Volunteering for heterogeneous tasks [1]

Nicolas Sahuguet

Université Libre de Bruxelles[2]

July 8, 2005

[1] I thank George Mailath and Steven Matthews for helpful conversations. I am aslo grateful to the associate editor and an anonymous referee for useful comments. All errors are mine.

[2] ECARES, 50 Av. Franklin Roosevelt, CP 114, 1050 - Brussels Belgium. E-mail: sahuguet@ulb.ac.be.

Abstract

We model the search for volunteers as a war of attrition. Every player is tempted to wait for someone else to volunteer for the tasks. When tasks are not equivalent, it may be optimal to volunteer quickly to perform an easy task. We analyze the trade-off between volunteering for an easy task and taking the risk of having to perform a more strenuous task in order to get the chance of avoiding all tasks. When the cost of waiting is borne by agents until every task has found a volunteer, we show that it may be optimal to volunteer for the difficult task even if an easier task is available, in order to speed up the process and reduce the costs of waiting.

Keywords: Public Goods, Volunteering, Games of Timing, Preemption Game, War of attrition.

JEL Classification Numbers: H41, D44, C73, D81

1 Introduction

Communities and institutions often rely on volunteers for the provision of public goods. Since many public services are performed most efficiently by a single individual, one person bears the entire cost of providing a service that benefits the whole community. The allocation of chores among a household is an example of this type of strategic interaction. Individuals have strong incentives to let someone accept the job, which often leads to a waiting game. The longer one waits, the more likely it is that someone else will do the job. However, until somebody has volunteered, everyone has to wait, which is a cost to the community. Bliss and Nalebuff (1984) analyze this type of waiting game using a war of attrition:[1] each individual decides how long to wait for someone else to volunteer before deciding to concede and provide the service himself. Bliss and Nalebuff show how time plays the role of a screening device in the presence of private information about the costs of providing the service. They derive a symmetric equilibrium in which individuals choose a waiting time that is positively related to their provision costs. However, their model analyzes the provision of only one service. In most situations that come to mind, it is natural to consider the case in which several services have to be provided. Revisiting the household example, cleaning rooms, buying the groceries, and fixing a broken device are common chores that need to be performed. Few papers consider the issue of providing multiple services. Bulow and Klemperer (1999) analyze a generalized war of attrition with multiple players and multiple prizes. Their analysis applies to the case of many agents who need to volunteer for similar tasks. They develop the example of the call for volunteers in a university department to serve on a committee. Lacasse et al. (2002) is another paper that looks at the provision of multiple public goods in the context of a

[1] Another similar model is Bilodeau and Slivinski (1996). They analyze a model with complete information and a finite horizon and derive a unique equilibrium in which the agent with the highest benefit/cost ratio volunteers immediately.

war of attrition. Agents can volunteer for more than one task. However, since agents can only perform one task at a time, volunteering is a credible commitment not to take on another task until they have completed the first. The main insight from that paper is the analysis of the strategic trade-off that the ability for commitment and the multiplicity of tasks provide.

Our focus is different. In most contexts in which several public goods must be provided voluntarily, it is natural to think that tasks are not equivalent, in the sense that the cost varies from task to task. In the university example, the head of the department may need one person to chair the graduate program, another to organize seminars, and perhaps several persons to serve on the recruiting committee. It is clear that these tasks require different amounts of effort. While the case of heterogeneity of tasks is mentioned in the literature, it is never explicitly introduced in the model. The present model introduces this new dimension in the strategic interaction between agents.

We analyze this trade-off in the simplest set-up. Three individuals have to volunteer to perform two public services. The tasks are not equivalent; one task is more difficult and thus more costly. It is possible to volunteer for either of the two tasks. Individuals have to wait until both tasks have found a volunteer and waiting is costly. The cost for an individual to perform each task is private information. This environment shares features with two well-known timing games analyzed in the economic literature: attrition and preemption. In a war of attrition, each player prefers others to act before him. In a game of preemption, the situation is reversed, with players preferring to act before others.[2].

In a set-up such as that just outlined, there are countervailing incentives. On the one hand, people want to wait and volunteer as late as possible to avoid the cost of providing a service. On the other hand, volunteering early reduces the risk of having to perform the most difficult task. Depending on assumptions about the relative costs of performing

[2] A well-known example of a preemption game is the game known as "grab the dollar" in which the first player to move gets a dollar, but players do not get anything if they move simultaneously.

tasks, we identify several equilibria:

- When the two tasks are almost identical, the agents behave as if the tasks were the same, and time plays the role of a screening device. That is, individuals with a high cost of performing the services wait a relatively long time before volunteering. The game comes to resemble a war of attrition.

- When the difference between the tasks is large enough (in a sense that is made clear later), there can be a frenzy to volunteer. It is more valuable to avoid the difficult task than to wait to try to avoid all tasks. The cost of volunteering early for the easy task is low compared with the possible cost of the difficult task. This frenzy to volunteer displays the features of a preemption game. Once one player has volunteered during the initial frenzy, the game becomes a war of attrition between the two remaining players.

- We also identify equilibria with partial frenzy, in which some players prefer to volunteer immediately while others prefer to wait. For some types, the preemption motive dominates and they are willing to volunteer immediately. For other types, preemption is not attractive; they prefer to wait and enter into a war of attrition. This depends on the relative costs of undertaking the tasks.

- Another interesting strategic behavior can arise in equilibrium. When the cost of waiting is borne by agents until every task has found a volunteer, it may be optimal to volunteer for the difficult task even if an easier task is available. The rationale behind this seemingly paradoxical behavior is that choosing the difficult task speeds up the volunteering process and reduces the costs of waiting for other players to volunteer.

Our analysis has application wider than the strategic provision of public goods. In industrial organization, the war of attrition has been used to analyze the theory of market exit (Fudenberg and Tirole, 1986). It is often the case that many firms compete in a natural oligopoly. The market can only support a few firms, and firms incur losses until the number of active firms has decreased to the point at which they make a profit. However, in many situations, it is possible to exit the main market early to concentrate on a niche in a less attractive but related market. The decision of an early exit in the hope of securing a profitable share in a related market is similar to the decision to volunteer early in our model. The case of the software industry is of particular interest. Ten years ago, there were over 300 general-purpose word processors on the market for Windows and the Mac; clearly too many for all manufacturers to show a profit. Now, almost all of those manufacturers have been forced out of the market, with only WordPerfect and MS Word remaining. However, a number of firms chose to give up the fight early, before they were forced out, choosing instead to enter the niche market of developing word processors with specialized uses, such as Scientific Word. The present analysis can help in the analysis of early exit behavior in hard fought battles for standards. Our results are also related to the phenomenon of entry in oligopoly with more than two firms. When there is a last mover advantage, the entry game is a war of attrition; but as in our game, being the first to move is often better than being second since the first entrant will enjoy a period of temporary monopoly. Furthermore, there is also the possibility that the first mover will be able to influence the remaining war of attrition between the other players, since the entry decision can incorporate an investment decision that influences future payoffs.

We believe our paper is the first example of a game that is neither just a preemption game nor a war of attrition. A paper, that is closely related to this aspect of our paper, is Park and Smith (2003). They independently developed a general theory of timing games that incorporate incentives for both attrition and preemption. They call this class of game "timing games with rank-order payoffs". Their analysis is more general but differs in several dimensions from the present paper. First, there is no private information about payoffs. Second, they analyze the case of unobservable actions, while in our model, players condition their actions on the past behavior of other players. Finally, our model is not formally a game with rank-order payoffs. The first player to volunteer is able to choose his task. The payoff is thus not directly related to the rank but comes, to a certain extent, endogenously from players' decisions. In that respect, the present model is not a pure stopping game and thus differs from the games analyzed by Park and Smith (2003). Their model offers additional insights into sudden rushes followed by relative quiet. In particular, their model admits equilibria with interior atoms. This interesting behavior cannot be generated in the framework presented in this paper. Their framework is well suited to develop further applications of such timing games in economics.

The paper is organized as follows. Section 2 presents the model. Section 3 reviews the war of attrition between two players and presents some preliminary results. Section 4 analyzes a monotonic equilibrium in which the first player to volunteer performs the easy task. Section 5 extends the analysis to the case in which the first player to volunteer rationally decides to perform the difficult task. Section 6 discusses other types of equilibria with partial or complete frenzy to volunteer. Section 7 places the present paper in context regarding the literature, in particular, with respect to analyses of decentralization versus authority. Section 8 concludes. Proofs omitted in the text can be found in the appendix.

2 The model

Consider a group of three individuals indexed by $i \in \{1, 2, 3\}$ who have to perform two[3] public services. These two tasks are not equivalent: the costs of performing them are different. Each agent has a private type θ, drawn independently from distribution $F(\theta)$, with $F(\theta_{\min}) = 0$ and $F(\theta_{\max}) = 1$. We assume $F(\cdot)$ has a derivative $f(\cdot)$, that is strictly positive and finite everywhere. It will be convenient to write $h(\theta)$ for the hazard rate

[3] Analyzing a model with more players would not add any major insights to the strategic trade-offs and would be more cumbersome.

$\frac{f(\theta)}{1-F(\theta)}$. Public services bring benefits to all individuals over a period of time. Let $B(\theta)$ denote the discounted benefits of the services. Performing a service necessitates effort over a period of time, and the total discounted cost of exerting effort, are respectively $c(\theta)$ and $C(\theta)$ for the easy and the difficult task. Agents can volunteer for either of the two tasks at any time[4]. As long as the allocation of tasks is not finished, agents pay an opportunity cost of waiting per unit of time. Following Bulow and Klemperer (1999), we normalize to 1 the cost of waiting of an individual who has yet to volunteer. He subsequently pays a cost of γ until another volunteer is found. This means that the first individual to volunteer pays a waiting cost until another volunteer is found for the remaining task. For most of the analysis, we will consider the case where $\gamma = 1$. This corresponds to the "standards case" in the Bulow and Klemperer analysis. As an example, consider a faculty meeting. The head of the department has called three junior members of faculty for a meeting, one evening. There are two administrative tasks to be performed in the following semester. He lets them volunteer for these tasks. Once both tasks have been allocated, the meeting is over and everybody can go home.

It is easy to recast this formulation in a simpler model in which players compete for two heterogeneous prizes. The gross payoff for performing the difficult task is $B(\theta) - C(\theta)$. If we normalize this to zero, the gross payoff for performing the easier task is $B(\theta) - c(\theta)$ and the gross payoff for performing no task is $B(\theta)$. Denoting $v(\theta) = C(\theta) - c(\theta)$ and $V(\theta) = C(\theta)$, we get the following correspondence between prizes and tasks. Getting no prize corresponds to performing the difficult task, getting the small prize corresponds to performing the easier task, and getting the large prize corresponds to performing no task. Hence, $V(\theta)$ and $v(\theta)$ are the payoffs (gross waiting costs) associated with the large prize and the small prize, respectively.

Players can decide at any time whether or not to concede, and which prize to choose if

[4] It is not possible to volunteer to do nothing!

they do. Their decision may depend on past actions. Information is continuously revealed in equilibrium. If no one has conceded at time t, players draw inferences about the type of the remaining players. However, this process is predictable. Players know that their decision to concede is relevant to the extent that they do not observe any concession by another player. However, they can predict in advance what they learn. Hence, a strategy for a player can be summarized by a (type-dependent) concession time in the three-player stage and the prize he would select and a concession time in every possible two-player subgame. Such subgames are characterized by the remaining prizes and the updated beliefs about the other player's type. It is useful to note that even if some histories are off the equilibrium path, beliefs can always be computed according to Bayes' rule. A deviation from the equilibrium strategies can only be observed if a player has conceded (either he conceded at the wrong time or he chose the wrong prize), but then he is not in the game anymore. The beliefs consist in an updating of the probability distribution of the type of the remaining players. Since they did not take any action, the updating will simply eliminate the types that should have conceded before the actual concession occurred.

We restrict attention to symmetric (Bayesian) equilibria. There exist, of course, asymmetric equilibria of this game, but they are quite unrealistic. We follow Farrell and Saloner (1988) and Bolton and Farrell (1990), who argue that asymmetric equilibria are unconvincing and inappropriate for the study of decentralized coordination mechanisms, such as the one analyzed in our framework. Moreover, symmetric equilibria capture the anonymity of play that is natural in many contexts.

3 Preliminary results: the two-player war of attrition

After a concession, two players remain and need to volunteer for the second task: this is a standard war of attrition. Equilibrium behavior in this game has been analyzed in the literature. We review here some results that constitute the building blocks for the characterization of equilibrium in the general model. Players compete for a prize of value $W(\theta)$, and $G(\theta)$ is the distribution of types. Lemma 1 describes equilibrium behavior: waiting times are increasing in type. Lemma 2 characterizes the expected profit of an agent of type θ. Lemma 3 calculates the expected length of the war of attrition. It corresponds to the cost associated with such a contest. Proofs are standard and are provided in the appendix.

Lemma 1 *The unique symmetric Bayesian Nash equilibrium of the two-player war of attrition is characterized by a stopping time $T(\theta)$:*

$$T(\theta) = \int_{\theta_{\min}}^{\theta} W(x)h(x)dx.$$

The interpretation is the following. At each moment, the marginal type θ has to be indifferent between conceding and waiting a little longer to outlast types between θ and $\theta + d\theta$. The cost of waiting more, which corresponds to $T'(\theta) \cdot d\theta$, must be equal to the benefit of waiting more, that is the value of winning $W(\theta)$ times the probability $\frac{f(\theta)}{1-F(\theta)} \cdot d\theta = h(\theta) \cdot d\theta$, that the other player concedes in this time interval.

Lemma 2 *The expected surplus of an individual of type θ in the two-player war of attrition is*

$$S(\theta) = \int_{\theta_{\min}}^{\theta} W'(x)F(x)dx.$$

Lemma 3 *The expected length $\bar{T}$ of the two-player war of attrition is equal to:*

$$\frac{1}{2}E[\min(W(\theta_1), W(\theta_2))] = \int_{\theta_{\min}}^{\theta_{\max}} W(x)f(x)(1-F(x))dx.$$

The two-player war of attrition is an optimal auction in the sense that the prize always goes to the highest type and that the surplus of the lowest type is zero. The Revenue Equivalence Theorem (See Myerson (1981) or Riley and Samuelson (1981)) applies. The expected cost per player must be the same in the war of attrition as in a second-price (Vickrey) auction. The expected cost per player in the war of attrition, which is exactly the expected duration of the war, is then equal to half the expected price paid by the winner in a second price auction. The price paid by the winner in the second-price auction is the expected value of the smaller bid or the expected value of the minimum of the players' valuation. It is useful to note that the more difficult the task is to perform, the more time it takes to find a volunteer.

4 A bird in hand is worth two in the bush

As mentioned in the introduction, many considerations enter into the players' decisions to volunteer. First, there is a trade-off between volunteering quickly for the easy task and waiting longer at the risk of having to perform the difficult task. It is natural to look for an equilibrium in which time plays the role of a screening device. After a concession, high types know they have a good chance of winning the subsequent war of attrition. Thus, they are willing to wait longer than low types. The second important decision is the choice of task once an agent has conceded. He can choose to perform the easy task or the difficult task. It may seem obvious that he would choose the easy task. However, if the difficult task has the obvious drawback of being more costly, choosing it speeds up the subsequent war of attrition , thus reducing the waiting costs.

In this section, we characterize a monotonic symmetric equilibrium in which the first individual to concede volunteers for the easy task, and we derive conditions under which such an equilibrium exists. In equilibrium, waiting times are strictly increasing in types. Thus, there is a one-to-one mapping between types and waiting times. Beliefs are updated in a simple way. After observing a concession, remaining players compute the type θ^* that corresponds to the observed retiring time. The updated belief about the type of the remaining player is characterized by the posterior distribution $F^*(\theta)$, which is the truncation of the initial distribution at the point corresponding to the type that has conceded:

$$F^{*}(\theta)=\begin{cases}0 \text{ if } \theta<\theta^{*} \\ \frac{F(\theta)-F(\theta^{*})}{1-F(\theta^{*})}\end{cases}$$

Proposition 1: When the difference between prizes is large enough $(V(\theta)\geq 2v(\theta))$ and no type is impatient $(v(\theta)\geq\int_{\theta}^{\theta_{\max}}v(x)f(x)\frac{(1-F(x))}{(1-F(\theta))^{2}}dx$, for any $\theta)$, the unique symmetric perfect Bayesian equilibrium is characterized by stopping time $t(\theta)$ associated with the choice of the small prize and a stopping time $T(\theta,\theta^{*})$ in the subgame following a concession at time $t(\theta^{*})$:

$$\begin{aligned} t(\theta) &= \int_{\theta_{\min}}^{\theta}(\gamma V(x)-2v(x))h(x)dx, \\ T(\theta,\theta^{*}) &= \int_{\theta^{*}}^{\theta}V(x)\,h(x)dx. \end{aligned}$$

Proof : In the Appendix.

To understand this theorem, let us consider the basic trade-off that a player faces in deciding whether or not to delay his concession.

Rewards and costs of delay

The first-order condition derived in the appendix yields:

$$t^{'}(\theta)=(\gamma V(\theta)-2v(\theta))\cdot h(\theta).$$

At each moment, the marginal individual of type θ has to be indifferent between conceding and waiting a little longer to outlast types between θ and $\theta+d\theta$. The left-hand side corresponds to the marginal cost of delay. The right hand-side corresponds to the strategic effects of delay. In the usual war of attrition, there is a strategic benefit of delay that comes from an increase in the probability of winning the prize. Here, two effects are at play.

The first effect $(-2h(\theta)\cdot v(\theta))$ is negative: it corresponds to the preemption motive; delay increases the probability that another player concedes and secures the small prize.

The second effect $(\gamma V(\theta) \cdot h(\theta))$ is positive. Delay reduces the expected length, and thus the cost, of the war of attrition for the prize V. This second effect comes from the fact that after an initial concession, there is less competition (only two players remain) and the incentive to concede to secure the small prize no longer exists. This leads to a slower rate of concession.

Note that there is no direct marginal benefit of delay that comes from a higher probability of getting the large prize. The reason is that at the margin, delaying one's concession is not sufficient to outlast the two other players.

Monotonicity

A monotonic equilibrium exists only if the marginal strategic benefits of delay are positive and compensate for the exogenous costs of delay. Inspecting $t(\cdot)$, a monotonic equilibrium obtains if $\gamma V(\theta) \geq 2v(\theta)$. This condition ensures that there is a large enough difference between the prizes. Returning to the interpretation in terms of tasks, this condition states that the cost of the easy task should not be too different from the cost of the difficult task. If the small task is too easy, agents do not take the risk of performing the difficult task and prefer to volunteer immediately for the easy task. To see this more clearly, consider the following limit cases. When $v = 0$, the monotonicity condition is satisfied. There is no gain in conceding quickly to secure the small prize; this is essentially a war of attrition for one prize, for which it is known that the equilibrium is monotonic. When $v(\theta) = V(\theta)$, monotonicity cannot be satisfied. If a player can be sure of gaining a prize equivalent to the large prize without waiting, time cannot be a screening device.

Patience

An important feature of the war of attrition is that the more valuable is the prize, the longer it takes, on average, for someone to concede. This is evident from the formula of the length of a two-player war of attrition derived in lemma 3. An agent influences, by his choice of task, the length of the subsequent war of attrition. Consequently, a trade-off exists between the gross payoff coming from the choice of prize and the waiting cost that is borne when waiting for one more person to volunteer. An individual of type θ, who has just conceded, prefers to take the small prize rather than no prize if the value of the prize is larger then the reduction of the cost of delay: $v(\theta) > \int_{\theta}^{\theta_{\max}} v(x) f(x) \frac{(1-F(x))}{(1-F(\theta))^2} dx$.

The interpretation of this condition is clear. The left-hand side $v(\theta)$ represents the immediate benefit of volunteering for the easy task rather than the difficult task. The right hand side represents the expected length reduction of a war of attrition in which the remaining players are fighting to avoid doing the easy task rather than fighting to avoid the difficult task. When this condition is not satisfied, some types prefer to volunteer for the difficult task (impatient types) while other prefer to volunteer for the easy task (patient types). Impatient types are types for which the opportunity cost of time is more important than the cost of performing the tasks. Impatient types exist when $v(\theta)$ is small for low types ($\theta_{\min}$ in particular) while it is relatively large for high types ($\theta_{\max}$) [5].

5 Impatient volunteers

Now consider an environment in which some types are impatient, i.e. they prefer to volunteer for the difficult task even when the easy task is available. The rationale behind this seemingly paradoxical behavior is that the loss in terms of disutility of effort is more than compensated for by the benefit of reducing the time waiting for the second volunteer, because a volunteer for an easy task will be found more quickly than for a difficult one. Note that impatience is defined by the task chosen after the initial concession. It does not depend on the behavior of other players in the three-player game. so we do not need to know the features of equilibrium to define impatient types.

The behavior of patient types is still characterized by the first-order conditions $t'(\theta) = (\gamma V(\theta) - 2v(\theta)) \cdot h(\theta)$. The behavior of impatient types is different since the trade-off

[5] The real comparison is between the value of the prize for the lowest type compared to half the second order statistic out of 2 draws from the distribution.

between the cost and benefits of delay is different. The first-order condition (derived in the appendix) is now characterized by $t'(\theta) = (\gamma(V(\theta) - v(\theta)) + 2v(\theta)) \cdot h(\theta)$.

In equilibrium, the rate at which types volunteer depends on two things: the hazard rate and the patience of the types that are supposed to concede at that time. The concession rate varies according to whether types are patient or impatient. The exact description of equilibrium can be complicated if there are many intervals of patient and impatient types. For concreteness, we derive a specialized example that illustrates the role of impatient types.

Example 1 : (Uniform distribution and linear valuation)

Suppose types are distributed uniformly on $[0,1]$ and suppose $V(\theta) = 3\theta$, $v(\theta) = \theta$ and $\gamma = 1$. A type is patient if $\theta \geq \int_\theta^1 x \frac{(1-x)}{(1-\theta)^2} dx$; that is, if $\theta \geq \frac{1}{4}$.

Note that since $V(\theta) \geq 2v(\theta)$, the condition for a monotonic equilibrium is fulfilled.

The only symmetric Perfect Bayesian Equilibrium is characterized by the following stopping times:

$$
\begin{aligned}
t(\theta) &= -4(\theta + \ln(1-\theta)) \text{ for } \theta \leq \frac{1}{4} \\
t(\theta) &= -3(\theta + \ln\left(\frac{3}{4}\right) - (\theta + \ln(1-\theta)) \text{ for } \theta \geq \frac{1}{4}.
\end{aligned}
$$

A patient type ($\theta \geq \frac{1}{4}$) volunteers for the easy task while impatient types ($\theta \leq \frac{1}{4}$) volunteer for the difficult task.

The stopping time after someone has volunteered for the difficult task is $T(\theta, \theta^*) = -2\left((\theta - \theta^*) + \ln\left(\frac{\theta}{\theta^*}\right)\right)$. The stopping time after someone has volunteered for the easy task is $T(\theta, \theta^*) = -3\left((\theta - \theta^*) + \ln\left(\frac{\theta}{\theta^*}\right)\right)$.

Impatient types volunteer more quickly in the sense that the rate at which types volunteer is faster than if the same types were patient (or forced to volunteer for the easy task). This speeds up the process. In addition, after a concession by an impatient type, a volunteer for the easy task can be found more quickly. Hence, the presence of impatient types has the consequence of speeding up the volunteering process. From the point of view of efficiency also, the presence of impatient volunteers is a good thing because when an impatient type volunteers, efficient screening obtains. The tasks are performed by those agents that have lower costs.

It is possible to extend the result derived in this specialized example to a more general setting. The difficulty is that when many intervals of patient/impatient types exist, the description of equilibrium is cumbersome. We thus restrict attention to the case in which the type space can be divided in two, between the low impatient types $\theta \leq \theta^p$ and the patient high types.

Proposition 2: When types $\theta \in \left[\theta_{\min}, \theta^P\right]$ are impatient, types $\theta \in [\theta^p, \theta_{\max}]$ are patient, and the monotonicity condition $V(\theta) > 2v(\theta)$ is satisfied for patient types. The unique symmetric perfect Bayesian Nash equilibrium is characterized by a stopping time $t(\theta)$ and stopping time $T(\theta, \theta^*)$ in the subgame following a concession at time $t(\theta^*)$. Impatient types choose no prize after a concession while patient types choose the small prize.

$$
\begin{aligned}
t(\theta) &= \int_{\theta_{\min}}^{\theta} ((\gamma V(x) - v(x)) + 2v(x))h(x)dx \text{ for } \theta \leq \theta^p, \\
t(\theta) &= \int_{\theta_{\min}}^{\theta^p} ((\gamma V(x) - v(x)) + 2v(x))h(x)dx + \int_{\theta^p}^{\theta} (\gamma V(x) - 2v(x))h(x)dx \text{ for } \theta \geq \theta^p,
\end{aligned}
$$

$$
T(\theta, \theta^*) = \int_{\theta^*}^{\theta} (W(x))h(x)dx \text{ with } W(x) = V(x) \text{ or } W(x) = V(x) - v(x).
$$

6 A frenzy to volunteer

So far, we have examined monotonic equilibria in which time plays the role of a screening device. It remains to study the interesting case in which monotonic equilibria do not exist. When the cost of performing the easy task is very low relative to that of performing the difficult task, volunteering for the easy task becomes very attractive. The behavior in such an environment is dominated by a motive for preemption. As in a "grab the dollar" game, players concede immediately to secure the available prize. This leads to a frenzy to volunteer. When every type of player participates in this frenzy, we have a pooling equilibrium: every individual volunteers immediately for the easy task. This frenzy can also be limited to low types who volunteer immediately, while higher types prefer to wait longer. In this semi-pooling equilibrium, if there is no immediate concession, the behavior is similar to the monotonic equilibria derived in the previous sections. To simplify the notation, we will consider throughout this section that $\gamma = 1$.

6.1 A complete frenzy: pooling equilibrium

When the cost of performing the easy task is low relative to the cost to perform the difficult task, volunteering for the easy task becomes very attractive. All players, regardless of their type, volunteer immediately. The equilibrium is characterized by the following stopping times:

$$\begin{aligned} t(\theta) &= 0, \\ T(\theta) &= \int_{\theta_{\min}}^{\theta} V(x)\, h(x) dx. \end{aligned}$$

Players volunteer immediately for the easy task. One of them is awarded the easy task, the two remaining players then enter into a war of attrition to decide who perform the difficult task. Since everybody behaves in the same way, players do not learn anything in the first part of the game: there is no updating of beliefs. In equilibrium, with probability 1/3, a player gets the small prize and waits the expected length of the continuation game, or with probability 2/3, he does not get the prize, and gets his type's expected surplus in the two-player war of attrition. His expected payoff is then

$$\frac{1}{3}(v(\theta) - \int_{\theta_{\min}}^{\theta_{\max}} V(x)\, f(x)(1 - F(x))dx) + \frac{2}{3}\int_{\theta_{\min}}^{\theta} V'(x)\, F(x)dx.$$

The only type of deviation that can be beneficial is not to concede immediately. This deviation yields a payoff equal to the surplus in the two-player war of attrition

$$\int_{\theta_{\min}}^{\theta} V'(x)\, F(x) dx.$$

Hence, for a pooling equilibrium to exist, the following incentive constraint needs to be satisfied:

$$\forall \theta,\ v(\theta) \geq \int_{\theta_{\min}}^{\theta} V'(x)\, F(x) dx + \int_{\theta_{\min}}^{\theta_{\max}} V(x)\, f(x)(1-F(x)) dx.$$

6.2 Limited frenzy: semi-pooling equilibrium

Volunteering for the easy task may not be very attractive for the highest types. They know that by waiting, they will be in a war of attrition in which they have a good chance of being the highest type and thus of winning the large prize. Inspecting the incentive constraint related to the pooling equilibrium, we see that if $V'(\theta)$ is larger than $v'(\theta)$, it can be the case that the constraint is satisfied for low types but not for high types. The logic behind a semi-pooling equilibrium is that the existence of a monotonic equilibrium depends on the condition $V(\theta) > 2v(\theta)$ for any θ. It is possible that the condition is satisfied for types larger than a type $\bar{\theta}$. In this case, a monotonic equilibrium is still possible when the appropriate lowest part of the distribution has been "eliminated". Therefore, a semi-pooling equilibrium exists when types lower than $\bar{\theta}$ volunteer immediately for the easy task, while higher types wait. If nobody volunteers immediately, beliefs are updated and equilibrium is similar to the monotonic equilibrium in a game with type distributions truncated from below at $\bar{\theta}$.

It becomes necessary to specify what happens when more than one player volunteers immediately. As before, one of the players who volunteers is chosen randomly(with equal probability) for the easy task. The remaining players keep on playing the game as if they had not conceded. What is important is the information revealed. We make the assumption, natural in the context of the game, that players observe whether a player remaining in the game wants to concede.

Proposed equilibrium strategies are as follows:

$$\begin{aligned}
\forall\theta &\leq \bar{\theta},\ t(\theta) = 0, \\
\forall\theta &\geq \bar{\theta},\ t(\theta) = \int_{\bar{\theta}}^{\theta} (V(x) - 2v(x))h(x)dx, \\
\forall\theta &\geq \theta^*,\ T(\theta, \theta^*) = \int_{\theta^*}^{\theta} V(x)\, h(x)dx.
\end{aligned}$$

There are two necessary equilibrium conditions. First, the monotonicity condition must hold for types larger than $\bar{\theta}$, $V(\theta) > 2v(\theta)$. Second, the incentive constraints for immediate volunteering must hold for types lower than $\bar{\theta}$. The payoff associated with immediate volunteering is:

$$F(\bar{\theta})^2 \left(\frac{v(\bar{\theta}) - T^-(\bar{\theta}) + 2S^-(\bar{\theta})}{3} \right) + F(\bar{\theta})\left(1 - F(\bar{\theta})\right) v(\bar{\theta}) + \left(1 - F(\bar{\theta})\right)^2 \left(v(\bar{\theta}) - T^+(\bar{\theta})\right)$$

The payoff associated with waiting is:

$$F(\bar{\theta})^2 V(\bar{\theta}) + \left(1 - F(\bar{\theta})\right)^2 \left(v(\bar{\theta}) - T^+(\bar{\theta})\right)$$

The incentive constraint can be simplified:

$$F(\bar{\theta})^2 \left(\frac{v(\bar{\theta}) - T^-(\bar{\theta}) + 2S^-(\bar{\theta})}{3} \right) + F(\bar{\theta})\left(1 - F(\bar{\theta})\right) \left(v(\bar{\theta})\right) \geq F(\bar{\theta})^2 V(\bar{\theta}).$$

with $T^-(\bar{\theta})$ $\left(T^+(\bar{\theta})\right)$denoting the expected length of a war of attrition when types are below (above) $\bar{\theta}$ and $S^-(\bar{\theta})$ denoting the expected profits of type $\bar{\theta}$ when types are below $\bar{\theta}$.

6.3 Some parametric examples

To illustrate this frenzy to volunteer and to demonstrate that the conditions derived previously are not “superfluous”, we present numerical examples of pooling and semi-pooling equilibria. In what follows, we assume that types are uniformly distributed on $[1,2]$.

Example 2 : (Pooling Equilibrium)

When individuals value the prizes respectively $V(\theta) = \theta$ and $v(\theta) = .7 + \frac{1}{2}(\theta-1)^2$[6], a pooling equilibrium exists. We need to check that the incentive constraint satisfied. We get from section 2 the expected length of a two-player war of attrition and the expected surplus of a type θ:

$$\begin{aligned} S(\theta) &= \frac{1}{2}\frac{(\theta-\theta_{\min})^2}{(\theta_{\max}-\theta_{\min})} = \frac{1}{2}(\theta-1)^2, \\ \bar{T} &= \frac{1}{6}(2\theta_{\min}+\theta_{\max}) = \frac{2}{3}. \end{aligned}$$

The incentive constraint is: $v(\theta) \geq \bar{T} + S(\theta)$ or $v(\theta) \geq \frac{2}{3} + \frac{1}{2}(\theta-1)^2$. This condition is obviously satisfied for the given functions V and v.

Example 3 : (Semi-pooling equilibrium)

When individuals value the prizes respectively $V(\theta) = \theta$ and $v(\theta) = \frac{3}{5}$, a semi-pooling equilibrium exists with $\bar{\theta} = 1.211$. The monotonicity constraint is satisfied since $V(\theta) \geq 2v(\theta) = \frac{6}{5}$, for $\theta \geq \bar{\theta}$. The incentive constraint for immediate volunteering imposes that $\bar{\theta}$ be indifferent between volunteering and waiting. Note that lower types strictly prefer immediate volunteering while higher types prefer to wait. The payoff for volunteering is: $(\bar{\theta}-1)^2(\frac{1}{3}(\frac{3}{5}-\frac{1+2\bar{\theta}}{6})+\frac{2}{3}(\frac{\bar{\theta}-1}{2}))+2(\bar{\theta}-1)(2-\bar{\theta})\frac{1}{2}\frac{3}{5}v+(2-\bar{\theta})^2(\frac{3}{5}-\frac{\bar{\theta}+4}{6})$.

The payoff to wait is $(\bar{\theta}-1)^2\bar{\theta}+(2-\bar{\theta})^2(\frac{3}{5}-\frac{\bar{\theta}+4}{6})$. Simple algebra yields $\bar{\theta} = 1.211$.

We have a semi-pooling equilibrium in which types below $\bar{\theta}$ concede immediately. If nobody has conceded immediately, then the remaining players know there is no type below $\bar{\theta}$ and starts playing the monotonic equilibrium for types uniformly distributed between $\bar{\theta}$ and 2.

[6] It is easy to check that in this particular example, all types are patient since $\int_{\theta}^{2}\frac{(2-x)}{(2-\theta)^2}v(\theta)\,dx < v(\theta)$ for $1<\theta<2$.

7 Decentralization vs authority

The trade-off between efficiency in the distribution of tasks and the time wasted for this allocation is well-known in the literature. In particular, Bolton and Farrell (1990) contrast the advantages of decentralization (in terms of finding low-cost solutions) with the potential costs of delay. They argue that, when speed is more important than the marginal gain of finding an efficient solution, the decision should be made and imposed by a central planner. In the case of the public provision of services, a planner is able to influence the nature of the interaction in a number of ways . Pure authority leads to a simple decision by the central planner about the allocation of tasks.[7] This solution entails no delay but the planner must make uninformed decisions. He can also call for volunteers, as in the present model, in an attempt to elicit information. In this case, the planner can influence strategic interactions by choosing the rules of the volunteering game. In particular, imposing the order of the tasks to be volunteered for can change equilibrium behavior. Similarly, allowing or an agent to leave (or prohibiting him from doings so) after having volunteered, (i.e. to lower the cost γ of waiting after volunteering) has important consequences. In particular, these rules modify the time spent waiting, as well as the allocation of tasks. Analyzing these different variations from the perspective of design is beyond the scope of this paper. However, it is worth noting that the "optimal" way of allocating tasks would depend on the objective function of the planner, as well as the information he has about the environment. A planner can value the welfare of the individuals and would therefore minimize their waiting cost and maximize an efficient allocation of tasks. However, the planner himself presumably bears a waiting cost, and as a consequence is interested in minimizing the length of the game rather than the sum of the waiting cost borne by the individuals. To design such a mechanism, the planner would need to know, not only the distribution of types, but also the *value* of the different tasks.

[7] The planner designates "volunteers".

It is therefore natural to think that very simple mechanisms, such as those described in this paper, are likely to emerge and be used in practice. We hope our results shed some light on the effects of changing the rules of a volunteering game on the allocation of tasks and on the time needed to reach this allocation.

In the present paper, we restrict attention to environments in which players are constrained by the absence of transfers or contracts. Of course, a mechanism designer could improve the allocation of tasks by designing a Vickrey-Clarkes-Groves mechanism that would implement efficient allocation through a message game and appropriate transfers. In fact, the mechanism would look like a multi-unit Vickrey auction in which players would submit bids in order to avoid the tasks. We follow the literature on the private provision of public goods in which time is the only available screening device. We believe that in many environments (household chores for example), the absence of transfers is a natural assumption.

Some of the features of our results are similar to the analysis of Jehiel and Moldovanu (1995) of the link between negative externalities and delays in negotiations. Here, the waiting time in the ensuing war of attrition can be interpreted as a negative externality that the two remaining players exert on the first volunteer. In a monotonic equilibrium with impatient types, the first player to volunteer chooses the difficult task. It could look more logical to volunteer immediately; however, each player would prefer the others to volunteer for the difficult task. The link between the externality imposed by the need to wait for the second volunteer and the behavior in the three-player game is subtle since it can influence both the delay and the actions of the players.

8 Conclusion

We have analyzed a model in which individuals must volunteer to undertake tasks. These tasks yield collective benefits and impose private costs on the individual who carries them out. In such a framework, everyone is tempted to simply wait for someone else to volunteer. When there is only one task, the situation reduces to a war of attrition. In this paper, we extend the model to analyze situations in which many tasks, heterogeneous by nature, have to be undertaken, and we focus on the case in which the first volunteer can choose the task for which he volunteers. The situation becomes a generalized timing game that possesses features of both a war of attrition and a preemption game. Being the first to volunteer has value since the player can choose to undertake the easier task. However, letting others players volunteer has even greater value, since it allows one to enjoy all the benefits without undertaking any costly task. The present study makes two contributions. (1) We analyze the trade-off between these countervailing incentives. With private information, time can play the role of a screening device. When the difference between the costs to undertake different tasks is small enough, high types have a lot to gain, and in equilibrium, they wait longer than low types. When the difference between costs is large, time cannot screen types, volunteering for the easy task is very attractive, and there is a frenzy to volunteer in equilibrium. (2) We highlight an interesting yet seemingly paradoxical behavior. We show that, in some environments, it is optimal for a player to volunteer for a difficult task even though an easier task is available. This happens when the cost of choosing a difficult task is more than compensated by the reduction in waiting costs that results from this concession. We believe that this idea is quite general and applies, for instance, to the study of multilateral negotiations. A player can help to speed up the negotiation process between the other players by making an important concession.

9 Appendix

Proof of Lemma 1: Given that the other player follows the strategy, the expected utility of a type θ who behaves as a type $\hat{\theta}$ is

$$U\left(\theta,\hat{\theta}\right)=-(1-F(\hat{\theta}))T(\hat{\theta})+\int_{\theta_{\min}}^{\hat{\theta}}f(x)(W(\theta)-T(x))dx,$$

in which the first term represents the utility of the player when he is the first to concede and the second term the expected utility of the player in case he wins the prize. A necessary condition for equilibrium is that the derivative of the preceding expression is equal to zero when evaluated at θ.

$$\begin{aligned} -(1-F(\theta))T^{'}(\theta)+f(\theta)W(\theta) &= 0, \\ T^{'}(\theta) &= \frac{f(\theta)W(\theta)}{(1-F(\theta))}=W(\theta)h(\theta). \end{aligned}$$

The second order conditions are satisfied since $sign\left(\frac{\partial U(\theta,\theta)}{\partial\hat{\theta}}\right)=sign(\theta-\hat{\theta})$, and using the boundary condition $T(\theta_{\min})=0$, we get the desired result.

Proof of Lemma 2: By definition, $S(\theta)=\max_{\hat{\theta}}EU(\theta,\hat{\theta})=EU(\theta,\theta)$. Since all the functions are continuously differentiable, we can use the Envelope Theorem to get:

$$S^{'}(\theta)=EU_1(\theta,\theta)=\int_{\theta_{\min}}^{\theta}W^{'}(\theta)f(x)dx=W^{'}(\theta)F(\theta).$$

Integration yields:

$$S(\theta)=\int_{\theta_{\min}}^{\theta}W^{'}(x)F(x)dx+S(\theta_{\min}).$$

The surplus of the lowest type $S(\theta_{\min})$ is 0 since he gives up immediately. This leads to the desired result.

Proof of Lemma 3: This is an application of Revenue Equivalence (See Myerson (1981) and Riley and Samuelson (1981)). A war of attrition is an optimal auction in the sense that the prize always goes to the highest type and that the surplus of the lowest type is zero. The Revenue Equivalence Theorem applies, the cost to agents is the same in the war of attrition as in a second-price (Vickrey) auction. The expected cost per player in the war of attrition, which is exactly the expected duration of the war, is then equal to half the expected price paid by the winner of a second price auction. The price paid by the winner in the second-price auction is the expected value of the smaller bid or the expected value of the minimum of the players' valuation.

$$E[R_{SPA}] = E[\min(W(\theta_1), W(\theta_2))] = E[R_{WA}] = 2\bar{T}.$$

Hence :

$$\bar{T} = \frac{1}{2}E[\min(W(\theta_1), W(\theta_2))].$$

Proof of proposition 1:

Consider the subgame that starts after a concession. It is a war of attrition for prize $V(\theta)$ between two players with beliefs $F^*(\theta)$. Using results of section 3, the expected length of the two-player war of attrition and the expected surplus of each player are easily characterized. From lemma 1, we get $T(\theta, \theta^*) = \int_{\theta^*}^{\theta} V(x)\, h(x)dx$.

The expected payoff $U(\theta, \hat{\theta})$ of an individual of type θ using type $\hat{\theta}$ strategy when the other players are following equilibrium strategies is:

$$\begin{aligned} U(\theta, \hat{\theta}) &= [1 - F(\hat{\theta})]^2[v(\theta) - t(\hat{\theta})] - \gamma \int_{\hat{\theta}}^{\theta_{\max}} V(x)\, f(x)(1 - F(x))dx \\ &+ \int_{\theta_{\min}}^{\hat{\theta}} [2f(x)(1 - F(x))][S(\theta, x) - t(x)]dx. \end{aligned}$$

With probability $[1-F(\hat{\theta})]^2$, he is the first to concede. After waiting $t(\hat{\theta})$, he gets the prize $v(\theta)$ and then has to wait until the ensuing war of attrition is over. With complementary probability, another player concedes before him at time $t(x)$. In this case he gets $S(\theta, x)$ the expected surplus of a type θ player in the subgame starting after a type x player has conceded. A necessary condition for $t(\theta)$ be an equilibrium is that it is optimal for type θ to follow strategy $t(\theta)$ rather than mimic any other type $\hat{\theta}$, which means that the derivative of the preceding expression with respect to $\hat{\theta}$ must be zero when evaluated at θ. Taking the derivative with respect to $\hat{\theta}$ yields :

$$\begin{aligned}\frac{\partial U(\theta,\hat{\theta})}{\partial\hat{\theta}} &= [2f(\hat{\theta})(1-F(\hat{\theta})][t(\hat{\theta})-v(\theta)]-[1-F(\hat{\theta})]^2t'(\hat{\theta}) \\ &\quad -\gamma V(\theta)\,f(\hat{\theta})(1-F(\hat{\theta})+2f(\hat{\theta})(1-F(\hat{\theta}))][S(\theta,\hat{\theta})-t(\hat{\theta})].\end{aligned}$$

Evaluating at $\hat{\theta}=\theta$, and setting the derivative equal to zero we get:

$$\begin{aligned}t'(\theta)[1-F(\theta)]^2 &= \gamma V(\theta)\,f(\theta)(1-F(\theta))-2v(\theta)\,f(\theta)(1-F(\theta)), \\ \text{or } t'(\theta) &= (\gamma V(\theta)-2v(\theta))h(\theta).\end{aligned}$$

Furthermore, $t(\theta_{\min})=0$ since a player of type $\theta_{\min}$ exits immediately.

$$t(\theta)=0+\int_{\theta_{\min}}^{\theta}t'(x)dx=\int_{\theta_{\min}}^{\theta}(\gamma V(x)-2v(x))h(x)dx.$$

For these stopping times to constitute an equilibrium, two additional conditions need to be checked. Stopping times need to be monotonic. This is the case if $\gamma V(\theta)\geq 2v(\theta)$.

The second condition is that the first player to concede prefers to choose the small prize rather than no prize.

The payoff of a player of type θ who concedes and chooses prize v is

$$v(\theta)-\gamma\int_{\theta}^{\theta_{\max}}V(x)f(x|x\geq\theta)(1-F(x|x\geq\theta)\,dx.$$

The payoff of a player who concedes and chooses no prize is

$$-\gamma\int_{\theta}^{\theta_{\max}}(V(x)-v(x))f(x|x\geq\theta)(1-F(x|x\geq\theta)\,dx.$$

The condition becomes:

$$\begin{aligned}v(\theta) &\geq \gamma\int_{\theta}^{\theta_{\max}}V(x)f(x|x\geq\theta)(1-F(x|x\geq\theta)\,dx, \\ v(\theta) &\geq \gamma\int_{\theta}^{\theta_{\max}}V(x)\frac{f(x)(1-F(x)}{(1-F(\theta))^2}dx.\end{aligned}$$

Behavior of impatient types

The derivation of comes from lemma $T(\theta,\theta^{*},V-v)$ comes from lemma 1. The expected utility of an agent of type θ behaving like a type $\hat{\theta}$ when the two other agents follow the equilibrium strategy:

$$-[1-F(\hat{\theta})]^{2}t(\hat{\theta})-\gamma\int_{\hat{\theta}}^{\theta^{\max}}f(x)(1-F(x))(V(x)-v(x))dx+$$
$$(1-(1-F(\hat{\theta}))^{2})v(\theta)+\int_{\theta_{\min}}^{\hat{\theta}}(2f(x)(1-F(x)))(S(\theta,x)-t(x))dx$$

A necessary condition for t to be an equilibrium is that it is optimal for type θ to volunteer at time $t(\theta)$ rather than mimic type $\hat{\theta}$. The derivative of the preceding expression with respect to $\hat{\theta}$ is:

$$[2f(\hat{\theta})(1-F(\hat{\theta})]t(\hat{\theta})-[1-F(\hat{\theta})]^{2}t^{'}(\hat{\theta})+\gamma\left(V\left(\hat{\theta}\right)-v\left(\hat{\theta}\right)\right)f(\hat{\theta})(1-F(\hat{\theta}))$$
$$+2f(\hat{\theta})(1-F(\hat{\theta}))v(\theta)+2f(\hat{\theta})(1-F(\hat{\theta}))[S(\theta,\hat{\theta})-t(\hat{\theta})].$$

It has to be equal to zero when evaluated at $\hat{\theta}=\theta$:

$$\begin{aligned} t^{'}(\theta)[1-F(\theta)]^{2} &= (\gamma(V(\theta)-v(\theta))+2v(\theta))f(\theta)(1-F(\theta)), \\ \text{or } t^{'}(\theta) &= [\gamma(V(\theta)-v(\theta))+2v(\theta)]h(\theta). \end{aligned}$$

10 References

Bilodeau, M., and Slivinski A., 1996. Toilet Cleaning and Department Chairing: Volunteering for a Public Service. *J. Public Econ.*, 59, 299-308.

Bliss, C., and Nalebuff B., 1984. Dragon-Slaying and Ballroom Dancing: The Private Supply of a Public Good. *J. Public Econ.*, 25, 1-12.

Bolton P., and Farrell J., 1990. Decentralization, Duplication and Delay. *J. Polit. Economy*, 98, 803-26.

Bulow, J., and Klemperer, P., 1999. The Generalized War of Attrition. *Amer. Econ. Rev.*, 89, 1-12.

Farrell J., and Saloner G., 1988. Coordination Through Committees and Markets. *RAND J. Econ.*, 19, 235-53.

Fudenberg, D., and Tirole, J., 1986. A Theory of Exit in Duopoly. *Econometrica*, 54, 943-960.

Jehiel P., and Moldovanu B., 1995. Negative Externalities May Cause Delay in Negotiation. *Econometrica*, 63, 1321-35.

Lacasse C., Ponsati C., and Barham V., 2002. Chores. *Games Econ. Behav.*, 39, 237-281.

Myerson, R., 1981. Optimal Auction Design. *Math. Oper. Res.*, 6, 58-73.

Riley, J., and Samuelson, W., 1981. Optimal Auctions. *Amer. Econ. Rev.*, 71, 381-392.

Park A., and Smith, L., 2003. Calling Number Five: Timing Games that Morph from One Form to Another, mimeo.