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Newsletter

*Groupe d'études et de recherche en analyse des décisions*

GERAD

# One of ours is the world's TOP!

According to a review by Pao-Nuan Hsieh and Pao-Long Chang, published in August 2009 in the *International Journal of Production Economics*, Gilbert Laporte, one of GERAD's founding members, is one of the world's most productive researchers (and for some criteria, the first) in the area of production and operations management. We are very proud of this accomplishment by one of our members. Congratulations Gilbert!

We also heard excellent news in the renewal of the GERAD grant by the Fonds québécois de la recherche sur la nature et les technologies (FQRNT), in the "Strategic clusters" program. GERAD will receive $2.34 million during the next six years. This support by FQRNT is an endorsement of our Centre and a motivation for us to continue our quest for excellence in research and training.

This issue presents the outlines of thirteen research projects that were reported-on in first-class journals. The selection shows the diversity of tools (mathematical programming, stochastic processes, statistics, game theory, and other operations research methods) and of application areas (scheduling, financial engineering, marketing, vehicle routing, energy, and health) in research projects undertaken at GERAD.

Enjoy the newsletter,
Georges Zaccour

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## SUMMARY



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*In this issue … articles in first-class journals …*

# Option Pricing under **GARCH** Processes by PDE Methods

Michèle Breton and Javier de Frutos

An option is a financial instrument whose value depends on the value of another underlying asset, giving the holder the right, but not the obligation, to trade the underlying asset at some pre-specified dates in the future, at a previously agreed price, until maturity. The value of an option depends essentially on the underlying asset's volatility, which affects the magnitude of possible future gains in case the option is exercised. The well-known Black and Scholes formula computes the value of an option that can only be exercised at maturity, when the volatility of the underlying asset is assumed constant over time.

However, it is well known that financial series models using constant, or even deterministic, volatility do not fit empirical observations very well. Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models, where the conditional volatility of a time series is updated at discrete times from observed realizations, were shown to be much better predictors of the evolution of financial asset prices over time. However, even in the simplest case, the valuation of options written on underlying assets following a GARCH-type process raises substantial theoretical and numerical issues.

In this paper, we propose a valuation approach based on the numerical solution of a partial differential equation describing the evolution over time of the option's value. Under the option pricing model, volatility is constant between two GARCH time steps, and jump conditions are imposed each time the volatility estimation is updated. In this case, an explicit solution by Fourier series exists between updates, and the option value can be represented by several functions of time and asset price, parametrized by volatility and by the price of the underlying asset at the last volatility update time.

Our algorithm is based on a spectral approximation of the partial differential equation describing the evolution of the option's value along with the jump conditions, and on a Fourier-Chebyshev interpolation of the function describing the option's value. The key idea is to exploit the properties of Fourier series representation of periodic solutions of partial differential equations. Thus, we use an even periodic extension of the option value, as a function of time and asset price, sufficiently smooth to allow for high precision using a relatively small number of Fourier coefficients. A spectral interpolation using Chebyshev polynomials is then used to approximate the option value as a function of volatility and of the asset price at the last update time. As a result, the option value (a function of four variables) is represented by its Fourier and Chebyshev coefficients.

Our method is very general and can be applied to any of the numerous GARCH specifications used in practice. We reach high precision in a few seconds of computing time, and exponential convergence with respect to the number of discretization points.

(*Operations Research*, to appear)

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