Pricing the CBOT T-bonds futures

Ramzi Ben-Abdallah, Hatem Ben-Ameur and Michèle Breton

A futures contract is an agreement between two parties to sell or to buy an asset at some given time in the future, called the delivery date, for a given price. Futures contracts include details about delivery arrangements and quality specifications and are normally traded on an exchange. The Treasury Bond futures traded on the Chicago Board of Trade (CBOT) is the most actively traded and widely used futures contract in the U.S.A. It calls for the delivery of $100,000 of a long-term governmental bond, and includes several delivery options. This paper proposes a numerical method to value the CBOT Treasury Bond futures contract in a stochastic interest-rate environment, and compares the theoretical prices obtained by this method with the actual prices observed on the CBOT between 1990 and 2008.

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The delivery options included in the contract apply to the timing and the quality of the delivery. The timing options allow the delivery of the bond at any time during the delivery month, even during some periods where the futures market is closed while the underlying bond market is open. The quality option gives the seller the right to deliver any governmental bond in a set of known eligible bonds.

A bond is a contract to repay the principal at maturity and fixed interest (coupon) at fixed intervals (say annually). For instance, a $100,000 bond maturing in 15 years with a 6% annual coupon is a promise to pay $6,000 each year during 15 years, and $100,000 in 15 years. The market value of such a bond depends on the current interest rate, but also on how the market expects the interest rate to evolve over time. The quality option gives the seller the right to sell for a fixed price any bond in a set of eligible bonds with different coupon rates and maturities, and therefore different values. In fact, to account for variations in quality, the price received by the seller is adjusted via a set of known conversion factors. The conversion factor system used by the CBOT is such that the value of all eligible bonds is the same when the interest rate is assumed to be 6% and constant over time. When this is not the case, all bonds are not equal for delivery, and there is a best choice for the seller (called the cheapest-to-deliver (CTD)).

To date, no work has been presented regarding the identification of optimal exercise strategies (when and what to deliver) and the pricing of this contract under stochastic interest rates when the interaction of all the delivery options is taken into account. Indeed, even under constant deterministic interest rates, the CTD changes over time during the delivery month, so that the delivery strategy cannot be characterized analytically. Adding uncertainty about the future evolution of the interest rate complicates the problem even further.

We propose a pricing algorithm in the general setting of a multifactor Markov diffusion model for the evolution of interest rates. Our pricing procedure is a backward numerical algorithm combining Dynamic Programming, approximation by finite elements, and fixed-point evaluation. The algorithm yields the value of the contract for the short trader (which is reset to 0 at the settlement dates), the futures prices at settlement dates, as well as the delivery strategy (deliver or not) on position days and the CTD on notice days, as a function of the futures price at the last settlement date and of the value of the state vector. ■

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Ramzi Ben-Abdallah
Hatem Ben-Ameur
Michèle Breton
Department of Mangement Science
HEC Montréal and GERAD