Information provision in financial markets

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Abstract

We set up a rational expectations model in which investors trade a risky asset based on a private signal they receive about the quality of the asset, and a public signal that represents a noisy aggregation of the private signals of all investors. Our model allows us to examine what happens to market performance (market depth and price efficiency) when regulators can induce improved information provision in one of two ways. Regulations can be designed that either provide investors with more accurate prior information by improving the precision of all private signals, or that enhance the transparency of the market by improving the quality of the public signal. In our rational expectations equilibrium, improving the quality of the public signal can be interpreted as a way of providing information about the anticipations and trading motives of all market participants. We find that both alternatives improve market depth. However, in the limit, we show that improving the precision of prior information is a more efficient way to do so. More accurate prior information decreases asymmetric information problems and consequently reduces the informativeness of prices, while a more accurate public signal increases price informativeness.

1 Introduction

In recent years, regulations have been implemented in financial markets throughout the world designed to induce improved information provision in order to reduce existing uncertainty about the fundamentals of publicly traded firms. There are two alternative means available to regulators trying to stimulate better information provision. The first involves tightening the standards of information dissemination of publicly traded firms. This is achieved by improving the precision of the prior information available to all investors about asset value. The Sarbanes-Oxley Act of 2002 is an example of a regulation designed to provide investors with more accurate accounting information.

The second way to improve information provision is to enhance the transparency of the market. The main argument for this alternative is that uncertainty is more about the actions of competing investors than about fundamentals, and so enhancing transparency can be achieved by providing all market participants access to information about the trading motives of their competitors. This can involve, for instance, giving access to more information about the order book or about all the transactions that have taken place in the market.

The objective of this paper is to determine the ways in which these two means of improving information provision affect market performance. We set up a rational expectations model in which investors trade a risky asset based on a private signal they receive about the quality of the asset, and a public signal that represents a noisy aggregation of the private signals of all investors. Our model allows us to examine what happens to market performance when regulators can decrease uncertainty about fundamentals by improving information provision in one of the two ways described above. In our setup, regulators can provide investors with more accurate prior information by improving the precision of all private signals, and/or can enhance the transparency of the market by improving the quality of the public signal. In our rational expectations equilibrium, improving the quality of the public signal can be interpreted as a way of providing information about the anticipations and trading motives of all market participants.

We consider the effect of these alternatives on two measures of market performance. We focus on market depth and price efficiency. Improving the precision of prior information or increasing the level of transparency in the market always increases market depth. However, the effect of increasing transparency is less important than the effect of increasing the precision of prior information. In the limit, while very precise prior information results in an infinitely deep market, increasing market transparency results in a level of market depth that is bounded from above. The reason is that the informational content of each of the alternatives is different. Improving information provision by increasing the precision of prior information directly affects the trading strategies of agents, while enhancing market transparency does so only indirectly by affecting the precision of their information about their competitors signals and the information they infer from the equilibrium price.

Turning to the effect of improved information provision on price efficiency we find that price efficiency is decreasing in improvements in the accuracy of prior information. More accurate private signals reduce price efficiency since the availability of higher quality information implies that the information transmitted by prices is less important. The opposite effect occurs when the market is made more transparent. The provision of the public signal compensates for the reduction of information asymmetries.

We are not the first to study these issues. In the market microstructure literature, researchers have examined the impact of increasing financial market transparency on market performance. Theoretical (Pagano and Röell, 1996, Madhavan, 1995 and 1996, and Baruch 2005), empirical (Gemmill, 1994, Porter and Weaver, 1998, and Madhavan et al., 2005) and experimental (Bloomfield and O'Hara, 1999 and Flood et al., 1999) studies reached mixed conclusions about the impact of transparency on financial markets. In all of these studies transparency is defined as the information that the market or some participants should disclose in order to increase the information shared between agents. This transparency can be either pre-trade (for instance information about the book) or post-trade (for instance information on past trades). All of this information will affect the trading behavior of agents because it allows information about the trading strategies of competitors to be shared. We are not interested in modeling transparency, but simply in determining its ultimate impact on market performance. Therefore, in our model, we capture the impact of transparency by simply allowing for the existence of a public signal that is a noisy aggregation of the signals held by all the players in the market and which, given our rational expectations equilibrium, provides information about the anticipations of all market participants. In contrast with the market microstructure literature on market transparency, since the precision of the public signal can be varied continuously, our model allow us to study the marginal impact of decreasing uncertainty by increasing transparency.

There have also been a number of papers that study the impact of reducing uncertainty directly by announcing a public signal that is independent of the private signals held by investors. Morris and Shin (2002) show that public information may be harmful for expected social welfare in a beauty-contest set up where the payoff of an agent decreases in the distance between his action and the actions of the others. The main intuition for their result is that in the beauty-contest set up, public information helps investors achieve coordination, but greater coordination between agents is assumed to be socially irrelevant.[1]

There are two important differences between our model and Morris and Shin (2002). First, there is no direct coordination effect in our model since we do not consider a beauty-contest set up. As a result, the positive effect of decreasing uncertainty would be related to a decrease in the adverse selection problem which would affect their trading behavior. Second, we consider a rational expectations equilibrium set up where investors are imperfectly competitive. So in some sense, we introduce the strategic behavior of investors and explore the way it is affected by the existence of a more valuable information.

The rest of the paper proceeds as follows. In the next section we describe the model. Section 3 characterizes the equilibrium. In Section 4 we analyse the equilibrium effects of improving information provision on market performance. We conclude in Section 5. All proofs are in the Appendix.

2 The model

Consider a market where Q units of a risky asset are traded. There are n investors who participate in this market. The value of the risky asset is denoted by θ and is unobserved by the participants. However, each investor has some information about θ. Based on the information available to them, each submits a demand function and in equilibrium total demand will equal the quantity supplied, Q.

[1] Svensson (2005) shows that this result holds only for special cases and that when the parameters of the Morris and Shin (2002) model take reasonable values the condition for public information to be welfare decreasing is violated. Other papers have challenged Morris and Shin's result by showing that the increase in public information announcements would increase the expected social welfare if sent only to a proportion of agents (Cornand and Heinemann, 2006), in economies with investment complementarity (Angeletos and Pavan, 2004), or in economies featuring monopolistic competition among heterogeneously informed firms.

As in Kyle (1989), we consider a rational expectations equilibrium with imperfect competition among agents in which investors submit downward sloping demand curves. The main difference between our model and Kyle's is that, in our model, investors not only observe a private signal about the value of θ, but also observe a common public signal, which is itself an imperfect signal of the sum of individual private signals.

2.1 Information structure

We assume that prior to trading each investor i receives two imperfect signals about the value of the risky asset, θ. The first signal is private for investor i, and is denoted by s_i. The second signal, denoted by S, is a public signal observed by all investors. Both signals are noisy and all random variables are assumed to be normally distributed. So, we suppose that

$$s_i = \theta + t_i \quad \text{where} \quad \theta \sim N(\bar{\theta}, \sigma_\theta^2) \quad \text{and} \quad t_i \sim N(0, \sigma_t^2), \tag{1}$$

and

$$S = \sum_j s_j + \mu = n\theta + \sum_j t_j + \mu \quad \text{where } \mu \sim N(0, \sigma_\mu^2). \tag{2}$$

From these expressions we can see the two means available to regulators for reducing uncertainty about the value of a financial asset.[2] One option is for standards on information dissemination of publicly traded assets to be tightened. In our setup this is captured by reducing σ_θ^2. The second alternative is to enhance the transparency of financial markets by giving access to more information about the order book or about all the transactions in the market. In our model this is captured by a decrease in σ_μ^2. Since S is a noisy signal of all private signals received by investors, if $\sigma_\mu^2 \to 0$, it is as if participants can observe the signals received by all other investors. This would correspond to the case where investors have access to all of the existing valuable information. Alternatively, if $\sigma_\mu^2 \to \infty$, we have $E(\theta|S, s_i) = E(\theta|s_i)$, so that investors can infer no additional information about the final value of the asset from observing S. By changing the value of σ_μ^2 we will explore the impact of information aggregation and market transparency on the market equilibrium and the bidding strategies of agents.

[2] Note that we do not focus on a trade-off between these two types of signals, but rather on their respective impacts on market performance. See Morris and Shin (2002) and Clark and Polborn (2006) for models in which agents weigh the relative benefits of the two types of information.

2.2 Agents

We suppose that agents have CARA preferences with r denoting their risk aversion coefficient. Given the assumption of normally distributed random variables, this implies that preferences can be denoted by the mean-variance representation. An agent who receives a quantity $q_i = x$ has expected utility of:

$$E_\theta \left[W + \theta(x + \epsilon_i) - px|\Omega\right] - \frac{r}{2} Var\left[\theta|\Omega\right](x + \epsilon_i)^2, \tag{3}$$

where W is some initial wealth and ϵ_i corresponds to an idiosyncratic liquidity shock that agent i receives before participating in the market and Ω denotes all information available to the investor. We assume that ϵ_i is also private information for i and follows distribution $N(0, \sigma_\epsilon^2)$. The introduction of ϵ_i adds a second motivation for trading as in Glosten (1989). Without these shocks, the rational expectations equilibrium would be fully revealing and so traders would have no incentive to gather information. The result would be a no-trade equilibrium as in Grossman and Stiglitz (1980).[3] With the inclusion of ϵ_i, higher demand by a particular agent may be interpreted as being the result either of good information or a large negative liquidity shock.

The precision of the liquidity shock will affect the existence of the linear equilibrium. We prove in what follows that unless this shock is sufficiently noisy, agents may refuse to take part in the market.

3 Equilibria with linear downward sloping demand

As in Kyle (1989) we focus on linear equilibria by assuming that investors submit linear demand functions. Under simplifying assumptions of normal and independent idiosyncratic shocks and a CARA utility function, we are able to compute a symmetric rational expectations equilibrium with downward sloping demand curves. The equilibrium is derived by maximizing each agent's expected utility against the residual demand curve. Indeed, in the spirit of rational expectations equilibria, after making conjectures about the optimal demand functions of his competitors, each agent will choose his optimal strategy by acting as a monopsonist with respect to a residual demand curve conditional on these conjectures.

Let us suppose that agent i conjectures that each agent $j \neq i$ has the following inverse demand function :

[3] The alternative is to introduce noise traders into the model.

$$P(x_i, S, \epsilon_i, s_i) = \beta_0 + \beta_1 s_i - \beta_2 \epsilon_i + \beta_3 S - \delta x_i \tag{4}$$

Since, the equilibrium price is the same for all agents, summing the demand functions for all $j \neq i$ and adding the market clearing condition yields the inverse residual demand function for agent i:

$$P(S, \boldsymbol{\epsilon}_{-j}, \boldsymbol{s}_{-j}, x_i) = \left[\beta_0 + \beta_1 \frac{\sum_{j\neq i} s_j}{(n-1)} - \beta_2 \frac{\sum_{j\neq i} \epsilon_j}{(n-1)} + \beta_3 S - \frac{\delta\,(Q - x_i)}{(n-1)}\right], \tag{5}$$

which we can rewrite as

$$P(S, y_i, x_i) = \left[\beta_0 + \beta_1 y_i + \beta_3 S - \frac{\delta\,(Q - x_i)}{(n-1)}\right], \tag{6}$$

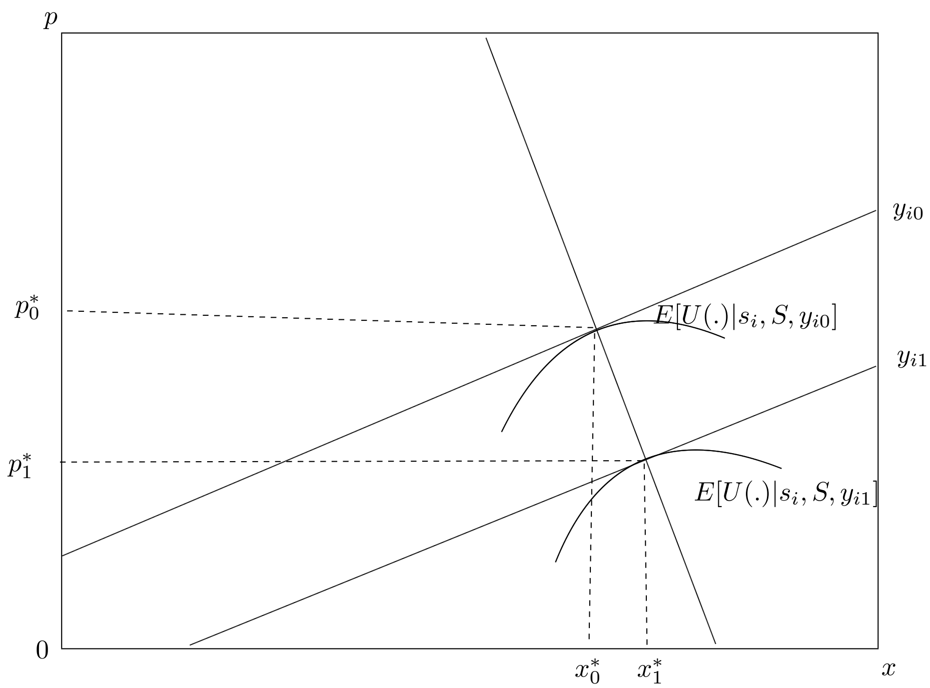
where $y_i \equiv \left[\frac{\sum_{j\neq i} s_j}{(n-1)} - \frac{\beta_2}{\beta_1}\frac{\sum_{j\neq i} \epsilon_j}{(n-1)}\right]$. y_i represents the indirect information that agent i can infer about the other investors' strategies given his demand function $x_i(\cdot)$ and the equilibrium price p. It is normally distributed and is correlated with the public signal S. Note that when the variance of ϵ_j, σ_ϵ^2, goes to zero then $y_i = \left[\frac{b}{(n-1)} \sum_{j\neq i} s_j\right]$ and the market equilibrium is fully informative.

Under the rational expectation equilibrium, we will assume that investor i selects his optimal demand function as if he were observing the y_i from the residual demand function. This is illustrated in Figure 1. Suppose that agent i faces the residual demand curve associated with some y_{i0}. He can select the pair (p_0^*, x_0^*) lying along this curve which maximizes his expected utility given s_i, S, and y_{i0}. Now suppose instead that $y_i = y_{i1}$. The agent can select a price and quantity pair (p_1^*, x_1^*) that lies along the new residual demand curve associated with $y_i = y_{i1}$ and that maximizes his expected utility given s_i, S, and y_{i1}. The optimal demand curve for agent i can, therefore, be constructed by connecting the optimal price-quantity pairs for each value of y_i. The optimal demand curve provides an optimal response for all values of s_i, S, and y_i even if y_i is not directly observed by the investor i.

Formally, for some S, y_i, and s_i, the investor problem consists of selecting a price and quantity pair (p, x) that solves the following problem

$$\begin{aligned} &\max_{(p,x)} W + E_\theta\left[\theta | S, y_i, s_i\right](x + \epsilon_i) - px - \frac{r}{2} Var\left[\theta | S, y_i, s_i\right](x + \epsilon_i)^2 \\ \text{subject to } p \quad = \quad &\left[\beta_0 + \beta_1 y_i + \beta_3 S - \frac{\delta\,(Q - x)}{(n-1)}\right]. \end{aligned}$$

FIGURE 1: CONSTRUCTION OF BID FUNCTIONS



Formally, for some S, y_i, and s_i, the investor problem consists of selecting a price and quantity pair (p, x) that solves the following problem

$$\begin{aligned} & \max_{(p,x)} W + E_\theta\left[\theta|S, y_i, s_i\right](x+\epsilon_i) - px - \frac{r}{2}Var\left[\theta|S, y_i, s_i\right](x+\epsilon_i)^2 \\ \text{subject to } p \quad = \quad & \left[\beta_0 + \beta_1 y_i + \beta_3 S - \frac{\delta\,(Q-x)}{(n-1)}\right]. \end{aligned}$$

The first-order condition is

$$E_\theta\left[\theta|S, y_i, s_i\right] - \left[\beta_0 + \beta_1 y_i + \beta_3 S - \delta\frac{(Q-x)}{(n-1)} + \delta\frac{x}{(n-1)}\right] - rVar\left[\theta|S, y_i, s_i\right](x+\epsilon_i) = 0, \qquad (7)$$

and the second-order condition is

$$\frac{2\delta}{(n-1)} + rVar(\theta|S, y_i, s_i) > 0. \qquad (8)$$

In order to further characterize the structure of the demand function, we state the following lemma:

Lemma 1 *Given the definitions of y_i, S and s_i, and the assumption that all random variables are distributed normally, we have the following*

$$E[\theta|S, y_i, s_i] = \left(\frac{k_0}{k}\right)\overline{\theta} + \left(\frac{k_1}{k}\right)y_i + \left(\frac{k_2}{k}\right)\frac{(S-s_i)}{(n-1)} + \left(\frac{k_3}{k}\right)s_i \tag{9}$$

$$\text{and } Var(\theta|S, y_i, s_i) = \sigma_t^2 k_3/k, \tag{10}$$

with

$$k_0 = \sigma_t^2 \tag{11}$$

$$k_1 = \frac{(n-1)\,\sigma_\theta^2\sigma_t^2\sigma_\mu^2}{\left[(X)^2\sigma_\mu^2 + \sigma_t^2\sigma_\mu^2 + (n-1)(X)^2\sigma_t^2\right]} \tag{12}$$

$$k_2 = \frac{(n-1)^2(X)^2\sigma_\theta^2\sigma_t^2}{\left[(X)^2\sigma_\mu^2 + \sigma_t^2\sigma_\mu^2 + (n-1)(X)^2\sigma_t^2\right]} \tag{13}$$

$$k_3 = \sigma_\theta^2 \tag{14}$$

$$\text{and } k = [k_0 + k_1 + k_2 + k_3] \tag{15}$$

where $X = \frac{\beta_2}{\beta_1}\sigma_\epsilon$.

Solving the first-order condition yields the optimal demand function which is of the form conjectured in Eq. (4) and which we characterize in the following proposition.

Proposition 1 *For strictly positive* $r, \sigma_\varepsilon^2, \sigma_\theta^2, \sigma_\mu^2,$ *and* σ_t^2 *, and* $n > 2$*, if a (linear) rational expectations equilibrium exists, then each agent must submit a strictly decreasing inverse demand curve of the form:*

$$P(x_i, S, \epsilon_i, s_i) = \beta_0 + \beta_1 s_i - \beta_2\epsilon_i + \beta_3 S - \delta x_i \tag{16}$$

where:

$$\beta_1 = \frac{k_3}{k} - \frac{k_2}{k(n-1)} + \frac{k_1}{k} \tag{17}$$

$$\delta = \frac{(n-1)\left(k_3 - \frac{k_2}{(n-1)} + k_1\right)r\sigma_t^2 k_3/k}{\left((n-2)\,k_3 - \frac{(n-2)k_2}{(n-1)} - 2k_1\right)} \tag{18}$$

$$\beta_0 = \left(\frac{k_0}{k}\right)\overline{\theta} + \frac{Qr\sigma_t^2}{(n-2)}\frac{k_3}{k}\frac{k_1}{\left((n-2)\,k_3 - \frac{(n-2)k_2}{(n-1)} - 2k_1\right)} \tag{19}$$

$$\beta_3 = \frac{k_2}{k(n-1)} \tag{20}$$

$$\beta_2 = r\sigma_t^2\frac{k_3}{k}\left[\frac{\left(k_3 - \frac{k_2}{(n-1)} + k_1\right)}{\left(k_3 - \frac{k_2}{(n-1)}\right)}\right] \tag{21}$$

and k, k_0, k_1, k_2 *and* k_3 *are defined above.*

In order to show that the equilibrium inverse demand function is well-defined, we must write the unknown values of the equilibrium inverse demand function (β_0, β_1, β_2, β_3, and δ) in terms of the parameters of the model (n, $r, \sigma_\varepsilon^2, \sigma_\theta^2$, σ_μ^2,and σ_t^2) and confirm that the conjectures on prices are satisfied. Since β_0, β_1, β_2, β_3, and δ depend on k, k_0, k_1, k_2, and k_3 which in turn depend on β_1 and β_2 through the value of $X = \frac{\beta_2}{\beta_1}\sigma_\epsilon$, the problem is somewhat complicated. Using the Eq. (17) and Eq. (21), we can write $X = \frac{\beta_2}{\beta_1}\sigma_\epsilon$ in terms of the parameters of the model (n, $r, \sigma_\varepsilon^2, \sigma_\theta^2$, σ_μ^2,and σ_t^2). We have:

$$X = \frac{\beta_2}{\beta_1}\sigma_\epsilon = r\sigma_t^2\sigma_\varepsilon \left[\frac{[(X)^2 \sigma_\mu^2 + \sigma_t^2\sigma_\mu^2 + (n-1)(X)^2 \sigma_t^2]}{[(X)^2 \sigma_\mu^2 + \sigma_t^2\sigma_\mu^2]} \right], \tag{22}$$

or alternatively,

$$\left[X - r\sigma_t^2\sigma_\varepsilon\right] = r\sigma_\varepsilon(n-1)\frac{X^2}{[X^2+\sigma_t^2]}\frac{\sigma_t^4}{\sigma_\mu^2}. \tag{23}$$

So if we can show that there exists a (unique) X that solves the above equation and that satisfies the second-order condition (Eq. (8)), then we can characterize the (unique) equilibrium with linear demand. The following lemma describes when X exists and is unique.

Lemma 2 *When $r^2\sigma_\varepsilon^2 \geq \left(\frac{n}{(n-2)\sigma_t^2}\right)$, the equilibrium defined in Proposition 1 is unique and is fully characterized by $X = \frac{\beta_2}{\beta_1}\sigma_\epsilon$. There is a unique positive X that solves Eq. (22) and that satisfies the second-order condition given by Eq. (8). When $r^2\sigma_\varepsilon^2 < \left(\frac{n}{(n-2)\sigma_t^2}\right)$, there are values of σ_μ^2 for which no equilibrium exists.*

The equilibrium existence condition is sufficient but not necessary. When this condition is violated, we can only show that an equilibrium does not exist for certain values of σ_μ^2. Note that the condition does not depend on σ_μ^2 or on σ_θ^2 which are the variables defining the alternatives available to regulators for reducing uncertainty in our model. This fact allows us to perform comparative statics without being constrained by bounded values for σ_μ^2 and σ_θ^2.

The equilibrium existence condition stems from the second-order condition of the optimality of the trading strategies of agents. Roughly speaking, it imposes that the lower bound on X given by Eq. (23) (which is equal to $r\sigma_t^2\sigma_\varepsilon$) is higher than the lower bound given by the second-order condition (which is equal to $\left(\frac{n}{(n-2)}\right)^{1/2}\sigma_t$ derived from Eq. (8)). Therefore changing the values of different parameters does not violate the second-order condition.

Intuitively, the equilibrium existence condition states that a linear equilibrium exists when (i) there is enough noise caused by liquidity shocks, which would make private signals more valuable and increase the likelihood of trading; (ii) agents are sufficiently risk averse and therefore willing to trade in the market in order to share risk; and/or (iii) the variance of private signals is sufficiently high that adverse selection problems are minimized.

In the following, we summarize the results about existence of equilibrium that will be helpful when we turn to equilibrium analysis in the following section.

Proposition 2 *A rational expectations equilibrium with linear demand functions exists for all values of σ_μ^2 and σ_θ^2, if and only if $r^2\sigma_\varepsilon^2 \geq \left(\frac{n}{(n-2)\sigma_t^2}\right)$.*

4 Equilibrium analysis

As mentioned above, we are interested in determining the effect of improved information provision on market performance. In particular, we would like to investigate the two alternatives available to regulators for reducing uncertainty about the value of financial assets. In this section we consider what happens when standards on information dissemination of publicly traded assets are tightened (ie. when σ_θ^2 is reduced), and what happens when more information about the order book or about all of the transactions in the market are provided (ie. when σ_μ^2 is reduced).

Our main focus is on the impact of changes in these variables on different measures of market performances. We concentrate our analysis on the effect of reducing σ_θ^2 and/or σ_μ^2 on market depth and market efficiency. As in Kyle (1985) we define market depth as the inverse of sensitivity of prices to changes in quantities. In our equilibrium setup market depth is captured by $1/\delta$. By market efficiency we mean the way prices aggregate the available information in the market. This is captured by $e = 1 - \frac{var(\theta/p)}{var(\theta)}$.

4.1 Market depth

Market depth is measured in our rational expectations equilibrium set up by $1/\delta$. The value δ measures the way prices change in response to quantity changes and is defined in Proposition 1. The lower is

δ, the deeper is the market.[4]

Using Eq. (18) we can express market depth as

$$\frac{1}{\delta}=\frac{k_0+k_1+k_2+k_3}{2r\sigma_t^2k_3}\left[\frac{(n-2)k_3-\frac{(n-2)k_2}{(n-1)}-2k_1}{(n-1)\left(k_3-\frac{k_2}{(n-1)}+k_1\right)}\right],$$

which, after substitution of the values of k, k_1, k_2, and k_3, can be written as

$$\frac{1}{\delta}=\frac{\left[(n-2)\,X^2-n\sigma_t^2\right]}{(n-1)r\sigma_t^2\left[X^2+n\sigma_t^2\right]}\left[\frac{\left(\sigma_t^2+n\sigma_\theta^2\right)}{\sigma_\theta^2}-\frac{(n-1)\,(X)^2\,\sigma_\mu^2}{(X)^2\,\sigma_\mu^2+\sigma_t^2\left[\sigma_\mu^2+(n-1)X^2\right]}\right].$$

Alternatively, substitution of σ_μ^2 from Eq. (22) gives

$$\frac{1}{\delta}=\frac{\left[(n-2)\,X^2-n\sigma_t^2\right]}{\left[X^2+n\sigma_t^2\right]}\left(\frac{\left(\frac{\sigma_t^2}{\sigma_\theta^2}+n\right)}{(n-1)r\sigma_t^2}-\frac{\sigma_\varepsilon X}{\left[X^2+\sigma_t^2\right]}\right). \tag{24}$$

Before analysing the impact of σ_μ^2 and σ_θ^2 on market depth. We first state the following results.

Lemma 3 *X has the following properties:*

(i) X does not depend on σ_θ^2

(ii) X is a decreasing function of σ_μ^2, it convergences to $r\sigma_t^2\sigma_\varepsilon$ as σ_μ^2 goes to infinity and it goes to infinity as σ_μ^2 converges to 0.

Note that $1/\delta$ varies with σ_μ^2 only indirectly through its dependence on X. Also, X does not depend on σ_θ^2. Hence, we can use Eq. (24) to compute the impact of σ_μ^2 and σ_θ^2.

Proposition 3 *(i) Market depth, is decreasing both in σ_θ^2 and σ_μ^2.*

(ii) As σ_θ^2 converge to zero, market depth converges to infinity. However, as σ_μ^2 converge to zero, market depth converges to a finite value, i.e.

$$\lim_{\sigma_\mu^2\to 0}\frac{1}{\delta}=\left(\frac{(n-2)\,n}{(n-1)r\sigma_t^2}\right).$$

(iii) The marginal effect of σ_θ^2 on market depth is increasing in σ_μ^2, i.e. $\frac{\partial^2\left(\frac{1}{\delta}\right)}{\partial\sigma_\theta^2\partial\sigma_\mu^2}>0$.

Making the market more transparent increases market depth. Increasing the precision of the prior information available to all investors about asset value also increases market depth, and the marginal

[4] Note also that in our set-up δ is a measure of the effective bid-ask spread.

effect is increasing in market transparency. This suggests that regulators should somehow link market transparency to an increase in the quality of prior information.

More interestingly, (ii) states that in the limit, as prior information becomes extremely precise, the market becomes infinitely deep, while an extremely transparent market would yield a level of market depth bounded from above. This demonstrates the difference between the two alternative ways of improving information provision. While, both generate an increase in market depth, they do so with different levels of efficiency. Increasing the precision of prior information is more efficient since doing so would have a direct impact on the trading behavior of agents by affecting the accuracy of all available information. On the other hand, enhancing transparency only influences agents' strategies by affecting the precision of their information about their competitors signals and the information they infer from the equilibrium price. Therefore, we argue that a further increase in the quality of prior information or of private signals has a more important impact on market depth than does a marginal increase in market transparency. Henceforth, working on rules that allow for an improvement in the quality of the information about firms (whether by increasing prior information or even the private signals) would be a more efficient way of increasing market depth than would rules that increase the quality of the information of the traders' behavior reflecting a higher market transparency.

In some sense we are related to Morris and Shin (2002) who show that public information may be harmful for the expected social welfare in a beauty-contest set up where the payoff of an agent decreases with the distance between his action and the actions of the others. In this context public information may lead to greater coordination by agents in order to increase their payoffs, which does not affect the social welfare. This is due to the fact that the coordination between agents is socially irrelevant. In our model, agents are imperfectly competitive and there are no coordination motives for them that appear directly in their utility functions. In our set-up we show that public information is always beneficial for market depth whether it involves enhancing the quality of prior information or improving market transparency.

4.2 Price efficiency

Here we consider the effect of changes in the two available alternatives on price efficiency. By price efficiency we mean the quality of information about the asset value transmitted by equilibrium price.

We consider the following measure of price efficiency:[5]

$$e = 1 - \frac{var(\theta|p)}{var(\theta)} = 1 - \frac{var(\theta|y_i, s_i, S)}{\sigma_\theta^2} = 1 - \frac{\sigma_t^2 k_3}{\sigma_\theta^2 k} = 1 - \frac{\sigma_t^2}{k}.$$

This measure lies in $[0, 1]$. When e is zero (one) p is completely uninformative (perfectly informative) about the final value θ. Using Proposition 1 and the equilibrium existence condition (defining X) we can see that

$$e = \frac{(X)^2 + n\sigma_t^2 + n(n-1)(X)^2 \frac{\sigma_t^2}{\sigma_\mu^2}}{\frac{\sigma_t^2}{\sigma_\theta^2}[(X)^2 + \sigma_t^2 + (n-1)(X)^2 \frac{\sigma_t^2}{\sigma_\mu^2}] + X^2 + n\sigma_t^2 + n(n-1)X^2 \frac{\sigma_t^2}{\sigma_\mu^2}}. \tag{25}$$

From Eq. (23), we can write

$$(n-1)(X)^2 \frac{\sigma_t^2}{\sigma_\mu^2} = \frac{\left[X - r\sigma_t^2\sigma_\varepsilon\right][X^2 + \sigma_t^2]}{r\sigma_\varepsilon\sigma_t^2}.$$

Introducing this into Eq. (25) and simplifying we obtain:

$$e = \frac{\sigma_\theta^2 \left(n - r\sigma_\varepsilon\sigma_t^2(n-1)\frac{X}{[X^2+\sigma_t^2]}\right)}{\sigma_t^2 + \sigma_\theta^2 \left(n - r\sigma_\varepsilon\sigma_t^2(n-1)\frac{X}{[X^2+\sigma_t^2]}\right)}. \tag{26}$$

Note that e varies with σ_μ^2 only indirectly through its dependence on X. Also, X does not depend on σ_θ^2. Hence, we can use Eq. (26) to compute the impact of σ_μ^2 and σ_θ^2 on e.

Proposition 4 *(i) Price efficiency is increasing in σ_θ^2. The value e convergences to 0 as σ_θ^2 goes to 0, and e convergences to 1 as σ_θ^2 goes to ∞.*

(ii) Price efficiency is decreasing in σ_μ^2. As σ_μ^2 goes to 0, X goes to infinity and e convergences to $\frac{n\sigma_\theta^2}{\sigma_t^2+n\sigma_\theta^2}$. As σ_μ^2 goes to infinity, X goes to $r\sigma_t^2\sigma_\varepsilon$ and e convergences to

$$\frac{\left([(r\sigma_t^2\sigma_\varepsilon)^2 + n\sigma_t^2]\right)}{\frac{\sigma_t^2}{\sigma_\theta^2}[(r\sigma_t^2\sigma_\varepsilon)^2 + \sigma_t^2] + \left([(r\sigma_t^2\sigma_\varepsilon)^2 + n\sigma_t^2]\right)}.$$

The first part of (i) may seem counter-intuitive in the sense that it states that increasing the quality of prior information reduces price efficiency. However, since price efficiency measures the ability of prices to transmit signals about the information available in the market, it makes sense that if the available information is of higher quality, prices are less efficient as information aggregators. Furthermore, a marginal increase in the quality of information would positively affect the informativeness of

[5] See Brown and Zhang (1997) for a discussion of the properties of this measure of price efficiency.

prices. On the other hand, (ii) states that enhancing market transparency by increasing the quality of the aggregate signal (decreasing σ_μ^2) positively affects the informativeness of the equilibrium price. However, the positive effect of transparency on market efficiency is bounded since, in the limit, as σ_μ^2 converges to zero or to infinity, price efficiency converges to finite values.

5 Concluding remarks

In this paper we have studied the impact on market performance of regulations that would improve the level of information provision in financial markets. In our rational expectations model we are able to examine what happens to market performance when regulators can provide investors with more accurate prior information by improving the precision of all private signals, and/or can enhance the transparency of the market. We show that the two alternative ways of decreasing uncertainty have the same effect on market depth, but opposite effects on market efficiency. Providing more precise prior private information increases market depth and decreases market efficiency since adverse selection effects are less relevant. Enhancing market transparency will both increase the market depth and market efficiency. The effect of increasing transparency is less important for market depth than the effect of increasing the precision of prior information since in the limit, while very precise prior information results in an infinitely deep market, increasing market transparency results in a level of market depth that is bounded from above.

Appendix

Proof of Lemma 1. All the variables θ, y_i, S, s_i have normal distribution form (see Hoel (1984)). Hence the expectation of θ given y_i, S and s_i is given by some linear function

$$E[\theta|y_i, S, s_i] = \left(\frac{k_0}{k}\right)\overline{\theta} + \left(\frac{k_1}{k}\right) y_i + \left(\frac{k_2}{k}\right)\frac{(S - s_i)}{(n-1)} + \left(\frac{k_3}{k}\right) s_i \tag{27}$$

$$\text{and } \; Var(\theta|y_i, s_i, S) = \sigma_t^2 k_3/k, \tag{28}$$

where

$$k_0 = \sigma_t^2 \tag{29}$$

$$k_1 = \frac{(n-1)(\beta_1)^2 \sigma_\theta^2 \sigma_t^2 \sigma_\mu^2}{[(\beta_2)^2 \sigma_\varepsilon^2 \sigma_\mu^2 + (\beta_1)^2 \sigma_t^2 \sigma_\mu^2 + (n-1)(\beta_2)^2 \sigma_\varepsilon^2 \sigma_t^2]} \tag{30}$$

$$k_2 = \frac{(n-1)^2 (\beta_2)^2 \sigma_\theta^2 \sigma_t^2 \sigma_\varepsilon^2}{[(\beta_2)^2 \sigma_\varepsilon^2 \sigma_\mu^2 + (\beta_1)^2 \sigma_t^2 \sigma_\mu^2 + (n-1)(\beta_2)^2 \sigma_\varepsilon^2 \sigma_t^2]} \tag{31}$$

$$k_3 = \sigma_\theta^2 \tag{32}$$

$$\text{and } \; k = [k_0 + k_1 + k_2 + k_3] \tag{33}$$

Substituting for $X = \beta_2 \sigma_\varepsilon / \beta_1$, we obtain the result in Lemma 1. ■

Proof of Lemma 1. Recall that we have:

$$p = \left[\beta_0 + \beta_1 y_i + \beta_3 S - \frac{\delta(Q - x)}{(n-1)}\right] \text{ and} \tag{34}$$

$$E[\theta|y_i, S, s_i] = \left(\frac{k_0}{k}\right)\overline{\theta} + \left(\frac{k_1}{k}\right) y_i + \left(\frac{k_2}{k}\right)\frac{(S - s_i)}{(n-1)} + \left(\frac{k_3}{k}\right) s_i \tag{35}$$

Hence, we can rewrite the first-order condition in Eq. (7) in order to find a relationship between x, a demand quantity, and, p, the equilibrium price which should not depend on y_i. Since $Var[\theta|y_i, S, s_i] = \sigma_t^2 \frac{k_3}{k}$, we obtain the following relationship:

$$\begin{aligned} 0 = & \left[\left(\frac{k_0}{k}\right)\overline{\theta} + \left(\frac{k_2}{k}\right)\frac{(S - s_i)}{(n-1)} + \left(\frac{k_3}{k}\right) s_i + \left(\frac{k_1}{k}\right)\left(\frac{p - \beta_0 - \beta_3 S + \frac{\delta(Q-x)}{(n-1)}}{\beta_1}\right)\right] \\ & - [p] - \left(\frac{\delta}{(n-1)} + r\sigma_t^2 \frac{k_3}{k}\right) x - r\sigma_t^2 \frac{k_3}{k}\epsilon_i \end{aligned} \tag{36}$$

Isolating p, we obtain the following inverse demand function:

$$\begin{aligned} p\left[1 - \left(\frac{k_1}{\beta_1 k}\right)\right] = & \left[\left(\frac{k_0}{k}\right)\overline{\theta} + \left(\frac{k_1}{\beta_1 k}\right)\frac{\delta Q}{(n-1)} - \left(\frac{k_1 \beta_0}{k \beta_1}\right)\right] + \left(\frac{k_3}{k} - \left(\frac{k_2}{(n-1)k}\right)\right) s_i - \left(r\sigma_t^2 \frac{k_3}{k}\right)\epsilon_i \\ & + \left[\frac{k_2}{(n-1)k} - \left(\frac{k_1}{k}\frac{\beta_3}{\beta_1}\right)\right] S - \left[\frac{\delta}{(n-1)} + r\sigma_t^2 \frac{k_3}{k} + \left(\frac{k_1}{\beta_1 k}\right)\frac{\delta}{(n-1)}\right] x \end{aligned} \tag{37}$$

Since by assumption:

$$P(x_i, S, \epsilon_i, s_i) = \beta_0 + \beta_1 s_i - \beta_2 \epsilon_i + \beta_3 S - \delta x_i$$

matching the arguments of the two above equations, we have:

$$\begin{aligned}
\beta_0 &= \left[\frac{\left(\frac{k_0}{k}\right)\overline{\theta} + \left(\frac{k_1}{\beta_1 k}\right)\frac{\delta Q}{(n-1)} - \left(\frac{k_1 \beta_0}{k \beta_1}\right)}{1 - \left(\frac{k_1}{\beta_1 k}\right)} \right] \\
\beta_1 &= \left[\frac{\left(\frac{k_3}{k} - \frac{k_2}{(n-1)k}\right)}{\left[1 - \left(\frac{k_1}{\beta_1 k}\right)\right]} \right] = \left(\frac{k_3}{k} - \frac{k_2}{(n-1)k} + \frac{k_1}{k} \right) \\
\beta_2 &= \frac{\left(r\sigma_t^2 \frac{k_3}{k}\right)}{\left[1 - \left(\frac{k_1}{\beta_1 k}\right)\right]} \\
\beta_3 &= \frac{\left[\frac{k_2}{(n-1)k} - \left(\frac{k_1}{k}\frac{\beta_3}{\beta_1}\right)\right]}{\left[1 - \left(\frac{k_1}{\beta_1 k}\right)\right]} \\
\delta &= \frac{\left[\frac{\delta}{(n-1)} + r\sigma_t^2 \frac{k_3}{k} + \left(\frac{k_1}{\beta_1 k}\right)\frac{\delta}{(n-1)}\right]}{\left[1 - \left(\frac{k_1}{\beta_1 k}\right)\right]}
\end{aligned}$$

By simplifying, we obtain the results of Proposition 1. ■

Proof of Lemma 2. Using the equations (11), (12), (13) and (14), we obtain an expression for X :

$$X = \frac{\beta_2}{\beta_1}\sigma_\epsilon = r\sigma_t^2 \sigma_\varepsilon \left[\frac{\left[(X)^2 \sigma_\mu^2 + \sigma_t^2 \sigma_\mu^2 + (n-1)(X)^2 \sigma_t^2\right]}{\left[(X)^2 \sigma_\mu^2 + \sigma_t^2 \sigma_\mu^2\right]} \right] \tag{38}$$

which may be written as:

$$\left[X - r\sigma_t^2 \sigma_\varepsilon\right] = r\sigma_\varepsilon (n-1)\frac{X^2}{\left[X^2 + \sigma_t^2\right]}\frac{\sigma_t^4}{\sigma_\mu^2} \tag{39}$$

Notice that if a positive solution exists for X, then the left side must be positive and we have $X > r\sigma_t^2\sigma_\varepsilon$. Evaluated at $X = r\sigma_t^2\sigma_\varepsilon$, the left side of Eq. (39) is 0 while the right side is positive. Conversely, when X goes to infinity, the left side goes to infinity while the right side is finite. By continuity, there exists at least one solution to this equation.

Finally, we must show that the obtained X satisfies the second-order condition. Rewriting the second-order condition, we obtain:

$$\begin{aligned}
\frac{2\delta}{(n-1)} + rVar(\theta | y, s_i, S) &= \sigma_t^2 \frac{k_3}{k} \left(\frac{2k_3 - \frac{2k_2}{(n-1)} + 2k_1}{(n-2)k_3 - \frac{(n-2)k_2}{(n-1)} - 2k_1} \right) + r\sigma_t^2 \frac{k_3}{k} \\
&= nr\sigma_t^2 \frac{k_3}{k}\left[\frac{\left[X^2 + \sigma_t^2\right]}{(n-2)\left[X^2\right] - n\sigma_t^2} \right] > 0
\end{aligned}$$

Hence, the second-order condition is verified for all $X^2 > \frac{n\sigma_t^2}{(n-2)}$. Notice that if a positive solution exists for X, then the left side must be positive and we have $X > r\sigma_t^2\sigma_\varepsilon$. If σ_μ^2 goes to infinity, X will converge to $r\sigma_t^2\sigma_\varepsilon$. So if $r^2\sigma_\varepsilon^2 > \frac{n}{\sigma_t^2(n-2)}$, then we must always have $X^2 > \frac{n\sigma_t^2}{(n-2)}$. Conversely, if $r^2\sigma_\varepsilon^2 < \frac{n}{\sigma_t^2(n-2)}$, then there exists a σ_μ^2 sufficiently large so that $X^2 < \frac{n\sigma_t^2}{(n-2)}$, and the second-order condition is not verified.

Uniqueness is guaranteed by the fact that (a) the right-side of Eq. (39) is linear in X, and (b) the left side is concave for all value $X^2 > \frac{\sigma_t^2}{3}$.[6] Under the condition of the lemma we have established that $X^2 > \frac{n\sigma_t^2}{(n-2)} > \frac{\sigma_t^2}{3}$. Hence, the left-side crosses the right side from above only once. ■

Proof of Lemma 3. The result in (i) follows immediately from the fact the equation that determines X (Eq. (39)) is independent of σ_θ^2. In order to prove the results in (ii) recall the same equation that defines X. Note that the left side is decreasing in σ_μ^2 and so as σ_μ^2 increases, the left curve shifts down and X must decrease. When σ_μ^2 convergences to infinity, $\left[X - r\sigma_t^2\sigma_\varepsilon\right]$ must converge to zero. Conversely, when σ_μ^2 goes to zero, $\left[X - r\sigma_t^2\sigma_\varepsilon\right]$ must converge to infinity. ■

Proof of Proposition 3. Recall Equation (24) that determines $\frac{1}{\delta}$. It has one argument that varies with σ_θ^2. The value $\frac{1}{\delta}$ is clearly decreasing in σ_θ^2. Equation (24) has two arguments that vary with X. The value $\frac{\left[(n-2)X^2 - n\sigma_t^2\right]}{\left[X^2 + n\sigma_t^2\right]}$ is increasing in X for all values such that $\left[(n-2)\,X^2 - n\sigma_t^2\right] > 0$. The second argument $\frac{\sigma_\varepsilon X}{\left[X^2 + \sigma_t^2\right]}$ is decreasing in X if and only if $X^2 > \sigma_t^2$. Recall that from the second-order condition, we have $X^2 > \frac{n\sigma_t^2}{(n-2)}$. Hence, the expression $\frac{1}{\delta}$ given by Eq. (24) is increasing in X. We have established that X is decreasing in σ_μ^2, hence $\frac{1}{\delta}$ is decreasing in σ_μ^2. This completes the proof of (i).

If we take the limit, as σ_θ^2 converges to 0, $\frac{1}{\delta}$ goes to ∞. From Lemma 3, when σ_μ^2 converges to 0, X goes to $r\sigma_t^2\sigma_\varepsilon$. Hence, we have:

$$
\begin{aligned}
\lim_{\sigma_\mu^2 \to \infty} \frac{1}{\delta} &= \lim_{X \to r\sigma_t^2\sigma_\varepsilon} \frac{1}{\delta} \\
&= \frac{\left[(n-2)\left(r^2\sigma_t^2\sigma_\varepsilon^2\right) - n\right]}{\left[(r^2\sigma_t^2\sigma_\varepsilon^2) + n\right]} \left(\frac{\left(\sigma_t^2 + n\sigma_\theta^2\right)}{(n-1)r\sigma_t^2\sigma_\theta^2} - \frac{r\sigma_\varepsilon^2(n-1)}{\left[(r^2\sigma_t^2\sigma_\varepsilon^2) + 1\right]} \right)
\end{aligned}
$$

[6] Indeed we have:

$$
\frac{d^2}{(dX)^2} \frac{X^2}{\left[X^2 + \sigma_t^2\right]} = \frac{2\sigma_t^2[\sigma_t^2 - 3X^2]}{[X^2 + \sigma_t^2]^3} < 0, \text{ for all } X^2 > \frac{\sigma_t^2}{3}
$$

This establishes (ii). Differentiation of Eq. (24) with respect to σ_θ^2 and X gives

$$\frac{\partial^2\left(\frac{1}{\delta}\right)}{\partial\sigma_\theta^2\partial X}=\frac{\partial}{\partial X}\left(\frac{\left[(n-2)\,X^2-n\sigma_t^2\right]}{\left[X^2+n\sigma_t^2\right]}\right)\frac{\partial}{\partial\sigma_\theta^2}\left(\frac{\left(\frac{\sigma_t^2}{\sigma_\theta^2}+n\right)}{(n-1)r\sigma_t^2}\right)<0.$$

From Lemma 3 we have $\frac{\partial X}{\partial\sigma_\mu^2}<0$, so

$$\frac{\partial^2\left(\frac{1}{\delta}\right)}{\partial\sigma_\theta^2\partial\sigma_\mu^2}>0$$

which shows (iii). ■

Proof of Proposition 4. From Eq. (26), we have:

$$e=\frac{\sigma_\theta^2\left(n-r\sigma_\varepsilon\sigma_t^2(n-1)\frac{X}{[X^2+\sigma_t^2]}\right)}{\sigma_t^2+\sigma_\theta^2\left(n-r\sigma_\varepsilon\sigma_t^2(n-1)\frac{X}{[X^2+\sigma_t^2]}\right)} \tag{40}$$

which is increasing in σ_θ^2. When σ_θ^2 goes to 0, e goes to 0. Conversely, if σ_θ^2 goes to ∞, then $\frac{1}{e}$ and e go to 1. This establishes (i).

Note that e is decreasing in $\frac{X}{[X^2+\sigma_t^2]}$, and that $\frac{X}{[X^2+\sigma_t^2]}$ is decreasing in X since we have $X^2>\sigma_t^2$. It follows that e is increasing in X. Since X decreases in σ_μ^2, market efficiency, e, must decrease in σ_μ^2. When σ_μ^2 goes to 0, X goes to infinity. Hence, we have:

$$\lim_{\sigma_\mu^2\to 0}e=\lim_{X\to\infty}e=\frac{n}{\frac{\sigma_t^2}{\sigma_\theta^2}+n}=\frac{\sigma_\theta^2 n}{\sigma_t^2+\sigma_\theta^2 n}$$

Also when σ_μ^2 goes to ∞, X goes to $r\sigma_\varepsilon\sigma_t^2$, hence the result presented in the Proposition. ■

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