Auction versus Dealership Markets*

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Abstract

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Abstract

In this paper, we compare two market structures, namely auction and dealership markets defined respectively as centralized order-driven and fragmented quote-driven markets. Our approach departs from previous works comparing these market mechanisms by considering both the timing of order submission (quote versus order-driven) and trading concentration (centralized versus fragmented) as distinctive features of these trading structures. We compare markets using measures of market viability, informational efficiency, price variance, informed trading aggressiveness and market liquidity. We show that auction markets are less sensitive to the asymmetric information problem and exhibit a higher level of informational efficiency than dealership markets. Moreover, we find that the relative magnitude (in both structures) of price variance, informed trading aggressiveness and market depth depend on the market thickness.

Keywords: Auction Markets, Dealership Markets, Market performance, concentration.

JEL Classification: D43, D44, D82

1 Introduction

Each financial market has its own functioning rules, which can be either structural or organizational. In spite of this diversity, each trading structure may be described as a hybrid version of three prototypical trading systems: call (or periodic) auctions, continuous auctions and dealership markets. Understanding the relative merits of each of these pure trading mechanisms is an important issue from a normative standpoint since it would allow to better design the specific hybrid version. Comparing these structures would also help to deal with the optimal structure issue.

This paper compares continuous pure trading systems: continuous auctions and dealership markets. Auction and dealership markets differ along several dimensions out of which two structural properties may be considered particularly distinctive. These properties are the degree of concentration of trading (centralized and fragmented) and the timing of order submission by liquidity providers (quote and order-driven markets). Auction markets are concentrated and order-driven while dealership markets are fragmented and quote-driven. As in Pagano and Röell (1996), auction markets are concentrated because each execution may require more than two orders from buy and sell sides of the market. We argue indeed that, at each moment, limit orders outstanding in the book will be executed against more than only one order. On the other hand, each submitted order may "hit" more than only one outstanding order. Dealership markets are fragmented instead since transactions occur after bilateral meetings between dealers and traders.

However, auction markets are also order-driven since liquidity is provided by limit order submitters, and dealership markets are quote-driven, since dealers (liquidity providers) begin the trading process by submitting their bid and ask quotations.

In the literature, the comparison between auction and dealership markets is generally based either on the timing of order submission (e.g., Pithyachariyakul, 1986, Shin, 1996, Bernhardt and Hughson, 1996, and Viswanathan and Wang, 2002), or on the concentration of trading (e.g., Mendelson, 1987, Pagano and Röell, 1996 and Biais, 1993). Madhavan (1992) compares continuous order-driven markets, quote-driven markets and periodic order-driven markets. He argues that continuous order-driven markets are "fragmented" in the sense that each trader's order is executed against different outstanding orders submitted by liquidity providers. He considers that the existence of more than only one order submitted by traders during a trading round is a feature of periodic auctions.

By using the number of traders in each trading round to distinguish between periodic and continuous auctions, Madhavan (1992) ignores the concentration feature of continuous markets. In this paper, we argue however that concentration is a common feature of continuous auction and periodic auction markets, and sustain that what distinguishes periodic from continuous auctions is the fact that orders are submitted sequentially in periodic markets. This adds a dynamic learning process in the trading behavior of agents in periodic auctions.

In this paper, we compare auction and dealership markets by considering both distinctive dimensions. In an asymmetric information framework, these dimensions affect differently the trading behavior of different market participants. The concentration of auction markets increases the level of information available to agents when they choose their trading strategies. Timing of order submission impinges on the degree of competition between liquidity providers. In dealership markets (quote-driven markets), dealers are engaged in price competition while limit-order submitters in auction markets (order-driven) compete on quantities.

Furthermore, this paper carries out a comparison between continuous auctions and dealership markets with respect to different measures of market quality like market viability (measured by the robustness of markets to information asymmetry), price variability, trading aggressiveness of informed traders and market liquidity. The fact that we use both concentration and timing of order submission as distinctive features of auction and dealership markets is mainly motivated by the opposite effects of these dimensions on participants' trading behavior, and consequently on market performance.

Our comparison allows us to reach two types of results. First, for some measures of market performance (i.e., market viability and market efficiency), the effect of concentration dominates the effect of the timing of order submission for almost all market conditions. For instance, we show that continuous auction markets are more robust to problems of asymmetric of information, and allow a higher informational transmission among market participants, hence generating higher informational efficiency. Second, for the other performance measures (i.e., informed agents' aggressiveness, price variance and market liquidity), we find that the relative dominance of a trading structure depends on market thickness.

The remainder of this paper is organized as follows: the next section spells out the model which is based on Glosten (1989) and Madhavan (1992). In Sections 3 and 4, equilibria in auction and dealership markets are derived, and some of their properties are described. Comparisons based on different performance measures are discussed in Section 5. We finally conclude with some remarks and possible extensions. All proofs are in the Appendix.

2 The model

We consider a simple one-period model in which agents liquidate their positions after trading.[1] There are two assets in the market: a risk-free asset (cash), and a risky asset with a stochastic liquidation value denoted by $\tilde{v}$.

Two types of agents participate in the market: Traders and liquidity providers (or market makers). Each of the N risk averse traders (indexed by $i = 1, ..., N$) chooses a trading strategy that maximizes his expected utility given his information set H_i. This set contains his private information that represents his trading motivations, public information and the information related to the trading structure. There are also M risk neutral[2] market makers (indexed by

[1] As in Madhavan (1992), we can easily extend this analysis to a multi-period model. We should however assume that agents cannot strategically choose the timing of entering the market. To simplify notation we omit this extension without loss of generality.

[2] As suggested by Pagano and Röell (1993) and Gould and Verrechia (1985), market makers should be suffi-

$m = 1, ..., M$) who provide liquidity to the market. They behave strategically and maximize their expected profits conditional on their information sets D_m. These sets contain public information and information related to the trading mechanism.

Each trader i is assumed to have a negative exponential utility function $U(W_i) = -e^{-\rho W_i}$, where ρ is the coefficient of risk aversion and W_i is his final wealth.

Trader i's private information is described by a vector (s_i, ω_i); s_i is his private signal about the final value of the risky asset and ω_i is his initial endowment of the risky asset.[3] For all agent i, endowment is normally distributed with mean 0 and precision π_ω. The private signal of trader i is a noisy observation of the final value:

$$\widetilde{s}_i = \widetilde{v} + \tilde{\epsilon}_i.$$

We assume that, for all i, $\tilde{\epsilon}_i$ are independently normally distributed with mean 0 and precision π_ϵ. It is publicly known that the final value of the risky asset is normally distributed with mean μ and precision π_v. We also assume that $\widetilde{v}$ and $\tilde{\epsilon}_i$ are independantly distributed for all i.

This structure of private information with two sources of uncertainty allows the existence of different trading motivations and the introduction of adverse selection problems in this model of asymmetric information. Indeed, when a liquidity provider observes a large purchase [respectively sell] order, he cannot know whether it comes from an information-based trader, i.e., a trader having a good signal (s_i is high) [respectively, s_i is low], or from a liquidity-based trader who trades for hedging reasons because of his initial endowment ($-\omega_i$ is large) [ω_i is large].

For each agent i, when q_i is the quantity of risky asset demanded (or offered) and p is the related unit price, his final wealth is $\tilde{W}_i = (q_i + \omega_i)\tilde{v} - pq_i$. If $q_i > 0$, the trader i is a buyer and if $q_i < 0$ he is a seller. Since his final wealth is normally distributed and he has an exponential

ciently less risk averse than traders so as to keep a certain level of market viability. Moreover, risk neutrality in this model avoids the inventory costs problems that risk averse market makers would face.

[3] We could also see ω_i as a quantity of the risky asset that agent i earns as a result of a liquidity shock.

utility function, the objective function of trader i is:

$$E[\tilde{W}_i|H_i] - \frac{\rho}{2} var[\tilde{W}_i|H_i]$$

where $E[\cdot|H_i]$ and $var[\cdot|H_i]$ are expectation and variance operators conditional on H_i which is his observed information set. So H_i contains private observation (s_i and ω_i), public information (market exogenous parameters - number of agents for each type, absolute risk aversion- the distributions of the asset's liquidation value, the noise about private observations and trader's initial endowments of risky assets) and information available to traders when they choose their strategies. This latter information depends on the trading structure.

2.1 Auction markets (or centralized order-driven markets)

We model the trading process in continuous auction markets as follows. First, informed traders choose their strategies given their information sets and submit their orders to be displayed on the screen. Second, after observing the traders' order flow, market makers determine their trading strategies and submit their orders. All orders are accumulated and executed at a single clearing price.

These markets are order-driven because trading is started up by traders' orders that are chosen before the price is fixed. This is similar to the way Madhavan (1992) describes auction markets. There is however a major difference. Madhavan (1992) considers that each execution in the continuous order-driven markets involves only one trader in the market facing orders by market makers. On the contrary, we assume in this work that auction markets are also concentrated. Hence, we argue that each execution may involve N different informed traders. Consequently, each trader should consider not only his own effect on the price but also the effects of his competitors' orders. This competitive environment for informed traders will affect their trading behavior.

Market makers in these markets are liquidity providers. They are institutional investors or intermediaries who respond to the order flow displayed on the screen. Because these markets are characterized by a higher degree of transparency, market makers are assumed to be symmetrically informed about market parameters and order flow. Each market maker submits a quantity-price schedule that maximizes his expected profits given his competitors' trading strategies. Therefore, competition between them is based on quantities and their effect on the equilibrium price is drawn through the effect of their orders on the market clearing condition.

An important aspect distinguishing trading structures is the information sets available for each market participant before choosing his trading strategy. Each agent i's observed information set H_i contains no information about markets since traders begin the trading process. For market makers, D_m contains the aggregate order flow Q submitted by traders.[4] Therefore:

$$H_i = \{\text{Public Information}, (s_i, \omega_i)\} \quad \text{for all } \; i = 1, ..., N$$

and

$$D_m = \{\text{Public Information}, Q\} \quad \text{for all } \; m = 1, ..., M.$$

We denote the vector of traders' demand functions by $\vec{Q} = \{q_1(p), ..., q_N(p)\}$ and the vector of dealers' demand functions by $\vec{d} = \{d_1(p), ..., d_M(p)\}$. For all i, let $\vec{Q}_{-i}$ be the $\mathbb{R}^{N-1}$ vector of all the other traders, i.e., $\vec{Q}_{-i} = \{q_1(p), ..., q_{i-1}(p), q_{i+1}(p), ..., q_N(p)\}$. We define $\vec{d}_{-m}$ for dealers in the same way. We define an equilibrium in auction markets as follows:

Definition 1 *The Bayesian Nash equilibrium in auction markets is defined by the set* $(p, \vec{Q}, \vec{d})$ *such that:*

(i) the equilibrium price p *satisfies the market clearing condition:*

$$\sum_{m=1}^{M} d_m(p) + \sum_{i=1}^{N} q_i(p) = 0 \tag{1}$$

(ii) for all $i \in \{1, ..., N\}$, $q_i(p)$ *is trader* i*'s strategy satisfying his optimality condition, given*

[4] This represents a low level of transparency for auction markets. However, we can imagine a more transparent auction market in which market makers observe orders separately. Within the present model (normal distributions, CARA utility function and rational expectations framework), this higher transparency has no effect on equilibrium outcomes.

the trading strategies of other market participants and given his information set:

$$q_i(p) \in argmax_{q(p)}\{E[\widetilde{W}_i|H_i, p, \vec{d}, \vec{Q}_{-i}] - \frac{\rho}{2}var[\widetilde{W}_i|H_i, p, \vec{d}, \vec{Q}_{-i}]\} \tag{2}$$

(iii) for all $m \in \{1, ..., M\}$*,* $d_m(p)$ *is the market maker's trading strategy satisfying the optimality of his expected profits conditional on the trading strategies of other market participants and his information set:*

$$d_m(p) \in argmax_{d_m(p)}\{E[(\tilde{v} - p)d_m(p)|D_m, p, \vec{Q}, \vec{d_{-m}}]\} \tag{3}$$

subject to non-negativity.

2.2 Dealership markets (or fragmented quote-driven markets)

We model the trading process in dealership markets as in Glosten (1989). Each trading period in dealership markets may be divided in two sub-periods. In the first sub-period, dealers begin by setting their bid-ask prices. Unlike market makers in auction markets, dealers compete in prices because of their status of price setters. Then, each trader chooses the best price for his unique order among dealers' quotations.[5]

Dealership markets are fragmented because trading occurs after bilateral meetings between dealers and traders. Hence, during the trading process, neither the dealer nor the trader is informed about other simultaneous trades on the market, if there are any. Because of this opacity, the price is affected by only one order.

Since they move first in the dealership trading process by setting their price-quantity schedules, dealers have no private information either about order flow or about the asset's value. This suggests that dealers, like market makers in auction markets, are symmetrically informed before they set their prices. For example, this may be caused by a Mandatory Last Trade Displaying rule. Obviously, we can consider another description of dealership markets with a higher de-

[5] Note that we assume that traders cannot split their orders among dealers. This may occur, for instance, because of higher fixed transaction fees. See Dennert (1993) and Biais, Martimort and Rochet (2000) for models of dealer competition where traders are allowed to split their orders among dealers.

gree of opacity when last trade publication is not mandatory[6] and where dealers cannot extract all private information from the pricing functions of informed dealers, or they cannot observe them.[7]

Since dealers compete in prices, using the standard argument for "Bertrand" games in these markets where risk neutral dealers are symmetrically informed, the unique Nash equilibrium for each of them is to set prices at the break-even level, i.e., prices equal the asset's expected value conditional on the order size and dealers' information set. In fact, if dealers set prices to make positive expected profits, it is always profitable for one dealer to undercut them.[8]

The information sets are $D_m = \{\text{Public Information}\}$ for dealers and $H_i = \{p(\cdot)$, Public Information, $(s_i, \omega_i)\}$ for traders. In dealership markets, the equilibrium is defined by the dealers' common pricing function $p(\cdot)$ and traders' orders $q(s_i, \omega_i)$ as follows:

Definition 2 *The equilibrium in dealership markets is a differentiable price function p and a corresponding demand $q(s_i, \omega_i)$ such that:*

(i) prices satisfy the zero-expected profit condition for dealers:

$$p(q) = E[\widetilde{v}|D_m, q] \tag{4}$$

(ii) each trader i maximizes his expected utility given the pricing function and his information set:

$$q(s_i, \omega_i) \in \arg\max_q \{E[\widetilde{W}_i|H_i, p] - \frac{\rho}{2} var[\widetilde{W}_i|H_i, p]\} \tag{5}$$

[6] See Madhavan (1995) for a theoretical analysis of the mandatory trade publication in fragmented and centralized markets. Gemmil (1996) provides an empirical evidence on the irrelevance of delayed publication on market liquidity by comparing the SEAQ's liquidity under three regimes of publication. He argues that competition between dealers prevents them from exploiting the potential advantage that a delay provides.

[7] See Biais, Martimort and Rochet (2000) for a model of competition between market makers in a mechanism design framework. In another context, Bernhardt and Hughson (1996) analyze the case of a competitive dealership market and consider the effect of the price tick size on the strategic behavior of dealers. See also Dennert (1993) for a game theoretical analysis of competition between dealers.

[8] More precisely, the price function is equal to the break-even level because of market opacity and because of the fact that traders cannot split their order among dealers. Under the same conditions of risk neutrality, symmetric information and price competition, when traders are allowed to split their orders and dealers are allowed to observe all trading strategies of traders with their competitors, Biais, Martimort and Rochet (2000) show that, with a finite number of dealers, equilibrium prices are different from the break-even level.

Now that equilibria in both markets heve been defined and trading structures have been presented, we derive these equilibria and establish their principal features.

3 Equilibrium in auction markets

We will focus on symmetric linear equilibria to make the comparison between auction and dealership mechanisms more tractable. As proposed in definition 1, the equilibrium is Bayesian-Nash. In this equilibrium, each trader determines his optimal trading strategy in order to maximize his expected profits given his conjectures about the trading strategies of the informed and liquidity traders. The conjecture of each informed trader must be correct conditional on his information.

Proposition 1 *Let* ψ *and* ψ_m *be defined as follows:* $\psi = \pi_v + \pi_\epsilon + (N-1)\pi$ *and* $\psi_m = \pi_v + N\pi$; *where* $\pi = \pi_\epsilon / (1 + \frac{\rho^2}{\pi_\epsilon \pi_\omega})$. *When*

$$\{\frac{\pi_\epsilon[(M+N-1)(2\psi - \psi_m) + \pi_v]}{[(M+N-1)\psi_m - \pi_v]} < \frac{\rho^2}{\pi_\omega}\} \tag{C1}$$

there exists an equilibrium in auction markets characterized by

(i)The strategy function of each market maker m *is:*

$$d_m(p) = \zeta(\mu - p), \tag{6}$$

(ii)The demand function of each trader i *is:*

$$q_i(s_i, \omega_i, p) = \alpha\mu + \beta s_i - \gamma\omega_i - \theta p, \tag{7}$$

(iii) The equilibrium price is:

$$p = \mu + \frac{1}{M\zeta} Q; \tag{8}$$

where $Q = \sum_{i=1}^{N} q_i$ *and* α, β, γ, θ *and* ζ *are positive constants defined in the Appendix. If Condition (C1) does not hold, there is no linear equilibrium and the market breaks down.*

In Proposition 1, Condition (C1) is necessary and sufficient for the existence of a (linear) equilibrium in auction markets. The left hand side may be interpreted as a measure of asymmetric information between traders and market makers. It indeed depends on ψ and ψ_m which are the adjusted precisions of the asset's value distribution for informed traders and market makers respectively. For each participant, his conjecture about each trader's strategy increases the precision of his information about the final asset's value by exactly π. So, for each trader, given his conjectures about the $(N-1)$ other traders, the precision of his information is the sum of his precision given his private signal $(\pi_v + \pi_\varepsilon)$ and the $(N-1)$ additional precisions (i.e. $(N-1)\pi$). On the other hand, for each market maker, given his conjectures about the traders' strategies, the precision of the his information is the additional precision inferred from theses strategies $(N\pi)$ and the ex ante precision π_v. Note that this measure of asymmetric information increases when π_ϵ is large relative to π_v, i.e., when traders' private information is precise compared to public information about the final asset's value. Conversely, when π_v is high and π_ϵ is low, i.e., when the private signal does not present a substantial improvement of information about the final asset's value, then $\frac{\pi_\epsilon[(M+N-1)(2\psi-\psi_m)+\pi_v]}{[(M+N-1)\psi_m-\pi_v]}$ is low, which may be interpreted as lower asymmetric information. Finally, we can easily see that the measure of asymmetric information is a decreasing function of N. Intuitively, when the number of traders on the market increases, market makers gather more information from observed variables (the precision of their learned information is $[\pi_v + N\pi]$) which decreases their informational disadvantage.

The right hand side of the equilibrium existence condition $(\frac{\rho^2}{\pi_\omega})$, may be seen as a measure of liquidity-motivated trading which depends on traders' risk aversion and the precision of liquidity shocks. When traders are more risk averse or when the variance of their initial endowments is high (π_ω is low), traders are more likely to be liquidity motivated. Hence, an equilibrium in the auction market exists when asymmetric information is small relative to non-information trading motivation. Otherwise, market makers' informational disadvantage relative to traders is so severe that they cannot avoid negative expected profits. This information disadvantage may be caused by a high precision of private signals relative to the public information about the final asset's value (i.e. π_ε is large relative to π_v).

Furthermore, in order to study the effects of ρ and π_ω on the occurence of Condition (C1), we need to study the variations of both measures with these parameters. As mentioned above, the non-information motivation measure is increasing in ρ and decreasing in π_ω. For the asymmetric information measure, substitution of the values of π, ψ and ψ_m gives

$$\frac{\pi_\epsilon[(M+N)\pi_v + (M+N-1)\pi_\epsilon(2+\frac{(N-2)\pi_\epsilon\pi_\omega}{\pi_\epsilon\pi_\omega+\rho^2})]}{(M+N-2)\pi_v + N(M+N-1)\frac{\pi_\epsilon^2\pi_\omega}{\pi_\epsilon\pi_\omega+\rho^2}},$$

or equivalently

$$\frac{2\pi_\epsilon[N(M+N-1)\pi_\epsilon + (M+2N-2)\pi_v]}{N[(M+N-2)\pi_v + N(M+N-1)\frac{\pi_\epsilon^2}{\pi_\epsilon+\frac{\rho^2}{\pi_\omega}}]} + (N-2)\frac{\pi_\epsilon}{N}.$$

Clearly, the asymmetric information measure is increasing in ρ and decreasing in π_ω. In Lemma 1 (see the Appendix), we study the effect of increasing liquidity motivation on the equilibrium existence condition in an auction market. We find that the marginal effect of an increase in the liquidity trading measure on the asymmetric information measure is lower than 1. This means that when we increase risk aversion or initial endowments variance, the liquidity measure increases more than the asymmetric information measure. Hence, increasing risk aversion or the variance of initial endowments weakens the equilibrium existence condition in continuous auction markets.

In the proof of Proposition 1, we also show that ζ is positive. Hence, market makers buy when prices are low and sell when prices are high. This means that liquidity providers have a stabilizing behavior, even in a model where they behave as profit-maximizers and not as social welfare maximizers.

How the number of participants affect the equilibrium existence condition? As mentioned above, the asymmetric information measure decreases in N. So, an increase in the number of traders decreases the informational disadvantage of market makers leading to a weaker equilibrium existence condition in auction markets.

As for the number of market makers, the measure of asymmetric information decreases with it. Indeed, a larger number of market makers decreases the marginal amount of asymmetric information supported by each of them. This reduces their reluctance to take the opposite side of the market and enhances market robustness to information asymmetry by weakening equilibrium existence condition in auction markets. The equilibrium in the limit case, i.e., when the number of market makers is extremely large, is characterized in Proposition 2.

Proposition 2 *With market maker's free entry, i.e. when $M \longrightarrow +\infty$, equilibrium in auction markets satisfies:*

(i) $\beta \rightarrow b_0$

(ii) $\alpha \rightarrow 0$

(iii) $\zeta \rightarrow 0$

(iv) $\gamma \rightarrow \frac{\rho b_0}{\pi_\epsilon}$

(v) $\theta \rightarrow b_0$ *where* b_0 *is the largest possible value of* β *given by*

$$b_0 = \frac{\pi_\varepsilon - \pi}{\rho} - \frac{\pi\psi}{\rho(\pi_v + (N-1)\pi)}$$

Note that Proposition 2 is similar to Proposition 7 in Madhavan (1992). Indeed, both propositions characterize the equilibrium for concentrated auction markets where market makers' competition converges to a Bertrand competition. As suggested by intuition, the equilibrium existence condition is weaker in this environment. Furthermore, Proposition 2 shows that market makers' free entry in auction markets raises traders' aggressiveness measured by the parameter β. Indeed, β takes its maximal value b_0. This occurs because of the centralization feature of auction markets. Indeed, since market makers are numerous, the marginal effect of each trader's order on the equilibrium price is sufficiently small to allow him to trade more aggressively according to his private information. Furthermore, as predicted, the quantity demanded by traders (which is finite) is shared between all market makers, and their individual order size tends towards zero.

In the following proposition, we derive some features of the equilibrium in auction markets, when it exists, in order to use them subsequently when we begin comparison between structures.

Proposition 3 *When a symmetric linear equilibrium exists for auction markets, then:*

(i) the equilibrium price is not semi-strong form efficient. This efficiency level is reached with market makers' free entry,

(ii) the variance of the equilibrium price tends towards zero when $N \to +\infty$*,*

(iii) with free entry, the quoted bid-ask spread tends to $\frac{2\pi}{b_o \pi_v}$*.*

In part (i) of Proposition 3 we show that the equilibrium price in auction markets is not semi-strong form efficient because of market makers' quantity-based competition which induces a difference between the equilibrium price and the expected value of the asset conditional on the extracted information from the price function and public information. This difference disappears with free entry because competition between market makers, even though it is quantity-based, becomes a perfect competition leading price to the semi-strong form efficiency level. In part (ii), it is shown that, when the number of traders becomes sufficiently large, market makers' precision of extracted information increases and the price tends towards the asset's value v. Finally, in part (iii), we show that the quoted bid-ask spread in auction markets (which is equal to $p(1) - p(-1)$) tends to its minimal value with free entry because, in this case, numerous market makers share the risk between them and the "marginal" informational risk borne by each of them is minimized.

4 Equilibrium in dealership markets

Equilibrium in these markets is derived as in Glosten (1989) and Madhavan (1992).[9] By using our notation we get the following proposition.

[9] See proposition 1 in Glosten (1989) and proposition 1 in Madhavan (1992).

Proposition 4 *In dealership markets, an equilibrium is defined by* $(p(q), q_i(s_i, \omega_i))$ *such that: for all* q

$$p(q) = \mu + \frac{\rho\pi_\epsilon\pi_\omega}{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}q \tag{9}$$

and

$$q_i(s_i, \omega_i) = \frac{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}{\rho[\pi_v\rho^2 + \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)]}[-\pi_\epsilon\mu + \pi_\epsilon s_i - \rho\omega_i]. \tag{10}$$

This equilibrium exists if $\pi_v\rho^2 > \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)$. *Otherwise, there is no equilibrium price schedule and the market breaks down.*

In Proposition 4, a necessary and sufficient condition for equilibrium existence is

$$\frac{\pi_\epsilon(\pi_\epsilon + \pi_v)}{\pi_v} < \frac{\rho^2}{\pi_\omega} .$$

If the traders' signal precision is sufficiently high, or the dealers' prior precision about the security's final value is sufficiently low, the market fails. Intuitively, when information asymmetry is large, then the dealers' informational disadvantage relative to traders is so severe that they cannot make non-negative expected profits. They will refuse to make transactions. Nevertheless, if the traders' non-information related motivations for trade are important, dealers are urged to take the opposite side of the market even with information asymmetry. Liquidity related motives for trade depend on the traders' risk aversion and the precision of their initial endowments.

Contrary to auction markets, note that the equilibrium outcomes in dealership markets depend neither on the number of traders nor on that of dealers. This is directly linked to the structure of the trading in these markets. Symmetric behavior of dealers and the fact that trading is involved in dealership markets after bilateral meetings between traders and dealers entail this independence between the equilibrium and the number of each type of market participants in the market. However, the extreme fragmentation considered in this model, where each dealer cannot receive more than one order, implies that this equilibrium outcome is realized for $M > N$.

As in auction markets, we derive in the following proposition some useful features of the equilibrium in dealership markets.

Proposition 5 *In a dealership market, when equilibrium exists, we have:*

(i) prices are semi-strong form efficient,

(ii) price variance is equal to $\frac{\pi}{(\pi_v+\pi)^2}$,

(iii) the quoted bid-ask spread is an increasing function of π_ϵ *and* π_ω *and a decreasing function of* π_v *and* ρ.

The explicit bid-ask spread in these markets is an increasing function of the trader's order size, reflecting the response of dealers to the asymmetric information problem they face. Furthermore, the quoted bid-ask spread is an increasing function of π_ϵ and π_ω, and a decreasing function of ρ and π_v. Thus, when the trader's private signal about the asset's value is more precise or when his initial endowment is less volatile, the dealers' belief that trading is information-motivated increase. This leads to an increase in the adverse selection problem, and consequently to an increase in the bid-ask spread. On the contrary, when the trader is more risk averse or when dealers have enough information about the risky asset (π_v is higher), asymmetric information is no longer severe and traders are more likely to be liquidity-motivated which induces a decrease of bid-ask spreads.

5 Market performance and trading structures

In this section, we compare trading equilibria in both structures. Different metrics are used to measure market performance. For each measure, we discuss the implications of our results on policy makers' and investors' decisions.

5.1 Market viability

Market viability reflects the ability of the trading structure to be less sensitive to asymmetric information between different participants. In particular, it measures the willingness of liquidity providers (market makers in auction markets and dealers in dealership markets) to take the opposite side of the market despite their informational disadvantage relative to informed traders.

From Proposition 1 and Proposition 4, market viability in both markets is measured by the equilibrium existence conditions. We say that one market is more viable when its equilibrium existence condition is satisfied each time an equilibrium exists in the other market.

Proposition 6 *Auction markets are always more viable than dealership markets; indeed, the equilibrium existence condition in auction markets is satisfied whenever trading in dealership markets exists. Moreover, if*

$$\pi_\epsilon < (1 - \frac{2}{M+N})\frac{\rho^2}{\pi_\omega},$$

and

$$\frac{M{\pi_\epsilon}^2\pi_\omega}{(M+N-2)\rho^2 - (M+N)\pi_\epsilon\pi_\omega} - (N-1)\frac{{\pi_\epsilon}^2\pi_\omega}{\rho^2 + \pi_\epsilon\pi_\omega} < \pi_v < \frac{{\pi_\epsilon}^2\pi_\omega}{\rho^2 - \pi_\epsilon\pi_\omega},$$

an equilibrium exists only in auction markets.

Madhavan (1992) shows in his Proposition 4 that fragmented quote-driven markets are more robust to problems of asymmetric information than fragmented order-driven markets. Moreover, in his Proposition 7, Madhavan (1992) states that concentrated auctions where market makers compete on prices[10] are more robust to problems of asymmetric information than continuous trading systems. In Madhavan (1992)'s propositions, the effects of concentration and timing of order submission on market viability are disentangled. Indeed, while Proposition 4 considers the unique effect of timing of order submission, Proposition 7 studies the sole effect of concentration. The principal lesson we can draw form these propositions, is that both the level of competition between liquidity providers (Proposition 4) and concentration (Proposition 7) allow higher market viability. However, since dealership markets feature a higher level of competition and continuous auction markets are characterized by a higher concentration, the results stated in Madhavan (1992) cannot inform us about the relative contribution of each dimension to market viability.

[10] As stated above, this environment is qualified in Madhavan (1992) as periodic auctions.

This issue is addressed in our Proposition 6. Indeed, we demonstrate that concentration dominates the timing of order submission by having a larger impact on market viability. This is directly attributable to the higher level of information conveyed in concentrated markets. This higher level of information decreases the market makers' informational disadvantage relative to dealers operating in more opaque markets. So, dealers will be incited to take the opposite side of the market in auction markets. For some values of private information precision, it is possible that the precision of public information is such that trading occurs only in auction markets.[11]

Several regulating incentives and decision rules for investors may be deduced from this result. First, consider a firm going public and having the possibility to choose to be quoted either in a dealership or an auction structure. It is commonly argued that the IPO environment is characterized by a high level of asymmetric information. Underpricing in IPOs is mainly explained by different arguments related to the level of asymmetric information between agents. Proposition 6 suggests that firms are better off going public in an auction structure since this will increase the probability of trading new shares in the secondary market. So, in order to increase the probability of success of trading of new shares in the secondary market, our result suggests that underpricing would be higher in dealership markets. This result is confirmed empirically in Falconieri, Murphy and Weaver (2003) where it is documented that the level of underpricing is higher in the NASDAQ (a dealership like structure) than in the NYSE (an auction like structure).

Second, market viability in practice is related to trading halts and circuit breakers. Proposition 6 suggests that a firm quoted in both structures will be less exposed to trading halts, caused

[11] Glosten (1994) gets a similar result about the viability of auction markets. In his model, he proved under more general conditions that auction markets (which have the same structure as the limit order book analyzed in that work) do not invite competition from third market dealers, while other trading institutions do. However, the comparison in Glosten (1994) is focused on the outcome for an agent who is in competition with the existing limit order book (with an infinite number of market makers) by offering a liquidity-providing service. It is shown that this agent will always earn negative expected profits. On the contrary, these expected profits may be positive if this agent would compete with another existing trading structure. This may be interpreted here by considering that another trading alternative enhances the market viability in dealership markets contrary to auction markets.

by asymmetric information problems, than a firm quoted only in a dealership structure.[12]

Third, from a policy making point of view, dealership markets need a higher standards of information disclosure about firms than auction markets in order to alleviate the asymmetric information problem.

5.2 Traders' aggressiveness

Trading aggressiveness of informed traders is measured in both markets by the marginal effect of increasing one trader's private signal on his trading strategy. This reflects the importance of private information for informed traders when they design their trading strategies. In auction markets it is equal to β and in dealership markets to:

$$\begin{aligned} \beta_D &= \frac{\pi_\epsilon[\pi_v \rho^2 - \pi_\epsilon \pi_\omega(\pi_\epsilon + \pi_v)]}{\rho[\pi_v \rho^2 + \pi_\epsilon \pi_\omega(\pi_\epsilon + \pi_v)]} \\ &= \frac{\pi_v(\pi_\epsilon - 2\pi) - \pi_\epsilon \pi}{\rho(\pi_v + \pi)} \end{aligned}$$

It is commonly argued that, because of opacity, informed traders trade more aggressively in fragmented quote-driven markets than in fragmented order-driven markets. If we consider concentrated order-driven markets, the relative opacity of dealership markets is more important, strengthening the argument of higher traders' aggressiveness in dealership markets. However, concentration has a positive effect on traders' aggressiveness in auction markets. Indeed, concentration allows a higher quantity of information inferred by uninformed market makers and consequently a reduction of informational advantage for informed traders. Traders may therefore prefer to trade more aggressively in auction markets in order to compensate for their reduced informational advantage. In accordance with this intuition, we prove the following:

Proposition 7 *For a finite M, when* $(N-1) > \frac{\pi_\epsilon(\pi_v+\pi)}{2\pi(\pi_\epsilon-\pi)}$, *traders in auction markets trade more aggressively.*

[12] Trading halts are activated for different reasons. Some of them are institutional like those related to regulatory rules of information disclosure and others may be strategically demanded by market makers (see Edelen and Gervais, 2003).

Thus, when the number of traders sufficiently high, the second (positive) effect dominates since strategies convey higher information and the marginal effect of each trader is sufficiently low. In this case, traders choose to be more aggressive in auction markets. With free entry, Proposition 2 proves that β tends towards b_0, which is higher than β_D. Thus, trading is more aggressive in auction markets in that case.

5.3 Price variance

Before receiving their private information and when they face the choice between trading systems, traders will be concerned with the ex ante price variance in both markets. Alternatively, for regulators, price variance may be considered as a measure of price volatility and could be an important argument for choosing the market structure featuring lower price variance. From the traders' point of view, expected prices are the same in both structures. Thus, because of their risk aversion the ex ante price variance may be considered as an important parameter of comparison between markets.

Comparing price variance in dealership and fragmented order-driven markets, Madhavan (1992) finds that price distortions caused by the trading strategies of market makers lead to higher price variance in order-driven markets. If we consider concentration in auction market, this price distortion effect is amplified since we in this case have $(M + N)$ agents affecting the equilibrium price. Nevertheless, an opposite effect on price variance arises. Indeed, concentration reduces the asymmetric information between traders and market makers. So, all agents learn more information from their competitors' strategies inducing lower price variance. In line with this intuition, we prove the following.

Proposition 8 *With a fixed M, if $N < (\frac{\pi_v}{\pi})^2$, price variance is higher in auction markets. However, when $(N - 1) > \frac{{\pi_\epsilon}^2(\pi_v+\pi)^2 - 4\pi^3(\pi_v+\pi_\epsilon)}{4\pi^4}$, prices in auction markets have lower variance.*

Hence, as suggested by the intuition, if N is sufficiently low, the price distortion effect dominates the informational effect of concentration on price variance. However, when N is sufficiently high, the information acquired by different market participants induces lower price variance in auction markets compared to dealership markets.

This result is supported by the empirical work in Jain (2002). In that work, it is documented that price volatility is higher in dealership markets than in auction markets. Market volatility in each market, is measured by the price variance of the most active firms for which it is more likely to have a larger N.

In conclusion, regulators preferring markets with lower price variability will opt for an auction structure if they expect their market to be relatively deep (N high) and for a dealership structure if they consider that their market will be relatively thin (N is low).

5.4 Informational efficiency

It follows from Proposition 3 and Proposition 5 that prices are semi-strong form efficient in dealership markets but not in auction markets. This result is directly linked to the difference of competition among liquidity providers. Price competition in dealership markets induces prices to reflect all publicly available information, whereas, in auction markets, since they are concentrated and market makers compete on quantities, each agent's strategy affects the equilibrium price leading to a non semi-strong form efficiency. With market making free entry, auctions markets attain this level of efficiency. In that case, quantity-based competition converges, in terms of efficiency, to price based competition.

Note, however, that the semi-strong form efficiency of prices in dealership markets in this model is due to the specific assumptions leading to the expected zero-profits condition. When we introduce asymmetric information between dealers (no mandatory last trade publication) or the possibility of splitting a trader's order among dealers,[13] this level of efficiency should disappear.

[13] In this case, when dealers are symmetrically informed and with a certain level of transparency for markets, Biais, Martimort and Rochet (2000) show that dealers' expected profits are strictly higher than zero.

An alternative measure of efficiency is the following:[14]

$$e = 1 - \frac{var(\widetilde{v}/p)}{var(\widetilde{v})}. \tag{11}$$

This measure takes its values in $[0, 1]$, and the extreme values represent complete informational inefficiency or efficiency. It is zero (one) when p is completely uninformative (perfectly informative) about the final value $\widetilde{v}$.

Transparency of auction markets leads to more information transmitted in the equilibrium price, and hence to a higher degree of efficiency. This is the intuition of the following proposition.

Proposition 9 *If we use the informational efficiency measure defined in equation (11), auction markets are more efficient than dealership markets.*

5.5 Market liquidity

Since the model we use in this work is static, we use price related measures of market liquidity.[15] Market depth or bid-ask spread are the appropriate measures. Because equilibria are linear, both measures lead to the same comparison. Let us then consider market depth as a measure of market liquidity. We say that one market is more liquid if its depth is higher. As in Kyle (1985), market depth in each market is measured by the inverse of the marginal effect of quantities on equilibrium price. Market depths in auction and dealership markets are denoted respectively by Δ_A and Δ_D, with

$$\Delta_A = M\zeta = \frac{(2-N)\pi_\epsilon + 2(N-1)\pi + (N-1)\rho\beta}{\pi_\epsilon(\pi_\epsilon - \pi - \rho\beta)}$$

and

$$\Delta_D = \frac{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}{\rho\pi_\epsilon\pi_\omega} = \frac{\pi_v(\pi_\epsilon - \pi) - \pi(\pi_\epsilon + \pi_v)}{\rho\pi}.$$

[14] See Brown and Zhang (1997) for a discussion of the properties of this measure of informational efficiency.

[15] In a dynamic framework, another dimension of liquidity is the one related to immediacy cost for trading over time (see for example Grossman and Miller, 1988).

Intuitively, because market depth is the effect of individual orders on prices, concentration leads to a lower marginal effect of each individual trader on prices. Then, concentrated markets seem to be deeper. But, if we consider the second dimension distinguishing pure structures, i.e., the timing of order submission, an opposite argument arises. Indeed, quote-driven markets are deeper because price competition leads to a lower price sensitivity to trader's orders than in order-driven markets where competition between market makers is based on quantities.[16]

Therefore, market participant's timing of action and concentration have opposite effects on market depth, and once again, the liquidity based comparison between trading structures is not obvious.

Because we do not have an explicit formulation of β, a direct comparison between Δ_A and Δ_D is not possible. Nevertheless, one can obviously see that for the bounds of β (i.e., zero and b_0), the difference between Δ_A and Δ_D is negative for the lower bound and positive for the upper bound. Since β is a continuous function of M, we conclude that if M is sufficiently large, concentrated order-driven markets are deeper than fragmented quote-driven markets. Intuitively, if quantity-based competition between market makers is greater, or sufficiently "efficient", the positive effect of concentration on depth dominates the negative effect generated by the timing of action of market participants.

From regulators' point of view, enhancing liquidity in auction markets should be done by encouraging market making, otherwise it is better for them to opt for a fragmented quote-

[16] This argument may be proved using the equilibria derived in Madhavan (1992). In that work, if Δ_Q^* and Δ_O^* denote market depths in both markets (Q for quote-driven and O for order-driven), we have:

$$\Delta_Q^* = \Delta_D = \frac{\pi_v \rho^2 - \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)}{\rho \pi_\epsilon \pi_\omega}$$

and

$$\Delta_O^* = \frac{(M-2)\pi_v \rho^2 - M \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)}{(M-1)\rho \pi_\epsilon \pi_\omega}.$$

So

$$\Delta_Q^* - \Delta_O^* = \frac{\pi_v \rho^2 + \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)}{(M-1)\rho \pi_\epsilon \pi_\omega} > 0.$$

driven structure where liquidity is greater. Similarly, investors and firms, looking "greedily" for liquidity when they face the choice between market structures, should study the level of competition between market makers in auction markets. If it is deemed sufficiently efficient, they will choose the auction structure. Otherwise, it is optimal for them to opt for dealership markets.

6 Concluding Remarks

In this work, we establish a comparison between two financial market structures using the fact that they differ with respect to two dimensions: concentration and timing of action for different market participants. It turns out that some previous results on relative market performance are no longer true when we consider both concentration and timing of action. For instance, we show that auction markets are less sensitive to asymmetric information than quote-driven markets. This result generalizes those of Madhavan (1992) by suggesting that the concentration effect on market viability dominates the impact of the level of competition between liquidity providers. In the same way, other results related to traders' aggressiveness, price variability and market liquidity are derived, confirming the importance of using both dimensions distinguishing markets. Some of these results are confirmed by several recent empirical works.

Nevertheless, this analysis lacks some features of financial market organization that could influence investors or regulators' choices. Among others, we omitted the effects of inventory costs on liquidity providers' strategies, and execution risk faced by limit orders' submitters in auction markets (while in dealership markets, market makers provide them with an insurance against this risk). For the latter, it is clear that introducing it as a parameter of choice for traders fosters dealership markets. So, depending on the importance that each investor attributes to this parameter relative to the measure of performance considered in our model, one can guess the choice of each trader. For investors and firms, there exist other regulating parameters which could be considered when they are choosing a trading structure; examples of such parameters are transaction fees and trading capacity of intermediaries.[17]

For inventory costs, strategies of liquidity providers would be different in both markets and their risk aversion measure will affect different market performance. However, it is not clear whether this will be in favor of auction or dealership markets. An interesting field of research is to compare these market structures when we consider both the adverse selection and the inventory costs paradigms.[18]

Moreover, in this work auction and dealership markets are compared in a context where they are assumed to be separate entities. So, the interaction between them when both of them exist is ignored. Several works[19] however argue that this point may have an important effect on the performance of these structures and hence on the investment strategies of investors and the market structure choice of regulators.

Financial markets are rarely organized as pure fragmented quote-driven or centralized order-driven. In fact, each one may be seen as an hybrid version of these extreme organizations. For instance, at the Paris Bourse, which is organized as an electronic auction market, some trades may be executed on the off-exchange markets[20] so that trading is conducted without being displayed on the screen (at least before its execution). On the New York Stock Exchange, there is a monopolistic specialist for each stock which is in competition with a limit order book. So, two natural questions arise: Is it possible to derive an optimal trading structure as a combined version of the basic market organizations studied in this work? And how could this optimal mechanism depend on market features and regulators' parameters of choice?

[17] See Röell (1990) and Fishman and Longstaff (1992).

[18] Brown and Zhang (1997) uses both paradigms to compare auction markets (termed limit order book markets) and another centralized order-driven market in which traders are compelled to submit market orders (termed dealer markets). Then, the difference between these markets is based on the higher information for dealers (observability of the traders' order flow) and the existence of the execution-price risk in their dealer markets. See also Hendershott and Mendelson (2000) for an analysis of the effects of introducing a competing trading structure on dealer markets, where both asymmetric information and inventory costs are introduced.

[19] See for example Grossman (1990), Seppi (1992) and Blume and Goldstein (1997) for a theoretical analysis of the interaction between trading structures.

[20] Such transactions are called *opérations de contrepartie.*

APPENDIX

Proof of Proposition 1:

This proposition is an extension of Madhavan (1992)'s fragmented order-driven equilibrium construction. The proof constructs the linear Bayesian Nash equilibrium for auction markets by solving for an agent's best response to the conjectured strategies adopted by other agents and then shows that these conjectures are consistent.

$*$ **Step 1 (Traders' best replies):** Suppose that a trader i with the information set $H_i = (s_i, \omega_i)$ will conjecture that:

1. the trading strategy for all $j \in \{1, ..., N\}$ and $j \neq i$ is:

$$q_j(p) = \alpha\mu + \beta s_j - \gamma\omega_j - \theta p \tag{12}$$

2. the trading strategy of market maker m, for all $m \in \{1, ..., M\}$ is:

$$d_m(p) = \zeta(\mu - p) \tag{13}$$

where β and γ are positive constants.

Then, if trader i chooses q_i, the conjectured market clearing condition is

$$M\zeta(\mu - p) + q_i + \sum_{j \neq i}(\alpha\mu + \beta s_j - \gamma\omega_j - \theta p) = 0.$$

Then, the equilibrium price satisfies the following equality:

$$p = [\frac{M\zeta + (N-1)\alpha}{\lambda}]\mu + \frac{1}{\lambda}q_i + \frac{1}{\lambda}\sum_{j \neq i}(\beta s_j - \gamma\omega_j) \tag{14}$$

where $\lambda = [M\zeta + (N-1)\theta]$. From equation (2) the trading strategy $q_i(s_i, \omega_i, p)$ satisfies:

$$q_i(s_i, \omega_i, p) \in \underset{q_i}{argmax}\{E[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}](\omega_i + q_i) - q_i p - \frac{\rho}{2}var[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}](\omega_i + q_i)^2\}.$$

Optimality conditions (the First and the Second Order Conditions) are

$$FOC: \quad E[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}] - p - \tfrac{\partial p}{\partial q_i} q_i - \rho var[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}](\omega_i + q_i) = 0 \tag{15}$$

$$SOC: \quad -2\tfrac{\partial p}{\partial q_i} - q_i \tfrac{\partial^2 p}{\partial {q_i}^2} - \rho var[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}] < 0. \tag{16}$$

From equation (14), we have $\frac{\partial p}{\partial q_i} = \frac{1}{\lambda}$ and $\frac{\partial^2 p}{\partial {q_i}^2} = 0$, so the optimal trading order for i is

$$q_i = \frac{E[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}] - p - \rho var[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}]\omega_i}{\rho var[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}] + \frac{1}{\lambda}}. \tag{17}$$

Let us turn to the computation of $E[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}]$ and $var[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}]$. Define z_i as follows:

$$z_i = \frac{1}{(N-1)\beta}[\lambda p - (M\zeta + (N-1)\alpha)\mu - q_i]. \tag{18}$$

From equation (14), z_i may also be written as follows:

$$z_i = \frac{1}{(N-1)\beta}[\sum_{j \neq i}(\beta s_j - \gamma \omega_j)].$$

Thus, z_i is a realization of a random variable $\tilde{z}_i = \tilde{v} + \widetilde{x}_i$ where $\widetilde{x}_i \sim \mathcal{N}(0, [(N-1)\pi]^{-1})$ and

$$\pi = \frac{1}{\frac{1}{\pi_\epsilon} + \frac{\gamma^2}{\beta^2 \pi_\omega}} = \frac{\pi_\epsilon \pi_\omega}{\pi_\omega + \frac{\gamma^2 \pi_\epsilon}{\beta^2}}. \tag{19}$$

Given the equilibrium price, trader i observes a realization of $\tilde{z}_i$; this allows him to adjust his beliefs about the final value of the asset. Since all variables are normally distributed and stochastically independent, the trader's conditional expectation of $\tilde{v}$ is:

$$E[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}] = E[\tilde{v}|s_i, z_i] = \frac{\pi_v \mu + \pi_\epsilon s_i + (N-1)\pi z_i}{\psi} \tag{20}$$

and the conditional variance is

$$var[\tilde{v}|s_i, p, \vec{d}, \vec{Q}_{-i}] = var[\tilde{v}|s_i, z_i] = \psi^{-1} \tag{21}$$

where $\psi = \pi_v + \pi_\epsilon + (N-1)\pi$. Substituting equations (20) and (21) into equation (17) gives

$$q_i(s_i, \omega_i, p) = \frac{1}{r}\{\mu[\beta\pi_v - \pi(M\zeta + (N-1)\alpha)] + \beta\pi_\epsilon s_i - \beta\rho\omega_i - p[\psi\beta - \lambda\pi]\} \tag{22}$$

where $r = \beta\rho + \pi + \beta\frac{\psi}{\lambda}$.

Then, the trading strategy of i has the conjectured form with α, β, γ, and θ satisfying the following equations:

$$\alpha = \frac{\beta\pi_v - \pi(M\zeta + (N-1)\alpha)}{r} \tag{23}$$

$$\beta = \frac{\beta\pi_\epsilon}{r} \tag{24}$$

$$\gamma = \frac{\beta\rho}{r} \tag{25}$$

$$\theta = \frac{\psi\beta - \lambda\pi}{r} \tag{26}$$

$*$ **Step 2 (Market makers' best replies):** Suppose that the market maker m conjectures that traders' strategies are described as in equation (12) for $j \in \{1, ..., N\}$ and that the trading strategies of his competitors are: $d_l(p) = \zeta(\mu - p)$ for all $l \in \{1, .., m-1, m+1, ..., M\}$.

For this market maker, if his optimal order is $d_m(p)$, then the market clearing condition is:

$$d_m(p) + (M-1)\zeta(\mu - p) + \sum_{j=1}^{N}(\alpha\mu + \beta s_j - \gamma\omega_j - \theta p) = 0.$$

Therefore,

$$p = \mu[\frac{(M-1)\zeta + N\alpha}{\lambda_m}] + \frac{d_m(p)}{\lambda_m} + \frac{1}{\lambda_m}\sum_{j=1}^{N}(\beta s_j - \gamma\omega_j) \tag{27}$$

with $\lambda_m = (M-1)\zeta + N\theta$. From equation (3), $d_m(p)$ satisfies:

$$d_m(p) \in argmax_{d_m} E[(\tilde{v} - p)d_m(p)|Q, p, \vec{Q}, \vec{d_{-m}}].$$

The optimality conditions are

$$FOC: \quad E[\tilde{v}|Q, p, \vec{Q}, \vec{d_{-m}}] - p - \tfrac{\partial p}{\partial d_m} d_m(p) = 0 \tag{28}$$

$$SOC: \quad -2\tfrac{\partial p}{\partial d_m} - d_m \tfrac{\partial^2 p}{\partial {d_m}^2} < 0 \tag{29}$$

From equation (27), we have: $\frac{\partial p}{\partial d_m} = \frac{1}{\lambda_m}$ and $\frac{\partial^2 p}{\partial {d_m}^2} = 0$. Thus, from equation (28), the optimal trading order of m is:

$$d_m(p) = \lambda_m[E[\tilde{v}|Q, p, \vec{Q}, \vec{d_{-m}}] - p]. \tag{30}$$

Given his conjectures, market maker m observes

$$Q = \sum_{j=1}^{N} q_j = N\alpha\mu - N\theta p + \sum_{j=1}^{N}(\beta s_j - \gamma\omega_j); \tag{31}$$

then, given p and this observation, m should observe

$$z_m = \frac{Q - N\alpha\mu + N\theta p}{N\beta}. \tag{32}$$

From equation (31), we have also that

$$z_m = \frac{1}{N}\sum_{j=1}^{N}(s_j - \frac{\gamma}{\beta}\omega_j).$$

Then, z_m is a realization of a random variable $\tilde{z}_m = \tilde{v} + \widetilde{y}_m$, where $\widetilde{y}_m$ is a centered normal random variable with variance $[N\pi]^{-1}$ where π is defined in equation (19).

Consequently,

$$E[\tilde{v}|Q, p, \vec{Q}, \vec{d_{-m}}] = E[\tilde{v}|z_m] = \frac{\pi_v\mu + N\pi z_m}{\psi_m}; \tag{33}$$

with $\psi_m = \pi_v + N\pi$. Substituting equation (33) into (30) gives:

$$d_m(p) = \{\frac{\pi_v\beta - \pi[(M-1)\zeta + N\alpha]}{\pi + \beta\frac{\psi_m}{\lambda_m}}\}\mu - \{\frac{\psi_m\beta - \pi[(M-1)\zeta + N\theta]}{\pi + \beta\frac{\psi_m}{\lambda_m}}\}p. \tag{34}$$

This solution takes the form of the market makers' conjectured strategies when both terms are equal to ζ. So we have

$$\zeta = \frac{\pi_v\beta - \pi[(M-1)\zeta + N\alpha]}{\pi + \beta\frac{\psi_m}{\lambda_m}} \tag{35}$$

and,

$$\zeta = \frac{\psi_m\beta - \pi[(M-1)\zeta + N\theta]}{\pi + \beta\frac{\psi_m}{\lambda_m}}. \tag{36}$$

$*$ **Step 3 (existence of the equilibrium):** The equilibrium exists when the equations system (23), (24), (25), (26), (35) and (36) for the five variables α, β, γ, θ and ζ has a solution.

First, from equations (24) and (25) we have $\frac{\gamma}{\beta} = \frac{\rho}{\pi_\epsilon}$. Substitution into equation (19) allows us to conclude that π does not depend on equilibrium parameters:

$$\pi = \frac{\pi_\epsilon^2 \pi_\omega}{\pi_\epsilon \pi_\omega + \rho^2}.$$

Moreover, from equations (35) and (36) we have: $\theta = \beta + \alpha$. Therefore, if we substitute values of θ and ζ (derived from (35) and (36)) into equations (23), (25) and (26), the underlying system is simplified and we have to solve a new system with only three variables:

$$M\zeta + (N-1)\theta = \frac{(\pi_v + (N-1)\pi)\beta - \pi_\epsilon \alpha}{\pi} \tag{37}$$

$$[(\pi_v + (N-1)\pi)\beta - \pi_\epsilon \alpha](\pi_\epsilon - \pi - \rho\beta) = \pi\beta\psi \tag{38}$$

$$\zeta[\pi + \beta\frac{\psi_m}{\lambda_m}] = \pi_v\beta - \pi[(M-1)\zeta + N\alpha] \tag{39}$$

In the following, we will endeavor to find conditions under which a solution for this system exists. From equation (38) we can write α as a function of β :

$$\alpha = \frac{\beta[(\pi_v + (N-1)\pi)(\pi_\epsilon - \pi - \rho\beta) - \pi\psi]}{\pi_\epsilon(\pi_\epsilon - \pi - \rho\beta)} \tag{40}$$

and from equation (37) we have,

$$\zeta = \frac{\pi_v\beta - \alpha(\pi_\epsilon + (N-1)\pi)}{M\pi}. \tag{41}$$

Then multiplying by $(M-1)$, adding $N\theta$ and considering the fact that $\theta = \alpha + \beta$ gives

$$[(M-1)\zeta + N\theta] = \frac{[((M-1)\pi_v + MN\pi)\beta - \alpha((M-1)\pi_\epsilon - (M+N-1)\pi)]}{M\pi}.$$

Substituting the value of α (in equation (40)) gives:

$$[(M-1)\zeta + N\theta] = \frac{\beta\psi[(2M+N-2)\pi_\epsilon - 2(M+N-1)\pi - \rho(M+N-1)\beta]}{M\pi_\epsilon(\pi_\epsilon - \pi - \rho\beta)}; \tag{42}$$

we denote

$$g(\beta) \stackrel{Def}{=} \frac{\beta\psi[(2M+N-2)\pi_\epsilon - 2(M+N-1)\pi - \rho(M+N-1)\beta]}{M\pi_\epsilon(\pi_\epsilon - \pi - \rho\beta)}.$$

We will then derive another relation between $[(M-1)\zeta + N\theta]$ and β using equation (39). From equation (37) we have:

$$[(M-1)\zeta + N\alpha] = \frac{\pi_v\beta - \alpha(\pi_\epsilon - \pi)}{\pi} - \zeta.$$

Substitution into the right hand side of equation (39) and simplification yield:

$$\frac{\zeta\beta\psi_m}{\lambda_m} = \alpha(\pi_\epsilon - \pi); \qquad (43)$$

substituting equation (41) and rearranging terms gives:

$$\lambda_m = \frac{\beta\psi_m[\pi_v\beta - \alpha(\pi_\epsilon + (N-1)\pi)]}{M\pi\alpha(\pi_\epsilon - \pi)}. \qquad (44)$$

Finally, substitution of (40) into (44) and simplification give the following second relation between λ_m and β:

$$[(M-1)\zeta + N\theta] = \frac{\beta\psi_m\psi[(2-N)\pi_\epsilon + 2(N-1)\pi + (N-1)\rho\beta]}{M(\pi_\epsilon - \pi)[(\pi_v + (N-1)\pi)(\pi_\epsilon - \pi - \rho\beta) - \pi\psi]} \stackrel{Def}{=} f(\beta). \qquad (45)$$

From the second order condition of the market makers' maximization problem, λ_m has to be strictly positive. Thus, an equilibrium exists when $f(\beta) > 0$ and $g(\beta) > 0$. This occurs when $\beta > \sup\{0, a_0\}$ and $\beta < b_0$ with:

$$a_0 = \frac{(N-2)\pi_\epsilon}{(N-1)\rho} - \frac{2\pi}{\rho}$$

and

$$b_0 = \frac{\pi_\epsilon - \pi}{\rho} - \frac{\pi\psi}{\rho(\pi_v + (N-1)\pi)}$$

(note that b_0 is always greater than a_0).

If $b_0 \leq 0$, $f(\cdot)$ is not positive and the equilibrium does not exist; so b_0 has to be positive. Let us derive a necessary and sufficient condition for the equilibrium existence.

We can write the following

$$(f-g)(\beta) = K_1(\beta) \times h(\beta),$$

with

$$K_1(\beta) = \frac{\beta\psi}{M\pi_\epsilon\rho(\pi_\epsilon - \pi - \rho\beta)(\pi_\epsilon - \pi)(\psi_m - \pi)(b_0 - \beta)}$$

and

$$h(\beta) = X_1\rho^2\beta^2 + X_2\rho\beta + X_3,$$

where

$$\begin{aligned} X_1 = & \; -\{(N-1)\pi_\epsilon\psi_m + (M+N-1)(\pi_\epsilon - \pi)(\psi_m - \pi)\} \\ X_2 = & \; [(2N-3)\pi_\epsilon - 3(N-1)\pi]\pi_\epsilon\psi_m + (\pi_\epsilon - \pi)\{(\psi_m - \pi) \\ & [(3M+2N-3)\pi_\epsilon - 3(M+N-1)\pi] - (M+N-1)\pi\psi]\} \\ X_3 = & \; (\pi_\epsilon - \pi)\{2\pi\psi[(2M+N-2)\pi_\epsilon - 2(M+N-1)\pi] - \\ & \pi_\epsilon\psi_m[(2M+2N-4)\pi_\epsilon - 2(M+2N-2)\pi]\}. \end{aligned} \tag{46}$$

Since, $K_1(\beta) > 0$ for all $\beta \in (0, b_0)$, we should study the sign of $h(\cdot)$ in order to state the equilibrium existence condition.

We can easily see that $X_1 < 0$. Then $h(\cdot)$ is a concave function over $\mathbb{R}$ and it reaches its maximum at b' such that: $b' = \frac{-X_2}{2\rho X_1}$. Since $b_0 > 0$, we can easily prove that $b' > b_0$.[21]

Therefore, $h(\cdot)$ is a strictly increasing function on $(0; b_0)$ and, the equilibrium exists if and only if $h(0) = X_3 < 0$. To see this, note first that $f(\beta) - g(\beta) \to +\infty$ when $\beta \to b_0$. Second, we have $h(b_0) > 0$. So, if $h(0) < 0$, then from the intermediate value theorem we can conclude that $h(\beta') = 0$ for some $\beta' \in (0; b_0)$ and therefore that $f(\beta') = g(\beta')$. Note also that by strict monotonicity of $h(\cdot)$, this equilibrium is the unique linear symmetric equilibrium.

[21] Indeed,

$$b' = \frac{(N-1)\pi_\epsilon\psi_m[\pi_\epsilon - \pi + \rho a_0]}{2\rho[(N-1)\pi_\epsilon\psi_m + (M+N-1)(\pi_\epsilon - \pi)(\psi_m - \pi)]} + \frac{(\pi_\epsilon - \pi)(\psi_m - \pi)[(2M+N-2)\pi_\epsilon - 2(M+N-1)\pi + (M+N-1)\rho b_0]}{2\rho[(N-1)\pi_\epsilon\psi_m + (M+N-1)(\pi_\epsilon - \pi)(\psi_m - \pi)]}$$

then,

$$b' - b_0 = \pi_\epsilon\left[\frac{\psi_m[(N-1)\pi\psi - \pi_\epsilon\pi_v] + (\pi_\epsilon - \pi)(\psi_m - \pi)[(M-1)\pi_v + MN\pi]}{2\rho(\psi_m - \pi)[(N-1)\pi_\epsilon\psi_m + (M+N-1)(\pi_\epsilon - \pi)(\psi_m - \pi)]}\right]$$

which is positive when b_0 is positive.

Now, rearranging the terms of equation (46) and considering the fact that $(\pi_\epsilon - \pi)(\psi_m - \pi) = -\pi\psi + \pi_\epsilon\psi_m$ yield the following equality

$$h(0) = X_3 = 2(\pi_\epsilon - \pi)^2\{[(2M + N - 2)\pi_\epsilon - 2(M + N - 1)\pi](\pi - \psi_m) + M\pi_\epsilon\psi_m\}.$$

Then, considering the fact that $[(2M + N - 2)\pi_\epsilon - 2(M + N - 1)\pi](\psi_m - \pi)$ is equal to

$$(M + N - 1)(\pi_\epsilon\psi_m - 2\pi\psi) + \pi_\epsilon((M - 1)\pi_v + MN\pi)$$

gives

$$h(0) = 2(\pi_\epsilon - \pi)^2\{\pi_\epsilon\pi_v - (M + N - 1)(\pi_\epsilon\psi_m - 2\pi\psi)\}.$$

Therefore, equilibrium in auction markets exists if and only if:

$$\pi_\epsilon\pi_v - (M + N - 1)(\pi_\epsilon\psi_m - 2\pi\psi) < 0.$$

Substituting the equation defining π as a function of π_ϵ, π_ω and ρ, this condition can be written as follows:

$$\frac{\pi_\epsilon[(M + N - 1)(2\psi - \psi_m) + \pi_v]}{[(M + N - 1)\psi_m - \pi_v]} < \frac{\rho^2}{\pi_\omega}.$$

Under this equilibrium existence condition, we can easily prove that the trader's second order conditions are satisfied and that ζ, α, γ and θ are positive. The equation defining p in Proposition 1 may be dervied by considering the market clearing condition. ■

Lemma 1 *: The first derivative of the measure of asymmetric information relative to liquidity measure is less than 1, i.e.,*

$$\frac{\partial[\frac{\pi_\epsilon[(M+N-1)(2\psi-\psi_m)+\pi_v]}{[(M+N-1)\psi_m-\pi_v]}]}{\partial(\frac{\rho^2}{\pi_\omega})} < 1.$$

Proof of Lemma 1:

We have,

$$\partial[\frac{\pi_\epsilon[(M+N-1)(2\psi-\psi_m)+\pi_v]}{[(M+N-1)\psi_m-\pi_v]}] \Big/ \partial(\frac{\rho^2}{\pi_\omega}) = \begin{array}{l} 2(M+N-1)\pi_\epsilon^3[N(M+N-1)\pi_\epsilon + \\ (M+2N-2)\pi_v]\{\pi_\epsilon[N(M+N-1)\pi_\epsilon + \\ (M+N-2)\pi_v] + (M+N-2)\pi_v\frac{\rho^2}{\pi_\omega}\}^{-2} \end{array}$$

Moreover, we can easily show that

$$\begin{gathered} \pi_\epsilon^2[N(M+N-1)\pi_\epsilon + (M+N-2)\pi_v]^2 - 2(M+N-1) \\ \pi_\epsilon^3[N(M+N-1)\pi_\epsilon + (M+2N-2)\pi_v] \\ = \\ \pi_\epsilon^2[(M+N-2)^2{\pi_v}^2 + N(N-2)(M+N-1)^2\pi_\epsilon^2 + 2M(N-2)(M+N-1)\pi_\epsilon\pi_v], \end{gathered}$$

which is positive. Thus,

$$\begin{gathered} 2(M+N-1)\pi_\epsilon^3[N(M+N-1)\pi_\epsilon + (M+2N-2)\pi_v] \\ < \\ \pi_\epsilon^2[N(M+N-1)\pi_\epsilon + (M+N-2)\pi_v]^2 \\ < \\ [\pi_\epsilon[N(M+N-1)\pi_\epsilon + (M+N-2)\pi_v] + (M+N-2)\pi_v\frac{\rho^2}{\pi_\omega}]^2. \end{gathered}$$

This ends the lemma's proof. ■

Proof of Proposition 2:

Consider $\beta_* = \lim_{M\to+\infty}\beta$ where $\beta_* \in [0, b_0]$ and β_* must be different from 0 since we consider that, at the equilibrium, $\beta > 0$ for all $M > 1$ and $N > 1$.[22]

We have, for the equilibrium solution β:

$$\lim_{M\to+\infty} f(\beta) = \lim_{M\to+\infty} g(\beta)$$

where,

$$\lim_{M\to+\infty} g(\beta) = \frac{\beta_*\psi(2\pi_\epsilon - 2\pi - \rho\beta_*)}{\pi_\epsilon(\pi_\epsilon - \pi - \rho\beta_*)} \tag{47}$$

and

$$\lim_{M\to+\infty} f(\beta) = \frac{\beta_*\psi\psi_m(-(N-2)\pi_\epsilon + 2(N-1)\pi + \rho\beta_*)}{\lim_{M\to+\infty} M(\pi_\epsilon - \pi)[(\pi_\epsilon - \pi - \rho\beta_*)(\pi_v + (N-1)\pi) - \pi\psi]}.$$

From equation (47), $0 < \lim_{M\to+\infty}(\beta) < +\infty$; it follows

$$\lim_{M\to+\infty}\{M(\pi_\epsilon - \pi)[(\pi_\epsilon - \pi - \rho\beta_*)(\pi_v + (N-1)\pi) - \pi\psi]\} \neq \infty.$$

[22] note that $\beta = 0$ is out of the equilibrium path since, in this case, traders will not trade using their private signal.

Therefore $\lim_{M\to+\infty}\{(\pi_\epsilon-\pi)[(\pi_\epsilon-\pi-\rho\beta_*)(\pi_v+(N-1)\pi)-\pi\psi]\}=0$ and from this equality we conclude that: $\beta_*=b_0$. (ii), (iii), (iv) and (v) follow directly from equations (38), (37) and (24). ■

Proof of Proposition 3:

(i) The equilibrium price is equal to $\mu+\frac{1}{M\zeta}Q$, so p and Q are equivalently informative for market makers, and we have:

$$E[\tilde{v}|p,Q]=E[\tilde{v}|p].$$

Yet, from (33),

$$E[\tilde{v}|p,Q]=E[\tilde{v}|z_m]=\frac{\pi_v\mu+N\pi z_m}{\psi_m}.$$

Substitution of z_m and Q respectively from equation (32) and the market clearing condition (equation (14)) yields the following equation of the conditional expectation of v:

$$E[\tilde{v}|p]=\frac{\pi_v\beta-\pi(M\zeta+N\alpha)}{\beta\psi_m}\mu+\frac{\pi(M\zeta+N\theta)}{\beta\psi_m}p.$$

Substitution of $(M\zeta+N\alpha)$ and $(M\zeta+N\theta)$ from equation (37) as functions of β gives

$$E[\tilde{v}|p]=\frac{\alpha(\pi_\epsilon-\pi)}{\beta\psi_m}\mu+\frac{\pi\psi(2\pi_\epsilon-2\pi-\rho\beta)}{\pi_\epsilon\psi_m(\pi_\epsilon-\pi-\rho\beta)}p$$

which is equal to p only when $M\to+\infty$.

(ii) From the market clearing condition we can derive the following equation of p:

$$p=[\frac{M\zeta+N\alpha}{M\zeta+N\theta}]\mu+\frac{\beta}{M\zeta+N\theta}[\sum_{j=1}^{N}(s_j-\frac{\rho}{\pi_\epsilon}\omega_j)].$$

Then after substituting the value of $M\zeta+N\theta$ from equation (37), we have:

$$var(p)=\frac{N{\pi_\epsilon}^2(\pi_\epsilon-\pi-\rho\beta)^2}{\pi\psi^2(2\pi_\epsilon-2\pi-\rho\beta)^2} \tag{48}$$

which is equivalent to

$$var(p)=\frac{N{\pi_\epsilon}^2}{\pi\psi^2[1+\frac{\pi_\epsilon-\pi}{\pi_\epsilon-\pi-\rho\beta}]^2}.$$

Since β is in $(0, b_0)$ then $var(p)$ satisfies the following property:

$$\frac{N\pi}{{\psi_m}^2} < var(p) < \frac{N{\pi_\epsilon}^2}{4\pi\psi^2}. \tag{49}$$

Thus, we can conclude that $\lim_{N\to+\infty} var(p) = 0$.

(iii) The quoted bid-ask spread is equal to $p(1) - p(-1) = \frac{2}{M\zeta}$. From equation (41), if we compute the limit of $M\zeta$ as $M \to +\infty$ by considering the fact that $\beta \to b_0$ we find the announced result. ■

Proof of Proposition 4:

This proof is similar to the proof of Proposition 1 in Madhavan (1992). For the sake of completeness and in order to use some results derived in the proof we present it.

The equilibrium in dealership markets is defined by the couple $(p(q), q)$. Considering that dealers set a differentiable price function $p(\cdot)$, then a trader i, with information (s_i, ω_i), chooses a trading strategy q_i satisfying his optimality condition

$$q_i \in argmaxE[U(\tilde{v}(\omega_i + q_i) - p(q_i)q_i)|p(\cdot), H_i] \tag{50}$$

which is equivalent to

$$q_i \in argmax\{E[\tilde{v}|s_i](\omega_i + q_i) - p(q_i)q_i - \frac{\rho}{2}(\omega_i + q_i)^2 var[\tilde{v}|s_i]\}.$$

The first and second order conditions are respectively

$$-p'(q_i)q_i - p(q_i) + E[\tilde{v}|s_i] - \rho(\omega_i + q_i)var[\tilde{v}|s_i] = 0 \tag{51}$$

$$-p''(q_i)q_i - 2p'(q_i) - \frac{\rho}{\pi_v + \pi_\epsilon} < 0 \tag{52}$$

Since, $E[\tilde{v}|s_i] = \frac{\pi_v\mu + \pi_\epsilon s_i}{\pi_v + \pi_\epsilon}$, substitution in (51) and rearrangement of terms give

$$\frac{\pi_v + \pi_\epsilon}{\pi_\epsilon}[p'(q_i)q_i + p(q_i)] - \frac{\pi_v}{\pi_\epsilon}\mu + \frac{\rho q_i}{\pi_\epsilon} = s_i - \frac{\rho}{\pi_\epsilon}\omega \tag{53}$$

Then, given his pricing function $p(\cdot)$, the dealer should observe a noisy valuation of the final asset's value $s_i - \frac{\rho}{\pi_\epsilon}\omega_i$. Consider $\tilde{z} = \tilde{s}_i - \frac{\rho}{\pi_\epsilon}\tilde{\omega}_i$; then $\tilde{z} = \widetilde{v} + \widetilde{y}_0$ with

$$\widetilde{y}_0 \sim \mathcal{N}(0, \pi),$$

where π, as in the proof of Proposition 1, is equal to $\pi = \frac{\pi_\epsilon^2 \pi_\omega}{\pi_\epsilon \pi_\omega + \rho^2}$.

Thus, from the dealer's point of view, conditional expectation is:

$$E[\tilde{v}|z] = \frac{\pi_v \mu + \pi z}{\pi_v + \pi} \tag{54}$$

and

$$var[\tilde{v}|z] = \frac{1}{\pi_v + \pi}.$$

Substitution of equation (53) and the value of π into (54) and introduction of the fact that $E[\tilde{v}|z] = E[\tilde{v}|q_i] = p(q_i)$ gives the following equation

$$p'(q_i) q_i \pi_\epsilon \pi_\omega (\pi_v + \pi_\epsilon) - p(q_i) \pi_v \rho^2 + \pi_v \rho^2 \mu + \rho \pi_\epsilon \pi_\omega q_i = 0, \tag{55}$$

or, equivalently:

$$p'(q_i) q_i = \frac{a}{b}(p(q_i) - \mu) - \frac{\rho \pi_\epsilon \pi_\omega}{b} q_i \tag{56}$$

where $a = \pi_v \rho^2$ and $b = \pi_\epsilon \pi_\omega (\pi_v + \pi_\epsilon)$.

- *First case:* $a = b$

In this case, equation (56) is:

$$p'(q_i) q_i = p(q_i) - \mu - \frac{\rho \pi_\epsilon \pi_\omega}{b} q_i.$$

This is a first-order differential equation in p with a second member. Its solution is

$$p(q_i) = \mu + C q_i - \frac{\rho}{\pi_v + \pi_\epsilon} q_i \ln |q_i|, \tag{57}$$

where C is the integration constant. If $C \leq 0$ then we have $p(-q_i) - p(q_i) = -2cq_i + 2\frac{\rho}{\pi_v + \pi_\epsilon} q_i \ln |q_i|$. Thus, for all $q_i > 1$ we have $p(-q_i) - p(q_i) > 0$. This represents an arbitrage opportunity which will be eliminated by inter-dealer trading. Then, this cannot be an equilibrium. If $C > 0$, substituting the value of $p(q_i)$ into the trader's second order condition gives the following:

$$-2C + 2\frac{\rho}{\pi_v + \pi_\epsilon}(\ln|q_i| + 1) < 0.$$

Then, the order size has to be lower than $\exp(\frac{C(\pi_v+\pi_\epsilon)}{\rho} - 1)$ to satisfy the dealer's second order condition, otherwise this condition will be violated. Thus, this cannot be an equilibrium.

- *Second case:* $a \neq b$

If we denote by

$$T(q_i) = p(q_i) - \mu + \frac{\rho\pi_\epsilon\pi_\omega}{b-a}q_i; \tag{58}$$

and write equation (56) as a differential equation in T, we get

$$T'(q_i)q_i = \frac{a}{b}T(q_i). \tag{59}$$

The solution to this differential equation is

$$T(q_i) = C_1 sign(q_i)|q_i|^{a/b}.$$

Then, from (58) we have:

$$p(q_i) = \mu - \frac{\rho\pi_\epsilon\pi_\omega}{b-a}q_i + C_1 sign(q_i)|q_i|^{a/b} \tag{60}$$

Substitution of this value of $p(q_i)$ into equation (52) gives the following

$$C_1\frac{a}{b}[\frac{a-b}{b} - 2\pi_\epsilon\pi_\omega]|q_i|^{(a-b)/b} > \rho\frac{a+b}{(\pi_v+\pi_\epsilon)(b-a)} \tag{61}$$

If $a < b$ and $C_1 \leq 0$ then $p(-q_i) > p(q_i)$ for all $q_i > 0$. Indeed

$$p(-q_i) - p(q_i) = \frac{2\rho\pi_\epsilon\pi_\omega}{b-a}q_i - 2C_1(q_i)^{a/(\pi_\epsilon+\pi_v)} > 0.$$

In this case we have an arbitrage opportunity and so this cannot be an equilibrium.

If $a < b$ and $C_1 > 0$ then from the trader's second order condition we can prove the existence of q_* and q^* such that this condition is violated for $q \notin [q_*, q^*]$. Then, this cannot be an equilibrium. Similarly, if $a > b$ and $C_1 < 0$ the trader's second order condition is violated for some values of q.

Finally, if $a > b$, and $C_1 \geq 0$, the second order condition is always satisfied, but these pricing functions are dominated by the linear pricing function where $C_1 = 0$. In fact, for all q_i we have:

$$\mu + \frac{\rho\pi_\epsilon\pi_\omega}{a-b}q_i < \mu + \frac{\rho\pi_\epsilon\pi_\omega}{a-b}q_i + C_1|q_i|^{\frac{a}{b}},$$

and then the equilibrium price function is :

$$p(q_i) = \mu + \frac{\rho\pi_\epsilon\pi_\omega}{a-b}q_i. \tag{62}$$

Given this pricing function we can easily derive the trader's strategy by substituting the value of $p(q_i)$ into equation (53) which gives:

$$q_i(s_i, \omega_i)) = \frac{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}{\rho[\pi_v\rho^2 + \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)]}[-\pi_\epsilon\mu + \pi_\epsilon s_i - \rho\omega_i].$$

■

Proof of Proposition 5:

(i) $E[\tilde{v}|p] = E[\tilde{v}|E(\tilde{v}/q)] = E[\tilde{v}|q] = p$, so prices in dealership market are semi-strong form efficient.

(ii) From the market clearing condition we have:

$$p(q) = \mu + [\frac{\pi_\epsilon^2\pi_\omega}{\pi_v\rho^2 + \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}]\{-\mu + s_i - \frac{\rho}{\pi_\epsilon}\omega_i\};$$

rearranging terms gives

$$p(q) = [\frac{\pi_v(\rho^2 + \pi_\epsilon\pi_\omega)}{\pi_v\rho^2 + \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}]\mu + [\frac{\pi_\epsilon^2\pi_\omega}{\pi_v\rho^2 + \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}]\{s_i - \frac{\rho}{\pi_\epsilon}\omega_i\}. \tag{63}$$

Then, the price variance is :

$$var(p) = [\frac{\pi_\epsilon^2\pi_\omega}{\pi_v\rho^2 + \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}]^2\frac{1}{\pi}. \tag{64}$$

After simplification we can write:

$$var(p) = \frac{\pi}{(\pi_v + \pi)^2}. \tag{65}$$

(iii) The quoted bid-ask spread is defined by $p(1) - p(-1)$. In dealership markets, it is equal to

$$\frac{2\rho\pi_\epsilon\pi_\omega}{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}. \tag{66}$$

Given this relation we can easily prove that partial derivatives of the quoted bid-ask spread equation relative to π_ϵ, π_ω, ρ and π_v are positive for the first and second argument and negative for the latter. ■

Proof of Proposition 6:

(i) Auction markets are more viable if their equilibrium exists each time the equilibrium in dealership markets exists. In other words, this is the case when:

$$\frac{\pi_\epsilon(\pi_\epsilon + \pi_v)}{\pi_v} < \frac{\rho^2}{\pi_\omega} \Longrightarrow \frac{[(M+N-1)\pi_\epsilon[2\psi - \psi_m] + \pi_v]}{(M+N-1)\psi_m - \pi_v} < \frac{\rho^2}{\pi_\omega}.$$

From the first condition we have

$${\pi_\epsilon}^2\pi_\omega(\pi_\epsilon + \pi_v) < \pi_v\pi_\epsilon\rho^2.$$

Therefore, we get

$$\pi_\epsilon\pi_v > \pi(2\pi_v + \pi_\epsilon), \tag{67}$$

dividing both terme by π_ϵ gives

$$(\pi_\epsilon - 2\pi) > \pi_\epsilon/\pi_v > 0. \tag{68}$$

Now, we should prove that under these conditions, the auction market's equilibrium existence condition is satisfied. We have:

$$(M+N-1)(\pi_\epsilon\psi_m - 2\pi\psi) - \pi_\epsilon\pi_v =$$
$$(M+N-2)\pi_\epsilon\pi_v + (M+N-1)[(N-2)\pi_\epsilon - \pi_v + 2(N-1)\pi].$$

Using equation (67), we get:

$$(M+N-1)(\pi_\epsilon\psi_m - 2\pi\psi) - \pi_\epsilon\pi_v >$$
$$\pi[(M+N-1)(N-1)(\pi_\epsilon - 2\pi) + (M+N-3)\pi_v].$$

Since the right hand member of this equation is positive from equation (68), we have:

$$(M+N-1)(\pi_\epsilon\psi_m - 2\pi\psi) - \pi_\epsilon\pi_v > 0.$$

(ii) Suppose that:

$$\pi_\epsilon < (1 - \frac{2}{M+N})\frac{\rho^2}{\pi_\omega}, \tag{69}$$

and

$$\frac{M{\pi_\epsilon}^2\pi_\omega}{(M+N-2)\rho^2 - (M+N)\pi_\epsilon\pi_\omega} - (N-1)\frac{{\pi_\epsilon}^2\pi_\omega}{\rho^2 + \pi_\epsilon\pi_\omega} < \pi_v < \frac{{\pi_\epsilon}^2\pi_\omega}{\rho^2 - \pi_\epsilon\pi_\omega}.$$

The second equation can be written as:

$$\frac{M\pi_\epsilon\pi}{(M+N-2)\pi_\epsilon - 2(M+N-1)\pi} - (N-1)\pi < \pi_v < \frac{\pi_\epsilon\pi}{\pi_\epsilon - 2\pi}. \tag{70}$$

By equation (69), we have $(M+N-2)\pi_\epsilon - 2(M+N-1)\pi > 0$, so $\pi_\epsilon - 2\pi > 0$.

Finally from these inequalities and equation (70), we have the following:

- $\pi_v[\pi_\epsilon - 2\pi] - \pi_\epsilon\pi < 0$, then dealership market breaks down.
- $(M+N-1)[\pi_\epsilon\psi_m - 2\pi\psi] - \pi_\epsilon\pi_v > 0$, then the equilibrium in auction markets exists. ■

Proof of Proposition 7:

First, we can easily verify that $0 < \beta_D < b_0$. Then, in order to compare trading aggressiveness in both markets, it is sufficient to compute $h(\beta_D)$ (see proof of Proposition 1). In fact, since $h(\cdot)$ is an increasing function on $]0, b_0[$ and $h(\beta) = 0$, if $h(\beta_D) > 0$ then $\beta_D > \beta$ and conversely if $h(\beta_D) < 0$.

The function $h(\cdot)$ may also be written as follows:

$$\begin{aligned} h(\beta) = \; & \pi_\epsilon\psi_m[\pi_\epsilon - \pi - \rho\beta][(2-N)\pi_\epsilon + 2(N-1)\pi + (N-1)\rho\beta] \\ & -(\pi_\epsilon - \pi)[(\pi_v + (N-1)\pi)(\pi_\epsilon - \pi - \rho\beta) - \pi\psi] \\ & [(2M+N-2)\pi_\epsilon - 2(M+N-1)\pi - (M+N-1)\rho\beta]; \end{aligned}$$

then,

$$\begin{aligned} h(\beta_D) = \; & \pi\{\pi_\epsilon\psi_m[2\pi_\epsilon + \pi_v - \pi][\pi_\epsilon(\pi_v + \pi) - 2(N-1)\pi(\pi_\epsilon - \pi)] \\ & -(\pi_\epsilon - \pi)[2(\pi_\epsilon - \pi)(\pi_v + (N-1)\pi) - \pi_\epsilon(\pi_v + \pi)] \\ & [(M-1)\pi_\epsilon(\pi_v + \pi) + 2(M+N-1)\pi(\pi_\epsilon - \pi)]\}. \end{aligned}$$

If we consider the fact that equilibrium conditions in both markets are satisfied, and that $\beta_D \in (0, b_0)$, we can argue that $h(\beta_D) < 0$ whenever[23]

$$\pi_\epsilon(\pi_v + \pi) - 2(N-1)\pi(\pi_\epsilon - \pi) < 0$$

which is equivalent to:

$$(N-1) > \frac{\pi_\epsilon(\pi_v + \pi)}{2\pi(\pi_\epsilon - \pi)}.$$

In this case $\beta_D < \beta$ and trading is more aggressive in auction markets. ■

Proof of Proposition 8:

From equation (49) we have

$$\frac{N\pi}{{\psi_m}^2} < var(p^A) < \frac{N{\pi_\epsilon}^2}{4\pi\psi^2}$$

and from equation (65), $var(p^D) = \frac{\pi}{(\pi_v + \pi)^2}$.

(i) Consider that $N < (\frac{\pi_v}{\pi})^2$. Then, $(N-1)[N\pi^2 - {\pi_v}^2] < 0$. This is equivalent to $(\pi_v + N\pi)^2 - N(\pi_v + \pi)^2 < 0$. Hence, $var(p^D) < \frac{N\pi}{(\pi_v + N\pi)^2} < var(p^A)$.

(ii) If $(N-1) > \frac{{\pi_\epsilon}^2(\pi_v+\pi)^2 - 4\pi^3(\pi_v+\pi_\epsilon)}{4\pi^4}$, then we have:

$$4\pi^3\psi > {\pi_\epsilon}^2(\pi_v + \pi)^2.$$

Multiplying both sides by N and considering that $N\pi < \psi$, we can write $4\pi^2\psi^2 - N{\pi_\epsilon}^2(\pi_v+\pi)^2 > 0$ which is equivalent to:

$$\frac{\pi}{(\pi_v + \pi)^2} > \frac{N{\pi_\epsilon}^2}{4\pi\psi^2}.$$

[23] Notice that this is just a sufficient condition for our result.

then $var(p^A) < \frac{N\pi_\epsilon^2}{4\pi\psi^2} < var(p^D)$. ■

Proof of Proposition 9:

The measure of efficiency for both markets is:

$$e_A = \frac{N\pi}{\pi_v + N\pi}$$

and

$$e_D = \frac{\pi}{\pi_v + \pi};$$

where the first equation is derived from the definition of e and the fact that[24] $E[\tilde{v}|p] = E[\tilde{v}|z_m] = \psi_m = \pi_v + N\pi$; and, the second equation is derived from the definition of e and equation (55).

A straightforward comparison between e_A and e_D gives the result. ■

[24] See the proof of proposition 3.

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