Monetary Policy Shifts and the Term Structure*

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First Version: 27 October 2007

JEL Classification: C13, C50, E43, E52, G12

Keywords: Quadratic term structure model; Monetary policy;
Interest rate risk; time-varying parameter model.

*The paper has benefited from discussions with Michael Johannes and Monika Piazzesi. We thank seminar participants at UT Austin and the Capital Group. We thank Rudy Loo-Kung for excellent research assistance. Andrew Ang and Jean Boivin acknowledge support from the NSF (SES-0137145 and SES-0518770).

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Abstract

We estimate the effect of shifts in monetary policy on the term structure of interest rates. In the no-arbitrage model, the short rate follows a version of the Taylor (1993) rule where the coefficients on inflation and output can vary over time. We identify the impact of changing monetary stances on inflation and output across the yield curve and also estimate the prices of risk that agents assign to unexpected changes in monetary policy. We find that monetary policy changed substantially over the last 50 years and that changes in the policy rule have a quantitatively important influence on the shape of the term structure of interest rates. In particular, the risk premia on long-term bonds would be 1.9% per annum higher compared to a world where agents assigned no value to the stabilizing effect of monetary policy.

1 Introduction

A large body of narrative and empirical evidence suggests that the conduct of monetary policy in the U.S. has changed in substantial ways over the last 50 years. The Volcker disinflation in the early 1980's is a well-known example of a drastic, and perhaps long-lasting, change in the way monetary policy is set in response to economic developments. The possibility of shifts in monetary policy has spurred a large body of empirical research attempting to document and quantify the importance of these changes.[1]

One important reason to be interested in these changes in monetary policy is that they can serve as "monetary policy experiments" that could help us better identify and measure the effect of systematic monetary policy on the economy. So far, the focus in the literature has been mainly on the impact of monetary policy on real activity and inflation. In particular, a lot of attention has recently been devoted to determining the role played by monetary policy in explaining the "Great Moderation," referring to the fact that the volatility of real activity and inflation, and other macro series, has decreased substantially since the mid-1980's.[2]

One aspect that has received little attention so far is the implications of these monetary policy changes for financial markets, in particular for the term structure of interest rates.[3] A growing number of studies that have employed macro factors in term structure models have found that macroeconomic fluctuations are an important source of uncertainty affecting bond risk premia (see, among others, Ang and Piazzesi, 2003; Ang, Bekaert and Wei, 2007). An unanswered question is what are the effects of the Fed's changing monetary policy stances vis-à-vis output or inflation on the term structure. Monetary policy changes affect the entire term structure because the actions of the Fed at the short end of the yield curve influence the dynamics of the long end of the yield curve through no-arbitrage restrictions. Consequently, the term structure of yields also provides valuable information in estimating monetary policy shifts.

It is unclear in what direction changing monetary policy affects long-term yields and the risk premia of long-term bonds. If monetary policy is entirely neutral, then agents would assign the same risk premia to long-term bonds in a world where monetary policy changed over time and

[1] See, among others, Clarida, Galí and Gertler (2000), Orphanides (2001), Cogley and Sargent (2001, 2005), Sims and Zha (2006), and Boivin (2006).

[2] See, for instance, Stock and Watson (2003), Boivin and Giannoni (2006), Sims and Zha (2006) and Justiniano and Primiceri (2006).

[3] One exception is Bikbov (2006), who examines the effect of discrete regime shifts in monetary policy rules on the term structure. Our monetary policy shifts are continuous and we also estimate the price of risk of monetary policy changes.

in a world where monetary policy was constant. If monetary policy is priced and agents dislike the uncertainty of policy changes, then part of the risk premium for holding long-term bonds is to compensate investors for their risk aversion to monetary policy shifts. On the other hand, if monetary policy has a stabilizing effect on macroeconomic fluctuations and this is valued by investors, then real-world risk premia would be lower compared to an economy where investors assigned no value to Fed policy changes. Similarly, it is not clear whether changes in monetary policy increase or decrease long-term yields and how these yield changes may differ across different parts of the yield curve.

The central goal of this paper is to determine the influence of monetary policy on the term structure of interest rates. The existence of historical shifts in monetary policy, or different monetary policy "experiments," provides an opportunity to statistically estimate the effects of changes in the policy rule on the term structure. To do so we estimate a quadratic term structure model, where the dynamics of the short rate follow a version of Taylor's (1993) rule. Our no-arbitrage model allows for the Fed response to inflation and output to potentially vary over time. In contrast to most existing empirical models, we do not impose that the time variation in the policy parameters is exogenous. For instance, we entertain the possibility that a high response to inflation today might be due to the fact that inflation was high in the recent past.

An additional advantage of estimating the Taylor rule jointly with a term structure model is that by including more information, it can potentially yield sharper estimates of the changes in the policy rule. This is potentially important since conflicting evidence has been reported in the literature on the importance of monetary policy shifts, and the evidence based on the estimation of single equation have been subject to considerable statistical uncertainty. Exploiting term structure information in this context could thus lead to more conclusive evidence on how monetary has evolved.

Using the estimated model, we perform a series of exercises. We first document the importance of the historical changes in monetary policy. We then investigate the effect of these changes on the term structure of interest rates by computing impulse responses and expected holding period returns. Our key findings can be summarized as follows. First, our estimates suggest that monetary policy changed substantially over the last 50 years. The Fed's sensitivity to inflation has changed markedly over time and our estimates are largely consistent with the evidence reported in Clarida, Galí and Gertler (2000), Cogley and Sargent (2005) and Boivin (2006). However, we find that shifts of monetary policy stances with regards to output shocks exhibit very small variation.

Second, the use of term structure information in the estimation of the policy rule leads to sharper parameter estimates, which statistically allow us to reject the hypothesis that the Taylor principle was satisfied throughout the 1970's. Estimates of policy changes without the information contained in the term structure of interest rates are quite different from our model. Using term structure information, we find that the Fed's inflation response is more aggressive and flexible, on average, compared to estimates that ignore the yield curve. While the Fed's response to output gap shocks have not changed much over the last 50 years in our model, estimates of the Fed's output gap sensitivity exhibit substantial time variation without using term structure information.

Finally, changes in monetary policy have a quantitatively important influence on the shape of the term structure. A shock to the Fed response to output gap fluctuations, ceteris paribus, raises short term rates and shrinks the term spread. Not surprisingly, as the Fed becomes more aggressive in combating inflation, short rates initially increase. However, long yields are quite stable and do not respond much to monetary policy changes with respect to inflation. Overall, we find that activist monetary policy leads to low risk premia on long-term bonds. Specifically, if monetary policy movements were not valued by investors and were assigned zero prices of risk, then holding period returns on long-term bonds would need to be approximately 1.9% per annum higher, or their Sharpe ratios approximately 1.8 times higher, to induce investors to hold these securities. This comes in part from the stabilizing influence of monetary policy on macroeconomic fluctuations and the fact that some policy shifts stem from an endogenous response to past economic development.

The rest of the paper is organized as follows. Section 2 describes the modelling framework. It first describes the short rate equation, specified as a time-varying policy reaction function, and then derives bond prices based on a quadratic, arbitrage-free, term structure model. Section 3 discusses the results and Section 4 concludes. The details of the bond pricing derivations and the Bayesian estimation technique can be found in the Appendix.

2 Model

We assume that the dynamics of the short end of the yield curve (the one-quarter short rate) follows a version of Taylor's (1993) rule where the monetary authority sets the short rate as a function of inflation and the output gap. Unlike a standard Taylor rule, we let the policy responses on output and inflation vary over time:

$$r_t = \delta_0 + (\bar{a} + a_t)g_t + (\bar{b} + b_t)\pi_t, \tag{1}$$

where r_t is the 1-quarter yield, g_t is the output gap, and π_t is inflation. In estimating equation (1) in our model, we also include a small orthogonal error term. The coefficient on the output gap, $(\bar{a} + a_t)$, measures how much the monetary authority adjusts the short rate to output gap shocks and consists of a base level, $\bar{a}$, and a zero-mean time-varying component, a_t. Similarly, the policy response to inflation consists of an average response, $\bar{b}$, and a zero-mean deviation around this mean level, b_t. If there has been no change to the Fed's policy reaction function, then $a_t = b_t = 0$, otherwise time-variation in a_t and b_t represent policy shifts in the relative importance of output gap or inflation shocks in setting short-term interest rates.

We collect the macro variables and the policy coefficients in the state vector $X_t = [g_t\ \pi_t\ a_t\ b_t]^\top$, which follows the stationary VAR:

$$X_t = \mu + \Phi X_{t-1} + \Sigma\varepsilon_t, \tag{2}$$

where $\varepsilon_t \sim$ IID $N(0, I)$. We order the macro variables first in the VAR. We parameterize Φ and Σ as

$$\Phi = \begin{pmatrix} \Phi_{gg} & \Phi_{g\pi} & \Phi_{ga} & 0 \\ \Phi_{\pi g} & \Phi_{\pi\pi} & 0 & \Phi_{\pi b} \\ \Phi_{ag} & 0 & \Phi_{aa} & 0 \\ 0 & \Phi_{b\pi} & 0 & \Phi_{bb} \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \Sigma_{gg} & 0 & 0 & 0 \\ \Sigma_{\pi g} & \Sigma_{\pi\pi} & 0 & 0 \\ 0 & 0 & \Sigma_{aa} & 0 \\ 0 & 0 & \Sigma_{ab} & \Sigma_{bb} \end{pmatrix}. \tag{3}$$

Without the time-varying policy coefficients a_t and b_t, the upper 2×2 matrix of Φ represents a regular VAR of output and inflation. The coefficients Φ_{ga} and $\Phi_{\pi b}$ allow the policy coefficients to influence the future path of output and inflation. A negative coefficient Φ_{ga} means that a more aggressive response to the output gap would reduce the output gap next period. Similarly, if $\Phi_{\pi b} < 0$, then future inflation reduces as the Fed tightens monetary policy.[4]

We treat the policy variables a_t and b_t as latent factors and are especially interested in their variation through the sample. In systems with latent factors, the same reduced-form model may often be produced by arbitrarily scaling or shifting the coefficients governing the dynamics of a_t and b_t in Φ or Σ. To identify a_t and b_t, we set the conditional correlations between them and the macro variables to be zero. However, the policy sensitivities to the output gap and inflation

[4] We do not allow the Fed's response to inflation to influence the future output gap or the Fed's output gap sensitivity to influence future inflation ($\Phi_{gb} = \Phi_{\pi a} = 0$). Estimates with non-zero Φ_{gb} and $\Phi_{\pi a}$ are hard to identify and resulted in VAR estimates that were non-stationary.

may potentially be correlated, and the macro variables may influence future policy stances. If Φ_{ag} or $\Phi_{\pi b}$ are positive, then the Fed responds to an environment with an increasing output gap or inflation by raising the policy responses to the output gap or inflation.

The time-varying policy rule in equation (1) can be written in the form of a standard time-invariant Taylor (1993) rule with a "policy shock," η_t, that depends explicitly on the level of the output gap and inflation, combined with a time-varying policy stance. For ease of exposition, we ignore the orthogonal error term that is included in estimating equation (1) and write equation (1) as:[5]

$$\begin{aligned} r_t &= \delta_0 + \bar{a} g_t + \bar{b} \pi_t + (a_t g_t + b_t \pi_t) \\ &= \delta_0 + \bar{a} g_t + \bar{b} \pi_t + \eta_t, \end{aligned} \tag{4}$$

where $\eta_t = (a_t g_t + b_t \pi_t)$. In the special case that a_t and b_t are uncorrelated with the output gap and inflation (or $\Phi_{ga} = \Phi_{\pi b} = \Phi_{ag} = \Phi_{b\pi} = 0$ in equation (3)), then the average policy responses $\bar{a}$ and $\bar{b}$ in equation (4) can be consistently estimated by OLS. However, if a_t or b_t are correlated with the output gap or inflation, then conventional estimates of a linear policy rule like equation (4) will produce biased estimates of the policy responses to macro variable shocks.

If policy shifts do occur over time in the form of equation (1), then we can interpret the traditional policy shock, η_t, in the linear Taylor setting (4) as comprising two components: policy reaction components a_t and b_t, and macro variable components, g_t and π_t. The residual η_t would also exhibit conditional heteroskedasticity. We can decompose a traditional linear policy shock into policy shifts by the Fed (a_t and b_t terms) and separate shocks to output gap and inflation components ($a_t g_t$ and $b_t \pi_t$, respectively). Previous research in affine models have found that a linear latent factor, η_t, is related to movements in macro variables and can represent a monetary policy shock (see, among others, Ang, Dong and Piazzesi, 2006; Bikbov and Chernov, 2006). In our set up, we can quantify the variation in short rates directly emanating from policy shifts versus shocks to macro variables.

[5] We also investigated an alternative policy shift model, where the short rate took the form:

$$\hat{r}_t = \delta_0 + (\bar{a} + a_t) g_t + (\bar{b} + b_t) \pi_t + f_t,$$

where f_t was an additional IID latent factor. This latent factor differs the measurement error u_t^1 put on the short rate in equation (14) because the observation errors are yield specific, whereas the latent factor f_t is also priced by all other yields. This model resulted in extremely small estimates of f_t with almost zero improvement in model fit (observation error standard deviations). A Bayes factor test also makes this model extremely unlikely compared to the benchmark quadratic model.

We assume that the time variation in the policy coefficients is a covariance stationary process, that is all the eigenvalues of Φ lie inside the unit circle. This is in contrast to previous approaches which model time variation in policy parameters using random walks (see, among others, Cooley and Prescott, 1976; Cogley and Sargent, 2001, 2005; Boivin, 2006; Cogley, 2005; Justiniano and Primiceri, 2006). While the random walk is a convenient framework to account for permanent changes in coefficients, inferring how the term structure reacts to policy shifts is better done with a stationary process for several reasons. First, since yields are intertemporal marginal rates of substitution, they should be stationary in well-defined exchange models with representative agents having utility over consumption. Second, random walk models cannot be used to attribute the variance of long-term yields to policy shift components and shocks to macro factors, as the unconditional variance is infinite in a random walk process. Similarly, since there is no well-defined long-run mean in a random walk system, it is hard to define the long-run effects of policy shifts on yields or macro factors.

The time-varying Taylor rule (1) is an example of a regression model with stochastically varying coefficients. Using only macro data and short rates, the system is asymptotically identified (see Pagan, 1980). However, it is hard to use only one observable variable, short rates, to identify two latent processes in small samples. Fortunately, it is not only the short rate that responds to policy shifts – we identify the variation in a_t and b_t by using information from the entire yield curve. A further advantage of using the entire term structure is that we can identify the prices of risk that agents assign to the policy authority's time-varying policy rules. Thus, we can infer the effect on long-term yields of a policy shift by the Fed on its inflation stance, as well as the traditional analysis of tracing through the effect of an inflation shock on the term structure. In contrast, the previous literature on macro-finance term structure models assumes that the policy coefficients are time invariant.

2.1 Bond Prices

To derive bond prices from the policy shift model of equation (1), we write the short rate as a quadratic function of the factors $X_t = [g_t \; \pi_t \; a_t \; b_t]^\top$:

$$\hat{r}_t = \delta_0 + \delta_1^\top X_t + X_t^\top \Omega X_t \tag{5}$$

where δ_0 is a scalar and $\delta_1 = [\bar{a} \; \bar{b} \; 0 \; 0]^\top$. We use the carrot notation for yields which are direct functions of the model to distinguish the model-implied short rate, $\hat{r}_t$, from the short rate in data, r_t. In the quadratic term $X_t^\top \Omega X_t$ in equation (5), Ω is specified as

$$\Omega = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}. \tag{6}$$

The short rate is linear in the observable macro variables and the quadratic form results from the interaction of the stochastic policy coefficients with the macro factors. If there are no policy shifts, then $\Omega = 0$, and the model simplifies to a standard affine term structure model.

To price long-term bonds, we specify the pricing kernel to take the standard form:

$$m_{t+1} = \exp\left(-\hat{r}_t - \frac{1}{2}\lambda_t^\top \lambda_t - \lambda_t \varepsilon_{t+1}\right), \tag{7}$$

with the time-varying prices of risk:

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{8}$$

for the 4×1 vector λ_0 and the 4×4 matrix λ_1. The prices of risk control the response of long-term yields to macro and policy shocks, and cause the expected holding period returns of long-term bonds to vary over time (see Dai and Singleton, 2002). Of particular interest are the risk premia parameters on the policy shift variables a_t and b_t. These have not been examined before because the prices of risk in equation (8) have almost exclusively been employed in traditional affine macro-term structure models where the policy coefficients are constant (see, for example, Ang and Piazzesi, 2003).

The pricing kernel prices zero-coupon bonds from the recursive relation:

$$\hat{P}_t^n = \mathrm{E}_t[m_{t+1}\hat{P}_{t+1}^{n-1}],$$

where $\hat{P}_t^n$ is the price of a zero-coupon bond of maturity n quarters at time t. Equivalently, we can solve the price of a zero-coupon bond as:

$$\hat{P}_t^n = \mathrm{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{i=0}^{n-1}\hat{r}_{t+i}\right)\right], \tag{9}$$

where $\mathrm{E}_t^{\mathbb{Q}}$ denotes the expectation under the risk-neutral probability measure $\mathbb{Q}$, under which the dynamics of the state vector X_t are characterized by the risk-neutral constant and autocorrelation matrix:

$$\begin{aligned} \mu^Q &= \mu - \Sigma\lambda_0 \\ \Phi^Q &= \Phi - \Sigma\lambda_1. \end{aligned} \tag{10}$$

The quadratic short rate (1) or (5), combined with the linear VAR in equation (2), and the pricing kernel (7) gives rise to a quadratic term structure model. We can write the bond price for maturity n implied by the model as:

$$\hat{P}_t^n = \exp(A_n + B_n^\top X_t + X_t^\top C_n X_t), \tag{11}$$

where the terms A_n, B_n, and C_n are given in Appendix A. Hence, if we denote the yield on a zero-coupon bond with maturity n quarters as $\hat{y}_t^n = -1/n \log \hat{P}_t^n$, yields are quadratic functions of X_t:

$$\hat{y}_t^n = a_n + b_n^\top X_t + X_t^\top c_n X_t, \tag{12}$$

where $a_n = -A_n/n$, $b_n = -B_n/n$, and $c_n = -C_n/n$. This analytical form enables the estimation of the model and allows us to investigate how the entire term structure responds to policy changes and macro shocks.

We define an excess holding period return as the return on holding a long-term bond in excess of the short rate:

$$xhpr_{t+1}^n = \log \frac{\hat{P}_{t+1}^{n-1}}{\hat{P}_t^n} - \hat{r}_t,$$

where the notation $xhpr_{t+1}^n$ denotes that the excess holding period return applies to a zero coupon bond of n periods today at time t. The conditional expected excess holding period return implied by the model is also given by a quadratic function:

$$\mathrm{E}_t[xhpr_{t+1}^n] = \bar{A}_n + \bar{B}_n^\top X_t + X_t^\top \bar{C}_n X_t, \tag{13}$$

where the coefficients $\bar{A}_n$, $\bar{B}_n$ and $\bar{C}_n$ are given in Appendix A. It also possible to compute out the conditional variance of excess holding period returns, which also has a quadratic form.

Since the yields are quadratic functions of the state variables, the model belongs to the class of quadratic term structure models developed by Longstaff (1989), Beaglehole and Tenney (1992), Constantinides (1992), Leippold and Wu (2002, 2003), and Ahn, Dittmar and Gallant (2002).[6] None of these authors incorporate observable macro factors or investigate policy shifts. Ahn, Dittmar and Gallant (2002) and Brandt and Chapman (2003) demonstrate that quadratic models have several advantages over the affine class in adding more flexibility to better match yield dynamics, particularly conditional moments. The non-linearity of yields also

[6] These quadratic models are members of the broader affine class of term structure models of Duffie and Kan (1996) as they have linear representations of yields involving factors X_t and second moments of factors, $vech(X_t X_t')$. The quadratic term itself follows an affine process, as shown by Filipovic and Teichmann (2002) and Gourieroux and Sufana (2003). Buraschi, Cieslak and Trojani (2007) show that the quadratic short rate process can be supported in a Cox, Ingersoll and Ross (1985) production economy with a representative agent.

aids in identifying prices of risk because there is an additional source of identification, through the non-linear mapping of state variables to yields, that is absent in an affine setting.

To estimate the model, we assume that all yields are measured with error. Specifically, we assume:

$$y_t^n = \hat{y}_t^n + u_t^n, \tag{14}$$

where $\hat{y}_t^n$ is the model-implied yield in equation (12), y_t^n is the yield observed in data, and u_t^n IID $\sim N(0, \sigma_n^2)$, are additive measurement errors for all yields n. The quadratic form of the yields implies that there is not a one-to-one correspondence between certain yields assumed to be observed without error and latent state variables. Thus, standard filtering techniques for estimating affine models cannot be used to estimate our quadratic term structure model. We employ a Bayesian filtering algorithm that requires no approximation to estimate the model, which we detail in Appendix B.

In our empirical results, we compare our policy shift model with a standard affine model with macro and latent factors common in the macro-finance literature. This model contains the output gap and inflation without any policy variation (so $a_t = b_t = 0$), but adds an additional latent factor f_t in the short rate:

$$\hat{r}_t = \delta_0 + \bar{a} g_t + \bar{b} \pi_t + f_t. \tag{15}$$

We detail this model in Appendix C.

3 Empirical Results

In Section 3.1, we describe the construction of the output gap and inflation and how the model matches macro variables and yields in data. Section 3.2 discusses the parameter estimates. Section 3.3 documents how the Fed reaction to output gap and inflation shocks have changed over time. In Section 3.4, we discuss how Fed policy is priced in the term structure of interest rates.

3.1 Data

All our data is at a quarterly frequency and the sample period is from June 1952 to December 2006. The output gap is constructed following Rudebusch and Svensson (2002) and is given by

$$g_t = \frac{1}{4} \frac{Q_t - Q_t^*}{Q_t^*}, \tag{16}$$

where Q_t is real GDP and Q_t^* is potential GDP. We obtain real GDP from the Bureau of Economic Analysis (BEA), which is produced using chained 2000 dollars. We use the measure of potential output published by the Congressional Budget Office (CBO) in the Budget and Economic Outlook using chained 1996 dollars. To make the BEA series comparable to the CBO series, we translate real GDP to 1996 dollars. Finally, we demean the output gap and divide the output gap by four to correspond to quarterly units. Since we will be using per quarter short rates, this allows us to read the coefficient on the output gap as an annualized number. Our series for inflation is the year-on-year GDP deflator expressed as a continuously compounded growth rate. This is also divided by four to be in per quarter units. In addition to the one-quarter short rate, our term structure of interest rates comprises take zero-coupon bond yields from CRSP of maturities 4, 8, 12, 16, and 20 quarters. These are all expressed as continuously compounded yields per quarter.

Figure 1 plots the output gap, inflation, and the short rate over our sample in annualized terms. The output gap decreases during all the NBER recessions and reaches a low of -7.1% during the 1981:Q3 to 1983:Q4 recession. The output gap strongly trends upwards during the expansions of the 1960's, the mid-1980's, and the 1990's.

Inflation is slightly negatively correlated with the output gap at -24.5%. Inflation rises to near 10% during the mid-1970's and early 1980's, but otherwise remains below 5%. In the data, the correlation between the output gap and the short rate is -15.6% and the correlation between inflation and the short rate is 70.2%. These correlations are matched closely by the model, with implied correlations of g_t and π_t with the short rate of -0.119 and 0.723, respectively.

As a benchmark, we report OLS estimates of simple Taylor (1993) rules where the short rate is a linear combination of macro factors and lagged inflation:

$$\begin{array}{ccccccccc} r_t & = & \underset{(0.001)}{0.001} & + & \underset{(0.060)}{0.020}\, g_t & + & \underset{(0.064)}{0.904}\, \pi_t & + & \varepsilon_t, \end{array} \qquad (17)$$

where standard errors are reported in parentheses. Adding lagged short rates we obtain:

$$\begin{array}{ccccccccccc} r_t & = & \underset{(0.001)}{0.001} & + & \underset{(0.028)}{0.071}\, g_t & + & \underset{(0.041)}{0.141}\, \pi_t & + & \underset{(0.031)}{0.873}\, r_{t-1} & + & \varepsilon_t, \end{array} \qquad (18)$$

which can be written in partial adjustment format as:

$$r_t = 0.001 + 0.873\, r_{t-1} + (1 - 0.873)(0.560\, g_t + 1.105\, \pi_t) + \varepsilon_t.$$

These estimates are very similar to those reported in the literature.

In our model, the coefficients on g_t and π_t from these simple OLS estimates do not correspond to the policy coefficients by the monetary authority on the output gap and inflation. While the short rate equation (4) also specifies the short rate as a linear combination of g_t and π_t, the OLS shock ε_t is assumed to be orthogonal to the macro factors. In contrast, when a_t and b_t are time varying and correlated with g_t and π_t, $\eta_t = (a_t g_t + b_t \pi_t)$ is correlated with g_t and π_t. Thus, using OLS will result in biased estimates of the long-run responses $\bar{a}$ and $\bar{b}$. Nevertheless, we expect that the OLS residuals ε_t will be correlated with η_t and we will explore this connection in the context of estimating ε_t using the whole term structure of yields rather than just the short rate similar to Ang and Piazzesi (2003).

In Table 1, we report summary statistics of the factors in data and implied by the estimated model. The factors and yields are expressed in percentage terms at a quarterly frequency. The model provides an excellent match to the data, with model-implied unconditional means and standard deviations very close to the moments in data. In Panel A, the unconditional moments of the output gap and inflation implied by the model are well within 95% confidence bounds of the data estimates. The VAR over-estimates the unconditional volatility of inflation (0.941 compared to 0.554 in the data) because of the well-known persistence of inflation, which is over 0.98 in the sample and is closely matched at 0.99 in the model. Inflation persistence also causes the model-implied posterior standard deviations of the unconditional inflation mean and standard deviation to be large. However, since we estimate a_t and b_t conditional on the observed sample of g_t and π_t and the conditional volatilities of g_t and π_t are precisely estimated (see below), this is unlikely to have a large effect on the inference of the time-varying policy coefficients.

Panel B of Table 1 compares the yields in data with the model-implied yields. All yields are expressed in percentage terms per quarter. We construct the posterior moments of the model-implied yields by using the generated latent factors in each iteration from the Gibbs sampler estimation. The tight posterior standard deviations indicate that the draws of the latent a_t and b_t factors in the estimation result in yields that very closely lie around the data yields. All of the model-implied estimates are almost identical to the data. Note that because we match the short rate exactly in the estimation, the mean of the short rate aligns exactly with the data mean by construction.

3.2 Parameter Estimates

We report the estimates of the model parameters in Table 2. We report posterior means, with posterior standard deviations in parentheses, of the model parameters. Any parameters with no standard errors are not estimated. The first panel of Table 2 reports the long-run responses of the Fed to the output gap and inflation. Unconditionally, the long-run response to the output gap is small, at 0.142, and the long-run response to inflation is slightly above one, at 1.147. These are larger than the simple Taylor rule estimates of 0.020 and 0.904 in equation (17) suggesting that the time variation of a_t and b_t play some role in determining the short rate and OLS estimates contain small bias. The posterior standard deviations of a_t and b_t across all sample paths in the estimation are 0.027 and 0.693, respectively, so the policy coefficients vary over time in important ways which we detail below.

Not surprisingly, Table 2 shows that in the companion form Φ, all the factors are highly autocorrelated, with the coefficients of Φ lying very near one along the diagonal. High inflation Granger-causes low economic activity next quarter ($\Phi_{g\pi} = -0.038$), but this effect is statistically insignificant, and high economic growth suggests that next-period inflation will accelerate ($\Phi_{\pi g} = 0.065$). These effects have been noted before in standard VAR macro models like Christiano, Eichenbaum and Evans (1996, 1999).

The parameter $\Phi_{ga} = 0.014$ is positive indicating that changes in the Fed's response to output gap shocks induce a small, but insignificant, influence over the path of next-period future economic activity. Similarly, $\Phi_{b\pi}$ is estimated to be almost zero, so changes to the Fed's inflation policy also do not Granger-cause changes to inflation next period. However, the fact that Φ_{ag} and $\Phi_{b\pi}$ are near zero does not mean that changes in the Fed's responses to output gap and inflation shocks do not affect the full future path of economic activity or inflation, since the responses are unconditionally correlated with the macro factors. Below, we show that a_t and b_t shocks generate significant effects after one period for the impulse responses for g_t and π_t.

In contrast to the small Granger effects on the macro factors from a_t and b_t, there is economically stronger Granger-causality from macro shocks to policy changes. The coefficient $\Phi_{ag} = -0.261$ with a posterior standard deviation of 0.080. This indicates that the Fed's sensitivity to output gap shocks tends to be counter-cyclical. When g_t is high during expansions, a_t is small, so the short rate response to output gap shocks is small. When g_t is negative during recessions, a_t is large and the Fed moves to reduce the short rate more aggressively to bad output shocks than if the same shocks were experienced during expansions when g_t is high.

The positive coefficient $\Phi_{b\pi} = 1.193$ is economically large, but has a large posterior standard deviation of 2.820. The positive sign of $\Phi_{b\pi}$ indicates that the Fed becomes more sensitive to inflation shocks when inflation is high. Conversely, in low inflation environments, the Fed's response to inflation also decreases. This active response to fighting inflation plays a role in determining the risk premia of nominal long-term bonds. The endogenous reaction of b_t to the past path of π_t in our model is not captured by studies using exogenous random walk specifications to model drifting policy coefficients like Cogley (2005), Cogley and Sargent (2005), and Boivin (2006).

The policy coefficients on output and inflation are conditionally positively correlated at 0.567, with a posterior standard deviation of 0.066. This high correlation may reflect a strong active monetary policy. The Fed is much more likely to cut interest rates by larger amounts in environments of deflation combined with low economic activity than when the Fed reacts to the same inflation shock in a period of strong economic growth.

In the time-varying risk premia parameter matrix λ_1, many of the coefficients corresponding to the rows of a_t and b_t are significant. This indicates that the policy actions of the Fed are priced and influence long-term bond prices. There is also strong evidence that a_t and b_t affect the risk prices of the macro variables, particularly with a_t affecting the price of output gap risk and b_t affecting the price of inflation risk. Thus, output and inflation shocks are priced on their own, but the effect of how the Fed reacts to output gap and inflation shocks also affects the pricing of long-term bonds.[7]

3.3 Policy Shifts in Output and Inflation Responses

Figure 2 displays the policy parameters, $\bar{a}+a_t$ and $\bar{b}+b_t$, over the sample. We plot the mean posterior estimates at each point in time of the Fed's response to output and inflation produced by the Gibbs sampler, along with two posterior standard deviations. These estimates lend support to the conjecture that the changes in monetary policy during this period were substantial.

The Fed's response to output gap shocks is centered around $\bar{a} = 0.142$ and is generally above 0.15 through the 1950's and early 1960's. From the late 1960's and through the 1970's, the loading on the output gap decreases to a low of 0.060 reached in 1981. There are pronounced decreases in the output gap sensitivity during the 1974 and 1980 recessions. Given the low output gaps during these periods (see Figure 1), the drops in $\bar{a} + a_t$ imply a lower propensity of

[7] The zero entries in the λ_1 matrix result from the companion form Φ taking the form of equation (3) under both the risk neutral and the real measure. The risk prices are inferred from equation (8). See the Appendix for further details.

the Fed to lower the short rate in response to these bad output shocks during these times, without the additional consideration of inflation. However, inflation was high and the Fed had relatively high loadings on inflation during these times. The post-Volcker era has been characterized by an increase in the output gap loading, with the exception of the most recent 2001 recession, which saw a decrease in the output gap sensitivity.

One notable feature of the output gap response is that its range is relatively narrow, with a minimum of 0.060 and a maximum of 0.1913. In contrast, the response to inflation varies between -0.312, during the 1953 recession, to a high of 2.88 during the late 1990's. The second highest inflation policy response is 2.53, which occurs in 1984 during the Volcker disinflation.[8] Overall, the time-series pattern of the inflation coefficient is roughly consistent with the evidence reported in Clarida, Galì and Gertler (2000), Cogley and Sargent (2005) and Boivin (2006).

The response to inflation was high and above one in the mid-1960's, decreased in the late 1960's and was generally below one throughout the 1970's. An appealing feature of these estimates is that, consistent with the narrative evidence (see, for example, Meltzer, 2005), it clearly shows that the response to inflation started to increase in 1979. Interestingly, and as in Boivin (2006), the sharpest increase in the inflation response was not in late 1979, as is often assumed because of the appointment of Volcker in July 1979, but after 1981. Recently, the response to inflation dipped well below one during the 2001 recession and the aftermath of the September 2001 terrorism acts, where the short rate declined from 4.25% in 2001:Q1 to 0.90% in 2003:Q5. From this low towards the end of 2006, the Fed response to inflation shocks increases sharply.

Overall, the most striking feature of the evolution of monetary policy since 1952 is the change in the Fed's response to inflation. The estimated increase from the 1970's to the 1980's is sizeable. In particular this increase starts from a level less than one in the 1970's, which does not satisfy the Taylor principle. That means that during most of the 1970's a unit increase in inflation translated into a less than unit increase in the nominal policy rate, which represents a decline in the real rate, and hence implies an easing of monetary policy. Whenever the Taylor principle is not satisfied it is possible for inflation expectations, and thus economic fluctuations, to be driven by non-fundamental sunspot shocks. A failure to rule out the presense of such shocks could thus have been responsible for the greater economic volatlity of the 1970's (see

[8] The posterior standard deviations of the best estimates of a_t and b_t are 0.027 and 0.693, respectively, over the sample period. The autocorrelations of the best estimates of a_t and b_t are 0.972 and 0.942, respectively.

the discussion by Taylor, 1999; and Clarida, Galí and Gertler, 2000). The importance and the direction of these shifts are overall consistent with the view that the conduct of monetary policy was not stable during the 1970's and has evolved under Volcker toward a more stabilizing conduct. Moreover, the timing of these changes are broadly consistent with the general decline in the volatility of the US economy, suggesting that monetary policy could have been in part responsible for the Great Moderation (see comments by Stock and Watson, 2003).

It is an interesting question to see what the yield curve would have looked like had the Fed not changed its inflation loading over the post-2001 period. Inflation during this time was low, possibly even below an implicit target (see Figure 1), so interest rates may have declined over this period even with unchanged policy coefficients. In Figure 3 we report the results of a counter-factual experiment where we hold the Fed weight on inflation at the average weight of $\bar{b} + b_t$ over 2000 and trace the effects on the yields post-2001. We allow the other factors to be take their sample values. Figure 3 plots the path of the short rate and term spread if the Fed had maintained the same inflation stance as in 2000 in the dashed lines and overlays the actual short rate and term spread in the solid lines.

The top panel of Figure 3 shows that had the Fed maintained the same inflation stance in 2000, short rates would indeed have been considerably higher post-2001 than in data. With the same inflation tolerance in 2000, the short rate in 2003:Q4 would have been 4.81% compared to 0.90% in data. In the bottom panel of Figure 3 we plot the term spread. The yield curve would also have been much steeper post-2001 without the more dovish stance of the Fed.

3.3.1 Comparisons with Estimations Using No Term Structure Information

Our model uses the entire term structure to identify the time-series of output and inflation policy responses. We now demonstrate that this leads to sharper estimates of the Taylor rule coefficients than models that omit term structure information. Intuitively this is because no-arbitrage restrictions, through the bond prices in equation (11), link policy actions on the short rate with movements in long-term bonds. The omission of term structure information does not only increase, sometimes substantially, the estimation error of the policy responses – it also results in estimates of policy paths that are different from the full model which incorporates long-term bond information.

The top panel of Figure 4 plots the output gap coefficient from our full estimation and is the same figure as the top panel of Figure 2, except on a different scale. Clearly, the output gap coefficient does not vary much around $\bar{a} = 0.142$ and is precisely estimated. We compare these estimates with the output gap response from two other models in the two lower panels of Figure 4.[9] In the middle panel, we estimate the same VAR as equation (2) except only information on g_t, π_t, and r_t is used, that is there is no term structure information. In the last panel, we estimate a model close to the models in the literature where a_t and b_t follow a random walk. In this case, we change the companion form Φ in equation (3) to:

$$\Phi = \begin{pmatrix} \Phi_{gg} & \Phi_{g\pi} & 0 & 0 \\ \Phi_{\pi g} & \Phi_{\pi\pi} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The last two panels of Figure 4 show that the posterior standard deviation bands of the models that do not use term structure information are much wider than the full model. The unit root estimation error is particularly large and exhibits much larger time variation. When term structure information is omitted from our full model in the middle panel, the resulting path of a_t is negatively correlated, at -0.429, with the best estimates of a_t in the first panel. In particular, a_t increases during the 1979-1981, 1991, and the 2001 recessions in the middle panel, but decreases in our full estimation (see also Figure 2). The estimates of a_t in the second and last panels of Figure 4 are positively correlated, at 0.282. Thus, the inference of time-varying policy coefficients is very much affected by whether long-term bond information is used.

In Figure 5, we plot the inflation policy responses from the full model (top panel), the full model estimated without yield curve information (middle panel), and the unit root model (bottom panel). (The top panel repeats the same curve as the bottom graph of Figure 2 except on a different scale.) In contrast to the output gap responses, the time series of $\bar{b} + b_t$ from all three specification are highly correlated. The correlations of $\bar{b} + b_t$ in the middle and bottom panels with the benchmark estimation are 0.943 and 0.895, respectively. The correlation of the middle and bottom panels is a very high 0.977. Again, the posterior standard error bands are tightest for the full model estimated with term structure information and are widest for the unit root specification. However, all three specifications allow us to reject the hypothesis that monetary policy satisfied the Taylor principle in the 1970's, similar to what Clarida, Galí and Gertler (2000) find.

While the general comovement of the inflation response is similar across the three models,

[9] Technically, these models can be estimated using a methodology similar to the full model described in the Appendix, except that no accept/reject draw for the latent factors a_t and b_t is required. There are also no accept/reject draws needed for the term structure likelihood for the VAR or short rate parameters.

there are important differences. First, the general level of the Fed's inflation sensitivity is generally lower for the models estimated without term structure information. In the full model, the unconditional inflation response $\bar{b} = 1.147$, and omitting yield curve information from the model results in $\bar{b} = 0.847$. In the unit root model, $\bar{b} = 0.857$. Third, the range of the inflation responses is higher in the benchmark specification. The lowest (highest) inflation response is -0.312 (2.880) in the benchmark model compared to -1.086 (2.208) and -0.988 (2.082) in the bottom two panels. This implies that using term structure information, the estimate of the inflation response is more aggressive, on average, and that the Fed exhibits a more flexible, active response compared to estimates of inflation policy that ignore the yield curve.

3.3.2 Factor Impulse Responses

Figure 6 reports the impulse responses of the factors on each other using a Cholesky decomposition based on the ordering $(g_t \, \pi_t \, a_t \, b_t)$. We consider factor shocks of one unconditional standard deviation and report the response of the variable on the y-axis in terms of that variable's unconditional standard deviation. For example, in the upper left panel $g \rightarrow g$, the initial response of a unit unconditional standard deviation shock in g_t in terms of g_t's unconditional standard deviation is 1. In the panel $g \rightarrow \pi$, the effect of a one unconditional standard deviation shock in g_t on π_t reaches a peak of 40% of π's unconditional standard deviation after 15 quarters. Similarly, the y-axis for the panel $g \rightarrow a$ is in terms of a_t's unconditional standard deviation. The x-axis units are quarters.

The factor impulses show that positive output shocks lead the Fed to lower its sensitivity to output gap fluctuations. While the response of a_t to an unconditional standard deviation shock in g_t reaches -30% of a_t's unconditional standard deviation, the variation of a_t is very small. Positive output shocks have small, but positive, effects on increasing the Fed's inflation loading. In the second row of Figure 6, positive inflation shocks lead to higher Fed sensitivities to both output gap and inflation variation, but the effect is small for b_t.

In the bottom two rows, we report the effect of Fed policy shifts to output and inflation shocks. The autocorrelation of a_t and b_t are 0.978 and 0.950, respectively, so not surprisingly the panels $a \rightarrow a$ and $b \rightarrow b$ show that changes in policy take over 40 quarters to die out. As the loading on the output gap increases, future output and inflation tend to increase. Increasing a_t is contemporaneously correlated with an increase in b_t. In contrast, an increase in b_t tends to lower the response to a_t, but this response is very small.

When the Fed's inflation sensitivity increases, there is no initial response of g_t and π_t because Φ_{gb} is set to zero in the model and $\Phi_{\pi b}$ is estimated to be zero. However, there are effects on g_t and π_t on subsequent periods from a shock to b_t. Making the Fed more inflation averse tends to increase output only up to 2% of the unconditional standard deviation of g_t after 20 quarters. A stronger response is seen for future inflation. An unconditional standard deviation shock in b_t lowers inflation by 10% of inflation's unconditional standard deviation. Hence, in the model successful disinflation can occur by increasing the policy loading on inflation shocks.[10]

3.4 How Policy Shifts Affect the Yield Curve

3.4.1 Short Rate Components

If there were no policy shifts, the short rate would be linear in g_t and π_t and a linear Taylor rule would hold. From the stance of a standard affine model like equation (15), the linear latent factor should have a high correlation with our policy component $(a_t g_t + b_t \pi_t)$.[11] This is true empirically, with a latent affine model factor having a correlation of 0.658 with $(a_t g_t + b_t \pi_t)$. The advantage of our policy shift model is that it is able to decompose the total shock into macro and time-varying policy components.

Figure 7 separates the output gap and inflation components of the short rate, $(\bar{a} + a_t)g_t$ and $(\bar{b}+b_t)\pi_t$, respectively. The policy factors are evaluated at the best estimates of a_t and b_t through the sample, together with the short rate in per quarter units. The correlation between the actual short rate and the fitted components is 0.986, indicating that movements in the macro variables and policy rule account for almost all of the variation in the short rate. The bottom panel of Figure 7 shows that most of the variation in the short rate comes from inflation components. Although the variation in the output gap and inflation are similar (see Table 1), the small policy response on output shocks and the relatively large response on inflation cause the inflation component to dominate in the short rate variance.

3.4.2 Impulse Responses of the Term Structure

In Figures 8 and 9, we plot the response of the yield curve to macro shocks and policy shifts, respectively. Since the yields are non-linear functions of macro and policy variables, we compute

[10] In contrast, standard affine models such as Ang, Dong and Piazzesi (2006) tend to suffer from price puzzles (see Sims, 1992; Christiano, Eichenbaum and Evans, 1996), so positive shocks to the short rate are often associated with increases, not decreases, in future inflation.

[11] Appendix C describes an affine model corresponding to equation (15).

the impulse responses numerically, which we detail in Appendix D. We graph the response of a one unconditional standard deviation shock with yields expressed in annualized terms.

Figure 8 shows that positive output shocks increase short rates. A one unconditional standard deviation shock to g_t initially increases the short rate by 37 basis points, reaching a peak of 1.08% in 12 quarters. The same shock causes the long 20-quarter yield to also increase, initially to almost the same level of 38 basis points, reaching a peak of 76 basis points after 10 quarters. Thus, at first there is little response of the term spread, with the term spread narrowing slightly by -35 basis points after 14 quarters.

In Figure 8, we also plot the response of yields in the case of zero risk premia in dashed lines. These would be the response of yields under the Expectations Hypothesis (EH) and are produced by setting the time-varying prices of risk $\lambda_1 = 0$. Clearly, there are differences in the response of yields under risk neutrality compared to the case with risk premia. Under the EH, long-term yields would be more responsive to output shocks, with the yield spread increasing initially by 52 basis points, rather than remaining flat. Part of the reason behind this is that investors recognize the stabilizing effect of the Fed, which causes the yield curve not to amplify the effects of output shocks as much the case where policy risk is not priced.

The right-hand column of Figure 8 traces the response of the yield curve to an unconditional one standard deviation inflation shock. This causes the short rate to increase by 2.46% and its effect is extremely persistent as it is still approximately 1% after 60 quarters. The long rate does not respond to the inflation shock causing the spread to become negative. The response of the yield curve under the EH is very different. Under the EH, the long rate moves almost 1:1 with the short rate causing the spread to remain approximately constant.[12] Investors value the changing monetary policy response of the Fed, particularly towards inflation shocks. They know that when inflation is very high, the Fed moves to increase b_t and the policy shift is priced by investors so that the long-term yield does not respond.

In Figure 9, we trace the effect of output and inflation policy shifts on the term structure of interest rates. An unconditional one standard deviation shock to a_t increases the short rate by 1.90%. This effect is almost permanent so that after 60 quarters, the short rate is still higher by 1.06%. Simultaneously, this change in the output gap response causes the long yield to fall so the term spread decreases by almost -3%. The picture is strikingly different under risk neutrality where the Fed movements are not valued by investors. Under the EH, the long yield moves almost 1:1 with the short rate so the term spread is relatively unaffected.

[12] In an affine model the spread would not move under the EH, except for some small Jensen's inequality terms.

The opposite response of the long rate compared to the short rate causes the term spread to increase when a_t decreases. The impulse response of the yield curve to changes in a_t may point to changing monetary policy as one cause for the tendency of the term spread to be low prior to recessions (see, among others, Estrella and Mishkin, 1998). Figure 2 shows that the Fed has lowered a_t in all recessions, except for the 1982 recession, where it was already very low after the 1980 recession. If the response to a_t is taken on its own, this would cause the yield curve to be strongly upward sloping during economic recessions. During the peaks of economic expansions, large positive changes in a_t would tend to decrease the term spread.

The right column graphs of Figure 9 show that the response of the yield curve to the Fed's inflation loading is very different to the response of the yield curve to changes in the Fed's output gap sensitivity. A one unconditional standard deviation shock to b_t increases the short rate by 2.24%, which decreases to zero after 20 quarters. The response of the short rate is very similar, leading to a very flat response of the term spread to a change in b_t. Under risk neutrality, the long yield responds less than 1:1 compared to the short rate causing the term spread to initially decrease around 1%. These responses are very similar to how the yield curve responds to inflation shocks in Figure 8. Changes in inflation responses and inflation shocks impact the short end of the yield curve. These short-term changes in b_t are valuable and are priced by investors so that at long horizons, the risk from inflation components is small. Thus, long-term bond yields are fairly stable to inflation shocks.

3.4.3 The Fed Policy Risk Premium

The different response of the yield curve in Figures 8 and 9 under the case where all factor risk is priced compared to the case under the EH indicate that investors value the role of active monetary policy. We can estimate the value that investors assign to policy shifts by computing the risk premia of long-term bonds under the full model compared to a scenario where investors do not assign a risk premia to the movements of the a_t and b_t factors. This is done in Figure 10.

Figure 10 plots the expected excess holding period return on long-term bonds given by equation (13). We compute the risk premia using equation (13) evaluated at the unconditional mean implied by the estimates of the model. Under the case with full risk premia, the risk premium on the 4-quarter bond is 0.81% per annum and increases to 1.68% per annum for the 20-quarter bond. These risk premia are graphed with circles in Figure 10.

We compute bond risk premia in the case where investors do not value changing monetary policy stances by zeroing out the third and fourth rows and columns of the λ_1 matrix in equation

(8), but leaving the upper 2×2 risk prices for g_t and π_t intact. This exercise keeps risk premia of only macro factors, but any risk premia of macro factors explicitly tied to monetary policy is set to zero. The expected excess holding period returns on long-term bonds when a_t and b_t have zero prices of risk are graphed with crosses in Figure 10.

When investors do not value monetary policy changes, bond risk premia increase to 1.59% and 3.54% per annum for the 4-quarter and 20-quarter bond, respectively, compared to 0.81% and 1.68% per annum when all variables have non-zero prices of risk. Thus, the value priced by the market of active monetary policy is a risk discount of $0.0354 - 0.0168 = 1.86\%$ per annum. Thus, if investors assigned zero value to monetary policy changes, investors would be demanding an extra 1.86% per annum to hold long-term bonds. We can also state the value of activist monetary policy in terms of conditional Sharpe ratios, evaluated at the unconditional mean of variables implied by the estimation. When all risk is priced, the annualized Sharpe ratio of the 20-quarter bond is 0.287 compared to a Sharpe ratio of 0.524 when changes in policy responses a_t and b_t are not valued. This Sharpe ratio increase of approximately 1.8 times is approximately the same magnitude for all maturities. In summary, bond risk premia would be significantly higher in an economy where agents did not price monetary policy shifts. In particular, part of the reason why risk premia on long-term bonds are low is because investors assign significant value to activist monetary policy.

4 Conclusion

Existing results suggest that monetary policy has changed in substantial ways over the last 50 years. While the implications of these changes for the business cycles dynamics has received considerable attention, little is known about the implications of these changes for financial markets, in particular for the term structure of interest rates.

In this paper we propose a quadratic term structure model where the coefficients of the short rate equation – which describe the behavior of monetary policy – can change over time. By exploiting term structure information, we are able to obtain sharper estimates of the changes in monetary policy compared to previous studies that do not use term structure information. Contrary to what is typically assumed, our empirical model does not impose that the shifts in monetary policy are exogenous. A particularly appealing feature of our framework is that it provides an estimate of the price of risk that financial market participants attribute to monetary policy shifts.

Our empirical results show that monetary policy has changed in important ways and the shifts we estimate line up largely with narrative accounts of monetary policy and with some existing empirical estimates. However, we find that monetary policy shifts in inflation loadings with term structure information show that monetary policy is more aggressive and flexible, on average, than estimations without long-term bonds. In contrast, policy shifts in output gap loadings exhibit little time series variation when estimated with yield curve information.

A central contribution of the paper is to show that monetary policy shifts are priced by investors. We find that market participants attribute an important value to activist monetary policy. In particular, if investors assigned no value to monetary policy shifts, then the risk premium on holding long-term bonds would be 1.9% per annum higher. This valuable contribution of the of monetary policy shifts arises through the stabilizing influence of monetary policy on macroeconomic fluctuations and, in particular, the endogenous response of monetary policy to past economic changes.

Appendix

A Bond Pricing

The price of a one-period zero-coupon bond is given by:

$$\begin{aligned}\hat{P}_t^1 &= \exp(-\hat{r}_t) = \exp(-\delta_0 - \delta_1^\top X_t - X_t^\top \Omega X_t) \\ &= \exp(A_1 + B_1^\top X_t + X_t^\top C_1 X_t), \end{aligned} \tag{A-1}$$

where $A_1 = -\delta_0$, $B_1 = -\delta_1 = -[\bar{a}\ \bar{b}\ 0\ 0]^\top$, and $C_1 = -\Omega$, with Ω given in equation (6).

Under measure $\mathbb{Q}$, the price of a n-period zero-coupon bond, $\hat{P}_t^n$, is:

$$\begin{aligned}\hat{P}_t^n &= \mathrm{E}_t^{\mathbb{Q}}(\exp(-\hat{r}_t) P_{t+1}^{n-1}) \\ &= \mathrm{E}_t^{\mathbb{Q}}(\exp\left(-\hat{r}_t + A_{n-1} + B_{n-1}^\top X_{t+1} + X_{t+1}^\top C_{n-1} X_{t+1}\right)) \\ &= \exp\left(-\hat{r}_t + A_{n-1} + B_{n-1}^\top(\mu^Q + \Phi^Q X_t) + (\mu^Q + \Phi^Q X_t)^\top C_{n-1}(\mu^Q + \Phi^Q X_t)\right) \\ &\quad \times \mathrm{E}_t^{\mathbb{Q}}(\exp\left((B_{n-1}^\top \Sigma + 2(\mu^Q + \Phi^Q X_t)^\top C_{n-1}\Sigma)\varepsilon_{t+1} + \varepsilon_{t+1}^\top \Sigma^\top C_{n-1}\Sigma\varepsilon_{t+1}\right)) .\end{aligned} \tag{A-2}$$

To take the expectation, note that the expectation of the exponential of a quadratic Gaussian variable is given by:

$$\mathrm{E}[\exp(A^\top \epsilon + \epsilon^\top \Gamma \epsilon)] = \exp\left(-\frac{1}{2}\ln\det\left(I - 2\Psi\Gamma\right) + \frac{1}{2}A^\top(\Psi^{-1} - 2\Gamma)^{-1}A\right)$$

for $\epsilon \sim N(0, \Psi)$. This can be derived by general properties of Gaussian quadratic forms (see Mathai and Provost, 1992; Searle, 1997).

After taking the expectation and equating the terms with

$$\hat{P}_t^n = \exp(A_n + B_n^\top X_t + X_t^\top C_n X_t),$$

the coefficients A_n, B_n, and C_n are given by the recursions:

$$\begin{aligned}A_n &= -\delta_0 + A_{n-1} + B_{n-1}^\top \mu^Q + \mu^{Q^\top} C_{n-1}\mu^Q - \frac{1}{2}\ln\det(I - 2\Sigma^\top C_{n-1}\Sigma) \\ &\quad + \frac{1}{2}(\Sigma^\top B_{n-1} + 2\Sigma^\top C_{n-1}\mu^Q)^\top (I - 2\Sigma^\top C_{n-1}\Sigma)^{-1}(\Sigma^\top B_{n-1} + 2\Sigma^\top C_{n-1}\mu^Q) \\ B_n^\top &= -\delta_1^\top + B_{n-1}^\top \Phi^Q + 2\mu^{Q^\top} C_{n-1}\Phi^Q + 2(\Sigma^\top B_{n-1} + 2\Sigma^\top C_{n-1}\mu^Q)^\top (I - 2\Sigma^\top C_{n-1}\Sigma)^{-1}\Sigma^\top C_{n-1}\Phi^Q \\ C_n &= -\Omega + \Phi^{Q^\top} C_{n-1}\Phi^Q + 2(\Sigma^\top C_{n-1}\Phi^Q)^\top (I - 2\Sigma^\top C_{n-1}\Sigma)^{-1}(\Sigma^\top C_{n-1}\Phi^Q)\end{aligned} \tag{A-3}$$

If the model were specified in continuous time, then the recursions in equation (A-3) are versions of the ordinary differential equations derived by Ahn, Dittmar and Gallant (2002) on the bond pricing coefficients.

To compute conditional excess holding period returns, we use the exponential quadratic form for zero-coupon bond prices in equation (11) to write:

$$\begin{aligned}xhpr_{t+1}^n &= \log\frac{\hat{P}_{t+1}^{n-1}}{\hat{P}_t^n} - r_t \\ &= A_{n-1} + B_{n-1}^\top X_{t+1} + X_{t+1}^\top C_{n-1} X_{t+1} - (A_n + B_n^\top X_t + X_t^\top C_n X_t) \\ &\quad -(A_1 + B_1^\top X_t + X_t^\top C_1 X_t).\end{aligned} \tag{A-4}$$

Since $X_{t+1} \sim N(\mu + \Phi X_t, \Sigma\Sigma^\top)$, we can write the expectation of a quadratic form, $\mathrm{E}_t(X_{t+1}^\top C X_{t+1})$, as:

$$\mathrm{E}_t(X_{t+1}^\top C X_{t+1}) = tr(C\Sigma\Sigma^\top) + (\mu + \Phi X_t)^\top C(\mu + \Phi X_t).$$

This allows us to compute the expectation as:

$$\mathrm{E}_t[xhpr^n_{t+1}] = \bar{A}_n + \bar{B}_n^\top X_t + X_t^\top \bar{C}_n X_t, \tag{A-5}$$

where

$$\begin{aligned}\bar{A}_n &= A_{n-1} - A_n - A_1 + tr(C_{n-1}\Sigma\Sigma^\top) + \mu^\top C_{n-1}\mu + B_{n-1}^\top \mu \\ \bar{B}_n &= \Phi^\top B_{n-1} - B_n - B_1 + 2\Phi^\top C_{n-1}\mu \\ \bar{C}_n &= \Phi^\top C_{n-1}\Phi - C_n - C_1.\end{aligned} \tag{A-6}$$

B Estimating the Model

The model is estimated using a Bayesian Gibbs sampling algorithm. While there are several examples of these types of estimations for affine models (see, among others, Lamoureux and Witte, 2002; Johannes and Polson, 2005; Ang, Dong and Piazzesi, 2006; Dong, 2006), these cannot be directly employed to estimate the quadratic model because in an affine setting, drawing the latent factors requires a Kalman filter. The Kalman filter assumes that yields are linear functions of state variables, whereas they are non-linear functions in the quadratic model. In this appendix, we develop an acceptance-rejection algorithm to draw the latent factors without approximation.

For ease of notation, we group the macro variables as $M_t = [g_t \, \pi_t]^\top$ and the latent factors as $L_t = [a_t \, b_t]^\top$ and rewrite the dynamics of $X_t = \left[M_t^\top \, L_t^\top\right]^\top$ in equation (2) as:

$$\begin{pmatrix} M_t \\ L_t \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \begin{pmatrix} M_{t-1} \\ L_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{M,t} \\ \varepsilon_{L,t} \end{pmatrix}, \tag{B-1}$$

where $\varepsilon_t = (\varepsilon_{M,t}^\top \, \varepsilon_{L,t}^\top)^\top \sim$ IID $N(0, \Sigma\Sigma^\top)$. We partition $\Sigma\Sigma^\top$ as:

$$\Sigma\Sigma^\top = \begin{pmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{22} \end{pmatrix}.$$

The parameters of the model are $\Theta = (\mu,\, \Phi,\, \Sigma,\, \delta_0,\, \delta_1,\, \Omega,\, \mu^Q,\, \Phi^Q,\, \sigma_u)$, where μ^Q and Φ^Q are parameters governing the state variable process under the risk neutral probability measure, and σ_u denotes the vector of observation error volatilities $\{\sigma_n\}$. We draw μ^Q and Φ^Q, but invert the prices of risk λ_0 and λ_1 using the relations:

$$\begin{aligned}\lambda_0 &= \Sigma^{-1}(\mu - \mu^Q) \\ \lambda_1 &= \Sigma^{-1}(\Phi - \Phi^Q).\end{aligned} \tag{B-2}$$

The latent factors $L_t = \{a_t \, b_t\}$ are generated in each iteration of the Gibbs sampler. Note that Ω is not a parameter, but is fixed from equation (6). We also do not draw δ_0, but set δ_0 in each iteration to match the sample mean of the short rate.

We now detail the procedure for drawing each of these variables. We denote the factors $X = \{X_t\}$ and the set of yields for all maturities in data as $Y = \{y_t^n\}$. Note that the model-implied yields $\hat{Y} = \{\hat{y}_t^n\}$ differ from the yields in data, Y, by observation error. By definition, $Y = \hat{Y} + u$, where $u = \{u_t^n\}$ is the set of all observation errors for all yields. This notation also implies that the short rate in data, r_t, is the same as y_t^1.

B.1 Drawing the Latent Factors

We use a single-move algorithm based on Jacquier, Polson and Rossi (1994, 2004) adapted to our model. We derive a draw from the distribution $P(L_t|Y, L_{-t}, M)$, where L_t is the t-th observation of the latent factors, L_{-t} denotes all the latent factors except the t-th observation, and Y and M are the complete time-series of yields and macro variables, respectively. We use the notation Y_t and M_t to denote the t-th observation of the set of yields and macro variables.

From the Markov structure of the model, we can write:

$$P(L_t|Y, L_{-t}, M) \propto P(L_t|L_{t-1}, M_{t-1})P(Y_t|L_t, M_t)P(L_{t+1}, M_{t+1}|L_t, M_t). \tag{B-3}$$

Each conditional distribution of the RHS of equation (B-3) is known. From equation (B-1), we have

$$\begin{aligned} P(L_t|L_{t-1}, M_{t-1}) &\propto \exp\left(-\frac{1}{2}(L_t - \mu_L)^\top (\Sigma_{22}\Sigma_{22}^\top)^{-1}(L_t - \mu_L)\right) \\ &\propto \exp\left(-\frac{1}{2}\left[L_t^\top (\Sigma_{22}\Sigma_{22}^\top)^{-1} L_t - 2\mu_L (\Sigma_{22}\Sigma_{22}^\top)^{-1} L_t\right]\right) \end{aligned} \tag{B-4}$$

where

$$\mu_L = \mu_2 + \Phi_{21} M_{t-1} + \Phi_{22} L_{t-1}.$$

Similarly, from the VAR in equation (B-1), we can write:

$$\begin{aligned} P(L_{t+1}, M_{t+1}|L_t, M_t) &\propto \exp\left(-\frac{1}{2}\left[(\hat{L}_{t+1} - \Phi_{22}L_t)^\top (\Sigma_{22}\Sigma_{22}^\top)^{-1}(\hat{L}_{t+1} - \Phi_{22}L_t)\right]\right) \\ &\quad \times \exp\left(-\frac{1}{2}\left[(\hat{M}_{t+1} - \Phi_{12}L_t)^\top (\Sigma_{11}\Sigma_{11}^\top)^{-1}(\hat{M}_{t+1} - \Phi_{12}L_t)\right]\right) \\ &\propto \exp\left(-\frac{1}{2}\left[L_t^\top \Phi_{22}^\top (\Sigma_{22}\Sigma_{22}^\top)^{-1}\Phi_{22}L_t - 2\hat{L}_{t+1}(\Sigma_{22}\Sigma_{22}^\top)^{-1}\Phi_{22}L_t \right.\right. \\ &\quad \left.\left. + L_t^\top \Phi_{12}^\top (\Sigma_{11}\Sigma_{11}^\top)^{-1}\Phi_{12}L_t - 2\hat{M}_{t+1}(\Sigma_{11}\Sigma_{11}^\top)^{-1}\Phi_{12}L_t\right]\right) \end{aligned} \tag{B-5}$$

where

$$\begin{aligned} \hat{L}_{t+1} &= L_{t+1} - \mu_2 - \Phi_{21}M_t \\ \hat{M}_{t+1} &= M_{t+1} - \mu_1 - \Phi_{11}M_t. \end{aligned}$$

We can factor the likelihood of bond yields, $P(Y_t|L_t, M_t)$, into:

$$P(y_t|L_t, M_t) = P(r_t|L_t, M_t) \prod_{n \neq 1} P(y_t^n|L_t, M_t), \tag{B-6}$$

where there are N yields used in the estimation. The likelihood of the short rate is:

$$\begin{aligned} P(r_t|L_t, M_t) &\propto \exp\left(-\frac{1}{2}\frac{(\hat{r}_t^* - \Gamma_t L_t)^2}{\sigma_1^2}\right) \\ &\propto \exp\left(-\frac{1}{2}\left[L_t^\top \Gamma_t^\top \sigma_1^{-2}\Gamma_t L_t - 2\hat{r}_t^* \sigma_1^{-2}\Gamma_t L_t\right]\right) \end{aligned} \tag{B-7}$$

where σ_1^2 is the observation error variance of the short rate, $\hat{r}_t^* = r_t - \delta_0 - [\bar{a}\ \bar{b}]^\top M_t$, and $\Gamma_t = M_t^\top$. The likelihood of the other yields take the form

$$P(y_t^n|X_t = [L_t^\top\ M_t^\top]^\top) \propto \exp\left(-\frac{1}{2}\left[\frac{(y_t^n - (a_n + b_n^\top X_t + X_t^\top c_n X_t))^2}{\sigma_n^2}\right]\right), \tag{B-8}$$

where the model-implied bond yield, $\hat{y}_t^n = a_n + b_n^\top X_t + X_t^\top c_n X_t$ is given in equation (12), and σ_n^2 is the observation error variance of the yield of maturity n.

With equations (B-4), (B-5), and (B-7), we can complete the square to obtain:

$$P(L_t|Y, L_{-t}, M) \propto \exp\left(-\frac{1}{2}(L_t - \mu_t^*)^\top (\Sigma_t^*)^{-1}(L_t - \mu_t^*)\right) \prod_{n=2}^{N} P(y_t^n|L_t, M_t) \tag{B-9}$$

where

$$\begin{aligned} (\Sigma_t^*)^{-1} &= \left((\Sigma_{22}\Sigma_{22}^\top)^{-1} + \Phi_{22}^\top (\Sigma_{22}\Sigma_{22}^\top)^{-1}\Phi_{22} + \Phi_{12}^\top (\Sigma_{11}\Sigma_{11}^\top)^{-1}\Phi_{12} + \Gamma_t^\top \sigma_1^{-2}\Gamma_t\right)^{-1} \\ \mu_t^* &= \Sigma_t^*(\mu_L(\Sigma_{22}\Sigma_{22}^\top)^{-1} + \hat{L}_{t+1}(\Sigma_{22}\Sigma_{22}^\top)^{-1}\Phi_{22} + M_{t+1}(\Sigma_{11}\Sigma_{11}^\top)^{-1}\Phi_{12} + \hat{r}_t\sigma_1^{-2}\Gamma_t)^\top. \end{aligned}$$

Since this distribution is not recognizable, we use a Metropolis draw. We draw a proposal from the distribution $N(\mu_t^*, \Sigma_t^*)$ and then the acceptance probability is based on the likelihood of $\prod_{n=2}^{N} P(y_t^n|L_t, M_t)$. Since we specify the mean of f to be zero for identification, we set each generated draw of f to have a mean of zero.

To generate initial values for the very first draw, we use the Carter and Kohn (1994) forward-backward algorithm to first run a Kalman filter forward taking M_t to be exogenous variables and then sample L_t backwards. The Kalman filter is constructed linearizing the yields at $L_{t|t-1}$. Note that this Kalman filter is only used to produce initial values for the draw; the steady-state distribution of L_t relies on the single-step accept/reject algorithm given above.

B.2 Drawing μ and Φ

We follow Johannes and Polson (2005) and explicitly differentiate between $\{\mu, \Phi\}$ under the real measure and $\{\mu^Q, \Phi^Q\}$ under the risk-neutral measure. As X_t follows a VAR in equation (2), we follow standard Gibbs sampling and use conjugate normal priors and posteriors for the draw of μ and Φ. We note that the posterior of μ and Φ conditional on X, Y and the other parameters is:

$$\begin{aligned} P(\mu, \Phi|\Theta_-, X, Y) &\propto P(Y|\Theta, X)P(X|\mu, \Phi, \Sigma)P(\mu, \Phi) && \text{(B-10)} \\ &\propto P(Y|\Sigma, \delta_0, \delta_1, \mu^Q, \Phi^Q, \sigma_\eta, X)P(X|\mu, \Phi, \Sigma)P(\mu, \Phi) \\ &\propto P(X|\mu, \Phi, \Sigma)P(\mu, \Phi), && \text{(B-11)} \end{aligned}$$

where Θ_- denotes the set of all parameters except μ and Φ, and $P(X|\mu, \Phi, \Sigma)$ is the likelihood function of the VAR, which is normally distributed from the assumption of normality for the errors in the VAR. The validity of going from the first line to the second line is ensured by the bond recursion in equation (A-3): given μ^Q and Φ^Q, the bond price is independent of μ and Φ. We specify the prior $P(\mu, \Phi)$ to be $N(0, 1000)$, which effectively represents an uninformative prior. We draw μ and Φ separately for each equation in the VAR system (2).

B.3 Drawing $\Sigma\Sigma^\top$

To draw $\Sigma\Sigma^\top$, we note that the posterior of $\Sigma\Sigma^\top$ conditional on X, Y and the other parameters is:

$$P(\Sigma\Sigma^\top|\Theta_-, X, Y) \propto P(Y|\Theta, X)P(X|\mu, \Phi, \Sigma)P(\Sigma\Sigma^\top), \tag{B-12}$$

where Θ_- denotes the set of all parameters except Σ. This posterior suggests an Independence Metropolis draw. We draw $\Sigma\Sigma^\top$ from the proposal density

$$q(\Sigma\Sigma^\top) = P(X \mid \mu, \Phi, \Sigma)P(\Sigma\Sigma^\top),$$

which is an Inverse Wishart (IW) distribution if we specify the prior $P(\Sigma\Sigma^\top)$ to be IW, so that $q(\Sigma\Sigma^\top)$ is an IW natural conjugate. The proposal draw $(\Sigma\Sigma^\top)^{m+1}$ for the $(m+1)$th draw is then accepted with probability α, where

$$\begin{aligned} \alpha &= \min\left\{ \frac{P((\Sigma\Sigma^\top)^{m+1} \mid \Theta_-, X, Y)}{P((\Sigma\Sigma^\top)^{m} \mid \Theta_-, X, Y)} \frac{q((\Sigma\Sigma^\top)^{m})}{q((\Sigma\Sigma^\top)^{m+1})}, 1 \right\} \\ &= \min\left\{ \frac{P(Y \mid (\Sigma\Sigma^\top)^{m+1}, \Theta_-, X)}{P(Y \mid (\Sigma\Sigma^\top)^{m}, \Theta_-, X)}, 1 \right\}, && \text{(B-13)} \end{aligned}$$

where $P(Y|\mu, \Phi, \Theta_-, X)$ is the likelihood function of all yields, including the short rate, which is normally distributed from the assumption of normality for the observation errors. From equation (B-13), α is just the ratio of the likelihoods of the new draw of $\Sigma\Sigma^\top$ relative to the old draw.

B.4 Drawing $\bar{a}$ and $\bar{b}$

We draw $\bar{a}$ and $\bar{b}$ jointly with a Random Walk Metropolis algorithm. We assume a flat prior. The accept/reject probability for the draws of $\bar{a}$ and $\bar{b}$ is the ratio of the likelihood of bond yields based on candidate and last draw of $\bar{a}$ and $\bar{b}$:

$$\begin{aligned} \alpha &= \min\left\{ \frac{P((\bar{a},\bar{b})^{m+1} \mid \Theta_-, X, Y)}{P((\bar{a},\bar{b})^{m} \mid \Theta_-, X, Y)} \frac{q((\bar{a},\bar{b})^{m})}{q((\bar{a},\bar{b})^{m+1})}, 1 \right\} \\ &= \min\left\{ \frac{P(Y \mid (\bar{a},\bar{b})^{m+1}, \Theta_-, X)}{P(Y \mid (\bar{a},\bar{b})^{m}, \Theta_-, X)}, 1 \right\}. \end{aligned} \tag{B-14}$$

B.5 Drawing μ^Q and Φ^Q

We draw μ^Q and Φ^Q with a Random Walk Metropolis algorithm assuming a flat prior. We draw each parameter separately in μ^Q, and each row in Φ^Q. The accept/reject probability for the draws of μ^Q and Φ^Q is the ratio of the likelihood of bond yields based on candidate and last draw of μ^Q and Φ^Q:

$$\begin{aligned} \alpha &= \min\left\{ \frac{P((\mu^Q,\Phi^Q)^{m+1} \mid \Theta_-, X, Y)}{P((\mu^Q,\Phi^Q)^{m} \mid \Theta_-, X, Y)} \frac{q((\mu^Q,\Phi^Q)^{m})}{q((\mu^Q,\Phi^Q)^{m+1})}, 1 \right\} \\ &= \min\left\{ \frac{P(Y \mid (\mu^Q,\Phi^Q)^{m+1}, \Theta_-, X)}{P(Y \mid (\mu^Q,\Phi^Q)^{m}, \Theta_-, X)}, 1 \right\}, \end{aligned} \tag{B-15}$$

In each iteration, we invert λ_0 and λ_1 and report the estimates of the prices of risk instead of μ^Q and Φ^Q.

B.6 Drawing σ_u

Drawing the variance of the observation errors, σ_u^2, is straightforward, because we can view the observation errors η as regression residuals from equation (14). We draw the observation variance $(\sigma_\eta^n)^2$ separately from each yield. We specify a conjugate prior $IG(0, 0.00001)$, so that the posterior distribution of σ_η^2 is a natural conjugate Inverse Gamma distribution. The prior information roughly translates into a 30bp bid-ask spread in Treasury securities, which is consistent with studies on the liquidity of spot Treasury market yields (see, for example, Fleming, 2000).

C An Affine Model

The state variables are given by $X_t = [g_t \ \pi_t \ f_t]^\top$, where g_t and π_t are the same output gap and inflation used in the benchmark model, and f_t is a latent factor. The variables follow the VAR:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \tag{C-1}$$

with no restrictions on Φ or Σ. All factors linearly enter the short rate:

$$r_t = \delta_0 + \delta_1^\top X_t. \tag{C-2}$$

For identification, the parameter corresponding to f_t in δ_1 is set to 1, and the mean of the short rate is matched in each iteration of the Gibbs sampler, so δ_0 is an inferred parameter. The pricing kernel takes the form of equation (7). This is the model estimated by Ang, Dong and Piazzesi (2006). Under this model, bond yields take the form of equation (11), which are the same recursions derived by Ang and Piazzesi (2003), with $\Omega = C_n = 0$. However, this model is not nested by our quadratic specification because our quadratic model does not employ latent factors that enter the short rate equation linearly. Parameter estimates for the affine model are available upon request.

D Impulse Responses

Since the yields are non-linear, we follow Gallant, Rossi and Tauchen (1993) and Potter (2000), among others, and compute the impulse response functions using simulation. We start with the sample series of data (g_t and π_t) and the posterior means of the latent factors (a_t and b_t) at each observation t. We term these points X_t^*. From the VAR in equation (2), we construct an orthogonalized error term ν_t by taking the Cholesky of $\Sigma\Sigma^\top$. To construct the impulse response for the jth variable of X_t, we first draw a shock v_t that represents a shock only to variable j from the error term distribution ν_t. From the points X_t^*, we construct a new series where each observation has been shocked by v_t, which we denote as $X_t^v = X_t^* + v_t$.

The impulse response functions are taken as the difference between the averaged response of the yields to the evolution of X_t^* without shocks to the evolution of the shocked X_t^v series:

$$\mathrm{E}(y_{t+k}^n | X_t^v) - \mathrm{E}(y_{t+k}^n | X_t^*).$$

Using the VAR in equation (2), we simulate out the value of X_{t+k}^v from X_t^v and the value of X_{t+k}^* from X_t^*. This is done at each observation t. Then, we construct the yields, y_{t+k}^n, from equation (12) corresponding to the state vectors X_{t+k}^v and X_{t+k}^*. We take values of $k = 1 \ldots 60$ quarters.

The impulse responses are computed at each observation by taking the average of the sample paths of y_{t+k}^n computed using X_{t+k}^v minus the average of the sample paths of y_{t+k}^n computed using X_{t+k}^*. We report the average of the impulse responses across all observations t. This procedure results in impulse responses that are identical to impulse responses computed for traditional VAR systems for large numbers of observations.

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Table 1: Summary Statistics

Panel A: Moments of Macro Factors

	Means (%)		Standard Deviations (%)		Autocorrelations	
	Data	Model	Data	Model	Data	Model
g	0.000	-0.015	0.589	0.568	0.930	0.917
	(0.082)	(0.193)	(0.057)	(0.146)	(0.034)	(0.024)
π	0.856	0.839	0.554	0.941	0.982	0.990
	(0.082)	(1.903)	(0.066)	(0.648)	(0.026)	(0.007)

Panel B: Moments of Yields

	$n = 1$	$n = 4$	$n = 8$	$n = 12$	$n = 16$	$n = 20$
Means (%)						
Data	1.275	1.363	1.411	1.452	1.482	1.501
	(0.103)	(0.102)	(0.101)	(0.098)	(0.098)	(0.103)
Model	1.275	1.344	1.411	1.454	1.481	1.501
	–	(0.002)	(0.002)	(0.001)	(0.001)	(0.002)
Standard Deviations (%)						
Data	0.710	0.701	0.691	0.674	0.667	0.657
	(0.087)	(0.077)	(0.076)	(0.074)	(0.074)	(0.075)
Model	0.738	0.698	0.685	0.677	0.667	0.654
	(0.012)	(0.005)	(0.002)	(0.001)	(0.001)	(0.002)
Autocorrelations						
Data	0.936	0.944	0.952	0.958	0.961	0.964
	(0.030)	(0.029)	(0.027)	(0.026)	(0.026)	(0.024)
Model	0.936	0.950	0.960	0.964	0.967	0.968
	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)

The table lists various moments of the factors in data and implied by the model. All the factors and yields are expressed at a quarterly frequency in percentage terms. All standard errors are reported in parentheses. Panel A lists moments of the output gap and inflation. For the model, we construct the posterior distribution of unconditional moments by computing the unconditional moments implied from the parameters in each iteration of the Gibbs sampler. Panel B reports data and model unconditional moments of n-quarter maturity yields. We compute the posterior distribution of the model-implied yields using the generated latent factors in each iteration of the Gibbs sampler. In Panels A and B, the data standard errors are computed using GMM. The sample period is June 1952 to December 2004.

Table 2: Parameter Estimates

Short Rate Parameters

δ_0	$\bar{a}$	$\bar{b}$
0.003	0.142	1.147
(0.001)	(0.007)	(0.093)

VAR Parameters

		Companion Form Φ				Volatility $\times$1000/Correlation Matrix			
	$\mu \times 1000$	g	π	a	b	g	π	a	b
g	0.296	0.885	-0.038	0.014	0	0.004	0.027	0	0
	(0.478)	(0.032)	(0.054)	(0.013)	–	(0.000)	(0.069)	–	–
π	0.105	0.065	0.989	0	-0.000	0.027	0.001	0	0
	(0.088)	(0.011)	(0.007)	–	(0.000)	(0.069)	(0.000)	–	–
a	-0.042	-0.261	0	0.987	0	0	0	0.039	0.567
	(0.428)	(0.080)	–	(0.010)	–	–	–	(0.005)	(0.066)
b	-6.040	0	1.193	0	0.940	0	0	0.567	55.68
	(28.91)	–	(2.820)	–	(0.023)	–	–	(0.066)	(7.552)

Risk Premia Parameters

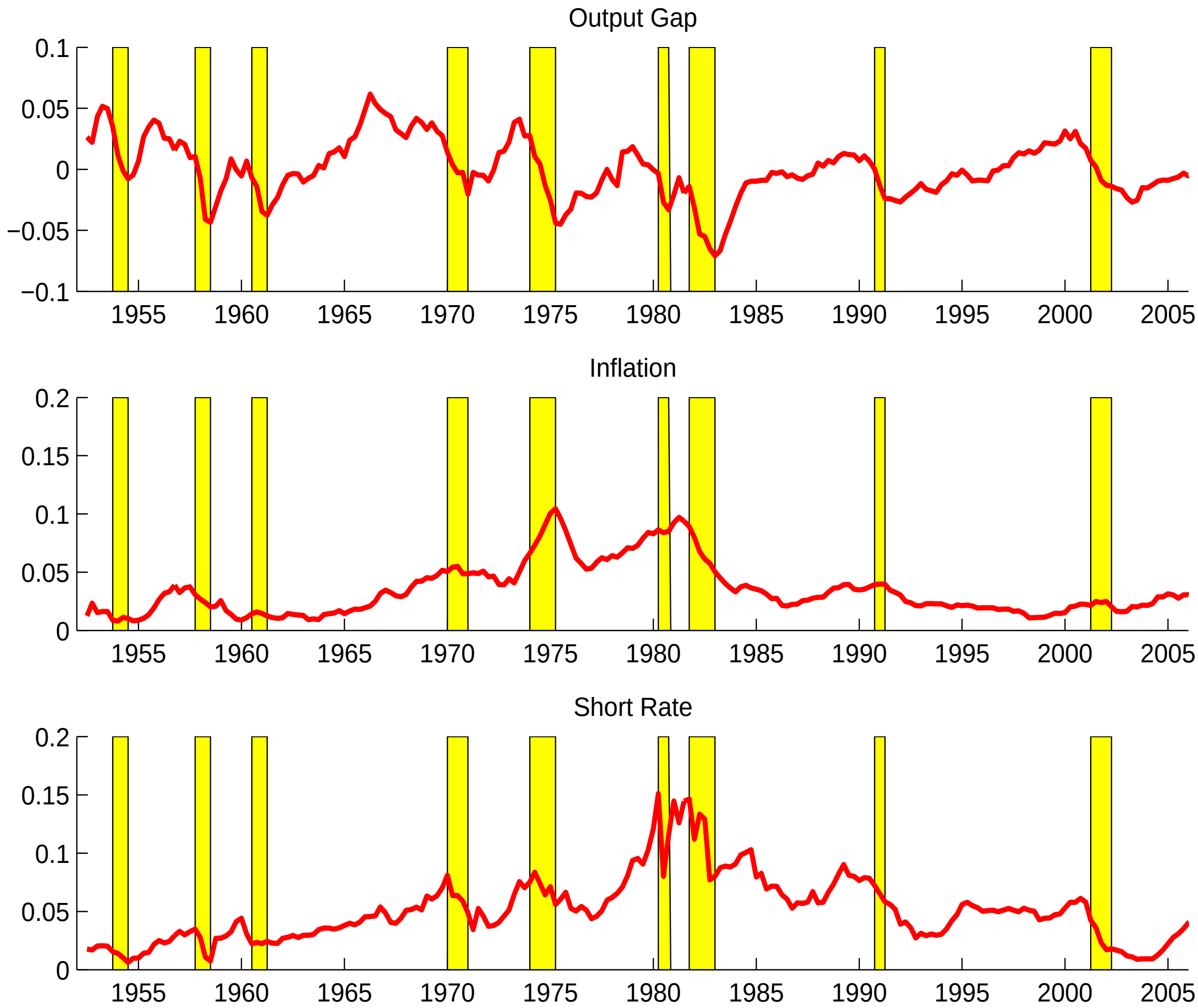
		λ_1			
	λ_0	g	π	a	b
g	-3.182	-45.71	195.1	216.0	0
	(0.517)	(18.46)	(42.42)	(19.56)	–
π	-2.318	57.56	168.8	-5.819	0.243
	(0.275)	(13.06)	(18.13)	(14.92)	(0.108)
a	0.203	-35.52	0	37.38	0
	(0.091)	(13.24)	–	(4.182)	–
b	0.170	24.95	6.467	-26.11	0.006
	(0.176)	(10.43)	(14.63)	(5.284)	(0.121)

Observation Error Standard Deviation

	$n = 1$	$n = 4$	$n = 8$	$n = 12$	$n = 16$	$n = 20$
σ_u^n	0.133	0.065	0.035	0.022	0.021	0.027
	(0.008)	(0.004)	(0.002)	(0.003)	(0.002)	(0.002)

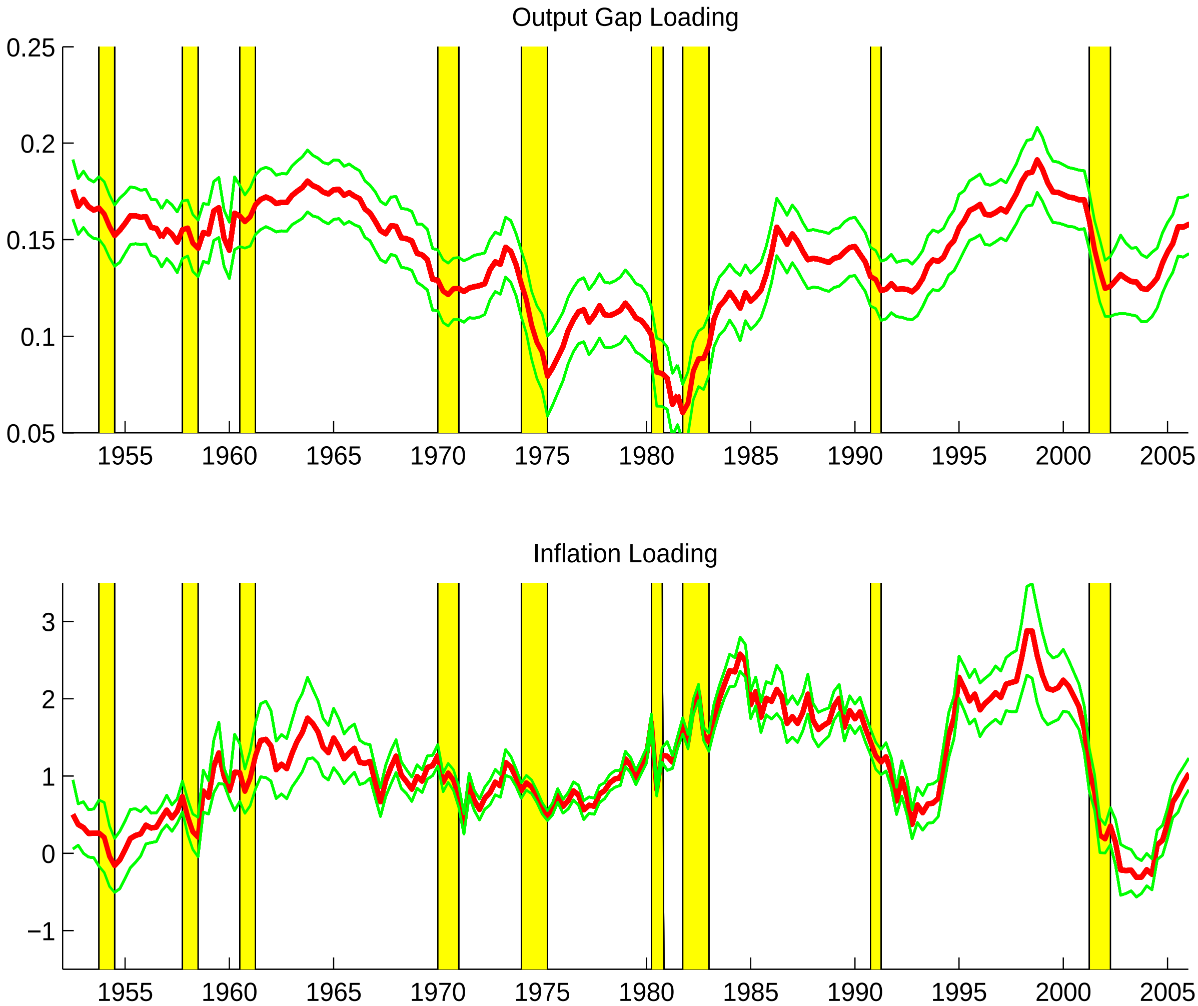
The table lists parameter values for the model in equations (2)-(8) and observation error standard deviations in equation (14) for yields of maturity n quarters. We estimate the model by Gibbs sampling using 50,000 simulations after a burn-in sample of 10,000. We report the posterior mean and posterior standard deviation (in parentheses) of each parameter. In the Volatility/Correlation matrix, we report standard deviations of each factor along the diagonal multiplied by 1000 and correlations between the factors on the off-diagonal elements. The sample period is June 1952 to December 2006 and the data frequency is quarterly.

Figure 1: Output Gap, Inflation, and the Short Rate



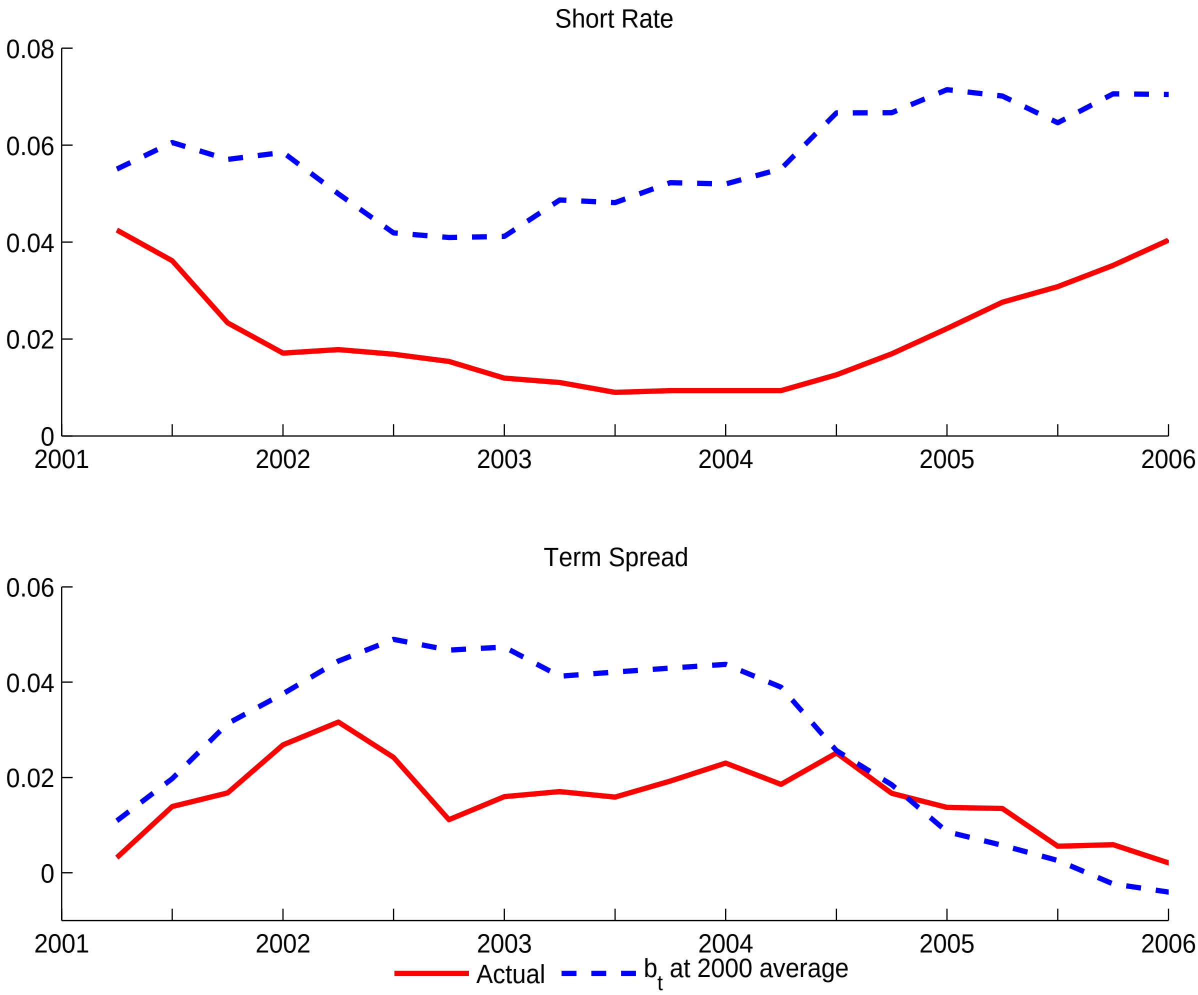
We plot the output gap, inflation, and the short rate. The output gap is defined as the proportional difference between actual and potential real GDP. Inflation is the year-on-year GDP deflator. The short rate is the 3-month T-bill yield. We overlay the NBER recession periods in shaded bars. The sample period is from June 1952 to December 2006 and the data frequency is quarterly. All data is annualized.

Figure 2: Time-Varying Policy Coefficients



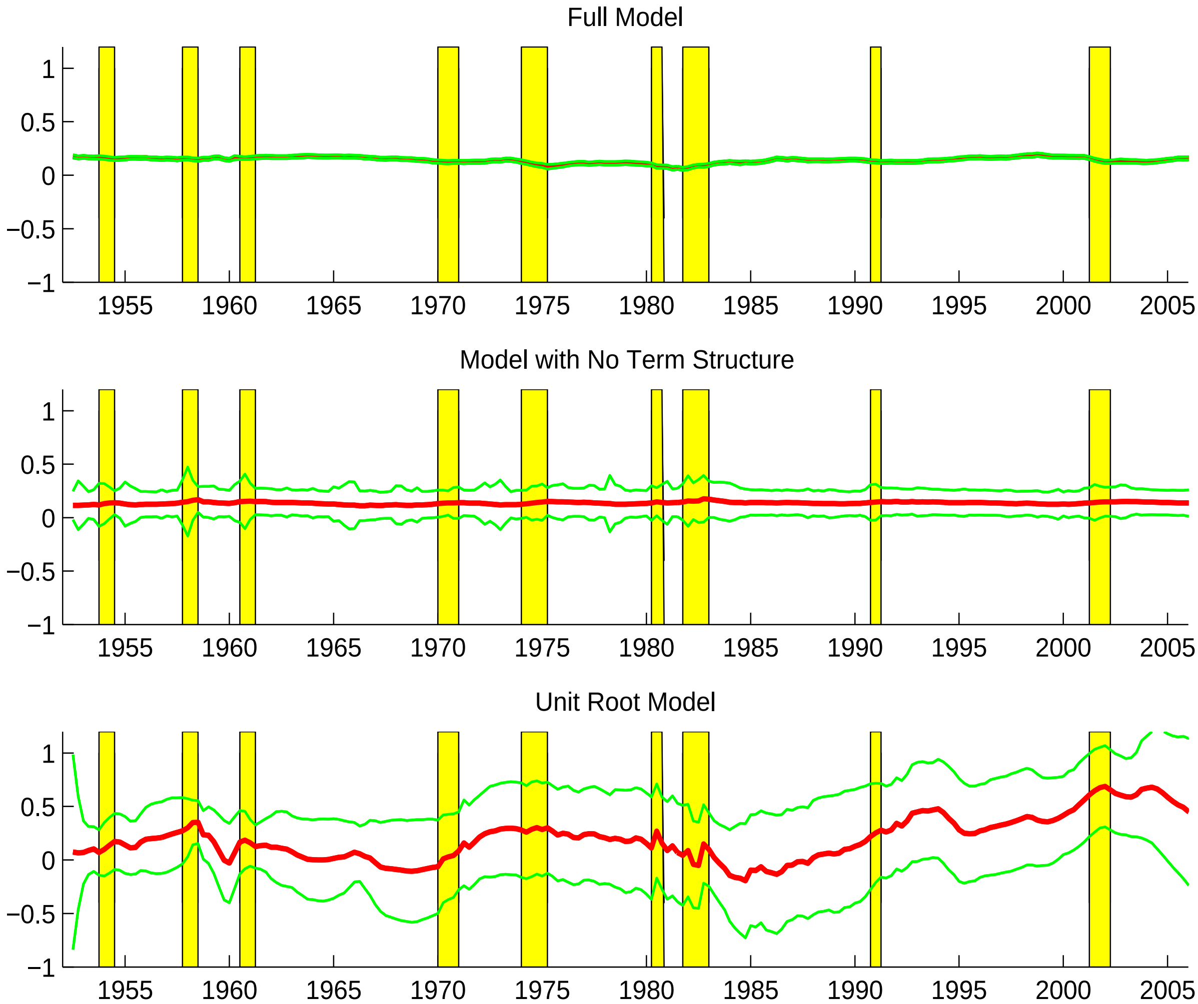
We plot the posterior mean of the time-varying coefficient $\bar{a} + a_t$ and $\bar{b} + b_t$ in the thick lines together with two posterior standard deviation bands in thin lines. We overlay the NBER recession periods in shaded bars. The sample period is from June 1952 to December 2006 and the data frequency is quarterly.

Figure 3: Counter-Factual Experiment



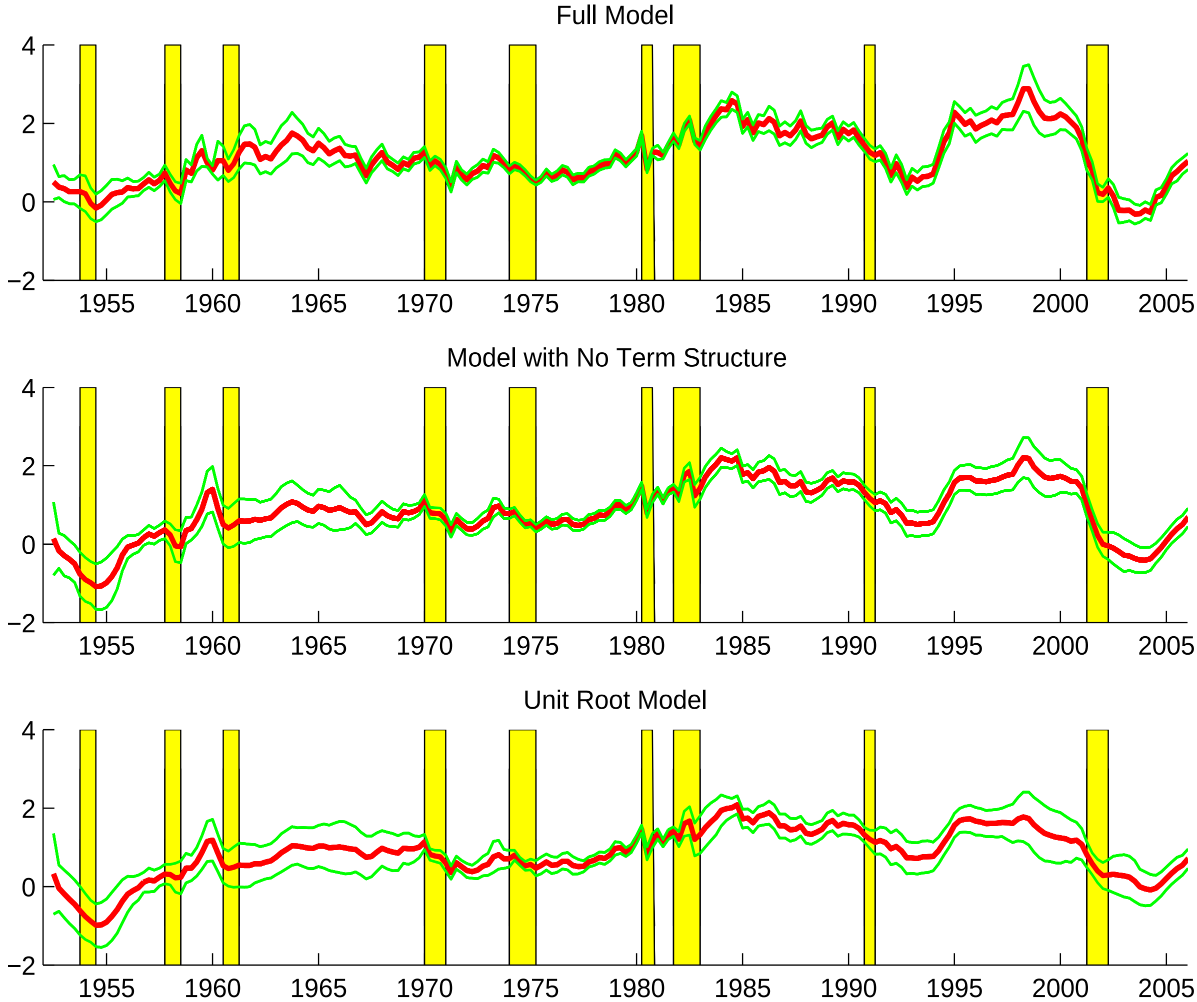
The figure plots the short rate (top panel) and the 5-year term spread (bottom panel), which is the 5-year yield minus the 3-month T-bill, from the results of a counter-factual experiment. We hold the Fed weight on inflation constant at its average level over 2000 and allow all other factors to take their sample values. We assume the posterior mean values for a_t. The figure plots the effect on the yield curve post-2001 in the dashed lines along with the actual paths of the yield curve in the solid lines. Units on the y-axis are annualized.

Figure 4: Output Gap Coefficients from Various Models



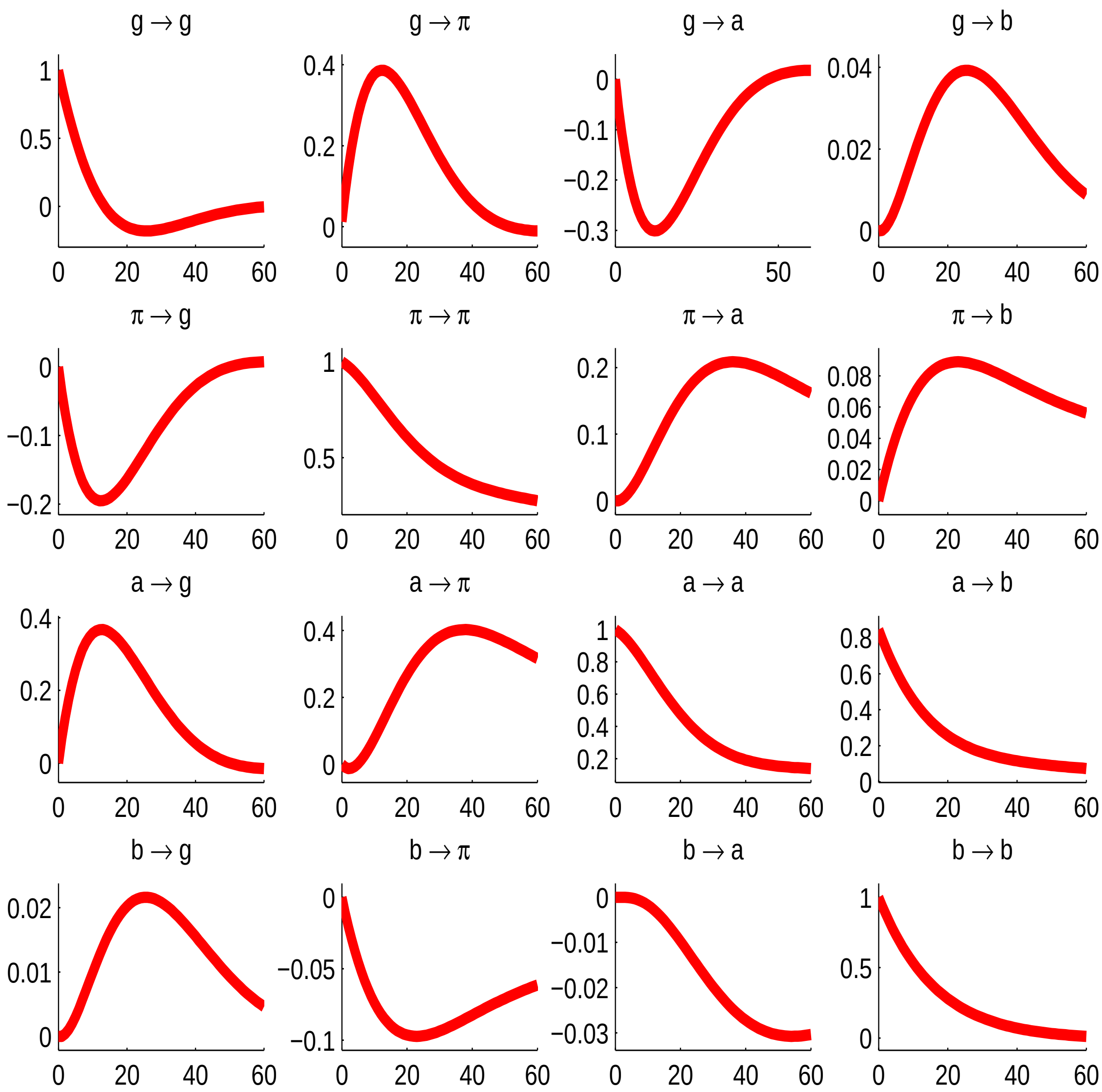
The figure plots the posterior mean of the time-varying output gap policy coefficient $\bar{a} + a_t$ implied by the full model (top panel), the model estimated without any yield curve information (middle panel), and a model where the policy coefficients follow random walks (bottom panel). Two posterior standard deviation bands are also drawn in thin lines. We overlay the NBER recession periods in shaded bars. The sample period is from June 1952 to December 2006 and the data frequency is quarterly.

Figure 5: Inflation Coefficients from Various Models



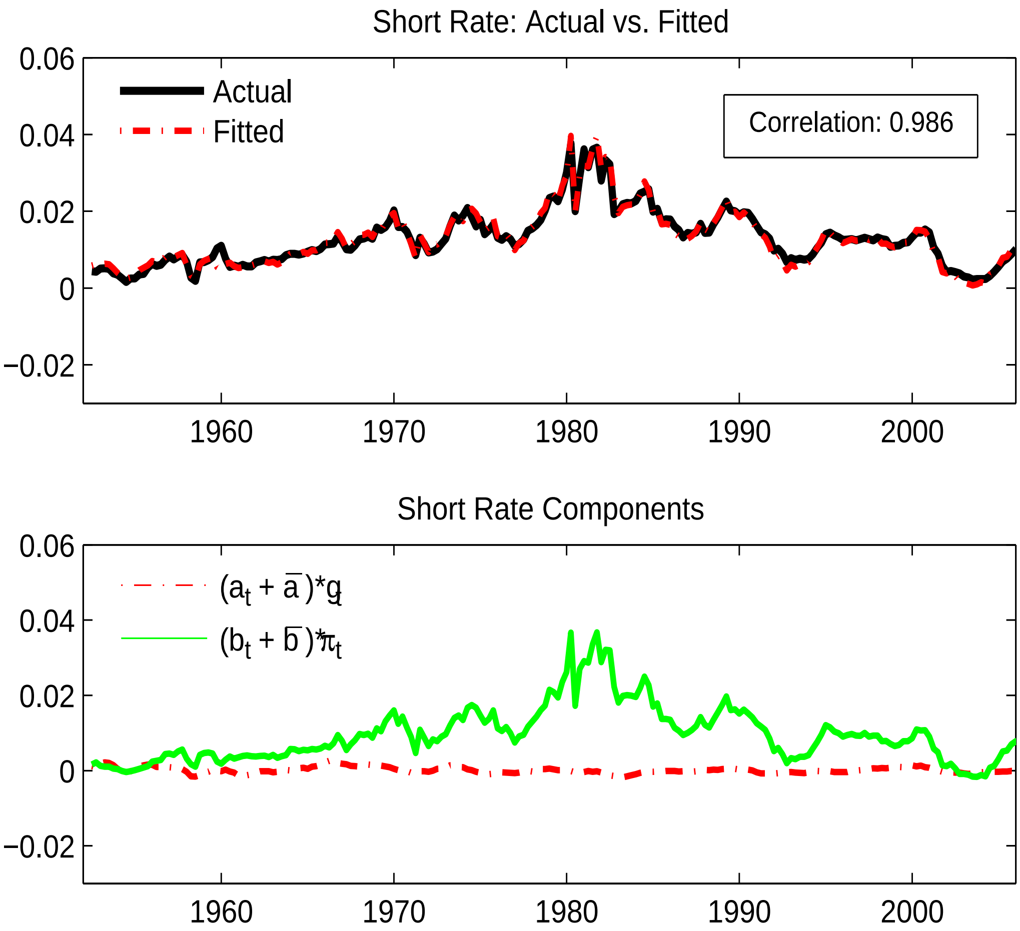
The figure plots the posterior mean of the time-varying inflation policy coefficient $\bar{b} + b_t$ implied by the full model (top panel), the model estimated without any yield curve information (middle panel), and a model where the policy coefficients follow random walks (bottom panel). Two posterior standard deviation bands are also drawn in thin lines. We overlay the NBER recession periods in shaded bars. The sample period is from June 1952 to December 2006 and the data frequency is quarterly.

Figure 6: Impulse Responses of Factors



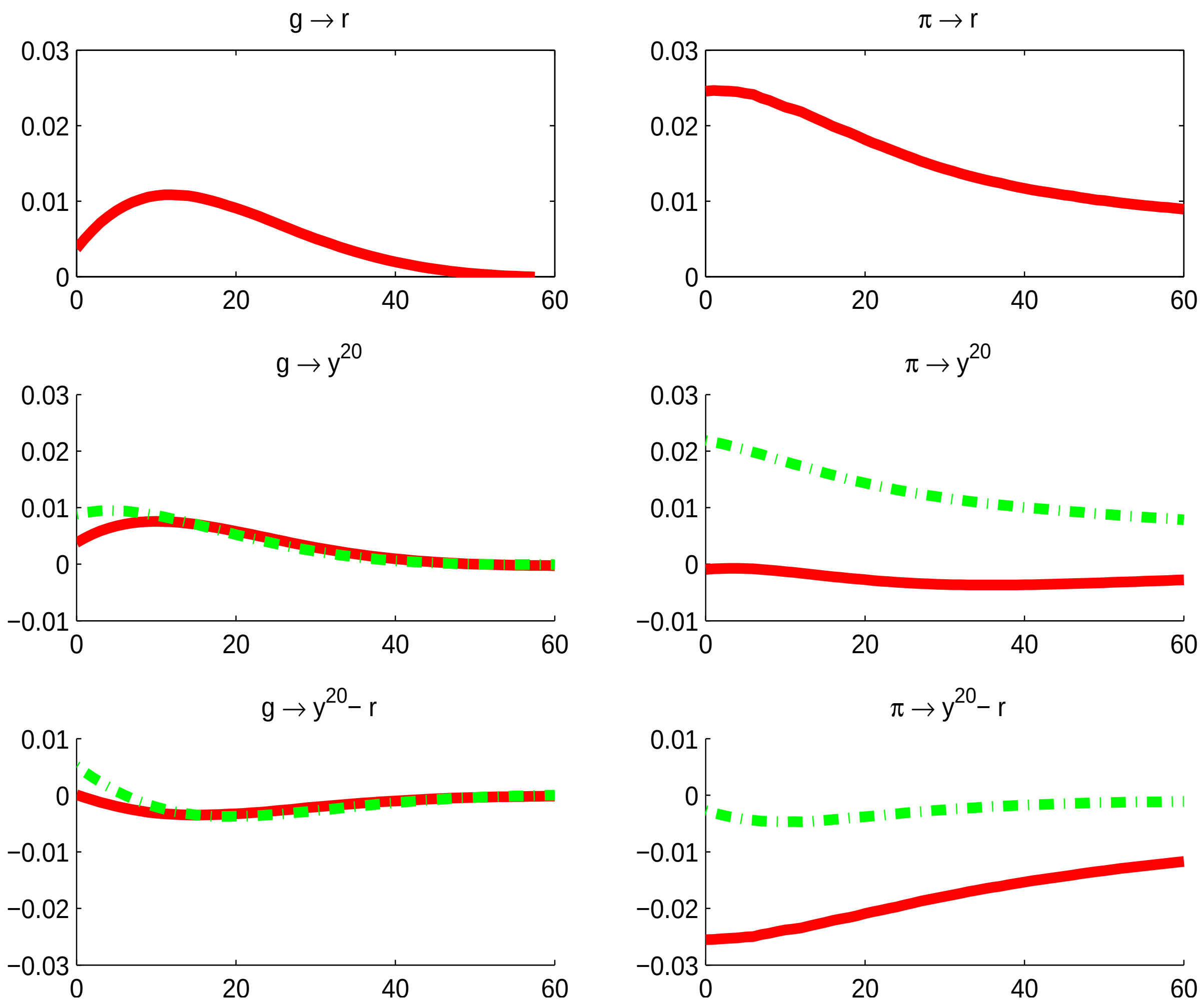
We plot the impulse responses of the factors to each other using a Cholesky decomposition based on the ordering $(g_t\ \pi_t\ a_t\ b_t)$. We consider factor shocks of one unconditional standard deviation and report the response of the variable on the y-axis in terms of that variable's unconditional standard deviation. For example, in the panel $g \longrightarrow g$, the initial response of a unit unconditional standard deviation shock in g in terms of g's standard deviation is 1. In the panel $g \longrightarrow \pi$, the effect of a unit unconditional standard deviation shock in g_t on π_t reaches a peak of 40% of π_t's unconditional standard deviation after 15 quarters. The x-axis units are quarters.

Figure 7: Components of the Short Rate



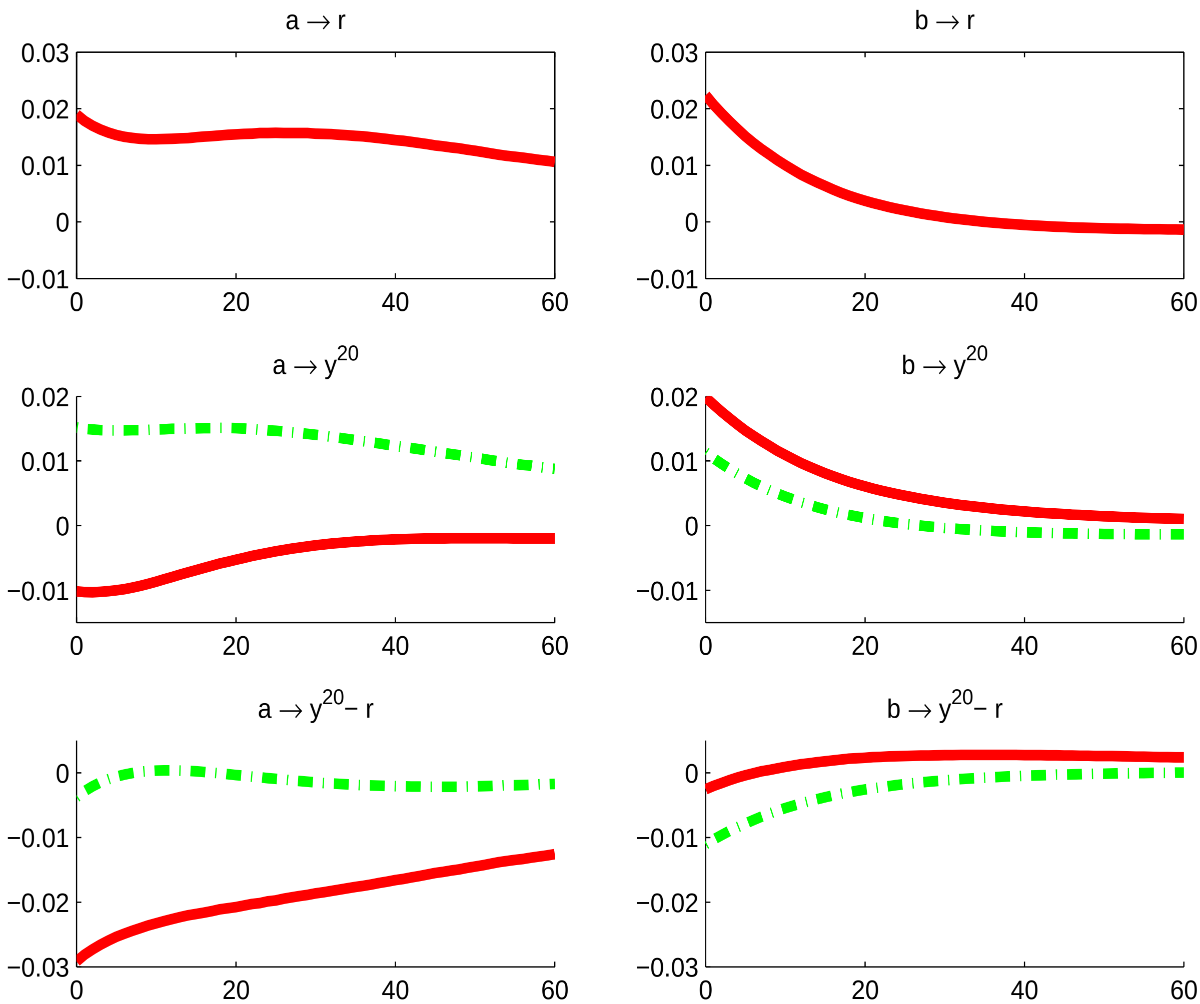
The top panel plots the short rate together with the fitted components $(\bar{a} + a_t)g_t + (\bar{b} + b_t)\pi_t$, where the policy factors a_t and b_t are evaluated at their posterior means at each observation from the Gibbs sampler. All variables are in per quarter units. The bottom panel plots each short rate component separately. The sample period is from June 1952 to December 2006 and the data frequency is quarterly.

Figure 8: Yield Curve Impulse Responses to Macro Shocks



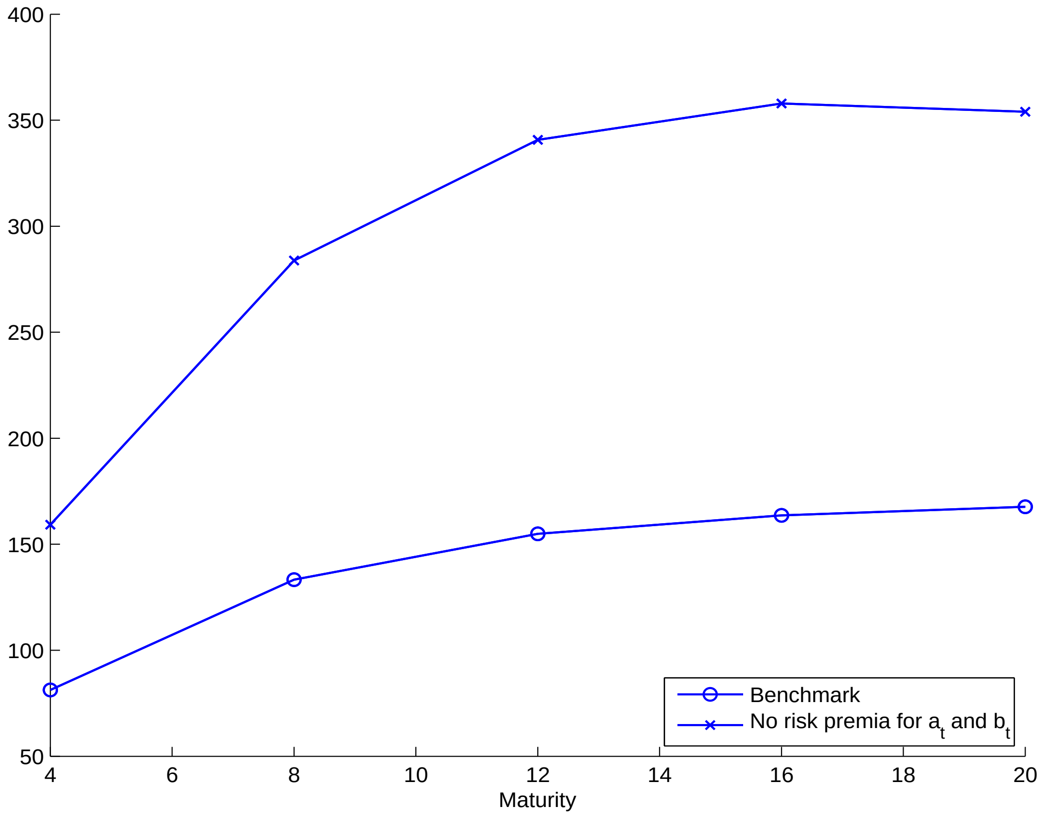
We plot the impulse responses of the short rate, r, the 20-quarter yield, y_t^{20}, and the yield spread, $y_t^{20} - r$, to a one unconditional standard deviation shocks in the output gap and inflation (g and π respectively). We overlay the responses of the yields if investors are risk neutral in dashed lines. We compute impulse responses following the method in Appendix D using a Cholesky decomposition based on the ordering $(g_t \; \pi_t \; a_t \; b_t)$. Units on the x-axis are in quarters and the responses of yields on the y-axis are annualized.

Figure 9: Yield Curve Impulse Responses to Policy Shifts



We plot the impulse responses of the short rate, r, the 20-quarter yield, y_t^{20}, and the yield spread, $y_t^{20} - r$, to a one unconditional standard deviation shocks in the output gap response and inflation response of the Fed (a_t and b_t, respectively). We overlay the responses of the yields if investors are risk neutral in dashed lines. We compute impulse responses following the method in Appendix D using a Cholesky decomposition based on the ordering $(g_t\ \pi_t\ a_t\ b_t)$. Units on the x-axis are in quarters and the responses of yields on the y-axis are annualized.

Figure 10: Risk Premia on Long-Term Bonds



The figure plots the annualized expected excess holding period return on long-term bonds given by equation (13) for various points on the yield curve under full risk premia and under the case where investors assign zero risk premia to the a_t and b_t factors. We compute the risk premia at the unconditional mean implied by the estimates of the model.