Tests of the Present-Value Model of the Current Account: A Note

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Abstract

Using a Monte Carlo approach, we evaluate the small-sample properties of four different tests of the present-value model (PVM) of the current account: the non-linear Wald, linear Wald, Lagrange multiplier, and likelihood ratio tests. We find that the non-linear Wald test is biased towards over-rejecting the cross-equation restrictions implied by the PVM, and that the test statistic is uncorrelated with the goodness of fit of the PVM. The three alternative tests are essentially equivalent and are more reliable in evaluating the PVM.

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1. Introduction

In its simplest form, the present-value model (PVM) of the current account predicts that a country's current account is determined by the discounted sum of expected future changes in net output (or cash flow). The widely used methodology to evaluate this prediction consists in testing the cross-equation restrictions that the PVM imposes on an unrestricted vector autoregression (VAR) composed of the current account and the change in net output.[1] Although, in principal, several statistical procedures can be applied to test these restrictions, the one most commonly used in the literature is a Wald test in which the test statistic involves a non-linear transformation of the VAR coefficients (henceforth called the non-linear Wald test). Two alternative tests that are sometimes used in the literature are the linear Wald test and the Lagrange Multiplier (LM) test. Both evaluate the null hypothesis using test statistics that are linear in the VAR coefficients.

The standard PVM is often statistically rejected by the non-linear Wald test. In many cases, however, a graphical comparison of the predicted and actual current-account series shows that the former tracks the latter reasonably well.[2] This observation hints at the possibility that the non-linear Wald test may be biased towards over-rejecting the cross-equation restrictions implied by the PVM.

In this note, we use a Monte Carlo approach to evaluate the performance of alternative tests of the PVM in finite samples. To do so, we exploit the cross-equation restrictions implied by the PVM to derive a restricted VAR which we use as a data-generating process. As a by-product of this approach, we obtain a Likelihood Ratio (LR) test that is straightforward to compute, and which is also compared to the three traditional tests discussed above. Results show that the linear Wald, LM, and LR tests have the correct size, whereas the size of the non-linear Wald test is distorted towards over-rejecting the PVM. In addition, we find no association between the non-linear Wald statistic and the goodness of fit of the PVM. These findings raise serious doubts about the usefulness of the non-linear Wald test as a criterion to judge the empirical plausibility of the PVM.

[1]See Sheffrin and Woo (1990), Otto (1992), Ghosh (1995), Bergin and Sheffrin (2000), and Nason and Rogers (2005).

[2]See Obstfeld and Rogoff (1996, chapter 2).

2. Tests of the Present-Value Model of the Current Account

The standard PVM implies that the current account is equal to the (negative) discounted sum of expected future changes in net output. That is,

$$CA_t = -\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i \mathbf{E}_t \Delta NO_{t+i}. \tag{1}$$

where CA_t is the current account in period t, NO_t is net output in period t, r is the constant real interest rate, and $\mathbf{E}_t$ is the mathematical expectation operator conditional on the information available at time t.

Present-value tests of (1) are based on the assumption that the joint distribution of the data is well approximated by an unrestricted VAR that includes the current account and changes in net output. More specifically, it is assumed that the bivariate vector $X_t = [\Delta NO_t \quad CA_t]'$ evolves according to the following pth order VAR:

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + \cdots + A_p X_{t-p} + \epsilon_t, \quad \epsilon_t \sim i.i.d.\ N(\mathbf{0}, \Sigma) \tag{2}$$

where A_i, $i = 1, 2, \cdots, p$ are 2×2 VAR coefficient matrices and $\epsilon_t = [\epsilon_t^1 \quad \epsilon_t^2]'$ is a 2×1 disturbance vector that is normally distributed with mean zero and variance-covariance matrix Σ. The VAR (2) can be written in more compact form as follows:

$$Y_t = AY_{t-1} + u_t \tag{3}$$

where $A = \begin{bmatrix} A_1 & A_2 & \cdots & A_p \\ & I_{2(p-1)} & & \mathbf{0}_{2\times 2} \end{bmatrix}$, $Y_t = \left[X_t' \ X_{t-1}' \ \ldots \ X_{t-p-1}'\right]'$, and $u_t = \left[\epsilon_t' \ \mathbf{0}_{1\times 2(p-1)}'\right]'$.

We now describe four alternative tests that can be used to evaluate the cross-equation restrictions implied by the PVM (1).

Non-linear Wald test Let e_i denote a $1\times 2p$ row vector in which the ith element is 1 but the rest of elements are zeros, i.e., $e_i = [0 \ \cdots \ 0 \ 1_{ith} \ 0 \cdots \ 0]$. Then the PVM (1) imposes the following non-linear cross-equation restrictions on the unrestricted VAR (3)

$$e_2 = -(1+r)^{-1} e_1 A [I_{2p} - (1+r)^{-1} A]^{-1}. \tag{4}$$

Define $\mathcal{H}(A) \equiv e_2 + (1+r)^{-1} e_1 A[I_{2p} - (1+r)^{-1}A]^{-1}$ and let $\hat{A}$ and $\hat{\Sigma}_A$ denote the OLS estimates of the matrix A and the variance-covariance matrix of the VAR coefficients, respectively. The Wald statistic $\mathcal{W}^n$ is then constructed as

$$\mathcal{W}^n = \mathcal{H}(\hat{A}) \left[\frac{\partial \mathcal{H}(\hat{A})}{\partial \hat{A}} \hat{\Sigma}_A \frac{\partial \mathcal{H}(\hat{A})}{\partial \hat{A}'} \right]^{-1} \mathcal{H}(\hat{A})'. \tag{5}$$

Under the null hypothesis $\mathcal{H}(A) = 0$, the $\mathcal{W}^n$ statistic is asymptotically distributed $\chi^2(2p)$. Note that, in addition to this formal test of the PVM, a predicted current-account series can be constructed as

$$\mathcal{CA}_t = -(1+r)^{-1} e_1 A [I_{2p} - (1+r)^{-1} A]^{-1} Y_t, \tag{6}$$

and compared with the actual series, thus allowing one to graphically appreciate the in-sample forecasting performance of the PVM. Under the null hypothesis $\mathcal{H}(A) = 0$, the actual and predicted current-account series must coincide, i.e., $\mathcal{CA}_t = CA_t$.

Linear Wald test Assuming that $I_{2p} - (1+r)^{-1}A$ is invertible, the non-linear cross-equation restrictions (4) can be rewritten as a linear function of the VAR coefficients

$$e_2 = (1+r)^{-1}(e_2 - e_1)A. \tag{7}$$

Define $\mathcal{I}(A) \equiv e_2 - (1+r)^{-1}(e_2 - e_1)A$. An alternative Wald statistic $\mathcal{W}^l$ can the be computed as

$$\mathcal{W}^l = \mathcal{I}(\hat{A}) \left[\frac{\partial \mathcal{I}(\hat{A})}{\partial \hat{A}} \hat{\Sigma}_A \frac{\partial \mathcal{I}(\hat{A})}{\partial \hat{A}'} \right]^{-1} \mathcal{I}(\hat{A})'. \tag{8}$$

Under the null hypothesis $\mathcal{I}(A) = 0$, the $\mathcal{W}^l$ statistic is asymptotically distributed $\chi^2(2p)$. Note that although the restrictions (4) and (7) are equivalent, $\mathcal{W}^l$ and $\mathcal{W}^n$ need not be identical, since, as is well known, the Wald statistic is not invariant to the formulation of the restrictions.

Lagrange Multiplier test The cross-equation restrictions (7) imply

$$\begin{aligned} &a(2,j)_k = a(1,j)_k \quad \text{for all} \quad j,k \quad \text{except} \\ &a(2,2)_1 = a(1,2)_1 + 1 + r \end{aligned}$$

where $a(i,j)_k$ is the (i,j) element of the kth VAR matrix A_k. Imposing these restrictions on the VAR and subtracting the first row from the second row yields

$$CA_t - \Delta NO_t = (1+r)CA_{t-1} + \epsilon_t^1 - \epsilon_t^2. \tag{9}$$

Let $D_t \equiv CA_t - \Delta NO_t - (1+r)CA_{t-1}$. Equation (9) then implies that, under the PVM, the variable D_t must be orthogonal to any past information, i.e., $\mathbf{E}_{t-1}D_t = 0$. This orthogonality condition implies the following LM statistic (equivalently, TR^2 statistic) to test (7)

$$\mathcal{LM} = TR^2 = TD'Y(Y'Y)^{-1}Y'D/D'D \tag{10}$$

where T is the sample size, $D = [D_2 \; D_3 \; \cdots \; D_T]'$, and $Y = [Y_1 \; Y_2 \; \cdots \; Y_{T-1}]'$, respectively. The $\mathcal{LM}$ statistic is asymptotically distributed $\chi^2(2p)$.

Likelihood Ratio test Let $X_t = A_1^r X_{t-1} + A_2^r X_{t-2} + \cdots + A_p^r X_{t-p} + \upsilon_t$ denote the restricted VAR on which the restrictions (7) are imposed. Let $\hat{L}^r$ and $\hat{L}$ denote the maximized log-likelihood of the restricted and unrestricted VARs, respectively. The linear restrictions (7) can be jointly tested using the Likelihood Ratio statistic

$$\mathcal{LR} = 2(\hat{L} - \hat{L}^r). \tag{11}$$

Under the null, the $\mathcal{LR}$ statistic is asymptotically distributed $\chi^2(2p)$.

Note that the linear Wald, Lagrange Multiplier, and Likelihood Ratio tests are asymptotically equivalent under the null hypothesis. In small and moderately sized samples, however, they may yield different results.

3. An Example

This section provides an example that illustrates the difference in results one might obtain by applying the four tests described above. The example evaluates the PVM using quarterly data from the United Kingdom (see the Appendix for a description of the data). The results are based on an unrestricted VAR of order 2, as determined by the Akaike information criterion. Figure 1 depicts the predicted current-account series constructed according to (6). The figure shows that the predicted series is somewhat smoother than the actual, but the overall fit is nonetheless rather good. Yet, the non-linear Wald test strongly rejects the cross-equation restrictions implied by the PVM at all levels of significance, as shown in panel A of Table 1. On the other hand, the linear Wald, Lagrange Multiplier, and Likelihood Ratio tests do not reject the PVM at the 5% level. This result raises some doubts on the inference based on the non-linear Wald test and on its usefulness as a criterion to gauge the goodness of fit of the PVM.

4. Monte Carlo Experiment and Results

To perform our Monte Carlo experiment, we impose the cross-equation restrictions (7) on the unrestricted VAR estimated using U.K. data. This yields a restricted VAR of the same order, which we use to generate 10,000 sequences of artificial series of net output and the current account. In each iteration, a sufficient number of observations are discarded to ensure that the results do not depend on initial conditions. The number of remaining observations corresponds to the size of the actual sample used to estimate the unrestricted VAR. Note that, by construction, the data-generating process (DGP) satisfies the null hypothesis and can therefore be used to compute the actual size of the different tests considered.

Panel B of Table 1 shows the frequency of making a Type 1 error (i.e., the frequency of a false rejection of the null hypothesis) by each test at the 5% and 1% critical values. The table shows that the actual size of the non-linear Wald test is 15.1% at the 5% level and 8.1% at the 1% level. This test, therefore, tends to over-reject the null hypothesis. In contrast, the linear Wald, Lagrange Multiplier, and Likelihood Ratio tests have the correct size both at the 5% and 1% levels.

In each replication, we also compute Theil's U-statistic, a measure of the goodness of fit of the predicted current-account series, in order to determine the extent to which the test statistics reflect the in-sample forecasting performance of the PVM.[3]

Figure 2 depicts a scatterplot of the (log of the) U-statistic and the p-value for each of the four tests. If a given statistic reflects the distance between the actual and predicted current-account series, then a strong negative association between the U-statistic and the p-value is to be expected. This is obviously not the case for the non-linear Wald test: the test statistic and the p-value seem to be unrelated in this case, as is also indicated by the correlation coefficient of -0.106 (see the last row of Table 1). On the other hand, each the three remaining Panels of Figure 2 displays a clear downward-sloping cloud of points, indicating a clear negative relationship between the U-statistic and the p-value. In all three cases, the correlation is higher than 0.42 in absolute value.

[3]The U-statistic is constructed as $\sqrt{\sum_t (\mathcal{CA}_t - CA_t)^2 / \sum_t CA_t^2}$.

5. Conclusion

This note has shown that the non-linear Wald test commonly used to evaluate the PVM of the current account (i) tends to over-reject the model in finite samples, and (ii) is outperformed by alternative procedures that instead test the linear restrictions implied by the PVM. These tests should therefore be preferred when evaluating standard and extended versions of the PVM in small and moderately sized samples.

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Appendix: Data Description and Construction

U.K. data are taken from the U.K. National Statistics database (http://www.statistics.gov.uk/). The sample is 1964:Q2 to 2003:Q4. The current-account series CA_t is constructed as net foreign interest payments plus net exports. Net foreign interest payments are measured by *Net income from abroad* (CAES). Net exports are measured by *Total Export* (IKBH) minus *Total Import* (IKBI). The net-output series, NO_t, is constructed as *GDP* (YBHA) minus *Total Gross Fixed Capital Formation* (NPQS) minus *Changes in Inventories* (CAEX) minus *Durable Goods* (UTIB) minus *Semidurable Goods* (UTIR) minus *General Government Final Consumption Expenditure* (NMRP). All series are seasonally adjusted at annual rates, converted to real terms using the GDP deflator, and divided by *Total Population* (GBRPOP). The latter series is taken from the OECD database.

Table 1: Results†

	Non-linear Wald	Linear Wald	Lagrange Multiplier	Likelihood Ratio
	A. Test Results for the U.K.			
Statistic	23.559	8.709	8.251	8.476
P-value	0.000	0.069	0.083	0.075
	B. Monte Carlo Results			
Frequency of rejection				
Nominal size = 5%	0.151	0.060	0.047	0.053
Nominal size = 1%	0.081	0.016	0.010	0.012
Corr(U-stat, p-value)	−0.106	−0.427	−0.427	−0.448

† Notes: Test results are based on a second-order VAR estimated using U.K. data. The sample size is 159 observations. All the test statistics are distributed $\chi^2(4)$. Monte Carlo results are based on 10,000 replications. The DGP is a restricted second-order VAR.

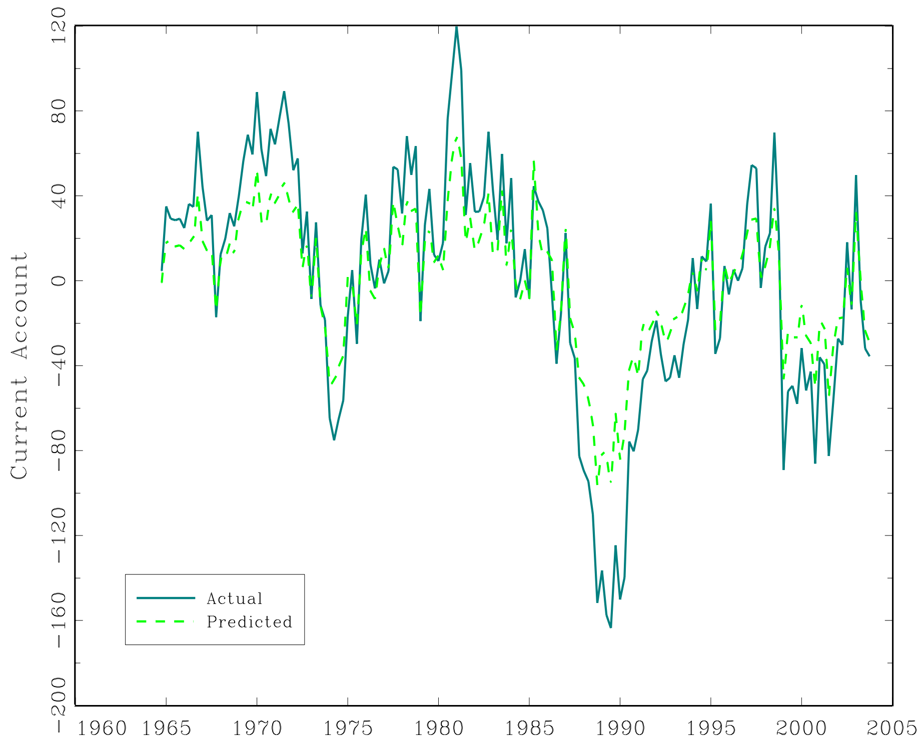

Figure 1: Actual and predicted current account for the United Kingdom

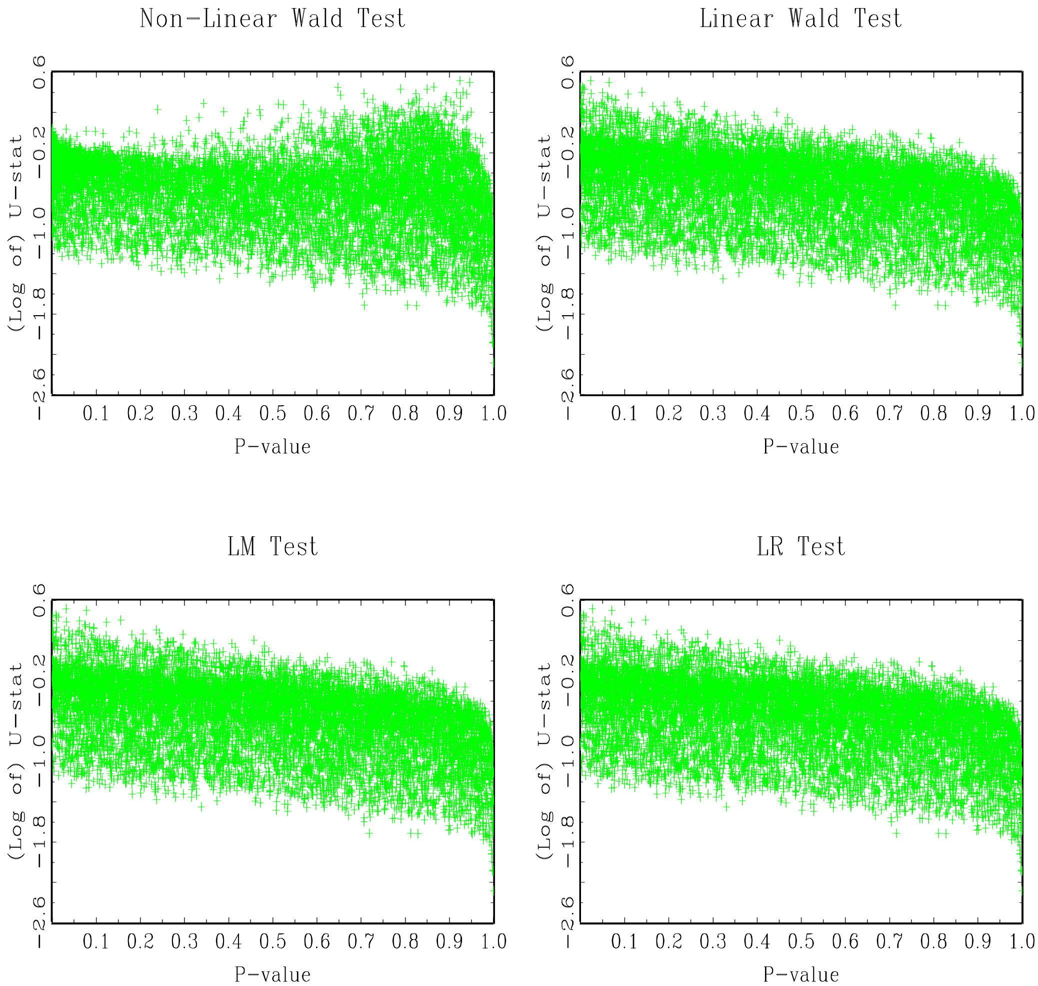

Figure 2: P-value and U-statistic