30 periods (years). The out of sample evaluation starts from 1987 to 2011 with a total of 25 forecasting periods. Values in bold show the model with the smallest MAE within each model class.

Table 3: **Summary Statistics of 6-Month-Ahead Forecast Errors**

	FM					**AR(1) - 40 Periods**					**AR(1) - 60 Periods**				
Company	**N Obs**	**Mean**	**Std Dev**	**Minimum**	**Maximum**	**N Obs**	**Mean**	**Std Dev**	**Minimum**	**Maximum**	**N Obs**	**Mean**	**Std Dev**	**Minimum**	**Maximum**
Coca Cola	55	-0.13	0.37	-0.93	0.63	55	0.02	0.23	-0.60	0.55	55	0.01	0.23	-0.60	0.52
Du Pont	55	0.08	0.16	-0.25	0.40	55	-0.01	0.20	-0.72	0.38	55	-0.01	0.19	-0.57	0.37
Exxon	55	-0.19	0.27	-0.88	0.50	55	-0.02	0.25	-0.72	0.52	55	0.00	0.25	-0.71	0.53
GE	55	-0.05	0.28	-0.65	0.69	55	-0.03	0.17	-0.56	0.32	55	-0.02	0.18	-0.56	0.39
IBM	55	0.07	0.33	-0.54	0.82	55	0.02	0.22	-0.60	0.42	55	0.01	0.22	-0.60	0.37
Chevron	55	-0.10	0.28	-0.54	0.57	55	0.01	0.26	-0.76	0.61	55	0.03	0.26	-0.78	0.66
United Tech	55	0.20	0.26	-0.58	0.76	55	0.02	0.20	-0.87	0.53	55	0.03	0.20	-0.84	0.51
P&G	55	-0.12	0.29	-0.83	0.52	55	0.02	0.24	-0.56	0.95	55	0.00	0.23	-0.59	0.91
Caterpillar	55	0.10	0.29	-0.50	0.68	55	-0.09	0.25	-0.64	0.45	55	-0.12	0.25	-0.68	0.35
Boeing	55	0.11	0.42	-0.71	1.20	55	0.02	0.27	-0.71	0.63	55	0.05	0.28	-0.66	0.65
Pfizer	55	-0.09	0.41	-0.93	0.93	55	0.04	0.25	-0.58	0.76	55	0.03	0.24	-0.59	0.77
J&J	55	0.00	0.29	-0.47	0.76	55	0.04	0.24	-0.34	0.81	55	0.03	0.25	-0.39	0.79
3M	55	-0.05	0.18	-0.41	0.40	55	0.01	0.16	-0.26	0.39	55	0.02	0.15	-0.25	0.37
Merck	55	-0.04	0.28	-0.46	0.87	55	0.00	0.23	-0.41	0.91	55	0.00	0.23	-0.36	0.90
Alcoa	55	0.20	0.29	-0.54	1.03	55	-0.04	0.26	-0.71	0.53	55	-0.03	0.27	-0.71	0.49

This table presents summary statistics of each company's sample of 55 forecasting errors from Fama-MacBeth and the the two best autoregressive models (AR(1) model with 40 and 60 periods of in-sample estimation). Forecasting error is measured as $error = \beta_{t+h|t} - \beta_{t+h}$. The out of sample evaluation starts from 1984:2 to 2011:2 with a total of 55 forecasting periods.

Table 4: **Summary Statistics of One-Year-Ahead Forecast Errors**

	FM					AR(1) - 30 Periods				
Company	**N Obs**	**Mean**	**Std Dev**	**Minimum**	**Maximum**	**N Obs**	**Mean**	**Std Dev**	**Minimum**	**Maximum**
Coca Cola	25	-0.10	0.37	-0.78	0.60	25	0.03	0.20	-0.50	0.32
Du Pont	25	0.08	0.13	-0.26	0.26	25	-0.01	0.15	-0.48	0.21
Exxon	25	-0.22	0.24	-0.79	0.32	25	-0.01	0.25	-0.47	0.46
GE	25	-0.05	0.29	-0.56	0.58	25	-0.02	0.18	-0.52	0.39
IBM	25	0.12	0.31	-0.30	0.78	25	0.03	0.16	-0.33	0.33
Chevron	25	-0.13	0.24	-0.51	0.41	25	0.02	0.24	-0.45	0.53
United Tech	25	0.19	0.25	-0.23	0.76	25	0.03	0.17	-0.51	0.47
P&G	25	-0.12	0.28	-0.77	0.46	25	0.01	0.21	-0.37	0.49
Caterpillar	25	0.08	0.29	-0.44	0.56	25	-0.13	0.26	-0.72	0.27
Boeing	25	0.05	0.38	-0.45	1.14	25	0.06	0.31	-0.49	0.66
Pfizer	25	-0.06	0.37	-0.72	0.88	25	0.04	0.21	-0.37	0.48
J&J	25	0.01	0.26	-0.41	0.54	25	0.07	0.23	-0.37	0.65
3M	25	-0.07	0.16	-0.40	0.22	25	0.02	0.15	-0.26	0.28
Merck	25	-0.01	0.26	-0.42	0.61	25	0.01	0.20	-0.43	0.52
Alcoa	25	0.20	0.24	-0.40	0.59	25	-0.07	0.29	-0.74	0.48

This table presents summary statistics of each company's sample of 25 forecasting errors from Fama-MacBeth and the the best autoregressive model (AR(1) model with 30 periods of in-sample estimation). Forecasting error is measured as $error = \beta_{t+h|t} - \beta_{t+h}$. The out of sample evaluation starts from 1987 to 2011 with a total of 25 forecasting periods.

Table 5: **Forecasting Bias (M-Z test) - 6-Month-Ahead Forecasts Fama-MacBeth and Constant Models**

	FM			6M			12M			18M			24M			48M			60M		
Company	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*
Coca Cola	**0.88**	**-0.05**	**0.0000**	**0.24**	**0.71**	**0.0163**	0.15	0.81	0.1810	0.11	0.86	0.3459	0.14	0.82	0.2456	0.20	0.74	0.1254	0.21	0.72	0.1114
Du Pont	0.27	0.70	**0.0007**	**0.84**	**0.23**	**0.0000**	**0.63**	**0.43**	**0.0061**	**0.57**	**0.49**	**0.0326**	**0.51**	**0.55**	0.0806	0.35	0.70	0.2941	0.47	0.59	0.2067
Exxon	**0.48**	0.53	**0.0000**	**0.46**	**0.43**	**0.0001**	**0.40**	**0.51**	**0.0044**	**0.35**	**0.57**	**0.0222**	**0.32**	**0.61**	**0.0493**	**0.44**	**0.45**	**0.0272**	**0.52**	**0.35**	**0.0194**
GE	**0.94**	**0.20**	**0.0000**	**0.45**	**0.61**	**0.0033**	**0.41**	**0.65**	**0.0332**	**0.44**	**0.62**	**0.0493**	**0.54**	**0.54**	**0.0373**	**1.20**	**-0.03**	**0.0029**	**1.64**	**-0.41**	**0.0004**
IBM	**0.83**	**0.15**	**0.0000**	**0.78**	**0.21**	**0.0000**	**0.89**	**0.10**	**0.0000**	**0.48**	**0.52**	0.0659	**0.51**	**0.48**	0.0566	0.44	**0.54**	0.1334	**0.52**	**0.46**	0.0719
Chevron	**0.44**	0.51	**0.0125**	**0.39**	**0.51**	**0.0008**	**0.36**	**0.55**	**0.0068**	**0.29**	**0.63**	**0.0401**	**0.28**	**0.65**	0.0732	**0.39**	**0.50**	**0.0240**	**0.44**	**0.43**	**0.0100**
United Tech	**1.06**	**-0.01**	**0.0000**	**0.77**	**0.26**	**0.0000**	**0.75**	**0.28**	**0.0002**	**0.59**	**0.44**	**0.0081**	**0.53**	**0.50**	**0.0216**	**0.63**	**0.40**	**0.0046**	**0.68**	**0.35**	**0.0022**
P&G	**0.50**	**0.44**	**0.0000**	**0.36**	**0.55**	**0.0014**	**0.34**	**0.57**	**0.0094**	**0.30**	**0.61**	**0.0274**	**0.28**	**0.63**	**0.0405**	**0.29**	**0.61**	**0.0423**	**0.31**	**0.59**	**0.0347**
Caterpillar	0.29	0.70	**0.0069**	**0.38**	**0.69**	**0.0125**	**0.37**	**0.70**	0.0619	**0.38**	**0.71**	0.1106	0.37	0.72	0.1455	0.27	0.85	0.0572	0.19	0.94	**0.0222**
Boeing	**1.04**	**-0.02**	**0.0000**	**0.53**	**0.49**	**0.0004**	**0.60**	**0.41**	**0.0009**	**0.76**	**0.27**	**0.0004**	**0.87**	**0.16**	**0.0004**	**1.43**	**-0.41**	**0.0000**	**1.02**	**0.00**	**0.0043**
Pfizer	**0.99**	**-0.14**	**0.0000**	**0.43**	**0.51**	**0.0005**	**0.34**	**0.61**	**0.0260**	**0.36**	**0.58**	**0.0495**	**0.43**	**0.51**	**0.0435**	**0.98**	**-0.11**	**0.0056**	**1.04**	**-0.18**	**0.0265**
J&J	**0.37**	**0.52**	**0.0033**	**0.29**	**0.62**	**0.0039**	**0.27**	**0.64**	**0.0253**	**0.22**	**0.70**	0.1037	0.16	0.78	0.2811	0.10	0.83	0.3093	0.08	0.84	0.2564
3M	**0.99**	**-0.06**	**0.0000**	**0.67**	**0.28**	**0.0000**	**0.85**	**0.09**	**0.0000**	**0.79**	**0.15**	**0.0011**	**0.71**	**0.25**	**0.0070**	**0.92**	**0.01**	**0.0015**	**0.83**	**0.11**	**0.0058**
Merck	**0.64**	**0.28**	**0.0001**	**0.52**	**0.40**	**0.0001**	**0.52**	**0.40**	**0.0021**	**0.59**	**0.32**	**0.0031**	**0.61**	**0.30**	**0.0078**	**1.03**	**-0.18**	**0.0011**	**1.30**	**-0.49**	**0.0002**
Alcoa	**0.29**	**0.65**	**0.0000**	**0.36**	**0.71**	**0.0233**	**0.33**	**0.74**	0.0812	0.27	0.79	0.2099	0.24	0.82	0.3157	0.18	0.89	0.3531	0.15	0.93	0.2331

This table presents the parameter estimates and p-values of the Mincer-Zarnowitz test of forecasting bias for each company for Fama-MacBeth and constant realized beta models. Columns $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ show the coefficients from M-Z regression with bold values indicating rejection of the null of individual tests $\alpha = 0$ and $\gamma = 1$ at the 5% two-tailed significance level. The columns $p - val$ show the p-values of the joint hypothesis test of systematic bias in the forecast $H_0 : \alpha = 0, \gamma = 1$ with bold values highlighting rejection of the null at the 5% two-tailed significance level. The out of sample evaluation starts from 1984:2 to 2011:2 with a total of 55 forecasting periods.

Table 6: **Forecasting Bias (M-Z test) - 6-Month-Ahead Forecasts Autoregressive Models (40 in-sample periods)**

	AR(1)			AR(2)			AR(3)			AR(4)			AR(5)		
Company	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*
Coca Cola	-0.01	0.99	0.7543	0.04	0.95	0.9180	0.06	0.92	0.8424	0.06	0.91	0.7028	0.08	0.89	0.7105
Du Pont	**1.46**	**-0.33**	**0.0218**	0.68	0.38	0.2168	**0.88**	**0.20**	0.1382	**0.84**	**0.23**	0.1184	**0.98**	**0.11**	0.0619
Exxon	0.22	0.74	0.5312	0.28	0.67	0.4069	0.30	0.64	0.2981	**0.38**	0.54	0.1416	**0.45**	**0.45**	0.0674
GE	-0.01	1.03	0.5465	0.05	0.98	0.5409	0.06	0.97	0.5557	0.04	0.99	0.5449	0.10	0.94	0.4751
IBM	0.67	0.32	0.2208	**0.86**	**0.12**	0.0700	0.36	0.62	0.2902	0.43	0.56	0.1803	**0.50**	**0.48**	0.0962
Chevron	0.22	0.71	0.3212	0.30	**0.61**	0.1365	0.27	0.65	0.1530	**0.32**	**0.59**	0.0733	**0.38**	**0.50**	0.0221
United Tech	**0.89**	**0.15**	**0.0472**	**1.00**	**0.04**	**0.0190**	**0.51**	**0.51**	0.1419	**0.57**	**0.45**	0.0845	**0.61**	**0.42**	0.0576
P&G	0.03	0.94	0.8681	0.18	0.77	0.6546	0.25	0.68	0.4665	0.31	0.60	0.2695	0.38	0.52	0.1223
Caterpillar	0.17	0.93	**0.0415**	0.21	0.88	0.0651	0.26	0.84	**0.0340**	0.28	0.83	**0.0355**	0.31	0.81	**0.0195**
Boeing	0.32	0.68	0.2964	0.35	0.66	0.2714	0.40	**0.60**	0.1358	**0.51**	**0.48**	**0.0193**	**0.61**	**0.39**	**0.0052**
Pfizer	0.17	0.78	0.4378	0.20	0.75	0.4079	0.31	0.63	0.2146	0.39	**0.55**	0.0884	**0.36**	**0.56**	**0.0119**
J&J	-0.13	1.10	0.3682	-0.08	1.05	0.4145	0.04	0.90	0.5502	0.12	0.83	0.5791	0.14	0.77	0.2494
3M	0.88	0.05	0.2150	**0.91**	**0.02**	0.0529	**0.70**	**0.24**	0.0988	**0.73**	**0.21**	0.0543	**0.67**	**0.27**	**0.0213**
Merck	0.24	0.73	0.7475	0.44	0.50	0.3600	0.51	0.41	0.2545	**0.61**	**0.30**	0.0957	**0.64**	**0.26**	0.0483
Alcoa	-0.11	1.13	0.3698	-0.06	1.09	0.4374	-0.02	1.05	0.5798	0.03	1.01	0.5989	0.21	0.85	0.4891

This table presents the parameter estimates and p-values of the Mincer-Zarnowitz test of forecasting bias for each company for autoregressive models with in-sample estimation size of 40 periods (half-years). Columns α and γ show the coefficients from M-Z regression with bold values indicating rejection of the null of individual tests $\boldsymbol{\alpha} = 0$ and $\boldsymbol{\gamma} = 1$ at the 5% two-tailed significance level. The columns $p - val$ show the p-values of the joint hypothesis test of systematic bias in the forecast $H_0 : \alpha = 0, \gamma = 1$ with bold values highlighting rejection of the null at the 5% two-tailed significance level. The out of sample evaluation starts from 1984:2 to 2011:2 with a total of 55 forecasting periods.

Table 7: **Forecasting Bias (M-Z test) - 6-Month-Ahead Forecasts Autoregressive Models (60 in-sample periods)**

	AR(1)			AR(2)			AR(3)			AR(4)			AR(5)		
Company	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*	α	γ	*p-val*
Coca Cola	-0.04	1.04	0.9369	-0.04	1.05	0.9426	-0.03	1.03	0.9607	-0.05	1.05	0.8751	-0.02	1.01	0.9712
Du Pont	1.28	-0.17	0.2706	0.27	0.76	0.8417	0.34	0.69	0.7600	0.43	0.61	0.6582	0.65	0.41	0.3419
Exxon	0.17	0.80	0.7717	0.23	0.72	0.6339	0.25	0.70	0.5097	0.32	0.61	0.2963	0.39	0.52	0.1616
GE	-0.14	1.15	0.4723	-0.06	1.08	0.5296	-0.08	1.09	0.4908	-0.08	1.09	0.4612	-0.06	1.07	0.5311
IBM	0.67	0.31	0.4833	0.95	0.04	0.1694	0.21	0.77	0.7129	0.25	0.74	0.6892	0.37	0.61	0.4664
Chevron	0.21	0.71	0.2061	0.21	0.71	0.2231	0.16	0.77	0.3512	0.18	0.75	0.2719	0.20	0.72	0.1692
United Tech	0.64	0.38	0.1746	0.71	0.31	0.1622	0.43	0.58	0.3563	0.53	0.48	0.1850	0.56	0.45	0.1606
P&G	-0.10	1.12	0.9154	-0.03	1.04	0.9793	0.01	0.99	0.9982	0.05	0.93	0.9576	0.07	0.91	0.9295
Caterpillar	0.17	0.95	**0.0043**	0.24	0.88	**0.0156**	0.25	0.87	**0.0088**	0.28	0.85	**0.0092**	0.30	0.83	**0.0051**
Boeing	0.33	0.65	0.1204	0.33	0.64	0.1153	0.38	**0.59**	**0.0367**	**0.43**	**0.54**	**0.0114**	**0.48**	**0.49**	**0.0038**
Pfizer	-0.15	1.13	0.5674	-0.01	0.98	0.7738	0.14	0.82	0.5246	0.15	0.80	0.5273	0.27	0.67	0.0994
J&J	-0.33	1.37	0.2874	-0.29	1.33	0.3631	-0.28	1.31	0.3601	-0.20	1.21	0.5254	-0.13	1.12	0.5485
3M	0.25	0.72	0.6146	0.18	0.79	0.6320	0.24	0.73	0.6393	0.40	0.56	0.3760	0.49	0.47	0.2047
Merck	0.07	0.92	0.9824	0.37	0.58	0.6260	0.66	0.25	0.2607	0.63	0.28	0.2671	0.67	0.24	0.2055
Alcoa	-0.26	1.24	0.2944	-0.24	1.23	0.3212	-0.18	1.17	0.4461	-0.18	1.18	0.4087	-0.03	1.04	0.7865

This table presents the parameter estimates and p-values of the Mincer-Zarnowitz test of forecasting bias for each company for autoregressive models with in-sample estimation size of 60 periods (half-years). Columns α and γ show the coefficients from M-Z regression with bold values indicating rejection of the null of individual tests $\boldsymbol{\alpha} = 0$ and $\boldsymbol{\gamma} = 1$ at the 5% two-tailed significance level. The columns $p - val$ show the p-values of the joint hypothesis test of systematic bias in the forecast $H_0 : \alpha = 0, \gamma = 1$ with bold values highlighting rejection of the null at the 5% two-tailed significance level. The out of sample evaluation starts from 1984:2 to 2011:2 with a total of 55 forecasting periods.

Table 8: **Forecasting Bias (M-Z test) - One-Year-Ahead Forecasts Fama-MacBeth and Constant Models**

	FM			12M			18M			24M			48M			60M		
Company	α	γ	***p-val***	α	γ	***p-val***	α	γ	***p-val***	α	γ	***p-val***	α	γ	***p-val***	α	γ	***p-val***
Coca Cola	**0.92**	**-0.13**	**0.0011**	0.19	**0.75**	0.1223	0.19	0.74	0.1421	0.22	0.71	0.1426	0.28	0.62	0.1100	0.26	0.64	0.1216
Du Pont	0.37	**0.62**	**0.0026**	**0.67**	**0.39**	**0.0205**	**0.53**	0.53	0.0819	**0.59**	**0.47**	0.1004	0.49	0.57	0.2807	0.52	0.54	0.2466
Exxon	0.28	0.91	**0.0007**	**0.48**	**0.42**	**0.0226**	**0.36**	**0.56**	0.1156	**0.40**	**0.51**	0.1062	**0.54**	**0.33**	0.0742	**0.63**	**0.23**	0.0645
GE	**1.03**	**0.11**	**0.0002**	**0.58**	**0.49**	**0.0355**	**0.67**	**0.42**	0.0836	**0.81**	**0.30**	**0.0277**	**1.50**	**-0.30**	**0.0090**	**1.82**	**-0.57**	**0.0072**
IBM	**0.82**	**0.14**	**0.0000**	**0.61**	**0.37**	**0.0117**	**0.51**	**0.47**	0.0599	0.43	**0.55**	0.1576	0.42	0.55	0.1760	0.49	0.48	0.1162
Chevron	0.04	1.13	0.0561	**0.35**	**0.57**	0.0769	0.30	0.64	0.1779	0.30	0.64	0.2263	0.36	0.57	0.2281	0.40	0.51	0.2092
United Tech	**1.15**	**-0.10**	**0.0000**	**0.69**	**0.33**	**0.0113**	**0.66**	**0.36**	**0.0175**	**0.68**	**0.34**	**0.0278**	**0.73**	**0.30**	**0.0173**	**0.79**	**0.24**	**0.0106**
P&G	**0.54**	**0.38**	**0.0000**	**0.33**	**0.58**	0.0609	**0.34**	**0.56**	**0.0341**	0.29	**0.61**	0.1070	0.25	0.65	0.1344	0.28	0.61	0.1094
Caterpillar	0.49	**0.57**	0.0639	**0.59**	**0.53**	**0.0454**	**0.60**	**0.53**	0.0919	0.55	0.57	0.1507	0.52	0.63	0.1799	0.42	0.72	0.1405
Boeing	**1.24**	**-0.21**	**0.0002**	**0.76**	**0.26**	**0.0055**	**0.95**	**0.07**	**0.0029**	**1.03**	**-0.01**	**0.0044**	**1.66**	**-0.67**	**0.0006**	**1.28**	**-0.28**	**0.0224**
Pfizer	**0.93**	**-0.09**	**0.0000**	**0.59**	**0.31**	**0.0073**	**0.66**	**0.23**	**0.0039**	**0.75**	**0.12**	**0.0055**	**1.44**	**-0.65**	**0.0004**	**1.59**	**-0.83**	**0.0019**
J&J	**0.40**	**0.46**	**0.0105**	**0.30**	**0.59**	0.0555	**0.29**	**0.60**	0.0976	0.25	0.64	0.1567	0.18	0.72	0.2756	0.14	0.76	0.2454
3M	**0.86**	**0.08**	**0.0056**	**0.90**	**0.03**	**0.0011**	**0.77**	**0.18**	**0.0287**	**0.85**	**0.09**	**0.0230**	**1.03**	**-0.11**	**0.0205**	**0.83**	**0.11**	0.0694
Merck	**0.73**	**0.17**	**0.0003**	**0.75**	**0.14**	**0.0011**	**0.78**	**0.10**	**0.0014**	**0.74**	**0.15**	**0.0135**	**1.18**	**-0.36**	**0.0024**	**1.36**	**-0.56**	**0.0018**
Alcoa	0.18	**0.73**	**0.0001**	**0.42**	0.68	0.1437	0.28	0.81	0.3600	0.28	0.81	0.3378	0.15	0.94	0.3411	0.12	0.97	0.2364

This table presents the parameter estimates and p-values of the Mincer-Zarnowitz test of forecasting bias for each company for Fama-MacBeth and constant realized beta models. Columns $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ show the coefficients from M-Z regression with bold values indicating rejection of the null of individual tests $\alpha = 0$ and $\gamma = 1$ at the 5% two-tailed significance level. The columns $p-val$ show the p-values of the joint hypothesis test of systematic bias in the forecast $H_0 : \alpha = 0, \gamma = 1$ with bold values highlighting rejection of the null at the 5% two-tailed significance level. The out of sample evaluation starts from 1987 to 2011 with a total of 25 forecasting periods.

Table 9: **Forecasting Bias (M-Z test) - One-Year-Ahead Forecasts Autoregressive Models**

	AR(1)			**AR(2)**			**AR(3)**			**AR(4)**			**AR(5)**		
Company	α	γ	***p-val***	α	γ	***p-val***	α	γ	***p-val***	α	γ	***p-val***	α	γ	***p-val***
Coca Cola	-0.23	1.24	0.5151	-0.28	1.29	0.3982	-0.19	1.16	0.4040	-0.12	1.09	0.5200	-0.08	1.04	0.6356
Du Pont	-0.19	1.18	0.9376	0.10	0.92	0.8790	0.12	0.90	0.8276	0.46	0.59	0.7551	0.32	0.72	0.8517
Exxon	0.14	0.83	0.9464	0.33	0.61	0.7143	0.50	0.38	0.4618	0.41	0.49	0.5206	0.61	0.25	0.2480
GE	0.05	0.97	0.9075	0.01	1.01	0.8369	-0.01	1.02	0.9438	0.06	0.96	0.9092	0.20	0.84	0.8349
IBM	0.19	0.78	0.5865	0.45	0.52	0.4598	0.82	0.16	0.1693	**1.00**	**-0.03**	0.0942	**1.07**	**-0.10**	**0.0470**
Chevron	0.15	0.79	0.6913	0.19	0.73	0.5092	0.23	0.68	0.3848	0.15	0.75	0.3176	0.21	0.70	0.3987
United Tech	0.83	0.19	0.3002	0.89	0.13	0.3317	0.90	0.13	0.3885	0.90	0.12	0.1697	0.93	0.10	0.1487
P&G	-0.19	1.21	0.8601	0.03	0.94	0.8703	0.36	0.55	0.5036	0.54	0.32	0.1835	**0.51**	**0.35**	0.0932
Caterpillar	0.29	0.85	0.0660	0.36	0.80	**0.0418**	0.48	0.69	**0.0357**	0.49	0.69	**0.0148**	**0.55**	0.66	**0.0029**
Boeing	**1.15**	**-0.13**	**0.0325**	**0.93**	**0.08**	**0.0116**	**0.98**	**0.04**	**0.0047**	**0.88**	**0.12**	**0.0303**	**0.84**	**0.17**	**0.0489**
Pfizer	0.45	0.45	0.2102	0.31	0.59	0.1546	0.33	0.57	0.1198	0.32	0.58	0.0935	0.36	0.55	0.1115
J&J	-0.08	1.01	0.3085	0.16	0.71	0.2233	0.07	0.84	0.4541	0.16	0.74	0.3943	0.24	0.64	0.2918
3M	0.76	0.18	0.5377	0.92	0.02	0.3806	0.90	0.04	0.4038	**1.24**	**-0.33**	0.1502	**1.25**	**-0.33**	**0.0376**
Merck	**1.25**	**-0.43**	0.0832	**1.38**	**-0.58**	0.0554	**1.26**	**-0.44**	**0.0300**	**1.29**	**-0.47**	**0.0082**	**1.29**	**-0.47**	**0.0027**
Alcoa	-0.21	1.23	0.4332	-0.04	1.10	0.5364	0.13	0.94	0.6887	0.24	0.85	0.5602	0.39	0.73	0.3961

This table presents the parameter estimates and p-values of the Mincer-Zarnowitz test of forecasting bias for each company for autoregressive models. Columns α and γ show the coefficients from M-Z regression with bold values indicating rejection of the null of individual tests $\boldsymbol{\alpha} = 0$ and $\boldsymbol{\gamma} = 1$ at the 5% two-tailed significance level. The columns $p-val$ show the p-values of the joint hypothesis test of systematic bias in the forecast $H_0 : \alpha = 0, \gamma = 1$ with bold values highlighting rejection of the null at the 5% two-tailed significance level. The out of sample evaluation starts from 1987 to 2011 with a total of 25 forecasting periods.

Table 10: **Summary Statistics of Net Present Value Calculations**

(a) One Year Betas

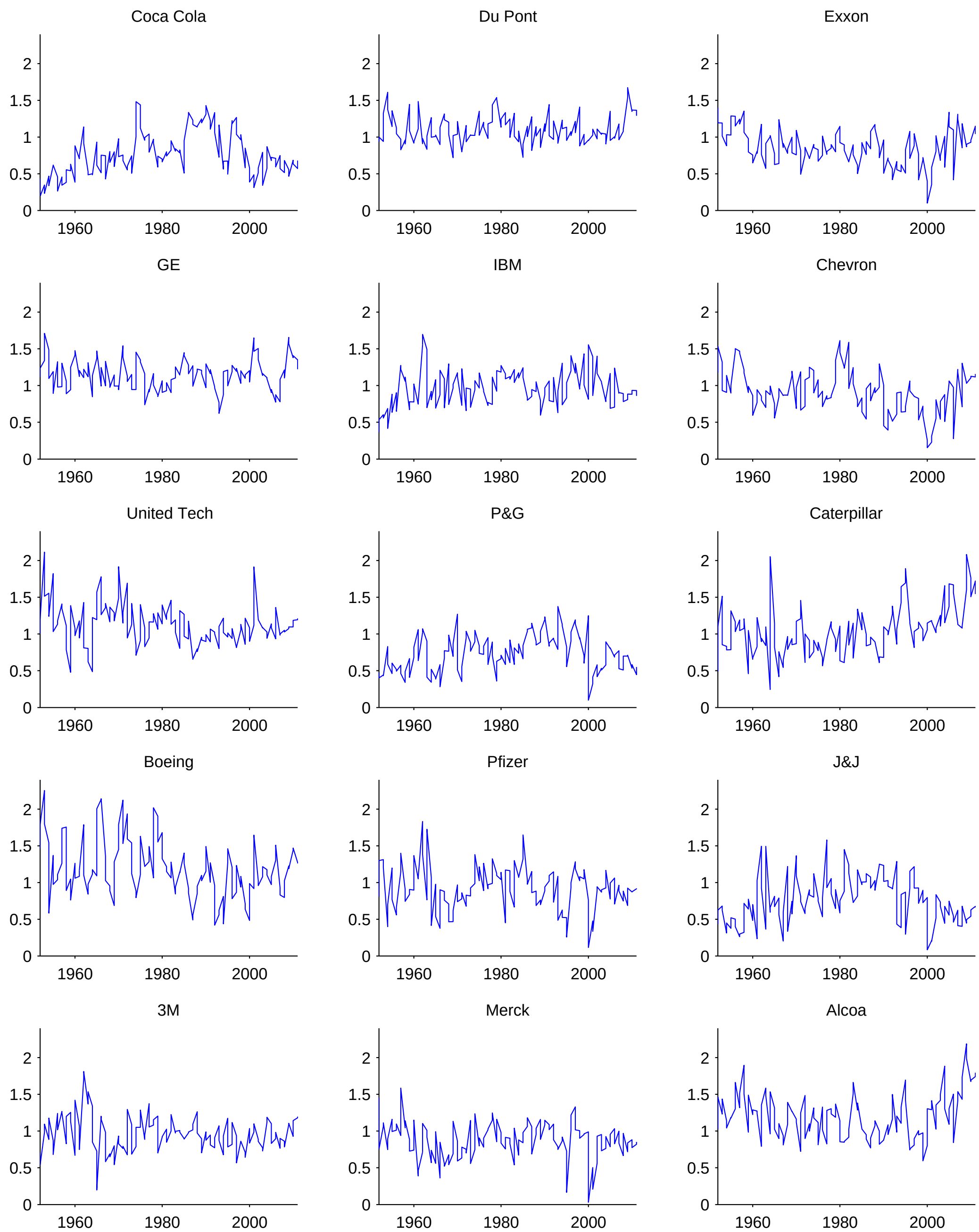
	5-year project A		**5-year project B**		**5-year project C**		**Perpetuity**	
	AR(1)	**FM**	**AR(1)**	**FM**	**AR(1)**	**FM**	**AR(1)**	**FM**
Mean	4,205	4,237	4,776	4,799	3,634	3,674	-961	-358
Std	301	612	245	500	356	725	1,201	3,011
Max	5,252	6,062	5,622	6,265	4,883	5,858	5,200	19,642
Min	2,815	2,406	3,628	3,283	2,003	1,529	-4,582	-5,211
25%	4,004	3,842	4,612	4,480	3,396	3,205	-1,792	-2,294
75%	4,386	4,640	4,923	5,130	3,848	4,151	-361	850

(b) 6 Month Betas

	5-year project A			**5-year project B**			**5-year project C**			**Perpetuity**		
	AR(1)-40p	**AR(1)-60p**	**FM**	**AR(1)-40p**	**AR(1)-60p**	**FM**	**AR(1)-40p**	**AR(1)-60p**	**FM**	**AR(1)-40p**	**AR(1)-60p**	**FM**
Mean	4,208	4,210	4,227	4,778	4,779	4,791	3,638	3,640	3,662	-917	-899	-423
Std	334	345	608	272	282	496	395	409	720	1,355	1,415	2,881
Max	5,375	5,382	6,062	5,720	5,726	6,265	5,030	5,039	5,858	6,462	6,547	19,642
Min	2,790	2,783	2,406	3,606	3,600	3,283	1,973	1,965	1,529	-4,625	-4,636	-5,211
25%	3,999	4,015	3,850	4,609	4,621	4,486	3,390	3,409	3,214	-1,806	-1,755	-2,271
75%	4,388	4,390	4,641	4,925	4,927	5,130	3,851	3,854	4,152	-350	-340	854

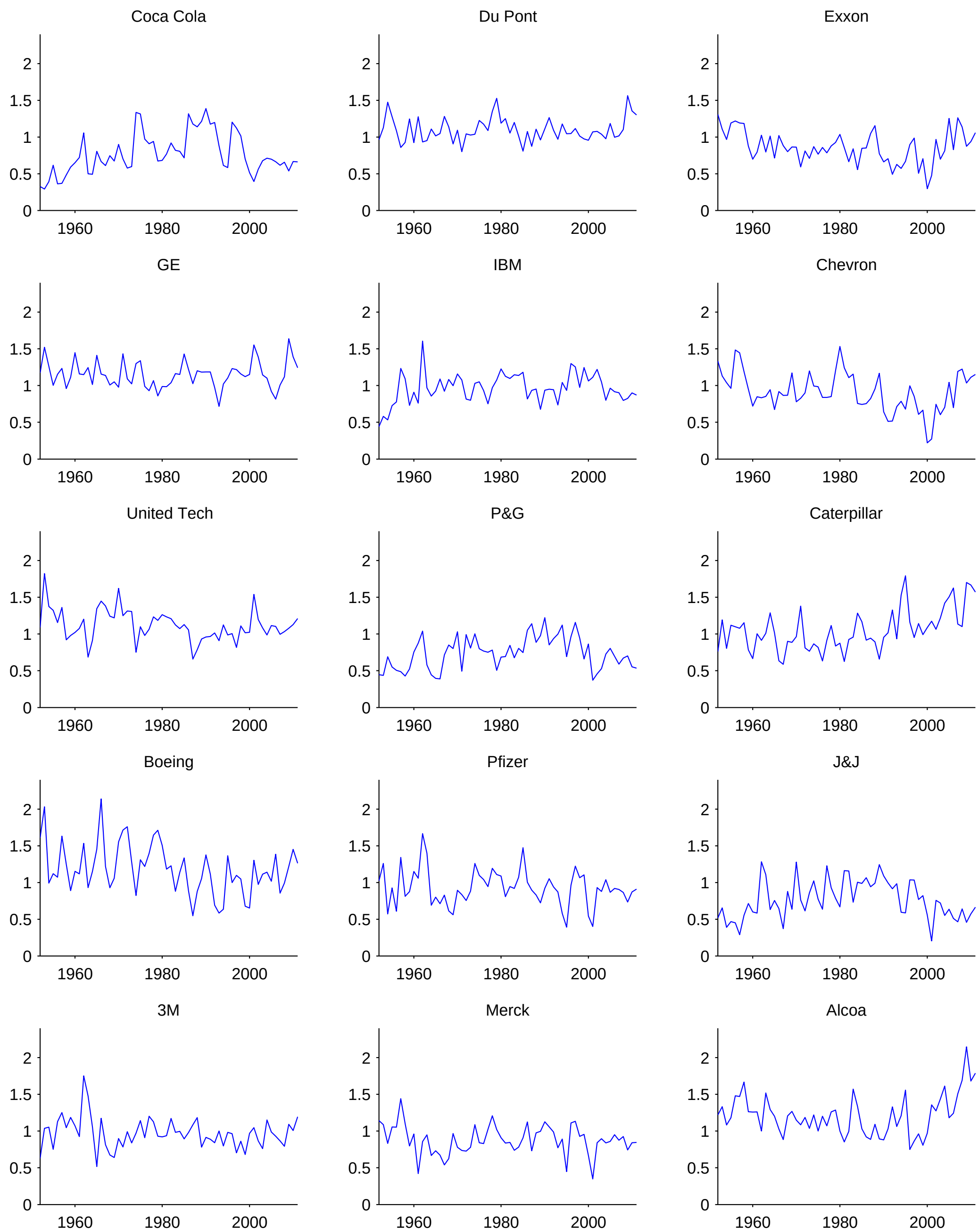
This table presents the summary statistics for net present values for four sample investment projects for all 25 (55) out-of-sample forecasting periods and all companies. Net present values are calculated with a discount rate given by the CAPM with beta being the one year (6 month) forecast value of AR(1) and FM betas, a risk free rate of 2% per annum and a market risk premium of 5.5%.

Figure 1: **Time-series of 6-month realized betas**



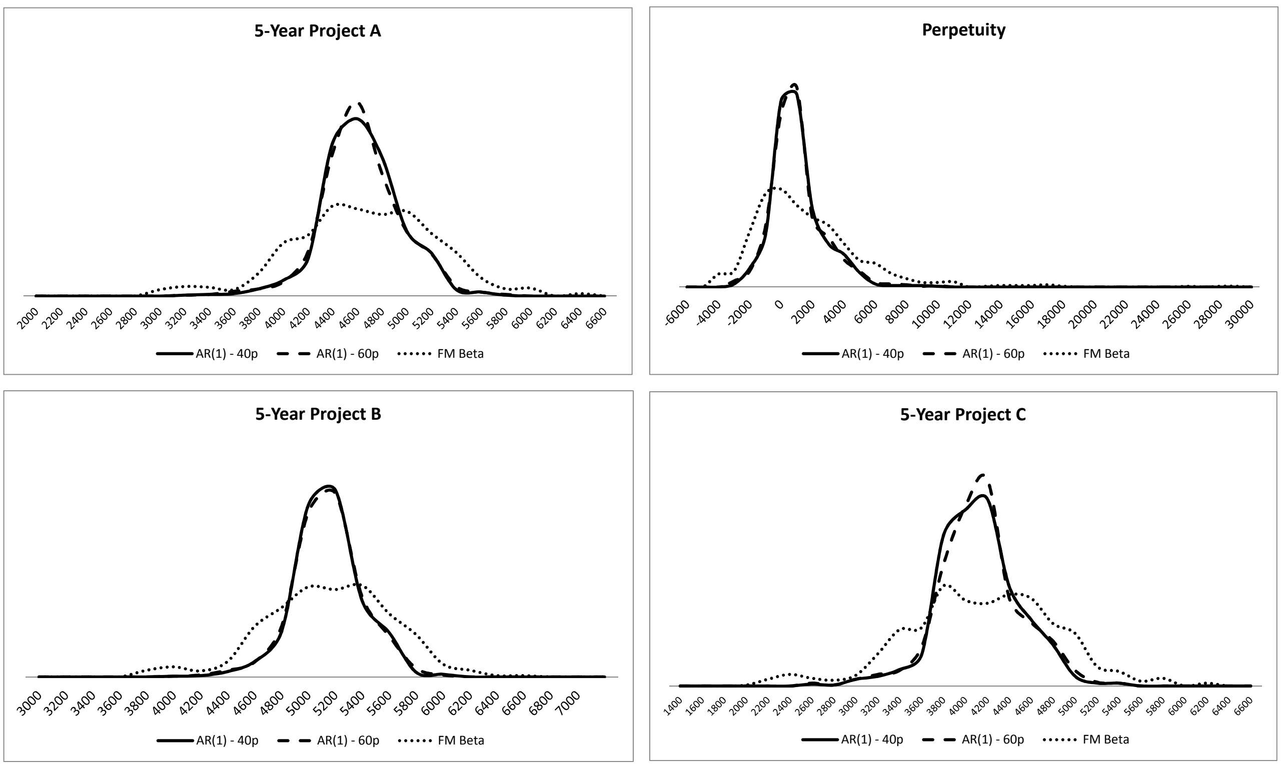
This figure shows the time-series of realized betas calculated every half-year from daily returns data. The betas are calculated at the end of the last trading day of the month of June and the last trading day of the year for each stock. Each individual time series is comprised of 120 data points from 1952:1 to 2011:2.

Figure 2: **Time-series of one year realized betas**



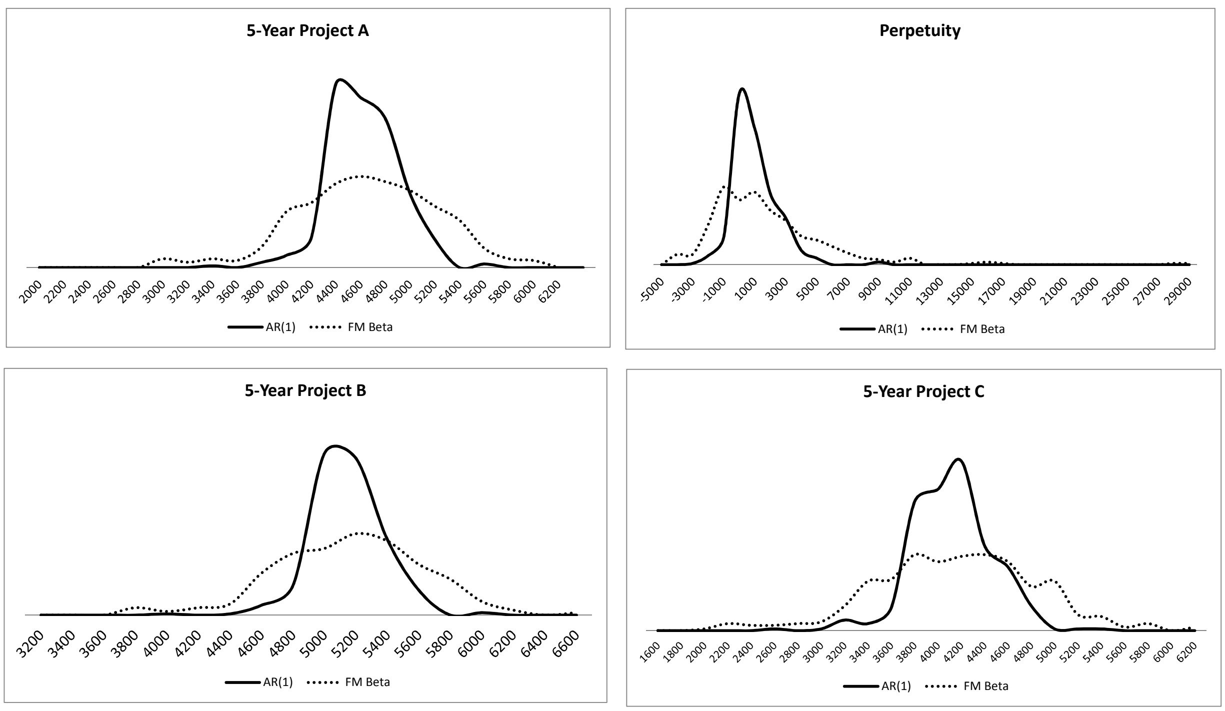
This figure shows the time-series of realized betas calculated every year from daily returns data. The betas are calculated at the end of the last trading day of the year for each stock. Each individual time series is comprised of 60 data points starting from 1952 to 2011.

Figure 3: **Net Present Value Distributions of Four Sample Investments - 6 Month Ahead Forecasts**



This figure shows the 6-month betas frequency distribution of net present values of four sample investment projects, calculated for all companies for all 55 out-of-sample forecasting periods. Net present values are calculated with a discount rate given by the CAPM with beta being the forecast value of AR(1) and FM betas, a risk free rate of 2% per annum and a market risk premium of 5.5%.

Figure 4: **Net Present Value Distributions of Four Sample Investments - One Year Ahead Forecasts**



This figure shows the one year betas frequency distribution of net present values of four sample investment projects, calculated for all companies for all 25 out-of-sample forecasting periods. Net present values are calculated with a discount rate given by the CAPM with beta being the forecast value of AR(1) and FM betas, a risk free rate of 2% per annum and a market risk premium of 5.5%.