The Reaction of Stock Returns to News about Fundamentals

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Stock returns react negatively to good news in good times. To account for this stylized fact, we consider a general equilibrium asset pricing model where the growth rate of dividends switches between a high and a low value. Investors never observe the true state of the dividend growth process but learn about it through two sources of information: dividend realizations and external signals. External signals are assumed to be more precise in good times than in bad times. Under this assumption, investors in our model assign a lower probability to the high growth state following not only negative news from external signals, as one would expect, but also following large positive news. Furthermore, the negative effect of positive news on investors' belief about the high growth state is not only more likely but also more pronounced in good than in bad times. Linear regression analysis of simulated data from our calibrated model shows that positive news has a negative effect on unexpected returns in good times and a positive effect in bad times.

1. Introduction

How stock prices react to news is of central importance to financial decision making. A key problem in analyzing the reaction of stock returns to news is, however, that it can be difficult to determine when such news arrives. Furthermore, it can also be difficult to accurately measure the information content of an announcement. Scheduled information releases such as macroeconomic announcements provide a good starting point. First of all, the timing of these information releases is generally known in advance by financial market participants. Secondly, investors' expectations about scheduled announcements can be quantified by employing either model- or survey-based measures. Hence, it is not surprising to find a large literature on the reaction of stock returns to macroeconomic news.

One of the stylized facts in this literature is that the sign of the reaction depends on the underlying state of the economy. Specifically, stock prices generally react negatively to "good" news, i.e. higher than expected economic activity, in "good" times, i.e. during periods of good economic conditions. For example, McQueen and Roley (1993) write "[...] when the economy is strong the stock market responds negatively to news about higher real economic activity.". Boyd et al. (2005) find that a lower than expected unemployment rate is actually bad news for stock returns in expansions. Andersen et al. (2007) and Cenesizoglu (2011) analyze the reaction of returns on different portfolios to a wide range of macroeconomic news over the business cycle and find similar empirical evidence. Gilbert (2011) provides further supporting evidence for this stylized fact based on the relation between stock returns and future revisions to macroeconomic variables.

This pattern of stock price responses to macroeconomic news seems surprising (see McQueen and Roley (1993) and Boyd et al. (2005)). It is also relatively hard to justify from a theoretical standpoint. One possibility is based on the monetary response function of the Federal Reserve. Following good news about real economic activity, the Federal Reserve is generally expected to raise interest rates to reduce inflationary pressures. This, in turn, should not only decrease stock prices but also increase future interest rates. In other words, if the stylized fact were due to the monetary response function of the Federal Reserve, one would expect stock and bond prices to respond in a similar fashion. However, this is not found to be supported empirically as argued by McQueen and Roley (1993) and Boyd et al. (2005). Another possibility is based on a state-dependent elasticity of intertemporal substitution, i.e. investors' willingness to substitute future for current consumption. Specifically, one needs to assume that the elasticity of intertemporal substitution (the inverse of the risk aversion in their power utility frameworks) is below one in good times and above one in bad times. However, the empirical evidence in Gordon and St-Amour (2000) and Brandt and Wang (2003) suggests that the elasticity of intertemporal substitution is systematically lower in recessions or bear markets than in expansions or bull markets, the opposite of what is required.

This paper proposes a third possibility based on state-dependent precision of signals. Specifically, we consider a general equilibrium asset pricing model where the growth rate of dividends switches between a high and a low value. The investor never observes the true state of the dividend growth process but learns about it through two sources of information: dividend realizations observed every period and external signals observed only on announcement days. We assume that external signals are more precise in the high growth state than in the low growth state. We then solve for the unexpected return on the risky asset and analyze its reaction to news from external signals. To do this, we first simulate daily returns from our model calibrated to U.S. data. Using simulated data only on announcement days, we then estimate a linear regression of unexpected returns on news from external signals while controlling for other factors such as news from dividend realizations. We distinguish between positive and negative news in good, normal and bad times, depending on the investor's beliefs prior to announcements. Matching the empirical results in the literature, we find that positive news can have a negative effect on unexpected returns in good times and a positive effect in normal and bad times.

The intuition for our results on the effect of news on unexpected returns follows from the investor's valuation of the risky asset in different states and how he updates his beliefs following news from an external signal. In a general equilibrium model where the aggregate stock market is treated as a claim on the aggregate dividend, a higher dividend growth rate does not only increase expected future dividends but also expected future discount rates. In other words, the dividend growth rate affects the price of the risky asset through two channels. The first effect increases the investor's demand for the risky asset and, hence, its price. The second effect lowers the price of the risky asset due to the investor's attempt to smooth his consumption financed by selling the risky asset. Which of these two effects dominates in equilibrium depends on the investor's elasticity of intertemporal substitution. Under our assumption of an elasticity of intertemporal substitution greater than one, the first effect dominates the second one. The investor values the risky asset more and is willing to pay a higher price per unit of dividend paid in the high growth state than in the low growth state. In other words, under this assumption, the price-dividend ratio of the risky asset is always higher in the high growth state than in the low growth state.

The assumption of higher external signal precision in the high growth state implies that the distribution of the external signal has fatter tails in the low growth state. In other words, the probability of observing large positive and negative news is higher in the low growth state than in the high growth state. Thus, the investor assigns a lower probability to the high growth state following not only negative news, as one would expect, but following also large positive news. On the other hand, following small positive news, the investor assigns a higher probability to the high growth state. More importantly, independent of the realized dividend news in the same period, the negative effect of positive news on the investor's belief about the high growth state is not only more likely but also more pronounced in good times than in bad times. The intuition behind this follows from the definition of good times as periods where the investor assigns a higher prior probability to the high growth state before observing the information revealed in a given period. This fact and the investor's valuation of the risky asset in different states imply that the negative effect of positive news on unexpected returns are not only more likely but also more pronounced in good times than in bad times. Furthermore, given that not all positive news decrease the probability that the investor assigns to the high growth state, one might observe in a linear regression framework that positive news has a negative effect on unexpected returns in good times and a positive effect in bad times.

Our assumption that external signals are more precise in the high growth state is supported by the empirical evidence in van Nieuwerburgh and Veldkamp (2006) who find that the median forecast error and the dispersion among forecasters' beliefs increase in recessions. They argue that this is evidence of lower precision of signals in recessions than in expansions.[1] Under the commonly used alternative assumption of constant external signal precision (e.g. Veronesi (2000)), the investor assigns a higher probability to the high growth state following positive news and a lower probability following negative news. Hence, one cannot account for the stylized fact in the literature in a model like ours under this alternative assumption.

David (1997) develops a general equilibrium model where the investor learns about unobserved profitability switches between different industries in the economy and finds that stock returns react more strongly to news about the relative profitability of different industries during periods of low confidence or high uncertainty. Veronesi (1999) analyzes the reaction of the aggregate stock market to news about the growth rate of dividends and finds that stock prices overreact to bad news when the growth rate of dividends is high and underreact to good news when it is low. Instead, we focus on the effect of news from external signals rather than news from dividend realizations. The closest paper to ours in terms of the model used is Veronesi (2000). Our paper differs from his in several aspects. First of all, we analyze the effect of news from external signals on returns in different periods depending on the investor's beliefs. On the other hand, Veronesi (2000) analyzes the relation between stock returns and the information quality of an external signal and finds that higher quality information leads to an increase in the risk premium. More importantly, our model can account for the stylized fact in the literature due to our assumption on state-dependent precision of external signals. As discussed above, this is not possible in Veronesi (2000)'s model due to his assumption of constant external signal precision.

The rest of the paper is organized as follows: Section 2 presents our model and solves for the unexpected return on the risky asset. Section 3.1 discusses the motivation behind certain assumptions of our model. Section 3.2 presents evidence that our model can account for the stylized fact in a linear regression framework. Section 3.3 discusses the intuition behind our results in Section 3.2 based on some theoretical results. Section 4 concludes.

[1] Veronesi (1999) and Patton and Timmermann (2010) also provide some evidence that the dispersion of forecasts is higher during recessions. Swanson and van Dijk (2006) find that there is a clear increase in the volatility of revisions to macroeconomic variables during recessions. We also provide some empirical evidence in support of this assumption in Section 3.1 using data releases about industrial production as our proxy for the external signal.

2. The Model

We consider a pure exchange economy (Lucas (1978)) in discrete time where the representative investor has recursive preferences over consumption as in Epstein and Zin (1989) and Weil (1990),

$$U(C_t) = \{(1-\beta)C_t^{(1-\gamma)/\theta} + \beta E_t[(U(C_{t+1}))^{1-\gamma}]^{1/\theta}\}^{\theta/(1-\gamma)} \tag{1}$$

where C_t denotes the investor's consumption in period t, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution (EIS), and $\theta = (1-\gamma)/(1-\psi^{-1})$. The investor's opportunity set consists of a risky asset whose supply is fixed and normalized to one and a risk-free asset. Dividends of the risky asset grow according to the following process:

$$\Delta d_t = \mu_{d,S_t} + \sigma_d \varepsilon_{d,t} \tag{2}$$

where $d_t = \log(D_t)$ is the log-dividend in period t, Δ denotes the first difference operator, i.e. $\Delta d_t = d_t - d_{t-1}$, and $\varepsilon_{d,t}$ is an independently and identically distributed Gaussian random variable with zero mean and unit variance. S_t is a latent state variable that determines the dividend growth rate in period t and follows a first-order two-state Markov chain with transition probability matrix $\mathbf{Q}$, i.e.

$$\{\Pr(S_t = j|S_{t-1} = i)\} = \{q_{ij}\} = \mathbf{Q} \quad \text{for } i,j = 1,2. \tag{3}$$

Without loss of generality, we assume that the dividend growth rate is higher in the first state, i.e. $\mu_{d,1} > \mu_{d,2}$. The investor never observes the true state variable but learns about it through not only dividend realizations but also imperfect external signals, x_m,

$$x_m = \mu_{x,S_t} + \sigma_{x,S_t}\varepsilon_{x,m}, \quad m = 1,2,\ldots \tag{4}$$

where $\varepsilon_{x,m}$ is an independently and identically distributed Gaussian random variable with zero mean and unit variance and is independent of $\varepsilon_{d,t}$ for all t and m. We assume that the investor observes dividend realizations, tracked by t, every period. Furthermore, the dividend growth process can also switch to a new state every period.[2] The external signals, tracked by m, are observed only on announcement periods, T_m^x, that are not necessarily regularly spaced.

2.1. Investor's Beliefs

In models like ours with learning, the investor's beliefs about the state of the dividend growth process play a central role. In this section, we characterize how the investor's beliefs evolve over time as new information about the state of the dividend growth process arrives.

[2] Our results continue to hold in a more general model where the dividend growth process is allowed to switch to a new state only on certain regime-switching periods.

Let $\tilde{\pi}_{j,t}$ denote the probability that the investor assigns to state j before observing the information revealed in period t (the dividend realization and possibly the external signal if t is an announcement period), i.e. his prior belief about state j. Similarly, let $\pi_{j,t}$ denote the probability that he assigns to state j after observing the information revealed in period t. The investor's information set in period t, $\mathcal{F}_t$, includes past and current dividend realizations and external signals as well as the information on whether t is an announcement period. Assuming that the investor has a given set of beliefs about the initial state of the dividend growth process before observing any dividend realizations or external signals, i.e. $\tilde{\pi}_{j,0}$ for $j = 1, 2$, the following lemma characterizes the investor's beliefs about the state variable:

LEMMA 1.

$$\pi_{j,t} = \begin{cases} \frac{\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_d})\tilde{\pi}_{j,t}}{\sum_{i=1}^{2} \phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_d})\tilde{\pi}_{i,t}} & \text{if } t \neq T_m^x \\ \frac{\frac{1}{\sigma_{x,j}}\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_d})\phi(\frac{x_m - \mu_{x,j}}{\sigma_{x,j}})\tilde{\pi}_{j,t}}{\sum_{i=1}^{2} \frac{1}{\sigma_{x,i}}\phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_d})\phi(\frac{x_m - \mu_{x,i}}{\sigma_{x,i}})\tilde{\pi}_{i,t}} & \text{if } t = T_m^x \end{cases} \quad \text{for } j = 1, 2 \text{ and for } m = 1, 2, \ldots \tag{5}$$

where $\tilde{\pi}_{j,t} = \sum_{i=1}^{2} \pi_{i,t-1} q_{ij}$ *and* $\phi(\cdot)$ *is the standard normal density function.*

Prior to observing the information revealed in a given period t, the investor knows that the dividend growth process might have switched to a new state according to the transition probability matrix. Hence, his prior beliefs about the new state variable, $\tilde{\pi}_{j,t}$, are weighted averages of his beliefs about the previous state variable, $\pi_{i,t-1}$, where the weights are the transition probabilities, q_{ij}. Given his prior beliefs for the state variable S_t, the investor then updates his beliefs according to Bayes' rule based on the additional information revealed by the dividend realization in period t (the first case of Equation (5)) as well as the external signal if t is an announcement period (the second case of Equation (5)).

2.2. Equilibrium Asset Prices and Returns

We characterize the price and the unexpected return of the risky asset in the following proposition:

PROPOSITION 1. *The price of the risky asset in period t is given by:*

$$\frac{P_t}{D_t} = \sum_{j=1}^{2} \lambda_j \pi_{j,t} \tag{6}$$

where λ_j *is the price-dividend ratio in state* j *and satisfies the fixed point equation for* $j = 1, 2$

$$\lambda_j = \beta \left(\sum_{i=1}^{2} q_{ij} (1 + \lambda_i)^{\theta} e^{a_i} \right)^{1/\theta} \tag{7}$$

where $a_i = (1-\gamma)\mu_{d,i} + (1-\gamma)^2 \sigma_d^2 / 2$.

Let r_t *denote the log return on the risky asset in period t, i.e.* $r_t = \log(\frac{P_t + D_t}{P_{t-1}})$*, then the unexpected log return on the risky asset in period t can be approximated by:*

$$r_t^* = r_t - \tilde{E}_t[r_t] \approx \frac{\lambda_1 - \lambda_2}{1 + \bar{\lambda}} (\pi_{1,t} - \tilde{\pi}_{1,t}) + \Delta d_t - \sum_{j=1}^{2} \mu_{d,j} \tilde{\pi}_{j,t} \tag{8}$$

where $\tilde{E}_t[\cdot]$ denotes the expectation conditional on the investor's prior beliefs before observing the dividend realization (and possibly the external signal) in period t. The long term average price-dividend ratio is $\overline{\lambda} = E[P_t/D_t] = \sum_{j=1}^{2} \Omega_j \lambda_j$ where $[\Omega_1, \Omega_2]'$ is the stationary distribution vector of the transition probability matrix $\mathbf{Q}$.

If the investor observes the true state variable, the price-dividend ratio takes one of the two values, λ_1 or λ_2, depending on the state variable. Given that the investor never observes the true state variable, the price-dividend ratio is a weighted average of λ_j's where the weights are the investor's beliefs about the state variable. Hence, the price-dividend ratio fluctuates between the two values as the investor receives additional information about the state variable and updates his beliefs. Similarly, the unexpected return on the risky asset is also determined by the unexpected dividend growth rate as well as the time variation in the investor's beliefs which , in turn, depend on the additional information revealed by the dividend realization and the external signal if it is an announcement period. These two sources of information affect unexpected returns differently. Dividend realizations do not only affect the investor's beliefs but also his consumption whereas external signals affect only his beliefs. Thus, we distinguish between additional information, i.e. news, revealed by dividend realizations and external signals. The news from the dividend realization in period t, $u_{d,t}$, is defined as the unexpected part of the realized dividend growth rate:

$$u_{d,t} = \Delta d_t - \tilde{\mu}_{d,t} \tag{9}$$

where $\tilde{\mu}_{d,t} = \sum_{j=1}^{2} \mu_{d,j} \tilde{\pi}_{j,t}$ is the expected dividend growth rate based on the investor's beliefs prior to observing the information in period t. Similarly, news from the external signal observed on the m^{th} announcement period, u_{x,T_m^x}, is defined as the unexpected part of the external signal:

$$u_{x,T_m^x} = x_m - \tilde{\mu}_{x,T_m^x} \tag{10}$$

where $\tilde{\mu}_{x,T_m^x} = \sum_{j=1}^{2} \mu_{x,j} \tilde{\pi}_{j,T_m^x}$ is the expected part of the external signal based on the investor's beliefs prior to observing the information in period T_m^x. In this paper, we are interested in the effect of news from external signals on unexpected returns. For the rest of the paper, the news variable refers to news from external signals and the return refers to unexpected returns, unless otherwise stated. We refer to news from dividend realizations as the dividend news variable.

3. The Reaction of Stock Returns to External Signals

In this section, we analyze the effect of news variables on returns while distinguishing between positive and negative news variables in different states and keeping other factors, such as the dividend news variable, fixed. To do so, we first calibrate the parameters of our model to US data and discuss the motivation behind the two main assumptions in our model. In a linear regression framework, we then present evidence based on simulated data that positive news variables can have a negative effect on returns in good times and a positive one in normal and bad times. Finally, based on some theoretical results, we discuss the intuition behind the mechanism that is driving our results.

3.1. Calibration

The effect of news variables on returns in our model depends closely on two parameters, the precision of the external signal in different states ($1/\sigma_{x,1}$ and $1/\sigma_{x,2}$) and the elasticity of intertemporal substitution (ψ). In this section, we first discuss the motivation behind our assumptions on these two parameters. We then present the calibration of other model parameters.

We assume that the external signal has a higher mean and a lower volatility in the high growth state of the dividend growth process, i.e. $\mu_{x,1} > \mu_{x,2}$ and $\sigma_{x,1} < \sigma_{x,2}$. In other words, we assume that the external signal is not only informative about the state variable but also less precise in the low growth state. The strongest empirical evidence in support of the hypothesis that the information quality and the precision of signals are lower during periods of weak economic activity is provided in van Nieuwerburgh and Veldkamp (2006). Specifically, they find that the median error and the dispersion of forecasts for future real GDP increase in recessions.[3] Here, we also provide some additional empirical evidence in support of our assumption. To this end, we use data releases about industrial production as our proxy for the external signal. We consider two approaches to proxy for the volatility of the external signal. The first is simply the in-sample standard deviation of the log growth rate of industrial production. The second is the standard deviation of news about the growth rate of industrial production.[4] The news variable is defined as the difference between the growth rate of industrial production, as it is first reported, and the median forecast from the Money Market Services (MMS) survey.[5] Table 1 presents the two proxies for the volatility of the external signal over business cycle phases as well as over the whole sample between 1980 and 2009. An observation is considered as a recession observation if its release date is in a recession period as defined by NBER. Our results based on several test statistics suggest that the volatility of the external signal is higher in recessions than in expansions. Results based on the first approach presented in Panel B of Table 1 also serve as our calibration for the mean and the precision of the external signal in different states.

It is well known that the elasticity of intertemporal substitution plays an important role in asset pricing models. Hence, it is not surprising that the implications of our model closely depend on this parameter. The boundary case is $\psi = 1$. In this paper, following Bansal and Yaron (2004) among others, we assume

[3] Veronesi (1999) and Patton and Timmermann (2010) provide some evidence that the dispersion of forecasts is higher during recessions. Swanson and van Dijk (2006) find that there is a clear increase in the volatility of revisions to macroeconomic variables during recessions.

[4] Our second approach is similar to that of van Nieuwerburgh and Veldkamp (2006) as we also use median forecasts from a panel of analysts. However, we focus on the volatility of the news variable over the business cycle rather than the dispersion among forecasters.

[5] We choose to use MMS data on industrial production mainly due to data availability reasons. Survey data for industrial production from MMS is available since 1980 at the monthly frequency resulting in a total of 360 observations. On the other hand, survey data for GDP from MMS is only available at the quarterly frequency since 1990 resulting in a total of 80 observations. One could also consider using survey data from the Survey of Professional Forecasters (SPF) at the Federal Reserve Bank of Philadelphia. Survey data on both industrial production and GDP from SPF is available at the quarterly frequency since 1968 resulting in a total of 165 observations.

that the elasticity of substitution is greater than one and calibrate it to 1.5.[6] As we discuss in further detail below, under this assumption, the income effect dominates the (intertemporal) substitution effect and the price-dividend ratio is higher in the high growth state. This assumption is crucial and allows us to focus on the implications of state-dependent external signal precision for the effect of the news variable on returns. To see this, note that, under the alternative assumption that the EIS is less than one, the price-dividend ratio is lower in the high growth state. Hence, in our model, one might still observe a negative effect of positive news variables on returns (although in both good and bad times) in a linear regression framework even under the assumption of constant external signal precision. Our assumption of an EIS greater than one rules out this possibility.

We now turn our attention to the calibration of other model parameters. Although still important, these parameters do not significantly affect our results in Section 3.2. In other words, we obtain coefficient estimates with the same sign but different magnitudes for a wide range of values for these parameters. We calibrate the parameters of the dividend growth process to the corresponding daily values of parameter estimates from a two-state Markov regime switching model for the log growth rate of quarterly real GDP between 1950 and 2009.[7] We assume that the external signal is observed every 21 periods, corresponding to monthly announcements under the assumption that there are 21 trading days in a month.[8] We assume that the investor has a coefficient of risk aversion of 7.5 and a daily time impatience parameter of 0.9998 corresponding to an annual value of 0.9508. Table 2 presents parameter values for the assumption that the precision of the external signal is higher in the high growth state than in the low growth state. We calibrate the volatility of the external signal to 0.7124% for the assumption that the precision of the external signal is constant.

3.2. Simulation Results

In this section, we analyze the effect of news variables on returns in our model in a linear regression framework. To this end, we first simulate data from our model based on the calibrated parameters. Specifically,

[6] This assumption is similar to assuming that the coefficient of risk aversion is less than one in the power utility framework.

[7] We estimate the following model via maximum likelihood: $\Delta\ln(GDP_t) = \mu_{d,S_t}^{\text{quarterly}} + \sigma_d^{\text{quarterly}}\nu_t$ where S_t is a two-state Markov chain with a transition probability matrix $\mathbf{Q}^{\text{quarterly}}$ and ν_t is an independently and identically distributed Gaussian random variable with zero mean and unit variance. GDP_t is the real GDP in quarter t and is available from the Federal Reserve of Bank of St. Louis. Following Hamilton (1989), we choose to use output rather than consumption or dividends. First of all, in a pure exchange general equilibrium asset pricing model like ours, dividends, consumption and output are identical. Secondly, evidence of regime-switching type behavior is more pronounced in real GDP than in consumption or dividends. The estimated means and standard deviation are $\mu_{d,1}^{\text{quarterly}} = 1.0838\%$, $\mu_{d,2}^{\text{quarterly}} = -0.2828\%$, $\sigma_d^{\text{quarterly}} = 0.8281\%$. The diagonal elements of the transition probability matrix are 0.9366 and 0.7403. All coefficient estimates are significant at the 5% level.

[8] Assuming that there are 63 trading days in a quarter, we use the following transformations to convert parameter estimates based on quarterly data to their corresponding daily values

$$\begin{aligned} \mu_{d,i}^{\text{daily}} &= \mu_{d,i}^{\text{quarterly}}/63 \text{ for } i = 1, 2, \\ \sigma_d^{\text{daily}} &= \sigma_d^{\text{quarterly}}/\sqrt{63}, \\ \mathbf{Q}^{\text{daily}} &= (\mathbf{Q}^{\text{quarterly}})^{1/63}. \end{aligned}$$

we simulate 100 samples with 21000 observations corresponding to 1000 announcement periods.[9] For each sample, we estimate the following linear regression model via OLS with heteroskedasticity consistent standard errors (White (1980)) using data only on announcement periods.

$$\begin{aligned} r^{*}_{T^x_m} &= \theta_1 + \theta_2 \tilde{\pi}_{1,T^x_m} + \theta_3 u_{d,T^x_m} + \theta_4 1_{\{u_{x,T^x_m} > 0\}} \\ &+ \theta_5 u_{x,T^x_m} 1_{\{u_{x,T^x_m} > 0\}} 1_{\{\tilde{\pi}_{1,T^x_m} > \pi_h\}} + \theta_6 u_{x,T^x_m} 1_{\{u_{x,T^x_m} \leq 0\}} 1_{\{\tilde{\pi}_{1,T^x_m} > \pi_h\}} \\ &+ \theta_7 u_{x,T^x_m} 1_{\{u_{x,T^x_m} > 0\}} 1_{\{\pi_l < \tilde{\pi}_{1,T^x_m} \leq \pi_h\}} + \theta_8 u_{x,T^x_m} 1_{\{u_{x,T^x_m} \leq 0\}} 1_{\{\pi_l < \tilde{\pi}_{1,T^x_m} \leq \pi_h\}} \\ &+ \theta_9 u_{x,T^x_m} 1_{\{u_{x,T^x_m} > 0\}} 1_{\{\tilde{\pi}_{1,T^x_m} \leq \pi_l\}} + \theta_{10} u_{x,T^x_m} 1_{\{u_{x,T^x_m} \leq 0\}} 1_{\{\tilde{\pi}_{1,T^x_m} \leq \pi_l\}} + \upsilon_{T^x_m} \end{aligned} \tag{11}$$

for $m = 1, 2, \ldots, 1000$ where $1_{\{\cdot\}}$ is the indicator function that takes the value of one if the condition in brackets is satisfied and zero otherwise. π_h and π_l are constants that allow us to distinguish between announcement periods in good, bad and normal times depending on the investor's beliefs prior to the announcement period. Specifically, any announcement period for which the investor's prior belief about the high growth state is above π_h (below π_l) can be considered to be an announcement period in good (bad) times. Any other announcement period can be considered to be an announcement period in normal times. In our empirical analysis, we consider two pairs of (π_h, π_l): $(0.75, 0.25)$ and $(0.90, 0.10)$.[10] The first pair roughly corresponds to the stationary probability distribution of the calibrated transition probability matrix. This empirical specification is similar to those considered in the literature. For example, to analyze the reaction of returns on the S&P 500 index to macroeconomic news, McQueen and Roley (1993) estimate a similar empirical specification where they distinguish between three states of the economy: good, normal and bad times.

Table 3 presents summary statistics for the coefficient estimates based on simulated data under the assumption of higher external signal precision in the high growth state. To remove the effect of random sampling on our empirical results, the same random shocks for $\varepsilon_{d,t}$ and $\varepsilon_{x,m}$ are used for corresponding samples in different panels of Table 3. Our results can be summarized as follows: First of all, coefficient estimates on $u_{x,T^x_m} 1_{\{u_{x,T^x_m} > 0\}} 1_{\{\tilde{\pi}_{1,T^x_m} > \pi_h\}}$ are almost always significantly negative and never significantly positive. This suggests that positive news variables generally have a negative effect on returns in good times when the investor's prior belief about the high growth is high. On the other hand, coefficient estimates on $u_{x,T^x_m} 1_{\{u_{x,T^x_m} > 0\}} 1_{\{\tilde{\pi}_{1,T^x_m} < \pi_l\}}$ are always significantly positive and never significantly negative. This suggests that positive news variables have a positive effect on returns in bad times when the investor's prior belief about the high growth is low. Furthermore, although not presented in Table 3, the effect of positive news variables in good times is always significantly different from that in bad times. In other words, a positive

[9] For each sample, we simulate 22600 observations. We remove the first 1260 periods from each sample to avoid any bias due to the initial state.

[10] We also considered an empirical specification without normal times, i.e. an empirical specification where $(\pi_h, \pi_l) = (0.50, 0.50)$. Our results are quite similar to those presented in Table 3.

news variable can have a negative effect on returns in good times and a significantly different positive one in bad times. In contrast to positive news variables, our results suggest that negative news variables always have a negative effect on returns independent of the investor's prior beliefs. This follows from the fact that coefficient estimates on negative news variables are always positive implying a negative effect on returns.[11] These results suggest that our model can generate empirical evidence similar to those in the literature when we consider the effect of news variables on returns in a linear regression framework.

As mentioned in Section 3.1 and discussed in further detail below, we argue that the assumption of higher external signal precision in the high growth state is the main mechanism driving our results in Table 3. To show that our results in Table 3 are not possibly due to our assumptions on other model parameters, we simulate data from our model under the assumption of constant external signal precision. Specifically, we keep all other parameters the same as above and calibrate the volatility of the external signal to 0.7124% for both states. Once again, to remove the effect of random sampling on our empirical results, we use the same random shocks for $\varepsilon_{d,t}$ and $\varepsilon_{x,m}$ as above. Table 4 presents summary statistics on the coefficient estimates based on simulated data under the constant external signal precision. Coefficient estimates on news variables are almost always significantly positive and never significantly negative. In other words, independent of the investor's prior beliefs, positive news variables have a positive effect on returns while negative news variables have a negative one. This suggests that our model can generate empirical evidence similar to those in the literature only under the assumption that the precision of the external signal is higher in the high growth state.

3.3. Theoretical Results

In this section, based on some theoretical results, we discuss the intuition behind our results in Section 3.2. To this end, we analyze how the return changes as a function of the news variable in good and bad times while controlling for (or keeping fixed) the effect of dividend news variables. Specifically, for a given investor's prior belief about the high growth state, one can consider the return on an announcement period T_m^x as a function of the two news variables: the dividend news variable (u_{d,T_m^x}) and the news variable of interest (u_{x,T_m^x}). Figure 1 presents $r^*_{T_m^x}$ as a function of u_{x,T_m^x} for three different values of u_{d,T_m^x} and $\tilde{\pi}_{1,T_m^x}$ under alternative assumptions for the precision of the external signal in different states. All the intuition behind our results in Section 3.2 can be readily explained by two observations that emerge from Panel A of Figure 1 where the external signal is assumed to be more precise in the high growth state. The first observation is that the return decreases following not only negative news variables, as one would expect, but also large positive news variables independent of the dividend news variable and the investor's prior beliefs. The second observation is that this negative effect of large positive news variable on the return is more pronounced in good times than in bad times. These two observations on the shape of the return as

[11] A positive coefficient estimate implies a negative effect on returns when considering negative news variables.

a function of the news variable explain why one might observe a negative coefficient estimate on positive news variables in good times but not in bad times. To see this, for a given dividend news variable, consider a linear approximation of the return as a function of the news variable only for positive news variables. It is easy to see from Panel A of Figure 1 that the slope of this linear approximation for positive news variables is more likely to be negative in good times than in bad times. Furthermore, given that the return is not a decreasing function for all positive news variables, it is possible to observe a positive slope for the linear approximation in bad times and a negative one in good times. It is also easy to see from Panel B of Figure 1 that these two observations do not hold true under the alternative assumption of constant external signal precision. In the rest of the section, we formalize these two observations on the shape of the return as a function of the news variable.

The following proposition formalizes the first observation that, only under the assumption of higher external signal precision in the high growth state, the return is a decreasing function of the news variable for large positive news variables independent of the dividend news variable and the investor's prior belief.

PROPOSITION 2. *Consider an announcement period T_m^x and a given investor's belief about the high growth state $\tilde{\pi}_{1,T_m^x}$ prior to observing the information revealed in period T_m^x.*
(a) If $\sigma_{x,1} < \sigma_{x,2}$, then $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ is positive for news variables smaller than $\delta_x(\tilde{\pi}_{1,T_m^x})$ and negative for news variables greater than $\delta_x(\tilde{\pi}_{1,T_m^x})$ where $\delta_x(\tilde{\pi}_{1,T_m^x}) = (\mu_{x,1} - \mu_{x,2})(\frac{\sigma^2_{x,2}}{\sigma^2_{x,2}-\sigma^2_{x,1}} - \tilde{\pi}_{1,T_m^x}) > 0$.
(b)If $\sigma_{x,1} = \sigma_{x,2}$, then $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ is positive for all news variables.

Part (a) of Proposition 2 shows that, under the assumption of higher external signal precision in the high growth state, the return is an increasing function of the news variable observed from the external signal for news variables smaller than $\delta_x(\tilde{\pi}_{1,T_m^x})$ and a decreasing function for news variables greater than $\delta_x(\tilde{\pi}_{1,T_m^x})$. This holds true independent of the dividend news variable observed in the same period. Given that $\delta_x(\tilde{\pi}_{1,T_m^x})$ is positive for all $\tilde{\pi}_{1,T_m^x}$, the implication of Part (a) of Proposition 2 for negative news variables is straightforward. For a given dividend news variable, part (a) of Proposition 2 implies that the return decreases as the news variable becomes more negative. On the other hand, for positive news variables, whether the return is an increasing or a decreasing function of the news variable depends on the investor's prior beliefs through $\delta_x(\tilde{\pi}_{1,T_m^x})$. Specifically, the return is an increasing function for small positive news variables ($0 < u_{x,T_m^x} < \delta_x(\tilde{\pi}_{1,T_m^x})$) and a decreasing function for large news variables ($u_{x,T_m^x} > \delta_x(\tilde{\pi}_{1,T_m^x})$). $\delta_x(\tilde{\pi}_{1,T_m^x})$ is the news variable for which the probability of observing it in the high growth state divided by the precision of the external signal in that state is equal to the same in the low growth state. The probability of observing any news variables greater than $\delta_x(\tilde{\pi}_{1,T_m^x})$ is higher in the low growth than in the high growth state after adjusting for the differences in the precision of the external signal in different states. Hence, the investor's belief about the high growth state as well as the return become decreasing functions of the news variables for large positive news variable. Part (b) of Proposition 2 implies that the return can be a decreasing function of the news variable for large positive news variables only under the assumption that the external signal is more precise in the high growth state. Specifically, it shows that the return is an increasing function of the news variable independent of the investor's prior beliefs under the assumption of constant external signal precision.

Several remarks on $\delta_x(\tilde{\pi}_{1,T^x_m})$ are in order. First of all, $\delta_x(\tilde{\pi}_{1,T^x_m})$ is itself a decreasing function of the investor's belief about the high growth state prior to the announcement. In other words, $\delta_x(\tilde{\pi}_{1,T^x_m})$ is closer to zero in good times when the investor assigns a higher prior probability to the high growth state than in bad times. This immediately implies that the return is a decreasing function of the news variable for a wider range of positive news variables in good times than in bad times. Under the assumption of higher external signal precision in the high growth state, this, in turn, implies that the return is more likely to be a decreasing function for positive variables in good times than in bad times. For example, under the calibrated model, when the investor assigns a prior probability of 10% to the high growth state, the probability of observing a positive news variable for which the return is a decreasing function is only 12%. This probability increases almost three times[12] to 32% when he assigns a prior probability of 90% to the high growth state. Secondly, $\delta_x(\tilde{\pi}_{1,T^x_m})$ is determined by the mean and precision of the external signal in different states and it can be made arbitrarily close to zero in the high growth state. For example, if the external signal reveals the true state variable in the high growth state, i.e. $\sigma_{x,1} = 0$, $\delta_x(\tilde{\pi}_{1,T^x_m})$ approaches zero as $\tilde{\pi}_{1,T^x_m}$ approaches one. Furthermore, if the mean of the external signal in the high growth state is also high enough, the probability of observing positive news variables for which the return is a decreasing function is one in the high growth state and zero in the low growth state. Finally, the difference between the values of $\delta_x(\tilde{\pi}_{1,T^x_m})$ for two given periods depends on the difference between the investor's beliefs about the high growth state in these two periods as well as the difference between the means of the external signal in the high and the low growth states. Hence, the distance between $\delta_x(\tilde{\pi}_{1,T^x_m})$ for two given periods can be arbitrarily large as we consider a larger difference between $\mu_{x,1}$ and $\mu_{x,2}$.

The intuition behind Proposition 2 follows from the investor's valuation of the risky asset in different states and how he updates his beliefs following a news variable. We start with how the investor updates his beliefs about the high growth state under alternative assumptions on the precision of the external signal in different states.

LEMMA 2. *Consider an announcement period T^x_m and a given investor's belief about the high growth state $\tilde{\pi}_{1,T^x_m}$ prior to observing the information revealed in period T^x_m*
(a) If $\sigma_{x,1} < \sigma_{x,2}$, $\frac{\partial \pi_{1,T^x_m}}{\partial u_{x,T^x_m}}$ is positive for news variables smaller than $\delta_x(\tilde{\pi}_{1,T^x_m})$ and negative for news variables greater than $\delta_x(\tilde{\pi}_{1,T^x_m})$.

[12] For the two probabilities considered in this example, i.e. 10% and 90%, the probability of observing large positive news variables in good times can be at most nine times higher than that in bad times. As we discuss below, this happens only when the probability of observing positive news variables for which the return is a decreasing function is one in the high growth state and zero in the low growth state.

(b) If $\sigma_{x,1} = \sigma_{x,2}$, $\frac{\partial \pi_{1,T_m^x}}{\partial u_{x,T_m^x}}$ *is positive for all news variables.*

Under the assumption that the precision of the external signal is higher in the high growth state, the investor's belief about the high growth state is an increasing function for negative news variables and positive news variables smaller than $\delta_x(\tilde{\pi}_{1,T_m^x})$ and a decreasing function for news variables greater than $\delta_x(\tilde{\pi}_{1,T_m^x})$. On the other hand, under the alternative assumption of constant external signal precision, it is always an increasing function for all news variables. Whether the investor's belief about the high growth state is an increasing or a decreasing function of the news variable depends on the probability ratio of observing a news variable in the low and the high growth states as well as on the precision of the external signal in different states. The assumption of higher external signal precision in the high growth state immediately implies that the distribution of the external signal has fatter tails in the low growth state compared to the high growth state. For a given set of prior beliefs and a news variable greater than $\delta_x(\tilde{\pi}_{1,T_m^x})$, the probability of observing this news variable divided by the precision of the external signal in the low growth state is always higher than the same in the high growth state. Hence, the investor's belief about the high growth state becomes a decreasing function of the news variable for these large positive news variables. On the other hand, the probability of observing negative news variables large in magnitude is always higher in the low growth state given that the distribution of the external signal does not only have fatter tails but also a lower mean in the low growth state. One does not observe a similar pattern under the assumption that the precision of the external signal does not depend on the state. This is due to the fact that the distribution of the external signal has similar tail behavior in both states and a higher mean in the high growth state. Thus, it is always more likely to observe large positive news variables in the high growth state than in the low growth state.

We now turn our attention to the investor's valuation of the risky asset in different states. The following lemma characterizes the relation between the price-dividend ratios of the risky asset in different states, i.e. λ_1 and λ_2.

LEMMA 3. *If* $\psi > 1$, *then* $\lambda_1 > \lambda_2$.

Lemma 3 shows that the price-dividend ratio is higher in the high growth state than in the low growth state if the investor's elasticity of substitution is greater than one. This result is similar to those in the literature and follows from the fact that a higher growth rate of dividends has two opposing effects on the investor's valuation of the risky asset in equilibrium: the income and the substitution effects. The income effect implies that a higher growth rate of dividends increases the investor's demand for the risky asset which, in turn, raises the current asset price and the price-dividend ratio. On the other hand, the substitution effect implies that a higher growth rate of dividends results in an increase in the investor's current consumption due to his consumption smoothing motives. This, in turn, decreases the current asset price due to the fact that the investor attempts to finance his consumption by selling the risky asset. In other words, a higher growth rate of dividends does not only imply higher expected future dividend realizations (the income effect) but also higher expected future discount rates (the substitution effect). Which of these two effects dominates in equilibrium depends on the investor's elasticity of intertemporal substitution. When his elasticity of intertemporal substitution is greater than one, i.e. $\psi > 1$, the income effect dominates the substitution effect and the price-dividend ratio of the risky asset is higher in the high growth state compared to the low growth state.

To help with the above intuition, the following lemma presents an alternative decomposition of the return based on the approach of Campbell and Shiller (1988a) and Campbell and Shiller (1988b).

LEMMA 4. *The return in period t can be expressed as follows:*

$$r_t^* = (E_t - \tilde{E}_t)(\sum_{\tau=1}^{\infty} \rho^\tau \Delta d_{t+\tau}) - (E_t - \tilde{E}_t)(\sum_{\tau=1}^{\infty} \rho^\tau r_{t+\tau}) + u_{d,t} \tag{12}$$

where $(E_t - \tilde{E}_t)(z) = E_t(z) - \tilde{E}_t(z)$ and ρ is defined in the appendix.

Lemma 4 shows that the return can be decomposed into three components: the change in the expectations of future dividend growth rates and future returns as well as the direct effect of the dividend news variable. One can see that the effect of a news variable on the return through the change in the expectation of future dividend growth rates (the income effect) is positive. In other words, a news variable that results in an increase in the expectation of future dividend growth rates will have a positive effect on the return. On the other hand, the effect of a news variable on the return through the change in the expectation of future returns (the substitution effect) is negative. To put it differently, a news variable that results in an increase in the expectation of future returns (or equivalently future discount rates) will have a negative effect on the return. The overall effect of a news variable observed from an external signal depends on which of these two effects dominates. As before, the income effect dominates the substitution effect for an investor with an intertemporal elasticity of substitution greater than one.

We now turn our attention to the second observation that the return is also generally lower in good times than in bad times following large positive news variables. The following proposition compares the return on similar announcement periods in good and bad times.

PROPOSITION 3. *Consider two announcement periods $T_m^{x,high}$ and $T_m^{x,low}$ with the same news variable observed from dividend realizations, i.e. $u_{d,T_m^{x,high}} = u_{d,T_m^{x,low}}$. Assume that the probabilities that the investor assigns to the high growth state prior to observing the information revealed on $T_m^{x,high}$ and $T_m^{x,low}$ are $\tilde{\pi}_{1,T_m^{x,high}}$ and $\tilde{\pi}_{1,T_m^{x,low}}$, respectively where $\tilde{\pi}_{1,T_m^{x,high}} = 1 - \tilde{\pi}_{1,T_m^{x,low}} > 0.5$. Then, $u_{x,T_m^{x,high}} = u_{x,T_m^{x,low}} > \delta_x(\tilde{\pi}_{1,T_m^{x,high}}) + \omega_x(\tilde{\pi}_{1,T_m^{x,high}})$ is a sufficient condition such that $r^*_{T_m^{x,high}} < r^*_{T_m^{x,low}}$ independent of the dividend news variable observed in the same period where $\omega_x(\tilde{\pi}_{1,T_m^{x,high}})$ is a positive-valued function defined in the appendix.*

In Proposition 3, we compare returns on two announcement periods with the same news variables observed from external signals and dividend realizations. We assume that these two announcement periods differ only in terms of the prior probability that the investor assigns to the high growth state. Specifically, $T_m^{x,high}$ represents an announcement period in good times when the investor assigns a prior probability higher than 0.5 to the high growth state while $T_m^{x,low}$ represents an announcement period in bad times when the investor assigns a prior probability smaller than 0.5 to the high growth state. Furthermore, we consider announcement periods in good and bad times that have prior probabilities symmetric around 0.5, i.e. $\tilde{\pi}_{1,T_m^{x,high}} = 1 - \tilde{\pi}_{1,T_m^{x,low}}$. For these announcement periods, Proposition 3 shows that the return is always lower in period $T_m^{x,high}$ than in period $T_m^{x,low}$ for all news variables greater than $\delta_x(\tilde{\pi}_{1,T_m^{x,high}}) + \omega_x(\tilde{\pi}_{1,T_m^{x,high}})$. This holds independent of the dividend news variable observed in the same period as long as they are the same for both announcement periods. In other words, following large positive news variables, the return on an announcement period in good times is guaranteed to be lower than the return on a similar announcement period in bad times. However, given that $\omega_x(\tilde{\pi}_{1,T_m^{x,high}})$ is positive, this does not hold for all positive news variables greater than $\delta_x(\tilde{\pi}_{1,T_m^{x,high}})$ for which the return is a decreasing function. On the other hand, we should note that the sufficient condition in Proposition 3 is quite conservative in the sense that $r^*_{T_m^{x,high}} < r^*_{T_m^{x,low}}$ holds for all prior probabilities and for all dividend news variables. In reality, whether the return following a news variable is lower in good times than in bad times depends on the dividend news variable and the definition of good and bad times. Hence, in most cases, the set of news variables for which the claim in Proposition 3 holds includes all positive variables greater than $\delta_x(\tilde{\pi}_{1,T_m^{x,high}})$. For example, for the cases considered in Panel A of Figure 1, the return on an announcement period in good times is almost always lower than the return on a similar announcement period in bad times for all positive variables greater than $\delta_x(\tilde{\pi}_{1,T_m^{x,high}})$. This suggests large positive news variables generally have a larger negative impact on the return in good times than in bad times.

The intuition behind this result follows from the four following facts: (1) the return is a function of the difference between the investor's belief about the high growth state before and after observing the information revealed on the announcement period; (2) the return is a decreasing function of the news variable for large positive news variables; (3) the lower limit for the investor's belief about the high growth state is zero; and (4) the prior probability of the high growth state is by definition higher in good times than in bad times. Thus, for large enough positive news variables ($u_{x,T_m^x} > \delta_x(\tilde{\pi}_{1,T_m^x}) + \omega_x(\tilde{\pi}_{1,T_m^x})$), the change in the investor's belief about the high growth state with respect to his prior belief is expected to be more negative in good times than in bad times. Given that $\lambda_1 > \lambda_2$, this implies that the return is expected to be lower in good times than in bad times for these large positive news variables.

3.4. Discussion and the Related Literature

In this section, we first summarize the relation between our empirical results in Section 3.2 and those in the literature. We then discuss how our findings in Section 3.3 can explain these results. Our results based on simulated data in Section 3.2 are closest to the empirical evidence in McQueen and Roley (1993). They are among the first to analyze the reaction of stock returns to macroeconomic news over different phases of the business cycle. Specifically, they distinguish between good, normal and bad states of the economy based on the growth rate of industrial production and analyze the reaction of daily returns on the S&P 500 index to macroeconomic news over different states. For some macroeconomic variables including the industrial production, they find that the daily returns on the S&P 500 index react negatively to good news, i.e. higher than expected real economic activity, in good times. Andersen et al. (2007) provide supporting evidence for this stylized fact from international equity markets, with good news having a negative impact on international stock returns during periods of expansions in the U.S. economy. Boyd et al. (2005) analyze the reaction of stock returns to employment numbers and find that the daily returns on the S&P 500 index react negatively to better than expected employment numbers in expansions. Cenesizoglu (2011) also finds that better than expected employment numbers is bad news for large and growth stocks in expansions but not in recessions. Gilbert (2011) provides further supporting evidence for this stylized fact based on the relation between stock returns and future revisions to macroeconomic variables.

To understand how our model can generate empirical results similar to those in the literature, one should first consider the following piecewise linear approximation of the return function, $r^*_{T^x_m} = r(u_{d,T^x_m}, u_{x,T^x_m}; \tilde{\pi}_{1,T^x_m})$, of the following form:

$$\begin{aligned}
r^*_{T^x_m} &= \left(r(0, u_x^{neg}; \tilde{\pi}_{1,T^x_m}) + \left.\frac{\partial r^*_{T^x_m}}{\partial u_{d,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{neg}} \times u_{d,T^x_m} + \left.\frac{\partial r^*_{T^x_m}}{\partial u_{x,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{neg}} \times (u_{x,T^x_m} - u_x^{neg}) \right) 1_{\{u_{x,T^x_m} \leq 0\}} \\
&+ \left(r(0, u_x^{pos}; \tilde{\pi}_{1,T^x_m}) + \left.\frac{\partial r^*_{T^x_m}}{\partial u_{d,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{pos}} \times u_{d,T^x_m} + \left.\frac{\partial r^*_{T^x_m}}{\partial u_{x,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{pos}} \times (u_{x,T^x_m} - u_x^{pos}) \right) 1_{\{u_{x,T^x_m} > 0\}} \\
&+ error_{T^x_m} \qquad (13)
\end{aligned}$$

where $u_{d,T^x_m} = 0$ is the point of approximation for all values of u_{d,T^x_m} while $u_{x,T^x_m} = u_x^{neg}$ and $u_{x,T^x_m} = u_x^{pos}$ are arbitrary points of approximation for negative and positive values of u_{x,T^x_m}, respectively.[13] The approximation can be rewritten by grouping certain terms together as follows:

$$\begin{aligned}
r^*_{T^x_m} &= \left(r(0, u_x^{neg}; \tilde{\pi}_{1,T^x_m}) - \left.\frac{\partial r^*_{T^x_m}}{\partial u_{x,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{neg}} \times u_x^{neg} \right) 1_{\{u_{x,T^x_m} \leq 0\}} \\
&+ \left(r(0, u_x^{pos}; \tilde{\pi}_{1,T^x_m}) - \left.\frac{\partial r^*_{T^x_m}}{\partial u_{x,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{pos}} \times u_x^{pos} \right) 1_{\{u_{x,T^x_m} > 0\}} \\
&+ \left(\left.\frac{\partial r^*_{T^x_m}}{\partial u_{d,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{neg}} 1_{\{u_{x,T^x_m} \leq 0\}} + \left.\frac{\partial r^*_{T^x_m}}{\partial u_{d,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{pos}} 1_{\{u_{x,T^x_m} > 0\}} \right) u_{d,T^x_m} \\
&+ \left.\frac{\partial r^*_{T^x_m}}{\partial u_{x,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{neg}} u_{x,T^x_m} 1_{\{u_{x,T^x_m} \leq 0\}} + \left.\frac{\partial r^*_{T^x_m}}{\partial u_{x,T^x_m}}\right|_{u_{d,T^x_m}=0, u_{x,T^x_m}=u_x^{pos}} u_{x,T^x_m} 1_{\{u_{x,T^x_m} > 0\}} \\
&+ error_{T^x_m} \qquad (14)
\end{aligned}$$

[13] We do not distinguish between positive and negative dividend news variables for two reasons. First of all, the dividend news variable is not the news variable of interest in this paper. More importantly, the return in our model is always an increasing function of the dividends news variable independent of the investor's prior beliefs and news variables observed from external signals.

Equation (14) suggests that the coefficient estimates on $u_{x,T_m^x}1_{\{u_{x,T_m^x}\leq 0\}}$ and $u_{x,T_m^x}1_{\{u_{x,T_m^x}>0\}}$ in a linear regression framework are determined by the partial derivative of the return with respect to the news variable evaluated at arbitrary negative and positive news variables used as approximation points, respectively. Proposition 2 shows that the partial derivative of the return with respect to the news variable is always positive for all negative news variables independent of the investor's prior beliefs. This immediately suggests that the coefficient estimate on $u_{x,T_m^x}1_{\{u_{x,T_m^x}\leq 0\}}$ would be positive independent of the investor's prior beliefs. This is exactly what we observe in our results in Section 3.2. On the other hand, for positive news variables, Proposition 2 suggests that the partial derivative of the return with respect to the news variable can be positive or negative depending on the point of approximation and its relation to $\delta_x(\tilde{\pi}_{1,T_m^x})$. Hence, the sign of the coefficient estimate on $u_{x,T_m^x}1_{\{u_{x,T_m^x}>0\}}$ depends on the investor's prior beliefs. Our two findings discussed in Section 3.3 explain why one might observe a negative coefficient estimate on positive news variables in good times but not in bad times in a linear regression framework. First of all, the partial derivative is more likely to be negative in good times than in bad times. Secondly, the negative effect of large positive news variables on the return is more pronounced in good times than in bad times. Given that the return is not a decreasing function for all positive news variables, it is possible to observe a positive coefficient estimate in bad times and a negative one in good times. Under the assumption of constant external signal precision, one does not observe a similar pattern for the coefficient estimates since the partial derivative of the return with respect to the news variable is always positive independent of the investor's prior beliefs. Under the commonly used alternative assumption of constant external signal precision, the investor always assigns a higher probability to the high growth state following positive news variables and a lower probability following negative news variables. Hence, one cannot account for the stylized fact in the literature in a model like ours under this alternative assumption.

4. Conclusion

In this paper, we propose a mechanism based on higher external signal precision in good times to account for the stylized fact that stock returns react negatively to good news in good times. Specifically, we analyze the effect of news on returns in a general equilibrium asset pricing model where the growth rate of dividends follow a Markov regime switching process. The investor never observes the true state of the dividend growth process but learns about it through two sources of information: external signals and dividend realizations. Under the assumption that the precision of the external signal is higher in the low growth state, the probability of observing large positive news from external signals is higher in the low growth state than in the high growth state. This is due to the fact that the distribution of the external signal does not only have a lower mean but also fatter tails in the low growth state compared to the high growth state. Hence, the investor assigns a lower probability to the high growth state following not only negative but also large positive news. More importantly, independent of news from the dividend realization in the same period, the decrease in the investor's belief about the high growth state following large positive news is not only more likely but also more pronounced in good times than in bad times. Given that the investor values the risky asset more in the high growth state, this implies that the negative effect of large positive news on unexpected returns is not only more likely but also more pronounced in good times than in bad times. Furthermore, not all positive news decrease the probability that the investor assigns to the high growth state. Hence, in a linear regression framework, one might observe that positive news has a negative effect on unexpected returns in good times and a positive effect in bad times.

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Table 1 Precision of the External Signal

Panel A: News about the Growth Rate of Industrial Production

	Expansions	Recessions	Total
Obs.	304	56	360
Mean	0.0138%	-0.1750%	-0.0156%
Std. Dev.	0.2748%	0.5494%	0.3387%
Levine's Test		26.8403 (0.0000)	
Bartlett's Test		58.9844 (0.0000)	
F Test		3.9958 (0.0000)	

Panel B: Realized Growth Rate of Industrial Production

	Expansions	Recessions	Total
Obs.	304	56	360
Mean	0.2982%	-0.6892%	0.1446%
Std. Dev.	0.5427%	0.9231%	0.7124%
Levine's Test		11.3298 (0.0008)	
Bartlett's Test		32.7583 (0.0000)	
F Test		2.8929 (0.0000)	

Note: Panel A presents the mean and the volatility of news about the growth of industrial production over the business cycle. The news variable is defined as the difference between the growth rate of industrial production, as it is first reported, and the median forecast from the Money Market Services (MMS) survey. Panel B presents the mean and standard deviation of the realized growth rate of industrial production over the business cycle. An observation is considered as a recession observation if its release date is in a recession period as defined by NBER. Levene's, Bartlett's and F test are test statistics testing the null hypothesis of equal volatility in recession and expansion periods. A rejection suggests that the volatilities of the variable of interest are significantly different from each other in recession and expansion periods. The corresponding p values are presented in parentheses.

Table 2 Calibrated Model Parameters

Parameter	Value
γ	7.5
ψ	1.5
β	0.9998
$\mu_{d,1}$	0.0172%
$\mu_{d,2}$	-0.0045%
σ_d	0.1043%
$\mu_{x,1}$	0.2982%
$\mu_{x,2}$	-0.6892%
$\sigma_{x,1}$	0.5427%
$\sigma_{x,2}$	0.9231%
$q_{1,1}$	0.9988
$q_{2,2}$	0.9950
$T_m^x - T_{m-1}^x$	21

Note: The table presents calibrated model parameters. $\mu_{d,1}$ and $\mu_{d,2}$ are the average dividend growth rates in states 1 and 2, respectively and σ_d is the volatility of the dividend growth process. Similarly, $\mu_{x,1}$ and $\mu_{x,2}$ are the average values for the external signal in states 1 and 2, respectively. $\sigma_{x,1}$ and $\sigma_{x,2}$ are the inverse of the precision of the external signal in states 1 and 2, respectively. The volatility of the external signal is calibrated to 0.7124% in cases where the precision of the external signal is assumed to be constant. $q_{1,1}$ and $q_{2,2}$ are the transition probabilities for states 1 and 2, respectively. $T_m^x - T_{m-1}^x$ is the number of periods between two consecutive announcements.

Table 3 Summary Statistics for Coefficient Estimates when $\sigma_{x,1} < \sigma_{x,2}$

Panel A: $(\pi_h, \pi_l) = (0.75, 0.25)$

Variable	Mean	Median	Std. Dev.	Percentage of samples with a significantly negative coefficient estimate	Percentage of samples with a significantly positive coefficient estimate
Constant	0.000	0.000	0.000	28%	4%
$\tilde{\pi}_{1,T_m^x}$	0.001	0.001	0.000	0%	98%
u_{d,T_m^x}	1.127	1.122	0.030	0%	100%
$1_{\{u_{x,T_m^x}>0\}}$	0.000	0.000	0.000	100%	0%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\tilde{\pi}_{1,T_m^x}>\pi_h\}}$	-0.030	-0.031	0.011	95%	0%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\tilde{\pi}_{1,T_m^x}>\pi_h\}}$	0.341	0.342	0.031	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\pi_l<\tilde{\pi}_{1,T_m^x}\leq\pi_h\}}$	0.317	0.317	0.048	0%	99%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\pi_l<\tilde{\pi}_{1,T_m^x}\leq\pi_h\}}$	0.419	0.410	0.055	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\tilde{\pi}_{1,T_m^x}\leq\pi_l\}}$	0.200	0.199	0.033	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\tilde{\pi}_{1,T_m^x}\leq\pi_l\}}$	0.125	0.117	0.032	0%	100%

Panel B: $(\pi_h, \pi_l) = (0.90, 0.10)$

Variable	Mean	Median	Std. Dev.	Percentage of samples with a significantly negative coefficient estimate	Percentage of samples with a significantly positive coefficient estimate
Constant	0.001	0.001	0.000	0%	85%
$\tilde{\pi}_{1,T_m^x}$	0.000	0.000	0.000	0%	59%
u_{d,T_m^x}	1.128	1.128	0.031	0%	100%
$1_{\{u_{x,T_m^x}>0\}}$	0.000	0.000	0.000	92%	0%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\tilde{\pi}_{1,T_m^x}>\pi_h\}}$	-0.030	-0.029	0.010	100%	0%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\tilde{\pi}_{1,T_m^x}>\pi_h\}}$	0.291	0.289	0.037	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\pi_l<\tilde{\pi}_{1,T_m^x}\leq\pi_h\}}$	0.200	0.202	0.029	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\pi_l<\tilde{\pi}_{1,T_m^x}\leq\pi_h\}}$	0.364	0.358	0.042	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\tilde{\pi}_{1,T_m^x}\leq\pi_l\}}$	0.097	0.096	0.026	0%	95%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\tilde{\pi}_{1,T_m^x}\leq\pi_l\}}$	0.129	0.116	0.038	0%	100%

Note: The table presents summary statistics for the coefficient estimates of the linear regression model in Equation (11) over 100 simulated samples. The significance level is 5% for one-sided tests in the last two columns.

Table 4 Summary Statistics for Coefficient Estimates when $\sigma_{x,1} = \sigma_{x,2}$

Panel A: $(\pi_h, \pi_l) = (0.75, 0.25)$

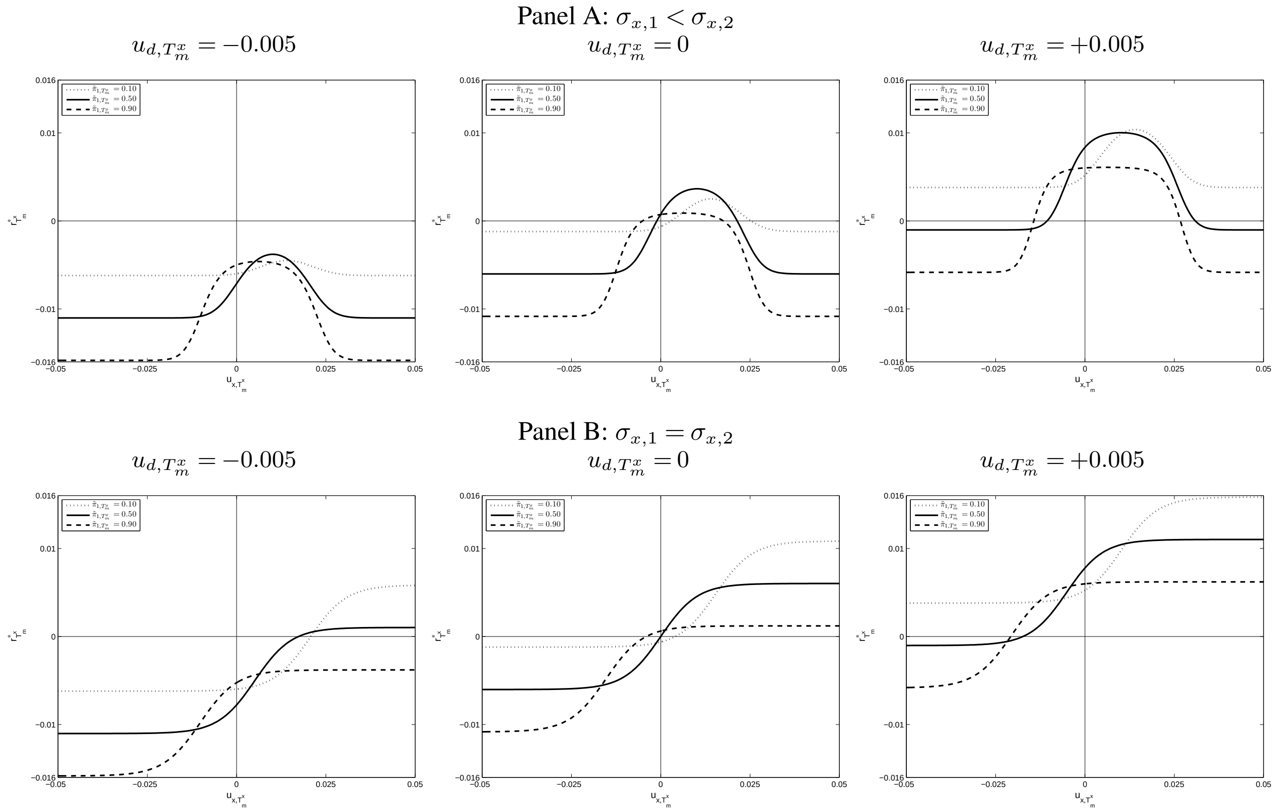
Variable	Mean	Median	Std. Dev.	Percentage of samples with a significantly negative coefficient estimate	Percentage of samples with a significantly positive coefficient estimate
Constant	-0.001	-0.001	0.000	98%	0%
$\tilde{\pi}_{1,T_m^x}$	0.002	0.002	0.000	0%	100%
u_{d,T_m^x}	1.135	1.131	0.025	0%	100%
$1_{\{u_{x,T_m^x}>0\}}$	0.000	0.000	0.000	100%	0%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\tilde{\pi}_{1,T_m^x}>\pi_h\}}$	0.016	0.016	0.007	0%	78%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\tilde{\pi}_{1,T_m^x}>\pi_h\}}$	0.209	0.208	0.019	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\pi_l<\tilde{\pi}_{1,T_m^x}\leq\pi_h\}}$	0.401	0.402	0.032	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\pi_l<\tilde{\pi}_{1,T_m^x}\leq\pi_h\}}$	0.372	0.370	0.031	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\tilde{\pi}_{1,T_m^x}\leq\pi_l\}}$	0.358	0.359	0.026	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\tilde{\pi}_{1,T_m^x}\leq\pi_l\}}$	0.080	0.074	0.024	0%	100%

Panel B: $(\pi_h, \pi_l) = (0.90, 0.10)$

Variable	Mean	Median	Std. Dev.	Percentage of samples with a significantly negative coefficient estimate	Percentage of samples with a significantly positive coefficient estimate
Constant	0.000	0.000	0.000	14%	15%
$\tilde{\pi}_{1,T_m^x}$	0.001	0.001	0.000	0%	89%
u_{d,T_m^x}	1.136	1.137	0.024	0%	100%
$1_{\{u_{x,T_m^x}>0\}}$	0.000	0.000	0.000	99%	0%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\tilde{\pi}_{1,T_m^x}>\pi_h\}}$	0.013	0.013	0.006	0%	75%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\tilde{\pi}_{1,T_m^x}>\pi_h\}}$	0.167	0.168	0.015	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\pi_l<\tilde{\pi}_{1,T_m^x}\leq\pi_h\}}$	0.287	0.289	0.025	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\pi_l<\tilde{\pi}_{1,T_m^x}\leq\pi_h\}}$	0.336	0.331	0.027	0%	100%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}>0\}} 1_{\{\tilde{\pi}_{1,T_m^x}\leq\pi_l\}}$	0.194	0.204	0.059	0%	96%
$u_{x,T_m^x} 1_{\{u_{x,T_m^x}\leq 0\}} 1_{\{\tilde{\pi}_{1,T_m^x}\leq\pi_l\}}$	0.097	0.089	0.033	0%	100%

Note: The table presents summary statistics for the coefficient estimates of the linear regression model in Equation (11) over 100 simulated samples. The significance level is 5% for one-sided tests in the last two columns.

Figure 1 Unexpected Return as a Function of News Variable Observed from External Signals



Note: The figure presents unexpected returns as a function of the news variable observed from external signals. The figure is based on the calibrated model parameters presented in Table 2.

Appendix

Proofs

[Proof of Lemma 1] We first characterize the investor's prior beliefs about the state variable of period t. Before observing the dividend realization (and possibly the external signal) in period t, the investor knows that the dividend growth process might have switched to a new state. Hence, his prior belief about state j is a weighted average of the transition probabilities into state j. The weights are his beliefs about the state variable of period $t-1$ after observing the dividend realization (and possibly the external signal) in period $t-1$. Hence, his prior belief about state j in period t is given by $\tilde{\pi}_{j,t} = \sum_{i=1}^{2} \pi_{i,t-1} q_{ij}$.

The investor's beliefs need to be characterized separately for announcement and non-announcement periods. We start with non-announcement periods. The only source of information about the state variable in between announcement periods is the dividend realizations. The investor updates his prior beliefs according to Bayes' rule based on the observed dividend realization. The probability that the investor assigns to state j, $\pi_{j,t} = \Pr(S_t = j|\mathcal{F}_t)$, can be expressed as

$$\begin{aligned}
\pi_{j,t} &= \Pr(S_t = j|\Delta d_t, \tilde{\mathcal{F}}_t) && (15)\\
&= \frac{\Pr(\Delta d_t|S_t = j, \tilde{\mathcal{F}}_t)\Pr(S_t = j|\tilde{\mathcal{F}}_t)}{\Pr(\Delta d_t|\tilde{\mathcal{F}}_t)} && (16)\\
&= \frac{\Pr(\Delta d_t|S_t = j, \tilde{\mathcal{F}}_t)\Pr(S_t = j|\tilde{\mathcal{F}}_t)}{\sum_{i=1}^{2}\Pr(\Delta d_t|S_t = i, \tilde{\mathcal{F}}_t)\Pr(S_t = i|\tilde{\mathcal{F}}_t)} && (17)\\
&= \frac{\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_d})\tilde{\pi}_{j,t}}{\sum_{i=1}^{2}\phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_d})\tilde{\pi}_{i,t}} && (18)
\end{aligned}$$

where $\phi(\cdot)$ is the standard normal density function. Equation (15) follows from the definition of the information set, $\mathcal{F}_t$, which can be decomposed into the dividend realization of period t, Δd_t, and the information set prior to observing the information revealed in period t, $\tilde{\mathcal{F}}_t$. $\tilde{\mathcal{F}}_t$ includes all past information and the fact that the process might have switched to a new state. Equations (16) and (17) follow from Bayes' rule and the law of total probability, respectively.[14] Equation (18) follows from the law of motion for the dividend growth process in Equation (2).

On an announcement period T_m^x, there are two sources of information about the state variable, the dividend realization and the external signal. The investor updates his prior beliefs according to Bayes' rule based on the observed dividend realization and external signal.

$$\begin{aligned}
\pi_{j,T_m^x} &= \Pr(S_{T_m^x} = j|\Delta d_{T_m^x}, x_m, \tilde{\mathcal{F}}_{T_m^x}) && (19)\\
&= \frac{\Pr(\Delta d_{T_m^x}|S_{T_m^x} = j, \tilde{\mathcal{F}}_{T_m^x})\Pr(x_m|S_{T_m^x} = j, \tilde{\mathcal{F}}_{T_m^x})\Pr(S_{T_m^x} = j|\tilde{\mathcal{F}}_{T_m^x})}{\Pr(\Delta d_{T_m^x}, x_m|\tilde{\mathcal{F}}_{T_m^x})} && (20)\\
&= \frac{\Pr(\Delta d_{T_m^x}|S_{T_m^x} = j, \tilde{\mathcal{F}}_{T_m^x})\Pr(x_m|S_{T_m^x} = j, \tilde{\mathcal{F}}_{T_m^x})\Pr(S_{T_m^x} = j|\tilde{\mathcal{F}}_{T_m^x})}{\sum_{i=1}^{2}\Pr(\Delta d_{T_m^x}|S_{T_m^x} = i, \tilde{\mathcal{F}}_{T_m^x})\Pr(x_m|S_{T_m^x} = i, \tilde{\mathcal{F}}_{T_m^x})\Pr(S_{T_m^x} = i|\tilde{\mathcal{F}}_{T_m^x})} && (21)\\
&= \frac{\frac{1}{\sigma_{x,j}}\phi(\frac{\Delta d_{T_m^x} - \mu_{d,j}}{\sigma_d})\phi(\frac{x_m - \mu_{x,j}}{\sigma_{x,j}})\tilde{\pi}_{j,T_m^x}}{\sum_{i=1}^{2}\frac{1}{\sigma_{x,i}}\phi(\frac{\Delta d_{T_m^x} - \mu_{d,i}}{\sigma_d})\phi(\frac{x_m - \mu_{x,i}}{\sigma_{x,i}})\tilde{\pi}_{i,T_m^x}} && (22)
\end{aligned}$$

[14] Recall that Bayes' rule is $\Pr(A|B,C) = \frac{\Pr(B|A,C)\Pr(A|C)}{\Pr(B|C)}$

The proof is similar to the one for the investor's beliefs on non-announcement periods. Equation (19) follows from the definition of the information set on the announcement period T_m^x, $\mathcal{F}_{T_m^x}$, which can be decomposed into the dividend realization, $\Delta d_{T_m^x}$, and the external signal, x_m, observed on the announcement period T_m^x and all past information, $\tilde{\mathcal{F}}_{T_m^x}$. Equations (20) and (21) follow from the independence of $\Delta d_{T_m^x}$ and x_m conditional on the state variable S_t. Equation (22) follows from the law of motion for the dividend growth rate in Equation (2) and the law of motion for the external signal in Equation (4).

[Proof of Proposition 1] The proof follows from Calvet and Fisher (2007). In a general equilibrium framework where dividends grow according to the process in Equation (2), the stochastic discount factor in period $t+1$, SDF_{t+1}, is given by

$$SDF_{t+1} = \beta^{\theta}\left(\frac{1+P_{t+1}/D_{t+1}}{P_t/D_t}\right)^{\theta-1}(\Delta d_{t+1})^{-\gamma}$$

The Euler equation can be expressed as

$$E_t\left[\beta^{\theta}\left(\frac{1+P_{t+1}/D_{t+1}}{P_t/D_t}\right)^{\theta}(\Delta d_{t+1})^{1-\gamma}\right] = 1$$

We conjecture a solution for the price-dividend ratio of the following form:

$$\frac{P_t}{D_t} = \lambda_{S_t}$$

Plugging in the conjectured solution in the Euler equation, we obtain Equation (6).
Log returns on the risky asset can be expressed as follows:

$$\begin{aligned} r_t &= \log(1+P_t/D_t) - \log(P_{t-1}/D_{t-1}) + \Delta d_t \\ &\approx \log(1+\overline{\lambda}) + \frac{1}{1+\overline{\lambda}}(P_t/D_t - \overline{\lambda}) - \log(\overline{\lambda}) - \frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1} - \overline{\lambda}) + \Delta d_t \end{aligned} \tag{23}$$

where Equation (23) follows from a first-order Taylor expansion of the log function around the long term average of the price-dividend ratio, $\bar{\lambda}$. The expectation of the log return in period t conditional on the investor's prior beliefs before observing the dividend realization (and possibly the external signal) in period t can be expressed as follows:

$$\tilde{E}_t[r_t] \approx \log(1+\overline{\lambda}) + \frac{1}{1+\overline{\lambda}}(\sum_{j=1}^{2}\lambda_j\tilde{\pi}_{j,t} - \overline{\lambda}) - \log(\overline{\lambda}) - \frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1} - \overline{\lambda}) + \sum_{j=1}^{2}\mu_{d,j}\tilde{\pi}_{j,t} \tag{24}$$

The unexpected log return on the risky asset in Equation (8) can be obtained as the difference between Equations (23) and (24). The long term average of the price-dividend ratio is the unconditional expectation of the price-dividend ratio as defined in Proposition 1.

[Proof of Proposition 2] The derivative of the return on an announcement period T_m^x with respect to the news variable

$$\frac{\partial r^*_{T_m^x}}{\partial u_{x,T_m^x}} = \left(\frac{\lambda_1-\lambda_2}{1+\overline{\lambda}}\right)\frac{\partial \pi_{1,T_m^x}}{\partial u_{x,T_m^x}}. \tag{25}$$

Proof of (a): If $\sigma_{x,1} < \sigma_{x,2}$, then π_{1,T_m^x} can be expressed as

$$\begin{aligned} \pi_{1,T_m^x} = \Bigg[1 &+ \frac{\sigma_{x,1}(1-\tilde{\pi}_{1,T_m^x})}{\sigma_{x,2}\tilde{\pi}_{1,T_m^x}}\exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma_d^2}\right)\exp\left(-\frac{(\mu_{x,1}-\mu_{x,2})^2}{2(\sigma_{x,2}^2-\sigma_{x,1}^2)}\right) \\ &\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_d^2}u_{d,T_m^x}\right)\exp\left(\frac{\sigma_{x,2}^2-\sigma_{x,1}^2}{2\sigma_{x,1}^2\sigma_{x,2}^2}\left(u_{x,T_m^x}-\delta_x(\tilde{\pi}_{1,T_m^x})\right)^2\right)\Bigg]^{-1} \end{aligned} \tag{26}$$

where $\delta_x(\tilde{\pi}_{1,T_m^x}) = \frac{(\mu_{x,1}-\mu_{x,2})\tilde{\sigma}^2_{x,T_m^x}}{\sigma^2_{x,2}-\sigma^2_{x,1}} = (\mu_{x,1}-\mu_{x,2})(\frac{\sigma^2_{x,2}}{\sigma^2_{x,2}-\sigma^2_{x,1}} - \tilde{\pi}_{1,T_m^x}) > 0$.
The derivative of π_{1,T_m^x} with respect to u_{x,T_m^x} is given by

$$\partial\pi_{1,T_m^x}/\partial u_{x,T_m^x} = f_1(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x})\left(\frac{\sigma^2_{x,1}-\sigma^2_{x,2}}{\sigma^2_{x,1}\sigma^2_{x,2}}\right)\left(u_{x,T_m^x} - \delta_x(\tilde{\pi}_{1,T_m^x})\right) \quad (27)$$

where

$$f_1(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}) = \kappa_1(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x})/(1+\kappa_1(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}))^2,$$

and

$$\begin{aligned}\kappa_1(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}) = {} & \frac{\sigma_{x,1}(1-\tilde{\pi}_{1,T_m^x})}{\sigma_{x,2}\tilde{\pi}_{1,T_m^x}} \exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma_d^2}\right) \\ & \exp\left(-\frac{(\mu_{x,1}-\mu_{x,2})^2}{2(\sigma^2_{x,2}-\sigma^2_{x,1})}\right)\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_d^2}u_{d,T_m^x}\right) \\ & \exp\left(\frac{\sigma^2_{x,2}-\sigma^2_{x,1}}{2\sigma^2_{x,1}\sigma^2_{x,2}}\left(u_{x,T_m^x}-\delta_x(\tilde{\pi}_{1,T_m^x})\right)^2\right).\end{aligned}$$

Note that κ_1 and f_1 are positive-valued functions, $\sigma_{x,1} < \sigma_{x,2}$ and $\delta_x(\tilde{\pi}_{1,T_m^x}) > 0$. Then, $\partial\pi_{1,T_m^x}/\partial u_{x,T_m^x} > 0$ if and only if $u_{x,T_m^x} < \delta_x(\tilde{\pi}_{1,T_m^x})$. Given that $\lambda_1 > \lambda_2$, this immediately implies that $\partial r^*_{T_m^x}/\partial u_{x,T_m^x} > 0$ if and only if $u_{x,T_m^x} < \delta_x(\tilde{\pi}_{1,T_m^x})$.
Proof of (b): If $\sigma_{x,1} < \sigma_{x,2}$, then π_{1,T_m^x} can be expressed as

$$\begin{aligned}\pi_{1,T_m^x} = {} & \left[1+\frac{(1-\tilde{\pi}_{1,T_m^x})}{\tilde{\pi}_{1,T_m^x}}\exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma_d^2}\right)\exp\left(\frac{(\mu_{x,1}-\mu_{x,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma^2_{x,1}}\right)\right. \\ & \left.\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_d^2}u_{d,T_m^x}\right)\exp\left(\frac{\mu_{x,2}-\mu_{x,1}}{\sigma^2_{x,1}}u_{x,T_m^x}\right)\right]^{-1}.\end{aligned} \quad (28)$$

The derivative of π_{1,T_m^x} with respect to u_{x,T_m^x} is given by

$$\partial\pi_{1,T_m^x}/\partial u_{x,T_m^x} = f_2(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x})\left(\frac{\mu_{x,1}-\mu_{x,2}}{\sigma^2_{x,1}}\right) \quad (29)$$

where

$$f_2(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}) = \kappa_2(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x})/(1+\kappa_2(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}))^2,$$

and

$$\begin{aligned}\kappa_2(u_{d,T_m^x}, u_{x,T_m^x}, {}_{1,T_m^x}) = {} & \frac{(1-\tilde{\pi}_{1,T_m^x})}{\tilde{\pi}_{1,T_m^x}}\exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma_d^2}\right)\exp\left(\frac{(\mu_{x,1}-\mu_{x,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma^2_{x,1}}\right) \\ & \exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_d^2}u_{d,T_m^x}\right)\exp\left(\frac{\mu_{x,2}-\mu_{x,1}}{\sigma^2_{x,1}}u_{x,T_m^x}\right).\end{aligned} \quad (30)$$

Note that κ_2 and f_2 are positive-valued functions. Then, $\partial\pi_{1,T_m^x}/\partial u_{x,T_m^x} > 0$. Given that $\lambda_1 > \lambda_2$, this immediately implies that $\partial r^*_{T_m^x}/\partial u_{x,T_m^x} > 0$.

[Proof of Lemma 2] See proof of Proposition 2

[Proof of Lemma 3] The proof follows from Calvet and Fisher (2007). Assume that the dividends of the risky asset grow according to $\Delta d_t = \mu_d + \sigma_d\varepsilon_{d,t}$ where $\varepsilon_{d,t}$ is an independently and identically distributed Gaussian random variable with zero mean and unit variance. Then, the price-dividend ratio, λ, is given by

$$\lambda = \frac{\beta\exp((1-\psi^{-1})\mu_d + (1-\gamma)(1-\psi^{-1})\sigma_d^2/2)}{1-\beta\exp((1-\psi^{-1})\mu_d + (1-\gamma)(1-\psi^{-1})\sigma_d^2/2)} \quad (31)$$

Note that λ is an increasing function of μ_d if and only if $\psi > 1$. Given that $\mu_{d,1} > \mu_{d,2}$, this immediately implies that $\lambda_1 > \lambda_2$ if and only if $\psi > 1$.

[Proof of Lemma 4] Using the Campbell and Shiller decomposition, the unexpected return can be expressed as:

$$\begin{aligned} r_t^* = r_t - \tilde{E}_t(r_t) &= (E_t - \tilde{E}_t)(\sum_{\tau=0}^{\infty}\rho^\tau \Delta d_{t+\tau}) - (E_t - \tilde{E}_t)(\sum_{\tau=1}^{\infty}\rho^\tau r_{t+\tau}) \\ &= (E_t - \tilde{E}_t)(\sum_{\tau=1}^{\infty}\rho^\tau \Delta d_{t+\tau}) + u_{d,t} \\ &- (E_t - \tilde{E}_t)(\sum_{\tau=1}^{\infty}\rho^\tau r_{t+\tau}) \\ &= (E_t - \tilde{E}_t)(\sum_{\tau=1}^{\infty}\rho^\tau \Delta d_{t+\tau}) - (E_t - \tilde{E}_t)(\sum_{\tau=1}^{\infty}\rho^\tau r_{t+\tau}) + u_{d,t} \end{aligned}$$

where $\rho = 1/(1+\exp(E[-\log(P_t/D_t)])) = 1/(1+exp(-\sum_{i=1}^{2} log(\lambda_i)\Omega_i))$.

[Proof of Proposition 3] Given that $\lambda_1 > \lambda_2$, $r^*_{T_m^{x,high}} - r^*_{T_m^{x,low}} < 0$ if and only if $\pi_{1,T_m^{x,high}} - \pi_{1,T_m^{x,low}} < \tilde{\pi}_{1,T_m^{x,high}} - \tilde{\pi}_{1,T_m^{x,low}}$. After some algebra, one can show that this condition is equivalent to

$$\begin{aligned} &\exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2}{\sigma_d^2}(\tilde{\pi}_{1,T_m^{x,high}} - \tilde{\pi}_{1,T_m^{x,low}})\right) \times \\ &\exp\left(\frac{\sigma_{x,1}^2 - \sigma_{x,2}^2}{2\sigma_{x,1}^2\sigma_{x,2}^2}(\mu_{x,1}-\mu_{x,2})(\tilde{\pi}_{1,T_m^{x,high}} - \tilde{\pi}_{1,T_m^{x,low}})\left(u_{x,T_m^x} - \frac{\delta_x(\tilde{\pi}_{1,T_m^{x,high}}) + \delta_x(\tilde{\pi}_{1,T_m^{x,low}})}{2}\right)\right) \\ &< 1 + (\tilde{\pi}_{1,T_m^{x,high}} - \tilde{\pi}_{1,T_m^{x,low}})\frac{\tilde{\pi}_{1,T_m^{x,low}}(1-\tilde{\pi}_{1,T_m^{x,high}})}{\tilde{\pi}_{1,T_m^{x,high}}(1-\tilde{\pi}_{1,T_m^{x,low}})}(\pi_{1,T_m^{x,low}}(1-\pi_{1,T_m^{x,high}}))^{-1} \end{aligned} \quad (32)$$

Note that the term $(\pi_{1,T_m^{x,low}}(1-\pi_{1,T_m^{x,high}}))^{-1}$ in Equation (32) is a function of the investor's beliefs about the high growth state after observing the dividend realization and the external signal. Hence, it is a function of the news variables observed from these two sources of information. Given that $(\pi_{1,T_m^{x,low}}(1-\pi_{1,T_m^{x,high}}))^{-1} > 1$, a sufficient condition that involves only the investor's prior beliefs and the news variable can be expressed as

$$u_{x,T_m^x} > \delta_x(\tilde{\pi}_{1,T_m^{x,high}}) + \omega_x(\tilde{\pi}_{1,T_m^{x,high}})$$

where

$$\begin{aligned} \omega_x(\tilde{\pi}_{1,T_m^{x,high}}) &= \frac{(\mu_{x,1}-\mu_{x,2})(\tilde{\pi}_{1,T_m^{x,high}} - \tilde{\pi}_{1,T_m^{x,low}})}{2} \\ &+ \frac{2\left(\frac{(\mu_{d,1}-\mu_{d,2})^2}{\sigma_d^2}(\tilde{\pi}_{1,T_m^{x,high}} - \tilde{\pi}_{1,T_m^{x,low}}) + \ln\left(1 + (\tilde{\pi}_{1,T_m^{x,high}} - \tilde{\pi}_{1,T_m^{x,low}})\frac{\tilde{\pi}_{1,T_m^{x,low}}(1-\tilde{\pi}_{1,T_m^{x,high}})}{\tilde{\pi}_{1,T_m^{x,high}}(1-\tilde{\pi}_{1,T_m^{x,low}})}\right)\right)\sigma_{x,1}^2\sigma_{x,2}^2}{(\sigma_{x,2}^2 - \sigma_{x,1}^2)(\mu_{x,1}-\mu_{x,2})(\tilde{\pi}_{1,T_m^{x,high}} - \tilde{\pi}_{1,T_m^{x,low}})}. \end{aligned}$$