The Reaction of Stock Returns to News about Fundamentals

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In good times, stock prices react negatively to good news and positively to bad news, while in bad times, they react positively to good news and negatively to bad news. To account for this stylized fact, we consider an asset pricing model where the dividend growth rate switches between different values depending on the underlying state of the economy. Investors never observe the true dividend growth rate but learn about it through not only its realizations but also external signals such as macroeconomic indicators. Under plausible assumptions, the differing precision of external signals across different states of the economy can change the sign of the market reaction to news from external signals in good and bad times.

1. Introduction

How stock prices react to news is of central importance to financial decision making. A key problem in analyzing the reaction of stock returns to news is, however, that it can be difficult to determine when such news arrives. Furthermore, it can also be difficult to accurately measure the information content of an announcement. Scheduled information releases such as macroeconomic announcements provide a good starting point. First of all, the timing of these information releases is generally known in advance by financial market participants. Secondly, investors' expectations about scheduled announcements can be quantified by employing either model- or survey-based measures. Hence, it is not surprising to find a large literature on the reaction of stock returns to macroeconomic news.

One of the stylized facts in this literature is that the sign of the reaction depends on the underlying state of the economy. Specifically, stock prices generally react negatively to "good" news, i.e. higher than expected economic activity, and positively to "bad" news, i.e. lower than expected economic activity, in "good" times, i.e. during periods of good economic conditions. On the other hand, they react positively to "good" news and negatively to "bad" news in "bad" times, i.e. during periods of bad economic conditions. For example, McQueen and Roley (1993) write "[...] when the economy is strong the stock market responds negatively to news about higher real economic activity.". Boyd et al. (2005) find that a lower than expected unemployment rate is actually bad news for stock returns in expansions. Andersen et al. (2007) and Cenesizoglu (2011) analyze the reaction of returns on different portfolios to a wide range of macroeconomic news over the business cycle and find similar empirical evidence. Gilbert (2011) provides further supporting evidence for this stylized fact based on the relation between stock returns and future revisions to macroeconomic variables.

This pattern of stock price responses to macroeconomic news seems surprising (see McQueen and Roley (1993) and Boyd et al. (2005)). It is also relatively hard to justify from a theoretical standpoint. One possibility is based on the monetary response function of the Federal Reserve. Following good news about real economic activity, the Federal Reserve is generally expected to raise interest rates to reduce inflationary pressures. This, in turn, should not only decrease stock prices but also increase future interest rates. In other words, if the stylized fact were due to the monetary response function of the Federal Reserve, one would expect stock and bond prices to respond in a similar fashion. However, this is not found to be supported empirically as argued by McQueen and Roley (1993) and Boyd et al. (2005). Another possibility is based on a state-dependent coefficient of relative risk aversion. Specifically, one can account for the stylized fact in a pure exchange economy with a representative investor with power utility by assuming that the coefficient of relative risk aversion is above one in good times and below one in bad times. However, the empirical evidence in Gordon and St-Amour (2000) and Brandt and Wang (2003) suggests that the coefficient of relative risk aversion is systematically higher in recessions or bear markets than in expansions or bull markets, the opposite of what is required.

This paper proposes a third possibility based on sign and state-dependent precision of signals. We consider an asset pricing model where the growth rate of dividends switches between four states: really good times, normal good times, normal bad times and really bad times. The investor can distinguish between good and bad times but cannot distinguish between really good (bad) and normal good (bad) times and, thus, never observes the true state of the dividend growth process. However, he learns about it through two sources of information: dividend realizations observed every period and external signals observed only on announcement days that are possibly less frequent than dividend realizations and not necessarily regularly scheduled. We distinguish between positive and negative shocks to the external signal and assume that they might have different precisions in each state. Specifically, we assume that (1) the external signal is more precise in really good times than in normal good times when the shock to the external signal is nonnegative and the opposite holds when the shock to the external signal is negative; (2) the external signal in really bad times is more precise than or equally precise as the external signal in normal bad times when the shock to the external signal is nonnegative and the opposite holds when the shock to the external signal is negative. We also assume that the mean parameter of the external signal is higher in really good times than in normal good times and is lower in really bad times than in normal bad times.

In this framework, we first solve for the price-dividend ratio of the risky asset and show that it is a weighted average of the price-dividend ratios in different states where the weights are the investor's beliefs about each state. We also show that the price-dividend ratio is higher in better states which in turn implies that the unexpected return on the risky asset increases as the investor assigns a higher probability to better states. We then analyze how the unexpected return on the risky asset reacts to the news variable, i.e. news observed from external signals, first in a simulation exercise then theoretically.

In the simulation exercise, we simulate daily returns from our model calibrated to U.S. data. Using simulated data only on announcement days, we consider good and bad times separately and estimate a linear regression of unexpected returns on news from external signals while controlling for other factors such as news from dividend realizations. Matching the empirical results in the literature, we find that returns can react negatively to good news and positively to bad news in good times while they can react positively to good news and negatively to bad news in bad times.

We then theoretically analyze how the return changes as a function of the news variable in good and bad times under our assumptions on the precision of the external signal. We show that (1) in good times, the return decreases as the investor receives better news variables when we consider large (in magnitude) positive news variables and it increases as he receives worse news variables when we consider large (in magnitude) negative news variables; (2) in bad times, the return always increases as he receives better news variables and decreases as he receives worse news variables. Based on these theoretical results, we then argue that the shape of the return as a function of the news variable explains how our model can account for the stylized fact. Specifically, one observes a negative reaction to good news and positive one to bad news due to the effects of large positive and negative news variables on returns in good times. However, in bad times, the return reacts positively to good news and negatively to bad news given that it is always an increasing function of the news variable.

The intuition on how our model can account for the stylized fact follows from how the investor updates his beliefs following news from an external signal under our assumptions on the precision of the external signal. First, consider the reaction of returns to good news in good times. Under our assumption about the precision of the external signal in good times, the distribution of the external signal has a fatter right tail in normal good times compared to really good times. As the investor observes better news variables, the probability ratio of observing the same news variable in really and normal good times eventually starts to decrease. As a result, the investor eventually starts to decrease the probability that he assigns to really good times, or equivalently starts to increase the probability that he assigns to normal good times, as he observes better news variables. Given that the return is an increasing function of the probability that the investor assigns to really good times, it also eventually starts to decrease as the investor observes better news variables. Thus, one can observe a negative reaction of the return to good news in good times in a linear regression framework.

Second, consider the reaction of returns to bad news in good times. The intuition is quite similar to the one discussed above and follows from our assumption that the external signal is more precise in normal good times compared to really good times when the shock to the external signal is negative. Specifically, this assumption implies that the distribution of the external signal has a fatter left tail in really good times compared to normal good times. As the investor observes worse news variables, the probability ratio of observing the same news variable in really and normal good times and the probability that he assigns to really good times eventually start to increase. This in turn implies that the return eventually starts to increase as he observes worse news variables and, thus, one can observe a positive reaction of the return to bad news in good times in a linear regression framework.

Third, consider the reaction of returns to news in bad times. Our assumptions on the precision of the external signal in bad times imply that the distribution of the external signal has a fatter (or similar) right tail and a thinner (or similar) left tail behavior in normal bad times compared to really bad times. This implies that the probability ratio of observing the same news variable in normal and really bad times as well as the probability that the investor assigns to normal bad times always increase as he observes better news variables. Given that the return is an increasing function of the probability that the investor assigns to normal bad times, it also increases as the investor observes better news variables. Thus, one can observe a positive reaction to good news and a negative one to bad news in bad times in a linear regression framework.

Finally, under the commonly made assumption of constant precision, the external signal has similar tail behavior in different states. Hence, the probability ratio of observing the same news variable in high- and low-growth states and, thus, the return always increase as the investor observes better news variables and always decrease as he observes worse news variables. As a result, one always observes a positive reaction to good news and a negative one to bad news in a linear regression framework regardless of the underlying state of the economy.

Our assumptions on the precision of the external signal are the main driving mechanism behind our results. There is some empirical evidence that the information quality and the precision of signals indeed vary over different cycles in the economy. Most studies find that information quality decreases and signals become less precise in bad times such as recessions.[1] However, these studies do not distinguish between positive and negative shocks to the external signal. To this end, we provide some preliminary empirical evidence. Specifically, we use the industrial production index as our proxy for the external signal and analyze its precision in different states while distinguishing between positive and negative shocks. We find that the precision of external signals might indeed be different in really good times but not necessarily in normal good times. More importantly, we find that the external signal is more precise in really good times than in normal good times when the shock to the external signal is nonnegative and the opposite holds when the shock to the external signal is negative. We also find that the precision of the external signal depends neither on the true underlying state nor on the sign of the shock to the external signal in bad times. This suggests that the external signal is equally precise in normal and really bad times independent of the sign of the shock to the external signal. Although we do not claim these preliminary evidence to be conclusive, they suggest that our assumptions are not inconsistent with the data.

Our paper is related to a theoretical literature on the reaction of stock returns to signals in different states. David (1997) develops a general equilibrium model where the investor learns about unobserved profitability switches between different industries in the economy and finds that stock returns react more strongly to news about the relative profitability of different industries during periods of low confidence or high uncertainty. Veronesi (1999) analyzes the reaction of the aggregate stock market to news about the growth rate of dividends and finds that stock prices overreact to bad news when the growth rate of dividends is high and underreact to good news when it is low. However, differently from our paper, he considers the effect of news from dividend realizations rather than news from external signals. More importantly, he does not consider how the sign of the reaction to news depends on the underlying state, which is the main focus of our paper. The closest paper to ours is Veronesi (2000). However, our paper differs from his in several aspects. First of all, the focus of our paper is quite different than his. We analyze the effect of news from external signals on returns in different periods depending on the investor's beliefs. On the other hand, Veronesi (2000)

[1] For example, van Nieuwerburgh and Veldkamp (2006) find that the median error and the dispersion of forecasts for future real GDP increase in recessions. Veronesi (1999) and Patton and Timmermann (2010) also provide some evidence that the dispersions of forecasts are higher during recessions. Similarly, Swanson and van Dijk (2006) find that there is a clear increase in the volatility of revisions to macroeconomic variables during recessions. Croushore (2011) also finds that there are some differences in the sizes of data revisions depending on the state of the business cycle. Dynan and Elmendorf (2001) find that revisions to real GDP growth are lower around business cycle peaks and higher around business cycle troughs.

analyzes the relation between stock returns and the information quality of an external signal and finds that higher quality information leads to an increase in the risk premium. Secondly, our model can account for the stylized facts in the literature due to our assumptions on sign and state-dependent precision of external signals. On the other hand, Veronesi (2000) assumes that the external signal has constant precision over time, an assumption crucial to derive the process for the investor's beliefs in continuous time. Finally, Veronesi (2000) considers a pure exchange economy with a power utility investor. Even under our assumptions on sign and state-dependent precision of external signals, one needs to further assume that the investor is less risk averse than a log utility investor, which might not be supported empirically, to account for the stylized facts in discrete time version of Veronesi (2000)'s framework. Differently, we model consumption and dividends separately and consider an investor with Epstein-Zin-Weil utility. This allows us to account for the stylized facts without having to take a stand on whether the risk aversion coefficient and the elasticity of intertemporal substitution are greater or less than one.

The rest of the paper is organized as follows: Section 2 presents our model and solves for the unexpected return on the risky asset. Section 3.1 discusses the motivation behind certain assumptions of our model. Section 3.2 presents evidence that our model can account for the stylized fact in a linear regression framework. Section 3.3 discusses the intuition behind our results. Section 4 concludes.

2. The Model

Consider a representative investor who has recursive preferences over consumption as in Epstein and Zin (1989) and Weil (1990),

$$U(C_t) = \{(1-\beta)C_t^{(1-\gamma)/\theta} + \beta E_t[(U(C_{t+1}))^{1-\gamma}]^{1/\theta}\}^{\theta/(1-\gamma)}, \tag{1}$$

where C_t denotes the investor's consumption in period t, β is the time impatience parameter, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution, and $\theta = (1-\gamma)/(1-\psi^{-1})$. The investor's consumption grows according to the following process:

$$\Delta c_t = \mu_c + \sigma_c \varepsilon_{c,t}, \tag{2}$$

where $c_t = \log(C_t)$ is the log-consumption in period t, Δ denotes the first difference operator, i.e. $\Delta c_t = c_t - c_{t-1}$, and $\varepsilon_{c,t}$ is an independently and identically distributed Gaussian random variable with zero mean and unit variance. The investor's opportunity set consists of a risky asset whose supply is fixed and normalized to one and a risk-free asset. Dividends of the risky asset grow according to the following process:

$$\Delta d_t = \mu_{d,S_t} + \sigma_d \varepsilon_{d,t}, \tag{3}$$

where $d_t = \log(D_t)$ is the log-dividend in period t and $\varepsilon_{d,t}$ is an independently and identically distributed Gaussian random variable with zero mean and unit variance and S_t is a latent state variable that determines the dividend growth rate in period t. We assume that ε_{c,t_1} is independent of ε_{d,t_2} and S_{t_2} for all t_1 and t_2. Furthermore, we assume that S_t follows a first-order four-state Markov chain with transition probability matrix $\mathbf{Q}$, i.e.

$$\{\Pr(S_t = j | S_{t-1} = i)\} = \{q_{ij}\} = \mathbf{Q} \quad \text{for } i, j = 1, \ldots, 4. \tag{4}$$

We also assume that $\mu_{d,1} > \mu_{d,2} > \mu_{d,3} > \mu_{d,4}$. Hence, $S_t = 1$ and $S_t = 2$ are referred to as really and normal good times, respectively, while $S_t = 3$ and $S_t = 4$ are referred to as normal and really bad times, respectively. We assume that the investor can distinguish between good ($S_t \in \{1, 2\}$) and bad ($S_t \in \{3, 4\}$) times but cannot distinguish between really good (bad) and normal good (bad) times and, thus, never observes the true state of the dividend growth process. However, he learns about it through not only dividend realizations but also imperfect external signals, x_m,

$$x_m = \mu_{x,S_t} + \sigma^+_{x,S_t}\varepsilon_{x,m}1_{\{\varepsilon_{x,m}\geq 0\}} + \sigma^-_{x,S_t}\varepsilon_{x,m}1_{\{\varepsilon_{x,m}<0\}}, \quad m = 1, 2, \ldots \tag{5}$$

where $1_{\{\cdot\}}$ is the indicator function that takes the value of one if the condition in brackets is satisfied and zero otherwise. $\varepsilon_{x,m}$ is an independently and identically distributed Gaussian random variable with zero mean and unit variance and is independent of $\varepsilon_{c,t}$ and $\varepsilon_{d,t}$ for all t and m. We refer to μ_{x,S_t} as the mean parameter and σ^+_{x,S_t} and σ^-_{x,S_t} as positive and negative variance parameters, respectively.[2] Equation (5) suggests that the external signal might have different precisions in each state depending on whether the shock, $\varepsilon_{x,m}$, is positive or negative. Finally, we assume that the investor observes dividend realizations, tracked by t, every period and the dividend growth process can switch to a new state every period. The external signals, tracked by m, are observed only on announcement periods, T^x_m, that are not necessarily regularly spaced.

2.1. Investor's Beliefs

In models like ours with learning, the investor's beliefs about the state of the dividend growth process play a central role. In this section, we characterize how the investor's beliefs evolve over time as new information about the state of the dividend growth process arrives.

Let $\tilde{\pi}_{j,t}$ denote the probability that the investor assigns to state j before observing the information revealed in period t (the dividend realization and possibly the external signal if t is an announcement period), i.e. his prior belief about state j. Similarly, let $\pi_{j,t}$ denote the probability that he assigns to state j after observing the information revealed in period t. The investor's information set in period t, $\mathcal{F}_t$, includes past and current dividend realizations, past (and current if t is an announcement period) external signals and whether $S_t \in \{1, 2\}$ or $S_t \in \{3, 4\}$. Assuming that the investor has a given set of beliefs about the initial state of the dividend growth process before observing any dividend realizations or external signals, i.e. $\tilde{\pi}_{j,0}$ for $j = 1, \ldots, 4$, the following lemma characterizes the investor's beliefs about the state variable:

[2] The mean and the variance of the external signal process depend on all three parameters of the external signal process, i.e. μ_{x,S_t}, σ^+_{x,S_t} and σ^-_{x,S_t}, as shown in the appendix. Hence, we cannot refer to these parameters as the mean or the variance of the external signal.

LEMMA 1.

$$\pi_{j,t} = \begin{cases} \dfrac{\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_d})\tilde{\pi}_{j,t}}{\sum_{i=1}^4 \phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_d})\tilde{\pi}_{i,t}} & \textit{if } t \neq T_m^x, \\ \dfrac{\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_d})\left(\frac{1}{\sigma_{x,j}^+}\phi(\frac{x_m - \mu_{x,j}}{\sigma_{x,j}^+})1_{\{x_m \geq \mu_{x,j}\}} + \frac{1}{\sigma_{x,j}^-}\phi(\frac{x_m - \mu_{x,j}}{\sigma_{x,j}^-})1_{\{x_m < \mu_{x,j}\}}\right)\tilde{\pi}_{j,t}}{\sum_{i=1}^4 \phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_d})\left(\frac{1}{\sigma_{x,i}^+}\phi(\frac{x_m - \mu_{x,i}}{\sigma_{x,i}^+})1_{\{x_m \geq \mu_{x,i}\}} + \frac{1}{\sigma_{x,i}^-}\phi(\frac{x_m - \mu_{x,i}}{\sigma_{x,i}^-})1_{\{x_m < \mu_{x,i}\}}\right)\tilde{\pi}_{i,t}} & \textit{if } t = T_m^x, \end{cases} \tag{6}$$

for $j = 1, \ldots, 4$ *and* $m = 1, 2, \ldots$ *where*

$$\tilde{\pi}_{j,t} = \begin{cases} \dfrac{\sum_{i=1}^4 \pi_{i,t-1} q_{ij}}{\sum_{i=1}^4 \pi_{i,t-1} q_{i1} + \sum_{i=1}^4 \pi_{i,t-1} q_{i2}} 1_{\{j=1,2\}} & \textit{if } S_t \in \{1,2\}, \\ \dfrac{\sum_{i=1}^4 \pi_{i,t-1} q_{ij}}{\sum_{i=1}^4 \pi_{i,t-1} q_{i3} + \sum_{i=1}^4 \pi_{i,t-1} q_{i4}} 1_{\{j=3,4\}} & \textit{if } S_t \in \{3,4\}, \end{cases} \tag{7}$$

and $\phi(\cdot)$ *is the standard normal density function.*

Prior to observing the information revealed in a given period t, the investor knows that the dividend growth process might have switched to a new state according to the transition probability matrix. He also observes whether $S_t \in \{1,2\}$ or $S_t \in \{3,4\}$. Hence, his prior beliefs about the new state variable, $\tilde{\pi}_{j,t}$, are weighted averages of his beliefs about the previous state variable, $\pi_{i,t-1}$, where the weights are the transition probabilities, q_{ij}, normalized by the sum of his prior beliefs about states 1 and 2 or 3 and 4. Given his prior beliefs for the state variable S_t, the investor then updates his beliefs according to Bayes' rule based on the additional information revealed by the dividend realization in period t (the first case of Equation (6)) as well as the external signal if t is an announcement period (the second case of Equation (6)). Note that the investor assigns zero prior probability to states 3 and 4 if he observes $S_t \in \{1,2\}$ and to states 1 and 2 if he observes $S_t \in \{3,4\}$. This in turn implies that he assigns zero probability to states 3 and 4 if he observes $S_t \in \{1,2\}$ and to states 1 and 2 if he observes $S_t \in \{3,4\}$, even after observing the information revealed in period t.

2.2. Asset Prices and Returns

We now characterize the price and the unexpected return of the risky asset in the following proposition:

PROPOSITION 1. *The price of the risky asset in period t is given by:*

$$\frac{P_t}{D_t} = \lambda_1 \pi_{1,t} + \lambda_2 \pi_{2,t} + \lambda_3 \pi_{3,t} + \lambda_4 \pi_{4,t}, \tag{8}$$

where λ_j *is the price-dividend ratio in state* j *given by*

$$\boldsymbol{\lambda} \equiv [\lambda_1, \ldots, \lambda_4]' = (\mathbf{I} - \mathbf{QH})^{-1}\mathbf{QG}, \tag{9}$$

and $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4 > 0$. $\mathbf{G}$ *and* $\mathbf{H}$ *are a* 4×1 *vector and a* 4×4 *diagonal matrix whose* j^{th} *element and* jj^{th} *diagonal elements are given by* $\alpha \exp(-\gamma\mu_c + \gamma^2\sigma_c^2/2)\exp(\mu_{d,j} + \sigma_d^2/2)$ *for* $j = 1, \ldots, 4$, *respectively, where* $\alpha = \beta \exp((\gamma - \psi^{-1})\mu_c + (1-\gamma)(\gamma - \psi^{-1})\sigma_c^2/2)$.

Let r_t denote the log return on the risky asset in period t, i.e. $r_t = \log(\frac{P_t+D_t}{P_{t-1}})$, then the unexpected log return on the risky asset in period t can be approximated by:

$$r_t^* = r_t - \tilde{E}_t[r_t] \approx \frac{1}{1+\overline{\lambda}} \sum_{j=1}^{4} \lambda_j(\pi_{j,t} - \tilde{\pi}_{j,t}) + \Delta d_t - \sum_{j=1}^{4} \mu_{d,j}\tilde{\pi}_{j,t}, \tag{10}$$

where $\tilde{E}_t[\cdot]$ denotes the expectation conditional on the investor's prior beliefs before observing the dividend realization (and possibly the external signal) in period t but after observing whether $S_t \in \{1,2\}$ or $S_t \in \{3,4\}$. The long term average price-dividend ratio is $\overline{\lambda} = E[P_t/D_t] = \sum_{j=1}^{4} \Omega_j\lambda_j$ where $[\Omega_1, \ldots, \Omega_4]'$ is the stationary distribution vector of the transition probability matrix $\mathbf{Q}$.

If the investor observes the true state variable, the price-dividend ratio takes one of the four values, λ_1, λ_2, λ_3 or λ_4, depending on the state variable. Note that the price-dividend ratio is always higher in states with higher growth rate independent of the risk aversion coefficient and elasticity of intertemporal substitution. We assume that the investor observes whether $S_t \in \{1,2\}$ or $S_t \in \{3,4\}$ but does not observe the true state variable. Hence, the price-dividend ratio is a weighted average of λ_1 and λ_2 if $S_t \in \{1,2\}$ or λ_3 and λ_4 if $S_t \in \{3,4\}$ where the weights are the investor's beliefs about the state variable. Furthermore, the unexpected return on the risky asset is also determined by the unexpected dividend growth rate as well as the time variation in the investor's beliefs which, in turn, depend on the additional information revealed by the dividend realization and the external signal if it is an announcement period. These two sources of information affect unexpected returns differently. Dividend realizations affect unexpected returns through their direct effect as well as their indirect effect through the investor's beliefs whereas external signals affect unexpected returns only through their effect on the investor's beliefs. Thus, we distinguish between additional information, i.e. news, revealed by dividend realizations and external signals. The news from the dividend realization in period t, $u_{d,t}$, is defined as the unexpected part of the realized dividend growth rate:

$$u_{d,t} = \Delta d_t - \tilde{\mu}_{d,t} \tag{11}$$

where $\tilde{\mu}_{d,t} = \sum_{j=1}^{4} \mu_{d,j}\tilde{\pi}_{j,t}$ is the expected dividend growth rate based on the investor's beliefs prior to observing the information in period t. Similarly, news from the external signal observed on the m^{th} announcement period, u_{x,T_m^x}, is defined as the unexpected part of the external signal:

$$u_{x,T_m^x} = x_m - \tilde{\mu}_{x,T_m^x} \tag{12}$$

where $\tilde{\mu}_{x,T_m^x} = \sum_{j=1}^{4}(\mu_{x,j} + (\sigma_{x,j}^+ - \sigma_{x,j}^-)/\sqrt{2\pi})\tilde{\pi}_{j,T_m^x}$ is the expected part of the external signal based on the investor's beliefs prior to observing the information in period T_m^x. In this paper, we are interested in the effect of news from external signals on unexpected returns. Thus, for the rest of the paper, the news variable refers to news from external signals and the return refers to unexpected returns, unless otherwise stated. We refer to news from dividend realizations as the dividend news variable.

3. The Reaction of Stock Returns to External Signals

In this section, we analyze the reaction of returns to news variables while distinguishing between positive and negative news variables in different states. We do this first in a simulation exercise. Specifically, we calibrate the parameters of our model to US data and discuss the motivation behind our assumptions on these parameters. We simulate our model based on this calibration and analyze the reaction of returns to news variables in a linear regression framework based on simulated data. Finally, we theoretically analyze the properties of returns as a function the news variable and discuss the intuition behind the mechanism that is driving our results.

3.1. Calibration

The effect of news variables on returns in our model depends closely on the relation between the precision of the external signal in different states ($1/\sigma^{+}_{x,j}$ and $1/\sigma^{-}_{x,j}$). Hence, in this section, we first present and discuss the motivation behind our assumptions on the precision of the external signal in different states. We then present the calibration of other model parameters.

We assume that the external signal is more precise in really good times than in normal good times when the shock to the external signal is nonnegative, i.e. $\sigma^{+}_{x,1} < \sigma^{+}_{x,2}$, and the opposite holds when the shock to the external signal is negative, i.e. $\sigma^{-}_{x,1} > \sigma^{-}_{x,2}$. As we discuss below, the first part of this assumption is crucial in generating the negative reaction of stock returns to good news in good times while the second part is crucial in generating the positive reaction of stock returns to bad news in good times. Second, we assume that the external signal in really bad times is more precise than or as precise as the external signal in normal bad times when the shock to the external signal is nonnegative, i.e. $\sigma^{+}_{x,3} \geq \sigma^{+}_{x,4}$, and the opposite holds when the shock to the external signal is negative, $\sigma^{-}_{x,3} \leq \sigma^{-}_{x,4}$. Once again, as we discuss below, the first part of this assumption is crucial in generating the positive reaction of stock returns to good news in bad times while the second part is crucial in generating the negative reaction of stock returns to bad news in bad times. Finally, we assume that the mean parameter of the external signal in really good times is higher than that in normal good times, i.e $\mu_{x,1} > \mu_{x,2}$, and the mean parameter of the external signal in normal bad times is higher than that in really bad times, i.e $\mu_{x,3} > \mu_{x,4}$.

There is empirical evidence that the information quality and the precision of signals vary over different cycles in the economy. For example, van Nieuwerburgh and Veldkamp (2006) find that the median error and the dispersion of forecasts for future real GDP increase in recessions. Veronesi (1999) and Patton and Timmermann (2010) also provide some evidence that the dispersions of forecasts are higher during recessions. Similarly, Swanson and van Dijk (2006) find that there is a clear increase in the volatility of revisions to macroeconomic variables during recessions. Croushore (2011) also finds that there are some differences in the sizes of data revisions depending on the state of the business cycle. Dynan and Elmendorf (2001) find that revisions to real GDP growth are lower around business cycle peaks and higher around business cycle troughs.

These studies, however, do not distinguish between the precision of positive and negative shocks to external signals in different states. To this end, we provide some preliminary empirical evidence that the precision of the external signal might indeed be different depending on the sign of its shock and the underlying state. To do this, we first estimate the four-state Markov regime switching process in Equation (3) using quarterly log-growth rate of real dividends of the S&P 500 Index between 1980 and 2011.[3] We then identify the four states based on a two-step approach using the smoothed probabilities. First, we distinguish between good (States 1 and 2) and bad times (States 3 and 4) based on the sum of the smoothed probabilities of states 1 and 2, or equivalently that of states 3 and 4. Specifically, we classify each quarter as good times if the sum of the smoothed probabilities of states 1 and 2 is greater than 0.5 and as bad times otherwise. Second, for a given quarter in good times, if the smoothed probability of state 1 is greater than that of state 2, we classify this quarter as really good times and otherwise it is classified as normal good times. Similarly, for a given quarter in bad times, if the smoothed probability of state 3 is greater than that of state 4, we classify this quarter as normal bad times and otherwise it is classified as really bad times. This two step classification approach is consistent with our assumption that the investor observes whether the dividend growth process is in good or bad times. Finally, we assume that the state of the dividend growth process does not change in a given quarter and is the same for all months in that quarter.

Having identified states of the dividend growth process in each month, we then estimate the parameters of the model for the external signal in Equation (5). Specifically, we use the log growth rate of monthly industrial production index between 1980 and 2011 obtained from the Federal Reserve Bank of St. Louis as our proxy for an external signal.[4] We estimate the parameters of the model via the method of moments and obtain their standard errors based on bootstrapping. The details of the estimation and bootstrapping approaches are given in the appendix.

Panel A of Table 1 presents results for good times. We reject the hypothesis that $\sigma_x^+ = \sigma_x^-$ in really good times but fail to do so in normal good times. This suggests that the variance parameter when the shock to the external signal is positive is significantly different than that when the shock to the external signal is negative only in really good times but not in normal good times. Hence, we re-estimate the variance of the external signal in normal good times without distinguishing between the variance parameters of positive and negative shocks in this state. These estimates are presented in Panel D of Table 2. We then test whether

[3] We obtain quarterly nominal dividends of the S&P 500 Index from Amit Goyal's website at HEC Lausanne. We calculate real dividends as nominal dividends divided by quarterly CPI obtained as quarterly averages of monthly CPI from the Federal Reserve Bank of St. Louis. We then estimate a four-state Markov regime switching model for the log growth rate of quarterly S&P 500 real dividends.

[4] We use revised rather than originally released data since we do not model the possibility of future revisions to a realization of the external signal. In other words, we assume that the investor observes the true realization of the external signal without any noise. Nevertheless, we also considered using originally released instead of revised data. Empirical evidence based on originally released data also provides some support for our assumptions on the precision of the external signal in different states and our results do not change significantly.

$\sigma^+_{x,1} = \sigma_{x,2}$ and $\sigma^-_{x,1} = \sigma_{x,2}$ and reject both of these hypotheses in favor of our assumptions that $\sigma^+_{x,1} < \sigma^+_{x,2}$ and $\sigma^-_{x,1} > \sigma^-_{x,2}$.[5]

Panel B of Table 1 presents results for bad times. We fail to reject the hypothesis that $\sigma^+_x = \sigma^-_x$ both in normal and really bad times suggesting that we might not need to distinguish between σ^+_x and σ^-_x in bad times. Hence, we re-estimate the variance of the external signal in normal and really bad times without distinguishing between positive and negative shocks. We then test whether $\sigma_{x,3} = \sigma_{x,4}$ and fail to reject it. In other words, we do not find any statistically significant evidence against our assumption that $\sigma^+_{x,3} \geq \sigma^+_{x,4}$, and $\sigma^-_{x,3} \leq \sigma^-_{x,4}$. This also suggests that we do not even need to distinguish between the precision of the external signal in normal and really bad times. Hence, we estimate the precision of the external signal without distinguishing between normal and really bad times. These estimates are presented in Panel D of Table 2.

Before proceeding to the calibration of other model parameters, several remarks are in order concerning our assumptions on the precision of the external signal and the empirical evidence provided above in their support. First of all, we do not claim the empirical evidence to be conclusive. Instead, we believe that it is suggestive and provides some support for our assumptions. Secondly, we do not claim our assumptions to be either the only or the most likely mechanism that can account for the observed empirical patterns in the reaction of stock returns to news. We argue that it is one of the possible mechanisms and the empirical evidence above suggests that it is not inconsistent with the data.

We now turn our attention to the calibration of other model parameters. Although still important, these parameters do not significantly affect our results in Section 3.2. In other words, we obtain coefficient estimates with the same sign but different magnitudes for a wide range of values for these parameters. We calibrate the parameters of the dividend growth process to the corresponding daily values of the parameter estimates from the four-state Markov regime switching model for the log growth rate of quarterly S&P 500 real dividends 1980 and 2011 as discussed above. We calibrate the parameters of the consumption process to the corresponding daily values of the mean and standard deviation of quarterly log real personal consumption expenditure growth rates between 1980 and 2011. We assume that the external signal is observed every 21 periods, corresponding to monthly announcements under the assumption that there are 21 trading days in a month.[6] Among others, Campbell and Viceira (1999), Barsky et al. (1997), Chen et al. (2012) and Bansal et al. (2012) provide empirical estimates of risk aversion and elasticity of intertemporal substitution coefficients. In this paper, following Bansal and Yaron (2004) among others, we assume that the elasticity of substitution is 1.5, the risk aversion coefficient is 7.5, and the daily time impatience parameter is 0.9998

[5] We continue to reject these hypotheses in favor of our assumptions even when we distinguish between the precision of positive and negative shocks to the external signal in normal good times.

[6] Assuming that there are 63 trading days in a quarter, we use the following transformations to convert parameter estimates based on quarterly data to their corresponding daily values $\mu_c^{\text{daily}} = \mu_c^{\text{quarterly}}/63$, $\sigma_c^{\text{daily}} = \sigma_c^{\text{quarterly}}/\sqrt{63}$, $\mu_{d,i}^{\text{daily}} = \mu_{d,i}^{\text{quarterly}}/63$ for $i = 1, \ldots, 4$, $\sigma_d^{\text{daily}} = \sigma_d^{\text{quarterly}}/\sqrt{63}$ and $\mathbf{Q}^{\text{daily}} = (\mathbf{Q}^{\text{quarterly}})^{1/63}$.

corresponding to an annual value of 0.9508. As shown in Proposition 1, the price dividend ratio is always higher in states with higher dividend growth rates. Hence, whether the risk aversion and the elasticity of substitution are greater or less than one does not significantly affect our simulation results and does not change our theoretical results. The calibrated parameter values are summarized in Table 2.

3.2. Simulation Results

In this section, we analyze the effect of news variables on returns in our model in a linear regression framework. To this end, we first simulate data from our model based on the calibrated parameters. Specifically, we simulate 100 samples with 21,000 observations corresponding to 1,000 announcement periods.[7] For each sample, we only consider data on announcement periods while distinguishing between good and bad times. We consider the two following linear regression models separately and estimate them via OLS with heteroskedasticity consistent standard errors (White (1980)):

$$\begin{aligned} r^*_{T^x_m} &= \theta_1 + \theta_2 \tilde{\pi}_{i,T^x_m} + \theta_3 u_{d,T^x_m} + \theta_4 1_{\{u_{x,T^x_m}>0\}} + \theta_5 u_{x,T^x_m} 1_{\{u_{x,T^x_m}>0\}} + \theta_6 u_{x,T^x_m} 1_{\{u_{x,T^x_m}\leq 0\}} + \nu_{T^x_m} \\ r^*_{T^x_m} &= \theta_1 + \theta_2 \tilde{\pi}_{i,T^x_m} + \theta_3 u_{d,T^x_m} + \theta_4 1_{\{u_{x,T^x_m}>0\}} + \theta_5 u_{x,T^x_m} + \nu_{T^x_m} \end{aligned}$$

for $m = 1, 2, \ldots, 1000$ and for $i = 1$ or 3 where $i = 1$ corresponds to good times and $i = 3$ corresponds to bad times. To remove the effect of random sampling on our empirical results, the same random shocks for $\varepsilon_{d,t}$ and $\varepsilon_{x,m}$ are used for corresponding samples in different panels of Table 3 and 4.

Table 3 presents summary statistics for the coefficient estimates in good times. Our results for the reaction of stock returns to news in good times can be summarized as follows: The coefficient estimates on $u_{x,T^x_m} 1_{\{u_{x,T^x_m}>0\}}$ are always significantly negative and never significantly positive. The coefficient estimates on $u_{x,T^x_m} 1_{\{u_{x,T^x_m}\leq 0\}}$ are also always significantly negative and never significantly positive. Similarly, the coefficient estimate on u_{x,T^x_m} presented in Panel B, where we do not distinguish between positive and negative news, is always negative. These results suggest that, in good times, positive news variables generally have a negative effect on returns while negative news variables generally have a positive one.

Table 4 presents summary statistics for the coefficient estimates in bad times. Our results for the reaction of stock returns to news in bad times can be summarized as follows: The coefficient estimates on $u_{x,T^x_m} 1_{\{u_{x,T^x_m}>0\}}$ are generally positive and never significantly negative. The coefficient estimates on $u_{x,T^x_m} 1_{\{u_{x,T^x_m}\leq 0\}}$ are also generally positive and never significantly negative. Similarly, the coefficient estimate on u_{x,T^x_m} presented in Panel B, where we do not distinguish between positive and negative news, is generally positive and never significantly negative. These results suggest that, in bad times, positive news variables generally have a positive effect on returns while negative news variables generally have a negative one.

[7] For each sample, we simulate 22,600 observations. We remove the first 1,260 periods from each sample to avoid any bias due to the initial state.

Several remarks are in order concerning our results. As we discuss below, the shape of the return as a function of the news variables is mainly determined by the precision of the external signal and does not significantly change with other parameters of our model. On the other hand, the significance of the coefficient estimates in a linear framework depends closely on some other parameters. For example, the transition probability matrix of the dividend process determines the number of good and bad times in a simulation exercise. Given that good times are relatively more persistent than bad times in our calibration of the transition probability matrix, there are not many observations in bad times compared to good times in our simulation exercise. This is reflected in the significance of the coefficient estimates in bad times. Similarly, the difference between the mean parameters of the external signal process in relation to its variance parameters also plays a role in the significance of the coefficient estimates. Generally speaking, the effect of news variables becomes clearer as the difference between the mean parameters of the external signal process in different states becomes larger in magnitude in relation to its variance parameters.

Our results based on simulated data are closest to the empirical evidence in McQueen and Roley (1993). They are among the first to analyze the reaction of stock returns to macroeconomic news over different phases of the business cycle. Specifically, they distinguish between good, normal and bad states of the economy based on the growth rate of industrial production and analyze the reaction of daily returns on the S&P 500 Index to macroeconomic news over different states. For some macroeconomic variables including the industrial production, they find that the daily returns on the S&P 500 Index react negatively to good news, i.e. higher than expected real economic activity, in good times. Andersen et al. (2007) provide supporting evidence for this stylized fact from international equity markets, with good news having a negative impact on international stock returns during periods of expansions in the U.S. economy. Boyd et al. (2005) analyze the reaction of stock returns to employment numbers and find that the daily returns on the S&P 500 Index react negatively to better than expected employment numbers in expansions. Cenesizoglu (2011) also finds that better than expected employment numbers is bad news for large and growth stocks in expansions but not in recessions. Gilbert (2011) provides further supporting evidence for this stylized fact based on the relation between stock returns and future revisions to macroeconomic variables.

3.3. Theoretical Results

In this section, we discuss the intuition behind our results in Section 3.2. To this end, we analyze how the return changes as a function of the news variable in good and bad times while controlling for (or keeping constant) the effect of dividend news variables. We analyze the reaction of the return to news variables in good and bad times separately. We can do this due to our assumption that the investor observes whether $S_t \in \{1, 2\}$ or $S_t \in \{3, 4\}$.

We start with good times before turning our attention to bad times. Consider the return on an announcement period T^x_m for a given investor's prior belief about really good times. It can be expressed as a function of the two news variables: the dividend news variable (u_{d,T_m^x}) and the news variable of interest (u_{x,T_m^x}). Panel A of Figure 1 presents $r^*_{T_m^x}$ as a function of u_{x,T_m^x} for three different values of u_{d,T_m^x} and $\tilde{\pi}_{1,T_m^x}$. All the intuition behind our results in Section 3.2 can be readily explained by three observations that emerge from Panel A of Figure 1.

The first observation concerns the right tail of the distribution of the news variable that corresponds to large (in magnitude) positive news variables. Panel A of Figure 1 shows that the return is a decreasing function of the news variable for large positive news variables. In other words, the return decreases (increases) as the investor receives better (worse) news variables that are large in magnitude.

The second observation concerns the left tail of the distribution of the news variable that corresponds to large (in magnitude) negative news variables. Panel A of Figure 1 shows that the return is a decreasing function of the news variable for large negative news variables. In other words, the return decreases (increases) as the investor receives better (worse) news variables that are large in magnitude.

The third and last observation concerns the center of the distribution of the news variable. For news variables between these two extremes of large positive and negative news variables, the return can be an increasing or a decreasing function of the news variable depending on the magnitude of the news variable and the investor's beliefs about the state of the dividend growth process. Specifically, as the investor receives better news variables, the return first decreases and then starts to increase. This change happens when the new variable is greater than a threshold which depends on the investor's beliefs. There is also a kink in the return as a function of the news variable when the external signal is exactly equal to its mean in really good times, i.e. $x_m = \mu_{x,1}$ or equivalently $u_{x,T_m^x} = \mu_{x,1} - \tilde{\mu}_{x,T_m^x}$. This happens due to the fact that the precision of the external signal in really good times is different depending on whether the shock to the external signal is positive or negative, i.e. $\sigma^+_{x,1} \neq \sigma^-_{x,1}$. As the investor continues to receive better news variables greater than $\mu_{x,1} - \tilde{\mu}_{x,T_m^x}$, the return continues to increase up to another threshold, which also depends on the investor's beliefs, and then starts to decrease.

Finally, Panel A of Figure 1 also shows that these three observations about the shape of the return as a function of the news variable hold independent of the dividend news variable observed in the same period. The following proposition formalizes these observations.

PROPOSITION 2. *Consider an announcement period T_m^x in good times and a given investor's belief about really good times $\tilde{\pi}_{1,T_m^x}$ prior to observing the information revealed in period T_m^x.*
(a) If $\sigma^+_{x,1} < \sigma^+_{x,2}$, then $\partial\pi_{1,T_m^x}/\partial u_{x,T_m^x}$ and $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ are negative for all news variables greater than $\delta^+_x(\tilde{\pi}_{1,T_m^x})$ where $\delta^+_x(\tilde{\pi}_{1,T_m^x}) = \frac{\mu_{x,1}(\sigma^+_{x,2})^2 - \mu_{x,2}(\sigma^+_{x,1})^2}{(\sigma^+_{x,2})^2 - (\sigma^+_{x,1})^2} - \tilde{\mu}_{x,T_m^x} > \mu_{x,1} - \tilde{\mu}_{x,T_m^x}$.
(b) If $\sigma^-_{x,1} > \sigma^-_{x,2}$, then $\partial\pi_{1,T_m^x}/\partial u_{x,T_m^x}$ and $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ are negative for all news variables smaller than $\delta^-_x(\tilde{\pi}_{1,T_m^x})$ where $\delta^-_x(\tilde{\pi}_{1,T_m^x}) = \frac{\mu_{x,1}(\sigma^-_{x,2})^2 - \mu_{x,2}(\sigma^-_{x,1})^2}{(\sigma^-_{x,2})^2 - (\sigma^-_{x,1})^2} - \tilde{\mu}_{x,T_m^x} < \mu_{x,2} - \tilde{\mu}_{x,T_m^x}$.
(c) $\partial\pi_{1,T_m^x}/\partial u_{x,T_m^x}$ and $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ are positive for all news variables between $\delta^-_x(\tilde{\pi}_{1,T_m^x})$ and $\delta^+_x(\tilde{\pi}_{1,T_m^x})$.

Parts (a) and (b) of Proposition 2 show that the investor's belief about really good times and, thus, the return are decreasing functions of the news variable for news variables greater than $\delta_x^+(\tilde{\pi}_{1,T_m^x})$ or smaller than $\delta_x^-(\tilde{\pi}_{1,T_m^x})$, respectively. To put it differently, the probability of really good times and the return decrease (increase) as the investor receives better (worse) news variables that are greater than $\delta_x^+(\tilde{\pi}_{1,T_m^x})$ or smaller than $\delta_x^-(\tilde{\pi}_{1,T_m^x})$. On the other hand, Part (c) of Proposition 2 shows that the investor's belief about really good times and, thus, the return are increasing functions of the news variable for any news variable between $\delta_x^-(\tilde{\pi}_{1,T_m^x})$ and $\delta_x^+(\tilde{\pi}_{1,T_m^x})$. In other words, the probability of really good times and the return increase (decrease) as the investor receives better (worse) news that are between $\delta_x^-(\tilde{\pi}_{1,T_m^x})$ and $\delta_x^+(\tilde{\pi}_{1,T_m^x})$. Finally, Proposition 2 shows that these hold true independent of the dividend news variable observed in the same period.

The intuition behind Proposition 2 follows from how the investor updates his beliefs following a news variable. Specifically, whether the investor's belief about really good times and the return are increasing or decreasing functions of the news variable depends on the probability ratio of observing the same news variable in really and normal good times, which, in turn, depends on the precision of the external signal in these two states.

First, consider Part (a) of Proposition 2. To understand the intuition behind this result, note first that the probability of observing a news variable greater than $\mu_{x,1} - \tilde{\mu}_{x,T_m^x}$ decreases in both really and normal good times as the investor receives better news variables. However, the assumption in Part (a), i.e. the external signal is more precise in really good times compared to normal good times when the shock to the external signal is positive, implies that the distribution of the external signal has a fatter right tail in normal good times compared to really good times. This in turn implies that the probability ratio of observing the same news variable in really and normal good times would eventually start to decrease as the investor observes better news variables. This happens at a news variable of magnitude $\delta_x^+(\tilde{\pi}_{1,T_m^x})$. Since the probability that the investor assigns to really good times is an increasing function of this ratio, he decreases the probability of really good times as he observes better news variables that are greater than $\delta_x^+(\tilde{\pi}_{1,T_m^x})$. This in turn implies that the return, an increasing function of the probability that the investor assigns to really good times, also decreases as he observes better news variable that are greater than $\delta_x^+(\tilde{\pi}_{1,T_m^x})$.

Now, consider Part (b) of Proposition 2. The intuition behind this result is similar to that behind Part (a) and can be better understood by considering how the investor updates his belief about really good times as he observes worse, rather than better, news variables. First note that the probability of observing a news variable smaller than $\mu_{x,2} - \tilde{\mu}_{x,T_m^x}$ decreases in both really and normal good times as the investor receives worse news variables. However, the assumption in Part (b), i.e. the external signal is more precise in normal good times compared to really good times when the shock to the external signal is negative, implies that the distribution of the external signal has a fatter left tail in really good times compared to normal good times. This in turn implies that the probability ratio of observing the same news variable in really and normal good times would eventually start to increase as the investor observes worse news variables. This happens at a news variable of magnitude $\delta_x^-(\tilde{\pi}_{1,T_m^x})$. Similar to the intuition above, this in turn implies that the probability of really good times and, thus, the return increase as he observes worse news variables that are smaller than $\delta_x^-(\tilde{\pi}_{1,T_m^x})$.

Finally, consider Part (c) of Proposition 2. Unlike large positive and negative news variables, the reaction of the return to news variables between $\delta_x^-(\tilde{\pi}_{1,T_m^x})$ and $\delta_x^+(\tilde{\pi}_{1,T_m^x})$ is not determined by the relation between the tail behaviors of the external signal in really and normal good times, but rather by the relation between its mean parameters in these states. Since the external signal has a higher mean parameter in really good times than in normal good times, the probability ratio of observing the same news variable between $\delta_x^-(\tilde{\pi}_{1,T_m^x})$ and $\delta_x^+(\tilde{\pi}_{1,T_m^x})$ in really and normal good times is an increasing function of the news variable. This in turn implies that both the probability of really good times and the return always increase as the investor observes better news variables that are between $\delta_x^-(\tilde{\pi}_{1,T_m^x})$ and $\delta_x^+(\tilde{\pi}_{1,T_m^x})$.

We now turn our attention to the reaction of the return to news variables in bad times. Similar to Panel A, Panel B of Figure 1 presents $r^*_{T_m^x}$ as a function of u_{x,T_m^x} for three different values of u_{d,T_m^x} and $\tilde{\pi}_{3,T_m^x}$. Once again, all the intuition behind our results in Section 3.2 can be readily explained by the observation that emerges from these graphs. Specifically, Panel B of Figure 1 shows that the return is an increasing function of the news variable independent of the investor's beliefs and the dividend news variable observed in the same period. This in turn implies that the return increases (decreases) as the investor receives better (worse) news variables. This observation holds under more general assumptions on the precision of the external signal in bad times than the ones underlying Panel B of Figure 1, i.e. $\sigma^+_{x,3} = \sigma^-_{x,3} = \sigma^+_{x,4} = \sigma^-_{x,4}$. The following proposition formalizes the reaction of returns to news variable in bad times under these more general assumptions.

PROPOSITION 3. *Consider an announcement period T_m^x in bad times and a given investor's belief about the normal good times $\tilde{\pi}_{3,T_m^x}$ prior to observing the information revealed in period T_m^x.*
(a) If $\sigma^+_{x,3} \geq \sigma^+_{x,4}$, then $\partial\pi_{3,T_m^x}/\partial u_{x,T_m^x}$ and $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ are positive for all news variables greater than $\mu_{x,3} - \tilde{\mu}_{x,T_m^x}$.
(b) If $\sigma^-_{x,3} \leq \sigma^-_{x,4}$, then $\partial\pi_{3,T_m^x}/\partial u_{x,T_m^x}$ and $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ are positive for all news variables smaller than $\mu_{x,4} - \tilde{\mu}_{x,T_m^x}$.
(c) $\partial\pi_{3,T_m^x}/\partial u_{x,T_m^x}$ and $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ are positive for a news variable, u_{x,T_m^x}, such that $\mu_{x,4} - \tilde{\mu}_{x,T_m^x} < u_{x,T_m^x} < \mu_{x,3} - \tilde{\mu}_{x,T_m^x}$.

Proposition 3 shows that, if our assumptions about the precision of the external signal in bad times hold, i.e. $\sigma^+_{x,3} \geq \sigma^+_{x,4}$ and $\sigma^-_{x,3} \leq \sigma^-_{x,4}$, then the investor's belief about normal bad times and, thus, the return are both increasing functions of the news variable except for news variables u_{x,T_m^x} such that $u_{x,T_m^x} = \mu_{x,3} - \tilde{\mu}_{x,T_m^x}$ and $u_{x,T_m^x} = \mu_{x,4} - \tilde{\mu}_{x,T_m^x}$, which correspond to the points of discontinuity of the external signal distribution where the derivatives are not defined. In other words, ignoring these points of discontinuity, the probability that the investor assigns to normal bad times and the return increase (decrease) as he receives better (worse) news variables.

Once again, the intuition behind Proposition 3 follows from how the investor updates his beliefs following a news variable. First, consider Part (a) of Proposition 3. To understand the intuition behind this result, note first that the probability of observing a news variable greater than $\mu_{x,3} - \tilde{\mu}_{x,T_m^x}$ decreases in both really and normal bad times as the investor receives better news variables. However, the assumption in Part (a), i.e. the external signal is more (or equally) precise in really bad times compared to normal bad times when the shock to the external signal is positive, implies that the distribution of the external signal has a fatter right tail (or similar right tail behavior) in normal bad times compared to really bad times. This in turn implies that the probability ratio of observing the same news variable greater than $\mu_{x,3} - \tilde{\mu}_{x,T_m^x}$ in normal and really bad times always increases as the investor observes better news variables. Since the probability that the investor assigns to normal bad times is an increasing function of this ratio, he increases the probability of normal bad times as he observes better news variables that are greater than $\mu_{x,3} - \tilde{\mu}_{x,T_m^x}$. This in turn implies that the return, an increasing function of the probability that the investor assigns to normal bad times, increases as he observes better news variable that are greater than $\mu_{x,3} - \tilde{\mu}_{x,T_m^x}$.

Now, consider Part (b) of Proposition 3. The intuition behind this result is similar to that behind Part (a) and, once again, can be better understood by considering how the investor updates his beliefs about normal bad times as he observes worse, rather than better, news variables. First note that the probability of observing a news variable smaller than $\mu_{x,4} - \tilde{\mu}_{x,T_m^x}$ decreases in both really and normal bad times as the investor receives worse news variables. However, the assumption in Part (b), i.e. the external signal is more (or equally) precise in normal bad times compared to really bad times when the shock to the external signal is negative, implies that the distribution of the external signal has a fatter (or similar) left tail in really bad times compared to normal bad times. This in turn implies that the probability ratio of observing the same news variable smaller than $\mu_{x,4} - \tilde{\mu}_{x,T_m^x}$ in normal and really bad times always decreases as the investor observes worse news variables. Similar to the intuition for Part (a) of Proposition 3, the probability of normal bad times and, thus, the return decrease as he observes worse news variables that are smaller than $\mu_{x,4} - \tilde{\mu}_{x,T_m^x}$.

Finally, consider Part (c) of Proposition 3. The reaction of the return to news variables between $\mu_{x,3} - \tilde{\mu}_{x,T_m^x}$ and $\mu_{x,4} - \tilde{\mu}_{x,T_m^x}$ is not determined by the relation between the tail behaviors of the external signal in normal and really bad times, but rather by the relation between its mean parameters in these states. Since the external signal has a higher mean parameter in normal bad times than in really bad times, the probability ratio of observing the same news variable between $\mu_{x,3} - \tilde{\mu}_{x,T_m^x}$ and $\mu_{x,4} - \tilde{\mu}_{x,T_m^x}$ in normal and really bad times is an increasing function of the news variable. This in turn implies that both the probability of normal bad times and the return always increase.

We can use our theoretical results for bad times to provide further intuition for those in good times and vice versa. We can do this mainly due to the symmetry between good and bad times and our assumption that the investor observes whether $S_t \in \{1, 2\}$ or $S_t \in \{3, 4\}$. The symmetry can be easily seen by noting that $S_t = 1$ and $S_t = 2$ are, respectively, the high and low growth states in good times while $S_t = 3$ and $S_t = 4$ have similar interpretation as the high and low growth states in bad times. Similarly, as mentioned above, it is also easy to see that our assumption allows us to analyze separately the reaction of the return to the news variable in good and bad times.

Proposition 3 implies the following for the reaction of the return to the news variable in good times: (1) If the external signal is more or equally precise in normal good times (the low growth state) compared to really good times (the high growth state) when the shock to the external signal is positive, then the return would react positively to large positive news variables in good times; (2) if the external signal is more or equally precise in really good times (the high growth state) compared to normal good times (the low growth state) when the shock to the external signal is negative, then the return would react negatively to large negative news variables in good times.

Similarly, Proposition 2 implies the following for the reaction of the return to the news variable in bad times: (1) If the external signal is more precise in normal bad times (the high growth state) compared to really bad times (the low growth state) when the shock to the external signal is positive, then the return would react negatively to large positive news variables in bad times; (2) if the external signal is more precise in really bad times (the low growth state) compared to normal bad times (the high growth state) when the shock to the external signal is negative, then the return would react positively to large negative news variables in bad times.

These implications under alternative assumptions are quite important. They suggest that one can no longer account for the stylized facts in the literature under alternative assumptions on the precision of the external signal. For example, under the commonly used alternative assumption of constant external signal precision, the return reacts positively to positive news variables and negatively to negative news variables in both good and bad times.

Although we discuss in this section the implications of our model under the most general assumptions, our model can generate similar implications under some further restrictions on the precision of the external signal in different states as we have done in our calibration exercise. Several remarks are in order concerning these implications.

First, to account for the empirical facts in good times, we need to assume that the external signal is more precise in really good times than in normal good times when the shock to the external signal is nonnegative and that the opposite holds when the shock to the external signal is negative. It is easy to see that this assumption requires the precision of positive and negative shocks to the external signals to be different in either really or normal good times but not necessarily in both. In other words, as long as this assumption holds, our model can account for the empirical facts if we simply assume that the precisions of positive and negative shocks are different only in, say, really good times but are equal in normal good times. As discussed above, the first part of this assumption is crucial in generating the negative reaction of stock returns to good news in good times while the second part is crucial in generating the positive reaction of stock returns to bad news in good times. Hence, one can generate in our framework only one of these implications by simply making the corresponding assumption.

Second, our results suggest that our model can also account for the negative reaction of stock returns to good news in good times if we simply assume that the precision of the external signal is higher in really good times than normal good times without distinguishing between the precision of positive and negative shocks to the external signal. However, our model cannot account for the positive reaction of stock returns to bad news in good times under this assumption where we do not distinguish between the precision of positive and negative shocks to the external signal.

Finally, to account for the empirical facts in bad times, we need to assume that the external signal in really bad times is more precise than or equally precise as the external signal in normal bad times when the shock to the external signal is nonnegative and the opposite holds when the shock to the external signal is negative. However, our model can account for the empirical facts even if we do not distinguish between the precision of positive and negative shocks in really and normal bad times as long as the precision of the external signal in really bad times is higher than or equal to the precision of the external signal in normal bad times.

4. Conclusion

In this paper, we propose a mechanism based on sign- and state-dependent precision of external signal shocks to account for the asymmetric reaction of stock returns to news in good and bad times. Specifically, we analyze the effect of news on returns in an asset pricing model where the growth rate of dividends follows a Markov regime switching process. The investor can distinguish between good and bad times but cannot distinguish between really good (bad) and normal good (bad) times and, thus, never observes the true state of the dividend growth process. However, he learns about it through two sources of information: dividend realizations observed every period and external signals observed only on announcement days.

In this framework, we distinguish between positive and negative shocks to the external signal and assume that they might have different precisions in each state. Specifically, in good times, we assume that the external signal is more precise in really good times than in normal good times when the shock to the external signal is nonnegative and the opposite holds when the shock to the external signal is negative. Under these assumptions in good times, the distribution of the external signal has fatter right and thinner left tails in normal good times compared to really good times. Thus, the investor decreases the probability that he assigns to really good times following large (in magnitude) positive news variables and increases it following large (in magnitude) negative news variables. This in turn implies that the return reacts negatively to large (in magnitude) positive news variables and positively to large (in magnitude) negative news variables in good times. On the other hand, we assume that the external signal in really bad times is more precise than or as precise as the external signal in normal bad times when the shock to the external signal is nonnegative and the opposite holds when the shock to the external signal is negative. Under these assumptions in bad times, the distribution of the external signal has a fatter (or similar) right tail and a thinner (or similar) left tail in normal bad times compared to really bad times. Thus, the investor always increases the probability that he assigns to normal bad times as he observes better news variables and decreases it as he observes worse news variables. This in turn implies that the return reacts positively to positive news variables and negatively to negative news variables in bad times.

References

Andersen, Torben G., Tim Bollerslev, Francis X. Diebold, Clara Vega. 2007. Real-time price discovery in global stock, bond and foreign exchange markets. *Journal of International Economics* **73** 251–277.

Bansal, Ravi, Dana Kiku, Amir Yaron. 2012. Risks for the long run: Estimation with time aggregation. Wharton Working Paper.

Bansal, Ravi, Amir Yaron. 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance* **59** 1481–1509.

Barsky, Robert B., F. Thomas Juster, Miles S. Kimball, Matthew D. Shapiro. 1997. Consumption and portfolio decisions when expected returns are time varying. *Quarterly Journal of Ecomomics* **112** 537–579.

Boyd, John H., Jian Hu, Ravi Jagannathan. 2005. The stock market's reaction to unemployment news: Why bad news is usually good for stocks. *Journal of Finance* **60**(2) 649–672.

Brandt, Michael W., Kevin Q. Wang. 2003. Time-varying risk aversion and unexpected inflation. *Journal of Monetary Economics* **50** 1457–1498.

Calvet, Laurent E., Adlai J. Fisher. 2007. Multifrequency news and stock returns. *Journal of Financial Economics* **86** 178212.

Campbell, John Y., Luis M. Viceira. 1999. Consumption and portfolio decisions when expected returns are time varying. *Quarterly Journal of Ecomomics* **114** 433–495.

Cenesizoglu, Tolga. 2011. Size, book-to-market ratio and macroeconomic news. *Journal of Empirical Finance* **18** 248–270.

Chen, Xiaohong, Jack Favilukis, Sydney C. Ludvigson. 2012. An estimation of economic models with recursive preferences. LSE Working Paper.

Croushore, Dean. 2011. Frontiers of real-time data analysis. *Journal of Economic Literature* **49** 72–100.

David, Alexander. 1997. Fluctuating confidence in stock markets: Implications for returns and volatility. *Journal of Financial and Quantitative Analysis* **32** 427–462.

Debreu, Gerard, I. N. Herstein. 1953. Nonnegative square matrices. *Econometrica* **21**(4) 597–607.

Dynan, Karen E., Douglas W. Elmendorf. 2001. Do provisional estimates of output miss economic turning points. Federal Reserve Board, FEDS Working Paper No. 2001-52, November 2001.

Epstein, Larry, Stanley Zin. 1989. Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica* **57** 937:968.

Gilbert, Thomas. 2011. Information aggregation around macroeconomic announcements: Revisions matter. *Journal of Financial Economics* **101** 114–131.

Gordon, Stephen, Pascal St-Amour. 2000. A preference regime model of bear and bull markets. *American Economic Review* **90** 1019–1033.

McQueen, Grant, V. Vance Roley. 1993. Stock prices, news, and business conditions. *Review of Financial Studies* **6**(3) 683–707.

Patton, Andrew J., Allan Timmermann. 2010. Why do forecasters disagree? Lessons from the term structure of cross-sectional dispersion. *Journal of Monetary Economics* **57** 803820.

Swanson, Norman R., Dick van Dijk. 2006. Are statistical reporting agencies getting it right? Data rationality and business cycle asymmetry. *Journal of Business and Economic Statistics* **24** 24–42.

van Nieuwerburgh, Stijn, Laura Veldkamp. 2006. Learning asymmetries in real business cycles. *Journal of Monetary Economics* **53** 753772.

Veronesi, Pietro. 1999. Stock market overreaction to bad news in good times: A rational expectations equilibrium model. *Review of Financial Studies* **12**(5) 975–1007.

Veronesi, Pietro. 2000. How does information quality affect stock returns? *Journal of Finance* **55**(2) 807–837.

Weil, Philippe. 1990. Nonexpected utility in macroeconomics. *Quarterly Journal of Economics* **105** 29–42.

White, Halbert. 1980. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica* **48** 817–838.

Table 1 Precision of the External Signal

Panel A: Good Times

	Really Good Times	Normal Good Times
Obs.	105	245
μ_x	0.440%	0.263%
σ_x^+	0.181%	0.617%
σ_x^-	0.966%	0.646%
p-value	0.020	0.426

Panel B: Bad Times

	Normal Bad Times	Really Bad Times
Obs.	21	11
μ_x	0.101%	-0.241%
σ_x^+	0.288%	0.783%
σ_x^-	1.179%	1.143%
p-value	0.335	0.213

Panel C: Tests of Equality

Hypothesis	p-value
$\sigma_{x,1}^+ = \sigma_{x,2}$	0.000
$\sigma_{x,1}^- = \sigma_{x,2}$	0.087
$\sigma_{x,3} = \sigma_{x,4}$	0.221

Note: Panels A and B present the parameter estimates of the external signal process in good and bad times, respectively. Obs. is the number of observations in each state. μ_x is the mean parameter of the external signal process. σ_x^+ and σ_x^- are the variance parameters (inverse of precision) of the external signal process when the shock to the external signal is positive and negative, respectively. The rows with the heading "p-value" present the bootstrapped p-values for the null hypothesis that $\sigma_x^+ = \sigma_x^-$ in each state. The parameters are estimated based on the method of moments approach and the p-values are based on the bootstrapping approach, both described in the appendix. Panel C presents bootstrapped p-values for the hypotheses presented in the column "Hypothesis" that correspond to our assumptions on the precision of the external signal in different states.

Table 2 Calibrated Model Parameters

Panel A: Utility Specification

Parameter	Value
γ	7.5
ψ	1.5
β	0.9998

Panel B: Consumption Process

Parameter	Value
μ_c	0.011%
σ_c	0.081%

Panel C: Dividend Process

State	μ_d	σ_d	$\mathbf{Q}$			
			Really Good Times	Normal Good Times	Normal Bad Times	Really Bad Times
Really Good Times	0.047%	0.091%	0.998	0.002	0.000	0.000
Normal Good Times	0.017%	0.091%	0.001	0.998	0.001	0.000
Normal Bad Times	-0.025%	0.091%	0.000	0.010	0.986	0.004
Really Bad Times	-0.094%	0.091%	0.000	0.000	0.005	0.995

Panel D: External Signal Process

State	μ_x	σ_x^+	σ_x^-
Really Good Times	0.440%	0.181%	0.966%
Normal Good Times	0.251%	0.633%	0.633%
Normal Bad Times	-0.255%	0.864%	0.864%
Really Bad Times	-0.385%	0.864%	0.864%

Note: The table presents calibrated model parameters. β is the daily time impatience parameter, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution. μ_c and σ_c are the mean and standard deviation of daily consumption growth rate, respectively. $\mu_{d,j}$ is the mean of the daily dividend growth rate in state j for $j = 1, \ldots, 4$ and σ_d is the standard deviation of daily dividend growth rate. $\mathbf{Q}$ is the daily transition probability matrix of the dividend growth process. Similarly, $\mu_{x,j}$ is the mean parameter of the external signal in state j for $j = 1, \ldots, 4$. $\sigma_{x,j}^+$ and $\sigma_{x,j}^-$ are the variance parameters in state j for $j = 1, \ldots, 4$ when the shock to the external signal is positive and negative, respectively. $T_m^x - T_{m-1}^x$, which is not presented in the table, is the number of periods between two consecutive announcements and we calibrate it to 21 days.

Table 3 Summary Statistics for Coefficient Estimates in Good Times

Panel A: $r^*_{T^x_m} = \theta_1 + \theta_2 \tilde{\pi}_{1,T^x_m} + \theta_3 u_{d,T^x_m} + \theta_4 1_{\{u_{x,T^x_m}>0\}} + \theta_5 u_{x,T^x_m} 1_{\{u_{x,T^x_m}>0\}} + \theta_6 u_{x,T^x_m} 1_{\{u_{x,T^x_m}\leq 0\}} + \nu_{T^x_m}$

Variable	Mean	Median	Std. Dev.	Percentage of samples with a significantly negative coefficient estimate	Percentage of samples with a significantly positive coefficient estimate
Constant	-0.005	-0.005	0.001	100%	0%
$\tilde{\pi}_{1,T^x_m}$	-0.004	-0.004	0.001	98%	0%
u_{d,T^x_m}	4.000	4.021	0.474	0%	100%
$1_{\{u_{x,T^x_m}>0\}}$	0.019	0.019	0.002	0%	100%
$u_{x,T^x_m} 1_{\{u_{x,T^x_m}>0\}}$	-2.295	-2.283	0.338	100%	0%
$u_{x,T^x_m} 1_{\{u_{x,T^x_m}\leq 0\}}$	-0.828	-0.801	0.150	100%	0%

Panel B: $r^*_{T^x_m} = \theta_1 + \theta_2 \tilde{\pi}_{1,T^x_m} + \theta_3 u_{d,T^x_m} + \theta_4 1_{\{u_{x,T^x_m}>0\}} + \theta_5 u_{x,T^x_m} + \nu_{T^x_m}$

Variable	Mean	Median	Std. Dev.	Percentage of samples with a significantly negative coefficient estimate	Percentage of samples with a significantly positive coefficient estimate
Constant	-0.008	-0.008	0.001	100%	0%
$\tilde{\pi}_{1,T^x_m}$	-0.004	-0.003	0.001	98%	0%
u_{d,T^x_m}	4.016	4.048	0.493	0%	100%
$1_{\{u_{x,T^x_m}>0\}}$	0.018	0.018	0.002	0%	100%
u_{x,T^x_m}	-1.413	-1.419	0.162	100%	0%

Note: The table presents summary statistics for the coefficient estimates of the linear regression models in good times over 100 simulated samples. The significance level is 5% for one-sided tests in the last two columns.

Table 4 Summary Statistics for Coefficient Estimates in Bad Times

Panel A: $r^*_{T^x_m} = \theta_1 + \theta_2 \tilde{\pi}_{3,T^x_m} + \theta_3 u_{d,T^x_m} + \theta_4 1_{\{u_{x,T^x_m}>0\}} + \theta_5 u_{x,T^x_m} 1_{\{u_{x,T^x_m}>0\}} + \theta_6 u_{x,T^x_m} 1_{\{u_{x,T^x_m}\leq 0\}} + \nu_{T^x_m}$

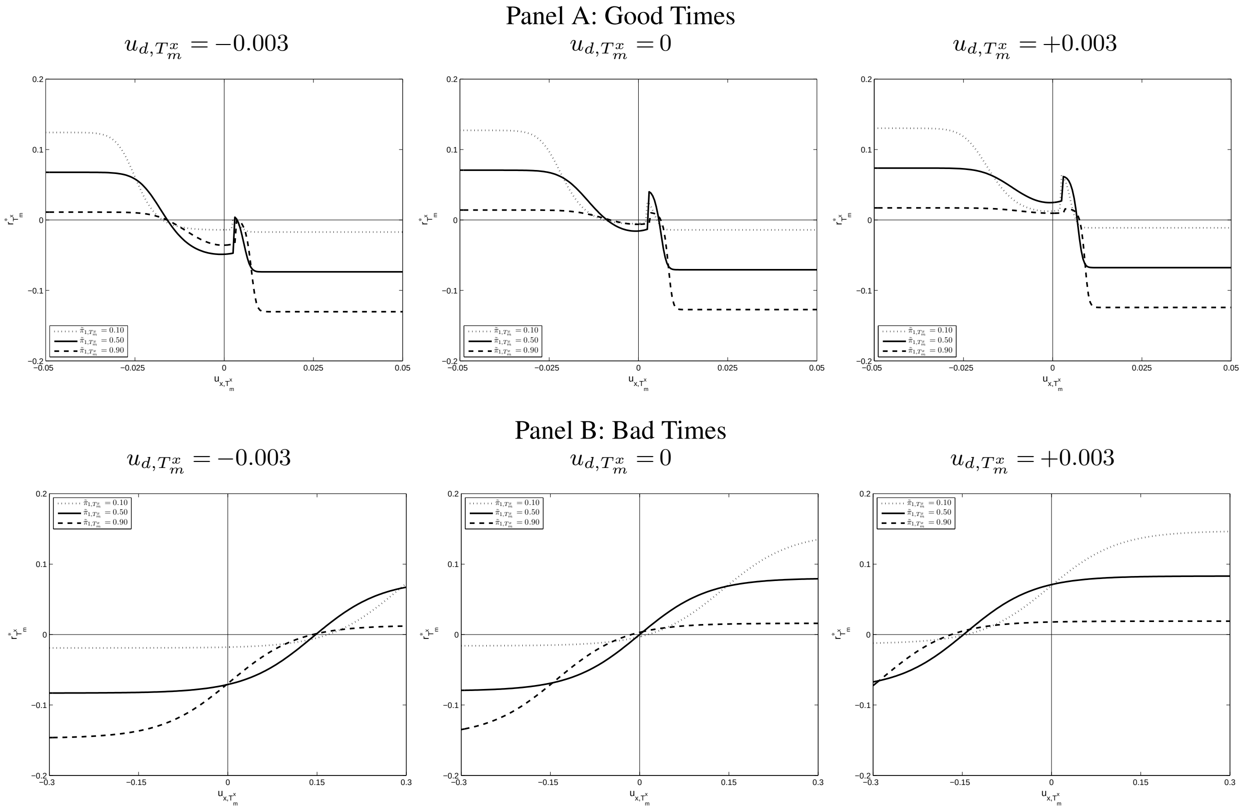
Variable	Mean	Median	Std. Dev.	Percentage of samples with a significantly negative coefficient estimate	Percentage of samples with a significantly positive coefficient estimate
Constant	-0.002	0.000	0.016	6%	4%
$\tilde{\pi}_{3,T^x_m}$	0.002	0.000	0.016	3%	11%
u_{d,T^x_m}	7.605	7.414	1.888	0%	100%
$1_{\{u_{x,T^x_m}>0\}}$	0.000	0.000	0.003	8%	2%
$u_{x,T^x_m} 1_{\{u_{x,T^x_m}>0\}}$	0.165	0.132	0.252	0%	18%
$u_{x,T^x_m} 1_{\{u_{x,T^x_m}\leq 0\}}$	0.178	0.124	0.274	0%	23%

Panel B: $r^*_{T^x_m} = \theta_1 + \theta_2 \tilde{\pi}_{3,T^x_m} + \theta_3 u_{d,T^x_m} + \theta_4 1_{\{u_{x,T^x_m}>0\}} + \theta_5 u_{x,T^x_m} + \nu_{T^x_m}$

Variable	Mean	Median	Std. Dev.	Percentage of samples with a significantly negative coefficient estimate	Percentage of samples with a significantly positive coefficient estimate
Constant	-0.002	0.000	0.016	7%	2%
$\tilde{\pi}_{3,T^x_m}$	0.002	0.000	0.016	3%	12%
u_{d,T^x_m}	7.608	7.438	1.897	0%	100%
$1_{\{u_{x,T^x_m}>0\}}$	0.000	0.000	0.003	6%	2%
u_{x,T^x_m}	0.166	0.135	0.155	0%	28%

Note: The table presents summary statistics for the coefficient estimates of the linear regression models in good times over 100 simulated samples. The significance level is 5% for one-sided tests in the last two columns.

Figure 1 Unexpected Return as a Function of News Variable Observed from External Signals



Note: The figure presents unexpected returns as a function of the news variable observed from external signals. The figure is based on the calibrated model parameters presented in Table 2.

Appendix

Proofs

[Proof of Lemma 1] We first characterize the investor's prior beliefs about the state variable of period t. Before observing the dividend realization (and possibly the external signal) in period t, the investor knows that the dividend growth process might have switched to a new state. Hence, his prior belief about state j is a weighted average of the transition probabilities into state j. The weights are his beliefs about the state variable of period $t-1$ after observing the dividend realization (and possibly the external signal) in period $t-1$. Given that he also observes whether $S_t \in \{1,2\}$ or $S_t \in \{3,4\}$, his prior belief about state j in period t is given by

$$\tilde{\pi}_{j,t} = \begin{cases} \frac{\sum_{i=1}^{4} \pi_{i,t-1} q_{ij}}{\sum_{i=1}^{4} \pi_{i,t-1} q_{i1} + \sum_{i=1}^{4} \pi_{i,t-1} q_{i2}} 1_{\{j=1,2\}} & \text{if } S_t \in \{1,2\} \\ \frac{\sum_{i=1}^{4} \pi_{i,t-1} q_{ij}}{\sum_{i=1}^{4} \pi_{i,t-1} q_{i3} + \sum_{i=1}^{4} \pi_{i,t-1} q_{i4}} 1_{\{j=3,4\}} & \text{if } S_t \in \{3,4\} \end{cases} \tag{13}$$

and $\phi(\cdot)$ is the standard normal density function.

The investor's beliefs need to be characterized separately for announcement and non-announcement periods. We start with non-announcement periods. After observing whether $S_t \in \{1,2\}$ or $S_t \in \{3,4\}$, the investor updates his beliefs about the true state of the dividend growth process according to Bayes' rule based on the observed dividend realization. The probability that the investor assigns to state j, $\pi_{j,t} = \Pr(S_t = j|\mathcal{F}_t)$, can be expressed as

$$\begin{aligned} \pi_{j,t} &= \Pr(S_t = j|\Delta d_t, \tilde{\mathcal{F}}_t) && (14) \\ &= \frac{\Pr(\Delta d_t|S_t = j, \tilde{\mathcal{F}}_t)\Pr(S_t = j|\tilde{\mathcal{F}}_t)}{\Pr(\Delta d_t|\tilde{\mathcal{F}}_t)} && (15) \\ &= \frac{\Pr(\Delta d_t|S_t = j, \tilde{\mathcal{F}}_t)\Pr(S_t = j|\tilde{\mathcal{F}}_t)}{\sum_{i=1}^{4}\Pr(\Delta d_t|S_t = i, \tilde{\mathcal{F}}_t)\Pr(S_t = i|\tilde{\mathcal{F}}_t)} && (16) \\ &= \frac{\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_d})\tilde{\pi}_{j,t}}{\sum_{i=1}^{4}\phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_d})\tilde{\pi}_{i,t}} && (17) \end{aligned}$$

where $\phi(\cdot)$ is the standard normal density function. Equation (14) follows from the definition of the information set, $\mathcal{F}_t$, which can be decomposed into the dividend realization of period t, Δd_t, and the information set prior to observing the information revealed in period t, $\tilde{\mathcal{F}}_t$, which includes information on whether $S_t \in \{1,2\}$ or $S_t \in \{3,4\}$ in addition to all past information. Equations (15) and (16) follow from Bayes' rule and the law of total probability, respectively.[8] Equation (17) follows from the law of motion for the dividend growth process in Equation (3).

On an announcement period T_m^x, there are two sources of information about the state variable, the dividend realization and the external signal. The investor updates his prior beliefs according to Bayes' rule based on the observed dividend realization and external signal.

$$\begin{aligned} \pi_{j,T_m^x} &= \Pr(S_{T_m^x} = j|\Delta d_{T_m^x}, x_m, \tilde{\mathcal{F}}_{T_m^x}) && (18) \\ &= \frac{\Pr(\Delta d_{T_m^x}|S_{T_m^x} = j, \tilde{\mathcal{F}}_{T_m^x})\Pr(x_m|S_{T_m^x} = j, \tilde{\mathcal{F}}_{T_m^x})\Pr(S_{T_m^x} = j|\tilde{\mathcal{F}}_{T_m^x})}{\Pr(\Delta d_{T_m^x}, x_m|\tilde{\mathcal{F}}_{T_m^x})} && (19) \\ &= \frac{\Pr(\Delta d_{T_m^x}|S_{T_m^x} = j, \tilde{\mathcal{F}}_{T_m^x})\Pr(x_m|S_{T_m^x} = j, \tilde{\mathcal{F}}_{T_m^x})\Pr(S_{T_m^x} = j|\tilde{\mathcal{F}}_{T_m^x})}{\sum_{i=1}^{4}\Pr(\Delta d_{T_m^x}|S_{T_m^x} = i, \tilde{\mathcal{F}}_{T_m^x})\Pr(x_m|S_{T_m^x} = i, \tilde{\mathcal{F}}_{T_m^x})\Pr(S_{T_m^x} = i|\tilde{\mathcal{F}}_{T_m^x})} && (20) \end{aligned}$$

[8] Recall that Bayes' rule is $\Pr(A|B,C) = \frac{\Pr(B|A,C)\Pr(A|C)}{\Pr(B|C)}$

$$= \frac{\phi(\frac{\Delta d_{T_m^x}-\mu_{d,j}}{\sigma_d})\left(\frac{1}{\sigma_{x,j}^+}\phi(\frac{x_m-\mu_{x,j}}{\sigma_{x,j}^+})1_{\{x_m\geq\mu_{x,j}\}}+\frac{1}{\sigma_{x,j}^-}\phi(\frac{x_m-\mu_{x,j}}{\sigma_{x,j}^-})1_{\{x_m<\mu_{x,j}\}}\right)\tilde{\pi}_{j,T_m^x}}{\sum_{i=1}^4\phi(\frac{\Delta d_{T_m^x}-\mu_{d,i}}{\sigma_d})\left(\frac{1}{\sigma_{x,i}^+}\phi(\frac{x_m-\mu_{x,i}}{\sigma_{x,i}^+})1_{\{x_m\geq\mu_{x,i}\}}+\frac{1}{\sigma_{x,i}^-}\phi(\frac{x_m-\mu_{x,i}}{\sigma_{x,i}^-})1_{\{x_m<\mu_{x,i}\}}\right)\tilde{\pi}_{i,T_m^x}} \tag{21}$$

The proof is similar to the one for the investor's beliefs on non-announcement periods. Equation (18) follows from the definition of the information set on the announcement period T_m^x, $\mathcal{F}_{T_m^x}$, which can be decomposed into the dividend realization, $\Delta d_{T_m^x}$, and the external signal, x_m, observed on the announcement period T_m^x and all past information, $\tilde{\mathcal{F}}_{T_m^x}$. Equations (19) and (20) follow from the independence of $\Delta d_{T_m^x}$ and x_m conditional on the state variable S_t. Equation (21) follows from the law of motion for the dividend growth rate in Equation (3) and the law of motion for the external signal in Equation (5).

[Proof of Proposition 1] Under Epstein and Zin (1989) and Weil (1990) preferences, the stochastic discount factor in period $t+1$, M_{t+1}, is given by

$$M_{t+1} = \left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right)^{\theta}\left(\frac{P_{t+1}^C+C_{t+1}}{C_t}\right)^{\theta-1}$$

where P_{t+1}^C is the price of the aggregate wealth portfolio, which pays a dividend in period $t+1$ equal to the consumption in that period. The first-order condition for the optimal portfolio consumption choice implies that the price-consumption ratio for the consumption claim satisfies

$$\left(\frac{P_t^C}{C_t}\right)^{\theta} = E_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\left(\frac{P_{t+1}^C}{C_{t+1}}+1\right)^{\theta}\right]$$

To solve for the $\frac{P_t^C}{C_t}$, we first conjecture that it is constant. We then plug in this conjecture in the above equation and verify that it is indeed constant. Specifically, let λ^C denote the constant price-consumption ratio, i.e. $\frac{P_t^C}{C_t}=\lambda^C$ for all t. Plugging this conjecture in the above equation yields

$$\begin{aligned}\left(\frac{\lambda^C}{1+\lambda^C}\right)^{\theta} &= \beta^{\theta}E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\\ &= \beta^{\theta}\exp((1-\gamma)\mu_c+(1-\gamma)^2\sigma_c^2/2)\end{aligned}$$

where the second equation follows from the assumption that the growth rates of log-consumption are independently and identically distributed Gaussian random variables. Solving for λ^C yields

$$\lambda^C = \frac{\alpha}{1-\alpha}$$

where $\alpha = \beta\exp((1-\psi^{-1})\mu_c+(1-\gamma)(1-\psi^{-1})\sigma_c^2/2) > 0$. Furthermore, we assume that the model parameters are such that $\alpha < 1$ which guarantees that the price-consumption ratio is positive and finite, i.e. $0<\lambda^C<\infty$.

Plugging the solution for the price-consumption ratio in the stochastic discount factor yields

$$M_{t+1} = \alpha\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$$

as in Calvet and Fisher (2007).

The first-order condition for the optimal portfolio consumption choice implies that the price-dividend ratio of the dividend claim satisfies

$$\frac{P_t}{D_t} = E_t\left[\alpha\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{D_{t+1}}{D_t}\left(\frac{P_{t+1}}{D_{t+1}}+1\right)\right].$$

Recursive substitution of future prices in the above equation yields

$$\frac{P_t}{D_t} = E_t\left[\sum_{\tau=1}^{\infty} \alpha^{\tau}\left(\frac{C_{t+\tau}}{C_t}\right)^{-\gamma}\frac{D_{t+\tau}}{D_t}\right]$$

From the definition of the conditional expectation, the price-dividend ratio can be expressed as a linear function of the investor's beliefs about the state variable:

$$\frac{P_t}{D_t} = E\left[\sum_{\tau=1}^{\infty} \alpha^{\tau}\left(\frac{C_{t+\tau}}{C_t}\right)^{-\gamma}\frac{D_{t+\tau}}{D_t}\middle| S_t = j\right]\pi_{j,t}$$

where $\pi_{j,t} = \Pr(S_t = j|\mathcal{F}_t)$ is the probability that he assigns to state j after observing the information revealed in period t. Let λ_j denote the price-dividend ratio in state j, i.e. $\lambda_j = E\left[\sum_{\tau=1}^{\infty} \alpha^{\tau}\left(\frac{C_{t+\tau}}{C_t}\right)^{-\gamma}\frac{D_{t+\tau}}{D_t}\middle| S_t = j\right]$. It is easy to see that the price-dividend ratio is positive in each state given that it is a sum of positive numbers. To guarantee that it is also finite in each state, we assume that model parameters are such that $\exp(-\gamma\mu_c + \gamma^2\sigma_c^2/2)\exp(\mu_{d,j} + \sigma_d^2/2) < 1$. Then, λ_j can be expressed as follows:

$$\begin{aligned}
\lambda_j &= E\left[\alpha\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{D_{t+1}}{D_t}\middle| S_t = j\right] \\
&+ E\left[\alpha\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{D_{t+1}}{D_t}\sum_{\tau=2}^{\infty}\alpha^{\tau-1}\left(\frac{C_{t+\tau}}{C_{t+1}}\right)^{-\gamma}\frac{D_{t+\tau}}{D_{t+1}}\middle| S_t = j\right] \\
&= \alpha\exp(-\gamma\mu_c + \gamma^2\sigma_c^2/2)\sum_{i=1}^{4}\exp(\mu_{d,i} + \sigma_d^2/2)q_{ji} \\
&+ \alpha\exp(-\gamma\mu_c + \gamma^2\sigma_c^2/2)\sum_{i=1}^{4}\exp(\mu_{d,i} + \sigma_d^2/2)\lambda_i q_{ji}
\end{aligned} \tag{22}$$

This yields a system of 4 equations which can be expressed as follows:

$$\boldsymbol{\lambda} = \mathbf{QG} + \mathbf{QH}\boldsymbol{\lambda} \tag{23}$$

where $\mathbf{G}$ and $\mathbf{H}$ are, respectively, a 4×1 vector and a 4×4 diagonal matrix whose j^{th} element and jj^{th} diagonal elements are given by $\alpha\exp(-\gamma\mu_c + \gamma^2\sigma_c^2/2)\exp(\mu_{d,j} + \sigma_d^2/2)$. Solving for λ yields

$$\boldsymbol{\lambda} = (\mathbf{I} - \mathbf{QH})^{-1}\mathbf{QG}$$

and the price-dividend ratio can be expressed as follows:

$$\frac{P_t}{D_t} = \sum_{j=1}^{4}\lambda_j\pi_{j,t}$$

To prove that $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$, it suffices to show that λ_i is an increasing function of $\mu_{d,i}$ for $i = 1, \ldots, 4$, i.e. $\partial\lambda_i/\partial\mu_{d,i} > 0$. To do this, first note the following facts:

(1) $\partial\alpha\exp(-\gamma\mu_c + \gamma^2\sigma_c^2/2)\exp(\mu_{d,i} + \sigma_d^2/2)/\partial\mu_{d,i} > 0$. Hence, the diagonal elements of $\partial\mathbf{H}/\partial\mu_i$ and all elements of $\partial\mathbf{G}/\partial\mu_i$ are positive.

(2) All elements of $(\mathbf{I} - \mathbf{QH})^{-1}$ are positive. To see this, first note that a matrix is irreducible if there exists some k such that all elements of the k^{th} power of the matrix are positive. One can prove that $\mathbf{QH}$ is an irreducible matrix since, for $k = 1$, $[(\mathbf{QH})^k]_{i,j} > 0$ for all i and j where $[\cdot]_{i,j}$ refers to the ij^{th} element of the matrix. According to Theorem III of Debreu and Herstein (1953), the elements of $(\mathbf{I} - \mathbf{QH})^{-1}$ are positive if and only if the maximal non-negative

characteristic root of $\mathbf{QH}$ is less than 1. To show this, let p denote the maximal non-negative characteristic root of $\mathbf{QH}$. We know that $\sum_{j=1}^{4}[\mathbf{QH}]_{i,j} = \alpha\exp(-\gamma\mu_c+\gamma^2\sigma_c^2/2)\exp(\mu_{d,i}+\sigma_d^2/2) < 1$ and $\min_i \sum_{j=1}^{4}[\mathbf{QH}]_{i,j} \le p \le \max_i \sum_{j=1}^{4}[\mathbf{QH}]_{i,j}$ (the latter follows from the Perron-Frobenius theorem for non-negative matrices). This immediately implies that $p<1$.

The partial derivative of $\boldsymbol{\lambda}$ with respect to $\mu_{d,i}$ can be expressed as follows:

$$\begin{aligned}
\frac{\partial\boldsymbol{\lambda}}{\partial\mu_{d,i}} &= \frac{\partial(\mathbf{I}-\mathbf{QH})^{-1}\mathbf{QG}}{\partial\mu_{d,i}} \\
&= \frac{\partial(\mathbf{I}-\mathbf{QH})^{-1}}{\partial\mu_{d,i}}\mathbf{QG}+(\mathbf{I}-\mathbf{QH})^{-1}\mathbf{Q}\frac{\partial\mathbf{G}}{\partial\mu_{d,i}} \\
&= -(\mathbf{I}-\mathbf{QH})^{-1}\frac{\partial(\mathbf{I}-\mathbf{QH})}{\partial\mu_{d,i}}(\mathbf{I}-\mathbf{QH})^{-1}\mathbf{QG}+(\mathbf{I}-\mathbf{QH})^{-1}\mathbf{Q}\frac{\partial\mathbf{G}}{\partial\mu_{d,i}} \\
&= (\mathbf{I}-\mathbf{QH})^{-1}\left[\frac{\partial\mathbf{H}}{\partial\mu_{d,i}}\mathbf{Q}\boldsymbol{\lambda}+\mathbf{Q}\frac{\partial\mathbf{G}}{\partial\mu_{d,i}}\right]
\end{aligned}$$

Facts (1) and (2) imply that $\frac{\partial\boldsymbol{\lambda}}{\partial\mu_{d,i}} > 0$ which in turn implies that λ_i is an increasing function of $\mu_{d,i}$ for $i=1,\ldots,4$.

Log returns on the risky asset can be expressed as follows:

$$\begin{aligned}
r_t &= \log(1+P_t/D_t)-\log(P_{t-1}/D_{t-1})+\Delta d_t \\
&\approx \log(1+\overline{\lambda})+\frac{1}{1+\overline{\lambda}}(P_t/D_t-\overline{\lambda})-\log(\overline{\lambda})-\frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1}-\overline{\lambda})+\Delta d_t
\end{aligned} \tag{24}$$

where Equation (24) follows from a first-order Taylor expansion of the log function around the long term average of the price-dividend ratio, $\bar{\lambda}$. The expectation of the log return in period t conditional on the investor's prior beliefs before observing the dividend realization (and possibly the external signal) in period t can be expressed as follows:

$$\tilde{E}_t[r_t] \approx \log(1+\overline{\lambda})+\frac{1}{1+\overline{\lambda}}(\sum_{j=1}^{4}\lambda_j\tilde{\pi}_{j,t}-\overline{\lambda})-\log(\overline{\lambda})-\frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1}-\overline{\lambda})+\sum_{j=1}^{4}\mu_{d,j}\tilde{\pi}_{j,t} \tag{25}$$

The unexpected log return on the risky asset in Equation (10) can be obtained as the difference between Equations (24) and (25). The long term average of the price-dividend ratio is the unconditional expectation of the price-dividend ratio as defined in Proposition 1.

[Proof of Proposition 2] The derivative of the return on an announcement period T_m^x in good times with respect to the news variable

$$\frac{\partial r^*_{T_m^x}}{\partial u_{x,T_m^x}} = \left(\frac{\lambda_1-\lambda_2}{1+\overline{\lambda}}\right)\frac{\partial\pi_{1,T_m^x}}{\partial u_{x,T_m^x}}. \tag{26}$$

Proof of (a): Note that for an external signal, x_m, such that $x_m > \mu_{x,1}$, or equivalently, for a news variable u_{x,T_m^x} such that $u_{x,T_m^x} > \mu_{x,1}-\tilde{\mu}_{x,T_m^x}$, the investor's beliefs about really good times, π_{1,T_m^x}, can be expressed as

$$\begin{aligned}
\pi_{1,T_m^x} = &\left[1+\frac{\sigma^+_{x,1}(1-\tilde{\pi}_{1,T_m^x})}{\sigma^+_{x,2}\tilde{\pi}_{1,T_m^x}}\exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma_d^2}\right)\exp\left(-\frac{(\mu_{x,1}-\mu_{x,2})^2}{2((\sigma^+_{x,2})^2-(\sigma^+_{x,1})^2)}\right)\right. \\
&\left.\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_d^2}u_{d,T_m^x}\right)\exp\left(\frac{(\sigma^+_{x,2})^2-(\sigma^+_{x,1})^2}{2(\sigma^+_{x,1})^2(\sigma^+_{x,2})^2}\left(u_{x,T_m^x}-\delta^+_x(\tilde{\pi}_{1,T_m^x})\right)^2\right)\right]^{-1}.
\end{aligned} \tag{27}$$

where $\delta^+_x(\tilde{\pi}_{1,T_m^x}) = \frac{\mu_{x,1}(\sigma^+_{x,2})^2-\mu_{x,2}(\sigma^+_{x,1})^2}{(\sigma^+_{x,2})^2-(\sigma^+_{x,1})^2} - \tilde{\mu}_{x,T_m^x}$ and $\tilde{\mu}_{x,T_m^x} = \sum_{j=1}^{2}(\mu_{x,j}+(\sigma^+_{x,j}-\sigma^-_{x,j})/\sqrt{2\pi})\tilde{\pi}_{j,T_m^x}$ and $\delta^+_x(\tilde{\pi}_{1,T_m^x}) > \mu_{x,1}-\tilde{\mu}_{x,T_m^x}$ since $\sigma^+_{x,1} < \sigma^+_{x,2}$.

Then, the derivative of π_{1,T_m^x} with respect to u_{x,T_m^x} is given by

$$\partial\pi_{1,T_m^x}/\partial u_{x,T_m^x} = f_1^+(u_{d,T_m^x},u_{x,T_m^x},\tilde{\pi}_{1,T_m^x})\left(\frac{(\sigma^+_{x,1})^2-(\sigma^+_{x,2})^2}{(\sigma^+_{x,1})^2(\sigma^+_{x,2})^2}\right)\left(u_{x,T_m^x}-\delta^+_x(\tilde{\pi}_{1,T_m^x})\right) \tag{28}$$

where

$$f_1^+(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}) = \kappa_1^+(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x})/(1+\kappa_1^+(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}))^2,$$

and

$$\begin{aligned}\kappa_1^+(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}) \;=\; & \frac{\sigma_{x,1}^+(1-\tilde{\pi}_{1,T_m^x})}{\sigma_{x,2}^+\tilde{\pi}_{1,T_m^x}} \exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma_d^2}\right) \\ & \exp\left(-\frac{(\mu_{x,1}-\mu_{x,2})^2}{2((\sigma_{x,2}^+)^2-(\sigma_{x,1}^+)^2)}\right)\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_d^2}u_{d,T_m^x}\right) \\ & \exp\left(\frac{(\sigma_{x,2}^+)^2-(\sigma_{x,1}^+)^2}{2(\sigma_{x,1}^+)^2(\sigma_{x,2}^+)^2}\left(u_{x,T_m^x}-\delta_x^+(\tilde{\pi}_{1,T_m^x})\right)^2\right).\end{aligned}$$

Note that κ_1^+ and f_1^+ are positive-valued functions. Then, given that $\lambda_1 > \lambda_2$ and $\sigma_{x,1}^+ < \sigma_{x,2}^+$, $\partial r_{T_m^x}^* / \partial u_{x,T_m^x}$ is negative for news variables $u_{x,T_m^x} > \delta_x^+(\tilde{\pi}_{1,T_m^x})$ and positive for news variables $\mu_{x,1} - \tilde{\mu}_{x,T_m^x} < u_{x,T_m^x} < \delta_x^+(\tilde{\pi}_{1,T_m^x})$.

Proof of (b): Note that for an external signal, x_m, such that $x_m < \mu_{x,2}$, or equivalently, for a news variable u_{x,T_m^x} such that $u_{x,T_m^x} < \mu_{x,2} - \tilde{\mu}_{x,T_m^x}$, the investor's beliefs about really good times, π_{1,T_m^x}, can be expressed as

$$\begin{aligned}\pi_{1,T_m^x} \;=\; & \Bigg[1+\frac{\sigma_{x,1}^-(1-\tilde{\pi}_{1,T_m^x})}{\sigma_{x,2}^-\tilde{\pi}_{1,T_m^x}} \exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma_d^2}\right)\exp\left(-\frac{(\mu_{x,1}-\mu_{x,2})^2}{2((\sigma_{x,2}^-)^2-(\sigma_{x,1}^-)^2)}\right) \\ & \exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_d^2}u_{d,T_m^x}\right)\exp\left(\frac{(\sigma_{x,2}^-)^2-(\sigma_{x,1}^-)^2}{2(\sigma_{x,1}^-)^2(\sigma_{x,2}^-)^2}\left(u_{x,T_m^x}-\delta_x^-(\tilde{\pi}_{1,T_m^x})\right)^2\right)\Bigg]^{-1}. \qquad (29)\end{aligned}$$

where $\delta_x^-(\tilde{\pi}_{1,T_m^x}) = \frac{\mu_{x,1}(\sigma_{x,2}^-)^2-\mu_{x,2}(\sigma_{x,1}^-)^2}{(\sigma_{x,2}^-)^2-(\sigma_{x,1}^-)^2} - \tilde{\mu}_{x,T_m^x}$ and $\tilde{\mu}_{x,T_m^x} = \sum_{j=1}^2(\mu_{x,j} + (\sigma_{x,j}^+ - \sigma_{x,j}^-)/\sqrt{2\pi})\tilde{\pi}_{j,T_m^x}$ and $\delta_x^-(\tilde{\pi}_{1,T_m^x}) < \mu_{x,2} - \tilde{\mu}_{x,T_m^x}$ since $\sigma_{x,1}^- > \sigma_{x,2}^-$.

Then, the derivative of π_{1,T_m^x} with respect to u_{x,T_m^x} is given by

$$\partial\pi_{1,T_m^x}/\partial u_{x,T_m^x} = f_1^-(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x})\left(\frac{(\sigma_{x,1}^-)^2-(\sigma_{x,2}^-)^2}{(\sigma_{x,1}^-)^2(\sigma_{x,2}^-)^2}\right)\left(u_{x,T_m^x}-\delta_x^-(\tilde{\pi}_{1,T_m^x})\right) \qquad (30)$$

where

$$f_1^-(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}) = \kappa_1^-(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x})/(1+\kappa_1^-(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}))^2,$$

and

$$\begin{aligned}\kappa_1^-(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{1,T_m^x}) \;=\; & \frac{\sigma_{x,1}^-(1-\tilde{\pi}_{1,T_m^x})}{\sigma_{x,2}^-\tilde{\pi}_{1,T_m^x}} \exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma_d^2}\right) \\ & \exp\left(-\frac{(\mu_{x,1}-\mu_{x,2})^2}{2((\sigma_{x,2}^-)^2-(\sigma_{x,1}^-)^2)}\right)\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_d^2}u_{d,T_m^x}\right) \\ & \exp\left(\frac{(\sigma_{x,2}^-)^2-(\sigma_{x,1}^-)^2}{2(\sigma_{x,1}^-)^2(\sigma_{x,2}^-)^2}\left(u_{x,T_m^x}-\delta_x^-(\tilde{\pi}_{1,T_m^x})\right)^2\right).\end{aligned}$$

Note that κ_1^- and f_1^- are positive-valued functions. Then, given that $\lambda_1 > \lambda_2$ and $\sigma_{x,1}^- > \sigma_{x,2}^-$, $\partial r_{T_m^x}^* / \partial u_{x,T_m^x}$ is negative for news variables $u_{x,T_m^x} < \delta_x^-(\tilde{\pi}_{1,T_m^x})$ and positive for news variables $\delta_x^-(\tilde{\pi}_{1,T_m^x}) < u_{x,T_m^x} < \mu_{x,2} - \tilde{\mu}_{x,T_m^x}$.

Proof of (c): Note that for an external signal, x_m, such that $\mu_{x,2} < x_m < \mu_{x,1}$, or equivalently, for a news variable u_{x,T_m^x} such that $\mu_{x,2} - \tilde{\mu}_{x,T_m^x} < u_{x,T_m^x} < \mu_{x,1} - \tilde{\mu}_{x,T_m^x}$, the investor's beliefs about really good times, π_{1,T_m^x}, can be expressed as

$$\begin{aligned}\pi_{1,T_m^x} \;=\; & \Bigg[1+\frac{\sigma_{x,1}^-(1-\tilde{\pi}_{1,T_m^x})}{\sigma_{x,2}^+\tilde{\pi}_{1,T_m^x}} \exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T_m^x})}{2\sigma_d^2}\right)\exp\left(-\frac{(\mu_{x,1}-\mu_{x,2})^2}{2((\sigma_{x,2}^+)^2-(\sigma_{x,1}^-)^2)}\right) \\ & \exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_d^2}u_{d,T_m^x}\right)\exp\left(\frac{(\sigma_{x,2}^+)^2-(\sigma_{x,1}^-)^2}{2(\sigma_{x,1}^-)^2(\sigma_{x,2}^+)^2}\left(u_{x,T_m^x}-\delta_x(\tilde{\pi}_{1,T_m^x})\right)^2\right)\Bigg]^{-1}. \qquad (31)\end{aligned}$$

where $\delta_x(\tilde{\pi}_{1,T^x_m}) = \frac{\mu_{x,1}(\sigma^+_{x,2})^2-\mu_{x,2}(\sigma^-_{x,1})^2}{(\sigma^+_{x,2})^2-(\sigma^-_{x,1})^2} - \tilde{\mu}_{x,T^x_m}$.
Then, the derivative of π_{1,T^x_m} with respect to u_{x,T^x_m} is given by

$$\partial\pi_{1,T^x_m}/\partial u_{x,T^x_m} = f_1(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m})\left(\frac{(\sigma^-_{x,1})^2-(\sigma^+_{x,2})^2}{(\sigma^-_{x,1})^2(\sigma^+_{x,2})^2}\right)\left(u_{x,T^x_m} - \delta_x(\tilde{\pi}_{1,T^x_m})\right) \tag{32}$$

where

$$f_1(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m}) = \kappa_1(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m})/(1+\kappa_1(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m}))^2,$$

and

$$\begin{aligned}\kappa_1(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m}) \;=\; & \frac{\sigma^-_{x,1}(1-\tilde{\pi}_{1,T^x_m})}{\sigma^+_{x,2}\tilde{\pi}_{1,T^x_m}}\exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T^x_m})}{2\sigma^2_d}\right)\\ & \exp\left(-\frac{(\mu_{x,1}-\mu_{x,2})^2}{2((\sigma^+_{x,2})^2-(\sigma^-_{x,1})^2)}\right)\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma^2_d}u_{d,T^x_m}\right)\\ & \exp\left(\frac{(\sigma^+_{x,2})^2-(\sigma^-_{x,1})^2}{2(\sigma^-_{x,1})^2(\sigma^+_{x,2})^2}\left(u_{x,T^x_m}-\delta_x(\tilde{\pi}_{1,T^x_m})\right)^2\right).\end{aligned}$$

Note that κ_1 and f_1 are positive-valued functions and $\lambda_1 > \lambda_2$. Then, for a news variable u_{x,T^x_m}, such that $\mu_{x,2} - \tilde{\mu}_{x,T^x_m} < u_{x,T^x_m} < \mu_{x,1} - \tilde{\mu}_{x,T^x_m}$, $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive independent of the relation between $\sigma^-_{x,1}$ and $\sigma^+_{x,2}$. To see this, consider the following three cases:

(1) If $\sigma^-_{x,1} < \sigma^+_{x,2}$ then $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is negative for news variables $u_{x,T^x_m} > \delta_x(\tilde{\pi}_{1,T^x_m})$ and positive for news variables $u_{x,T^x_m} < \delta_x(\tilde{\pi}_{1,T^x_m})$. However, $\delta_x(\tilde{\pi}_{1,T^x_m}) > \mu_{x,1} - \tilde{\mu}_{x,T^x_m}$ given that $\sigma^-_{x,1} < \sigma^+_{x,2}$. Hence, for a news variable u_{x,T^x_m} such that $\mu_{x,2} - \tilde{\mu}_{x,T^x_m} < u_{x,T^x_m} < \mu_{x,1} - \tilde{\mu}_{x,T^x_m}$, $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive.

(2) Similarly, if $\sigma^-_{x,1} > \sigma^+_{x,2}$, then $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is negative for news variables $u_{x,T^x_m} < \delta_x(\tilde{\pi}_{1,T^x_m})$ and positive for news variables $u_{x,T^x_m} > \delta_x(\tilde{\pi}_{1,T^x_m})$. However, $\delta_x(\tilde{\pi}_{1,T^x_m}) < \mu_{x,2} - \tilde{\mu}_{x,T^x_m}$ given that $\sigma^-_{x,1} > \sigma^+_{x,2}$. Hence, for a news variable u_{x,T^x_m} such that $\mu_{x,2} - \tilde{\mu}_{x,T^x_m} < u_{x,T^x_m} < \mu_{x,1} - \tilde{\mu}_{x,T^x_m}$, $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive.

(3) If $\sigma^-_{x,1} = \sigma^+_{x,2}$, then π_{1,T^x_m} can be expressed as

$$\begin{aligned}\pi_{1,T^x_m} \;=\; & \Bigg[1+\frac{(1-\tilde{\pi}_{1,T^x_m})}{\tilde{\pi}_{1,T^x_m}}\exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T^x_m})}{2\sigma^2_d}\right)\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma^2_d}u_{d,T^x_m}\right)\\ & \exp\left(\frac{\mu_{x,2}-\mu_{x,1}}{(\sigma^-_{x,1})^2}\left(u_{x,T^x_m}-(\frac{\mu_{x,1}+\mu_{x,2}}{2}-\tilde{\mu}_{x,T^x_m})\right)\right)\Bigg]^{-1}.\end{aligned} \tag{33}$$

The derivative of π_{1,T^x_m} with respect to u_{x,T^x_m} is given by

$$\partial\pi_{1,T^x_m}/\partial u_{x,T^x_m} = f_2(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m})\left(\frac{\mu_{x,1}-\mu_{x,2}}{(\sigma^-_{x,1})^2}\right) \tag{34}$$

where

$$f_2(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m}) = \kappa_2(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m})/(1+\kappa_2(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m}))^2,$$

and

$$\begin{aligned}\kappa_2(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{1,T^x_m}) \;=\; & \frac{(1-\tilde{\pi}_{1,T^x_m})}{\tilde{\pi}_{1,T^x_m}}\exp\left(\frac{(\mu_{d,1}-\mu_{d,2})^2(1-2\tilde{\pi}_{1,T^x_m})}{2\sigma^2_d}\right)\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma^2_d}u_{d,T^x_m}\right)\\ & \exp\left(\frac{\mu_{x,2}-\mu_{x,1}}{(\sigma^-_{x,1})^2}\left(u_{x,T^x_m}-(\frac{\mu_{x,1}+\mu_{x,2}}{2}-\tilde{\mu}_{x,T^x_m})\right)\right).\end{aligned} \tag{35}$$

Note that κ_2 and f_2 are positive-valued functions. Then, $\partial\pi_{1,T^x_m}/\partial u_{x,T^x_m} > 0$. Given that $\lambda_1 > \lambda_2$, this immediately implies that $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive.

[Proof of Proposition 3] The derivative of the return on an announcement period T^x_m in bad times with respect to the news variable

$$\frac{\partial r^*_{T^x_m}}{\partial u_{x,T^x_m}} = \left(\frac{\lambda_3 - \lambda_4}{1+\bar{\lambda}}\right)\frac{\partial \pi_{3,T^x_m}}{\partial u_{x,T^x_m}}. \tag{36}$$

Proof of (a): First, consider the case where $\sigma^+_{x,3} > \sigma^+_{x,4}$. Note that for an external signal, x_m, such that $x_m > \mu_{x,3}$, or equivalently, for a news variable u_{x,T^x_m} such that $u_{x,T^x_m} > \mu_{x,3} - \tilde{\mu}_{x,T^x_m}$, the investor's beliefs about normal bad times, π_{3,T^x_m}, can be expressed as

$$\begin{aligned}\pi_{3,T^x_m} = &\left[1 + \frac{\sigma^+_{x,3}(1-\tilde{\pi}_{3,T^x_m})}{\sigma^+_{x,4}\tilde{\pi}_{3,T^x_m}} \exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T^x_m})}{2\sigma^2_d}\right)\exp\left(-\frac{(\mu_{x,3}-\mu_{x,4})^2}{2((\sigma^+_{x,4})^2-(\sigma^+_{x,3})^2)}\right)\right.\\ &\left.\exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma^2_d}u_{d,T^x_m}\right)\exp\left(\frac{(\sigma^+_{x,4})^2-(\sigma^+_{x,3})^2}{2(\sigma^+_{x,3})^2(\sigma^+_{x,4})^2}\left(u_{x,T^x_m} - \delta^+_x(\tilde{\pi}_{3,T^x_m})\right)^2\right)\right]^{-1}.\end{aligned} \tag{37}$$

where $\delta^+_x(\tilde{\pi}_{3,T^x_m}) = \frac{\mu_{x,3}(\sigma^+_{x,4})^2-\mu_{x,4}(\sigma^+_{x,3})^2}{(\sigma^+_{x,4})^2-(\sigma^+_{x,3})^2} - \tilde{\mu}_{x,T^x_m}$ and $\tilde{\mu}_{x,T^x_m} = \sum_{j=3}^4(\mu_{x,j} + (\sigma^+_{x,j} - \sigma^-_{x,j})/\sqrt{2\pi})\tilde{\pi}_{j,T^x_m}$ and $\delta^+_x(\tilde{\pi}_{3,T^x_m}) < \mu_{x,3} - \tilde{\mu}_{x,T^x_m}$ since $\sigma^+_{x,3} > \sigma^+_{x,4}$.

Then, the derivative of π_{3,T^x_m} with respect to u_{x,T^x_m} is given by

$$\partial\pi_{3,T^x_m}/\partial u_{x,T^x_m} = f^+_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m})\left(\frac{(\sigma^+_{x,3})^2-(\sigma^+_{x,4})^2}{(\sigma^+_{x,3})^2(\sigma^+_{x,4})^2}\right)\left(u_{x,T^x_m} - \delta^+_x(\tilde{\pi}_{3,T^x_m})\right) \tag{38}$$

where

$$f^+_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}) = \kappa^+_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{13,T^x_m})/(1+\kappa^+_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}))^2,$$

and

$$\begin{aligned}\kappa^+_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}) = &\frac{\sigma^+_{x,3}(1-\tilde{\pi}_{3,T^x_m})}{\sigma^+_{x,2}\tilde{\pi}_{3,T^x_m}}\exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T^x_m})}{2\sigma^2_d}\right)\\ &\exp\left(-\frac{(\mu_{x,3}-\mu_{x,4})^2}{2((\sigma^+_{x,4})^2-(\sigma^+_{x,3})^2)}\right)\exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma^2_d}u_{d,T^x_m}\right)\\ &\exp\left(\frac{(\sigma^+_{x,4})^2-(\sigma^+_{x,3})^2}{2(\sigma^+_{x,3})^2(\sigma^+_{x,4})^2}\left(u_{x,T^x_m} - \delta^+_x(\tilde{\pi}_{3,T^x_m})\right)^2\right).\end{aligned}$$

Note that κ^+_3 and f^+_3 are positive-valued functions. Then, given that $\lambda_3 > \lambda_4$ and $\sigma^+_{x,3} > \sigma^+_{x,4}$, $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive for news variables $u_{x,T^x_m} > \delta^+_x(\tilde{\pi}_{3,T^x_m})$. Given that $\delta^+_x(\tilde{\pi}_{3,T^x_m}) < \mu_{x,3} - \tilde{\mu}_{x,T^x_m}$, $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive for a news variable u_{x,T^x_m} such that $u_{x,T^x_m} > \mu_{x,3} - \tilde{\mu}_{x,T^x_m}$.

Now, consider the case where $\sigma^+_{x,3} = \sigma^+_{x,4}$. Note that for an external signal, x_m, such that $x_m > \mu_{x,3}$, or equivalently, for a news variable u_{x,T^x_m} such that $u_{x,T^x_m} > \mu_{x,3} - \tilde{\mu}_{x,T^x_m}$, the investor's beliefs about normal bad times, π_{3,T^x_m}, can be expressed as

$$\begin{aligned}\pi_{3,T^x_m} = &\left[1 + \frac{(1-\tilde{\pi}_{3,T^x_m})}{\tilde{\pi}_{3,T^x_m}}\exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T^x_m})}{2\sigma^2_d}\right)\exp\left(\frac{\mu_{d,3}-\mu_{d,4}}{\sigma^2_d}u_{d,T^x_m}\right)\right.\\ &\left.\exp\left(\frac{\mu_{x,4}-\mu_{x,3}}{(\sigma^+_{x,3})^2}\left(u_{x,T^x_m} - \left(\frac{\mu_{x,3}+\mu_{x,4}}{2} - \tilde{\mu}_{x,T^x_m}\right)\right)\right)\right]^{-1}.\end{aligned} \tag{39}$$

The derivative of π_{3,T^x_m} with respect to u_{x,T^x_m} is given by

$$\partial\pi_{3,T^x_m}/\partial u_{x,T^x_m} = f_4(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m})\left(\frac{\mu_{x,3}-\mu_{x,4}}{(\sigma^+_{x,3})^2}\right) \tag{40}$$

where

$$f_4(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}) = \kappa_4(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m})/(1+\kappa_4(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}))^2,$$

and

$$\begin{aligned}\kappa_4(u_{d,T^x_m}, u_{x,T^x_m}, {}_{3,T^x_m}) = \; & \frac{(1-\tilde{\pi}_{3,T^x_m})}{\tilde{\pi}_{3,T^x_m}} \exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T^x_m})}{2\sigma_d^2}\right) \exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma_d^2} u_{d,T^x_m}\right) \\ & \exp\left(\frac{\mu_{x,4}-\mu_{x,3}}{(\sigma^+_{x,3})^2}\left(u_{x,T^x_m} - (\frac{\mu_{x,3}+\mu_{x,4}}{2} - \tilde{\mu}_{x,T^x_m})\right)\right). \end{aligned} \tag{41}$$

Note that κ_4 and f_4 are positive-valued functions. Then, $\partial \pi_{3,T^x_m}/\partial u_{x,T^x_m} > 0$. Given that $\lambda_3 > \lambda_4$, this immediately implies that $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive for a news variable u_{x,T^x_m} such that $u_{x,T^x_m} > \mu_{x,3} - \tilde{\mu}_{x,T^x_m}$.

Proof of (b): First, consider the case where $\sigma^-_{x,3} < \sigma^-_{x,4}$. Note that for an external signal, x_m, such that $x_m < \mu_{x,4}$, or equivalently, for a news variable u_{x,T^x_m} such that $u_{x,T^x_m} < \mu_{x,4} - \tilde{\mu}_{x,T^x_m}$, the investor's beliefs about normal bad times, π_{3,T^x_m}, can be expressed as

$$\begin{aligned}\pi_{3,T^x_m} = & \Bigg[1 + \frac{\sigma^-_{x,3}(1-\tilde{\pi}_{3,T^x_m})}{\sigma^-_{x,4}\tilde{\pi}_{3,T^x_m}} \exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T^x_m})}{2\sigma_d^2}\right) \exp\left(-\frac{(\mu_{x,3}-\mu_{x,4})^2}{2((\sigma^-_{x,4})^2-(\sigma^-_{x,3})^2)}\right) \\ & \exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma_d^2} u_{d,T^x_m}\right) \exp\left(\frac{(\sigma^-_{x,4})^2-(\sigma^-_{x,3})^2}{2(\sigma^-_{x,3})^2(\sigma^-_{x,4})^2}\left(u_{x,T^x_m} - \delta^-_x(\tilde{\pi}_{3,T^x_m})\right)^2\right)\Bigg]^{-1}. \end{aligned} \tag{42}$$

where $\delta^-_x(\tilde{\pi}_{3,T^x_m}) = \frac{\mu_{x,3}(\sigma^-_{x,4})^2 - \mu_{x,4}(\sigma^-_{x,3})^2}{(\sigma^-_{x,4})^2-(\sigma^-_{x,3})^2} - \tilde{\mu}_{x,T^x_m}$ and $\tilde{\mu}_{x,T^x_m} = \sum_{j=3}^4 (\mu_{x,j} + (\sigma^-_{x,j} - \sigma^-_{x,j})/\sqrt{2\pi})\tilde{\pi}_{j,T^x_m}$ and $\delta^-_x(\tilde{\pi}_{3,T^x_m}) > \mu_{x,4} - \tilde{\mu}_{x,T^x_m}$ since $\sigma^-_{x,3} < \sigma^-_{x,4}$.

Then, the derivative of π_{3,T^x_m} with respect to u_{x,T^x_m} is given by

$$\partial \pi_{3,T^x_m}/\partial u_{x,T^x_m} = f^-_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m})\left(\frac{(\sigma^-_{x,3})^2-(\sigma^-_{x,4})^2}{(\sigma^-_{x,3})^2(\sigma^-_{x,4})^2}\right)\left(u_{x,T^x_m} - \delta^-_x(\tilde{\pi}_{3,T^x_m})\right) \tag{43}$$

where

$$f^-_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}) = \kappa^-_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{13,T^x_m})/(1+\kappa^-_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}))^2,$$

and

$$\begin{aligned}\kappa^-_3(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}) = \; & \frac{\sigma^-_{x,3}(1-\tilde{\pi}_{3,T^x_m})}{\sigma^-_{x,2}\tilde{\pi}_{3,T^x_m}} \exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T^x_m})}{2\sigma_d^2}\right) \\ & \exp\left(-\frac{(\mu_{x,3}-\mu_{x,4})^2}{2((\sigma^-_{x,4})^2-(\sigma^-_{x,3})^2)}\right) \exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma_d^2} u_{d,T^x_m}\right) \\ & \exp\left(\frac{(\sigma^-_{x,4})^2-(\sigma^-_{x,3})^2}{2(\sigma^-_{x,3})^2(\sigma^-_{x,4})^2}\left(u_{x,T^x_m} - \delta^-_x(\tilde{\pi}_{3,T^x_m})\right)^2\right). \end{aligned}$$

Note that κ^-_3 and f^-_3 are positive-valued functions. Then, given that $\lambda_3 > \lambda_4$ and $\sigma^-_{x,3} < \sigma^-_{x,4}$, $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive for news variables $u_{x,T^x_m} < \delta^-_x(\tilde{\pi}_{3,T^x_m})$. Given that $\delta^-_x(\tilde{\pi}_{3,T^x_m}) > \mu_{x,4} - \tilde{\mu}_{x,T^x_m}$, $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive for a news variable u_{x,T^x_m} such that $u_{x,T^x_m} < \mu_{x,4} - \tilde{\mu}_{x,T^x_m}$.

Now, consider the case where $\sigma^-_{x,3} = \sigma^-_{x,4}$. Note that for an external signal, x_m, such that $x_m < \mu_{x,4}$, or equivalently, for a news variable u_{x,T^x_m} such that $u_{x,T^x_m} < \mu_{x,4} - \tilde{\mu}_{x,T^x_m}$, the investor's beliefs about normal bad times, π_{3,T^x_m}, can be expressed as

$$\begin{aligned}\pi_{3,T^x_m} = & \Bigg[1 + \frac{(1-\tilde{\pi}_{3,T^x_m})}{\tilde{\pi}_{3,T^x_m}} \exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T^x_m})}{2\sigma_d^2}\right) \exp\left(\frac{\mu_{d,3}-\mu_{d,4}}{\sigma_d^2} u_{d,T^x_m}\right) \\ & \exp\left(\frac{\mu_{x,4}-\mu_{x,3}}{(\sigma^-_{x,3})^2}\left(u_{x,T^x_m} - (\frac{\mu_{x,3}+\mu_{x,4}}{2} - \tilde{\mu}_{x,T^x_m})\right)\right)\Bigg]^{-1}. \end{aligned} \tag{44}$$

The derivative of π_{3,T_m^x} with respect to u_{x,T_m^x} is given by

$$\partial\pi_{3,T_m^x}/\partial u_{x,T_m^x} = f_4(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{3,T_m^x})\left(\frac{\mu_{x,3}-\mu_{x,4}}{(\sigma_{x,3}^-)^2}\right) \tag{45}$$

where

$$f_4(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{3,T_m^x}) = \kappa_4(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{3,T_m^x})/(1+\kappa_4(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{3,T_m^x}))^2,$$

and

$$\begin{aligned}\kappa_4(u_{d,T_m^x}, u_{x,T_m^x}, _{3,T_m^x}) \;=\; & \frac{(1-\tilde{\pi}_{3,T_m^x})}{\tilde{\pi}_{3,T_m^x}}\exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T_m^x})}{2\sigma_d^2}\right)\exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma_d^2}u_{d,T_m^x}\right)\\ & \exp\left(\frac{\mu_{x,4}-\mu_{x,3}}{(\sigma_{x,3}^-)^2}\left(u_{x,T_m^x} - \left(\frac{\mu_{x,3}+\mu_{x,4}}{2} - \tilde{\mu}_{x,T_m^x}\right)\right)\right).\end{aligned} \tag{46}$$

Note that κ_4 and f_4 are positive-valued functions. Then, $\partial\pi_{3,T_m^x}/\partial u_{x,T_m^x} > 0$. Given that $\lambda_3 > \lambda_4$, this immediately implies that $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ is positive for a news variable u_{x,T_m^x} such that $u_{x,T_m^x} < \mu_{x,4} - \tilde{\mu}_{x,T_m^x}$.

Proof of (c): Note that for an external signal, x_m, such that $\mu_{x,4} < x_m < \mu_{x,3}$, or equivalently, for a news variable u_{x,T_m^x} such that $\mu_{x,4} - \tilde{\mu}_{x,T_m^x} < u_{x,T_m^x} < \mu_{x,3} - \tilde{\mu}_{x,T_m^x}$, the investor's beliefs about really good times, π_{3,T_m^x}, can be expressed as

$$\begin{aligned}\pi_{3,T_m^x} \;=\; & \left[1+\frac{\sigma_{x,3}^-(1-\tilde{\pi}_{3,T_m^x})}{\sigma_{x,4}^+\tilde{\pi}_{3,T_m^x}}\exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T_m^x})}{2\sigma_d^2}\right)\exp\left(-\frac{(\mu_{x,3}-\mu_{x,4})^2}{2((\sigma_{x,4}^+)^2-(\sigma_{x,3}^-)^2)}\right)\right.\\ & \left.\exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma_d^2}u_{d,T_m^x}\right)\exp\left(\frac{(\sigma_{x,4}^+)^2-(\sigma_{x,3}^-)^2}{2(\sigma_{x,3}^-)^2(\sigma_{x,4}^+)^2}\left(u_{x,T_m^x}-\delta_x(\tilde{\pi}_{3,T_m^x})\right)^2\right)\right]^{-1}.\end{aligned} \tag{47}$$

where $\delta_x(\tilde{\pi}_{3,T_m^x}) = \frac{\mu_{x,3}(\sigma_{x,4}^+)^2-\mu_{x,4}(\sigma_{x,3}^-)^2}{(\sigma_{x,4}^+)^2-(\sigma_{x,3}^-)^2} - \tilde{\mu}_{x,T_m^x}$.

Then, the derivative of π_{3,T_m^x} with respect to u_{x,T_m^x} is given by

$$\partial\pi_{3,T_m^x}/\partial u_{x,T_m^x} = f_3(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{3,T_m^x})\left(\frac{(\sigma_{x,3}^-)^2-(\sigma_{x,4}^+)^2}{(\sigma_{x,3}^-)^2(\sigma_{x,4}^+)^2}\right)\left(u_{x,T_m^x}-\delta_x(\tilde{\pi}_{3,T_m^x})\right) \tag{48}$$

where

$$f_3(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{3,T_m^x}) = \kappa_3(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{3,T_m^x})/(1+\kappa_3(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{3,T_m^x}))^2,$$

and

$$\begin{aligned}\kappa_3(u_{d,T_m^x}, u_{x,T_m^x}, \tilde{\pi}_{3,T_m^x}) \;=\; & \frac{\sigma_{x,3}^-(1-\tilde{\pi}_{3,T_m^x})}{\sigma_{x,4}^+\tilde{\pi}_{3,T_m^x}}\exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T_m^x})}{2\sigma_d^2}\right)\\ & \exp\left(-\frac{(\mu_{x,3}-\mu_{x,4})^2}{2((\sigma_{x,4}^+)^2-(\sigma_{x,3}^-)^2)}\right)\exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma_d^2}u_{d,T_m^x}\right)\\ & \exp\left(\frac{(\sigma_{x,4}^+)^2-(\sigma_{x,3}^-)^2}{2(\sigma_{x,3}^-)^2(\sigma_{x,4}^+)^2}\left(u_{x,T_m^x}-\delta_x(\tilde{\pi}_{3,T_m^x})\right)^2\right).\end{aligned}$$

Note that κ_3 and f_3 are positive-valued functions and $\lambda_3 > \lambda_4$. Then, for a news variable u_{x,T_m^x}, such that $\mu_{x,4} - \tilde{\mu}_{x,T_m^x} < u_{x,T_m^x} < \mu_{x,3} - \tilde{\mu}_{x,T_m^x}$, $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ is positive independent of the relation between $\sigma_{x,3}^-$ and $\sigma_{x,4}^+$. To see this, consider the following three cases:

(1) If $\sigma_{x,3}^- < \sigma_{x,4}^+$ then $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ is negative for news variables $u_{x,T_m^x} > \delta_x(\tilde{\pi}_{3,T_m^x})$ and positive for news variables $u_{x,T_m^x} < \delta_x(\tilde{\pi}_{3,T_m^x})$. However, $\delta_x(\tilde{\pi}_{3,T_m^x}) > \mu_{x,3} - \tilde{\mu}_{x,T_m^x}$ given that $\sigma_{x,3}^- < \sigma_{x,4}^+$. Hence, for a news variable u_{x,T_m^x} such that $\mu_{x,4} - \tilde{\mu}_{x,T_m^x} < u_{x,T_m^x} < \mu_{x,3} - \tilde{\mu}_{x,T_m^x}$, $\partial r^*_{T_m^x}/\partial u_{x,T_m^x}$ is positive.

(2) Similarly, if $\sigma^-_{x,3} > \sigma^+_{x,4}$, then $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is negative for news variables $u_{x,T^x_m} < \delta_x(\tilde{\pi}_{3,T^x_m})$ and positive for news variables $u_{x,T^x_m} > \delta_x(\tilde{\pi}_{3,T^x_m})$. However, $\delta_x(\tilde{\pi}_{3,T^x_m}) < \mu_{x,4} - \tilde{\mu}_{x,T^x_m}$ given that $\sigma^-_{x,3} > \sigma^+_{x,4}$. Hence, for a news variable u_{x,T^x_m} such that $\mu_{x,4} - \tilde{\mu}_{x,T^x_m} < u_{x,T^x_m} < \mu_{x,3} - \tilde{\mu}_{x,T^x_m}$, $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive.

(3) If $\sigma^-_{x,3} = \sigma^+_{x,4}$, then π_{3,T^x_m} can be expressed as

$$\begin{aligned}\pi_{3,T^x_m} = & \left[1 + \frac{(1-\tilde{\pi}_{3,T^x_m})}{\tilde{\pi}_{3,T^x_m}} \exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T^x_m})}{2\sigma_d^2}\right) \exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma_d^2} u_{d,T^x_m}\right)\right. \\ & \left.\exp\left(\frac{\mu_{x,4}-\mu_{x,3}}{(\sigma^-_{x,3})^2}\left(u_{x,T^x_m} - (\frac{\mu_{x,3}+\mu_{x,4}}{2} - \tilde{\mu}_{x,T^x_m})\right)\right)\right]^{-1}. \end{aligned} \tag{49}$$

The derivative of π_{3,T^x_m} with respect to u_{x,T^x_m} is given by

$$\partial \pi_{3,T^x_m}/\partial u_{x,T^x_m} = f_4(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m})\left(\frac{\mu_{x,3}-\mu_{x,4}}{(\sigma^-_{x,3})^2}\right) \tag{50}$$

where

$$f_4(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}) = \kappa_4(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m})/(1+\kappa_4(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}))^2,$$

and

$$\begin{aligned}\kappa_4(u_{d,T^x_m}, u_{x,T^x_m}, \tilde{\pi}_{3,T^x_m}) = & \frac{(1-\tilde{\pi}_{3,T^x_m})}{\tilde{\pi}_{3,T^x_m}} \exp\left(\frac{(\mu_{d,3}-\mu_{d,4})^2(1-2\tilde{\pi}_{3,T^x_m})}{2\sigma_d^2}\right) \exp\left(\frac{\mu_{d,4}-\mu_{d,3}}{\sigma_d^2} u_{d,T^x_m}\right) \\ & \exp\left(\frac{\mu_{x,4}-\mu_{x,3}}{(\sigma^-_{x,3})^2}\left(u_{x,T^x_m} - (\frac{\mu_{x,3}+\mu_{x,4}}{2} - \tilde{\mu}_{x,T^x_m})\right)\right). \end{aligned} \tag{51}$$

Note that κ_4 and f_4 are positive-valued functions. Then, $\partial \pi_{3,T^x_m}/\partial u_{x,T^x_m} > 0$. Given that $\lambda_3 > \lambda_4$, this immediately implies that $\partial r^*_{T^x_m}/\partial u_{x,T^x_m}$ is positive.

Estimation

Note that the first three central moments of the external signal process in state j are given by:

$$\begin{aligned} E[x_m - \mu_{x,j}] &= \frac{\sigma^+_{x,j} - \sigma^-_{x,j}}{\sqrt{2\pi}} \\ E[(x_m - \mu_{x,j})^2] &= \frac{(\sigma^+_{x,j})^2 + (\sigma^-_{x,j})^2}{\sqrt{2}} \\ E[(x_m - \mu_{x,j})^3] &= \frac{2}{\sqrt{\pi}}(\sigma^+_{x,j} - \sigma^-_{x,j})((\sigma^+_{x,j})^2 + (\sigma^-_{x,j})^2 + \sigma^+_{x,j}\sigma^-_{x,j}) \end{aligned}$$

To estimate the parameters of the external signal process in state j, we use the sample analogs of the first three central moments of the external signal process given above and solve numerically for the three unknown parameters, $\mu_{x,j}$, $\sigma^+_{x,j}$ and $\sigma^+_{x,j}$ while imposing the restriction that $\sigma^+_{x,j}$ and $\sigma^-_{x,j}$ are strictly positive. Parameter estimates based on the method of moments tend to be somewhat sensitive to outliers. Hence, we estimate the parameters after removing the minimum observation in good times and the maximum observation in bad times.

We estimate the standard errors of the parameters of the external signal process in state j based on bootstrapping. Specifically, we create a bootstrapped sample of the same size as the original sample of external signals in state j by randomly drawing (with replacement) one observation at a time from the original sample. We then estimate the parameters of the external signal process based on the bootstrapped sample. We repeat this 1,000 times. The standard error is then calculated as the sample standard deviation of the distribution of the parameter estimates based on the 1,000 bootstrapped samples. The hypothesis tests are also based on the same bootstrapping approach.