Repeat Advertising

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Abstract

This paper formalizes the idea that advertising can be understood as a way to coordinate consumers' buying decisions in markets with consumption externalities. Information conveyed by advertising is not limited to product characteristics. Advertising, and in particular the intensity of advertising, is also informative about the number of consumers who know about the product. More advertising shifts expectations towards a larger consumer base, increasing the willingness to buy the product when there are consumption externalities. A simple model is developed. The producer advertises its product, but the intensity of advertising is uncertain. Each consumer observes a random number of ads, which increases with the intensity of the campaign. Upon observing ads, consumers form expectations about the number of other consumers who will purchase the good. The more advertising a consumer sees, the more likely he is to buy the good.

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1 Introduction

Advertising has a very important role in modern economic life. Consumers are faced with advertising in a growing number of media (Television, radio, newspaper, the internet etc.). In 2003, firms spent more than $235 billion on advertising in the United States. Despite extensive research, it is not clear why and how advertising works; many important effects of advertising remain poorly understood. Starting with Stigler (1961) and Nelson (1974), economists have focused on the informational aspect of advertising. This view argues that advertising primarily affects demand by conveying useful information to consumers on the existence and on the characteristics of the product. The information can directly provided to consumers (Butters (1977), Grossman and Shapiro (1984) and Robert and Stahl (1993)) or indirectly when advertising has a signalling role. However, most of the advertising is made for products and firms that are well-known to consumers. It is thus often argued that many advertisements have almost no explicit content, and thus provide no information to consumers (See for example Tesler (1964)). This has led to alternative theories of advertising. In particular, several recent papers have explored the role of advertising as a coordination device in markets with consumption externalities. This literature builds on Bagwell and Ramey (1994), who show that firms can coordinate consumers' purchases through advertising. In many markets, consumers' purchasing decisions depend on the purchasing decisions of other consumers, which gives rise to a coordination problem. The reason can be technological, as in the case of network externalities (See Katz and Shapiro (1985)). The reason can also be social; Becker (1991) shows that people often want to consume what is popular. Chwe (1999) also shows that, for reasons of social standing, people want to wear the right clothes or to consume the right products. For instance, watching a movie brings more utility if other people have watched it and it is possible to discuss and share opinions about it.

Even for products like beer or soft drink, buying a popular brand is preferred if one expects that their friends are more likely to like it.

Pastine and Pastine (2002) and Clark and Horstmann (2004, 2005) develop models with consumption externalities in which advertising plays the role of a coordination device for consumers. The models differ in their details but the basic specification is similar. Consumers are perfectly informed about the existence and characteristics of the product. The utility that consumers derive from consumption depends on the number of other consumers that purchase that product. They consider two firms which compete through advertising and prices. To summarize their analysis, the consumption externality creates multiple equilibria. If consumers believe that other consumers are going to choose one of the products, it becomes rational to purchase that same product. They show that advertising can coordinate consumers expectations. Consumers go for the most intensively advertised product[1]. Advertising plays the role of a coordination device that enables consumers to coordinate on a "self-fulfilling" focal equilibrium: consumers purchase the product that is most heavily advertised. Advertising is a self-fulfilling prophecy. If everybody believes that people react to advertising, it is then rational to react to advertising. But advertising is just a costly signal in the sense that any other instrument could be used. The very nature of advertising plays no particular role in the analysis.

In this paper, we argue that the coordination role of advertising can be understood not as a mere coordination device but as the consequence of the informativeness of the scale of advertising campaigns. Consumers learn not only about the characteristics of the product, but also about the number of other potential consumers. More advertising increases the awareness that many consumers know about the product. This shifts expectations towards a larger externality. Chwe (2001) documents this important aspect of advertising. In his words: "By observing the

[1] Their advetising technology differs from the one we use in the present model.

campaign vast's scale alone, each person could surmise that others were seeing the ads also." this also means that the informational content of an ad can be different from consumer to consumer. A consumer seeing an ad for the first time learns about the existence of the product and its characteristics. A consumer seeing the same ad for the nth time does not learn anything new about the characteristics of the product but learns something about the intensity of the campaign. Advertising contributes to create common knowledge among consumers that the product exists. Of course, knowing that more consumers are aware of the product is not enough to shift expectations about purchasing decisions. But if some consumers may decide to purchase the good even if their expectations about the sales are low, they would derive enough utility from the intrinsic value of the good.

The analysis reveals the importance of scale effects in an advertising campaign. Repeat advertising can have a multiplier effect on consumers decisions. But the ingredients needed to fully use the multiplier effect are numerous. The firm's reputation and past advertising campaigns matter. The choice of advertising channel and the type of media is also shown to be very important. In particular, advertising that creates improved common knowledge among consumers about the existence of the product can have tremendous impact.

The role of common knowledge in coordination games has recently attracted much attention in the economic literature following the pioneering work on global games by Carlsonn and Vandamme (1993), and the recent application of this concept to many economic applications by Morris and Shin. The present paper borrows the technique from the global games literature and applies it to advertising for goods with social externalities. This modeling approach leads to a unique equilibrium in the modified game. This gives a more solid foundation to the coordinating role of advertising. Advertising is not just coordination device that enables consumers to focus on an equilibrium with a large network size. Consumers react optimally given their expectations. The coordinating role of advertising can be generated by a novel feature of the informative nature of advertising. Advertising (and in particular repeat advertising) does not necessarily inform consumers about prices or about the quality of the product. Advertising can be effective when informative about the number of other consumers that are aware of the product. If seeing a lot of advertising makes you believe it is more likely that your neighbor is aware of the product, it can shape your expectation about the number of people who are going to purchase the good.

Chwe (2001,b) is an empirical analysis of advertising in markets with consumption externalities. In a very simple model, he shows the value for the advertisers of consumers being aware that other consumers are seeing the ad. Chwe's analysis goes further than his simple model. Informally, he argues that advertising generates common knowledge and leads to coordination. He also documents that social brands advertise more heavily in popular shows. It is natural that firms which sell goods with consumptions externalities are ready to pay a premium for slots on popular shows. The present paper is complementary to Chwe's informal analysis. It models in a precise way the idea that advertising conveys information about the intensity of advertising and as a result of the number of consumers who have seen the advertising.

Section 2 introduces the coordination game between consumers purchasing a good with consumption externalities. By introducing incomplete information on the intrinsic value of the good, we show that the game has a unique equilibrium. Advertising technology in described in section 3. Section 4 solves for a simple model with two consumers and at most two ads, that conveys most of the intuition of the results. Section 5 introduce the firms's decision to advertise. Section 6concludes. We analyze in the appendix a more general model with a continuum of consumers and a more complete advertising structure, and we show that the results obtained in section 4 generalize to this more general set-up.

2 Buying a social good: a coordination game

We analyze a simple model of consumption externalities. Two[2] consumers can purchase a new product. This product displays consumption externalities. A consumer's utility is larger if the other consumer buys the product as well. Let θ denote the intrinsic value of the good, let c denote the cost of the purchase and let N denote the consumption externality, or bandwagon effect.

Payoffs are summarized in the following matrix.

	Buy	Don't buy
Buy	$\theta - c + N, \theta - c + N$	$\theta - c, 0$
Don't buy	$0, \theta - c$	$0, 0$

As a function of the value of the parameters, there are three cases to consider in the analysis of the Nash equilibria of this coordination game:

- If $\theta > c$, **Buy** is a dominant strategy for consumers. The intrinsic value of the product is large enough for consumers to buy the product even if they believed that the other consumer was not purchasing.

- If $\theta < c - N$, **Don't Buy** is a dominant strategy for consumers. The value of the product is below c even if a consumer believes that the other consumer is buying.

- If $c - N < \theta < c$, there are two pure-strategy Nash equilibria[3] Consumers play a coordi-

[2]The assumption that there are only two potential consumers is made to simplify the analysis. In section 4, we relax this assumption and allow for an infinite number of buyers. The two models are, in reality, closely related, since we can reinterpret the probabilty that the other consumer purchases the good as the proportion of the population which purchases the good. This would not be the case if the externality was not linear in the number of other consumers.

[3]Which implies that there is also a mixed strategy equilibrium.

nation game. Each one wants to buy the good only if the other is also buying it.

We now introduce incomplete information about the value of the good θ.[4] We assume that θ is randomly drawn from the real line[5] with each realization equally likely. Potential consumers observe a private signal $x_i = \theta + \varepsilon_i$. Each ε_i is independently normally distributed[6] with mean 0 and standard deviation σ. This implies that a consumer observing signal x to update his belief an to consider θ to be distributed normally with mean x and standard deviation σ.

A buying strategy is now a mapping specifying the action $a \in \{0, 1\}$ as a function of the signal received. Action 1 (0) represents the (no) purchase decision. We consider switching strategies in which a consumer buys the good when the signal he receives is above some cutoff point x^*:

$$a(x) = \begin{cases} 1, \text{ if } x > x^* \\ 0, \text{ if } x \leq x^* \end{cases}$$

If a consumer observes a signal x and believes that the other consumer is using the same strategy with cutoff point x^*, he then assigns probability $1 - \Phi\left(\frac{1}{\sqrt{2}\sigma}(x^* - x)\right)$ that the other buys the good consumer (or that he observed a signal above the cutoff point). In particular, after observing a signal equal to the cutoff point x^*, he assigns a probability of $\frac{1}{2}$, to the other consumer buying. At an equilibrium, he is indifferent between buying and not buying.

[4] We follow the literature on global games initiated by Carlsson-van Damme (1993) and further developed in various papers by Morris and Shin.

[5] Improper priors allow us to concentrate on the updated beliefs of consumers conditional on their signals. This technique has been used by Morris and Shin in the context of currency crises models. See Morris and Shin (2000) for a discussion of this approach.

[6] The normality of the noise term is not a restrictive assumption. Same results would obtain if we had $x_i = \theta + \sigma\varepsilon_i$ with ε_i distributed according to any smooth symmetric cdf F(.).

Equalizing the payoffs, we get :

$$\frac{1}{2}\left(x^{*}-c+N\right)+\frac{1}{2}\left(x^{*}-c\right)=0.$$

Thus an equilibrium is characterized by a cutoff $x^{*}=c-\frac{N}{2}$.

Consumers' expectations play a central role in their purchasing decisions. Based on the signal they receive, they form beliefs about the probability that the other consumer is going to buy. This means that a higher signal has two effects. First, there is a direct increase in the private value of the good. Second, there is an indirect effect. A higher signal informs the consumer that the other consumer is likely to have received a high signal too: this bandwagon effect increases the value of the externality.

The introduction of incomplete information yields a unique equilibrium in the coordination game. How can advertising play a role in this framework? There is no real need for coordination. The point is that consumers' behavior depends on their expectations about the market size. The role of advertising is to shape these expectations. In the next section, we introduce a micro-founded model of advertising in which the intensity of the campaign shapes consumers beliefs about the market size. A vast campaign leads to a large informed pool of potential consumers. We show then that the more ads a consumer sees, the lower his signal needs to be for him to purchase the product. The first advertisement he sees informs him about the private value of the good, additional advertisement is informative of the network externality.

3 Advertising technology

In the previous section, we assumed that all consumers were aware of the product existence and received a signal about its quality. All this was common knowledge among consumers.

Recently, the role of common knowledge in coordination games has been investigated in several papers. Morris and Shin (2001,b) discuss how the media influence public outcomes and relate the importance of shared knowledge in coordination games such as currency crises and bank runs. They analyze in particular the importance of public announcement and the link between public and private information. In the context of consumption externalities, it is natural to analyze the role of advertising as a way to coordinate consumers' purchasing decisions. Chwe (2001) discusses how advertising plays a role of coordination in the markets for social goods. Advertising is usually understood as informative. Seeing an advertisement provides a consumer with information about the characteristics of a new product. In the context of our model, advertising provides consumers with the signal about the value of the good. If this was the only effect of advertising, consumers would have no use of additional advertising. But advertising also has an impact on the expected consumption externality. A consumer does not know how many consumers are aware of the products, since he does not know how many consumers have received an advertisement. Hence, he does not know either the level of the consumption externality for a given signal. Repeat advertising creates a bandwagon effect and thus has an indirect effect on purchasing decisions through this channel. Seeing a lot of advertising convinces a consumer that the scale of the advertising campaign is large and that the probability that other consumers have received an ad is high.

We present a simple way to formally model this aspect of advertising as coordinating purchase decisions. By seeing an ad, a player is made aware of the existence of the product and receives a signal θ_i about his valuation for this good. If he does not observe an ad, a player is not aware of its existence and thus does not make a purchase. This corresponds to the usual informational role of advertising that is analyzed in the literature. Seeing additional advertisement for the product does not provide any other information about the product to the consumer, and thus is of no use to the consumers in these usual models. Here, repeat advertising is used to infer about the intensity of the advertising campaign.

Advertising is made through the media. The firm chooses the intensity of advertising, and buys a certain number of ads in the newspapers, on the radio and on television. A consumer does not know the intensity of the advertising campaign, but he is exposed to the media and randomly observes advertisements drawn from the probability distribution corresponding to the intensity chosen by the firm.

To simplify things, suppose a firm can choose between two advertising strategies (high or low intensity). Consumers believe that the firm uses the high intensity campaign with probability λ and the low-intensity with probability $(1-\lambda)$. In section 5, we introduce the firm's advertising decision and endogenize λ. The distribution functions $p(\cdot|L)$ and $p(\cdot|H)$ describe the effect of such campaign in terms of consumers impressions: $p(k|H)$ (respectively $p(k|L)$)represents the probability that a consumer observes k ads ($k = 0, 1, 2, ...n.$) when the intensity is high (low)[7]. With the first ad observed, a consumer learns about the new product and gets a signal θ_i,which represents his private value of the good.

We impose the Monotone Likelihood Ratio Property on the advertising technology distribution.

Definition 1 *The distribution $p(\cdot|\cdot)$ satisfies the Monotone Likelihood Ratio Property (MLRP) if and only if :*

$$\frac{p(k|H) - p(k|L)}{p(k|H)} \text{ is increasing in } k.$$

By Bayes' rule, the posterior probability of a high intensity campaign after observing k ads

[7]Butters (1977) models advertising in a similar way. A firm chooses the intensity λ of advertising. A given consumer will the receive advertisng according to a Poisson distribution of parameter λ. We view our advertising technology as a simplified version of Butters' model. Also see Hertzendorf (1993).

is :

$$\Pr(H|k) = \frac{\lambda p(k|H)}{\lambda p(k|H) + (1-\lambda)\, p(k|L)}.$$

Thus, after a consumer observes k ads, he updates his beliefs about the campaign intensity to $(\Pr(H|k), 1 - \Pr(H|k))$. Let π_i^k denotes the probability that another consumer sees i ads when a given consumer sees k ads. We have:

$$\pi_i^k = \left[\Pr(H|k) \cdot p\,(i|H) + (1 - \Pr(H|k)) \cdot p\,(i|L)\right].$$

We now derive some properties of these conditional; probabilities π_i^k derived from the Monotone Likelihood Ratio Property. The MLRP condition implies that after observing a larger number of ads, a consumer thinks that it is more likely that the firm has used the high intensity advertising strategy.

Lemma 2 *If $p(\cdot|\cdot)$ satisfies MLRP, then observing a higher number of ads increases the posterior belief of the high intensity campaign:* $\Pr(H|k) > \Pr(H|j)$ *when* $k > j$*.*

Proof of lemma 1

To see this, suppose $k > j$, by MLRP we have

$$\frac{p(k|H) - p(k|L)}{p(k|H)} > \frac{p(j|H) - p(j|L)}{p(j|H)}.$$

Rearranging yields:

$$\frac{p(j|L)}{p(j|H)} > \frac{p(k|L)}{p(k|H)}.$$

Multiplying both sides by $\frac{1-\lambda}{\lambda}$ and adding 1 yields :

$$1+\frac{1-\lambda}{\lambda}\frac{P(j|L)}{p(j|H)}>1+\frac{1-\lambda}{\lambda}\frac{p(k|L)}{p(k|H)},$$

which is equivalent to:

$$\frac{\lambda p(k|H)}{\lambda p(k|H)+(1-\lambda)\,P(k|L)}>\frac{\lambda p(j|H)}{\lambda p(j|H)+(1-\lambda)\,p(j|L)},$$

which proves that $\Pr(H|k)>\Pr(H|j)$.■

Lemma 3 *If $p(\cdot|\cdot)$ satisfies MLRP, then the conditional probabilities π_i^k also satisfy MLRP property in i, that is .$\left(\pi_i^k-\pi_i^{k-1}\right)/\pi_i^k$ is increasing in i.*

Proof of lemma 2

We need to show that $\left(\pi_i^k-\pi_i^{k-1}\right)/\pi_i^k$ is increasing in i.

$$\frac{\pi_i^k-\pi_i^{k-1}}{\pi_i^k}>\frac{\pi_{i-1}^k-\pi_{i-1}^{k-1}}{\pi_{i-1}^k}\text{ is equivalent to }\frac{\pi_{i-1}^{k-1}}{\pi_{i-1}^k}>\frac{\pi_i^k}{\pi_i^k}.$$

Developing yields :

$$\frac{\pi_i^{k-1}}{\pi_i^k}=\frac{xs+(1-x)}{ys+(1-y)}=\phi\left(s\right)$$

with $x=\Pr(H|k)$, $y=pr\left(H|k-1\right)$ and $s=\frac{\Pr(k|H)}{\Pr(k|L)}$.

Since $p\left(.|.\right)$ satisfies the MLRP, $\Pr\left(i|H\right)/\Pr\left(i|L\right)$ is increasing in i.

We also have that:

$$\begin{aligned}\phi'(s) &= \frac{x(ys+1-y)-y(xs+1-x)}{(ys+1-y)^2} \\ &= \frac{x-y}{(ys+1-y)^2}>0.\end{aligned}$$

which completes the proof.■

Lemma 4 *If $p(\cdot|\cdot)$ satisfies MLRP, $\pi_i^k > \pi_i^{k+1}$ only if $p(i|H) < p(i|L)$ and $\pi_i^k < \pi_i^{k+1}$ if $p(i|H) > p(i|L)$.*

Proof of lemma 3

$$\begin{aligned}\Pr(H|k)p(i|H)+\Pr(L|k)p(i|L) &> \Pr(H|k+1)p(i|H)+\Pr(L|k+1)p(i|L) \\ \Leftrightarrow \Pr(H|k)+\Pr(L|k)\frac{p(i|L)}{p(i|H)} &> \Pr(H|k+1)+\Pr(L|k+1)\frac{p(i|L)}{p(i|H)}.\end{aligned}$$

Since Pr$(H|k)$ is increasing in k, the result is immediate.■

These lemmas characterize the beliefs of a consumer about the intensity of advertising perceived by the other consumer. The result (that follows our assumptions of MLRP) that the more advertising a consumer sees, the more optimistic he is about the other consumer has received a lot of ads, is quite natural if we believe that advertising is random but uniform across the population. In particular, this assumption would not be satisfied if there was a total number of ads sent by mail and that an ad received by one consumer means that it is not received by another consumer. For such a view of advertising, in which consumers are "competing" for received ads, see Hertzendorf (1993).

We now incorporate the advertising technology just analyzed in a market with consumption externalities introduced in section 2.

4 A two-buyer, two-ad model of repeat advertising

We now go back to the model developed in section 2 and introduce the advertising technology presented. In this section, to develop some intuitions about the way things work, we simplify even further the model.

The advertising campaign is of high intensity with probability λ or of low-intensity with probability $1-\lambda$) We assume further that a consumer can observe three different levels of advertising. For concreteness, suppose he can observe zero, one or two ads. The following table summarizes the effect of the intensity advertising campaign in terms of consumers impressions. It gives the probability that a given consumer observes no ad, one ad or two ads as a function of the campaign intensity

	Low-Intensity	High Intensity
No ad	$p(0\|L)$	$p(0\|H)$
One ad	$p(1\|L)$	$p(1\|H)$
Two ads	$p(2\|L)$	$p(2\|H)$

A strategy for a consumer specifies his purchasing decision as a function of his signal and the number of ads he has seen. We consider switching strategies, in which a consumer buys the product if the signal received is above a cutoff point that may depend on the number of ads seen. By assumption, a consumer does not buy when he does not observe an ad, since he is not aware that the product exists. Let's call x_1^* (x_2^*) the cutoff corresponding to the signals above which a consumer makes a purchase when he has received one (two) ads.

Suppose consumer 1 believes that consumer 2 is using such a strategy with cutoff points (x_1^*, x_2^*). When he observes two ads, he updates his beliefs about the campaign intensity to

$(\Pr(H|n=2), 1-\Pr(H|n=2))$. The probability that consumer 1 then assigns to player 2 having observed a signal above the cutoff point, given player 1 got a signal x is thus:

$$\pi_2^2 \cdot \left(1 - \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_2^* - x\right)\right)\right) + \pi_1^2 \cdot \left(1 - \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_1^* - x\right)\right)\right)$$

where π_j^i denotes the probability that consumer 2 sees j ads when player 1 sees i ads. Consumer 1 observing 2 ads and getting a signal x_2^* gets, this probability becomes :

$$l\left(2\right) = \pi_2^2 \cdot \frac{1}{2} + \pi_1^2 \cdot \left(1 - \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_1^* - x_2^*\right)\right)\right)$$

This corresponds to the probability that consumer 2 buys given that consumer 1 has observed two ads and got the cutoff signal x_1^*. Thus $l\left(2\right) \cdot N$ corresponds to the expected bandwagon effect.

Similarly when consumer 1 receives one ad, he updates his beliefs about the campaign intensity to $(\Pr(H|n=1), 1-\Pr(H|n=1))$. The probability he assigns to the other consumer having observed a signal above the cutoff point is as follows:

$$\pi_2^1 \cdot \left(1 - \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_2^* - x\right)\right)\right) + \pi_1^1 \left(1 - \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_1^* - x\right)\right)\right).$$

When consumer 1 observes 1 ad and gets a signal x_1^*, this probability becomes :

$$l\left(1\right) = \pi_2^1 \cdot \left(1 - \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_2^* - x_1^*\right)\right)\right) + \pi_1^1 \cdot \frac{1}{2}$$

An equilibrium obtains when a player seeing n ads and getting a signal of x_n^*, is indifferent between buying and not buying.

Summing up, an equilibrium is characterized by the two equations:

$$\begin{aligned} l(2) \cdot (x_2^* - c + N) + (1 - l(2))(x_2^* - c) &= 0 \\ l(1) \cdot (x_1^* - c + N) + (1 - l(1))(x_1^* - c) &= 0 \end{aligned}$$

We can rewrite this system as:

$$x_2^* = c - l(2) N \tag{1}$$

$$x_1^* = c - l(1) N \tag{2}$$

Note that the thresholds are both in the multiple equilibria region of the coordination game with no advertising. We now show that there exists a unique equilibrium in which the thresholds x_n^* are decreasing in n.

Proposition 5 *There exists a unique symmetric equilibrium in switching strategies with* $x_2^* < x_1^*$.

Proof. Let's first prove the existence of such an equilibrium. Consider the system of equations

$$\begin{aligned} x_2^* &= c - l(2) N \\ x_1^* &= c - l(1) N. \end{aligned}$$

We know that an equilibrium is a pair (x_1^*, x_2^*) solving this system. We know that $c - N \leq x_i^* \leq c$ since $l(i)$ represents the probability that the other player buys and so is between 0 and 1. $[c - N, c]$ is a compact set. $F(\mathbf{x}) = \begin{bmatrix} c - l(2) N \\ c - l(1) N \end{bmatrix}$ is a continuous function on $[c - N, c]$. By

Brouwer's fixed point theorem, we know there exists a solution. ■

Let's now show that in such an equilibrium the cutoff points are ordered $x_2^* < x_1^*$. Suppose towards a contradiction that $x_1^* < x_2^*$. We have:

$$\begin{aligned} l\,(2) &= \pi_2^2 \cdot \frac{1}{2} + \pi_1^2 \cdot \left(1 - \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_1^* - x_2^*\right)\right)\right) \\ l\,(1) &= \pi_2^1 \cdot \left(1 - \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_2^* - x_1^*\right)\right)\right) + \pi_1^1 \cdot \frac{1}{2} \\ &= \pi_2^1 \cdot \left(\Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_1^* - x_2^*\right)\right)\right) + \pi_1^1 \cdot \frac{1}{2} \end{aligned}$$

Hence,

$$l\,(2) - l\,(1) = \frac{1}{2}\left(\pi_2^2 - \pi_1^1\right) + \left(\pi_1^2 - \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_1^* - x_2^*\right)\right)\left(\pi_1^2 + \pi_2^1\right)\right)$$

$x_1^* < x_2^*$ implies that $\Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_1^* - x_2^*\right)\right) < \frac{1}{2}$. So

$$\begin{aligned} l\,(2) - l\,(1) &> \frac{1}{2}\left(\pi_2^2 - \pi_1^1\right) + \left(\pi_1^2 - \frac{1}{2}\left(\pi_1^2 + \pi_2^1\right)\right) \\ &= \frac{1}{2}\left(\pi_2^2 + \pi_1^2 - \pi_1^1 - \pi_2^1\right) \\ &= \frac{1}{2}\left(\pi_0^1 - \pi_0^2\right) > 0 \text{ by MLRP.} \end{aligned}$$

But if $l\,(2) > l\,(1)$, we can not have $x_1^* < x_2^*$.

Finally, we prove that the equilibrium in switching strategies is unique. Suppose there exist two equilibria (x_1^*, x_2^*) and (x_1^{**}, x_2^{**}). From equations $(1, 2)$, we know that $\Phi^* = \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_1^* - x_2^*\right)\right)$ can not be equal to $\Phi^{**} = \Phi\left(\frac{1}{\sqrt{2}\sigma}\left(x_1^{**} - x_2^{**}\right)\right)$. So suppose $\Phi^* > \Phi^{**}$. Equations $(1, 2)$ imply directly that $l^*\,(1) > l^{**}\,(1)$ and $l^*\,(2) < l^{**}\,(2)$. But that would imply also $x_1^* < x_1^{**}$ and $x_2^* > x_2^{**}$. But then we have $x_1^* - x_2^* < x_1^{**} - x_2^{**}$ and $\Phi^* < \Phi^{**}$ which leads to a contradiction.■

This proposition shows that repeat advertising is useful in convincing consumers to purchase the good. The difference $(x_1^* - x_2^*)$ represents the impact of repeat advertising. A potential consumer who is subject to repeat advertising has a lower reservation signal than if he was subject to only one advertising impression. Similarly, $\Phi\left(\frac{1}{\sqrt{2}\sigma}(x_1^* - x_2^*)\right)$ represents the increase in the probability that a given consumer makes a purchase due to repeat advertising. Repeat advertising works through the change in expectations about the consumption externality. Additional advertising informs the viewer that the product is intensely advertised and that other consumers are informed about the product. In a way, repeat advertising helps the coordination of the purchasing decisions. But, better coordination is not obtained because advertising is a coordination device in the usual sense. In this paper, the Nash equilibrium is unique : advertising brings information to consumer not only about the product but also about the intensity of the campaign. Repeat advertising is a convincing way to prove to consumers that the product is heavily advertised and that the coordination problem can be solved.

The impact of repeat advertising comes from the informational content of an additional advertisement. It depends obviously on the number of ads received but also on many other important variables. In particular, the heterogeneity in consumers' tastes, summarized by parameter σ, plays an important role. The prior λ about the likelihood of an intense advertising campaign as well as the advertising distribution function are also very important. Additional advertising is effective when it convinces consumers that the advertising campaign is intense.

Since both parameter λ and the advertising technology are part of the prior beliefs that consumers have about the characteristics of the firm, one cannot fully understand the impact of (repeat) advertising without taking into account the "reputation" of the firm. A firm with a reputation for heavy advertising of new products will more easily convince consumers that their new campaign is intense and that many consumers are aware of the new product. If we were to extend the analysis to the case of several competing firms , it is not clear that the most heavily advertised firm would necessarily be more successful. The efficiency of advertising depends on other factors. Even if the products are of equal quality, the difference between firms in terms of past advertising can matter.

The role of the advertising technology and consumers heterogeneity can be better understood when we make the parameter σ very small. This is usual in the global game literature in which the noise is usually a way to select a unique equilibrium. In our setting, consumers heterogeneity appears to be a natural assumption, and the equilibrium with a large σ has meaningful economic interpretation. However, taking σ to zero leads to some interesting observations. Looking back at the system of equations characterizing equilibrium thresholds, we see that $l\,(1)$ and $l\,(2)$ are the values we need to consider.

$$
\begin{aligned}
l\,(1) &= \pi_2^1 \cdot \left(1 - \Phi\left(\frac{1}{\sqrt{2\sigma}}\left(x_2^* - x_1^*\right)\right)\right) + \pi_1^1 \cdot \frac{1}{2} \\
l\,(2) &= \pi_2^2 \cdot \frac{1}{2} + \pi_1^2 \cdot \left(1 - \Phi\left(\frac{1}{\sqrt{2\sigma}}\left(x_1^* - x_2^*\right)\right)\right)
\end{aligned}
$$

There are two possible cases. Suppose that $\lim_{\sigma\to 0}\left(x_1^* - x_2^*\right) > 0$, then $\lim_{\sigma\to 0} l\,(1) = \pi_2^1 + \pi_1^1 \cdot \frac{1}{2}$ and $\lim_{\sigma\to 0} l\,(2) = \pi_2^2 \cdot \frac{1}{2}$. Since we need $l\,(2) > l\,(1)$, a necessary condition for repeat advertising to have an impact, when $\sigma \to 0$, is that $2\pi_2^1 + \pi_1^1 < \pi_2^2$. When $2\pi_2^1 + \pi_1^1 > \pi_2^2$, it must be the case that $\lim_{\sigma\to 0}\left(x_1^* - x_2^*\right) = 0$ and repeat advertising has no impact. This shows that consumers heterogeneity is very important for repeat advertising but is not necessary. Repeat advertising can be effective even with very little heterogeneity when π_2^2 is large enough. The way to interpret this result is in terms of scale effects. There is a return to run an advertising campaign in which consumers that see additional ads react very strongly in terms of updating their beliefs about the intensity of the campaign. To link this with Chwe's analysis, we can interpret a large π_2^2 as advertising that generates common knowledge among consumers. The number of ads can also be reinterpreted as the quality of the media on which the ad is run. If one sees a television advertising in a very popular show, he is more inclined to believe that many other consumers have seen this ad. This is what Chwe calls the Superbowl effect. "It is likely that people watching a media event know that a vast audience is in attendance. Such awareness is part of the event's appeal, and the media are generally eager to report the estimated worldwide audience". Advertisement during the Superbowl is very effective for goods with consumption externalities. Our analysis shows that there can be a qualitative difference between ads with large potential for common knowledge generation (high π_2^2) and others type of advertisement. Since the prior λ of high intensity campaign enters in π_2^2, we see that the nature of the firm advertising its product and its reputation in the eyes of consumers is very important for the multiplier effect of repeat advertising.

5 Conclusion

Advertising is a complex and important economic phenomenon. In this paper, we analyzed a model in which advertising informs consumers about the size of the market through the intensity of the campaign. We show that in the context of a market with consumption externalities, that is whenever consumers care about what other people consume, repeat advertising has an influence on purchasing decisions.

This analysis also reveals that the impact of advertising depends on many factors such as the type of media used, the reputation of the firm through brand name and past campaigns. This can shed some light on why firms need to continue to advertise intensively their products even after these products are well-known by consumers. Repeat advertising helps firms to maintain their brand goodwill.

The importance of consumption externalities shows that advertising plays an important and special role in markets for products that have the characteristic that what other people think matters in the purchasing decision. In particular, advertising plays a role in the creation of fads and fashion. But analyzing such phenomena would require a dynamic model, which is beyond the scope of this paper.

6 Appendix: A general model

We allow for a more general advertising technology with an arbitrary number of possible impressions per consumers. We show that the main result of the previous section carries over to this set-up: there is a unique equilibrium with increasing thresholds. Even in this more complicated setting, repeat advertising works. Each additional advertising a consumer sees increases his probability of purchase.

There is a continuum of potential buyers. A given consumer observes a random number of ads drawn from $\mathcal{I} = [0, 1, ...n]$, and gets a signal θ_i if he observes at least one advertisement. A strategy now specifies consumers' actions given the signal received and also the number of ads seen. Formally it is a map $s_i : \mathcal{R} \times \mathcal{I} - \{0\} \to \{0, 1\}$. By assumption, a consumer does not buy the good if he does not receive any advertisement. Let $\mathcal{S}$ be the space of all strategy profiles. A strategy is said to be a switching strategy if, for any k, there exists $\theta_k^* \in \mathcal{R}$ such that $s_i(\theta, k) = 0$ if $\theta < \theta_k^*$ and $s_i(\theta, k) = 1$ if $\theta > \theta_k^*$. A strategy s is said to be monotone if s is a switching strategy with ordered cut-points $\theta_n^* \leq \theta_{n-1}^* \leq .. \leq \theta_1^*$.

Let's consider for now that every consumer follows a switching strategy s, that can be summarized by a vector of cutoff points, $(\theta_k^*)_{k=1}^n$. When a consumer receives k ads, he updates his beliefs about the campaign intensity to $(\Pr(H|n = k), 1 - \Pr(H|n = k))$. Given a signal θ, he believes that another consumer buys the good with the following probability:

$$l\left(k,\theta\right)=\sum_{i=1}^{k}\pi_{i}^{k}\cdot\left(1-\Phi\left(\frac{1}{\sqrt{2\sigma}}\left(\theta_{i}^{*}-\theta\right)\right)\right),$$

where π_i^k denotes the probability that player 2 sees i ads when player 1 sees k ads.

$$\pi_{i}^{k}=\left[\Pr(H|n=k)\cdot p\left(i|H\right)+\left(1-\Pr(H|n=k)\right)\cdot p\left(i|L\right)\right].$$

$l\left(k,\theta\right)$ also represents the proportion of consumers who buy the good. To form an equilibrium, the strategy profile s must be such that each consumer's buying decision is optimal given his beliefs. In particular, a consumer observing k ads and receiving the signal θ_k^* needs to be indifferent between buying or not, $\theta_k^*-c+l\left(k,\theta_k^*\right)N=0$. This yields the following system of equations that characterize an equilibrium :

$$\theta_{k}^{*}=c-N\Sigma_{j=1}^{n}\pi_{j}^{k}\left(1-\Phi\left(\frac{1}{2\sqrt{\sigma}}\left(\theta_{j}^{*}-\theta_{k}^{*}\right)\right)\right).$$

Proposition 6 *There exists a unique equilibrium. The unique equilibrium is in monotone strategies.*

Let's first prove the existence of such an equilibrium.

Consider the system of equations

$$\theta_{k}^{*}=c-N\Sigma_{j=1}^{n}\pi_{j}^{k}\left(1-\Phi\left(\frac{1}{2\sqrt{\sigma}}\left(\theta_{j}^{*}-\theta_{k}^{*}\right)\right)\right)$$

We know that an equilibrium is a vector $\{\theta_k^*\}_{k=1}^n$ solving this system. We know that $.c-N\leq\theta_k^*\leq c$ since $l\left(k,\theta\right)$ represents the proportion of customers who decide to purchase the good, and so is between 0 and 1. $[c-N, c]$ is a compact set. $F(\mathbf{x}) = \left[c - N\Sigma_{j=1}^{n}\pi_j^k\left(1-\Phi\left(\frac{1}{2\sqrt{\sigma}}\left(x_j - x_k\right)\right)\right)\right]$ is a continuous function on $[c-N, c]^n$ The existence of a solution follows Brouwer's fixed point theorem..

It remains to show that there exists a unique equilibrium in monotone strategies, and that all equilibria are of this form.

Let's define on $\mathcal{S}$, the set of strategy profiles, the best response operator B$B : \mathcal{S} \to \mathcal{S}$ so that :

$$B(s_i) = \arg\max_{a\in\{0,1\}} E_{\theta_i}, k_i U\left(a, \theta_i, k_i, s_{-i}\right).$$

Define $\mathcal{S}^0$ to be the space of monotone strategy profiles such that are undominated. Clearly it is a dominant strategy to buy whenever $\theta > c$ and it is a dominant strategy not to buy whenever $\theta < c - N$. So we know that $\mathcal{B}$:$\mathcal{S}^0 \to \mathcal{S}^0$. If we start the highest possible strategy profile $\underline{s}$ in $\mathcal{S}^0$, that is a strategy that prescribes to buy whenever the signal is higher than $c-N$, and apply the best response operator, we obtain a decreasing sequence of monotone profiles- each of which becomes an upper bound on iteratively undominated strategies. Similarly if we start with the smallest possible strategy profile $\overline{s}$ in $\mathcal{S}^0$, that is the strategy that prescribes to buy whenever the signal is higher than c, and apply the best response operator, we obtain an increasing sequence of monotone profiles- each of which becomes a lower bound on the set of rationalizable profiles. We now show that this process converges to a uniquely iteratively undominated profile s^*, which also is a Nash Equilibrium.

To do this, we show that the best-response operator is a contraction. To do that, we use the following metric on strategies.

$$\rho\left(s, s^{'}\right) = \min_{\varepsilon}\left[\varepsilon : s + \varepsilon > s^{'} \text{ and } s^{'} + \varepsilon > s\right]$$

with $(s+\varepsilon)(\theta,k) = s(\theta+\varepsilon,k)$. Intuitively, the distance between two strategies correspond to the minimum shift in signal that would make a strategy larger than another. If a strategy is represented by cut-points $\{\theta_k^*\}_{k=1}^n$, the distance between the two strategies corresponds to the sup-norm: $\rho\left(s,s^{'}\right) = \sup_k \left|\theta_k^* - \theta_k^{'*}\right|$. $\left(\mathcal{S}^0,\rho\right)$ is a complete metric space. To show that B is a contraction, we must show that $\rho\left(Bs, B\left(s+\varepsilon\right)\right) < \delta\rho\left(s,s+\varepsilon\right)$.

The system of equations characterizing the best-response s^* characterized by cut-points $\{\theta_k^*\}_{k=1}^n$ to a strategy s characterized by cut-points $\{\theta_k\}_{k=1}^n$is:

$$\theta_k^* = c - N\Sigma_{j=1}^n \pi_j^k \left(1 - \Phi\left(\frac{1}{2\sqrt{\sigma}}\left(\theta_j - \theta_k^*\right)\right)\right).$$

Let $H\left(\theta,\theta^*\left(\theta\right)\right) = \theta_k^* - c + N\Sigma_{j=1}^n \pi_j^k \left(1 - \Phi\left(\frac{1}{2\sqrt{\sigma}}\left(\theta_j - \theta_k^*\right)\right)\right)$. By the Implicit Function theorem, we have :

$$\frac{\partial\theta^*}{\partial\theta} = -\frac{\partial H\left(\theta,\theta^*\right)}{\partial\theta} / \frac{\partial H\left(\theta,\theta^*\right)}{\partial\theta^*}.$$

Hence,

$$\frac{\partial\theta_k^*}{\partial\theta} = \frac{N\Sigma_{j=1}^n \pi_j^k \left(\frac{1}{2\sqrt{\sigma}}\phi\left(\frac{1}{2\sqrt{\sigma}}\left(\theta_j - \theta_k^*\right)\right)\right)}{1 + N\Sigma_{j=1}^n \pi_j^k \left(\frac{1}{2\sqrt{\sigma}}\phi\left(\frac{1}{2\sqrt{\sigma}}\left(\theta_j - \theta_k^*\right)\right)\right)}.$$

It is immediate that $0 < \frac{\partial\theta_k^*}{\partial\theta} < N\frac{N\Sigma_{j=1}^n \pi_j^k \left(\frac{\phi(0)}{2\sqrt{\sigma}}\right)}{1+N\Sigma_{j=1}^n \pi_j^k \left(\frac{\phi(0)}{2\sqrt{\sigma}}\right)} < 1$. Calling $\beta = N\frac{N\Sigma_{j=1}^n \pi_j^k \left(\frac{\phi(0)}{2\sqrt{\sigma}}\right)}{1+N\Sigma_{j=1}^n \pi_j^k \left(\frac{\phi(0)}{2\sqrt{\sigma}}\right)}$, we have that $\rho\left(Bs, B\left(s+\varepsilon\right)\right) < \beta\rho\left(s,s+\varepsilon\right)$, which proves that the best-response operator is a contraction..

This proves that the process of iterative deletion of strictly dominated strategies converges to a unique strategy profile s^*. Furthermore, this strategy profile is monotone. To see that, recall that we start the process with $\overline{s}$ and $\underline{s}$, which are monotone profiles (since the cut-points are equal). A given buyer believing that the other buyers follow a monotone strategy has a network externality that is non-decreasing in the number of ads he sees. This comes from the MLRP of the probability distribution of advertising. This leads the best response cutoff to be ordered and thus the best-response profile to be monotone.

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