

CAPM, Components of Beta and the Cross Section of Expected Returns

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Abstract

This paper demonstrates that a conditional version of the Capital Asset Pricing Model (CAPM) explains the cross section of expected returns, just as well as the three factor model of Fama and French. This is achieved by measuring beta (systematic risk) with short-, medium- and long-run components. The short-run component of beta is computed from daily returns over the prior year, while the medium-run beta component is from daily returns over the prior 5 years, and the long-run component from monthly returns over the prior 10 years.

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1 Introduction

It is well known that the unconditional (or static) version of CAPM fails to account for the cross-sectional variation in expected returns, especially expected returns on portfolios based on firm characteristics. However, it is still possible, at least theoretically, that a conditional version of CAPM with time-varying betas explains the cross-sectional variation in expected returns (see Jagannathan and Wang (1996)). Empirically, the success of such an approach depends critically on successfully capturing the time-variation of conditional betas through an appropriate econometric specification.

In this paper, we model the conditional beta of an asset as a weighted average of short-, medium- and long-run betas estimated over different periods using different frequency data. This modification to the conditional CAPM is based on assuming that agents form expectations of risk on information from varying time horizons. We then analyze whether the cross-section of expected returns can be explained by conditional CAPM with such a component structure for the conditional beta, which is still a single factor model where the only factor is the return on the market portfolio. The main finding of this paper is that this relatively parsimonious econometric specification for beta, leads to validation of the conditional CAPM.

Our approach is motivated by two empirical observations. First, there is growing empirical evidence that both volatilities and correlations in equity markets have more than one component. For example, Rangel and Engle (2012) and Engle and Rangel (2010) provide empirical evidence that a model with low and high frequency volatility and correlation components captures the dynamics of returns in equity markets better than a single component model. This, in turn, suggests that betas themselves might have more than one component. Second, there are numerous studies on the relationship of beta with a variety of firm characteristics and macroeconomic variables, see e.g., Hamada (1972), Rubinstein (1973), Shanken (1990), Ferson and Harvey (1993, 1999), Berk, Green, and Naik (1999) and Andersen, Bollerslev, Diebold, and Wu (2005), which tend to move over time at different frequencies, with relatively slowly changing macroeconomic variables associated with longer run periods. This in turn suggests that a single time-varying beta estimated based on a single data window and frequency might not be able to capture changes in risk which

happen at different frequencies.

From a theoretical point of view, the main motivation for a component structure in conditional betas follows from the structure of information flow in financial markets. For example, Andersen and Bollerslev (1997) and Calvet and Fisher (2007) show that information in financial markets does not only arrive at various frequencies but also has different degrees of persistence and, thus, affects the dynamics of returns differently at different frequencies. A component structure in conditional betas is also related to the heterogeneity across investors in financial markets, especially with respect to their investment horizons. For instance, Brennan and Zhang (2013) generalize CAPM to a framework where investors have stochastic investment horizons or portfolio liquidation dates and show that expected returns are determined by a weighted sum of betas times the market risk premium for different horizons. Our three component beta model can be considered as a nonparametric approach to capture the structure of information flow and heterogeneity across investors in measuring conditional betas.

We show that the conditional CAPM with our three component beta model can account for most of the cross-sectional variation in expected returns. Specifically, we analyze the performance of our three component beta model relative to the three factor model of Fama and French (1993, 1996) with constant factor loadings in accounting for the cross-sectional variation in expected monthly returns on the 25 size and book-to-market cross sorted portfolios. To do this, for each month, we first estimate the short and medium-term beta components using daily data over the previous one and five year periods, respectively, and the long-run beta component using monthly data over the previous ten year period. This leads to a decomposition of the market risk premium into its short-, medium- and long-run components. We then calculate the average pricing error as the average of the residuals from the estimation of the cross-sectional regression in each month as in Fama and MacBeth (1973). Differently from our three component beta model, the factor loadings on the three Fama-French factors are estimated only once based on the whole sample period. In this framework, our three component beta model outperforms the Fama-French three factor model. Specifically, it does not only achieve lower overall pricing errors as measured by sum of squared pricing errors (SSPE) but also for each size and book-to-market quintile. Furthermore, it only fails to account for the expected return on the small-growth portfolio, which is known to

be the most difficult portfolio to correctly price, compared to three mispriced assets under the Fama-French three factor model.

To understand the intuition behind the empirical success of this three component beta model relative to the Fama-French three factor model, we decompose its overall performance by analyzing the performances of its components. Specifically, we first analyze the performance of one component beta models, which are simply CAPM with time-varying betas estimated over different periods using different frequency data. This allows us to analyze the explanatory power of each component separately in accounting for the cross-sectional variation in expected returns. The one component beta model with the medium-term beta performs better than the one with the short-term beta which is in turn better than the one with the long-term beta. These results suggest that the overall performance of our three component beta model is mostly due to short and medium term betas with the long-term beta contributing only slightly. We then turn our attention to the two component beta models. Comparing two and three component models allows us to analyze the pure contribution of a specific beta to the overall performance of the three component beta model while controlling for relatively moderate levels of correlations between beta components. Specifically, we calculate the percentage decrease in the SSPE due to the inclusion of a specific component. This percentage decrease in the SSPE can be interpreted as a partial R^2 since it is the change in the variation explained by the part of that component orthogonal to the other two components. The short and medium term beta components decrease the overall SSPE by 37% and 48%, respectively, while the long-term beta component decreases it by 18%. Once again, these results suggest that the overall performance of our three component beta model is mostly due to short and medium term betas with the long-term beta contributing only slightly. However, these results do not necessarily imply that betas based on low frequency data have no use. As we discuss below, they capture a different dimension of the time variation in betas that cannot be captured by the short and medium run beta components.

To understand the economic intuition behind the relative success of the three component beta model, we analyze the relation between the components of beta and the determinants of risk, such as economic conditions and portfolio characteristics. To this end, we analyze the correlations between annual changes in each beta component and the lagged annual change in the book-to-market

ratio of the portfolio as well as the Treasury bill rate at the beginning of the year. The lagged annual change in the book-to-market ratio of the portfolio captures the change in one of the important portfolio characteristics. The short-run beta components of portfolios, especially those with high book-to-market ratios, have highly significant positive correlations with the lagged annual changes in their book-to-market ratios. Overall, there is a pattern of increasing correlations as the book-to-market ratio of the portfolio increases, demonstrating that an increase in the portfolio's risk characteristics due to its book-to-market ratio, often results in significant increases in the short-run beta component. The medium- and long-run beta components have mostly insignificant correlations, showing that these beta components are less sensitive to the more fast moving risk dynamics of assets. On the other hand, the Treasury bill rate at the beginning of a year is chosen to measure the economic conditions over the next 12 months. The annual changes in medium- and long-term beta components are significantly negatively correlated with the Treasury bill rate, especially for portfolios with high book-to-market ratios. This suggests that in recessions when the Treasury bill rate is low, the medium- and long-run beta components for portfolios with high book-to-market ratios, tend to rise over the year. Furthermore, the short-run beta components do not seem to be significantly correlated with the Treasury bill rate suggesting that the dynamics of the short-run beta components are not dominated by the business cycle. These results suggest that the impact of the business cycle on asset returns in our three component beta model is primarily captured through the medium- and long-run beta components. This is in line with Adrian and Rosenberg (2008) who also find that the business cycle is correlated with a long-run component of risk in their factor pricing model containing a short- and long-run volatility component. Overall, these results suggest that the more immediate changes in risk such as changes in portfolio characteristics are captured in the short-run beta component while the medium- and long-run beta components capture more slowly changing risk which we find to be correlated with the business cycle. In addition, we also analyze the performance of our three component beta model over phases of the business cycle and find that it has similar pricing errors to the Fama-French three factor model during expansions, and lower pricing errors during recessions. This in turn suggests that the empirical success of our three component beta model relative to the Fama-French three factor is due to its success in capturing time-variation in risk of portfolios in recessions.

Mindful of the Lewellen, Nagel, and Shanken (2010) critique of standard empirical methods used in the asset pricing literature and other potential problems, we perform a number of robustness checks. First, we should note that betas estimated with different windows and/or frequencies of data are, not surprisingly, correlated. However, these correlations are at relatively moderate levels suggesting that the relative performance of our three component models are not due to correlations between different beta components. Second, we conduct simulation experiments where we demonstrate that the success of our three component beta models, is unlikely to have been generated by chance. Third, we extend the number of test portfolios in our analysis to include momentum-sorted portfolios and industry portfolios. In this setting our three component beta model performs similar to the Fama-French three factor model. Fourth, most of our study is focused on analyzing the performance of the models in explaining the cross-sectional variation in monthly returns, though as part of our robustness checks, we also analyze the performance of the models in explaining the cross-sectional variation in quarterly returns where we again find that our three component beta model to perform just as well as the Fama-French three factor model. In addition, our three component beta model performs better than the Fama-French three factor model over different sample periods. Finally, our three component beta model continues to perform as well as the Fama-French three factor model when we control for nonsynchronous trading in estimating the short and medium-run beta components.

A number of empirical approaches for beta measurement have been proposed in the literature. The most common approach is to estimate betas based on a rolling window of monthly observations over the last 5 years as in Fama and MacBeth (1973). One can also model betas as linear functions of instruments observed at low frequencies as in Harvey (1989), Shanken (1990), Ferson and Harvey (1991, 1993, 1999), Cochrane (1996) and Jagannathan and Wang (1996). Another approach, is to use higher frequency data such as daily returns, for realized beta measurement as in Andersen, Bollerslev, Diebold, and Wu (2006), Lewellen and Nagel (2006), Ghysels and Jacquier (2006) and Hooper, Ng, and Reeves (2008). Finally, in contrast to the approaches based on historical data, option-implied beta estimates can be computed using equity and index options, as proposed by Chang, Christoffersen, Jacobs, and Vainberg (2012).

Our three component beta model can be considered as a mixed-frequency approach as the

short- and medium-run beta components are computed from daily returns, whereas, the long-run beta component is computed from monthly returns. It differs from the approaches in the previous literature in several aspects. First of all, most approaches, such as estimating betas based on a rolling window of monthly returns over the last 5 years as in Fama and MacBeth (1973) or estimating realized betas based on daily returns as in Andersen, Bollerslev, Diebold, and Wu (2005, 2006), Lewellen and Nagel (2006), Ghysels and Jacquier (2006) and Hooper, Ng, and Reeves (2008), need to make an explicit ad hoc choice on the estimation window and data frequency in order to estimate conditional betas. Instead, we allow conditional betas to be determined jointly by betas estimated over different periods based on data with different frequency. Secondly, Harvey (2001) shows that the approach based on instruments is relatively sensitive to the set of instruments used. We use a data-driven non-parametric approach to capture time variation in betas rather than parametric and instrument-based approaches such as those in Harvey (1989), Shanken (1990), Ferson and Harvey (1991, 1993, 1999), Cochrane (1996), Jagannathan and Wang (1996), Ang and Chen (2007), Boguth, Carlson, Fisher, and Simutin (2011) and Rangel and Engle (2012). We do not consider more than three components to beta, so to ensure that we have a parsimonious specification. This follows the component volatility literature which has typically either modeled with two components, such as in the Engle and Lee (1999) two component GARCH model, or as three components, such as in the Corsi (2009) three component realized volatility model.

The remainder of the paper is organized as follows. The empirical specification for three component beta models are introduced in Section 2. The benchmark models are discussed briefly in Section 3. Measures utilized to compare the asset pricing performance of different models are presented in Section 4. Test portfolios are described in Section 5. Asset pricing performance is presented in section 6. Further analysis of the performance of our three component beta models is discussed in Section 7. Robustness checks are presented in Section 8. Our conclusion is presented in section 9.

2 Three Component Beta Models

In this section, we first discuss the details of the three component beta model specification. We then discuss the empirical and theoretical motivations behind this specification. Finally, we discuss how we estimate the conditional CAPM based on the three component beta model.

2.1 Specification

Assuming that the hedging motives of risk averse investors in a dynamic economy are negligible, Jagannathan and Wang (1996) define the conditional version of Black (1972) CAPM as:

$$E_{t-1}[R_{i,t}] := E[R_{i,t}|\mathcal{F}_{t-1}] = \lambda_{0,t-1} + \beta_{i,t-1}\lambda_{m,t-1}, \quad (1)$$

where $R_{i,t}$ is the (simple) return on asset i in period t , $\mathcal{F}_{t-1}$ denotes the common information set of investors available at the end of period $t - 1$. The conditional beta of asset i based on information set $\mathcal{F}_{t-1}$, $\beta_{i,t-1}$, is defined as:

$$\beta_{i,t-1} = \frac{cov_{t-1}(R_{i,t}, R_{m,t})}{var_{t-1}(R_{m,t})} \quad (2)$$

where $R_{m,t}$ is the (simple) return on the market portfolio. In this framework, $\lambda_{0,t-1}$ and $\lambda_{m,t-1}$ are the conditional expected return on a “zero-beta” portfolio and the conditional market risk premium, respectively.

As we motivate below in the next sub-section, we argue that conditional beta of asset has a component structure with short-, medium- and long-run components. To capture this component structure, we model the conditional beta of an asset as a weighted average of betas measured over different time periods and frequencies. Specifically, following the notation in Ghysels and Jacquier (2006), let $\beta_{x,y,i,t}$ denote the conditional beta of asset i in period t estimated via OLS using x periods of data up to but not including any data from period t based on returns sampled at y frequency.¹ For example, assuming that t tracks months, $\beta_{6m,d,i,t}$ denotes the conditional beta of asset i in month t estimated using daily data over six months of data prior to period t , i.e. between the beginning of month $t - 6$ and the end of month $t - 1$. Then, our three component beta model

¹We also consider including data from period t when estimating $\beta_{x,y,i,t}$, our results do not change significantly.

is expressed as follows:

$$\hat{\beta}_{i,t} = w_{1,t}\hat{\beta}_{x_1,y_1,i,t} + w_{2,t}\hat{\beta}_{x_2,y_2,i,t} + w_{3,t}\hat{\beta}_{x_3,y_3,i,t} \quad (3)$$

where $w_{j,t}$ for $j = 1, 2, 3$ are the weights of each component.

We choose the pairs of (x_j, y_j) for $j = 1, 2, 3$ to capture the time variation in the beta over different horizons and frequencies. Specifically, we assume that the first component captures the fast moving short term component of beta and consider $(1m, d)$, $(3m, d)$, $(6m, d)$ or $(12m, d)$ for (x_1, y_1) . We consider either $(12m, d)$ or $(60m, d)$ for (x_2, y_2) so that the second component can capture the medium term movements in beta. Finally, we consider $(120m, m)$ as the only specification for (x_3, y_3) that is designed to capture the slow moving long term component of beta. We use 120 months rather than the usual choice of 60 months which is already considered for (x_2, y_2) .

We assume that the weights sum up to one for each asset in each time period, i.e. $\sum_{j=1}^3 w_{j,t} = 1$. This assumption ensures that the conditional beta of the market portfolio is one and the conditional risk premium in this framework is equal to the difference between the expected return on the market portfolio and the risk free rate (or the return on the “zero-beta” portfolio). To see this, note that each beta component for the market portfolio is equal to one by definition, i.e. $\hat{\beta}_{x_j,y_j,i,t}$ for $j = 1, 2, 3$. Hence, the conditional beta of the market portfolio is simply the sum of each component’s weight, which is equal to one by assumption. This in turn implies that $\lambda_{m,t-1} = E_{t-1}[R_{m,t}] - \lambda_{0,t-1}$ given that the conditional CAPM in Equation 1 holds for each asset including the market portfolio.

Several remarks are in order. First of all, the conditional CAPM in Equation 1 combined with the beta specification in Equation 3 still implies a single factor model where the only factor is the return on the market portfolio. However, differently from the previous literature, we assume that the loadings of each asset to this factor, i.e. the market portfolio, has a component structure with short-, medium- and long-run components. The assumption that the weights sum up to one for each asset and time period guarantees that one obtains the same conditional risk premium as the standard conditional CAPM. The unconditional risk premium can then be obtained as the time-

series average of the conditional risk premia, as in the standard Fama-MacBeth approach. Second, it is flexible enough to include common time-varying beta specifications in the literature as special cases. For example, the standard Fama-MacBeth betas can be obtained as a special case of the specification in Equation 3 by considering $(x_3, y_3) = (60m, m)$ and restricting $w_{1,t} = w_{2,t} = 0$ and $w_{3,t} = 1$ for all i and t . Finally, the weights are allowed to change over time with changing economic conditions. For example, the medium-term beta might be more important in determining the systematic risk of an asset, and thus, have a relatively higher weight during recessions.

2.2 Theoretical and Empirical Motivation

The main theoretical motivation behind our three component beta model is based on Andersen and Bollerslev (1997) and Calvet and Fisher (2007). Specifically, Andersen and Bollerslev (1997) find that volatility measured using high frequency data over short time spans is informative about volatility measured using lower frequency data over longer time spans and there are multiple volatility components in high frequency data. They argue that this component structure of volatility along with its long-memory characteristics is consistent with the fact that different types of information in financial markets tend to have different degrees of persistence. Similarly, Calvet and Fisher (2007) also argue that prices in equity markets are driven by news with different degrees of persistence, ranging from intraday to decades. They view corporate news, thus changes in firm characteristics, as high frequency information affecting the short-run dynamics of returns. On the other hand, they view macroeconomic variables as lower frequency information affecting long-run dynamics of returns. They then develop a framework based on Markov-switching multifractal models to capture news that arrive at different frequencies.

These papers suggest that different types of information in financial markets not only arrive at different frequencies but also have different degrees of persistence and, thus, affect the dynamics of returns at different frequencies differently. Motivated by this view of information flow in financial markets, we model the systematic risk of an asset as a weighted average of betas measured over different time periods and frequencies. This approach allows us to capture information arriving at different frequencies with different degrees of persistence in a rather nonparametric and data-

driven fashion.

Although we consider the above-mentioned papers as our main theoretical motivations, our three component beta model can also be motivated by the heterogeneity across investors in financial markets, specifically with respect to their investment horizons. For example, Brennan and Zhang (2013) generalize CAPM to a framework where investors have stochastic investment horizons or portfolio liquidation dates. They argue that this framework captures the fact that investors do not trade continuously as a result of transaction costs, but instead have some trading horizon that is unknown both to them and to the econometrician. In this framework, they show that the sum of expected excess returns for different horizons weighted by the marginal utility of wealth weighted probability can be expressed as the same probability weighted sum of betas times the market risk premium for each horizon. This implication of their framework is similar to our three component beta model in Equation 3 where the beta and, thus, expected excess return of an asset is a weighted average of different betas measured over different horizons and frequencies. Corsi (2009) also motivates his cascade model for realized volatility with a similar argument based on the heterogeneity of investment horizon across traders, from dealers, market makers, and intraday speculators with short horizons to institutional investors with relatively longer horizons. He argues that the existence of investors different time horizons results in a component structure in realized volatility. Similarly, Kamara, Korajczyk, Lou, and Sadka (2013) also argue that relevant asset risk depends on investment horizon and find that different risk factors are priced at different horizons.

Empirically, our three component beta model is similar in essence to component models for volatilities and correlations in financial markets. For example, Rangel and Engle (2012) and Engle and Rangel (2010) show that a model with low and high frequency volatility and correlation components capture the dynamics of returns in equity markets better than a single component model. Similarly, Corsi (2009) shows that a model with three components measured over different time horizons captures the dynamics of volatility better than models with a single component. Our model is also related to a growing literature using mixed frequency data in modeling financial market data, especially volatility. For example, based on the mixed data sampling (MIDAS) approach of Ghysels, Santa-Clara, and Valkanov (2002), Engle, Ghysels, and Sohn (2013) model stock market volatility with with long- and short-run components where the long-term component

is driven by macroeconomic variables. They find that this two component GARCH-MIDAS model performs better in capturing the relation between stock market volatility and macroeconomic variables. These studies among others point to a component structure in volatilities and correlations, which in turn suggests that betas should also have component structures, with at least two and possibly three components. The specification in Equation 3 captures this component structure in betas in a parsimonious and nonparametric way by modeling betas as weighted averages of short-, medium-, and long-run components measured based on different horizons and data frequencies.

2.3 Estimation

Plugging the beta specification of Equation 3 in the conditional CAPM yields:

$$\begin{aligned} E_{t-1}[R_{i,t}] &= \lambda_{0,t-1} + \lambda_{m,t-1}(w_{1,t}\hat{\beta}_{x_1,y_1,i,t} + w_{2,t}\hat{\beta}_{x_2,y_2,i,t} + w_{3,t}\hat{\beta}_{x_3,y_3,i,t}) \\ &= \lambda_{0,t-1} + \lambda_{m,t-1}w_{1,t}\hat{\beta}_{x_1,y_1,i,t} + \lambda_{m,t-1}w_{2,t}\hat{\beta}_{x_2,y_2,i,t} + \lambda_{m,t-1}w_{3,t}\hat{\beta}_{x_3,y_3,i,t} \end{aligned} \quad (4)$$

Equation 4 forms the basis of our empirical analysis. Specifically, we analyze whether the conditional CAPM specification in Equation 4 can account for the cross-sectional variation in expected returns. We do this based on the two pass regression approach of Fama and MacBeth (1973). To this end, we first obtain all three components of beta via OLS for each asset and time period as discussed above. We then run the following cross-sectional regression every time period:

$$R_{i,t} = b_{0,t} + b_{1,t}\hat{\beta}_{x_1,y_1,i,t} + b_{2,t}\hat{\beta}_{x_2,y_2,i,t} + b_{3,t}\hat{\beta}_{x_3,y_3,i,t} + \alpha_{i,t} \quad (5)$$

where $\alpha_{i,t}$ is the pricing error of asset i in period t with respect to the conditional CAPM with the three component beta specification.

As mentioned above, the conditional CAPM specification in Equation 4 is still a single factor model where the only factor is the return on the market portfolio. Thus, there is only one risk premium and the assumption that the component weights sum up to one guarantees that this risk premium is the market risk premium, i.e. the expected return on the market portfolio in excess of the risk free rate. The estimates of $b_{j,t}$ cannot be interpreted as the conditional risk premia on

different factors but they can rather be interpreted as the decomposition of the market risk premium into its short-, medium- and long-run components. Unconditional estimation is computed from the time-series average of $\hat{b}_{j,t}$.

In this paper, we are mainly interested in the average pricing errors, which can be simply obtained as the time series sample averages of $\alpha_{i,t}$ as:

$$\bar{\hat{\alpha}} = \frac{1}{T} \sum_{t=1}^T \hat{\alpha}_t \quad (6)$$

where $\hat{\alpha}_t = [\hat{\alpha}_{1,t}, \hat{\alpha}_{2,t}, \dots, \hat{\alpha}_{N,t}]'$ is the vector of estimated pricing errors. The variance-covariance matrix of $\bar{\hat{\alpha}}$, $cov(\bar{\hat{\alpha}})$ and its version corrected for autocorrelation, $\widetilde{cov}(\bar{\hat{\alpha}})$, can then be obtained as:

$$cov(\bar{\hat{\alpha}}) = \frac{1}{T^2} \sum_{t=1}^T (\hat{\alpha}_t - \bar{\hat{\alpha}})(\hat{\alpha}_t - \bar{\hat{\alpha}})' \quad (7)$$

$$\widetilde{cov}(\bar{\hat{\alpha}}) = \frac{1}{T^2} \sum_{t=1}^T (\hat{\alpha}_t - \bar{\hat{\alpha}})(\hat{\alpha}_t - \bar{\hat{\alpha}})' + \frac{1}{T^2} \sum_{j=1}^q \sum_{t=j+1}^T \left(1 - \frac{j}{q+1}\right) (\hat{\alpha}_t - \bar{\hat{\alpha}})(\hat{\alpha}_{t-j} - \bar{\hat{\alpha}})' \quad (8)$$

where we set $q = \lfloor (4(T/100)^{2/9}) \rfloor$ and $\lfloor x \rfloor$ denotes largest integer not greater than x .

3 Benchmark Models

In this paper, we consider the three factor model of Fama and French (1993, 1996) with constant factor loadings as the main benchmark model. For completeness, we also present results for the unconditional version of CAPM with constant betas.

We consider three different approaches to estimate the benchmark models. One obtains numerically identical average pricing errors from these three approaches when the betas are assumed constant (see Cochrane (2001)). These different approaches allow us to correct the standard errors of average pricing errors for different econometric problems, such as possible conditional heteroskedasticity of errors from the time series regressions and the well known errors in variables problem in the cross-sectional regressions due to the fact that betas are not known but estimated instead. Although it is possible to correct the standard errors for these econometric problems in a GMM framework when betas are assumed constant as discussed in Cochrane (2001), it is relatively

difficult to do so when betas are assumed time-varying as in our three component beta models. Here, we follow Cochrane's (2001) suggestion of comparing the standard errors from these different approaches for the benchmark models with constant factor loadings. This comparison would provide us some intuition about the potential effect that econometric problems might have on our statistical results in our three component beta models where betas are assumed time-varying. For example, large differences between the standard errors from these three different estimations of benchmark models would be a warning sign for potential problems in the standard errors of our three-component beta models.

In all these three approaches, we assume that the betas are constant and estimate them only once via OLS for each asset separately based on the following time-series regression using the full sample of time-series observations:

$$R_{i,t} = a_i + \beta_i' \mathbf{f}_t + \varepsilon_{i,t} \quad (9)$$

where $\mathbf{f}_t$ is the $K \times 1$ vector of factors in period t and β_i is the $K \times 1$ vector of factor loadings for asset i .

In the first approach, similar to the one discussed above, we estimate the following cross-sectional regression in each period:

$$\mathbf{R}_t = \hat{\beta} \boldsymbol{\lambda}_t + \boldsymbol{\alpha}_t \quad (10)$$

where $\hat{\beta} = [1 \quad \hat{\beta}'_1; \dots; 1 \quad \hat{\beta}'_N]$ is a $N \times (K + 1)$ matrix that includes a $N \times 1$ vector ones in its first column and estimated betas in other columns. $\boldsymbol{\lambda}_t = [\lambda_{0,t} \lambda_{1,t} \dots \lambda_{K,t}]'$ is a $(K + 1) \times 1$ vector where $\lambda_{0,t}$ is the conditional zero-beta rate and $\lambda_{k,t}$ is the conditional risk premium on the k^{th} factor in period t . The average pricing errors can be obtained as the sample averages of the pricing errors, as in Equation 6, from the estimation of the cross-sectional regression in each period. The uncorrected and corrected covariance matrices of the average pricing errors can then be estimated based on Equations (7) and (8), respectively.

In the second approach, we obtain the average pricing errors as the residuals from the estimation of a single cross-sectional regression of expected returns (calculated as the average returns

over the whole sample) on factor loadings via OLS:

$$\bar{\mathbf{R}} = \hat{\boldsymbol{\beta}}\boldsymbol{\lambda} + \boldsymbol{\alpha} \quad (11)$$

where $\bar{\mathbf{R}} = 1/T \sum_{t=1}^T \mathbf{R}_t$ is the $N \times 1$ vector of expected returns; $\hat{\boldsymbol{\beta}}$ and $\boldsymbol{\lambda}$ are as defined above.

The uncorrected covariance matrix of average pricing errors are then given by

$$\text{cov}(\bar{\boldsymbol{\alpha}}) = \frac{1}{T}(\mathbf{I}_N - \hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}})^{-1}\hat{\boldsymbol{\beta}}')\hat{\boldsymbol{\Sigma}}(\mathbf{I}_N - \hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}})^{-1}\hat{\boldsymbol{\beta}}') \quad (12)$$

where $\hat{\boldsymbol{\Sigma}} = 1/T \sum_{t=1}^T \hat{\boldsymbol{\epsilon}}_t \hat{\boldsymbol{\epsilon}}_t'$ is an estimate of the covariance matrix of the vector of residuals from the time-series regressions of asset returns on market returns, $\hat{\boldsymbol{\epsilon}}_t$. The covariance matrix of average pricing errors corrected for the errors in variable problem à la Shanken (1992) can then be estimated as:

$$\widetilde{\text{cov}}(\bar{\boldsymbol{\alpha}}) = \frac{1}{T}(\mathbf{I}_N - \hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}})^{-1}\hat{\boldsymbol{\beta}}')\hat{\boldsymbol{\Sigma}}(\mathbf{I}_N - \hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}})^{-1}\hat{\boldsymbol{\beta}}')(1 + \hat{\boldsymbol{\lambda}}'\hat{\boldsymbol{\Sigma}}_f\hat{\boldsymbol{\lambda}}) \quad (13)$$

where $\hat{\boldsymbol{\Sigma}}_f$ is the covariance matrix of factors.

In the third approach, we estimate the time-series and cross-sectional regressions simultaneously in a GMM framework with OLS factor loadings serving as the weighting matrix as in Section 12.2 of Cochrane (2001). Specifically, let $g(\boldsymbol{\theta})$ denote the moment conditions implied by the time-series and cross-sectional regressions:

$$\mathbf{g}(\boldsymbol{\theta}) = \begin{bmatrix} E[\mathbf{R}_t - \mathbf{a} - \boldsymbol{\beta}'\mathbf{f}_t] \\ E[(\mathbf{R}_t - \mathbf{a} - \boldsymbol{\beta}'\mathbf{f}_t) \otimes \mathbf{f}_t] \\ E[\mathbf{R}_t - \boldsymbol{\beta}\boldsymbol{\lambda}] \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (14)$$

where $\otimes$ denotes Kronecker product, $\boldsymbol{\theta} = [\mathbf{a}'\text{vec}(\boldsymbol{\beta})'\boldsymbol{\lambda}']'$ is the vector of parameters and vec is the vector operator that creates a column vector from a matrix by stacking its column vectors. Let $\mathbf{g}_T(\boldsymbol{\theta})$ denote the sample analogs of the moment conditions. The GMM estimate of $\boldsymbol{\theta}$ is then defined as the set of parameters that set some linear combination of sample means of the moment conditions, $\text{cg}_T(\boldsymbol{\theta})$, to zero. To obtain the same estimates as the ones from the first and second

approaches, we use the following weighting matrix:

$$\mathbf{c} = \begin{bmatrix} \mathbf{I}_{N(K+1)} & \mathbf{0} \\ \mathbf{0} & \hat{\boldsymbol{\beta}}' \end{bmatrix} \quad (15)$$

where $\hat{\boldsymbol{\beta}}$ is the matrix of OLS factor loadings that also includes a vector of ones as its first column as defined above. The pricing errors can then be obtained as the sample analogs of the last N moment conditions evaluated at the estimated parameter values. The covariance matrix of the pricing errors is the part of the covariance matrix of the moment conditions that corresponds to the last N moment conditions. The covariance matrix of the moment conditions is given by

$$\text{cov}(\mathbf{g}_T(\hat{\boldsymbol{\theta}})) = \frac{1}{T}(\mathbf{I}_{N(K+2)} - \mathbf{d}(\mathbf{c}\mathbf{d})^{-1}\mathbf{c})\mathbf{S}(\mathbf{I}_{N(K+2)} - \mathbf{d}(\mathbf{c}\mathbf{d})^{-1}\mathbf{c})' \quad (16)$$

where

$$\mathbf{d} = - \begin{bmatrix} \mathbf{I}_N & (\boldsymbol{\mu}_f \otimes \mathbf{I}_N)' & \mathbf{0}_{N,K+1} \\ \boldsymbol{\mu}_{1,f} \otimes \mathbf{I}_N & \boldsymbol{\mu}_{2,f} \otimes \mathbf{I}_N & \mathbf{0}_{NK,K+1} \\ \mathbf{0}_{N,N} & ([\lambda_1 \dots \lambda_K]' \otimes \mathbf{I}_N)' & \hat{\boldsymbol{\beta}} \end{bmatrix} \quad (17)$$

and $\boldsymbol{\mu}_{1,f} = 1/T \sum_{t=1}^T \mathbf{f}_t$, $\boldsymbol{\mu}_{2,f} = 1/T \sum_{t=1}^T \mathbf{f}_t \mathbf{f}_t'$ and $\mathbf{S}$ is the long-run covariance matrix of moment conditions and can be consistently estimated via the Barlett estimate as in Newey and West (1987).

Cochrane (2001) shows that the uncorrected errors from the first and second approach are identical. Hence, we consider four sets of standard errors when analyzing the statistical significance of the pricing errors of the benchmark models. Comparing the corrected and uncorrected standard errors from the first approach allows us have an idea about the effect of possible autocorrelation on the standard errors of pricing errors. Similarly, comparing the corrected and uncorrected standard errors from the second approach reveals whether an errors in variables problem has an important impact on the statistical significance of the pricing errors. Finally, the standard errors from the third approach are the most general and control for not only the errors in variables problem but also for the possibility that errors from the time series regressions might not be iid, conditionally homoskedastic and independent of the factors. Thus, comparing the uncorrected standard errors from the first approach and those from the third approach might reveal the effect of these potential

econometric problems on the statistical errors of the pricing errors. More importantly, as mentioned above, large differences between these standard errors might signal the unreliability of the standard errors of the average pricing errors from the three component beta models which are not corrected for potential econometric problems except autocorrelation.

4 Performance Measures

To compare the performance of different models in accounting for the cross-sectional variation in returns, we consider four metrics. The first one is the number of mispriced assets at 1% and 5% significance levels based on the variance-covariance matrix of $\bar{\tilde{\alpha}}$ corrected for autocorrelation in Equation 8.² As we discuss below, we also take a closer look at the pricing errors for individual assets, which allow us to analyze which assets are mispriced across different models. The second and third metrics are the sum of square pricing errors (SSPE) and root mean square pricing errors (RMSPE) (see Adrian and Rosenberg (2008)):

$$\text{SSPE} = \bar{\tilde{\alpha}}' \bar{\tilde{\alpha}} \quad (18)$$

$$\text{RMSPE} = (\text{SSPE}/N)^{1/2} \quad (19)$$

Finally, we consider adjusted R^2 (see Jagannathan and Wang (1996) and Lettau and Ludvigson (2001)):

$$\text{Adj. } R^2 = 1 - \frac{(T-1)(1-R^2)}{(T-K-1)} \quad (20)$$

$$R^2 = \frac{\text{var}_c(\bar{\mathbf{R}}) - \text{var}_c(\bar{\tilde{\alpha}})}{\text{var}_c(\bar{\mathbf{R}})} \quad (21)$$

where $\bar{\mathbf{R}} = 1/T \sum_{t=1}^T \mathbf{R}_t$ and var_c denotes a cross-sectional variance. We should note here that R^2 as defined in Equation 21 implicitly assumes that the cross-section variance between average fitted returns, $\bar{\tilde{\mathbf{R}}} = \bar{\mathbf{R}} - \bar{\tilde{\alpha}}$, and average pricing errors is zero. This is true when betas are as-

²We also consider the number of mispriced assets based on the uncorrected version of the variance-covariance matrix of $\tilde{\alpha}$ in Equation 7. The number of mispriced assets for a model is almost always the same regardless of whether the standard errors are corrected for autocorrelation or not.

sumed constant and guarantees that the R^2 in this framework takes on values between zero and one consistent with the usual definition of R^2 . However, this is not case when betas are allowed to change over time. In other words, the cross-sectional covariance between average fitted returns and average pricing errors can be different than zero and the R^2 as defined in Equation 21 is no longer guaranteed to take on values between zero and one. Although we are aware of this problem, we still choose to present this performance measure for two reasons. First of all, the R^2 never takes on negative values for the models and the sets of test portfolios considered in this paper. Second and more importantly, it is one of the most commonly used performance measures and allows us to compare our results to those in the literature.

5 Data

We analyze the performance of different models in accounting for the cross-sectional variation in monthly excess returns on the Fama and French's (1993) 25 size and book-to-market cross-sorted portfolios. The returns on these portfolios and the value-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks in the CRSP database, which we use as the proxy for the market portfolio, as well as the risk-free rate are all available from the website of Kenneth French at <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>. This data set is now rather standard in asset pricing tests so summary statistics of the monthly returns, in excess of the Treasury bill rate, of these portfolios are not displayed in this paper. In section 8, we also use 10 portfolios formed on momentum and 30 industry portfolios as additional test assets, which are also available from the same website. All portfolio returns are value-weighted and measured at both the daily and monthly frequency. An important advantage of working with these test assets is that individual stock betas do not need to be calculated. Instead, betas are calculated from portfolio returns which are less prone to delayed price reactions that can cause measurement error in beta, see Lo and MacKinlay (1990) and Gilbert, Hrdlicka, Kalodimos, and Siegel (2014).

6 Asset Pricing Performance

We now analyze the performance of the models in explaining the cross-sectional variation in monthly returns on the 25 size and book-to-market sorted portfolios.

We start with the performance of three component beta models, relative to our two benchmark models (the three factor model of Fama and French (1993, 1996) with constant factor loadings and the unconditional version of CAPM with constant betas) over two sample periods; the period between 1970 and 2010 and an extended period between 1937 and 2010. We will then focus mostly on one of the three component beta models and the sample period between 1970 and 2010, due to data availability and quality as accurate return measurement at the daily frequency requires relatively liquid markets.

Performance measures discussed in Section 4 are displayed in Table 1. In the sample between 1970 and 2010, the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ has the lowest RMSPE and SSPE and highest adjusted R^2 . All three of these performance measures are favorable for this component model, relative to the Fama-French three factor model and CAPM with constant beta. The adjusted R^2 for the component model is 0.7536, compared with 0.7134 for the Fama-French model and 0.1972 for CAPM. Other three component specifications that perform better than the Fama-French model all have the medium-run component set at $\beta_{60m,d}$, providing strong support for this component in explaining the cross-sectional variation in monthly returns. The performance of the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ is very similar to that of the three component beta model with $\beta_{6m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$, suggesting that either $\beta_{6m,d}$ or $\beta_{12m,d}$ is suitable in capturing the short-run component of beta. Table 1 also presents the number of mispriced assets at the 1% and 5% significance levels, based on Fama-MacBeth standard errors with Newey-West correction. Again the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ or $\beta_{6m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ is the best performing model with only one asset mispriced at the 1% significance level. While the Fama-French three factor model has three assets mispriced and the CAPM has six assets mispriced.

In the extended sample period, results are similar and are displayed in Table 2. The three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ has a RMSPE of 0.1089, a SSPE of 0.2966,

an adjusted R^2 of 73% and misprices three and five out of 25 portfolios at 1% and 5% significance levels, respectively. On the other hand, the Fama-French three factor model has a RMSPE of 0.1079, a SSPE of 0.2908, an adjusted R^2 of 73% and misprices four and seven out of 25 portfolios at 1% and 5% significance levels, respectively. Once again, other three component models with medium- and long-run captured by $\beta_{60m,d}$ and $\beta_{120m,m}$, respectively, have similar performances. In the rest of the paper, we will mostly focus on the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$.

Table 3 presents the risk premia estimates from CAPM and the Fama French three factor model in Panel (a) and the decomposition of the market risk premium based on the three component beta model $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ in Panel (b). As discussed in Section 3, four sets of standard errors are presented for the benchmark models while only two sets of standard errors are available for the three component beta model. The Fama-MacBeth standard errors with or without Newey-West correction are quite similar to standard errors based on the Shanken correction or the GMM estimation. This suggests that correcting for the errors in variable problem does not significantly affect the standard errors, at least for the benchmark models.³ In other words, we can conclude that the Fama-MacBeth standard errors, which do not correct for the errors in variables problem, are reliable enough to base our statistical testing for our three component beta models.

The coefficient on the market beta in the standard CAPM is positive but is not significantly different than zero. On the other hand, it is significantly negative when estimated in the Fama-French three factor model with constant factor loadings. Regarding the other factors in the Fama-French model, the size factor has a positive but insignificant coefficient estimate while the book-to-market factor has a significantly positive coefficient estimate. These results are quite similar to those in the literature.

Our three component beta model allows us to decompose the market risk premium into its short-, medium- and long-run components. First of all, the coefficient on the short-run component of beta is positive and significant. Second, the coefficient on the medium-run component is significantly negative and is much bigger in magnitude than the coefficient on the short-run component.

³This is in line with the discussion of the Shanken correction in Section 12.2.3 of Cochrane (2001). He argues that the multiplicative correction term is quite small at the monthly frequency and ignoring it makes little difference.

This is evidence that in periods with negative market returns, there is a relatively high weight on the medium-run beta component. Finally, the coefficient on the long-run component is positive but is not significantly different than zero.

Table 4 reports the average pricing errors along with their standard errors for each asset for the sample period between 1970 and 2010. The Fama-French three factor model fails to explain the return on three portfolios at the 1% significance level: the small-growth portfolio, the fourth size quintile in growth portfolios and the fourth book-to-market quintile in large portfolios. In contrast, the only mispriced asset at the 1% significance level with our three component beta model is the small-growth portfolio, which is known to be the most difficult portfolio to correctly price. Even for this extreme portfolio, the average pricing error based on our three component beta model is smaller in absolute value than that based on the Fama-French three factor model. Furthermore, the returns on all other 25 size and book-to-market portfolios are accounted for at the 1% significance level by our three component beta model. Table 4 also reports the SSPE for each size and book-to-market quintile. As is well known, the large pricing errors for the CAPM with constant beta are found to be concentrated in the small, large, growth and value quintiles. The three factor Fama French model does better in accounting for the cross-sectional variation in these extreme quintile portfolios. More importantly, our three component beta model performs better, in terms of SSPE, than the Fama-French three factor model in accounting for the cross-sectional variation in all the extreme quintile portfolios, except the value portfolios where its performance is only slightly worse than that of the Fama-French three factor model.

Pricing performance can also be examined by comparing the average monthly fitted excess return against the average monthly realized excess return for each asset and model. This is displayed in Figure 1 as a plot of fitted versus realized return. Again the deficiencies in the CAPM with constant beta are evident with asset returns deviating sometimes substantially from the 45 degree line. In contrast, the three factor Fama French and three component beta models have asset returns relatively close to the 45 degree line, except for one asset, the small-growth portfolio.

7 Understanding the Asset Pricing Performance

7.1 Decomposition of Performance

In this section, we provide further analysis on why the three component beta model performs well. To this end, we analyze the performances of one and two component beta models which allow us to decompose the overall performance of three component beta models. The one component beta models are simply CAPM with betas estimated with different windows and/or frequencies of data. The two component beta models assume that the beta of an asset is a weighted average of two betas estimated with different windows and/or frequencies of data.

Panel (a) of Table 5 presents different performance measures of one component beta models. First of all, one component beta models with $\beta_{1m,d}, \dots, \beta_{60m,d}$ perform much better than CAPM with constant betas regardless of the performance measure considered. For example, the sum of squared pricing errors for the one component beta model with $\beta_{60m,d}$ is half that of CAPM while its adjusted R^2 is almost threefold that of CAPM. These results suggest that one can explain more than half of the cross-sectional variation in returns by estimating time varying betas based on high frequency data. Secondly, although the one component beta model with $\beta_{120m,m}$, a commonly used approach of capturing time variation in betas, also performs better than CAPM with constant beta, its performance is far less impressive. However, this does not necessarily imply that betas based on low frequency data are of no use. As we discuss below, they capture a different dimension of the time variation in betas that cannot be captured by betas based on high frequency data. Finally, one can easily analyze the contribution of each beta to the overall performance of the three component beta model by comparing the performance measures of one component beta models to those of three component beta models. Consider the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ as an example. Among the three one component beta models, the one with $\beta_{60m,d}$ performs better than the one with $\beta_{12m,d}$ which is in turn better than the one with $\beta_{120m,m}$. This suggests that the overall performance of this three component beta model is mostly due to $\beta_{60m,d}$ and $\beta_{12m,d}$, with $\beta_{120m,m}$ contributing only slightly.

As discussed in the next section, betas estimated with different windows and/or frequencies of data are, not surprisingly, correlated, although at relatively moderate levels. Hence, the perfor-

mance measures of one component beta models do not reveal the pure contribution of a specific beta to the overall performance of the three component beta model. Two component beta models allow us to understand the pure contribution of each beta to the overall performance of three component beta models. First, note that two component models perform better than one component models regardless of how betas are estimated. This comparison illustrates the advantages of combining different betas in accounting for the cross-sectional variation in returns. Secondly, two component models where both beta components are estimated using daily data perform better than those where one of the beta components is estimated using monthly data. The best performing two component model, the one with $\beta_{12m,d}$ and $\beta_{60m,d}$, performs almost as well as the Fama-French three factor model. This again demonstrates the advantages of using high frequency data in estimating betas. More importantly, one can easily analyze the pure contribution of each beta to the overall performance of the three component beta model by comparing the performance measures of two component beta models to those of three component beta models. For example, the performance measures of the two component beta model with $\beta_{60m,d}$ and $\beta_{120m,m}$ reveal the pure contribution of $\beta_{12m,d}$ to the overall performance of the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$: (1) The number of assets mispriced at 5% significance levels decreases by one from five to four; (2) the RMSPE and SSPE decrease from 0.1466 and 0.5372 to 0.1162 and 0.3376, respectively; (3) the adjusted R^2 increases by almost 15% from 60.88% to 75.36%.

To better understand the contribution of each component to the overall performance of three component beta models, we take a closer look at the best performing three component beta model, i.e. the one with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$. To this end, Table 6 presents the percentage decrease in the SSPE due to the inclusion of a specific component. The percentage decrease in the SSPE can be interpreted as a partial R^2 since it is the change in the variation explained by the part of that component orthogonal to the other two components. $\beta_{12m,d}$ and $\beta_{60m,d}$ decrease the overall SSPE by 37% and 48%, respectively. This is mostly due to their explanatory power for portfolios with high market capitalizations (2nd quintile and above) and book-to-market ratios (3rd quintile and above). They do not significantly decrease the SSPE of portfolios with low book-to-market ratios. The inclusion of $\beta_{12m,d}$ in the three component beta model actually increases the SSPE of the 2nd quintile of book-to-market sorted portfolios. On the other hand, $\beta_{120m,m}$ decreases the

overall SSPE only by 18%. It performs relatively poorly in explaining the returns on portfolios of small and large and has a mixed performance in explaining the returns on book-to-market sorted portfolios.

7.2 Dynamics of the Components of Beta

To further our understanding of the asset pricing performance of our three component beta model, we now also study the time variation of beta measurements, $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$.

Table 7 reports the mean and standard deviation of these beta components for the 25 size and book-to-market sorted portfolios over the sample period between January 1970 to December 2010. In regard to the means of these betas, there is a strong pattern in all three beta measurements, in the form of a decreasing mean beta as the book-to-market ratio of a portfolio increases. This suggests that the mean betas cannot possibly account for the fact that portfolios with higher book-to-market ratios also have higher mean returns. Although not presented, this is also in line with the pattern observed in constant betas estimated once using the whole sample. On the other hand, there is no clear pattern in the mean of all three beta components as functions of the market capitalization of a portfolio. Similar to their constant betas, the mean of the long term beta component, $\beta_{120m,m}$, increases as the market capitalization of a portfolio decreases. This suggests that the mean of the long term beta component might account for the fact that portfolios with smaller market capitalizations have higher mean returns. However, the same cannot be said about the mean of short- and medium-term beta components. There is no clear pattern in the mean of short- and medium-term beta components. More importantly, if there is any pattern, it tends to work in the opposite direction with large firms having higher betas than small firms. To summarize, these results suggest that variation in the means of different beta components cannot possibly account for the observed patterns in mean returns. Instead, results on the variability of beta will contribute to providing an explanation to the success of our three component beta model.

To see this, Table 7 presents the standard deviation of different beta components. The standard deviation of $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ increases as the market capitalization of a portfolio decreases and its book-to-market ratio increases. For example, for the short-run beta component,

the standard deviation of the large-growth portfolio is 0.1407, compared with 0.2590 for the small-value portfolio. For the long-run beta component, the standard deviation for the large-growth and small-value portfolios are 0.0408 and 0.1872, respectively. In addition, summary statistics on the annual changes in $\beta_{12m,d}$ for the 25 size and book-to-market sorted portfolios over the January 1970 to December 2010 are presented in Table 8. The variability of these beta changes in respect to standard deviation, minimum, maximum and range provide insights into the relationship between the dynamics of beta and returns. In particular, as the book-to-market ratio of a portfolio increases, the maximum annual beta change tends to rise. For example, the maximum annual change for the small-growth portfolio is 0.5590, whereas the maximum annual change for the small-value portfolio is 0.7736. For the large-growth and large-value portfolios, the maximum annual change is 0.2169 and 0.8132, respectively. A pattern in the variability of beta changes also exists in relation to the market capitalization of a portfolio. As the market capitalization of a portfolio increases, the range of annual beta changes (maximum - minimum) increases. For example, the range of the small-growth portfolio is 1.2147, whereas the range of the large-growth portfolio is 0.5264. The recent financial crisis also provides further insights and Table 9 reports the mean and standard deviation of $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ for the 25 size and book-to-market sorted portfolios over the period, January 2009 to December 2010. During this period there is a dramatic change in the pattern of mean betas for $\beta_{12m,d}$ and $\beta_{60m,d}$. In particular for $\beta_{12m,d}$ there is now a strong pattern of an increasing mean beta as the market capitalization of the portfolio decreases and its book-to-market ratio increases. For example, the mean of $\beta_{12m,d}$ is 0.7914 for the large-growth portfolio and 1.1699 for the small-value portfolio. This reversal in the pattern of beta, relative to that over the full sample, demonstrates the dramatic changes in beta during the turmoil of the financial crisis. These summary statistics suggest that the variability in beta is related to the size and value premia that are evident in the mean returns of the 25 size and book-to-market sorted portfolios. Part of the success of our three component beta component model can be attributed to capturing these beta dynamics.

We next investigate the annual change in each beta component, $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$, in relation to the business cycle. Table 10 displays the correlations of the annual beta changes with the Treasury bill rate, for our 25 size and book-to-market sorted portfolios over the January 1970

to December 2010. The Treasury bill rate at the beginning of the 12 month period is chosen for measuring economic conditions over the next 12 months. Panel a displays the correlations for the annual changes of $\beta_{12m,d}$ which are mostly insignificant, indicating that the dynamics of the short-run component are not dominated by the business cycle. In contrast, panel b and c display a number of significant correlations for annual changes of $\beta_{60m,d}$ and $\beta_{120m,m}$ with the Treasury bill rate. A strong pattern is present where correlations become significantly more negative as the book-to-market ratio of the portfolio increases, with the most significant negative correlations being approximately -0.5. Thus, in recessions when the Treasury bill rate is low, the medium- and long-run beta components for portfolios with high book-to-market ratios, tend to rise over the year. It is widely acknowledged that the business cycle has significant impacts on asset returns, and in our three component beta model this is primarily captured through the medium- and long-run beta components.

It is not too surprising that the business cycle is not primarily driving the short-run beta component, as typically the dynamics of the business cycle are slowly changing, whereas the short-run beta component is designed to capture fast moving beta dynamics. For example, changes in a portfolio's specific risk characteristics are likely to be more immediate in nature and have a more rapid impact on the portfolio's short-run beta component. To illustrate this, Table 11 displays the correlations of 11 month changes in betas with the prior 12 month change in the value-weighted average book-to-market ratio of the portfolio. The 11 month change in an asset's beta component is measured from 1 July to 1 June so that the asset composition remains unchanged, as rebalancing of portfolios occurs at the beginning of July. The prior 12 month change in the value-weighted average book-to-market ratio of the portfolio is from 1 July and is obtained from the Kenneth French Data Library. This 12 month change in the book-to-market ratio of the portfolio captures the change in one of the important portfolio characteristics. Panel a of Table 11, displays correlations for the short-run beta component of the 25 size and book-to-market sorted portfolios, showing highly significant positive correlations for the value portfolios with the highest correlation being 0.6329. Overall, there is a pattern of increasing correlation as the book-to-market ratio of the portfolio increases. Demonstrating that an increase in the portfolio's risk characteristics due to an increase in its book-to-market ratio, often results in significant increases in the short-run beta

component. Panels b and c of Table 11 display correlations for the medium- and long-run beta components with mostly insignificant correlations, showing that these beta components are less sensitive to the more fast moving risk dynamics of assets.

Finally, Table 12 presents summary statistics on the correlations between the short-, medium- and long-run beta components for the 25 size and book-to-market sorted portfolios. These correlations are typically not too high, with the highest mean correlation being 0.6823 which is between the short- and medium-run beta components, and the lowest mean correlation being 0.1220 which is between the short- and long-run beta components. These relatively moderate levels of correlation between different components of beta suggest that they do indeed capture different frequency variations in betas.

8 Robustness Checks

8.1 Some Simulation Results

Lewellen, Nagel, and Shanken (2010) argue that it is relatively easy to come up with factors that explain most of the cross-sectional variation in the 25 size and book-to-market portfolios due to the strong factor structure in these portfolios. Specifically, they generate different types of artificial factors and show that these artificial factors can explain the cross-sectional variation in the 25 size and book-to-market portfolios as well as the Fama-French three factor model. They provide several suggestions to remedy this problem. We implement some of these suggestions to show that the performance of our three component beta model relative to the Fama-French three factor model is not due to this strong factor structure in the 25 size and book-to-market portfolios. Before considering their suggestions, we start in this section with a simulation exercise similar to theirs and analyze whether the relative success of our three component beta model is due to the possibility that it might be exploiting the strong factor structure present in the 25 size and book-to-market portfolio.

In our framework, instead of several factors with constant loadings, there is only one factor, the return on the market portfolio, whose loading is assumed to be time-varying and measured

as a weighted average of loadings estimated over different window of observations with different frequencies. Hence, we generate only two artificial factors replacing the daily and monthly observations of the return on the market portfolio. Specifically, let ω_t denote a 3×1 vector of random weights drawn from a standard normal distribution. We generate artificial factors as $P_t = \omega_t' F_t + v_t$ where F_t is a 3×1 vector of either daily or monthly Fama-French factors and v_t is another random variable independent of ω_t' and F_t . We then estimate the three component beta model with $\beta_{12m,d,t}$, $\beta_{60m,d,t}$ and $\beta_{120m,m,t}$ based on the time series of daily and monthly artificial factors. We repeat this exercise 5,000 times and report the summary statistics for different performance measures in Table 13.

First of all, there are, on average, five to six (two to three) mispriced assets at the 5% (1%) level when we use artificial factors to account for the cross-sectional variation in the 25 size and book-to-market portfolios. Secondly, the RMSPE and SSPE based on artificial factors are, on average, 0.1555 and 0.6118, respectively. Thirdly, the average adjusted R^2 based on artificial factors is 0.5536, suggesting that artificial factors can explain a little more than half of the cross-sectional variation in the 25 size and book-to-market portfolios. These results are consistent with those in Lewellen, Nagel, and Shanken (2010) who also find that artificial factors can explain a non-negligible part of the cross-sectional variation in the 25 size and book-to-market portfolios. However, the explanatory power of artificial factors are still, on average, well below that of our three component beta models. For example, the three component beta model with $\beta_{12m,d,t}$, $\beta_{60m,d,t}$ and $\beta_{120m,m,t}$ has an RMSPE of 0.1162, a SSPE of 0.3376 and an adjusted R^2 of 0.7536. More importantly, the performance measures of our three component beta models are outside the 5% and 95% quantiles of these performance measures based on artificial factors. In other words, the probability that the relative success of our three component beta models is due to luck is less than 5%.

8.2 Additional Test Portfolios

We now turn our attention to analyzing the performance of the models in explaining the cross-sectional variation in monthly returns with additional test portfolios, namely the 10 momentum

sorted portfolios and 30 industry portfolios, from the Kenneth French Data Library. Performance of the three component beta models as well as the two benchmark models, CAPM with constant beta and the Fama-French three factor model, is again evaluated for the period January 1970 to December 2010. Performance measures for the pricing of the 25 size and book-to-market cross sorted portfolios, 10 momentum sorted portfolios and 30 industry portfolios are displayed in Table 14.

These results firstly show that when all these assets are priced, there is a substantial increase in pricing errors across all models, relative to the pricing of just the 25 size and book-to-market cross sorted portfolios. For example, the adjusted R^2 for the best performing three component beta model, Fama-French three factor model and CAPM is 0.3692, 0.2699 and 0.0792, respectively. Whereas, for the pricing of just the 25 size and book-to-market cross sorted portfolios, the adjusted R^2 for the models are 0.7536, 0.7134 and 0.1972, respectively. This substantial increase in pricing errors is largely attributed to the inclusion of the industry portfolios, which exhibit relatively little cross sectional dispersion in average returns as shown in Santos and Veronesi (2006). As discussed in Lewellen, Nagel, and Shanken (2010), assessing the performance of asset pricing models on additional portfolios to the size and book-to-market cross sorted portfolios, raises the hurdle in evaluating model performance. In this regard, our three component beta models start to demonstrate outperformance, relative to the Fama-French three factor model. The best performing three component beta model is with $\beta_{3m,d}$, $\beta_{12m,d}$ and $\beta_{120m,m}$ based on RMSPE, SSPE and adjusted R^2 . The number of assets that this model misprices at the 1% significance level is 6. While the Fama-French three factor model misprices 8.

The performance of the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ is again very similar to that of the three component beta model with $\beta_{6m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ and the number of assets that these models misprice at the 1% significance level is also 6. The RMSPE, SSPE and adjusted R^2 of these two models is comparable with the Fama-French three factor model, however, they do not outperform the three component beta model with $\beta_{3m,d}$, $\beta_{12m,d}$ and $\beta_{120m,m}$. Thus, the inclusion of these additional portfolios, favors beta component models with short- and medium-run components measured over a shorter period. This is consistent with substantial time variation in industry betas, which has been found in earlier studies such as Ferson and Harvey

(1991) and Braun, Nelson, and Sunier (1995).

8.3 Alternative Frequencies

In this section, we analyze whether the three component beta models continue to perform just as well as the Fama-French three factor model in accounting for the cross-sectional variation in quarterly returns on the 25 size and book-to-market cross sorted portfolios. To this end, we use monthly returns on these portfolios to calculate their quarterly returns. We then estimate the three component beta models as well as the benchmark models using quarterly instead of monthly returns.

Table 15 presents the performance measures for the three component beta models and the benchmark models. Before comparing the performances of the three component beta models to those of the Fama-French three factor model, we should first note that the RMSPE and SSPE are higher than those based on monthly returns, regardless of the model considered, since quarterly returns are higher on average than monthly returns. Furthermore, compared to the results based on monthly returns, there is a deterioration in the performance of CAPM with constant betas when we consider quarterly returns, while the other models continue to perform similarly. For example, the adjusted R^2 of the CAPM with constant betas based on quarterly returns is only 6% compared to an adjusted R^2 of 19% based on monthly returns.

More importantly, the results in Table 15 suggest that the three component beta models continue to perform as well as or even sometimes better than the Fama-French three factor model in accounting for the cross-sectional variation in quarterly returns on the 25 size and book-to-market cross sorted portfolios. First of all, the best performing three component beta model for monthly returns, namely the one with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$, performs as well as the Fama-French three factor model based on RMSPE, SSPE and adjusted R^2 and outperforms it based on the number of significantly mispriced assets. Secondly, two similar three component beta models where we replace $\beta_{12m,d}$ with either $\beta_{6m,d}$ or $\beta_{3m,d}$ actually outperform the Fama-French three factor model based on all measures. These results suggest that three component beta models are as good of a model or even a better one than the Fama-French three factor model in accounting for the cross-sectional variation in both monthly and quarterly returns.

8.4 Alternative Sample Periods

In this section, we analyze how the best performing three component beta model fairs against the benchmark models especially the Fama-French three factor model over different sample periods. To this end, we analyze the performances of these models over an expanding window of sample periods starting with the first sample period between January 1970 and January 1990. Given that an increasing number of observations generally improves precision in estimating sample means, we consider an expanding rather than rolling window of observations in estimating average pricing errors, which are simply the sample averages of conditional alphas. Figure 2 presents the SSPE of the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ and the two benchmark models for the period between January 1990 and December 2010. The SSPEs of the three component beta model and the Fama-French three factor model are quite stable over different sample periods, whereas that of the CAPM model is less stable. Specifically, CAPM performs better when we consider sample periods between 1970 and the second half of the 90s, but its performance decreases, i.e. its SSPE increases, as we add data from the 2000s. More importantly, the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ performs always better than the Fama-French three factor model, which in turn performs better than the CAPM with constant betas, as we consider an expanding window of observations. To summarize, the three component beta model does not only have a stable performance in accounting for the cross-sectional variation in returns but also its success relative to the Fama-French three factor model is not due to the specific sample period considered in Sections 6 and 7.

We also analyze the performances of these models over expansion and recession periods during the business cycle. We classify a month as either part of the expansion or recession phase through the NBER classification. Table 16 presents the SSPE of the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ and the two benchmark models for each portfolio quintile, over both the expansion periods (displayed in panel a), and over the recession periods (displayed in panel b.) During expansions, the SSPE of the three component beta model is very similar to that of the Fama-French three factor model, over all quintiles. However, during recessions (when pricing errors rise for all the models) the SSPE of the three component beta model is lower relative to that

of the Fama-French three factor model. The largest differences in SSPE between these two models in recessions occurs in the growth and value quintiles. For the growth quintile, the SSPE's for the two models are 0.3197 and 0.5173, respectively. And for the value quintile, the SSPE's for the two models are 0.2048 and 0.4000, respectively. The three component beta model in recessions also produces substantially lower SSPE's for the small and large quintiles. This suggests that the three component beta model does a relatively good job in capturing dynamics during recessions. In contrast, the two benchmark models have inferior performance. The SSPE for CAPM with constant betas in expansions and recessions is 1.0112 and 2.1113, respectively. The SSPE for the Fama-French three factor model in expansions and recessions is 0.3496 and 1.1480, respectively. Whereas, the SSPE for the three component beta model in expansions and recessions is 0.3561 and 0.8037, respectively.

8.5 Nonsynchronous Trading

Lo and MacKinlay (1990) point out that nonsynchronous price movements in stocks can occur due to small stocks having delayed price reactions. While these effects can be important for individual stocks, the impact for broadly diversified value-weighted portfolios is much less. However, to ensure our results are not biased, due to nonsynchronous trading, we conduct two further robustness tests. Firstly, we simply remove the 5 test portfolios from the small quintile from the 25 size and book-to-market cross sorted portfolios. Performance measures of the models for the pricing of the remaining 20 size and book-to-market cross sorted portfolios for the period January 1970 to December 2010 are displayed in Table 17. Overall, the pricing performance of the Fama-French three factor model and the the three component beta model with $\beta_{12m,d,t}$, $\beta_{60m,d,t}$ and $\beta_{120m,m,t}$ remain comparable over the different performance measures. Both models have lower pricing error from the remove of these 5 test portfolios. The Fama-French three factor model adjusted R^2 rises from 0.7134 to 0.8050. While the three component beta model adjusted R^2 rises from 0.7536 to 0.8195. Thus, we can conclude that the 5 small test portfolios are not highly influential in the performance measurements of the three component beta model.

In addition, to further examine potential bias from nonsynchronous trading, we follow the

approach of Dimson (1979) and Lewellen and Nagel (2006) and estimate our short- and medium-run beta components from daily returns using the following regression equation

$$R_{i,t} = \alpha_i + \beta_{i0}R_{m,t} + \beta_{i1}R_{m,t-1} + \beta_{i2}[(R_{m,t-2} + R_{m,t-3} + R_{m,t-4})/3] + u_{i,t} \quad (22)$$

with $\beta_i = \beta_{i0} + \beta_{i1} + \beta_{i2}$. We then re-run our analysis of the component beta models over the 25 size and book-to-market cross sorted portfolios, with performance measures displayed in Table 18, showing relatively little change due to this correction for nonsynchronous trading. For example, the adjusted R^2 for the the three component beta model with $\beta_{12m,d,t}$, $\beta_{60m,d,t}$ and $\beta_{120m,m,t}$ only falls from 0.7536 to 0.7325.

9 Conclusions

Empirical asset pricing typically measures systematic risk with a factor model. The most commonly utilized econometric specifications follow Fama and MacBeth (1973) with a one factor model and Fama and French (1993) with a three factor model. In this paper we have proposed an econometric specification that allows a component structure to systematic risk in the one factor market model, with results providing empirical validation of the conditional CAPM. This econometric specification is consistent with agents forming expectations of risk from information over varying time horizons.

Related literature motivates our use of a component structure, both from theoretical and empirical perspectives, and our results demonstrate that our component model can explain the cross section of expected returns, just as well as the Fama-French three factor model. We empirically test a conditional version of CAPM by proposing a model for the conditional beta of an asset which is a weighted average of three betas estimated over different periods using different frequency data. Our three component beta model can be considered as a mixed-frequency approach as the short- and medium-run beta components are computed from daily returns, whereas, the long-run beta component is computed from monthly returns.

We demonstrate that much can be gained when daily and monthly returns are utilized to mea-

sure components of systematic risk in an asset pricing framework. Most of the gain occurs through the use of daily returns and thus an important implication of this study is that when accurate daily returns are available, they should be utilized in CAPM applications. Our results also suggest that the more immediate changes in risk such as changes in portfolio characteristics are captured in the short-run beta component while the medium- and long-run beta components capture more slowly changing risk which we find to be correlated with the business cycle. The empirical success of our three component beta model relative to the Fama-French three factor model is partly due to its success in capturing time-variation in risk of portfolios in recessions.

An important implication of this paper for future research in asset pricing, is that at least some caution should be exercised in interpreting an asset pricing anomaly from beta estimation with only a single component. The voluminous literature in asset pricing anomalies has typically focused on beta estimation with one component, often from returns only measured at the monthly frequency, which suggests that re-examination of parts of this literature with a component structure for beta is warranted. This avenue of research is left for future work.

Finally, it is hoped that the investment management industry finds components to beta useful for abnormal return measurement. Given that components to beta are calculated from the market factor, they are not associated with substantial transaction costs. This is in contrast to the Fama-French size and book-to-market factors where relatively high transaction costs, coupled with difficulty in accurately measuring these portfolio costs, often limit their usefulness in abnormal return measurement of investment funds.

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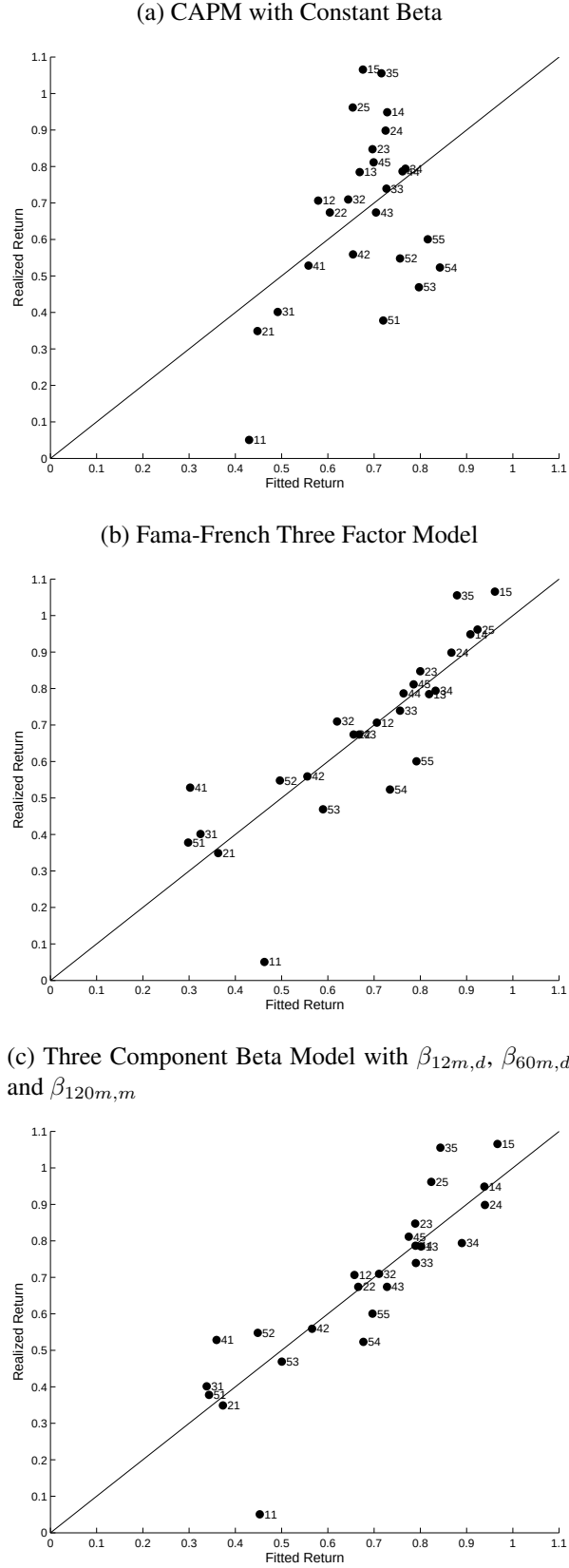
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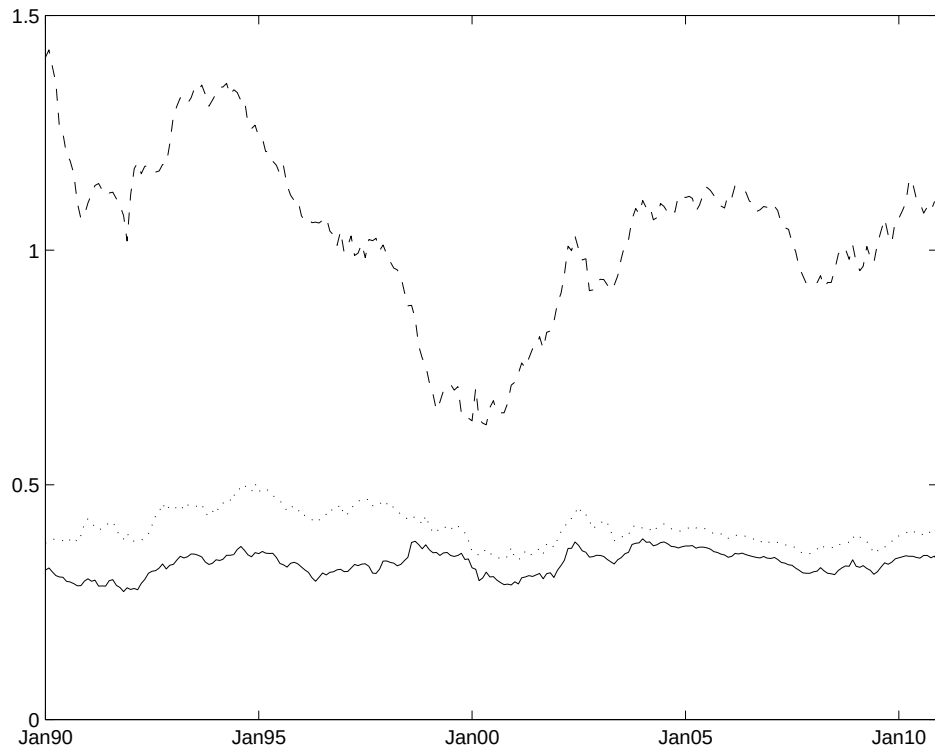
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Figure 1: Realized vs Fitted Returns



Note: This figure plots average monthly realized excess returns on the 25 size and book-to-market returns in the y-axis against their fitted values from three asset pricing models in the x-axis. The numbers on the graphs refer to individual portfolios with the first number denoting the size and the second number denoting the book-to-market quintile. The sample period is between 1970 and 2010.

Figure 2: SSPE of Asset Pricing Models for Alternative Sample Periods



Note: This figure presents the SSPE for 25 size and book-to-market portfolios from asset pricing models over an expanding window of observations starting with the first window between January 1970 and January 1990 and ending with the last window between January 1970 and December 2010. The dashed, dotted and solid lines correspond to the SSPEs of the CAPM with constant betas, the Fama-French three-factor model and the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$, respectively.

Table 1: Performance Measures of Asset Pricing Models between 1970 and 2010

	Number of Mispriced Assets at 1% Level	Number of Mispriced Assets at 5% Level	RMSPE	SSPE	Adj. R^2
Three Comp. Beta Model with					
$\beta_{1m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	3	4	0.1263	0.3985	0.7092
$\beta_{3m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	2	4	0.1285	0.4128	0.6988
$\beta_{6m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	3	4	0.1321	0.4365	0.6815
$\beta_{1m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	2	3	0.1209	0.3655	0.7333
$\beta_{3m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	2	3	0.1230	0.3783	0.7239
$\beta_{6m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	1	4	0.1167	0.3404	0.7516
$\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	1	4	0.1162	0.3376	0.7536
Fama-French Three Factor Model	3	4	0.1253	0.3927	0.7134
CAPM with constant β	6	11	0.2102	1.1047	0.1972

Note: This table presents performance measures for asset pricing models for the recent sample period between 1970 and 2010. Number of mispriced assets is the number of assets out of 25 size and book-to-market portfolios with an average pricing error significantly different than zero at 1% and 5% levels based on Fama-MacBeth standard errors with Newey-West correction. RMSPE, SSPE and Adj. R^2 are root mean square pricing error, sum of square pricing errors and adjusted R^2 , respectively.

Table 2: Performance Measures of Asset Pricing Models between 1937 and 2010

	Number of Mispriced Assets at 1% Level	Number of Mispriced Assets at 5% Level	RMSPE	SSPE	Adj. R^2
Three Comp. Beta Model with					
$\beta_{120m,m}$ & $\beta_{12m,d}$ & $\beta_{1m,d}$	4	6	0.1202	0.3610	0.6665
$\beta_{120m,m}$ & $\beta_{12m,d}$ & $\beta_{3m,d}$	4	5	0.1157	0.3345	0.6910
$\beta_{120m,m}$ & $\beta_{12m,d}$ & $\beta_{6m,d}$	3	5	0.1154	0.3330	0.6924
$\beta_{120m,m}$ & $\beta_{60m,d}$ & $\beta_{1m,d}$	4	4	0.1061	0.2814	0.7401
$\beta_{120m,m}$ & $\beta_{60m,d}$ & $\beta_{3m,d}$	4	5	0.1060	0.2810	0.7404
$\beta_{120m,m}$ & $\beta_{60m,d}$ & $\beta_{6m,d}$	3	4	0.1029	0.2645	0.7556
$\beta_{120m,m}$ & $\beta_{60m,d}$ & $\beta_{12m,d}$	3	5	0.1089	0.2966	0.7260
Three Factor Fama-French Model	4	7	0.1079	0.2908	0.7314
CAPM with constant β	11	15	0.2081	1.0825	0.0023

Note: This table presents performance measures for asset pricing models for the longer sample period between 1937 and 2010. Number of mispriced assets is the number of assets out of 25 size and book-to-market portfolios with an average pricing error significantly different than zero at 1% and 5% levels based on Fama-MacBeth standard errors with Newey-West correction. RMSPE, SSPE and Adj. R^2 are root mean square pricing error, sum of square pricing errors and adjusted R^2 , respectively.

Table 3: Risk Premia Estimates

(a) CAPM and Fama-French Three Factor Model with Constant Factor Loadings

Sample Period	CAPM	Fama-French Three Factor Model		
	MKT	MKT	SMB	HML
1937-2010	0.0853	-0.6970**	0.1220	0.4138***
	(0.3582)	(0.3064)	(0.1040)	(0.1008)
	(0.3638)	(0.3109)	(0.1056)	(0.1112)
	[0.3583]	[0.3121]	[0.1042]	[0.1010]
	{0.3617}	{0.3037}	{0.1108}	{0.1103}

(b) Three Component Beta Model with $\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$

Sample Period	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
1937-2010	0.5806*	-1.1329***	0.1926
	(0.2906)	(0.3888)	(0.3269)
	(0.3184)	(0.4156)	(0.3319)

Note: This table presents risk premia estimates from CAPM and Fama-French three factor model with constant factor loadings in Panel (a) and the three component beta model with $\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$ in Panel (b) for the longer sample period between 1937 and 2010. The uncorrected Fama-MacBeth standard errors are presented in parentheses immediately below the estimates. The second set of standard errors in parentheses are the Fama-MacBeth standard errors corrected for autocorrelation à la Newey and West (1987). The standard errors corrected for the errors in variables problem à la Shanken (1992) are presented in square brackets while the GMM standard errors are presented in curly brackets. ***, ** and * denote significance at, respectively, 1%, 5% and 10% levels based on corrected Fama-MacBeth standard errors.

Table 4: Average Pricing Errors

(a) CAPM with Constant Beta

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large	SSPE
Growth	-0.3791** (0.0823) (0.0814) [0.0832] {0.0931}	-0.0989 (0.0630) (0.0621) [0.0637] {0.0668}	-0.0903 (0.0793) (0.0860) [0.0802] {0.0873}	-0.0297 (0.0977) (0.1029) [0.0987] {0.1023}	-0.3417* (0.1362) (0.1566) [0.1377] {0.1560}	0.2793
2 nd Quintile	0.1273 (0.0973) (0.0882) [0.0983] {0.0964}	0.0696 (0.0616) (0.0623) [0.0623] {0.0626}	0.0658 (0.0674) (0.0670) [0.0681] {0.0698}	-0.0955 (0.0816) (0.0801) [0.0824] {0.0818}	-0.2082* (0.0967) (0.1010) [0.0977] {0.1068}	0.0779
3 rd Quintile	0.1159 (0.0988) (0.0972) [0.0998] {0.1002}	0.1508* (0.0664) (0.0670) [0.0672] {0.0673}	0.0126 (0.0602) (0.0646) [0.0609] {0.0635}	-0.0303 (0.0783) (0.0749) [0.0791] {0.0731}	-0.3278** (0.0953) (0.0968) [0.0963] {0.1071}	0.1447
4 th Quintile	0.2203* (0.1129) (0.1106) [0.1141] {0.1139}	0.1737* (0.0738) (0.0710) [0.0746] {0.0730}	0.0262 (0.0612) (0.0609) [0.0619] {0.0626}	0.0251 (0.0660) (0.0658) [0.0667] {0.0664}	-0.3193** (0.0745) (0.0786) [0.0753] {0.0828}	0.1820
Value	0.3895** (0.1128) (0.1239) [0.1140] {0.1336}	0.3078** (0.1028) (0.1185) [0.1038] {0.1210}	0.3394** (0.0929) (0.1043) [0.0939] {0.1100}	0.1124 (0.1083) (0.1045) [0.1095] {0.1040}	-0.2157 (0.1099) (0.1145) [0.1111] {0.1149}	0.4209
SSPE	0.3736	0.1623	0.1286	0.0242	0.4160	1.1047

(b) Fama-French Three Factor Model

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large	SSPE
Growth	-0.4122** (0.0810) (0.0805) [0.0828] {0.0962}	-0.0139 (0.0506) (0.0479) [0.0517] {0.0571}	0.0768 (0.0541) (0.0521) [0.0553] {0.0631}	0.2261** (0.0542) (0.0553) [0.0554] {0.0676}	0.0801 (0.0681) (0.0699) [0.0695] {0.0716}	0.2335
2 nd Quintile	0.0004 (0.0549) (0.0537) [0.0561] {0.0618}	0.0185 (0.0598) (0.0611) [0.0611] {0.0617}	0.0898 (0.0646) (0.0662) [0.0660] {0.0720}	0.0034 (0.0611) (0.0680) [0.0624] {0.0691}	0.0515 (0.0594) (0.0590) [0.0607] {0.0703}	0.0111
3 rd Quintile	-0.0343 (0.0414) (0.0413) [0.0423] {0.0477}	0.0476 (0.0561) (0.0564) [0.0573] {0.0665}	-0.0169 (0.0606) (0.0634) [0.0619] {0.0649}	0.0062 (0.0620) (0.0674) [0.0633] {0.0713}	-0.1205 (0.0713) (0.0680) [0.0729] {0.0805}	0.0183
4 th Quintile	0.0404 (0.0392) (0.0409) [0.0401] {0.0499}	0.0312 (0.0565) (0.0559) [0.0577] {0.0598}	-0.0389 (0.0594) (0.0554) [0.0607] {0.0688}	0.0229 (0.0646) (0.0650) [0.0660] {0.0740}	-0.2113** (0.0664) (0.0652) [0.0679] {0.0775}	0.0493
Value	0.1042 (0.0608) (0.0699) [0.0622] {0.0724}	0.0380 (0.0616) (0.0612) [0.0630] {0.0670}	0.1761* (0.0701) (0.0769) [0.0716] {0.0877}	0.0260 (0.0676) (0.0658) [0.0691] {0.0739}	-0.1912 (0.1074) (0.1100) [0.1097] {0.1150}	0.0806
SSPE	0.1835	0.0052	0.0468	0.0524	0.1048	0.3927

(c) Three Comp. Beta Model with $\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large	SSPE
Growth	-0.4022** (0.0828) (0.0964)	-0.0242 (0.0548) (0.0572)	0.0635 (0.0599) (0.0616)	0.1690* (0.0670) (0.0816)	0.0350 (0.0847) (0.0916)	0.1962
2 nd Quintile	0.0489 (0.0798) (0.0803)	0.0083 (0.0621) (0.0608)	-0.0010 (0.0706) (0.0765)	-0.0069 (0.0684) (0.0747)	0.0994 (0.0628) (0.0599)	0.0124
3 rd Quintile	-0.0167 (0.0597) (0.0620)	0.0584 (0.0537) (0.0562)	-0.0512 (0.0593) (0.0607)	-0.0540 (0.0843) (0.0766)	-0.0314 (0.0761) (0.0660)	0.0102
4 th Quintile	0.0102 (0.0648) (0.0670)	-0.0413 (0.0586) (0.0565)	-0.0958 (0.0610) (0.0587)	-0.0029 (0.0672) (0.0661)	-0.1538* (0.0695) (0.0760)	0.0347
Value	0.0985 (0.0751) (0.0828)	0.1380 (0.0755) (0.0823)	0.2119* (0.0854) (0.0909)	0.0365 (0.0937) (0.0920)	-0.0962 (0.1025) (0.1024)	0.0842
SSPE	0.1742	0.0248	0.0607	0.0329	0.0450	0.3376

Note: This table presents average pricing errors from asset pricing models over the sample period between 1970 and 2010. The uncorrected Fama-MacBeth standard errors are presented in parentheses immediately below the average pricing errors. The second set of standard errors in parentheses are the Fama-MacBeth standard errors corrected for autocorrelation à la Newey and West (1987). The standard errors corrected for the errors in variables problem à la Shanken (1992) are presented in square brackets while the GMM standard errors are presented in curly brackets. ** and * denote significance at, respectively, 1% and 5% levels based on corrected Fama-MacBeth standard errors. SSPE is the sum of square pricing errors.

Table 5: Performance Measures of One and Two Component Beta Models

(a) One Component Beta Model

	Number of Mispriced Assets at 1% Level	Number of Mispriced Assets at 5% Level	RMSPE	SSPE	Adj. R^2
One Comp. Beta Model with					
$\beta_{1m,d}$	2	4	0.1631	0.6650	0.5168
$\beta_{3m,d}$	2	2	0.1568	0.6147	0.5533
$\beta_{6m,d}$	1	3	0.1586	0.6286	0.5432
$\beta_{12m,d}$	1	4	0.1615	0.6523	0.5259
$\beta_{60m,d}$	1	3	0.1535	0.5888	0.5721
$\beta_{120m,m}$	5	10	0.1972	0.9725	0.2933

(b) Two Component Beta Model

	Number of Mispriced Assets at 1% Level	Number of Mispriced Assets at 5% Level	RMSPE	SSPE	Adj. R^2
Two Comp. Beta Model with					
$\beta_{1m,d}$ & $\beta_{12m,d}$	1	3	0.1408	0.4953	0.6393
$\beta_{3m,d}$ & $\beta_{12m,d}$	2	2	0.1461	0.5334	0.6116
$\beta_{6m,d}$ & $\beta_{12m,d}$	1	3	0.1474	0.5430	0.6046
$\beta_{1m,d}$ & $\beta_{60m,d}$	1	3	0.1321	0.4364	0.6822
$\beta_{3m,d}$ & $\beta_{60m,d}$	2	4	0.1448	0.5245	0.6181
$\beta_{6m,d}$ & $\beta_{60m,d}$	1	3	0.1377	0.4743	0.6546
$\beta_{12m,d}$ & $\beta_{60m,d}$	1	3	0.1282	0.4109	0.7008
$\beta_{1m,d}$ & $\beta_{120m,m}$	2	5	0.1517	0.5753	0.5810
$\beta_{3m,d}$ & $\beta_{120m,m}$	2	4	0.1440	0.5186	0.6224
$\beta_{6m,d}$ & $\beta_{120m,m}$	4	4	0.1478	0.5464	0.6021
$\beta_{12m,d}$ & $\beta_{120m,m}$	4	5	0.1524	0.5810	0.5769
$\beta_{60m,d}$ & $\beta_{120m,m}$	3	5	0.1466	0.5372	0.6088

Note: This table presents performance measures for one and two component beta models over the sample period between 1970 and 2010. The one component beta models are simply CAPM with betas estimated with different windows and/or frequencies of data. The two component beta models assume that the beta of an asset is a weighted average of two betas estimated with different windows and/or frequencies of data. Number of mispriced assets is the number of assets out of 25 size and book-to-market portfolios with an average pricing error significantly different than zero at 1% and 5% levels based on Fama-MacBeth standard errors with Newey-West correction. RMSPE, SSPE and Adj. R^2 are root mean square pricing error, sum of square pricing errors and adjusted R^2 , respectively.

Table 6: Percentage Decrease in the SSPE due to Individual Beta Components

	$\beta_{12m,d}$	$\beta_{60m,d}$	$\beta_{120m,m}$
Small	17%	23%	2%
2 nd Quintile	45%	54%	14%
3 rd Quintile	56%	52%	33%
4 th Quintile	45%	35%	58%
Large	48%	64%	-32%
Growth	9%	12%	5%
2 nd Quintile	-44%	3%	-105%
3 rd Quintile	79%	83%	69%
4 th Quintile	34%	47%	-69%
Value	60%	62%	42%
Total	37%	42%	18%

Note: This table presents the percentage decrease in the sum of square pricing errors that can be attributed to the beta component presented in column headings. The percentage decrease in the SSPE is calculated as the difference between the SSPEs of the three component beta model with $\beta_{12m,d}$, $\beta_{60m,d}$ and $\beta_{120m,m}$ and the two component beta model that excludes the component presented in the column heading divided by the SSPE of the same two component beta model. A positive number suggests a decrease in the SSPE while a negative number an increase. The sample period is between 1970 and 2010.

Table 7: Summary Statistics for Beta Components, January 1970 to December 2010

(a) $\beta_{12m,d}$

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	0.9794 (0.2740)	1.1263 (0.2193)	1.1251 (0.1654)	1.1198 (0.1514)	1.1208 (0.1407)
2 nd Quintile	0.8317 (0.2560)	0.9013 (0.2246)	0.8946 (0.1532)	0.9174 (0.0902)	1.0229 (0.1130)
3 rd Quintile	0.7303 (0.2473)	0.7966 (0.2646)	0.7901 (0.1846)	0.8606 (0.1454)	0.9515 (0.1401)
4 th Quintile	0.6575 (0.2561)	0.7530 (0.2828)	0.7668 (0.2094)	0.8308 (0.1720)	0.9062 (0.1658)
Value	0.6680 (0.2590)	0.8506 (0.3037)	0.8647 (0.2525)	0.9230 (0.2548)	0.9683 (0.2088)

(b) $\beta_{60m,d}$

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	0.9908 (0.2332)	1.1213 (0.1630)	1.1294 (0.1332)	1.1209 (0.0972)	1.1321 (0.0927)
2 nd Quintile	0.8252 (0.2131)	0.8782 (0.1459)	0.8774 (0.0817)	0.9115 (0.0534)	1.0143 (0.0951)
3 rd Quintile	0.7131 (0.1963)	0.7732 (0.1852)	0.7648 (0.1214)	0.8429 (0.1043)	0.9398 (0.1090)
4 th Quintile	0.6401 (0.1922)	0.7154 (0.1759)	0.7427 (0.1325)	0.8060 (0.1291)	0.8916 (0.1324)
Value	0.6420 (0.1898)	0.8085 (0.2018)	0.8437 (0.1654)	0.9018 (0.1920)	0.9390 (0.1404)

(c) $\beta_{120m,m}$

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	1.4558 (0.1565)	1.4426 (0.0819)	1.3696 (0.0779)	1.2571 (0.0645)	1.0071 (0.0408)
2 nd Quintile	1.2374 (0.1441)	1.1801 (0.1204)	1.1150 (0.0830)	1.0764 (0.1002)	0.9485 (0.0919)
3 rd Quintile	1.0862 (0.1675)	1.0356 (0.1543)	0.9822 (0.1219)	0.9900 (0.1054)	0.8573 (0.0608)
4 th Quintile	1.0019 (0.1862)	0.9807 (0.1485)	0.9144 (0.1302)	0.9191 (0.1190)	0.7912 (0.1056)
Value	1.0325 (0.1872)	1.0648 (0.1614)	0.9931 (0.1651)	0.9958 (0.1898)	0.8320 (0.1193)

Note: This table presents mean and standard deviation in parenthesis of selected beta components.

Table 8: Summary Statistics for $\beta_{12m,d,t} - \beta_{12m,d,t-12}$

(a) Mean

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	-0.0136	-0.0121	-0.0122	-0.0065	-0.0096
2 nd Quintile	-0.0068	0.0008	0.0017	-0.0014	0.0009
3 rd Quintile	-0.0002	0.0053	0.0040	0.0085	0.0010
4 th Quintile	0.0058	0.0130	0.0132	0.0075	0.0064
Value	0.0110	0.0182	0.0134	0.0115	0.0126

(b) Standard Deviation

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	0.2257	0.2028	0.1669	0.1538	0.0981
2 nd Quintile	0.1920	0.1612	0.1356	0.1006	0.1014
3 rd Quintile	0.1714	0.1576	0.1268	0.1097	0.1236
4 th Quintile	0.1888	0.1801	0.1519	0.1268	0.1280
Value	0.1890	0.1920	0.2190	0.1903	0.1908

(c) Max

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	0.5590	0.5552	0.3908	0.5880	0.2169
2 nd Quintile	0.5765	0.5010	0.3334	0.3114	0.3222
3 rd Quintile	0.6175	0.5428	0.2921	0.3590	0.3216
4 th Quintile	0.7761	0.6311	0.3825	0.5203	0.4844
Value	0.7336	0.5739	0.5482	0.5241	0.8132

(d) Min

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	-0.6557	-0.6318	-0.5328	-0.5253	-0.3095
2 nd Quintile	-0.4929	-0.4182	-0.3788	-0.2492	-0.3399
3 rd Quintile	-0.3905	-0.4612	-0.3765	-0.3488	-0.3951
4 th Quintile	-0.4041	-0.4348	-0.4659	-0.3697	-0.2814
Value	-0.4995	-0.4432	-0.5146	-0.4449	-0.3696

Note: This table presents summary statistics for the annual change in $\beta_{12m,d}$. The sample period is between 1970 and 2010.

Table 9: Summary Statistics for Beta Components, January 2009 to December 2010

(a) $\beta_{12m,d}$

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	1.0639 (0.0615)	0.9940 (0.0751)	0.9807 (0.0263)	0.9230 (0.0437)	0.7914 (0.0620)
2 nd Quintile	1.0154 (0.0842)	1.0417 (0.0680)	0.9600 (0.0614)	0.9987 (0.0256)	0.9437 (0.0510)
3 rd Quintile	1.0210 (0.1174)	1.1160 (0.0972)	1.0199 (0.0803)	1.1531 (0.0463)	1.0213 (0.0645)
4 th Quintile	1.0548 (0.1370)	1.1782 (0.1356)	1.1300 (0.1404)	1.0991 (0.0455)	1.1618 (0.0530)
Value	1.1699 (0.1735)	1.4356 (0.2318)	1.2131 (0.1810)	1.2996 (0.0974)	1.1991 (0.1286)

(b) $\beta_{60m,d}$

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	1.0419 (0.0040)	1.0015 (0.0141)	1.0120 (0.0112)	0.9418 (0.0110)	0.8517 (0.0152)
2 nd Quintile	0.9803 (0.0054)	1.0427 (0.0067)	0.9482 (0.0020)	0.9685 (0.0025)	0.9293 (0.0058)
3 rd Quintile	0.9539 (0.0133)	1.0850 (0.0045)	0.9866 (0.0047)	1.1013 (0.0097)	1.0661 (0.0073)
4 th Quintile	0.9491 (0.0224)	1.0969 (0.0124)	1.0150 (0.0157)	1.0714 (0.0062)	1.1309 (0.0122)
Value	1.0114 (0.0315)	1.2388 (0.0351)	1.0481 (0.0263)	1.1644 (0.0255)	1.0880 (0.0210)

(c) $\beta_{120m,m}$

	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	1.5266 (0.0634)	1.3525 (0.0383)	1.2950 (0.0344)	1.2307 (0.0508)	0.9310 (0.0152)
2 nd Quintile	1.2376 (0.0364)	1.0640 (0.0143)	1.0345 (0.0179)	0.9853 (0.0213)	0.7898 (0.0220)
3 rd Quintile	0.9883 (0.0130)	0.9612 (0.0342)	0.9104 (0.0390)	0.9923 (0.0419)	0.8334 (0.0345)
4 th Quintile	0.9155 (0.0240)	0.9717 (0.0481)	0.9026 (0.0649)	0.9682 (0.0383)	0.7630 (0.0630)
Value	1.1079 (0.0528)	1.1541 (0.0661)	0.9821 (0.0663)	1.0447 (0.0624)	0.9066 (0.0673)

Note: This table presents mean and standard deviation in parenthesis of selected beta components during the financial crisis.

Table 10: Correlation of 12 Month Change in Betas with Treasury Bill Rate

(a) $\beta_{12m,d,t} - \beta_{12m,d,t-12}$					
	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	-0.1536 (0.1603)	-0.1547 (0.1603)	0.1391 (0.1606)	0.0335 (0.1621)	0.2436 (0.1573)
2 nd Quintile	-0.2478 (0.1572)	-0.1763 (0.1597)	-0.1519 (0.1603)	-0.1541 (0.1603)	-0.1311 (0.1608)
3 rd Quintile	-0.2739 (0.1560)	-0.2350 (0.1577)	-0.1587 (0.1602)	-0.1127 (0.1612)	-0.0349 (0.1621)
4 th Quintile	-0.2301 (0.1579)	-0.1658 (0.1600)	-0.1024 (0.1614)	-0.0132 (0.1622)	0.0160 (0.1622)
Value	-0.2520 (0.1570)	-0.2417 (0.1574)	-0.1689 (0.1599)	-0.1100 (0.1612)	-0.1186 (0.1611)
(b) $\beta_{60m,d,t} - \beta_{60m,d,t-12}$					
	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	-0.1075 (0.1613)	-0.0631 (0.1619)	0.0717 (0.1618)	0.1679 (0.1599)	0.3120 (0.1541)
2 nd Quintile	-0.1805 (0.1596)	-0.2242 (0.1581)	-0.1420 (0.1606)	-0.1557 (0.1602)	0.0619 (0.1619)
3 rd Quintile	-0.2022 (0.1589)	-0.1862 (0.1594)	-0.2430 (0.1574)	-0.2862 (0.1554)	0.1243 (0.1610)
4 th Quintile	-0.2173 (0.1583)	-0.3039 (0.1545)	-0.3920 (0.1492)	-0.3177 (0.1538)	-0.2094 (0.1586)
Value	-0.3767 (0.1503)	-0.5182 (0.1387)	-0.4563 (0.1443)	-0.4943 (0.1410)	-0.3884 (0.1495)
(c) $\beta_{120m,m,t} - \beta_{120m,m,t-12}$					
	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	-0.0645 (0.1619)	-0.0020 (0.1622)	0.0469 (0.1620)	0.1927 (0.1592)	0.2508 (0.1570)
2 nd Quintile	-0.1695 (0.1599)	-0.1955 (0.1591)	-0.1756 (0.1597)	-0.1110 (0.1612)	0.0867 (0.1616)
3 rd Quintile	-0.2203 (0.1582)	-0.2586 (0.1567)	-0.2296 (0.1579)	-0.1764 (0.1597)	-0.0369 (0.1621)
4 th Quintile	-0.2956 (0.1550)	-0.3830 (0.1499)	-0.3378 (0.1527)	-0.3349 (0.1529)	-0.2021 (0.1589)
Value	-0.4516 (0.1447)	-0.4034 (0.1484)	-0.3916 (0.1493)	-0.4317 (0.1463)	-0.3893 (0.1494)

Note: This table presents correlation of 12 month changes in beta components and the Treasury bill rate at the beginning of the 12 month period, with standard errors presented in parenthesis. The sample period is between 1970 and 2010.

Table 11: Correlation of 11 Month Change in Betas with 12 Month Change in Value-Weighted Average of Portfolio Book-to-Market Ratio

(a) $\beta_{12m,d,t} - \beta_{12m,d,t-11}$					
	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	-0.1057 (0.1613)	-0.2518 (0.1570)	-0.2287 (0.1579)	-0.2712 (0.1561)	-0.3063 (0.1544)
2 nd Quintile	-0.0845 (0.1616)	0.0763 (0.1617)	-0.0530 (0.1620)	-0.0859 (0.1616)	-0.3015 (0.1547)
3 rd Quintile	0.0437 (0.1621)	-0.0858 (0.1616)	-0.0017 (0.1622)	0.0828 (0.1617)	-0.0158 (0.1622)
4 th Quintile	0.2030 (0.1588)	0.3155 (0.1539)	0.3988 (0.1488)	0.1987 (0.1590)	0.2885 (0.1553)
Value	0.4304 (0.1464)	0.5193 (0.1386)	0.6329 (0.1256)	0.4237 (0.1469)	0.3524 (0.1518)

(b) $\beta_{60m,d,t} - \beta_{60m,d,t-11}$					
	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	-0.2406 (0.1575)	-0.3013 (0.1547)	-0.2676 (0.1563)	-0.1309 (0.1608)	-0.0287 (0.1622)
2 nd Quintile	-0.2344 (0.1577)	-0.1529 (0.1603)	-0.2016 (0.1589)	-0.1972 (0.1590)	-0.0981 (0.1614)
3 rd Quintile	-0.1517 (0.1603)	-0.1737 (0.1598)	-0.2770 (0.1559)	-0.0721 (0.1618)	-0.1686 (0.1599)
4 th Quintile	-0.0534 (0.1620)	-0.0122 (0.1622)	0.0439 (0.1621)	-0.0265 (0.1622)	-0.0001 (0.1622)
Value	0.0378 (0.1621)	0.0142 (0.1622)	0.1626 (0.1601)	0.1090 (0.1613)	0.0539 (0.1620)

(c) $\beta_{120m,m,t} - \beta_{120m,m,t-11}$					
	Small	2 nd Quintile	3 rd Quintile	4 th Quintile	Large
Growth	-0.2952 (0.1550)	-0.3665 (0.1509)	-0.2608 (0.1566)	-0.2602 (0.1566)	0.1275 (0.1609)
2 nd Quintile	-0.3149 (0.1540)	-0.2380 (0.1576)	-0.2211 (0.1582)	-0.2377 (0.1576)	-0.0854 (0.1616)
3 rd Quintile	-0.1811 (0.1595)	-0.1736 (0.1598)	-0.2313 (0.1578)	-0.1276 (0.1609)	-0.1474 (0.1604)
4 th Quintile	-0.0983 (0.1614)	-0.0619 (0.1619)	-0.0532 (0.1620)	-0.0963 (0.1615)	0.0618 (0.1619)
Value	0.0967 (0.1615)	0.0079 (0.1622)	0.0174 (0.1622)	0.0795 (0.1617)	-0.0655 (0.1619)

Note: This table presents correlation of 11 month changes in beta components and 12 month change in the value-weighted average of portfolio book-to-market ratio, with standard errors presented in parenthesis. The sample period is between 1970 and 2010.

Table 12: Correlations Between Beta Components

	Mean	Median	Std. Dev.	Min	Max
$\beta_{12m,d}$ and $\beta_{60m,d}$	0.6823	0.6968	0.0881	0.3547	0.8463
$\beta_{12m,d}$ and $\beta_{120m,m}$	0.1220	0.1620	0.2592	-0.2608	0.5566
$\beta_{60m,d}$ and $\beta_{120m,m}$	0.4426	0.4795	0.2410	-0.0568	0.7876

Note: This table presents summary statistics on the correlations between the short-, medium- and long-run beta components for the 25 size and book-to-market portfolios. The sample period is between 1970 and 2010.

Table 13: Summary Statistics for Performance Measures of the Three Component Beta Model based on Simulated Artificial Factors

	Number of Mispriced Assets at 1% Level	Number of Mispriced Assets at 5% Level	RMSPE	SSPE	Adj. R^2
Mean	2.5944	5.4892	0.1555	0.6118	0.5536
Median	2.0000	5.0000	0.1550	0.6005	0.5618
Std. Dev.	1.3844	1.9791	0.0174	0.1362	0.0994
5% Quantile	1.0000	2.0000	0.1276	0.4068	0.3778
95% Quantile	5.0000	9.0000	0.1847	0.8527	0.7032

Note: We generate daily and monthly artificial factors replacing the daily and monthly observations of the return on the market portfolio. The three component beta model is then estimated based on the time series of daily and monthly artificial factors. We repeat this exercise 5,000 times and this table reports the summary statistics for performance measures from these simulations.

Table 14: Performance Measures of Asset Pricing Models for 25 Size and Book-to-Market, 10 Momentum and 30 Industry Portfolios

	Number of Mispriced Assets at 1% Level	Number of Mispriced Assets at 5% Level	RMSPE	SSPE	Adj. R^2
Three Comp. Beta Model with					
$\beta_{1m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	7	17	0.1921	2.3977	0.3316
$\beta_{3m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	6	12	0.1866	2.2628	0.3692
$\beta_{6m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	7	14	0.1911	2.3728	0.3385
$\beta_{1m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	7	16	0.2052	2.7381	0.2367
$\beta_{3m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	6	16	0.2106	2.8835	0.1962
$\beta_{6m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	6	14	0.2028	2.6746	0.2544
$\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	6	15	0.2074	2.7967	0.2204
Fama-French Three Factor Model	8	10	0.2007	2.6191	0.2699
CAPM with constant β	9	17	0.2259	3.3164	0.0792

Note: This table presents performance measures for asset pricing models in accounting for the cross-sectional variation in monthly returns on 25 size and book-to-market, 10 momentum and 30 industry portfolios, over the sample period between 1970 and 2010. Number of mispriced assets is the number of assets out of 25 size and book-to-market portfolios with an average pricing error significantly different than zero at 1% and 5% levels based on Fama-MacBeth standard errors with Newey-West correction. RMSPE, SSPE and Adj. R^2 are root mean square pricing error, sum of square pricing errors and adjusted R^2 , respectively.

Table 15: Performance Measures of Asset Pricing Models for Quarterly Returns on 25 Size and Book-to-Market Portfolios

	Number of Mispriced Assets at 1% Level	Number of Mispriced Assets at 5% Level	RMSPE	SSPE	Adj. R^2
Three Comp. Beta Model with					
$\beta_{1m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	3	6	0.3816	3.6405	0.7063
$\beta_{3m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	4	4	0.3894	3.7908	0.6941
$\beta_{6m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	3	5	0.4019	4.0371	0.6743
$\beta_{1m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	2	5	0.3729	3.4756	0.7196
$\beta_{3m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	2	3	0.3503	3.0686	0.7524
$\beta_{6m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	1	3	0.3424	2.9314	0.7635
$\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	1	3	0.3551	3.1517	0.7457
Fama-French Three Factor Model	3	4	0.3549	3.1497	0.7459
CAPM with constant β	7	11	0.6863	11.7743	0.0617

Note: This table presents performance measures for asset pricing models in accounting for the cross-sectional variation in quarterly returns on 25 size and book-to-market portfolios, over the sample period between 1970 and 2010. Number of mispriced assets is the number of assets out of 25 size and book-to-market portfolios with an average pricing error significantly different than zero at 1% and 5% levels based on Fama-MacBeth standard errors with Newey-West correction. RMSPE, SSPE and Adj. R^2 are root mean square pricing error, sum of square pricing errors and adjusted R^2 , respectively.

Table 16: SSPE of Asset Pricing Models over the Business Cycle

(a) Expansions

	CAPM with Constant Beta	Three Factor Fama-French Model	Three Component Beta Model with $\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$
Small	0.3832	0.1654	0.1726
2 nd Quintile	0.1226	0.0062	0.0227
3 rd Quintile	0.0857	0.0319	0.0491
4 th Quintile	0.0317	0.0503	0.0359
Large	0.3880	0.0959	0.0760
Growth	0.2673	0.1895	0.1833
2 nd Quintile	0.0641	0.0200	0.0200
3 rd Quintile	0.1213	0.0136	0.0166
4 th Quintile	0.1787	0.0538	0.0539
Value	0.3797	0.0726	0.0823
Total	1.0112	0.3496	0.3561

(b) Recessions

	CAPM with Constant Beta	Three Factor Fama-French Model	Three Component Beta Model with $\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$
Small	0.4115	0.3763	0.2597
2 nd Quintile	0.5357	0.1597	0.0879
3 rd Quintile	0.4827	0.2489	0.2489
4 th Quintile	0.0561	0.1419	0.1261
Large	0.6253	0.2212	0.0811
Growth	0.3854	0.5173	0.3197
2 nd Quintile	0.2858	0.0976	0.1172
3 rd Quintile	0.3027	0.0787	0.1178
4 th Quintile	0.2267	0.0545	0.0440
Value	0.9108	0.4000	0.2048
Total	2.1113	1.1480	0.8037

Note: This table presents the SSPE for monthly returns on quintiles of size and book-to-market portfolios over the business cycle. A month is classified as either an expansion or recession phase of the business cycle through the NBER classification. The sample period is between 1970 and 2010.

Table 17: Performance Measures of Asset Pricing Models for Monthly Returns on 25 Size and Book-to-Market Portfolios Excluding Small Firms

	Number of Mispriced Assets at 1% Level	Number of Mispriced Assets at 5% Level	RMSPE	SSPE	Adj. R^2
Three Comp. Beta Model with					
$\beta_{1m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	1	3	0.1068	0.2283	0.6924
$\beta_{3m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	1	3	0.1071	0.2293	0.6911
$\beta_{6m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	1	4	0.1034	0.2140	0.7116
$\beta_{1m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	2	2	0.1029	0.2118	0.7147
$\beta_{3m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	2	3	0.1029	0.2119	0.7145
$\beta_{6m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	2	2	0.0866	0.1498	0.7981
$\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	2	2	0.0818	0.1340	0.8195
Fama-French Three Factor Model	2	4	0.0851	0.1447	0.8050
CAPM with constant β	6	9	0.1848	0.6829	0.0837

Note: This table presents performance measures for asset pricing models in accounting for the cross-sectional variation in monthly returns on 25 size and book-to-market portfolios excluding the five small quintile portfolios, over the sample period between 1970 and 2010. Number of mispriced assets is the number of assets out of 20 size and book-to-market portfolios with an average pricing error significantly different than zero at 1% and 5% levels based on Fama-MacBeth standard errors with Newey-West correction. RMSPE, SSPE and Adj. R^2 are root mean square pricing error, sum of square pricing errors and adjusted R^2 , respectively.

Table 18: Performance Measures of Asset Pricing Models for Monthly Returns on 25 Size and Book-to-Market Portfolios with Short and Medium Term Betas Corrected for Nonsynchronous Trading

	Number of Mispriced Assets at 1% Level	Number of Mispriced Assets at 5% Level	RMSPE	SSPE	Adj. R^2
Three Comp. Beta Model with					
$\beta_{1m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	2	4	0.1216	0.3695	0.7304
$\beta_{3m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	2	3	0.1203	0.3617	0.7361
$\beta_{6m,d}$ & $\beta_{12m,d}$ & $\beta_{120m,m}$	1	5	0.1221	0.3725	0.7282
$\beta_{1m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	3	4	0.1299	0.4219	0.6922
$\beta_{3m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	3	3	0.1212	0.3674	0.7319
$\beta_{6m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	2	4	0.1195	0.3567	0.7397
$\beta_{12m,d}$ & $\beta_{60m,d}$ & $\beta_{120m,m}$	3	4	0.1211	0.3666	0.7325
Fama-French Three Factor Model	3	4	0.1253	0.3927	0.7134
CAPM with constant β	6	11	0.2102	1.1047	0.1972

Note: This table presents performance measures for asset pricing models in accounting for the cross-sectional variation in monthly returns on 25 size and book-to-market portfolios where betas estimated using daily data are corrected for potential nonsynchronous trading. The sample period is between 1970 and 2010. Number of mispriced assets is the number of assets out of 25 size and book-to-market portfolios with an average pricing error significantly different than zero at 1% and 5% levels based on Fama-MacBeth standard errors with Newey-West correction. RMSPE, SSPE and Adj. R^2 are root mean square pricing error, sum of square pricing errors and adjusted R^2 , respectively.