Return Decomposition over the Business Cycle

Tolga Cenesizoglu

HEC Montréal

February 18, 2014

Abstract

To analyze the determinants of the observed variation in stock prices, Campbell and Shiller (1988) have suggested decomposing unexpected stock returns into unexpected changes in investors' beliefs about future cash flows (cash flow news) and discount rates (discount rate news). Based on a generalization of this approach to a framework with regime-switching parameters and variances, we analyze the decomposition of the conditional variance of returns on the S&P 500 index over the business cycle. The cash flow news is relatively more important than discount rate news in determining the conditional variance of returns in expansions. The conditional variances of returns and its components increase in recessions. However, the conditional variance of discount rate news increases more than that of cash flow news and, thus, the discount rate news becomes relatively more important than cash flow news in determining the conditional variance of returns in recessions. In contrast to the standard Campbell and Shiller approach with constant parameters and variances, cash flow news becomes more important than discount rate news in determining the unconditional variance of returns when we allow parameters and variances to vary over the business cycle. We show that these results are broadly consistent with the implications of a stylized asset pricing model in which the growth rates of dividends and consumption take on different values depending on the underlying state of the economy.

Key words: return decomposition, business cycle, unconditional and conditional variances, time-varying parameters, time-varying variances, asset pricing model, learning, regime switching fundamentals.

1 Introduction

Stock prices depend on investors' expectations about future cash flows and discount rates. Thus, stock prices vary as a result of changes in investors' expectations about these factors. A natural question to ask is whether the observed variation in stock prices is mostly due to changes in investors' expectations about future cash flows or discount rates. Although this is an empirical question, it also has important implications for understanding and modeling how financial markets work. Hence, it is not surprising that this question has been a central issue in finance and is still a hotly debated topic.

To address this issue, Campbell and Shiller (1988) suggest decomposing stock returns into two components: (1) changes in investors' expectations about discount rate, which is commonly referred to as the discount rate news, and (2) changes in investors' expectations about future dividend growth rates, which is commonly referred to as the cash flow news. One can then simply analyze the relative importance of each component in determining the observed variation in stock prices by considering their contribution to the overall unconditional variance of stock returns.

However, neither discount rate nor cash flow news can be directly observed. Hence, one has to find empirical proxies to analyze their relative contribution to the observed variation in stock prices. The standard approach in the literature is to model the short-run dynamics of expected returns in a vector autoregressive (VAR) system, obtain an empirical proxy for the discount rate news based on forecasts from the estimated VAR system and back out cash flow news as residual from the decomposition of returns. This approach has several advantages. First of all, one needs to understand only the short-run dynamics of expected returns and not that of cash flows, which can be relatively difficult to model. Secondly, it has been easier to forecast returns than dividends, at least in the last 50 years. Last but not least, it is a very straightforward and easy-to-implement approach as it only requires the estimation of a simple VAR. Hence, it is not surprising to find a large literature implementing the standard approach to answer different questions in finance, macroeconomics and accounting.[1] However, the standard approach depends heavily on the predictability of returns. Given the growing empirical evidence against the predictability of returns (e.g. Welch and Goyal (2008)), it has also recently come under some criticism.[2]

Most studies in the literature focus on the decomposition of the unconditional variance of stock returns based on the standard approach with linear VAR models and constant parameters. However, there is growing empirical evidence that both variances and the predictability of returns are time-varying. First of all, it is a well-known empirical fact that the conditional variances of stock returns and most of the standard predictor variables are time-varying and change with changing market conditions. For example, most financial variables, including but not limited to stock returns, tend to be much more volatile in recessions than expansions. Secondly, there is growing recent empirical evidence that the predictive power of certain variables for returns is also time-varying and depends on underlying business and economic conditions (see Dangl and Halling (2011), Henkel, Spencer, and Nardari (2011) and references therein).

[1] In macroeconomics and finance, the list of articles using the standard approach is long and includes but not limited to Campbell (1991), Campbell (1993), Campbell and Ammer (1993), Campbell and Mei (1993), Campbell (1996), Campbell and Vuolteenaho (2004a), Campbell and Vuolteenaho (2004b), Bernanke and Kuttner (2005) and Campbell, Polk, and Vuolteenaho (2010). There are also few articles in accounting using the standard approach, e.g. Callen and Segal (2004), Callen, Hope, and Segal (2005), and Callen, Livnat, and Segal (2006).

[2] Chen and Zhao (2009) show that the empirical results based on the standard return decomposition approach tend to be sensitive to the set of predictor variables and the time period.

Hence, one needs to keep these empirical facts in mind when implementing any return decomposition approach since results based on an approach that captures these empirical facts might be completely different than those based on the standard approach. Furthermore, one also needs to distinguish the decomposition of unconditional variance from that of conditional variance, which might be changing over time as the economy and financial markets go through periods of tranquility and turbulence.

In this paper, we are mainly interested in the decomposition of the conditional variance of returns on the S&P 500 index over the business cycle. Our main assumption is that both parameters and variances are time-varying and depend on the underlying state of the economy. To this end, we first provide some empirical evidence that the VAR parameters (thus, the predictive power of variables for returns) and residual variance matrix do indeed change over the business cycle. Specifically, we find that (1) the predictor variables are less persistent in recessions than expansions and the whole sample, although implying stationary processes in both recessions and expansions; (2) the variances and covariances of VAR residuals are much higher (in magnitude) in recessions than expansions; (3) returns are much more predictable in recessions than expansions as suggested by higher adjusted R^2 and more parameters with statistically significant estimates. We then decompose the returns in expansions and recessions based on the standard approach under alternative assumptions about the VAR parameters and residual variance matrix. We find that the decomposition of returns changes dramatically between expansions and recessions mostly due to time-varying VAR parameters and less so due to time-varying residual variance matrix.

These results provide some preliminary empirical evidence that the decomposition of returns might be changing over the business cycle. However, they only correspond to hypothetical situations since the standard approach cannot capture in a consistent manner the empirical fact that the economy switches between expansion and recession periods. In this paper, we do this by modeling the short-run dynamics of returns and predictive variables in a Markov regime switching vector autoregressive model (MSVAR) where both the VAR parameters and residual variance matrix are assumed to switch between different values based on the underlying state of the economy. We then generalize the standard return decomposition approach to this framework and show that the conditional variances of cash flow and discount rate news as well as their conditional covariance can be expressed in closed-from when the state variable is observable and can be calculated numerically based on simulations otherwise.

Based on this framework with regime-switching VAR parameters and residual variance matrix, we decompose the returns on the S&P 500 index over the business cycle. We start with the decomposition of the unconditional variance based on the time-varying approach. This allows us to compare our results to those based on the standard approach. The unconditional variances of unexpected returns and discount rate news as well as the unconditional covariance between discount rate and cash flow news are smaller in magnitude while the unconditional variance of cash flow news is higher. This in turn implies an increase in the relative contribution of cash flow news to the unconditional variance of returns, compared to the standard approach. Specifically, the cash flow news explains 46% (compared to 29% in the standard approach), the discount rate news explains 40% (compared to 43% in the standard approach) and the covariance between them explains 14% (compared to 28% in the standard approach) of the unconditional variance of returns. These results suggest that the cash flow news becomes more important in determining the unconditional variance of returns when one takes into account the time-varying nature of predictive relations and variances over the business cycle.

Turning our attention to the decomposition of the conditional variance reveals how the relative importance of each component changes over the business cycle. First of all, the conditional variance of unexpected returns as well as its components are generally higher in recessions than expansions. Second, they also tend to be relatively stable within each regime, maybe with the exception of the recent financial crisis. Last but not least, the relative importance of each component in determining the conditional variance of returns changes over the business cycle. In expansions, the conditional variance of cash flow news is higher than that of discount rate news, and thus, contributes more to the conditional variance of returns. The opposite holds in recessions. Specifically, the conditional variance of cash flow news explains, on average, between 40% and 60% of conditional variance of returns in expansions. This ratio decreases in recessions (with the exception of the 2001 recession) to between 20% and 40%. The conditional variance of discount rate news explains, on average, between 30% and 40% of conditional variance of returns in expansions. This ratio increases in recessions to between 50% and 90%. The contribution of the conditional covariance between cash flow and discount rate news to the conditional variance of returns is between 30% and -30% in expansions and this contribution generally decreases and becomes more negative in recessions.

For our main empirical results, we focus on the decomposition of monthly returns on the S&P 500 index over NBER business cycles between January 1960 and December 2010 using term spread, dividend yield and value spread as additional state variables in the VAR. Chen and Zhao (2009) show that the empirical results based on the standard return decomposition approach tend to be sensitive to the set of state variables and the sample period. To this end, we also analyze the robustness of our main empirical results based on the time-varying approach and find that they are mostly robust to using a longer sample period between June 1927 and December 2010, using the first four principal components of a large number of known predictor variables as an alternative set of state variables and using an alternative definition of the business cycle based on the smoothed state probabilities obtained from the estimation of a two-state Markov regime switching model for the log growth rate of monthly industrial production index. More importantly, these results suggest that taking the time-varying nature of return predictability into account has the potential to address the criticism of the standard approach by Chen and Zhao (2009) based on the lack of return predictability.

To understand the intuition behind our empirical results, we consider a stylized asset pricing model and analyze its implications for the decomposition of returns over the business cycle. Specifically, we consider a pure exchange economy (Lucas (1978)) in discrete time where the preferences of a representative investor are modeled by a constant relative risk aversion utility over consumption. Assuming that investors have access to implicit labor income, we model the (log) growth rates of dividend and consumption as a Markov regime switching vector autoregressive model. We derive the data generating process of returns in closed form as a function of unexpected dividend growth rates and changes in investor's beliefs which in turn depend on unexpected dividend and consumption growth rates. Given that investors observe the true data generating process of returns, we can directly apply the return decomposition approach of Campbell and Shiller without the need for a forecasting model such as a VAR. We obtain cash flow and discount rate news as defined by the Campbell and Shiller approach in closed from as functions of unexpected dividend growth rates and changes in investor's beliefs. We also show that the unconditional and conditional variances and covariances of cash flow and discount rate news can be expressed in closed form when the state variable is assumed observable and can be calculated based on simulations otherwise. We then derive the implications of this model for the decomposition of returns over the business cycle. To do this, we calibrate the model parameters to US data and simulate monthly observations from the model assuming that the states correspond to the NBER business cycles. We then decompose the simulated returns based on the Campbell and Shiller approach using the true data generating process as the forecasting model.

In this framework, we first argue that the investors' risk aversion parameter is the main driving factor behind the unconditional variance of returns and its decomposition. To see this, note that the marginal rate of substitution and, thus, the stochastic discount factor depend on investors' risk aversion in asset pricing models like the one considered in this paper where investors have power utility over consumption. As investors become more risk averse, the stochastic discount factor and, thus, discount rate news become more volatile. On the other hand, the coefficients multiplying investors' beliefs in the definition of cash flow news in this framework becomes smaller and, thus, the cash flow news becomes less volatile. The covariance between the two components is always positive and increases with increasing investors' risk aversion. In the Campbell and Schiller decomposition, an increase in the variance of either discount rate or cash flow news increases the variance of returns while an increase in their covariance decreases it. For low levels of risk aversion, the variance of returns decreases as investors become more risk averse. This is due to the fact that the increase in the variance of discount rate news is dominated by the decreases in the variance of cash flow news and (-2 times) the covariance between the two components. For high levels of risk aversion, the opposite holds and the variance of returns increases as investors become more risk averse. We also show that the decomposition of the unconditional variance of returns observed in the data is in line with what this stylized asset pricing model implies for reasonable model parameters. Specifically, this stylized asset pricing model can match the empirical facts about the decomposition of the unconditional variances of returns for a risk aversion parameter of 7.5, which is similar to values considered in the literature, see for example Bansal and Yaron (2004).

We then show that this stylized asset pricing model predicts the following regarding the decomposition of the conditional variance of returns over the business cycle: (1) the conditional variance of unexpected returns from our model are higher in recessions than expansions; (2) the conditional variances of both cash flow and discount rate news are also significantly higher in recessions than expansions; (3) the conditional covariance between cash flow and discount rate news is positive and higher in recessions than expansions; (4) the conditional variances and covariances are constant within each regime; (5) the relative importance of cash flow news is lower in recessions than expansions; (6) the relative importance of discount rate news is higher in recessions than expansions; (7) the contribution of the conditional covariance between cash flow and discount rate news is negative in expansions and recessions and increases in magnitude in recessions; (8) cash flow news is relatively more important than discount rate news in expansions while the opposite holds in recessions. The observed empirical facts are mostly in line these implications with the exception of the one about the conditional covariance between cash flow and discount rate news, which is, on average, negative in the data.

The main driving factor behind these predictions of this stylized model is the transition probability matrix. We calibrate the transition probability matrix to match the monthly transition probabilities of the NBER business cycles between 1960 and 2010. Expansion periods as defined by the NBER tend to be longer than recession periods and thus also more persistent. Hence, the probability that the economy switches from a recession to an expansion is higher than the probability that the economy switches from an expansion to a recession. This fact makes investors' beliefs more volatile in recessions than expansions, which in turn implies the conditional variance of returns, the conditional variance of its components and the conditional covariance between its components are higher in recessions than expansions. In this stylized model, cash flow news depend on investors' beliefs as well as the unexpected dividend growth rate while discount rate news depends only on investors' beliefs. This in turn implies that the conditional variance of discount rate news is much more sensitive to any changes in the volatility of investors' beliefs than that of cash flow news. Thus, the increase in the volatility of investors' beliefs in recessions results in a bigger increase in the conditional variance of discount rate news relative to that of cash flow news, making discount rate news relatively more important in recessions.

The paper closest to ours is Bianchi (2010) which also considers decomposing returns based on a MSVAR. In this framework, he identifies a 1930s regime and argues that rare events during the Great Depression and its aftermath shaped the way agents think about financial markets. He then reconsiders the two beta model of Campbell and Vuolteenaho (2004a) and shows that its performance depends on including the 1930s regime. Although our return decomposition approach based on a MSVAR is similar to his framework, our paper differs from his in several aspects. First of all, we use this framework to identify expansion and recession periods of the business cycle while he uses it to identify the Great Depression period. Secondly, we focus mostly on the decomposition of the conditional variance of returns and show how it changes over the business cycle while he focuses mostly on the decomposition of the unconditional variance and its implications for the two beta model of Campbell and Vuolteenaho (2004a). Third, from a technical point of view, our solution and estimation approaches are quite different than his. Last but not least, we derive closed-form formulas for the decomposition of returns in a stylized asset pricing framework and use this framework to provide intuition behind our empirical results.

Our paper is related to a growing literature analyzing the relative importance of discount rate and cash flow news from some alternative perspectives. For example, Vuolteenaho (2002) uses an accounting-based present-value formula that uses return on equity instead of dividend growth as the basic cash flow fundamental. Larrain and Yogo (2008) suggest using net payout, which is the sum of dividends, interest, equity repurchase net of issuance, and debt repurchase net of issuance, as the proxy for the total cash outow from the corporate sector. Chen, Da, and Zhao (2013) propose using direct expected cash flow measures based on the firm-specific implied cost of equity. Most of these studies find cash flow news to be more important than previously thought, especially in determining the variation in prices of individual stocks. This is similar to what we find based on the time-varying approach.

The rest of the paper is organized as follows: Section 2 presents the standard approach for comparison purposes. Section 3 provides some preliminary empirical evidence on the time-varying nature of the decomposition of returns.

Section 4 introduces the time-varying approach and presents the decomposition of returns over the business cycle. Section 5 presents the implications of a stylized asset pricing model for the decomposition of returns over the business cycle. Section 6 concludes.

2 The Standard Return Decomposition Approach

In this section, we present some empirical results based on the standard approach to serve as a benchmark. To this end, we first briefly describe the basic framework of Campbell and Shiller (1988) and discuss the standard empirical approach employed to implement it. We then present empirical results on the decomposition of the unconditional variance of S&P 500 returns based on the standard approach.

2.1 Return Decomposition

Campbell and Shiller (1988) show that log stock returns, r_{t+1}, can be expressed as a linear approximation of the log dividend-price ratio around its long term mean:

$$r_{t+1} \approx k + \rho p_{t+1} + (1-\rho)d_{t+1} - p_t$$

where d_{t+1} and p_{t+1} are log dividend and price in period $t+1$, respectively, ρ and k are parameters of linearization defined as $\rho = 1/(1+\exp(\overline{d-p}))$ and $k = -\log(\rho) - (1-\rho)\log(1/\rho - 1)$ and $\overline{d-p}$ is the long term mean of the log dividend-price ratio, $d_{t+1} - p_{t+1}$. Assuming that a transversality condition holds, Campbell and Shiller (1988) show that unexpected return in period $t+1$ can be decomposed as follows:

$$\begin{aligned} r^*_{t+1} &= r_{t+1} - E_t[r_{t+1}] \\ &= E_{t+1}\left[\sum_{j=0}^{\infty}\rho^j \Delta d_{t+1+j}\right] - E_t\left[\sum_{j=0}^{\infty}\rho^j \Delta d_{t+1+j}\right] \\ &- \left(E_{t+1}\left[\sum_{j=1}^{\infty}\rho^j r_{t+1+j}\right] - E_t\left[\sum_{j=1}^{\infty}\rho^j r_{t+1+j}\right]\right) \\ &= CF_{t+1} - DR_{t+1}. \end{aligned} \quad (1)$$

where DR_{t+1}, referred to as the discount rate news, is the change in investors' expectations in period $t+1$ about discounted sum of future excess returns or, equivalently, future discount rates and CF_{t+1}, referred to as the cash flow news, is the change in investors' expectations in period $t+1$ about discounted sum of future dividend growth rates or, equivalently, future cash flows.

2.2 Unconditional Variance Decomposition

Based on the decomposition in Equation 1, the unconditional variance of unexpected stock returns can be decomposed into three components: the unconditional variances of cash flow and discount rate news and the unconditional covariance between the two components as follows:

$$var(r^*_{t+1}) = var(CF_{t+1}) + var(DR_{t+1}) - 2covar(CF_{t+1}, DR_{t+1}). \quad (2)$$

The relative importance of each component in determining the observed variation in stock returns can then be analyzed based on the relative contribution of each component to the overall unconditional variance of stock returns, i.e. $var(CF_{t+1})/var(r^*_{t+1})$, $var(DR_{t+1})/var(r^*_{t+1})$ and $covar(CF_{t+1})/var(r^*_{t+1})$.

2.3 Empirical Implementation

Given that neither discount rate nor cash flow news can be directly observed, one needs to find empirical proxies for them. Campbell and Shiller (1988) suggest modelling the short-run dynamics of expected returns to obtain forecasts of future expected returns and, thus, a proxy for discount rate news and back out cash flow news as the sum of unexpected returns and discount rate news. Hence, the standard practice in the literature has been to model the short-run dynamics of expected returns in a vector autoregressive (VAR) system with some other state variables that have predictive power for future returns:

$$\mathbf{X}_{t+1} = \boldsymbol{\phi} + \boldsymbol{\Phi}\mathbf{X}_t + \boldsymbol{\epsilon}_{t+1} \quad (3)$$

where $\mathbf{X}_{t+1} = [r_{t+1}, \mathbf{Z}'_{t+1}]'$ is an $N \times 1$ vector of excess stocks returns (r_{t+1}) and predictor variables ($\mathbf{Z}_{t+1}$). $\boldsymbol{\Phi}$ is an $N \times N$ matrix, $\boldsymbol{\phi}$ is an $N \times 1$ vector and $\boldsymbol{\epsilon}_{t+1} \sim N(\mathbf{0}, \boldsymbol{\Upsilon})$ is a $N \times 1$ vector of VAR residuals. We use bold symbols to denote vectors and matrices and non-bold symbols to denote scalars for the rest of the paper unless otherwise stated.

The forecasting model in Equation 3 is estimated using, generally, monthly data on excess stock returns and predictor variables. Choosing a value for ρ, one can obtain a proxy for the current discount rate news as the change in the expected future stock returns based on the forecasts from the estimated VAR system and then back out the current cash flow news as the sum of current unexpected return and discount rate news as follows:

$$\begin{aligned}
\widehat{DR}_{t+1} &= \mathbf{e}'_1(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1}\rho\hat{\boldsymbol{\Phi}}(\mathbf{X}_{t+1} - \hat{\boldsymbol{\phi}} - \hat{\boldsymbol{\Phi}}\mathbf{X}_t) = \mathbf{e}'_1(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1}\rho\hat{\boldsymbol{\Phi}}\hat{\boldsymbol{\epsilon}}_{t+1} \\
\widehat{CF}_{t+1} &= \mathbf{e}'_1(\mathbf{I} + \rho\hat{\boldsymbol{\Phi}}(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1})(\mathbf{X}_{t+1} - \hat{\boldsymbol{\phi}} - \hat{\boldsymbol{\Phi}}\mathbf{X}_t) = \mathbf{e}'_1(\mathbf{I} + \rho\hat{\boldsymbol{\Phi}}(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1})\hat{\boldsymbol{\epsilon}}_{t+1}
\end{aligned}$$

The unconditional variance of returns can then be decomposed into its components as in Equation 2. $var(DR_{t+1})$, $var(CF_{t+1})$ and $cov(CF_{t+1}, DR_{t+1})$ can be obtained as the sample variances of CF_{t+1} and DR_{t+1} and their sample covariance, respectively. Or, equivalently, they can be obtained based on the sample variance matrix of the VAR residuals, $\hat{\boldsymbol{\Upsilon}}$, as follows:

$$\begin{aligned}
\widehat{var}(DR_{t+1}) &= (\mathbf{e}'_1\rho\hat{\boldsymbol{\Phi}}(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1})\hat{\boldsymbol{\Upsilon}}(\mathbf{e}'_1\rho\hat{\boldsymbol{\Phi}}(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1})' \\
\widehat{var}(CF_{t+1}) &= (\mathbf{e}'_1(\mathbf{I} + \rho\hat{\boldsymbol{\Phi}}(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1}))\hat{\boldsymbol{\Upsilon}}(\mathbf{e}'_1(\mathbf{I} + \rho\hat{\boldsymbol{\Phi}}(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1}))' \\
\widehat{cov}(CF_{t+1}, DR_{t+1}) &= (\mathbf{e}'_1(\mathbf{I} + \rho\hat{\boldsymbol{\Phi}}(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1}))\hat{\boldsymbol{\Upsilon}}(\mathbf{e}'_1\rho\hat{\boldsymbol{\Phi}}(\mathbf{I} - \rho\hat{\boldsymbol{\Phi}})^{-1})'
\end{aligned}$$

2.4 Empirical Choices

In this paper, we are interested in decomposing the market return. To this end, we use the continuously compounded monthly returns on the S&P 500 index, including dividends, from Center for Research in Security Prices (CRSP) in excess of the log risk-free rate to proxy for the excess return on the market index (r_t) between January 1960 and December 2010. Following the literature, we set ρ to 0.997 in monthly data which correspond to an annual average dividend-price ratio of around 4%.

As for the other state variables in the VAR, we consider term spread (tms_t), dividend yield (dy_t) and value spread (vs_t). The term spread is the difference between the long term yield on government bonds and the Treasury bill. The dividend yield is the log ratio of dividends to lagged prices. The value spread is the difference between the log book-to-market of small value stocks and that of small growth stocks. Data on excess returns, term spread and dividend yield are from Amit Goyal's website. The value spread is calculated based on the six size and book-to-market sorted portfolios from Ken French's website. We also use these as the state variables in our estimations for the rest of the paper. We discuss the robustness of our results to using an alternative sets of state variables in Section 4.6.

2.5 Empirical Results

In this section, we present the decomposition of returns based on the standard approach. Panel (a) of Table 1 presents the estimates of the VAR parameters and the adjusted R^2 for each variable. First of all, the adjusted R^2 of the equation for returns is extremely low at 0.68%. This is not surprising as it is well known that most predictive variables, including the ones considered in Table 1, do not have much power in forecasting returns. Second, only the term spread has a significant coefficient estimate in the equation for the returns suggesting that other variables do not have any significant predictive power for returns. Third, all predictive variables are persistent with significant coefficients on their own lagged values. However, the eigenvalues of the matrix $\mathbf{\Phi}$ in Equation 3 all lie inside the unit circle suggesting that the VAR is stationary. Panel (b) of Table 1 presents the residual variance matrix. The first diagonal element is the unconditional variance of monthly excess returns on the S&P 500 index which we decompose into the variance of cash flow and discount rate news and their covariance. Panel (c) presents the decomposition of the unconditional variance of returns. The unconditional variance of discount rate news constitutes 43% of the unconditional variance of returns. On the other hand, 29% of the unconditional variance of returns can be attributed to the unconditional variance of cash flow news. The remaining 28% is due to the unconditional covariance between the two components. These results suggest that discount rate news is, on average, relatively more important than cash flow news in determining the unconditional variance of returns on the S&P 500 index. These results are also generally consistent with those in Campbell and Ammer (1993) and Chen and Zhao (2009).

3 Time-Varying Parameters and Variances

Our time-varying approach is motivated by the growing empirical evidence that both the variances and predictive power of certain variables for returns are time-varying. In this section, we first provide some empirical evidence that the VAR parameters (thus, the predictive power of variables for returns) and residual variance matrix do indeed change over the business cycle. To this end, we first distinguish between expansion and recession periods as defined by NBER. We estimate the VAR parameters and the residual variance matrix in expansions and recessions, separately, via weighted least squares. Specifically, we estimate the VAR model in expansions (recessions) assuming that the weight of an observation is one if the economy is in an expansion (recession) period and zero if it is in a recession (expansion) period. We then decompose the returns in expansions and recessions based on the standard approach under alternative assumptions about the VAR parameters and residual variance matrix.

Panels (a) of Tables 2 and 3 present the VAR parameters in expansions and recessions, respectively. First of all, the predictor variables are less persistent in recessions compared to expansions. However, the VAR parameter estimates in both recessions and expansions imply stationary processes. More importantly, the adjusted R^2 in expansions is only 0.40% and lower than the adjusted R^2 over the whole sample. On the other hand, the adjusted R^2 in recessions is slightly higher than 10%, which is generally considered a quite high explanatory power in the literature on forecasting returns. Finally, none of the variables in the equation for returns is statistically significant in expansions while they are all statistically significant in recessions with the exception of the value spread.

Panels (b) of Tables 2 and 3 present the residual variance matrix in expansions and recessions, respectively. As it is well known, the variance of unexpected returns in recessions is higher than (almost twice of) that in expansions. The variances of the residuals of predictor variables are also higher in recessions than expansions. Furthermore, the covariances also vary between expansions and recessions and mostly increase in magnitude in recessions.

Panels (c) of Tables 2 and 3 present the decomposition of unconditional variance of returns in expansions and recessions based on their corresponding VAR parameters and residual variance matrices. Before proceeding to the discussion of these results, we should first note that these decompositions of returns in expansions and recessions based on the standard approach correspond to hypothetical situations. To see this, note that the standard approach assumes that the economy will stay in the same state till infinity. This is due to the fact that the standard approach cannot capture in a consistent manner the fact that the economy switches between expansion and recession periods. Nevertheless, these results provide some intuition on how the decomposition of returns might be changing over the business cycle.

We start with the decomposition of returns in expansions and compare it to that based on the whole sample period. The variance of cash flow news increases almost fivefold and that of discount rate news decreases while the covariance between cash flow and discount rate news changes sign and increases in magnitude. As a result, the relative importance of cash flow news increases almost fivefold and that of discount rate news remains almost the same while the covariance term has a large negative contribution to the overall variance of returns compared to its modest positive contribution in the whole sample.

We now turn attention to the decomposition of returns in recessions and compare it to that based on the whole sample period. The variance of discount rate news increases almost sixfold and that of cash flow news increases only slightly while the covariance between cash flow and discount rate news changes sign and increases in magnitude. As a result, the relative importance of discount rate news increases and and that of cash flow news remains almost the same while the covariance term has a large negative contribution to the overall variance of returns compared to its modest positive contribution in the whole sample. These results suggest that the cash flow news are more important than discount rate news while the opposite holds in recessions. Furthermore, the covariance between the two plays a more important role in determining observed variation in stock prices both in expansions and recessions compared to the whole sample period.

The decompositions of returns in expansions and recessions presented in Tables 2 and 3 are based on the assumption that both VAR parameters and residual variance matrices are time-varying. To understand how these two empirical assumptions affect the decomposition of returns, one can consider them separately as we do in Table 4. We consider in Panel (a) of Table 4 the assumption that the VAR parameters are time-varying and identical to those presented in Panels (a) of Tables 2 and 3 with a constant residual variance matrix estimated over the whole sample based on time-varying VAR parameters. These results are similar to those presented in Panels (c) of Tables 2 and 3. Specifically, under the assumption of time-varying parameters but constant variance, the cash flow news are more important than discount rate news while the opposite holds in recessions. In Panel (b) of Table 4, we consider the assumption that the VAR parameters are constant and identical to those estimated over the whole sample but the residual variance matrix are estimated over expansions and recessions separately. The variances of cash flow and discount rate news as well as their covariance increase in recessions compared to expansions. This increase is more pronounced for discount rate news than cash flow news. The relative importance of each component also changes between expansions and recessions but not as dramatically as under the assumption of time-varying parameters. Overall, the results in Table 4 suggest that the dramatic change in the decomposition of returns between expansions and recessions presented in Panels (c) of Tables 2 and 3 is mostly due to time variation in the VAR parameters. Time-varying residual variance matrices also contribute to this change but in a somewhat less pronounced fashion.

4 Time-Varying Return Decomposition Approach

Our results in Section 3 suggest that time variation in the VAR parameters and residual variance matrix over the business cycle might have important implications for the decomposition of returns over the business cycle. However, as mentioned above, the evidence presented in Section 3 correspond to hypothetical situations due to the fact that the standard approach implicitly assumes that the economy stays in the same state till infinity. In this section, we analyze the decomposition of returns assuming that the economy switches between expansions and recessions. To do this, we first generalize the standard decomposition approach to a framework where both the VAR parameters and residual variance matrix are assumed to switch between different values based on the underlying state of the economy. We then analyze the decomposition of both unconditional and conditional variances of returns over the business cycle.

4.1 Forecasting Model

To capture the time-variation of the VAR parameters and residual variance matrix over the business cycle, we model the dynamics of returns and predictive variables in a Markov regime switching vector autoregression (MSVAR) as follows:

$$\mathbf{X}_{t+1} = \boldsymbol{\alpha}_{S_{t+1}} + \mathbf{A}_{S_{t+1}}\mathbf{X}_t + \boldsymbol{\epsilon}_{t+1} \tag{4}$$

where $\mathbf{X}_{t+1} = [r_{t+1}, \mathbf{Z}'_{t+1}]'$ is an $N \times 1$ vector of excess stocks returns and predictive variables, as before. $\mathbf{A}_i$ is an $N \times N$ matrix, $\boldsymbol{\alpha}_i$ is an $N \times 1$ vector for $i = 1, 2, \ldots, M$ and $\boldsymbol{\epsilon}_{t+1} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{S_{t+1}})$ is a $N \times 1$ vector of error terms. The state variable S_t follows a first order M-state Markov chain with transition probability matrix $\mathbf{Q}$ whose ij^{th} element $q_{i,j} = Prob(S_{t+1} = j|S_t = i)$.

Before characterizing the unexpected return and its decomposition, the following lemma derives the expected value of $\mathbf{X}_{t+\tau}$ based on the information set at time t.

Lemma 1.

$$E_t[\mathbf{X}_{t+\tau}] \quad = \quad (\mathbf{1}_M \otimes \mathbf{I}_N)'\Big(\mathbf{f}_1(\tau)(\boldsymbol{\Pi}_t \otimes \mathbf{1}_N) + \mathbf{f}_2(\tau)(\boldsymbol{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t\Big) \tag{5}$$

where $\mathbf{1}_M$ is a $M \times 1$ vector of ones and $\mathbf{I}_N$ is the $N \times N$ identity matrix. $\mathbf{f}_1(\tau)$ and $\mathbf{f}_2(\tau)$ are matrices defined in the appendix. $\boldsymbol{\Pi}_t$ is the $M \times 1$ vector of probabilities associated with each state conditional on the information set in period t, $\mathcal{F}_t$, i.e. $\boldsymbol{\Pi}_t = [Prob(S_t = 1|\mathcal{F}_t), Prob(S_t = 2|\mathcal{F}_t), \ldots, Prob(S_t = M|\mathcal{F}_t)]'$.

Lemma 1 shows that the expectation about the future values of $\mathbf{X}_{t+\tau}$ conditional on the information set in period t does not only depend on the values of the variables in period t, $\mathbf{X}_t$, as in the standard approach, but also on the probabilities associated with each state conditional on the information set in period t. We refer to the information set $\mathcal{F}$ as investors' information set. Thus, expectations correspond to investors' expectations and state probabilities correspond to investors' beliefs about the state variable.

4.2 Return Decomposition

The following proposition presents the decomposition of unexpected return in period $t+1$ into discount rate and cash flow news based on the MSVAR in Equation 4 as the forecasting model.

Proposition 1. *Assume that the $\mathbf{X}_t$ follows the process in Equation 4. The unexpected return on the risky asset in period $t+1$ can be expressed as follows:*

$$\begin{aligned} r^*_{t+1} &= \mathbf{e}'_1\Big(\mathbf{X}_{t+1} - E_t[\mathbf{X}_{t+1}]\Big) \\ &= \mathbf{e}'_1\Big(\mathbf{X}_{t+1} - (\mathbf{1}_M \otimes \mathbf{I}_N)'(\mathbf{f}_1(1)(\boldsymbol{\Pi}_t \otimes \mathbf{1}_N) + \mathbf{f}_2(1)(\boldsymbol{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t)\Big) \end{aligned} \tag{6}$$

and can be decomposed into cash flow and discount rate news as in Equation 1 with

$$DR_{t+1} = \mathbf{e}'_1(\mathbf{1}_M \otimes \mathbf{I}_N)'[\mathbf{B}_{1,1}(\boldsymbol{\Pi}_{t+1} \otimes \mathbf{1}_N) + \mathbf{B}_{2,1}(\boldsymbol{\Pi}_{t+1} \otimes \mathbf{X}_{t+1}) - \mathbf{B}_{1,2}(\boldsymbol{\Pi}_t \otimes \mathbf{1}_N) - \mathbf{B}_{2,2}(\boldsymbol{\Pi}_t \otimes \mathbf{X}_t)] \tag{7}$$

and

$$
\begin{aligned}
CF_{t+1} &= \mathbf{e}_1'\mathbf{X}_{t+1} + \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'[\mathbf{B}_{1,1}(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N) + \mathbf{B}_{2,1}(\mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})] \\
&- \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'[(\mathbf{f}_1(1) + \mathbf{B}_{1,2})(\mathbf{\Pi}_t \otimes \mathbf{1}_N) + (\mathbf{f}_2(1) + \mathbf{B}_{2,2})(\mathbf{\Pi}_t \otimes \mathbf{X}_t)] \qquad (8)
\end{aligned}
$$

where $\mathbf{B}_{i,j}$ *for* $i, j = 1, 2$ *are matrices defined in the appendix.*

4.3 Unconditional and Conditional Variance Decomposition

In this section, we discuss how to decompose the unconditional as well as the conditional variance of returns based on the forecasting model in Equation 4. The unconditional variance of unexpected returns can be decomposed into its components as in Equation 2. Similarly, the conditional variance of returns can be decomposed into conditional variance of cash flow and discount rate news and their conditional covariance as follows:

$$var_t(r^*_{t+1}) = var_t(CF_{t+1}) + var_t(DR_{t+1}) - 2cov_t(CF_{t+1}, DR_{t+1}) \qquad (9)$$

where $var_t(\cdot)$ and $cov_t(\cdot)$ denote variance and covariance, respectively, conditional on investors' information set in period t. This is the conditional analog of the decomposition of unconditional variance in Equation 2.

The following proposition characterizes the conditional variance of returns and its components under the forecasting model in Equation 4:

Proposition 2. *The conditional variance of returns is given by*

$$var_t(r^*_{t+1}) = \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'(\mathbf{\Omega}_t - \mathbf{Z}_t\mathbf{Z}_t')(\mathbf{1}_M \otimes \mathbf{I}_N)\mathbf{e}_1 \qquad (10)$$

where $\mathbf{\Omega}_t$ *is a* $NM \times NM$ *block diagonal matrix whose diagonal elements are* $\mathbf{\Omega}_{i,t} = (\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i' + \boldsymbol{\alpha}_i(\mathbf{A}_i\mathbf{X}_t)' + (\mathbf{A}_i\mathbf{X}_t)\boldsymbol{\alpha}_i' + (\mathbf{A}_i\mathbf{X}_t)(\mathbf{A}_i\mathbf{X}_t)' + \mathbf{\Sigma}_i)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t)$ *and* $\mathbf{Z}_t = [\mathbf{Z}_{1,t}', \ldots, \mathbf{Z}_{M,t}']'$ *is a* $NM \times 1$ *vector where* $\mathbf{Z}_{i,t} = (\boldsymbol{\alpha}_i + \mathbf{A}_i\mathbf{X}_t)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t)$.

The conditional variances of discount rate and cash flow news and their conditional covariance are given by

$$\begin{aligned} var_t(DR_{t+1}) &= \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'\Big(\mathbf{B}_{1,1}(var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)\mathbf{B}_{1,1}' + 2\mathbf{B}_{1,1}cov_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})\mathbf{B}_{2,1}' \\ &+ \mathbf{B}_{2,1}var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})\mathbf{B}_{2,1}'\Big)(\mathbf{1}_M \otimes \mathbf{I}_N)\mathbf{e}_1 \qquad (11) \\ var_t(CF_{t+1}) &= \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'\Big(\mathbf{\Omega}_t - \mathbf{Z}_t\mathbf{Z}_t' + \mathbf{B}_{1,1}(var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)\mathbf{B}_{1,1}' \\ &+ 2\mathbf{B}_{1,1}cov_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})\mathbf{B}_{2,1}' + \mathbf{B}_{2,1}var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})\mathbf{B}_{2,1}'\Big)(\mathbf{1}_M \otimes \mathbf{I}_N)\mathbf{e}_1 \\ &+ 2\mathbf{e}_1'\Big(cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)\mathbf{B}_{1,1}' + cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})\mathbf{B}_{2,1}'\Big)(\mathbf{1}_M \otimes \mathbf{I}_N)\mathbf{e}_1 \qquad (12) \\ cov_t(DR_{t+1}, CF_{t+1}) &= \mathbf{e}_1'\Big(cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)\mathbf{B}_{1,1}' + cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})\mathbf{B}_{2,1}'\Big)(\mathbf{1}_M \otimes \mathbf{I}_N)\mathbf{e}_1 \\ &+ \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'\Big(\mathbf{B}_{1,1}(var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)\mathbf{B}_{1,1}' + 2\mathbf{B}_{1,1}cov_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})\mathbf{B}_{2,1}' \\ &+ \mathbf{B}_{2,1}var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})\mathbf{B}_{2,1}'\Big)(\mathbf{1}_M \otimes \mathbf{I}_N)\mathbf{e}_1 \qquad (13) \end{aligned}$$

Furthermore, if the state variable is observable, then $var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)$, $var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})$, $cov_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})$, $cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)$ *and* $cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})$ *can be expressed in closed form as follows:*

$$\begin{aligned} var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N) &= ((\mathbf{\Pi}_t'\mathbf{Q} \otimes \mathbf{1}_M) \odot (\mathbf{I}_M - ((\mathbf{\Pi}_t'\mathbf{Q} \otimes \mathbf{1}_M)') \otimes (\mathbf{1}_N' \otimes \mathbf{1}_N) \\ var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1}) &= \mathbf{\Omega}_t - \mathbf{Z}_t\mathbf{Z}_t' \\ cov_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1}) &= \mathbf{\Gamma}_t - (\mathbf{Q}'\mathbf{\Pi}_t \otimes \mathbf{1}_N)(\boldsymbol{\alpha}(\mathbf{Q}' \otimes \mathbf{1}_N)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{1}_N + \mathbf{A}(\mathbf{Q}' \otimes \mathbf{1}_N)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t)' \\ cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N) &= \Upsilon_t - (\mathbf{1}_M \otimes \mathbf{I}_N)'\{\boldsymbol{\alpha}(\mathbf{Q}' \otimes \mathbf{1}_N)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{1}_N \\ &+ \mathbf{A}(\mathbf{Q}' \otimes \mathbf{1}_N)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t\}(\mathbf{Q}'\mathbf{\Pi}_t \otimes \mathbf{1}_N)' \\ cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1}) &= \mathbf{\Lambda}_t - (\mathbf{1}_M \otimes \mathbf{I}_N)'\{\boldsymbol{\alpha}(\mathbf{Q}' \otimes \mathbf{1}_N)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{1}_N + \mathbf{A}(\mathbf{Q}' \otimes \mathbf{1}_N)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t\} \\ &\times (\boldsymbol{\alpha}(\mathbf{Q}' \otimes \mathbf{1}_N)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{1}_N + \mathbf{A}(\mathbf{Q}' \otimes \mathbf{1}_N)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t)' \end{aligned}$$

where $\mathbf{\Gamma}_t$ *is block diagonal matrix whose ith diagonal element is given by* $\mathbf{\Gamma}_{i,t} = (\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t)(\mathbf{1}_N(\boldsymbol{\alpha}_i + \mathbf{A}_i\mathbf{X}_t)')$ *for* $i = 1, \ldots, M$, $\Upsilon_t = [\mathbf{\Gamma}_{1,t}', \ldots, \mathbf{\Gamma}_{M,t}']$ *and* $\Lambda_t = [\mathbf{\Omega}_{1,t}, \ldots, \mathbf{\Omega}_{M,t}]$.

Proposition 2 shows that the conditional variance of returns can be expressed in closed form as a function of investors' beliefs about the state variable and the current values of VAR variables. Furthermore, Proposition 2 shows that the conditional variances of discount rate and cash flows news and their conditional covariance can be expressed as functions of conditional variances of investors' beliefs and VAR variables as well as their conditional covariances. Proposition 2 also shows that these conditional variances can be calculated analytically when the state variable is assumed observable. This is due to two facts: (1) investors' beliefs and VAR variables in period $t+1$ are independent conditional on the state variable in period $t+1$ and (2) the distribution of investors' beliefs in period $t+1$ conditional on the information set in period t is a multinomial distribution with associated probabilities given by $\mathbf{Q}'\mathbf{\Pi}_t$. On the other hand, this no longer holds when the state variable is unobservable and these terms depend on the underlying law of motion of investors' beliefs. Thus, one needs to evaluate the quantities numerically based on simulations as described in the appendix.

4.4 Empirical Implementation

Similar to the standard approach, to operationalize the time-varying return decomposition approach, we first need to choose a value for ρ and a set of predictor variables. We also need to obtain a proxy for investors' beliefs about the underlying state variable of interest and estimate the VAR parameters and residual variance matrix in different states based on investors' beliefs. We can then simply plug the estimates and investors' beliefs in Equations 7 and 8 to obtain cash flow and discount rate news and in Equations 11, 12 and 13 to obtain their conditional variances and covariance.

For our main empirical results, we set ρ to 0.997 and use the same set of variables as in Section 3. Given that our main focus in this paper is on the decomposition of returns over the business cycle, we assume that there are two states, expansions and recessions as defined by the NBER. Furthermore, we assume that the state variable is observable so that investors would assign a probability of one to the observed state and zero to the other one. Hence, $\mathbf{\Pi}_t$ would be a unit vector with one in its element that corresponds to the observed state and zero in the other element. The transition probability matrix can also be directly estimated from the observed states. We then estimate the VAR parameters and the variance matrix of the VAR residuals via WLS as in Section 3 and obtain the same estimates presented in Tables 2 and 3.

Several remarks are in order concerning our empirical choices. First of all, we also considered modeling ρ so that it also changes with the underlying state variable like other model parameters. This does not change our results significantly. Second, as mentioned above, results based on the standard approach tend to be sensitive to the set of predictor variables used. We discuss the robustness of our results to using alternative alternative sets of predictive variables in Section 4.6. Finally, we can also consider alternative definitions of the business cycle. For example, we can assume that the business cycle corresponds to the state of the growth rate of industrial production. In this case, investors never observe the true state of the economy but form their beliefs based on the evolution of the growth rate of industrial production. One can then use filtered or smoothed state probabilities obtained from the estimation of, say, a two-state Markov regime switching model for the growth of industrial production to proxy for investors' beliefs about the state of the economy. We discuss the robustness of our results to using alternative definitions of the business cycle in Section 4.6.

4.5 Empirical Results

Table 5 presents the unconditional variance decomposition of stock returns. Compared to the decomposition of unconditional variance based on the standard approach presented in Table 1, the unconditional variance of unexpected returns is somewhat smaller based on our time-varying approach. Furthermore, the unconditional variance of cash flow news is relatively higher, the unconditional variance of discount rate news decreases slightly and the unconditional covariance is somewhat smaller in magnitude. More importantly, compared to the standard approach, the relative importance of cash flow news increases and that of discount rate news decreases slightly. Specifically, the cash flow news explains 46% (compared to 29% in the standard approach), the discount rate news explains 40% (compared to 43% in the standard approach) and the covariance between them explains 14% (compared to 28% in the standard approach) of the unconditional variance of returns. These results suggest that the cash flow news becomes more important in determining the unconditional variance of returns when one takes into account the time-varying nature of predictive relations and variances over the business cycle.

We now turn our attention to the decomposition of the conditional variance. Panel (a) of Figure 1 presents the conditional variance of unexpected returns as well as the conditional variances of its components and the conditional covariance between them. Not surprisingly, the conditional variance of unexpected returns are higher in recessions than expansions. Conditional variances of both cash flow and discount rate news are also significantly higher in recessions than expansions. Furthermore, the conditional covariance of cash flow and discount rate news also generally increase in magnitude in recessions. These conditional variances and covariances are relatively stable within each regime, maybe with the exception of the recent financial crisis. Panel (b) of Figure 1 presents relative importance of each component in determining the conditional variance of unexpected returns. The conditional variance of cash flow news explains, on average, between 40% and 60% of conditional variance of returns in expansions. This ratio decreases in recessions (with the exception of the 2001 recession) to between 20% and 40%. The conditional variance of discount rate news explains, on average, between 30% and 40% of conditional variance of returns in expansions. This ratio increases in recessions to between 50% and 90%. The contribution of the conditional covariance between cash flow and discount rate news to the conditional variance of returns is between 30% and -30% in expansions and this contribution generally decreases and becomes more negative in recessions. These results suggest that (1) in expansions, the conditional variance of cash flow news is higher than that of discount rate news, and thus, contributes more to the conditional variance of returns; (2) in recessions, conditional variances of both cash flow and discount rate news (as well as their conditional covariance) increase but the conditional variance of discount rate news increases more than that of cash flow news, and thus, contributes more to the conditional variance of returns. To better understand the magnitude of the change in the relative importance of these two components over the business cycle, Figure 2 presents the ratio of conditional variance of cash flow news to that of discount rate news. A ratio above one suggests that the cash flow news is more important than discount rate news. In expansions, the cash flow news is almost 1.5 times more important than the discount rate news in determining the conditional variance of returns. In recessions, on the other hand, the opposite holds and the discount rate news is almost 1.5 times more important that the cash flow news in determining the conditional variance of returns.

4.6 Robustness Checks

As mentioned above, Chen and Zhao (2009) show that the empirical results based on the standard return decomposition approach tend to be sensitive to the time period and the set of predictor variables used. In this section, we analyze whether our results in Section 4.5 based on the time-varying approach are robust to using an alternative time period, set of predictor variables and definition of business cycle.

4.6.1 Alternative State Variables

We start with the robustness of our results to using alternative set of state variables. To this end, instead of considering different sets of state variables, we follow one of the suggestions in Chen and Zhao (2009) and use the principal components of a set of state variables that are known to have some predictive power for returns. Specifically, we obtain the first four principal components of a set of state variables that includes default premium, one year price-earnings ratio, book-to-market ratio, book-to-market spread, stock market variance and net equity issuance in addition to term spread, value spread and dividend yield in our main empirical results in Section 4.5.[3]

Panel (a) of Table 6 presents the unconditional variance decomposition based on the time-varying approach. Although there are some differences, the decomposition of the uncondition variance is similar to that in our benchmark case with term spread, value spread and dividend yield as state variables presented in Table 5. The cash flow news is relatively more important than discount rate news in determining the unconditional variance of unexpected returns. Both cash flow and discount rate are much more volatile compared to those in the benchmark case. Furthermore, the covariance term contributes negatively to unconditional variance of unexpected returns, in contrast to a positive contribution in our benchmark case. These results suggest that the decomposition of the unconditional variance, especially the contribution of the covariance term, can be somewhat sensitive to the choice of state variables even in the time varying approach where we allow both the VAR parameters and variance of the residuals to change over time. However, Figure 3 shows that our findings on the decomposition of conditional variance in our benchmark case continue to hold with some differences when we consider the first four principle components of a set of state variables. Specifically, the conditional variance of unexpected returns and its components all increase significantly in recessions, although they are higher than those in the benchmark case. The relative importance of cash flow and discount rate news as well as that of the covariance term also increase in magnitude in recessions, although the relative importances are relatively bigger in magnitude than those in the benchmark case, especially during the financial crisis. More importantly, similar to the benchmark case, the cash flow news is relatively more important than discount rate news in determining the conditional variance of unexpected returns in expansions but not in recessions. However, we should not that the ratio of conditional variances of cash flow and discount rate is relatively higher than the benchmark case. Finally, although not presented, we also find that our results on the conditional variance decomposition of unexpected returns are mostly robust to using other alternative sets of state variables.

4.6.2 Alternative Sample Period

We now consider the robustness of our results to using a longer sample period. Similar to Section 4.5, we decompose the returns on the S&P 500 index based on the time-varying approach over the phases of NBER business cycles between July 1927 and December 2010. To do this, we consider the same predictor variables as in Section 4.5.

[3]These are the same variables considered in Chen and Zhao (2009)) except the 10 year PE ratio. We find that the vector autoregressive process of returns and the principal components obtained from a set of state variables that includes the 10 year PE ratio tend to be non-stationary in one or both of the states, mainly due to its highly persistent nature. Chen and Zhao (2009) find that their results are most sensitive to the inclusion or exclusion of the 10 year PE ratio. Hence, for these reasons, we choose to exclude the 10 year PE ratio from the set of state variables in our analysis.

Panel (b) of Table 6 presents the decomposition of unconditional variance based on the time-varying approach. The unconditional variance of returns is higher in the longer sample period compared to the short sample period between January 1960 and December 2010. This is not surprising since the longer sample period includes the Great Depression. This increase in the unconditional variance of returns in the longer sample period can mostly be attributed to an increase in the unconditional variance of discount rate news. Specifically, the unconditional variance of discount rate news in the longer sample period is almost 3 times higher than that in the shorter sample period. On the other hand, the unconditional variance of cash flow news does not change dramatically when we consider the longer sample period. More importantly, these results hold independent of the approach used and imply that the discount rate news is relatively more important than the cash flow news in explaining the unconditional variance of returns over 1927:07 and 2010:12. Figure 6 presents the decomposition of conditional variance over the business cycles in the longer sample. The results are similar to those based on the shorter sample period. Specifically, the conditional variance of unexpected returns are higher in recessions than expansions. Conditional variances of both cash flow and discount rate news are also significantly higher in recessions than expansions. Furthermore, the conditional covariance of cash flow and discount rate news also increase in magnitude in recessions. These conditional variances and covariances are relatively stable within each regime, with the exception of the Great Depression. The conditional variance of cash flow news explains, on average, between 20% and 30% of conditional variance of returns. This ratio increases only slightly in recessions with the exception of the Great Depression and the recession in late 30s when it increases to above 50%. The conditional variance of discount rate news explains, on average, between 60% of conditional variance of returns in expansions. This ratio increases in recessions to between 100% and 150% and to 200% during the Great Depression. The contribution of the conditional covariance between cash flow and discount rate news to the conditional variance of returns is between 30% and -30% in expansions and this contribution decreases and becomes more negative in recessions. Finally, the ratio of conditional variance of cash flow news to that of discount rate news is always below one, suggesting that the discount rate news is always relatively more important than cash flow news in determining the conditional variance of returns. More importantly, similar to our main results, this ratio decreases in recessions, suggesting that the discount rate news becomes even more important in recessions. To sum up, these results suggests that the empirical evidence presented in Section 4.5 is robust to using a longer sample period.

4.6.3 Alternative Business Cycle Definition

Finally, we consider the robustness of our results to using an alternative definition of the business cycle. To this end, we consider the following two state Markov regime switching process for the log growth rate of monthly industrial production index:

$$\Delta \log(IP_t) = \delta_{n_t} + \omega_{n_t}\nu_t \tag{14}$$

where n_t is a two state Markov chain and ν_t is an independently and identically distributed Gaussian random variable with zero mean and unit variance. δ_i and ω_i for $i = 1, 2$ are, respectively, the mean and standard deviation of the log growth rate of monthly industrial production index (IP_t) in state i. We estimate the model in Equation 14 using monthly data on the industrial production index between January 1960 and December 2010 from the Federal Reserve

Bank of St. Louis. The log growth rate of monthly industrial production index has a mean of 0.38% and a standard deviation of 0.55% in the first state while it has a mean of -0.52% and a standard deviation of 1.16% in the second state. These results suggest that the first state with a higher and less volatile growth rate of industrial production can be considered as corresponding to expansion periods while the second state with a lower and more volatile growth rate of industrial production can be considered as corresponding to recession periods. This can also be seen from Figure 5 which presents the smoothed probabilities of the second state against NBER recession periods. The probability of the second state increases to almost one in all NBER recession periods and decreases to zero in most NBER expansion periods with some exceptions.[4] Hence, instead of using weights based on NBER business cycles, we use the smoothed probabilities of these two states as weights and estimate the VAR in Equation 4 via WLS with the same variables in Section 4.5 between January 1960 and December 2010. We then decompose the monthly returns on the S&P 500 index based on the time-varying approach over the business cycle as determined by the smoothed state probabilities.

Panel (c) of Table 6 presents the decomposition of the unconditional variance of returns. The results are very similar to our main results presented in Table 5. Specifically, the cash flow news is relatively more volatile and, thus, relatively more important in determining the unconditional variance of returns than discount rate news, which, in turn, is slightly more important than the covariance term.

Figure 4 presents the decomposition of conditional variance of returns. As mentioned in Section 4.3, we cannot calculate the conditional variances of cash flow and discount rate news and their conditional covariance in closed form when the state variable is unobservable. Instead, we calculate these quantities based on a simulation approach described in detail in the appendix. Our results are very similar to those presented in Section 4.5. Specifically, the conditional variance of returns, cash flow and discount rate news as well as the conditional covariance between the two components increase in magnitude in recessions. The relative importance of cash flow news decreases in recessions (with the exception of the 2001 recession) while that of discount rate news increases in recessions. The contribution of the conditional covariance term generally decreases and becomes negative in some recession periods. Finally, the ratio of conditional variance of cash flow news to that of discount rate news decreases in recessions and goes below one in some recessions. Overall, these results suggest that our main empirical results are robust to using alternative definitions of business cycle.

5 Return Decomposition in a Structural Framework

So far, we have provided empirical evidence that the conditional variances of both cash flow and discount rate news increase in recessions while their ratio generally decreases in recessions. However, we have not answered whether these results are in line with what asset pricing theory implies. In this section, we analyze whether our findings are in line with the empirical implications for the relative importance of cash flow and discount rate news from a stylized asset pricing model.

[4] The probability of the second state increases during several episodes which are not identified as recession periods by the NBER. This is not surprising since it is based solely on the industrial production index while the NBER defines a recession as "a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales".

5.1 The Model

We consider a pure exchange economy (Lucas (1978)) in discrete time where the preferences of a representative investor are modeled by a constant relative risk aversion utility over consumption,

$$U(C_t) = \begin{cases} \frac{C_t^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(C_t) & \text{if } \gamma = 1 \end{cases} \tag{15}$$

where C_t denotes investors' consumption in period t and γ is his coefficient of relative risk aversion. Investors' opportunity set consists of a risky asset whose supply is fixed and normalized to one and a risk-free asset. We assume that investors have access to implicit labor income. We model the dynamics of dividends and consumption as a Markov regime switching vector autoregression of the following form:

$$\mathbf{y}_t = \begin{bmatrix} \Delta c_t \\ \Delta d_t \end{bmatrix} = \boldsymbol{\mu}_{S_t} + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{S_t})$$

where $\Delta d_t = \log(D_t/D_{t-1})$ and $\Delta c_t = \log(C_t/C_{t-1})$ are, respectively, the (log) growth rates of dividend and consumption in period t. $\boldsymbol{\mu}_{S_t} = \begin{bmatrix} \mu_{c,S_t} \\ \mu_{d,S_t} \end{bmatrix}$ and $\boldsymbol{\Sigma}_{S_t} = \begin{bmatrix} \sigma^2_{c,S_t} & \sigma_{cd,S_t} \\ \sigma_{cd,S_t} & \sigma^2_{d,S_t} \end{bmatrix}$ are, respectively, the mean and variance of the growth rate process as functions of the latent state variable S_t. We assume that the state variable S_t follows a first-order N-state Markov chain with transition probability matrix $\mathbf{Q}$, i.e.

$$\{\Pr(S_t = j | S_{t-1} = i)\} = \{q_{ij}\} = \mathbf{Q} \quad \text{for } i, j = 1, \ldots, N. \tag{16}$$

5.2 Investor's Beliefs

In models like ours with learning, investors' beliefs about the underlying state of the economy play a central role. In this section, we characterize how investor's beliefs evolve over time as they learn about the underlying state of the economy.

Let $\tilde{\pi}_{j,t}$ denote the probability that investors assign to state j before observing the realizations for dividend and consumption in period t. We refer to $\tilde{\pi}_{j,t}$ as investors' prior beliefs about state j in period t. Similarly, let $\pi_{j,t}$ denote the probability that they assign to state j after observing the information revealed in period t. Investors' information set in period t, $\mathcal{F}_t$, includes past and current dividend and consumption realizations. Assuming that investors have a given set of beliefs about the initial state before observing any information, i.e. $\tilde{\pi}_{j,0}$ for $j = 1, 2, \ldots, N$, the following lemma characterizes investors' beliefs about the state variable:

Lemma 2.

$$\pi_{j,t} = \frac{\phi(\mathbf{y}_t, \boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)\tilde{\pi}_{j,t}}{\sum_{i=1}^{N} \phi(\mathbf{y}_t, \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)\tilde{\pi}_{i,t}} \qquad \textit{for } j = 1, 2, \ldots, N. \tag{17}$$

where $\tilde{\pi}_{j,t} = \sum_{i=1}^{N} \pi_{i,t-1} q_{ij}$ *and* $\phi(x, \boldsymbol{\mu}, \boldsymbol{\Sigma})$ *is the multivariate normal density function with mean* $\boldsymbol{\mu}$ *and variance matrix* $\boldsymbol{\Sigma}$*.*

Prior to observing the information revealed in a given period t, investors know that the growth process might have switched to a new state according to the transition probability matrix. Hence, their prior beliefs about the new state variable, $\tilde{\pi}_{j,t}$, are weighted averages of their beliefs about the previous state variable, $\pi_{i,t-1}$, where the weights are the transition probabilities, q_{ij}. Given their prior beliefs for the state variable S_t, investors then update their beliefs according to Bayes' rule based on the additional information revealed in period t.

5.3 Equilibrium Asset Prices and Returns

We characterize the price and the unexpected return of the risky asset in the following proposition:

Proposition 3. *The price of the risky asset in period* t *is given by:*

$$\frac{P_t}{D_t} = \boldsymbol{\lambda}' \boldsymbol{\Pi}_t \tag{18}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_N]'$*,* λ_j *is the price-dividend ratio in state* j *and is given by*

$$\lambda_j = [(\mathbf{I} - \mathbf{QH})^{-1} \mathbf{QG}]_j > 0 \quad \textit{for } j = 1, 2, \ldots, N \tag{19}$$

where the operator $[\cdot]_j$ *refers to the* j^{th} *element of a vector.* $\mathbf{I}$ *is the* $N \times N$ *identity matrix and* $\mathbf{Q}$ *is the transition probability matrix defined in Equation 16.* $\mathbf{G} = (g_1, g_2, \ldots, g_N)'$ *is a* $N \times 1$ *vector and* $\mathbf{H}$ *is a* $N \times N$ *diagonal matrix whose* i^{th} *diagonal element is* g_i *where* $g_i = \beta \exp(\mu_{d,i} - \gamma \mu_{c,i} + \frac{1}{2}(\gamma^2 \sigma_{c,i}^2 - 2\gamma \sigma_{cd,i} + \sigma_{d,i}^2))$*.*
Let r_t *denote the log return on the risky asset in period* t*, i.e.* $r_t = \log(\frac{P_t + D_t}{P_{t-1}})$*, then the unexpected log return on the risky asset in period* t *can be approximated by:*

$$r_t^* \equiv r_t - E_{t-1}[r_t] \approx \rho \boldsymbol{\lambda}' (\boldsymbol{\Pi}_t - \mathbf{Q}' \boldsymbol{\Pi}_{t-1}) + \Delta d_t - \bar{\mu}_{d,t-1} \tag{20}$$

where $E_{t-1}[\cdot]$ *denotes conditional expectation based on information set in period* $t-1$*,* $\mathcal{F}_{t-1}$*.* $\bar{\mu}_{d,t-1} = \boldsymbol{\mu}_d' \mathbf{Q}' \boldsymbol{\Pi}_{t-1}$ *is the expected dividend growth rate for period* t *based on information set* $\mathcal{F}_{t-1}$*.* $\rho = 1/(1 + \bar{\lambda})$ *and* $\bar{\lambda} = \boldsymbol{\lambda}' \bar{\boldsymbol{\Pi}}$ *is the long term average price-dividend ratio where* $\bar{\boldsymbol{\Pi}} = [\bar{\pi}_1, \bar{\pi}_2, \ldots, \bar{\pi}_N]'$ *is the stationary distribution vector of the transition probability matrix* $\mathbf{Q}$*.*

Given that investors never observe the true state variable, the price-dividend ratio is a weighted average of λ_j's where the weights are investors' beliefs about the state variable. Hence, the price-dividend ratio fluctuates as investors receive additional information and update their beliefs about the state variable. Similarly, the unexpected return on the risky asset is also determined by the unexpected dividend growth rate as well as the time variation in investors' beliefs which, in turn, depend on the additional information revealed by dividend and consumption realization in each period.

The following corollary characterizes the conditional variance of returns:

Corollary 1. *The conditional variance of unexpected returns based on the information set at time t can be expressed as follows:*

$$var_t(r^*_{t+1}) \quad = \quad \rho^2 \boldsymbol{\lambda}' var_t(\boldsymbol{\Pi}_{t+1})\boldsymbol{\lambda} + 2\rho\boldsymbol{\lambda}' cov_t(\Delta d_{t+1}, \boldsymbol{\Pi}_{t+1}) + var_t(\Delta d_{t+1}) \tag{21}$$

where $var_t(\Delta d_{t+1}) = \sum_{i=1}^{N}(\mu^2_{d,i} + \sigma^2_{d,i})(\mathbf{e}'_i\mathbf{Q}'\boldsymbol{\Pi}_t) - \sum_{i=1}^{N}(\mu_{d,i}\mathbf{e}'_i\mathbf{Q}'\boldsymbol{\Pi}_t)^2$.

Furthermore, if the state variable is observable, then $var_t(\boldsymbol{\Pi}_{t+1})$ *and* $cov_t(\Delta d_{t+1}, \boldsymbol{\Pi}_{t+1})$ *can be expressed in closed form as follows:*

$$\begin{aligned} var_t(\boldsymbol{\Pi}_{t+1}) &= (\boldsymbol{\Pi}'_t\mathbf{Q} \otimes \mathbf{1}_N) \odot (\mathbf{I}_N - (\mathbf{Q}'\boldsymbol{\Pi}_t \otimes \mathbf{1}'_N)) \\ cov_t(\Delta d_{t+1}, \boldsymbol{\Pi}_{t+1}) &= (\boldsymbol{\mu}_d - \boldsymbol{\mu}'_d\mathbf{Q}'\boldsymbol{\Pi}_t) \odot (\mathbf{Q}'\boldsymbol{\Pi}_t) \end{aligned}$$

where $\odot$ *is the element-by-element multiplication.*

Corollary 1 shows that the conditional variance of returns depends on the conditional variances of investors' beliefs and the dividend growth rate as well as their conditional covariance. Corollary 1 also shows that all these terms can be calculated analytically when the state variable is assumed observable. Similar to Section 4.3, this is due to two facts: (1) investors' beliefs and the dividend growth rate in period $t+1$ are independent conditional on the state variable in period $t+1$ and (2) the distribution of investors' beliefs in period $t+1$ conditional on the information set in period t is a multinomial distribution with associated probabilities given by $\mathbf{Q}'\boldsymbol{\Pi}_t$. On the other hand, this no longer holds when the state variable is unobservable and these terms depend on the underlying law of motion of investors' beliefs. Thus, one needs to evaluate the quantities numerically based on simulations as described in the appendix.

5.4 Return Decomposition

In this framework, the return is determined by the unexpected dividend growth rate and the change investors' beliefs which in turn depend on unexpected dividend and consumption growth rates. Hence, investors can obtain forecasts of future returns based on the dividend and consumption growth processes. In other words, one can directly apply the return decomposition approach of Campbell and Shiller in our framework without the need for a forecasting model such as a VAR. The following proposition decomposes the unexpected log return of the risky asset into its cash flow and discount rate components.

Proposition 4. *The cash flow component (CF_t) and the discount rate (DR_t) of the unexpected log return on the risky asset in our model are given as follows:*

$$CF_t \quad = \quad \boldsymbol{\mu}'_d(\mathbf{I} - (1-\rho)\mathbf{Q}')^{-1}(1-\rho)\mathbf{Q}'(\boldsymbol{\Pi}_t - \mathbf{Q}'\boldsymbol{\Pi}_{t-1}) + \Delta d_t - \bar{\mu}_{d,t-1} \tag{22}$$

$$DR_t \quad = \quad (\boldsymbol{\mu}'_d(\mathbf{I} - (1-\rho)\mathbf{Q}')^{-1}(1-\rho)\mathbf{Q}' - \rho\boldsymbol{\lambda}')(\boldsymbol{\Pi}_t - \mathbf{Q}'\boldsymbol{\Pi}_{t-1}) \tag{23}$$

As it can be easily seen, the cash flow news in our model does not only depend on the unexpected dividend growth rate but also on the change in investors' beliefs about the underlying state of the economy. This is not surprising since any change in investors' beliefs would result in a change in investors' expectations about the discounted sum of future dividends, the definition of cash flow news in the Campbell and Shiller decomposition approach. However, we should note that investors' beliefs about the underlying state of the economy depends on both unexpected dividend and consumption growth rates. Hence, the cash flow news based on the Campbell and Shiller approach in our framework depends on the unexpected dividend growth rate, through its direct linear effect and its indirect nonlinear effect through investors' beliefs, as well as the unexpected consumption growth rate through its effect on investors' beliefs. On the other hand, the discount rate news depends on the unexpected consumption growth rate, which determines the stochastic discount factor in our framework, and the unexpected dividend growth rate only through their effects on investors' beliefs.

Proposition 4 suggests that the regime switching dynamics of the fundamentals in our framework plays an important role in determining the cash flow and discount rate news based on the Campbell and Shiller approach in our framework. To see this, consider an asset pricing model similar to ours in Section 5.1 where the growth rates of fundamentals have constant, instead of time-varying, means and variances. In this special case of our model, investors know the true state of the economy and the fact that it will not switch to another state. This in turn implies that the cash flow news based on the Campbell and Shiller approach will be equal to the unexpected dividend growth rate while the discount rate news will be always equal to zero. In other words, in this special case of our framework, the variance of unexpected returns would be completely explained by the variance of cash flow news. This is not surprising since expected returns in this special case are constant and the Campbell and Shiller decomposition approach is based on the assumption of time-varying expected returns. One way to generate time-varying expected returns in our framework is to assume that the growth rates of fundamentals depend on an underlying state variable. Hence, the regime switching dynamics of the fundamentals is crucial in having a nontrivial decomposition in our framework.

5.5 Unconditional and Conditional Variance Decomposition

The unconditional and conditional variances of unexpected returns can be decomposed into their components as in Equations 2 and 9, respectively. The following proposition characterizes the conditional variance of cash flow and discount rate news as well as their conditional in this framework:

Proposition 5. *The conditional variances of cash flow and discount rate news and their conditional covariance are given by*

$$\begin{aligned} var_t(CF_{t+1}) &= \mathbf{m}_d var_t(\mathbf{\Pi}_{t+1})\mathbf{m}_d' + 2\mathbf{m}_d cov_t(\Delta d_{t+1}, \mathbf{\Pi}_{t+1}) + var_t(\Delta d_{t+1}) \\ var_t(DR_{t+1}) &= (\mathbf{m}_d - \rho\boldsymbol{\lambda}')var_t(\mathbf{\Pi}_{t+1})(\mathbf{m}_d - \rho\boldsymbol{\lambda}')' \\ cov_t(CF_{t+1}, DR_{t+1}) &= \mathbf{m}_d var_t(\mathbf{\Pi}_{t+1})(\mathbf{m}_d - \rho\boldsymbol{\lambda}')' + (\mathbf{m}_d - \rho\boldsymbol{\lambda}')cov_t(\Delta d_{t+1}, \mathbf{\Pi}_{t+1}) \end{aligned}$$

where $var_t(\Delta d_{t+1}) = \sum_{i=1}^{N}(\mu_{d,i}^2 + \sigma_{d,i}^2)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t) - \sum_{i=1}^{N}(\mu_{d,i}\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t)^2$ *and* $\mathbf{m}_d = \boldsymbol{\mu}_d'(\mathbf{I} - (1-\rho)\mathbf{Q}')^{-1}(1-\rho)\mathbf{Q}'$*. If the state variable is observable, then* $var_t(\mathbf{\Pi}_{t+1})$ *and* $cov_t(\Delta d_{t+1}, \mathbf{\Pi}_{t+1})$ *can be expressed in closed form as in*

Proposition 2.

Not surprisingly, the three driving factors behind the conditional variance of returns are also the driving factors behind the conditional variance of its components and the conditional covariance between them. Specifically, the conditional variance of cash flows news is determined by all three factors while the conditional variance discount rate news depends only by the conditional variance of investors' beliefs. The conditional covariance between cash flow and discount rate news is determined by the conditional variance of investors' beliefs and its conditional covariance with the dividend growth rate. As we will discuss below, these observations play an important role about the implications of this stylized asset pricing model for the decomposition of the conditional variance of returns. Once again, $var_t(\mathbf{\Pi}_{t+1})$ and $cov_t(\Delta d_{t+1}, \mathbf{\Pi}_{t+1})$ can be expressed in closed form only when the state variable is observable and need to be evaluated numerically based on simulations as described in the appendix when the state variable is not observable.

5.6 Calibration and Simulation

We analyze the implications of our model for the decomposition of returns based on simulations. To this end, we first calibrate the parameters of our model. We then simulate data from the calibrated model and analyze the decomposition of unconditional and conditional variances of unexpected returns. In this section, we discuss our approach to calibrate and simulate our model.

We are interested in matching the empirical observations presented in Section 4.5. Hence, we simulate our model at monthly frequency for a total of 612 observations which corresponds to the number of monthly observations for the period considered in Section 4.5, i.e. between 1960 and 2010. Rather than simulating the state variable, we assume that it is observable and corresponds to expansions ($S_t = 1$) and recessions ($S_t = 2$) as defined by the NBER between 1960 and 2010. This allows us to directly match the results based on simulated data from our model to the empirical observations presented in Section 4.5. We also calibrate $\mathbf{Q}$ to match the monthly transition probabilities the NBER business cycles between 1960 and 2010.

To calibrate the parameters of the consumption and dividend processes, similar to Bansal and Yaron (2004), we use annual data on real per-capita personal consumption expenditures on nondurables and services and real dividends paid on the S&P 500 index to proxy for dividends between 1929 and 2010. We use annual, rather than monthly, data to avoid any problems associated with the seasonality of dividends. We use the longer sample period between between 1929 and 2010, rather than the sample period of interest between 1960 and 2010, to have the maximum number of observation in both expansion and recession periods. Data on nominal per-capita personal consumption expenditures on nondurables and nominal S&P 500 dividends are from Bureau of Economic Analysis (BEA) and Amit Goyal's website, respectively. These nominal annual quantities are deflated using the average annual Consumer Price Index for All Urban Consumers (All Items) from the Bureau of Labor Statistics (BLS). We assume that the economy was in recession in a given year if it has been so for more than 6 months of that year as defined by the NBER. We estimate the average (log) growth rates of consumption and dividends in expansion and recession periods. We then convert these growth rates from annual to monthly frequency by dividing them by 12 and use these monthly growth rates as our calibration for $\boldsymbol{\mu}_{S_t}$ for $S_t = 1, 2$. We assume that $\mathbf{\Sigma}_1 = \mathbf{\Sigma}_2$ and estimate it over the whole sample between 1929 and

2010. We then convert it from annual to monthly frequency by dividing it by 12.

Finally, we assume a monthly time impatience parameter of 0.9957, corresponding to an annual value of 0.95, and a coefficient of relative risk aversion of 7.5. Table 7 presents the calibrated model parameters.

5.7 Empirical Predictions

In this section, we first present the implications of our model for the decomposition of conditional variance of returns based on simulated data. We then compare these implications to what is observed in data based on the time-varying approach.

Figure 7 presents the conditional variance of simulated unexpected returns as well as the conditional variances of its components and the conditional covariance between them in Panel (a) and the relative importance of each component in determining the conditional variance of simulated unexpected returns in Panel (b). Figure 8 presents the ratio of conditional variance of cash flow news to that of discount rate news, i.e. the importance of cash flow news relative to that of discount rate news. The implications of our model based on Figures 7 and 8 can be summarized as follows:

1. The conditional variance of unexpected returns from our model are higher in recessions than expansions;
2. The conditional variances of both cash flow and discount rate news are also significantly higher in recessions than expansions;
3. The conditional covariance of cash flow and discount rate news is positive and higher in recessions than expansions;
4. The conditional variances and covariances are constant within each regime;
5. The relative importance of cash flow news is lower in recessions than expansions;
6. The relative importance of discount rate news is higher in recessions than expansions;
7. The contribution of the conditional covariance between cash flow and discount rate news is negative in expansions and recessions and increases in magnitude in recessions.
8. Cash flow news is relatively more important than discount rate news in expansions while the opposite holds in recessions.

Before comparing these implications of our model to what is observed in the data, we first discuss whether our calibrated model can match the decomposition of unconditional variance observed in the data based on the time-varying approach. First of all, the unconditional variance of cash flow and discount rate news from simulations have similar magnitudes to those observed in data presented in Table 8. Specifically, the unconditional variance (based percentage returns) of cash flow and discount rate news from simulations are 10.00 and 8.33 compared to 8.49 and 7.25, respectively. However, our model fails to match the magnitude and sign of the unconditional variance between cash flow and discount rate news observed in data. Specifically, $-2cov(CF, DR)$ is -6.36 in simulated data while it is 2.55 in the data based on our time-varying approach. In other words, it contributes negatively to the unconditional variance of returns in our model rather than positively as observed in the data. As a result, (1) the unconditional variance of simulated returns is somewhat lower than that observed in data presented in Table 5; (2) the relative importances of both cash flow and discount rate news in determining the unconditional variance of returns from simulations are slightly higher than those observed in Table 5. Nevertheless, the ratio of the relative importance of cash flow news to that of discount rate news is 1.19 in the simulated data which is quite similar to 1.17 observed in the data. These results suggest that our model is able to match the decomposition of the unconditional covariance of returns with some minor discrepancies.

We now turn our attention to whether the implications of our model for the decomposition of conditional variance of returns can match what is observed in data based on the time-varying approach. The first, second and fourth implications match closely what is observed in the data while the third implications is somewhat different. Specifically, Figure 1 shows that the conditional covariance is negative on average before the 2001 recession where it becomes and stays mostly positive. Concerning the fifth to seventh implications of our model, our results can be summarized as follows: The conditional variance of cash flow news explains 67% and 87% of conditional variance of returns in recessions and expansions, respectively. These percentages are somewhat higher than what is observed in the data. Once again, this is mainly due to the lower conditional variances of simulated returns compared to that observed in the data. More importantly, matching what is observed in the data, the relative importance of cash flow news decreases by 20% in recessions. The conditional variance of discount rate news explains 130% and 50% in recessions and expansions, respectively. These numbers are relatively higher than what is observed in the data due to the same reason mentioned above. However, the relative importance of discount rate news increases by 80% in recessions similar to what is observed in the data. The conditional covariance between cash flow and discount rate news contributes -98% and 38% to the conditional variance of simulated unexpected returns, in recessions and expansions, respectively. This is opposite of what is observed in data before the 2001 recession. However, conditional covariance between cash flow and discount rate news contributes almost -100% during the 2001 recession and almost -50% during the recession caused by the financial crisis. Finally, the eighth implication of our model replicates almost perfectly what is observed in the data. Specifically, the cash flow news is almost 1.5 times more important than the discount rate news in determining the conditional variance of returns In expansions. On the other hand, the opposite holds and the discount rate news is almost 1.5 times more important that the cash flow news in determining the conditional variance of returns in recessions.

5.8 Discussion

In this section, we discuss the intuition and the driving factors behind the implications of our model. We start with the decomposition of unconditional variance before turning our attention to that of conditional variance.

Given a calibration of the dividend and consumption processes, investors' risk aversion is the key variable driving our results on the unconditional variance of returns and its decomposition. Specifically, as investors become more risk averse, the discount rate and cash flow news become more and less volatile, respectively, while their covariance increases. Recall that an increase in the variance of either discount rate or cash flow news increases the variance of returns while an increase in their covariance decreases it. For low levels of risk aversion, the variance of returns decreases as investors become more risk averse. This is due to the fact that the increase in the variance of discount rate news is dominated by the decreases in the variance of cash flow news and (-2 times) the covariance between the two components. For high levels of risk aversion, the opposite holds and the variance of returns increases as investors become more risk averse.[5] This also implies that, as investors become more risk averse, the relative importance of discount rate news increases while those of cash flow news and the covariance term first increases then decreases (in magnitude). These results are quite intuitive. More precisely, in frameworks like ours with power utility, the risk aversion coefficient affects the marginal rate of substitution and thus the stochastic discount factor. As investors become more risk averse, the stochastic discount factor and thus, discount rate news become more volatile. To see this, note that the price dividend ratio decreases as investors become more risk averse in framework like ours. This in turn implies that the impact of the coefficients multiplying investors' beliefs in the discount rate news increases. Thus, for a given investors' beliefs, the discount rate news becomes more volatile as investors become more risk averse. On the other hand, the opposite holds for the cash flow news and the impact of the coefficients multiplying investors' beliefs in the cash flow news decreases. Thus, for a given investors' beliefs, the cash flow news becomes less volatile as investors become more risk averse. Given the calibration of the dividend and consumption processes discussed in Section 5.6, we choose investors' risk aversion coefficient to successfully match the decomposition of the unconditional variance of returns.

We now turn our attention to the decomposition of conditional variance of returns. The transition probability matrix is the driving mechanism behind all the empirical implications of our model for the decomposition of conditional variance of returns listed in Section 5.7. We calibrate the transition probability matrix to match the monthly transition probabilities of the NBER business cycles between 1960 and 2010. Expansion periods as defined by the NBER tend to be longer than recession periods and thus also more persistent. Hence, the probability that the economy switches from a recession to an expansion is higher than the probability that the economy switches from an expansion to a recession. This fact makes investors' beliefs more volatile in recessions than expansions, which in turn implies the conditional variance of returns, the conditional variance of its components and the conditional covariance between its components are higher in recessions than expansions. In this stylized model, cash flow news depend on investors' beliefs as well as the unexpected dividend growth rate while discount rate news depends only on investors' beliefs. This in turn implies that the conditional variance of discount rate news is much more sensitive to any changes in the volatility of investors' beliefs than that of cash flow news. Thus, the increase in the volatility of investors' beliefs in recessions results in a bigger increase in the conditional variance of discount rate news relative to that of cash flow news, making discount rate news relatively more important in recessions.

[5] Under the calibration of our model in Section 5.6, the variance of returns is an increasing function of any coefficient of relative risk aversion greater than 2.7.

6 Conclusion

In this paper, we analyze the decomposition of unconditional and conditional variances of returns on the S&P 500 index over the business cycle. To do this, we first generalize the standard return decomposition approach based on Campbell and Shiller (1988) to a framework where we model the short-run dynamics of returns and predictive variables in a Markov regime switching vector autoregressive model (MSVAR) where both the VAR parameters and residual variance matrix are assumed to switch between different values based on the underlying state of the economy. We then show that the conditional variances of cash flow and discount rate news as well as their conditional covariance can be expressed in closed-from when the state variable is observable and can be calculated numerically based on simulations otherwise. In contrast to the standard approach, we find that the cash flow news is more important than discount rate news in determining the unconditional variance of returns. More importantly, we find that the decomposition of the conditional variance of returns depends on the underlying state of the economy. Specifically, the cash flow news is relatively more important than discount rate news in determining the conditional variance of returns in expansions. The conditional variances of returns and its components increase in recessions. However, the conditional variance of discount rate news increases more than that of cash flow news and, thus, the discount rate news becomes relatively more important than cash flow news in determining the conditional variance of returns in recessions. Finally, we show that these results are broadly consistent with the implications of a stylized asset pricing model in which the growth rates of dividends and consumption take on different values depending on the underlying state of the economy.

References

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Bernanke, Ben S., and Kenneth N. Kuttner, 2005, What explains the stock market's reaction to Federal Reserve policy?, *Journal of Finance* 60, 1221–1257.

Bianchi, Francesco, 2010, Rare events, financial crises, and the cross-section of asset returns, working paper, Duke University.

Callen, J., O. Hope, and D. Segal, 2005, Domestic and foreign earnings, stock return variability, and the impact of investor sophistication, *Journal of Accounting Research* 43, 377–412.

Callen, J., J. Livnat, and D. Segal, 2006, The information content of sec filings and information environment: A variance decomposition analysis, *Accounting Review* 81, 10171043.

Callen, J., and D. Segal, 2004, Do accruals drive firm-level stock returns? a variance decomposition analysis, *Journal of Accounting Research* 42, 527560.

Campbell, John Y., 1991, A variance decomposition for stock returns, *Economic Journal* 101, 157–179.

———, 1993, Intertemporal asset pricing without consumption data, *American Economic Review* 83, 487–512.

———, 1996, Understanding risk and return, *Journal of Political Economy* 104, 298345.

———, and John Ammer, 1993, What moves the stock and bond markets? A variance decomposition for long-term asset returns, *Journal of Finance* 48, 3–37.

Campbell, John Y., and Jianping Mei, 1993, Where do betas come from? Asset price dynamics and the sources of systematic risk, *Review of Financial Studies* 6, 567–592.

Campbell, John Y., Christopher Polk, and Tuomo Vuolteenaho, 2010, Growth or glamour? fundamentals and systematic risk in stock returns, *Review of Financial Studies* 23, 305–344.

Campbell, John Y., and Robert J. Shiller, 1988, Stock prices, earnings, and expected dividends, *Journal of Finance* 43, 661–676.

Campbell, John Y., and Tuomo Vuolteenaho, 2004a, Bad beta, good beta, *American Economic Review* 94, 1249–1275.

———, 2004b, Inflation illusion and stock prices, *American Economic Review* 94, 19–23.

Chen, Long, Zhi Da, and Xinlei Zhao, 2013, What drives stock price movements?, forthcoming Review of Financial Studies.

Chen, Long, and Xinlei Zhao, 2009, Return decomposition, *Review of Financial Studies* 22, 5213–5249.

Dangl, Thomas, and Michael Halling, 2011, Predictive regressions with time-varying coefficients, forthcoming Journal of Financial Economics.

Henkel, S. J., J. S. Spencer, and F. Nardari, 2011, Time-varying short-horizon return predictability, *Journal of Financial Economics* 99, 560–580.

Larrain, B., and M. Yogo, 2008, Does firm value move too much to be justified by subsequent changes in cash flow?, *Journal of Financial Economics* 87, 200–226.

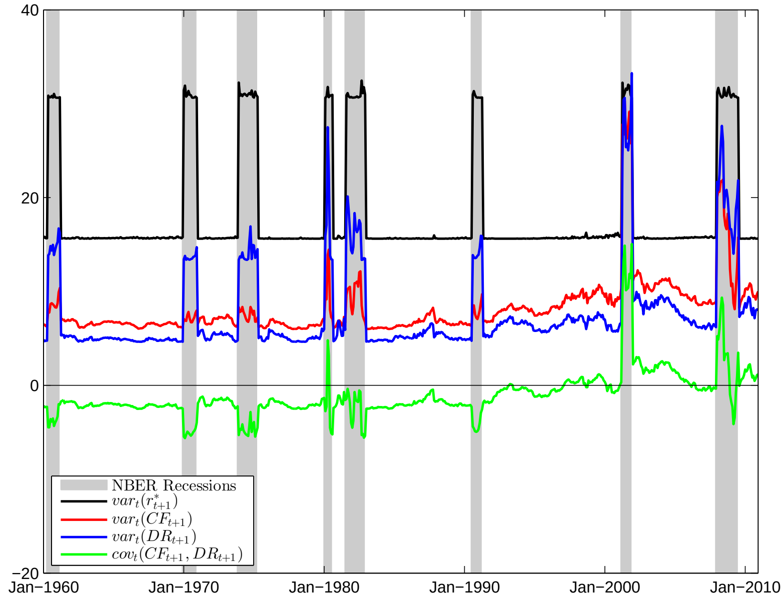
Lucas, Robert E., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.

Vuolteenaho, Tuomo, 2002, What drives firm-level stock returns?, *Journal of Finance* 57, 233–264.

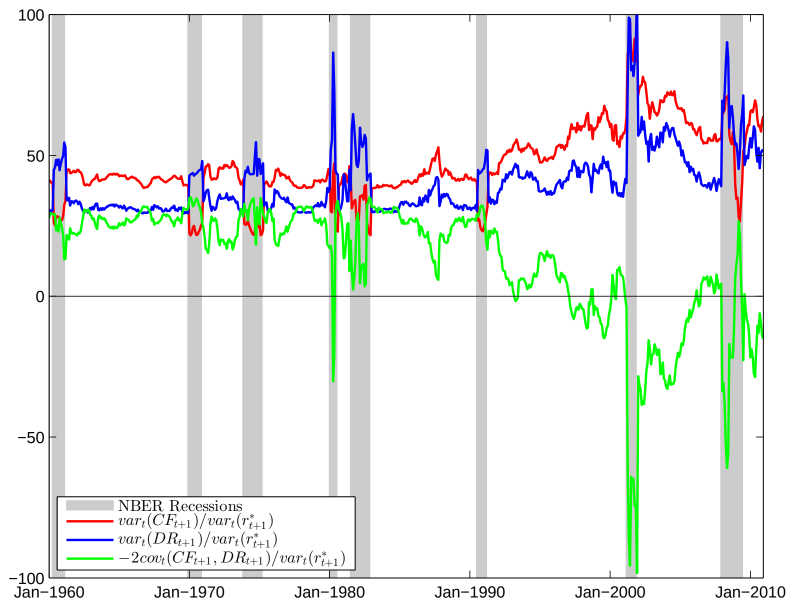
Welch, Ivo, and Amit Goyal, 2008, A comprehensive look at the empirical performance of equity premium prediction, *Review of Financial Studies* 21, 1455–1508.

Figure 1: Decomposition of the Conditional Variance of Returns over the Business Cycle

(a) Conditional Variances

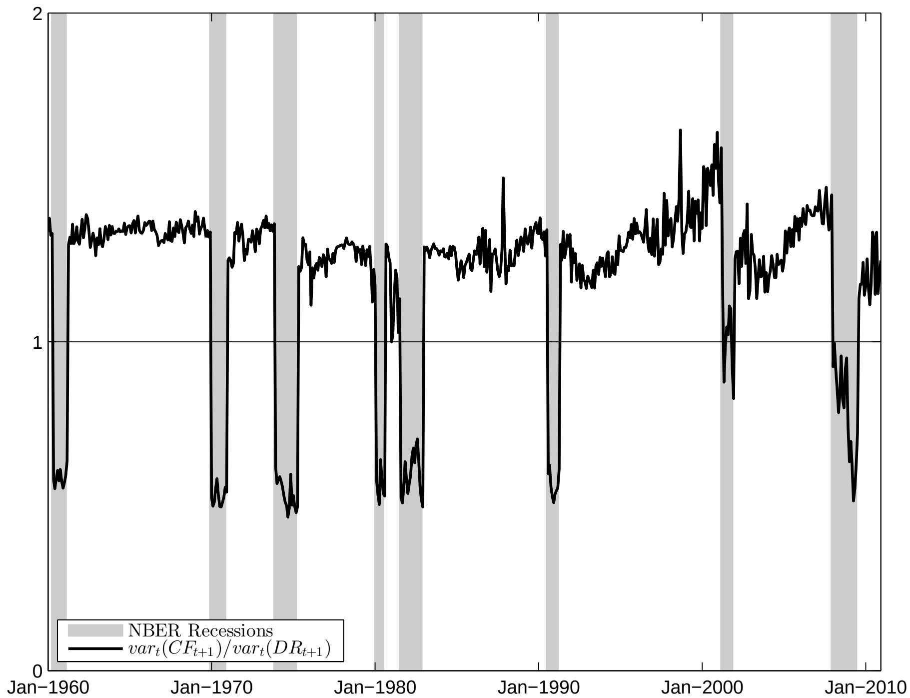


(b) Conditional Relative Importance



Note: The figure presents the decomposition of conditional variance of monthly unexpected returns on the S&P 500 index (in percentage points) over NBER business cycles. The decomposition is based on the proposed time-varying return decomposition approach using returns, term spread, dividend yield and value spread as state variables in the VAR. Panel (a) presents the conditional variance of unexpected returns (black line) and its decomposition into the conditional variances of cash flow news (red line) and discount rate news (blue line) and their conditional covariance (green line). Panel (b) presents the relative importance of each component in determining the conditional variance of unexpected returns, i.e. the ratio of the conditional variance of cash flow and discount rate news as well as their conditional covariance to the conditional variance of unexpected returns. The sample period is between January 1960 and December 2010. The shaded regions are the NBER recession periods.

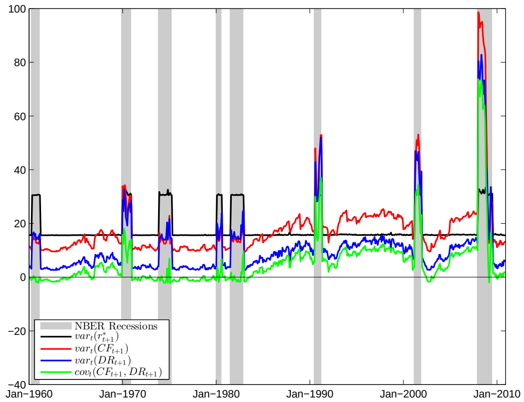
Figure 2: Ratio of Conditional Variances of Cash Flows and Discount News



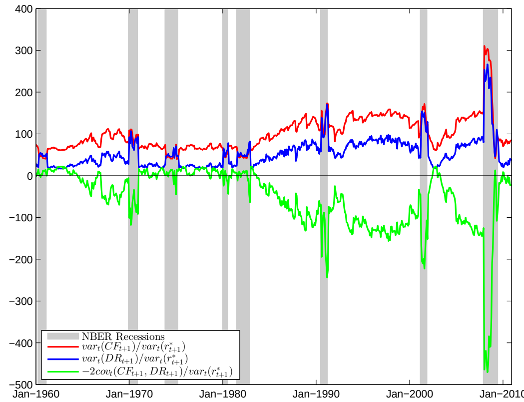
Note: The figure presents the ratio of conditional variances of cash flow and discount rate news. The decomposition is based on the proposed time-varying return decomposition approach using returns, term spread, dividend yield and value spread as state variables in the VAR. The sample period is between January 1960 and December 2010. The shaded regions are the NBER recessions.

Figure 3: Decomposition of the Conditional Variance of Returns over the Business Cycle - Alternative Set of Predictor Variables

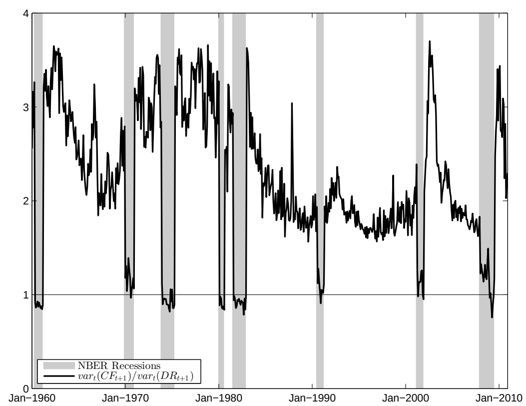
(a) Conditional Variances



(b) Conditional Relative Importances

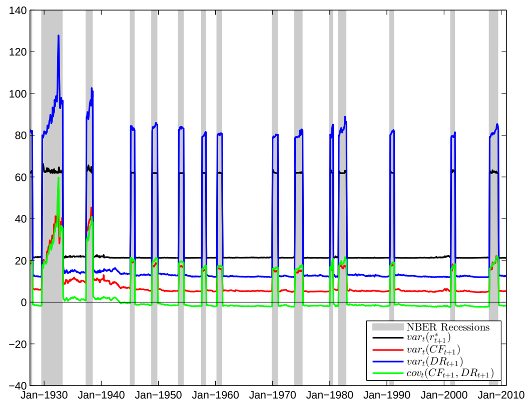


(c) Ratio of Conditional Variances of Cash Flows and Discount News

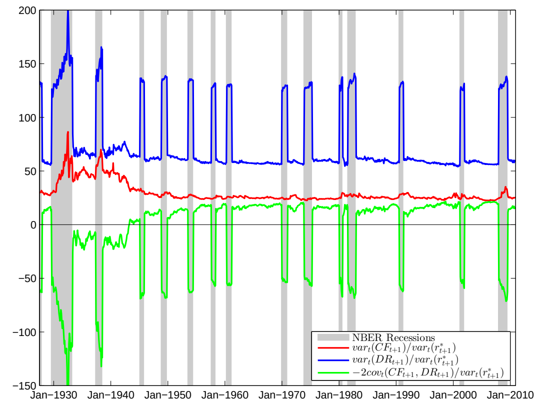


Note: The figure presents the decomposition of conditional variance of monthly unexpected returns on the S&P 500 index (in percentage points) over NBER business cycles. The decomposition is based on the proposed time-varying return decomposition approach when we consider the first four principle components from a large set of predictor variables described in Section 4.6 as state variables in the VAR in addition to returns. Panel (a) presents the conditional variance of unexpected returns in percentage points (black line) and its decomposition into the conditional variances of cash flow news (red line) and discount rate news (blue line) and their conditional covariance (green line). Panel (b) presents the relative importance of each component in determining the conditional variance of unexpected returns, i.e. the ratio of the conditional variance of cash flow and discount rate news as well as their conditional covariance to the conditional variance of unexpected returns. Panel (c) presents the ratio of conditional variances of cash flow and discount rate news. The sample period is between January 1960 and December 2010. The shaded regions are the NBER recessions.

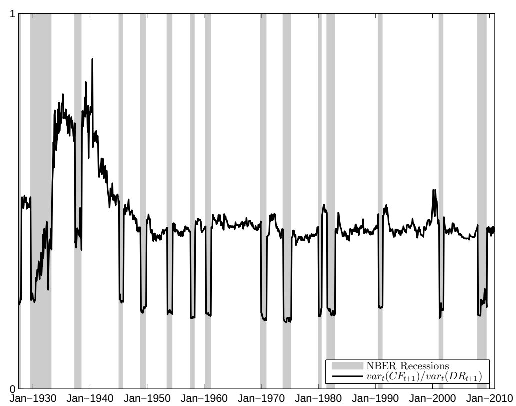
Figure 4: Decomposition of the Conditional Variance of Returns over the Business Cycle - Alternative Sample Period



(a) Conditional Variances



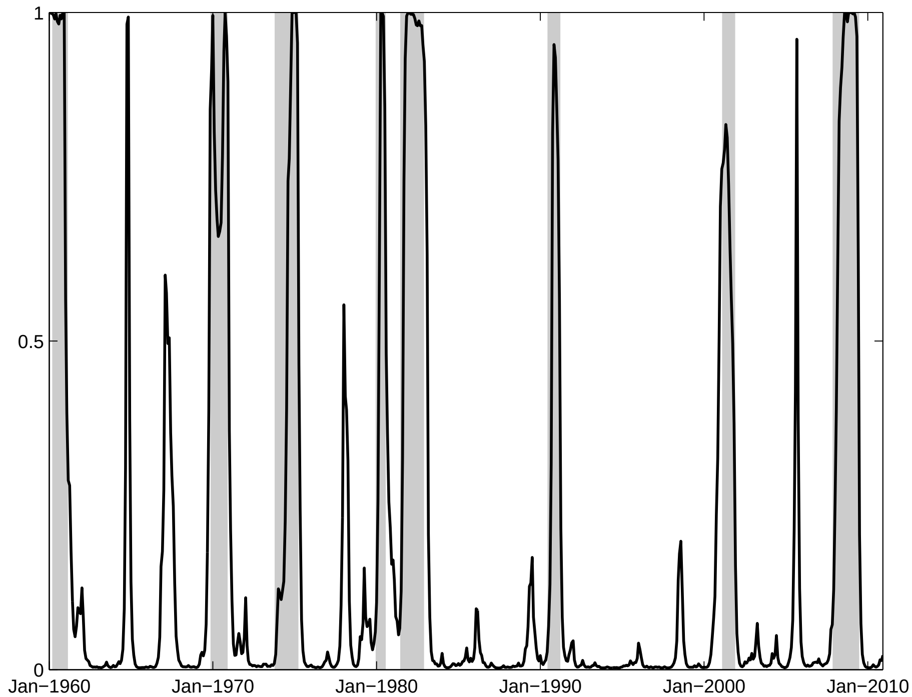
(b) Conditional Relative Importances



(c) Ratio of Conditional Variances of Cash Flows and Discount News

Note: The figure presents the decomposition of conditional variance of monthly unexpected returns on the S&P 500 index over NBER business cycles between June 1927 and December 2010. The decomposition is based on the proposed time-varying return decomposition approach using returns, term spread, dividend yield and value spread as state variables in the VAR. Panel (a) presents the conditional variance of unexpected returns in percentage points (black line) and its decomposition into the conditional variances of cash flow news (red line) and discount rate news (blue line) and their conditional covariance (green line). Panel (b) presents the relative importance of each component in determining the conditional variance of unexpected returns, i.e. the ratio of the conditional variance of cash flow and discount rate news as well as their conditional covariance to the conditional variance of unexpected returns. Panel (c) presents the ratio of conditional variances of cash flow and discount rate news. The shaded regions are the NBER recessions.

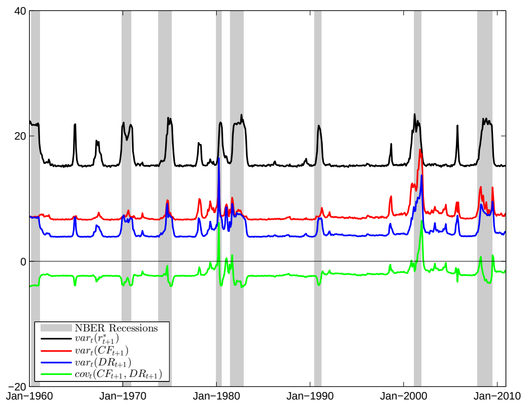
Figure 5: Smoothed Recession Probabilities



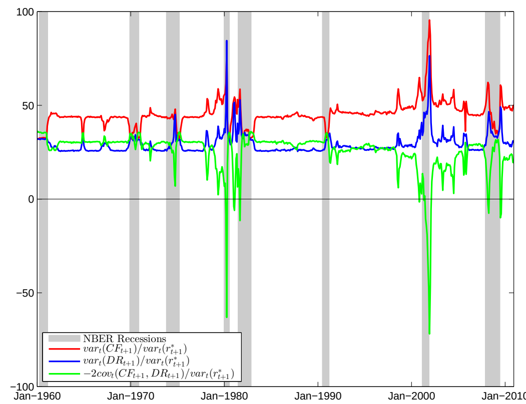
Note: The figure presents the smoothed probabilities of the state with lower and more volatile growth rate of industrial production. The smoothed transition probabilities are obtained from the estimation of the two-state Markov regime switching model in Equation 14 for the log growth rate of monthly industrial production index between January 1960 and December 2010.

Figure 6: Decomposition of the Conditional Variance of Returns over the Business Cycle - Alternative Business Cycle Definitions

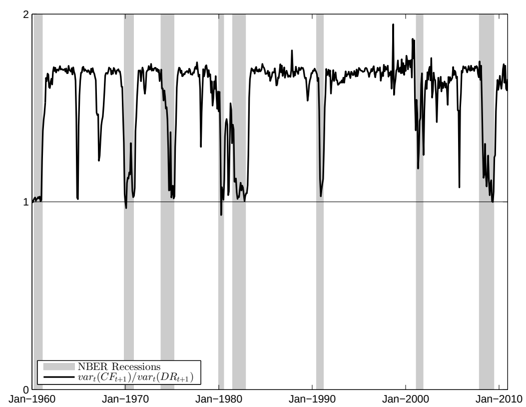
(a) Conditional Variances



(b) Conditional Relative Importances



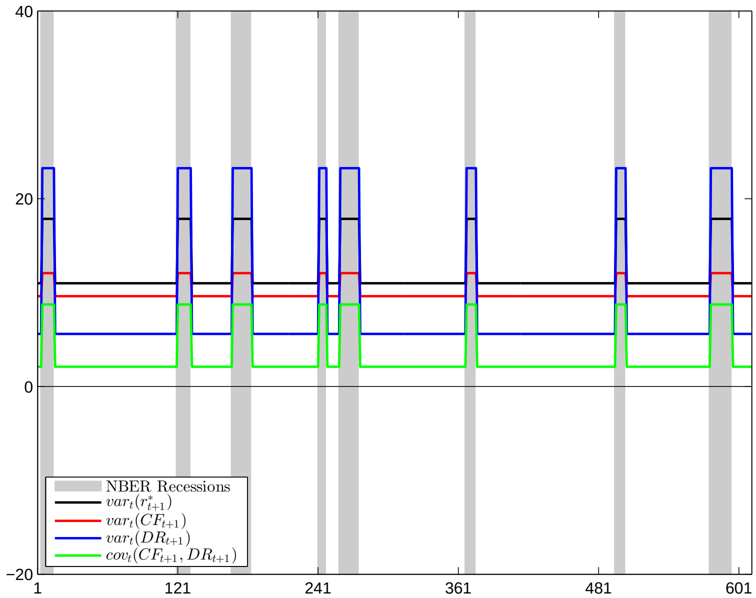
(c) Ratio of Conditional Variances of Cash Flows and Discount News



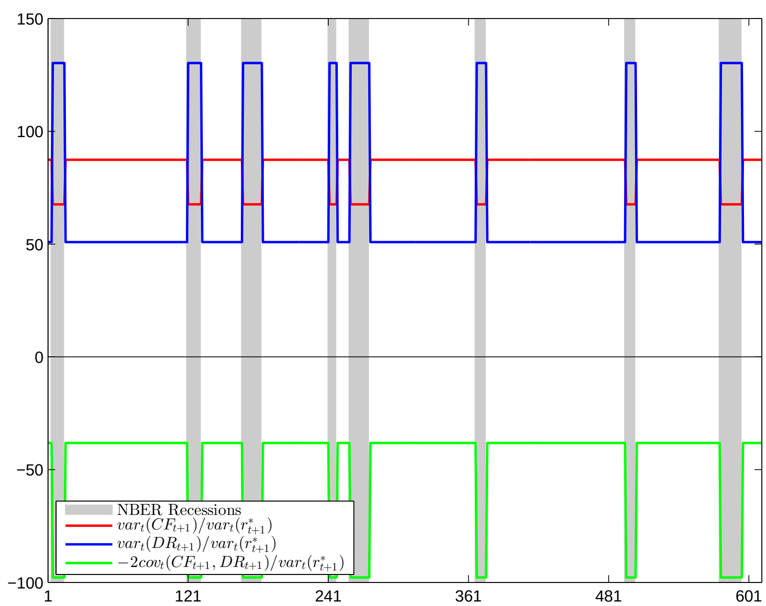
Note: The figure presents the decomposition of conditional variance of monthly unexpected returns on the S&P 500 index (in percentage points) over business cycles. The decomposition is based on the proposed time-varying return decomposition approach using returns, term spread, dividend yield and value spread as state variables in the VAR. Business cycles are defined by the smoothed state probabilities obtained from the estimation of the two-state Markov regime switching model in Equation 14 for the log growth rate of monthly industrial production index. Panel (a) presents the conditional variance of unexpected returns in percentage points (black line) and its decomposition into the conditional variances of cash flow news (red line) and discount rate news (blue line) and their conditional covariance (green line). Panel (b) presents the relative importance of each component in determining the conditional variance of unexpected returns, i.e. the ratio of the conditional variance of cash flow and discount rate news as well as their conditional covariance to the conditional variance of unexpected returns. Panel (c) presents the ratio of conditional variances of cash flow and discount rate news. The sample period is between January 1960 and December 2010. The shaded regions are the NBER recessions.

Figure 7: Decomposition of the Conditional Variance of Simulated Returns

(a) Conditional Variances

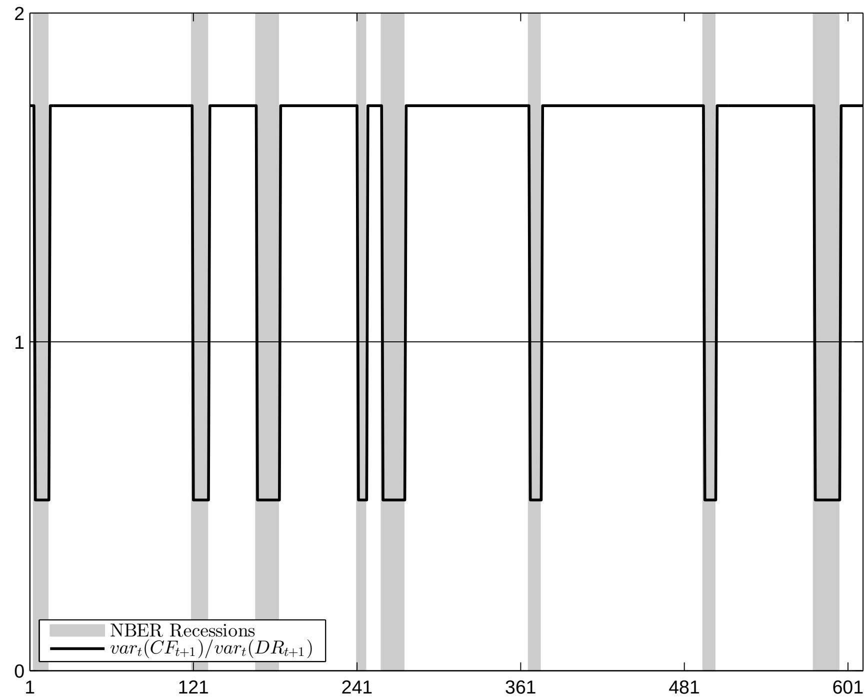


(b) Conditional Relative Importance



Note: We simulate the asset pricing model in Section 5.1 at monthly frequency for a total of 612 observations which corresponds to the number of monthly observations for the period between 1960 and 2010. To directly match the empirical observations presented in Section 5.7, we assume that the state variable in our simulation exercise corresponds to expansions ($S_t = 1$) and recessions ($S_t = 2$) as defined by the NBER. We then decompose the conditional variance of simulated returns (in percentage points) into its components as discussed in Section 4.3. Panel (a) presents the conditional variance of simulated unexpected returns (black line) and its decomposition into the conditional variances of cash flow news (red line) and discount rate news (blue line) and their conditional covariance (green line). Panel (b) presents the relative importance of each component in determining the conditional variance of simulated unexpected returns, i.e. the ratio of the conditional variance of cash flow and discount rate news as well as their conditional covariance to the conditional variance of simulated unexpected returns.

Figure 8: Ratio of Conditional Variances of Simulated Cash Flows and Discount News



Note: The figure presents the ratio of conditional variances of cash flow and discount rate news from the decomposition of simulated unexpected returns as discussed in Section 4.3. The sample period is between January 1960 and December 2010. The shaded regions are the NBER recessions.

Table 1: Unconditional Variance Decomposition based on the Standard Approach

(a) Estimates of the VAR Parameters

	α	r_{t-1}	tms_{t-1}	dy_{t-1}	vs_{t-1}	$\bar{R}^2$
r_t	0.5469	0.0486	0.2438**	0.2331	-0.8614	0.68%
tms_t	-0.0955	0.0039	0.9558***	0.0097	0.0988	91.07%
dy_t	0.1047	-0.0019	-0.0114***	0.9875***	-0.0323	98.24%
vs_t	0.1058***	-0.0003	-0.0002	-0.0021	0.9331***	88.11%

(b) Estimate of the Variance Matrix of the VAR Residuals

	r_t	tms_t	dy_t	vs_t
r_t	18.7419	0.0385	-0.6079	0.0416
tms_t	0.0385	0.1974	-0.0025	-0.0014
dy_t	-0.6079	-0.0025	0.0227	-0.0014
vs_t	0.0416	-0.0014	-0.0014	0.0024

(c) Unconditional Variance Decomposition of Returns

	Value (Ratio)
$var(CF)$	5.43 (28.99%)
$var(DR)$	8.14 (43.44%)
$-2cov(CF, DR)$	5.17 (27.57%)
$var(r)$	18.74 (100.00%)

Note: Panels (a) and (b) present the estimates of the parameters and the residual variance matrix of the VAR model in Equation 3. r_t is the continuously compounded monthly return on the S&P 500 index (in percentage points), including dividends, in excess of the log risk-free rate. tms_t is the term spread defined as the difference between the long term yield on government bonds and the Treasury bill. dy is the dividend yield defined as the log ratio of dividends to lagged prices. vs is the value spread defined as the difference between the log book-to-market of small value stocks and that of small growth stocks. The VAR model is estimated via OLS with HAC standard errors. ***, **, * denote parameter estimates that are significantly different than zero at 1%, 5% and 10% levels, respectively. $\bar{R}^2$ is the adjusted R^2 of the regression. Panel (c) presents the decomposition of the unconditional variance of r_t based on the standard approach described in Section 2 using the estimates of the VAR model in Equation 3 presented in Panels (a) and (b). $var(CF)$ and $var(DR)$ are the unconditional variances of cash flow and discount rate news, respectively, and $-2cov(CF, DR)$ is -2 time the unconditional variance between cash flow and discount rate news. The values for $var(CF)$, $var(DR)$ and $-2cov(CF, DR)$ sum up to the unconditional variance of returns ($var(r)$). The numbers in parenthesis are the relative importance of each component in determining the unconditional variance of returns and sum up to 100%. The relative importance of a component is defined as the ratio of the unconditional variance of that component to that of returns.The sample period is between January 1960 and December 2010.

Table 2: Unconditional Variance Decomposition based on the Standard Approach in Expansions

(a) Estimates of the VAR Parameters

	α	r_{t-1}	tms_{t-1}	dy_{t-1}	vs_{t-1}	$\bar{R}^2$
r_t	0.4647	-0.0448	0.1460	0.1303	-0.3119	0.40%
tms_t	0.1597	0.0050	0.9758***	-0.0306*	-0.0444	94.79%
dy_t	0.0923	0.0007	-0.0069**	0.9917***	-0.0422	99.39%
vs_t	0.1115***	-0.0005	-0.0022*	-0.0016	0.9305***	99.44%

(b) Estimate of the Variance Matrix of the VAR Residuals

	r_t	tms_t	dy_t	vs_t
r_t	15.3599	-0.0623	-0.4523	0.0401
tms_t	-0.0623	0.1409	0.0035	-0.0016
dy_t	-0.4523	0.0035	0.0153	-0.0012
vs_t	0.0401	-0.0016	-0.0012	0.0021

(c) Unconditional Variance Decomposition of Returns

	Value (Ratio)
$var(CF)$	25.90 (168.64%)
$var(DR)$	6.90 (44.93%)
$-2cov(CF, DR)$	-17.44 (-113.57%)
$var(r)$	15.36 (100.00%)

Note: Panels (a) and (b) present the estimates of the parameters and the residual variance matrix of the VAR model in Equation 3 in expansions as defined by the NBER. r_t is the S&P 500 return, tms is the term spread, dy is the dividend yield and vs is the value spread. We refer the reader to the note to Table 1 for detailed variable definitions. The VAR model is estimated via WLS where the weight of an observation is one if the observation corresponds to a month in a recession period as defined by the NBER and zero otherwise. ***, **, * denote parameter estimates that are significantly different than zero at 1%, 5% and 10% levels, respectively. $\bar{R}^2$ is the adjusted R^2 of the regression. Panel (c) presents the decomposition of the unconditional variance of r_t in expansions based on the standard approach described in Section 2 using the estimates of the VAR model in Equation 3 presented in Panels (a) and (b). This decomposition corresponds to a hypothetical situation where the economy is expected to stay in expansion till infinity. Furthermore, it also ignores recession periods in the estimation of VAR parameters and residual variance matrix. $var(CF)$ and $var(DR)$ are the unconditional variances of cash flow and discount rate news, respectively, and $-2cov(CF, DR)$ is -2 times the unconditional variance between cash flow and discount rate news. The values for $var(CF)$, $var(DR)$ and $-2cov(CF, DR)$ sum up to the unconditional variance of returns ($var(r)$). The numbers in parenthesis are the relative importance of each component in determining the unconditional variance of returns and sum up to 100%. The relative importance of a component is defined as the ratio of the unconditional variance of that component to that of returns. The sample period is between January 1960 and December 2010.

Table 3: Unconditional Variance Decomposition based on the Standard Approach in Recessions

(a) Estimates of the VAR Parameters

	α	r_{t-1}	tms_{t-1}	dy_{t-1}	vs_{t-1}	$\bar{R}^2$
r_t	0.4261	0.1699***	0.7463***	1.0404***	-1.8159	10.71%
tms_t	0.0287**	0.0143***	0.8470***	0.0270	0.7336***	90.11%
dy_t	0.3204	-0.0051***	-0.0377***	0.9621***	0.0494	99.59%
vs_t	0.0000***	-0.0003	0.0127***	-0.0051**	0.8605***	99.82%

(b) Estimate of the Variance Matrix of the VAR Residuals

	r_t	tms_t	dy_t	vs_t
r_t	31.9502	0.8037	-1.2739	0.0391
tms_t	0.8037	0.4029	-0.0415	0.0025
dy_t	-1.2739	-0.0415	0.0561	-0.0020
vs_t	0.0391	0.0025	-0.0020	0.0033

(c) Unconditional Variance Decomposition of Returns

	Value (Ratio)
$var(CF)$	9.40 (29.42%)
$var(DR)$	47.05 (147.26%)
$-2cov(CF, DR)$	-24.50 (-76.68%)
$var(r)$	31.95 (100.00%)

Note: Panels (a) and (b) present the estimates of the parameters and the residual variance matrix of the VAR model in Equation 3 in recessions as defined by the NBER. r_t is the S&P 500 return, tms is the term spread, dy is the dividend yield and vs is the value spread. We refer the reader to the note to Table 1 for detailed variable definitions. The VAR model is estimated via WLS where the weight of an observation is one if the observation corresponds to a month in a recession period as defined by the NBER and zero otherwise. ***, **, * denote parameter estimates that are significantly different than zero at 1%, 5% and 10% levels, respectively. $\bar{R}^2$ is the adjusted R^2 of the regression. Panel (c) presents the decomposition of the unconditional variance of r_t in recessions based on the standard approach described in Section 2 using the estimates of the VAR model in Equation 3 presented in Panels (a) and (b). This decomposition corresponds to a hypothetical situation where the economy is expected to stay in recession till infinity. Furthermore, it also ignores recession periods in the estimation of VAR parameters and variance matrix of VAR residuals. $var(CF)$ and $var(DR)$ are the unconditional variances of cash flow and discount rate news, respectively, and $-2cov(CF, DR)$ is -2 times the unconditional variance between cash flow and discount rate news. The values for $var(CF)$, $var(DR)$ and $-2cov(CF, DR)$ sum up to the unconditional variance of returns ($var(r)$). The numbers in parenthesis are the relative importance of each component in determining the unconditional variance of returns and sum up to 100%. The relative importance of a component is defined as the ratio of the unconditional variance of that component to that of returns. The sample period is between January 1960 and December 2010.

Table 4: Unconditional Variance Decomposition based on the Standard Approach under Different Assumptions

(a) Time-Varying VAR Parameters and Constant Variance of the VAR Residuals

	Expansions	Recessions
$var(CF)$	34.87 (192.69%)	4.37 (24.15%)
$var(DR)$	9.66 (53.38%)	17.54 (96.90%)
$-2cov(CF, DR)$	-26.43 (-146.07%)	-3.81 (-21.05%)
$var(r)$	18.10 (100.00%)	18.10 (100.00%)

(b) Constant VAR Parameters and Time-Varying Variance of the VAR Residuals

	Expansions	Recessions
$var(CF)$	4.98 (31.98%)	7.75 (22.25%)
$var(DR)$	6.10 (39.22%)	18.45 (52.98%)
$-2cov(CF, DR)$	4.48 (28.80%)	8.63 (24.77%)
$var(r)$	15.56 (100.00%)	34.83 (100.00%)

Note: The table presents the decomposition of the unconditional variance of monthly unexpected returns on the S&P 500 index under alternative assumptions about parameters and residual variance matrix of the VAR model in Equation 3. The decomposition is based on the standard approach using returns, term spread, dividend yield and value spread as state variables in the VAR. Panel (a) is based on the assumption that the VAR parameters are time-varying and identical to those presented in Panels (a) of Tables 2 and 3 with a constant variance matrix of residuals estimated over the whole sample based on time-varying VAR parameters. Panel (b) is based on the assumption that the VAR parameters are constant and identical to those estimated over the whole sample in Table 1. $var(CF)$ and $var(DR)$ are the unconditional variances of cash flow and discount rate news, respectively, and $-2cov(CF, DR)$ is -2 times the unconditional variance between cash flow and discount rate news. The values for $var(CF)$, $var(DR)$ and $-2cov(CF, DR)$ sum up to the unconditional variance of returns ($var(r)$). The numbers in parenthesis are the relative importance of each component in determining the unconditional variance of returns and sum up to 100%. The relative importance of a component is defined as the ratio of the unconditional variance of that component to that of returns. The sample period is between January 1960 and December 2010.

Table 5: Unconditional Variance Decomposition based on the Time-Varying Approach over the Business Cycle

	Value (Ratio)
$var(CF)$	8.49 (46.41%)
$var(DR)$	7.25 (39.64%)
$-2cov(CF, DR)$	2.55 (13.95%)
$var(r)$	18.29 (100.00%)

Note: The table presents the decomposition of the unconditional variance of monthly unexpected returns on the S&P 500 index over business cycles as defined by the NBER. The decomposition is based on the proposed time-varying approach described in 4 using returns, term spread, dividend yield and value spread as state variables in the VAR. We estimate the VAR via WLS twice using expansion and recession weights as discussed in Section 3 and obtain the same estimates of parameters and residual variance matrix presented in Panels (a) and (b) of Tables 2 and 3. $var(CF)$ and $var(DR)$ are the unconditional variances of cash flow and discount rate news, respectively, and $-2cov(CF, DR)$ is -2 times the unconditional variance between cash flow and discount rate news. The values for $var(CF)$, $var(DR)$ and $-2cov(CF, DR)$ sum up to the unconditional variance of returns ($var(r)$). The numbers in parenthesis are the relative importance of each component in determining the unconditional variance of returns and sum up to 100%. The relative importance of a component is defined as the ratio of the unconditional variance of that component to that of returns. The sample period is between January 1960 and December 2010.

Table 6: Unconditional Variance Decomposition based on the Time-Varying Approach over the Business Cycle - Robustness Checks

(a) Alternative Set of Predictor Variables

	Value (Ratio)
$var(CF)$	17.94 (97.71%)
$var(DR)$	10.01 (54.51%)
$-2cov(CF, DR)$	-9.59 (-52.23%)
$var(r)$	18.36 (100.00%)

(b) Alternative Sample Period

	Value (Ratio)
$var(CF)$	8.67 (29.21%)
$var(DR)$	25.89 (87.21%)
$-2cov(CF, DR)$	-4.87 (-16.42%)
$var(r)$	29.69 (100.00%)

(c) Alternative Business Cycle Definition

	Value (Ratio)
$var(CF)$	7.79 (42.41%)
$var(DR)$	5.35 (29.12%)
$-2cov(CF, DR)$	5.23 (28.47%)
$var(r)$	18.38 (100.00%)

Note: The table presents decomposition of the unconditional variance of monthly unexpected returns on the S&P 500 index over business cycles to using alternative set of predictor variables, sample periods and definition of business cycles. The decompositions are all based on the proposed time-varying approach described in 4. In Panel (a), the state variables in the VAR are returns and the first four principal components of a set of state variables that includes default premium, one year price-earnings ratio, book-to-market ratio, book-to-market spread, stock market variance and net equity issuance in addition to term spread, value spread and dividend yield. The business cycles are as defined by the NBER and the sample period is between January 1960 and December 2010. In Panel (b), the state variables in the VAR are returns, term spread, dividend yield and value spread. The business cycles are as defined by the NBER and the sample period is between June 1927 and December 2010. In Panel (c), the state variables in the VAR are returns, term spread, dividend yield and value spread. The business cycles are defined by the smoothed state probabilities obtained from the estimation of the two-state Markov regime switching model in Equation 14 for the log growth rate of monthly industrial production index.

Table 7: Calibrated Model Parameters

(a) Utility Specification

Parameter	Value
γ	7.5
β	0.9957

(b) Dividend and Consumption Process

Parameter	$i = 1$	$i = 2$
$\mu_{d,i}$	0.204%	-0.556%
$\mu_{c,i}$	0.297%	0.008%
$\sigma_{d,i}$	2.972%	2.972%
$\sigma_{c,i}$	0.693%	0.693%
$\rho_{cd,i}$	0.391	0.391
q_{ii}	0.983	0.925

Note: The table presents calibrated model parameters. γ is the coefficient of relative risk aversion and β is the daily time impatience parameter. $\mu_{c,i}$ and $\mu_{d,i}$ for $i = 1, 2$ are the mean consumption and dividend growth rates in different states, respectively. $\sigma_{c,i}$ and $\sigma_{d,i}$ for $i = 1, 2$ are the standard deviations of the consumption and dividend growth rates in different states, respectively. $\rho_{cd,i}$ for $i = 1, 2$ is the correlation coefficient between the consumption and dividend growth rates in different states. q_{ii} is the transition probability from state i to state i.

Table 8: Unconditional Variance Decomposition based on Simulated Data

	Value (Ratio)
$var(CF)$	10.00 (83.54%)
$var(DR)$	8.33 (69.62%)
$-2cov(CF, DR)$	-6.36 (-53.16%)
$var(r)$	11.97 (100.00%)

Note: The table presents the decomposition of the unconditional variance of unexpected simulated returns. We simulate the asset pricing model in Section 5.1 at monthly frequency for a total of 612 observations which corresponds to the number of monthly observations for the period between 1960 and 2010. To directly match the empirical observations presented in Section 5.7, we assume that the state variable in our simulation exercise corresponds to expansions ($S_t = 1$) and recessions ($S_t = 2$) as defined by the NBER. We then decompose the conditional variance of simulated returns (in percentage points) into its components as discussed in Section 4.3. $var(CF)$ and $var(DR)$ are the unconditional variances of cash flow and discount rate news, respectively, and $-2cov(CF, DR)$ is -2 times the unconditional variance between cash flow and discount rate news. The values for $var(CF)$, $var(DR)$ and $-2cov(CF, DR)$ sum up to the unconditional variance of returns ($var(r)$). The numbers in parenthesis are the relative importance of each component in determining the unconditional variance of returns and sum up to 100%. The relative importance of a component is defined as the ratio of the unconditional variance of that component to that of returns.

Proofs

Proof. [**Proof of Lemma 1**] After some matrix algebra, one can show that $\mathbf{X}_{t+\tau}$ can be expressed as follows:

$$\begin{aligned}\mathbf{X}_{t+\tau} &= \boldsymbol{\alpha}_{S_{t+\tau}} + \sum_{i=1}^{\tau}(\prod_{j=1}^{i}\mathbf{A}_{S_{t+\tau+1-j}})\boldsymbol{\alpha}_{S_{t+\tau-i}} + (\prod_{j=1}^{\tau}\mathbf{A}_{S_{t+\tau+1-j}})\mathbf{X}_t \\ &+ \boldsymbol{\epsilon}_{t+\tau} + \sum_{i=1}^{\tau}(\prod_{j=1}^{i}\mathbf{A}_{S_{t+\tau+1-j}})\boldsymbol{\epsilon}_{t+\tau-i} \end{aligned} \tag{24}$$

Taking expectations of both sides conditional on the information at time t,

$$\begin{aligned}E_t[\mathbf{X}_{t+\tau}] &= \sum_{i=0}^{\tau-1}(\mathbf{1}_M \otimes \mathbf{I}_N)'(\mathbf{A}(\mathbf{Q}' \otimes \mathbf{I}_N))^i \boldsymbol{\alpha}(\mathbf{Q}' \otimes \mathbf{I}_N))^{\tau-i}(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{1}_N \\ &+ (\mathbf{1}_M \otimes \mathbf{I}_N)'(\mathbf{A}(\mathbf{Q}' \otimes \mathbf{I}_N))^{\tau}(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t \end{aligned} \tag{25}$$

Note that the summation in Equation 25 is a Slyvester Equation and can be rewritten using Kronecker products and vec operator as follows:

$$\begin{aligned}E_t[\mathbf{X}_{t+\tau}] &= \sum_{i=1}^{\tau-1}((\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{1}_N)' \otimes (\mathbf{1}_M \otimes \mathbf{I}_N)'\Big[((\mathbf{Q}' \otimes \mathbf{I}_N)')^{-1} \otimes (\mathbf{A}(\mathbf{Q}' \otimes \mathbf{I}_N))\Big]^i vec(\boldsymbol{\alpha}(\mathbf{Q}' \otimes \mathbf{I}_N)^{\tau}) \\ &+ (\mathbf{1}_M \otimes \mathbf{I}_N)'(\mathbf{A}(\mathbf{Q}' \otimes \mathbf{I}_N))^{\tau}(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t \end{aligned} \tag{26}$$

Rewriting the sum, one obtains the equation in the lemma

$$E_t[\mathbf{X}_{t+\tau}] = (\mathbf{1}_M \otimes \mathbf{I}_N)'\Big(\mathbf{f}_1(\tau)(\mathbf{\Pi}_t \otimes \mathbf{1}_N) + \mathbf{f}_2(\tau)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t\Big) \tag{27}$$

where

$$\begin{aligned}\mathbf{f}_1(\tau) &= vec^{-1}\Big([\mathbf{I}_{M^2N^2} - ((\mathbf{Q}' \otimes \mathbf{I}_N)')^{-1} \otimes (\mathbf{A}(\mathbf{Q}' \otimes \mathbf{I}_N))]^{-1} \\ &\quad [\mathbf{I}_{M^2N^2} - (((\mathbf{Q}' \otimes \mathbf{I}_N)')^{-1} \otimes (\mathbf{A}(\mathbf{Q}' \otimes \mathbf{I}_N)))^{\tau}] vec(\boldsymbol{\alpha}(\mathbf{Q}' \otimes \mathbf{I}_N)^{\tau})\Big) \\ \mathbf{f}_2(\tau) &= (\mathbf{A}(\mathbf{Q}' \otimes \mathbf{I}_N))^{\tau} \end{aligned}$$

where vec^{-1} is the inverse vec operator that turns a $M^2N^2 \times 1$ vector into $MN \times MN$ matrix.

□

Proof. [**Proof of Proposition 1**] Based on Lemma 1, unexpected return in period $t+1$ can be expressed as follows:

$$\begin{aligned}r^*_{t+1} &= \mathbf{e}_1'\Big(\mathbf{X}_{t+1} - E_t[\mathbf{X}_{t+1}]\Big) \\ &= \mathbf{e}_1'\Big(\mathbf{X}_{t+1} - (\mathbf{1}_M \otimes \mathbf{I}_N)'(\mathbf{f}_1(1)(\mathbf{\Pi}_t \otimes \mathbf{1}_N) + \mathbf{f}_2(1)(\mathbf{\Pi}_t \otimes \mathbf{X}_t))\Big) \end{aligned} \tag{28}$$

Recall the definition of the discount rate news:

$$DR_{t+1} = \mathbf{e}_1'\Big(E_{t+1}\Big[\sum_{j=1}^{\infty}\rho^j \mathbf{X}_{t+1+j}\Big] - E_t\Big[\sum_{j=1}^{\infty}\rho^j \mathbf{X}_{t+1+j}\Big]\Big). \tag{29}$$

The expectations in the above equation can be calculated using Lemma 1 as follows:

$$\begin{aligned}
E_{t+1}\Big[\sum_{j=1}^{\infty}\rho^j \mathbf{X}_{t+1+j}\Big] &= [((\mathbf{\Pi}_{t+1}\otimes \mathbf{I}_N)\mathbf{1}_N)'\otimes(\mathbf{1}_M\otimes\mathbf{I}_N)'][\mathbf{I}_{M^2N^2} - ((\mathbf{Q}'\otimes\mathbf{I}_N)')^{-1}\otimes(\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))]^{-1} \\
&\times \Big(\sum_{j=1}^{\infty}(\rho^j(\mathbf{Q}\otimes\mathbf{I}_N)^{j+1}\otimes\mathbf{I}_{MN}) - \rho^j(\mathbf{I}_{MN}\otimes(\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))^{j+1})\Big)vec(\boldsymbol{\alpha}) \\
&- \sum_{j=1}^{\infty}\rho^j(\mathbf{1}_M\otimes\mathbf{I}_N)'(\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))^{j+1}(\mathbf{\Pi}_{t+1}\otimes\mathbf{I}_N)\mathbf{X}_{t+1} \\
&= (\mathbf{1}_M\otimes\mathbf{I}_N)'[\mathbf{B}_{1,1}(\mathbf{\Pi}_{t+1}\otimes\mathbf{1}_N) + \mathbf{B}_{2,1}(\mathbf{\Pi}_{t+1}\otimes\mathbf{X}_{t+1})]
\end{aligned}$$

and

$$\begin{aligned}
E_t[\sum_{j=1}^{\infty}\rho^j \mathbf{X}_{t+1+j}] &= ((\mathbf{\Pi}_{t}\otimes \mathbf{I}_N)\mathbf{1}_N)'\otimes(\mathbf{1}_M\otimes\mathbf{I}_N)'[\mathbf{I}_{M^2N^2} - ((\mathbf{Q}'\otimes\mathbf{I}_N)')^{-1}\otimes(\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))]^{-1} \\
&\times \Big(\sum_{j=1}^{\infty}(\rho^j(\mathbf{Q}\otimes\mathbf{I}_N)^{j+1}\otimes\mathbf{I}_{MN}) - \rho^j(\mathbf{I}_{MN}\otimes(\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))^{j+1})\Big)vec(\boldsymbol{\alpha}) \\
&- \sum_{j=1}^{\infty}\rho^j(\mathbf{1}_M\otimes\mathbf{I}_N)'(\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))^{j+1}(\mathbf{\Pi}_{t}\otimes\mathbf{I}_N)\mathbf{X}_{t} \\
&= (\mathbf{1}_M\otimes\mathbf{I}_N)'[\mathbf{B}_{1,2}(\mathbf{\Pi}_{t}\otimes\mathbf{1}_N) + \mathbf{B}_{2,2}(\mathbf{\Pi}_{t}\otimes\mathbf{X}_{t})]
\end{aligned}$$

where $\mathbf{B}_{1,i}$ and $\mathbf{B}_{2,i}$ for $i = 1, 2$ are

$$\begin{aligned}
\mathbf{B}_{1,i} &= vec^{-1}\Big([\mathbf{I}_{M^2N^2} - ((\mathbf{Q}'\otimes\mathbf{I}_N)')^{-1}\otimes(\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))]^{-1} \\
&\times [\rho(\mathbf{Q}\otimes\mathbf{I}_N)^i(\mathbf{I}_{MN} - \rho(\mathbf{Q}\otimes\mathbf{I}_N))^{-1}\otimes\mathbf{I}_{MN} - \mathbf{I}_{MN}\otimes\rho(\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))^i(\mathbf{I}_{MN} - \rho\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))^{-1}]vec(\boldsymbol{\alpha})\Big) \\
\mathbf{B}_{2,i} &= \rho(\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))^i(\mathbf{I}_{MN} - \rho\mathbf{A}(\mathbf{Q}'\otimes\mathbf{I}_N))^{-1}
\end{aligned}$$

Plugging these expectations in the definition of discount rate news, we obtain

$$DR_{t+1} = \mathbf{e}_1'(\mathbf{1}_M\otimes\mathbf{I}_N)'[\mathbf{B}_{1,1}(\mathbf{\Pi}_{t+1}\otimes\mathbf{1}_N) + \mathbf{B}_{2,1}(\mathbf{\Pi}_{t+1}\otimes\mathbf{X}_{t+1}) - \mathbf{B}_{1,2}(\mathbf{\Pi}_{t}\otimes\mathbf{1}_N) - \mathbf{B}_{2,2}(\mathbf{\Pi}_{t}\otimes\mathbf{X}_{t})] \tag{30}$$

The cash flow news is the sum of unexpected returns in Equation 28 and discount rate news in Equation 30:

$$
\begin{aligned}
CF_{t+1} &= r^*_{t+1} + DR_{t+1} \\
&= \mathbf{e}_1'\mathbf{X}_{t+1} + \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'[\mathbf{B}_{1,1}(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N) + \mathbf{B}_{2,1}(\mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})] \\
&- \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'[(\mathbf{f}_1(1) + \mathbf{B}_{1,2})(\mathbf{\Pi}_t \otimes \mathbf{1}_N) + (\mathbf{f}_2(1) + \mathbf{B}_{2,2})(\mathbf{\Pi}_t \otimes \mathbf{X}_t)] \qquad (31)
\end{aligned}
$$

□

Proof of Proposition 2. First note that

$$
\begin{aligned}
E_t[\mathbf{X}_{t+1}\mathbf{X}_{t+1}'] &= E_t[(\boldsymbol{\alpha}_{S_{t+1}} + \mathbf{A}_{S_{t+1}}\mathbf{X}_t + \boldsymbol{\varepsilon}_{t+1})(\boldsymbol{\alpha}_{S_{t+1}} + \mathbf{A}_{S_{t+1}}\mathbf{X}_t + \boldsymbol{\varepsilon}_{t+1})'] \\
&= E_t[\boldsymbol{\alpha}_{S_{t+1}}\boldsymbol{\alpha}_{S_{t+1}}'] + E_t[\boldsymbol{\alpha}_{S_{t+1}}(\mathbf{A}_{S_{t+1}}\mathbf{X}_t)'] \\
&+ E_t[(\mathbf{A}_{S_{t+1}}\mathbf{X}_t)\boldsymbol{\alpha}_{S_{t+1}}'] + E_t[(\mathbf{A}_{S_{t+1}}\mathbf{X}_t)(\mathbf{A}_{S_{t+1}}\mathbf{X}_t)'] \\
&+ E_t[\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}_{t+1}'] \\
&= \sum_{i=1}^{M}(\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i' + \boldsymbol{\alpha}_i(\mathbf{A}_i\mathbf{X}_t)' + (\mathbf{A}_i\mathbf{X}_t)\boldsymbol{\alpha}_i' + (\mathbf{A}_i\mathbf{X}_t)(\mathbf{A}_i\mathbf{X}_t)' + \mathbf{\Sigma}_i)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t) \qquad (32)
\end{aligned}
$$

and

$$
\begin{aligned}
E_t[\mathbf{X}_{t+1}] &= E_t[\boldsymbol{\alpha}_{S_{t+1}} + \mathbf{A}_{S_{t+1}}\mathbf{X}_t + \boldsymbol{\varepsilon}_{t+1}] \\
&= \sum_{i=1}^{M}(\boldsymbol{\alpha}_i + \mathbf{A}_i\mathbf{X}_t)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t) \qquad (33)
\end{aligned}
$$

The conditional variance of unexpected return in period $t+1$ based on information set in period t is given by:

$$
\begin{aligned}
var_t(r^*_{t+1}) &= var_t\Big(\mathbf{e}_1'(\mathbf{X}_{t+1} - (\mathbf{1}_M \otimes \mathbf{I}_N)'(\mathbf{f}_1(1)(\mathbf{\Pi}_t \otimes \mathbf{1}_N) + \mathbf{f}_2(1)(\mathbf{\Pi}_t \otimes \mathbf{X}_t)))\Big) \\
&= \mathbf{e}_1' var_t(\mathbf{X}_{t+1})\mathbf{e}_1 \\
&= \mathbf{e}_1'\Big(E_t[\mathbf{X}_{t+1}\mathbf{X}_{t+1}'] - E_t[\mathbf{X}_{t+1}]E_t[\mathbf{X}_{t+1}']\Big)\mathbf{e}_1 \qquad (34)
\end{aligned}
$$

Plugging Equations 32 and 33 into Equation 34 yields

$$
\begin{aligned}
var_t(r^*_{t+1}) &= \mathbf{e}_1'\Big(\sum_{i=1}^{M}(\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i' + \boldsymbol{\alpha}_i(\mathbf{A}_i\mathbf{X}_t)' + (\mathbf{A}_i\mathbf{X}_t)\boldsymbol{\alpha}_i' + (\mathbf{A}_i\mathbf{X}_t)(\mathbf{A}_i\mathbf{X}_t)' + \mathbf{\Sigma}_i)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t)\Big)\mathbf{e}_1 \\
&- \mathbf{e}_1'\Big(\sum_{i=1}^{M}(\boldsymbol{\alpha}_i + \mathbf{A}_i\mathbf{X}_t)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t)\sum_{i=1}^{M}(\boldsymbol{\alpha}_i' + (\mathbf{A}_i\mathbf{X}_t)')(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t)\Big)\mathbf{e}_1 \qquad (35)
\end{aligned}
$$

After some matrix algebra one can show that Equation 35 can be written as Equation 10.

Given the definitions of DR_{t+1} and CF_{t+1} in Equations 7 and 8, respectively, it is easy to see that their conditional variances and covariance can be expressed as in Equations 11, 12 and 13. This completes the first part of the proof.

To prove the second part, note that the following holds when the state variable is observable:

$$\begin{aligned}
E_t[\Pi_{i,t+1}] &= E_t[\Pi^2_{i,t+1}] = \mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t \\
E_t[\Pi_{i,t+1}\Pi_{j,t+1}] &= 0 \quad \text{for } i \neq j \\
E_t[\Pi_{i,t+1}\mathbf{X}_{t+1}] &= E_t[\Pi^2_{i,t+1}\mathbf{X}_{t+1}] = (\boldsymbol{\alpha}_i + \mathbf{A}_i\mathbf{X}_t)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t) \\
E_t[\Pi_{i,t+1}\Pi_{j,t+1}\mathbf{X}_{t+1}] &= \mathbf{0}_N \quad \text{for } i \neq j \\
E_t[\Pi_{i,t+1}\mathbf{X}_{t+1}\mathbf{X}_{t+1}'] &= E_t[\Pi^2_{i,t+1}\mathbf{X}_{t+1}\mathbf{X}_{t+1}'] \\
&= (\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i' + \boldsymbol{\alpha}_i(\mathbf{A}_i\mathbf{X}_t)' + (\mathbf{A}_i\mathbf{X}_t)\boldsymbol{\alpha}_i' + (\mathbf{A}_i\mathbf{X}_t)(\mathbf{A}_i\mathbf{X}_t)' + \mathbf{\Sigma}_i)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t) \\
E_t[\Pi_{i,t+1}\Pi_{j,t+1}\mathbf{X}_{t+1}\mathbf{X}_{t+1}'] &= \mathbf{0}_N \otimes \mathbf{0}_N' \quad \text{for } i \neq j
\end{aligned}$$

where $\mathbf{0}_N$ is a $N \times 1$ vector of zeros.

Plugging these into the definitions of $var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)$, $var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})$, $cov_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})$, $cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)$ and $cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})$ yields the equations in Proposition 2.

□

Proof of Lemma 2. We first characterize investors' prior beliefs about state j in period t, $\tilde{\pi}_{j,t}$, i.e. the probability that investors assign to state j before observing the realizations for dividend and consumption in period t. Prior to observing the information revealed in a given period t, investors know that the growth process might have switched to a new state according to the transition probability matrix. Hence, their prior beliefs about the new state variable are weighted averages of his beliefs about the previous state variable, $\pi_{i,t-1}$, where the weights are the transition probabilities, q_{ij}, i.e. $\tilde{\pi}_{j,t} = \sum_{i=1}^{N} \pi_{i,t-1} q_{ij}$.

Investors then update their prior beliefs according to Bayes' rule based on the realizations of dividend and consumption processes. Recall that the probability that investors assign to state j, $\pi_{j,t} = \Pr(S_t = j|\mathcal{F}_t)$.

$$\begin{aligned}
\pi_{j,t} &= \Pr(S_t = j|\mathbf{y}_t, \mathcal{F}_{t-1}) && (36) \\
&= \frac{\Pr(\mathbf{y}_t|S_t = j, \mathcal{F}_{t-1})\Pr(S_t = j|\mathcal{F}_{t-1})}{\Pr(\mathbf{y}_t|\mathcal{F}_{t-1})} && (37) \\
&= \frac{\Pr(\mathbf{y}_t|S_t = j, \mathcal{F}_{t-1})\Pr(S_t = j|\mathcal{F}_{t-1})}{\sum_{i=1}^{N}\Pr(\mathbf{y}_t|S_t = i, \mathcal{F}_{t-1})\Pr(S_t = i|\mathcal{F}_{t-1})} && (38) \\
&= \frac{\phi(\mathbf{y}_t; \boldsymbol{\mu}_j, \mathbf{\Sigma}_j)\tilde{\pi}_{j,t}}{\sum_{i=1}^{N}\phi(\mathbf{y}_t; \boldsymbol{\mu}_i, \mathbf{\Sigma}_i)\tilde{\pi}_{i,t}} && (39)
\end{aligned}$$

where $\phi(\mathbf{x}; \boldsymbol{\mu}, \mathbf{\Sigma})$ is the multivariate normal density function with mean $\boldsymbol{\mu}$ and variance matrix $\mathbf{\Sigma}$.

Equation 36 follows from the definition of the information set, $\mathcal{F}_t$, which can be decomposed into the realization of dividend and consumption processes in period t, $\mathbf{y}_t$, and all past information, $\mathcal{F}_{t-1}$. Equations 37 and 38 follow from Bayes' rule and the law of total probability, respectively.[6] Equation 39 follows from the law of motion for the dividend and consumption process in Equation 16. □

[6]Recall that Bayes' rule is $\Pr(A|B,C) = \frac{\Pr(B|A,C)\Pr(A|C)}{\Pr(B|C)}$

Proof of Proposition 3. By recursive substitution of future prices into Euler equation, the price of the risky asset can be expressed as the conditional expectation of the discounted sum of future dividends where the discount factor is the intertemporal marginal rate of substitution:

$$P_t = E_t\Big[\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{U'(C_{t+\tau})}{U'(C_t)}D_{t+\tau}\Big] \tag{40}$$

where $E_t[\cdot]$ denotes the conditional expectation based on the information set in period t after investors observe the realization of dividend and consumption processes.

Substituting the functional form for the utility function and rearranging the terms, the price-dividend ratio in period t can be expressed as follows:

$$\begin{aligned}\frac{P_t}{D_t} &= E_t\Big[\sum_{\tau=1}^{\infty}\beta^{\tau}\Big(\frac{C_{t+\tau}}{C_t}\Big)^{-\gamma}\Big(\frac{D_{t+\tau}}{D_t}\Big)\Big]\\ &= \sum_{i=1}^{N}E_t\Big[\sum_{\tau=1}^{\infty}\beta^{\tau}\Big(\frac{C_{t+\tau}}{C_t}\Big)^{-\gamma}\Big(\frac{D_{t+\tau}}{D_t}\Big)\Big|S_t=i\Big]\pi_{i,t}\end{aligned} \tag{41}$$

where the second equation follows from the law of total probability. Let λ_i denote the price-dividend ratio in state i, i.e. $\lambda_i = E\Big[\sum_{\tau=1}^{\infty}\beta^{\tau}\Big(\frac{C_{t+\tau}}{C_t}\Big)^{-\gamma}\Big(\frac{D_{t+\tau}}{D_t}\Big)\Big|S_t=i\Big]$. It is easy to see that the price-dividend ratio is positive in each state given that it is a sum of positive numbers. To guarantee that it is also finite in each state, we assume that model parameters are such that $g_j = \beta\exp(\mu_{d,j} - \gamma\mu_{c,j} + \frac{1}{2}(\sigma_{d,j}^2 - 2\gamma\sigma_{cd,j} + \gamma^2\sigma_{c,j}^2)) < 1$ for $j = 1,\ldots,N$. Then, λ_i can be expressed as follows:

$$\begin{aligned}\lambda_i &= E\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma}\Big(\frac{D_{t+1}}{D_t}\Big)\Big|S_t=i\Big] + E\Big[\sum_{\tau=2}^{\infty}\beta^{\tau}\Big(\frac{C_{t+\tau}}{C_t}\Big)^{-\gamma}\Big(\frac{D_{t+\tau}}{D_t}\Big)\Big|S_t=i\Big]\\ &= \sum_{j=1}^{N}q_{ij}E\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma}\Big(\frac{D_{t+1}}{D_t}\Big)\Big|S_{t+1}=j\Big] + \sum_{j=1}^{N}q_{ij}E\Big[\sum_{\tau=2}^{\infty}\beta^{\tau}\Big(\frac{C_{t+\tau}}{C_t}\Big)^{-\gamma}\Big(\frac{D_{t+\tau}}{D_t}\Big)\Big|S_{t+1}=j\Big]\\ &= \sum_{j=1}^{N}q_{ij}E\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma}\Big(\frac{D_{t+1}}{D_t}\Big)\Big|S_{t+1}=j\Big]\\ &+ \sum_{j=1}^{N}q_{ij}E\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma}\Big(\frac{D_{t+1}}{D_t}\Big)\Big|S_{t+1}=j\Big]E\Big[\sum_{\tau=2}^{\infty}\beta^{\tau-1}\Big(\frac{C_{t+\tau}}{C_{t+1}}\Big)^{-\gamma}\Big(\frac{D_{t+\tau}}{D_{t+1}}\Big)\Big|S_{t+1}=j\Big]\\ &= \sum_{j=1}^{N}q_{ij}g_j + \sum_{j=1}^{N}q_{ij}g_j\lambda_j\end{aligned} \tag{42}$$

for $i = 1,\ldots,N$. This yields a system of 4 equations which can be expressed as follows:

$$\boldsymbol{\lambda} = \mathbf{QG} + \mathbf{QH}\boldsymbol{\lambda} \tag{43}$$

where $\mathbf{G} = (g_1, g_2, \ldots, g_N)'$ is a $N \times 1$ vector and $\mathbf{H}$ is a $N \times N$ diagonal matrix whose j^{th} diagonal element is g_j.

Solving for $\boldsymbol{\lambda}$ yields

$$\boldsymbol{\lambda} = (\mathbf{I}_N - \mathbf{QH})\mathbf{QG} \tag{44}$$

and the price-dividend ratio can be expressed as follows:

$$\frac{P_t}{D_t} = \sum_{i=1}^{N} \lambda_i \pi_{i,t} = \boldsymbol{\lambda}'\boldsymbol{\Pi}_t \tag{45}$$

Log returns on the risky asset can be expressed as follows:

$$\begin{aligned} r_t &= \log(1 + P_t/D_t) - \log(P_{t-1}/D_{t-1}) + \Delta d_t \\ &\approx \log(1+\overline{\lambda}) + \frac{1}{1+\overline{\lambda}}(P_t/D_t - \overline{\lambda}) - \log(\overline{\lambda}) - \frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1} - \overline{\lambda}) + \Delta d_t \end{aligned} \tag{46}$$

where Equation 46 follows from a first-order Taylor expansion of the log function around the long term average of the price-dividend ratio, $\bar{\lambda}$. The expectation of the log return in period t conditional on investors' prior beliefs before observing the dividend realization (and possibly the external signal) in period t can be expressed as follows:

$$\tilde{E}_t[r_t] \approx \log(1+\overline{\lambda}) + \frac{1}{1+\overline{\lambda}}(\sum_{j=1}^{N} \lambda_j \tilde{\pi}_{j,t} - \overline{\lambda}) - \log(\overline{\lambda}) - \frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1} - \overline{\lambda}) + \sum_{j=1}^{N} \mu_{d,j}\tilde{\pi}_{j,t} \tag{47}$$

The unexpected log return on the risky asset in Equation 20 can be obtained as the difference between Equations 46 and 47. The long term average of the price-dividend ratio is the unconditional expectation of the price-dividend ratio as defined in Proposition 3.

□

Proof of Corollary 1. First note that the variance of the dividend growth rate in period $t+1$ conditional on the information set in period t is given by

$$\begin{aligned} var_t(\Delta d_{t+1}) &= E_t[\Delta d_{t+1}^2] - E_t[\Delta d_{t+1}]^2 \\ &= E_t[\mu_{d,S_{t+1}}^2 + 2\mu_{d,S_{t+1}}\sigma_{d,S_{t+1}}\varepsilon_{d,t+1} + \sigma_{d,S_{t+1}}^2\varepsilon_{d,t+1}^2] - E_t[\mu_{d,S_{t+1}} + \sigma_{d,S_{t+1}}\varepsilon_{d,t+1}]^2 \\ &= \sum_{i=1}^{N}(\mu_{d,i}^2 + \sigma_{d,i}^2)(\mathbf{e}_i\mathbf{Q}'\boldsymbol{\Pi}_t) - (\sum_{i=1}^{N}\mu_{d,i}(\mathbf{e}_i\mathbf{Q}'\boldsymbol{\Pi}_t))^2 \end{aligned} \tag{48}$$

Then, it is easy to see that the conditional variance of unexpected returns is given by Equation 21, given its law of motion in Equation 20. This completes the first part of the proof.

To prove the second part, note that the following holds when the state variable is observable:

$$\begin{aligned} E_t[\Pi_{i,t+1}] &= E_t[\Pi_{i,t+1}^2] = \mathbf{e}_i'\mathbf{Q}'\boldsymbol{\Pi}_t \\ E_t[\Pi_{i,t+1}\Pi_{j,t+1}] &= 0 \quad \text{for } i \neq j \\ E_t[\Pi_{i,t+1}\Delta d_{t+1}] &= \mu_{d,i}(\mathbf{e}_i'\mathbf{Q}'\boldsymbol{\Pi}_t)^2 \end{aligned}$$

Plugging these into the definitions of $var_t(\mathbf{\Pi}_{t+1})$ and $cov_t(\Delta d_{t+1}, \mathbf{\Pi}_{t+1})$ yields the equations in Corollary 1. $\square$

Proof of Proposition 4. Given the law of motion for returns in Equation 46, note that the following holds:

$$
\begin{aligned}
E_t\Big[\sum_{\tau=1}^{\infty}(1-\rho)^{\tau} r_{t+1+\tau}\Big] &= \sum_{\tau=1}^{\infty}(1-\rho)^{\tau} E_t[\log(1+\overline{\lambda}) + \frac{1}{1+\overline{\lambda}}(\frac{P_{t+1+\tau}}{D_{t+1+\tau}} - \overline{\lambda}) - \log(\overline{\lambda}) - \frac{1}{\overline{\lambda}}(\frac{P_{t+\tau}}{D_{t+\tau}} - \overline{\lambda}) + \Delta d_{t+1+\tau}] \\
&= \sum_{\tau=1}^{\infty}(1-\rho)^{\tau} E_t[\kappa + \frac{1}{1+\overline{\lambda}}\boldsymbol{\lambda}'\mathbf{\Pi}_{t+1+\tau} - \frac{1}{\overline{\lambda}}\boldsymbol{\lambda}'\mathbf{\Pi}_{t+\tau} + \Delta d_{t+1+\tau}] \\
&= \sum_{\tau=1}^{\infty}(1-\rho)^{\tau}(\kappa + \frac{1}{1+\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau+1}\mathbf{\Pi}_t - \frac{1}{\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau}\mathbf{\Pi}_t + \boldsymbol{\mu}_d'(\mathbf{Q}')^{\tau+1}\mathbf{\Pi}_t) \qquad (49)
\end{aligned}
$$

where $\kappa = \log(1+1/\overline{\lambda}) + \overline{1}/(1+\overline{\lambda})$. Similarly,

$$
E_{t+1}\Big[\sum_{\tau=1}^{\infty}(1-\rho)^{\tau} r_{t+1+\tau}\Big] = \sum_{\tau=1}^{\infty}(1-\rho)^{\tau}(\kappa + \frac{1}{1+\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau}\mathbf{\Pi}_{t+1} - \frac{1}{\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau-1}\mathbf{\Pi}_{t+1} + \boldsymbol{\mu}_d'(\mathbf{Q}')^{\tau}\mathbf{\Pi}_{t+1}) \qquad (50)
$$

Recall that discount rate news in the Campbell and Shiller framework is defined as

$$
DR_{t+1} = E_{t+1}\Big[\sum_{j=1}^{\infty}(1-\rho)^{j} r_{t+1+j}\Big] - E_t\Big[\sum_{j=1}^{\infty}(1-\rho)^{j} r_{t+1+j}\Big]
$$

Plugging in the above equations, the discount rate news can be expressed as follows:

$$
\begin{aligned}
DR_{t+1} &= \sum_{\tau=1}^{\infty}(1-\rho)^{\tau}(\kappa + \frac{1}{1+\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau}\mathbf{\Pi}_{t+1} - \frac{1}{\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau-1}\mathbf{\Pi}_{t+1} + \boldsymbol{\mu}_d'(\mathbf{Q}')^{\tau}\mathbf{\Pi}_{t+1} \\
&- (\kappa + \frac{1}{1+\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau+1}\mathbf{\Pi}_t - \frac{1}{\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau}\mathbf{\Pi}_t + \boldsymbol{\mu}_d'(\mathbf{Q}')^{\tau+1}\mathbf{\Pi}_t)) \\
&= \sum_{\tau=1}^{\infty}(1-\rho)^{\tau}(\frac{1}{1+\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau}(\mathbf{\Pi}_{t+1} - \mathbf{Q}'\mathbf{\Pi}_t) - \frac{1}{\overline{\lambda}}\boldsymbol{\lambda}'(\mathbf{Q}')^{\tau-1}(\mathbf{\Pi}_{t+1} - \mathbf{Q}'\mathbf{\Pi}_t) \\
&+ \boldsymbol{\mu}_d'(\mathbf{Q}')^{\tau}(\mathbf{\Pi}_{t+1} - \mathbf{Q}'\mathbf{\Pi}_t)) \\
&= (\mu_d'(\mathbf{I}_M - (1-\rho)\mathbf{Q}')^{-1}(1-\rho)\mathbf{Q}' - \rho\boldsymbol{\lambda}')(\mathbf{\Pi}_{t+1} - \mathbf{Q}'\mathbf{\Pi}_t)
\end{aligned}
$$

Given the definition of unexpected return in Equation 20, cash flow news can be expressed as:

$$
\begin{aligned}
CF_{t+1} &= r^*_{t+1} + DR_{t+1} \\
&= \rho\boldsymbol{\lambda}'(\mathbf{\Pi}_t - \mathbf{Q}'\mathbf{\Pi}_{t-1}) + \Delta d_t - \bar{\mu}_{d,t-1} + (\mu_d'(\mathbf{I}_M - (1-\rho)\mathbf{Q}')^{-1}(1-\rho)\mathbf{Q}' - \rho\boldsymbol{\lambda}')(\mathbf{\Pi}_{t+1} - \mathbf{Q}'\mathbf{\Pi}_t) \\
&= (\mu_d'(\mathbf{I}_M - (1-\rho)\mathbf{Q}')^{-1}(1-\rho)\mathbf{Q}')(\mathbf{\Pi}_{t+1} - \mathbf{Q}'\mathbf{\Pi}_t) + \Delta d_t - \bar{\mu}_{d,t-1}
\end{aligned}
$$

$\square$

Proof of Proposition 5. Given the formulas for CF_{t+1} and DR_{t+1}, it is easy to see that their conditional variances and covariance are given as in Proposition 5. $\square$

Simulation Approach to Calculate Conditional Quantities

In this section, we describe our Monte Carlo simulation approach to calculate conditional variances and covariances discussed in the text. We do this for a generic MSVAR of order one whose special cases correspond to the ones considered in the text. Specifically, consider a $K \times 1$ vector $\mathbf{Y}_{t+1}$ whose law of motion is given by

$$\mathbf{Y}_{t+1} = \mathbf{a}_{U_{t+1}} + \mathbf{b}_{U_{t+1}} \mathbf{Y}_t + \boldsymbol{\xi} \tag{51}$$

where state variable U_{t+1} follows a first order M-state Markov chain with transition probability matrix $\mathbf{Q}$ whose ij^{th} element $q_{i,j} = Prob(U_{t+1} = j | U_t = i)$ and $\boldsymbol{\xi} \sim N(\mathbf{0}_N, \boldsymbol{\Psi}_{S_{t+1}})$. When we set $\mathbf{Y}_{t+1} = \mathbf{X}_{t+1}$ and let U_{t+1} denote the NBER business cycles, we obtain the case in Section 4.5 for which we can calculate the conditional variances and covariances in closed form as in Proposition 2. We still consider this case as it allows us to verify the validity of our simulation approach. When we set $\mathbf{Y}_{t+1} = \Delta \log(IP_{t+1})$ and $U_{t+1} = n_{t+1}$, we obtain the case in Section 4.6.3. Finally, when we set $\mathbf{Y}_{t+1} = [\Delta d_{t+1}, \Delta c_{t+1}]$ and let $U_{t+1} = S_{t+1}$, which is the underlying state of the dividend and consumption process, we obtain the case in Section 5.

Let $\boldsymbol{\varphi}_{t+1} = [\varphi_{1,t+1}, \ldots, \varphi_{K,t+1}]'$ where $\varphi_{i,t+1} = Prob(U_{t+1} = i | \mathcal{F}_{t+1})$ for $i = 1, \ldots, K$ and $\mathcal{F}_{t+1}$ is the information set that includes $\mathbf{Y}_j$ and U_j for $j = 1, \ldots, t+1$ if U_j's are observable and includes only $\mathbf{Y}_j$ for $j = 1, \ldots, t+1$ otherwise. Then, note that the following holds under our assumptions for $\mathbf{Y}_{t+1}$ and U_{t+1}:

1. U_{t+1}, conditional on the information set in period t, has a multinomial distribution with associated probabilities given by $\widetilde{\boldsymbol{\varphi}}_{t+1} = \mathbf{Q}' \boldsymbol{\varphi}_t$.

2. $\mathbf{Y}_{t+1}$, conditional on the information set in period t and the state variable in period $t+1$, has a normal distribution mean $\mathbf{a}_{U_{t+1}} + \mathbf{b}_{U_{t+1}} \mathbf{Y}_t$ and variance matrix $\boldsymbol{\Psi}_{U_{t+1}}$.

3. $\boldsymbol{\varphi}_{t+1}$ can then be calculated as:

$$\varphi_{i,t+1} = \frac{\phi(\mathbf{Y}_{t+1}, \mathbf{a}_i + \mathbf{b}_i \mathbf{Y}_t, \boldsymbol{\Psi}_i) \widetilde{\varphi}_{i,t+1}}{\sum_{j=1}^{K} \phi(\mathbf{Y}_{t+1}, \mathbf{a}_j + \mathbf{b}_j \mathbf{Y}_t, \boldsymbol{\Psi}_j) \widetilde{\varphi}_{j,t+1}} \tag{52}$$

   where $\phi(x, \boldsymbol{\mu}, \boldsymbol{\Sigma})$ is the multivariate normal density function.

For each period $t+1$, we first draw the state variable U_{t+1} from the multinomial distribution with associated probabilities given by $\widetilde{\boldsymbol{\varphi}}_{t+1}$. Based on the state variable, we draw $\mathbf{Y}_{t+1}$ from the normal distribution with mean $\mathbf{a}_{U_{t+1}} + \mathbf{b}_{U_{t+1}} \mathbf{Y}_t$ and variance matrix $\boldsymbol{\Psi}_{U_{t+1}}$. We then calculate $\boldsymbol{\varphi}_{t+1}$ based on the Equation 52. We repeat these steps 1,000,000 times and calculate the conditional quantities of interest as the sample averages of the corresponding quantities from the simulations.