Why Does Private Consumption Rise After a Government Spending Shock?*

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Abstract

Some recent empirical evidence suggests that private consumption is crowded-in by government spending. This outcome violates neoclassical macroeconomic theory, according to which the negative wealth effect brought about by a rise in public expenditure should decrease consumption. In this paper, we develop a simple real business cycle model where preferences depend on private and public spending, and households are habit forming. The model is estimated by the maximum-likelihood method using U.S. data. Estimation results indicate a strong Edgeworth complementarity between private and public spending. This feature enables the model to generate a positive response of consumption following a government spending shock. In addition, the impulse-response functions generated by the estimated model are generally consistent with those obtained from a benchmark vector autoregression.

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1. Introduction

In the last few years, there has been a renewed interest in the effects of government spending on private consumption. Several recent empirical studies based on vector autoregressions (VARs) find that an increase in public spending leads to a significant and persistent increase in private consumption (e.g. Fatás and Mihov 2001; Blanchard and Perotti 2002; Perotti 2004; and Galí, López-Salido, and Vallés 2005). This crowding-in effect violates neoclassical macroeconomic theory, according to which government spending decreases consumption. As explained by Baxter and King (1993), the standard real business cycle (RBC) model predicts that an increase in government spending creates a negative wealth effect by lowering the households’ permanent income. To prevent a large drop in consumption, households increase their labour supply, but this substitution effect is typically not strong enough to offset the wealth effect. As a result, consumption decreases in equilibrium. This prediction led some authors to conclude that the neoclassical model may not be an appropriate framework to study the macroeconomic implications of fiscal policy shocks.

Departing from the frictionless neoclassical paradigm, Galí, López-Salido, and Vallés (2005) recently developed an elaborate New Keynesian model that is capable of generating a positive effect of government spending on consumption. The key features of their model are price stickiness, the presence of non-Ricardian (or rule-of-thumb) households, who consume their current disposable income, and a non-competitive labour market in which wages are set by an economy-wide union, and households are willing to meet the firms’ demand for labour given the wage set by the union. The mechanism by which public spending crowds-in private consumption is the following: Under price stickiness, an increase in government spending translates into higher aggregate demand, to which firms respond by increasing their demand for labour. Owing to the monopolistic nature of the labour market, this leads to a sharp increase in the real wage, which in turn raises the income of non-Ricardian households and stimulates their consumption. If the weight of those households in the population is large enough, aggregate consumption will also increase. The empirical relevance of this explanation has been questioned, however, by Coenen and Straub (2005) who estimate a

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1As discussed in Section 2, however, this result is not robust across all identification schemes: Studies that identify government spending shocks using the narrative approach proposed by Ramey and Shapiro (1998) find that the response of private consumption to an increase in public spending is negative and statistically insignificant.

2See, for example, Galí, López-Salido, and Vallés (2005).

3As shown by Linne mann and Schabert (2003), to the extent that monetary policy is not excessively accommodative, price stickiness cannot by itself give rise to a crowding-in effect of public spending on consumption. On the other hand, with sticky prices but a competitive labour market, one needs the fraction of rule-of-thumb consumers to be implausibly high to obtain an increase in aggregate consumption.
similar model to that of Galí, López-Salido, and Vallés using data from the Euro area. Their results indicate that the estimated fraction of non-Ricardian households is relatively small and that, as a consequence, it is unlikely that the mechanism described above yields a positive comovement of public and private spending.

In this paper, we propose an alternative explanation for the crowding-in effect documented in the VAR literature, and we provide an assessment of its empirical plausibility. Our explanation, which does not require a non-Ricardian environment, emphasizes the complementarity between public and private spending. We formalize this idea within a fully optimizing RBC model augmented with two important features. First, consumer preferences depend on government spending, and second, households are habit forming. Simulation results show that a strong Edgeworth complementarity between public and private spending is necessary to generate a positive effect of government spending on consumption. Intuitively, when the two variables are Edgeworth complements, government spending increases the marginal utility of consumption, providing an additional motive for households to work more, which in turn mitigates the negative wealth effect. When the complementarity effect is sufficiently strong, consumption rises in equilibrium. Without habit formation, however, the consumption response to a government spending shock is monotonic and not as persistent as in the data. Habit formation enables the model to generate a persistent and non-monotonic consumption response similar to that obtained from the VAR. Intuitively, habit-forming households smooth both the absolute level of consumption and its rate of change. As a result, the consumption response to shocks is typically smaller on impact and more gradual under habit formation than under time-separable preferences.

The question of whether private consumption and public spending are complements or substitutes has been examined by several studies such as those by Aschauer (1985), Karras (1994), Ni (1995), Amano and Wirjanto (1998), and Okubo (2003). All these studies have in common that they use a partial-equilibrium approach based on Euler equations to estimate the degree of substitutability between private and public spending. The empirical results,
however, are mixed and inconclusive. Although, in general, it is difficult to determine what might account for the differences in results, Ni (1995) shows that the nature of the relationship between private and public spending critically depends on the specification of the utility function, and on the measurement of interest rates. It remains that the existing literature does not seem to provide much guidance as to the magnitude of the elasticity of substitution between private and public spending. Hence, rather than using calibration to pin down the value of this parameter, we estimate it jointly with the remaining structural parameters by imposing on the data the cross-equation restrictions implied by the model. This general-equilibrium approach ensures that the parameter estimates are mutually consistent, and allows one to evaluate the overall fit of the model.

The model is estimated by the maximum-likelihood (ML) method using U.S. data. The results yield plausible and significant estimates of the structural parameters. The ML estimate of the elasticity of substitution between private consumption and public spending indicates that they are Edgeworth complements. Overall, the impulse-response functions based on the estimated parameters are consistent with their VAR-based counterparts. In particular, the estimated model predicts that a government spending shock leads to a non-monotonic and persistent increase in consumption. In addition, the model replicates the unconditional moments of the data better than a standard RBC model.

The rest of this paper is organized as follows. Section 2 constructs a benchmark VAR to illustrate the crowding-in effect documented by earlier studies. Section 3 develops the theoretical model. Section 4 discusses the model’s implications. Section 5 describes the estimation methodology. Section 6 reports and discusses the empirical results. Section 7 concludes.

2. Empirical Evidence

The purpose of this section is to construct and estimate a benchmark VAR to illustrate the macroeconomic effects of a government spending shock. Our benchmark VAR is given by

\[ Z_t = A + B(L)Z_{t-1} + u_t, \]  

where \( Z_t \) is a vector of endogenous variables that includes government spending, consumption, hours, output, investment, and the real wage; \( A \) is a vector that contains the constant terms; \( B(L) \) is a finite-order vector polynomial in non-negative powers of the lag operator \( L \); and \( u_t \) is a vector of serially uncorrelated shocks.

We use quarterly U.S. data taken from the Federal Reserve Bank of Saint Louis’ FRED database and from Citibase. The sample period is 1948Q1–2005Q4. Government spending
is measured by the sum of local, state, and federal real spending. Consumption is measured by real private spending on non-durable goods and services. Hours are measured by the total number of hours worked in the nonfarm business sector. Output is measured by real GDP. Investment is measured by real private non-residential investment. The real wage is measured by compensation per hour in the nonfarm business sector, divided by the nonfarm business deflator. All variables, except the real wage, are converted to per-capita terms by dividing them by the civilian population, age 16 and over. The variables are logged and detrended using a one-sided Hodrick-Prescott (H-P) filter. We adopt this strategy to be consistent with the solution of the model presented in Section 3, where variables are expressed as percentage deviations from their steady-state values.

We estimate the model (1) using the least-squares method. The Akaike criterion suggests that the optimal number of lags in the VAR is 2. Following Fatás and Mihov (2001) and Gál, López-Salido, and Vallés (2005), we identify the government spending shock using a Cholesky decomposition, where government spending is ordered first. This implies that innovations in government spending affect the remaining variables contemporaneously, but not the opposite.

Figure 1 depicts the impulse responses of consumption, hours, output, investment, and the real wage to a 1 per cent increase in government spending. These responses are represented with solid lines, while dotted lines delimit their 68 per cent confidence intervals. The shock triggers an increase in private consumption. That is, consumption is crowded-in by government spending. The response of consumption is non-monotonic and persistent, reaching its peak one quarter after the shock and lasting for more than four years. However, it is only statistically significant on impact. Hours worked, output, and the real wage also increase following the shock. The response of hours worked changes sign twice within the first eight quarters and becomes statistically insignificant after the first quarter. Output reaches its maximum response on impact before returning gradually to its initial level. In contrast, the response of the real wage is hump-shaped, attaining its highest level three quarters after the shock. Finally, the increase in public spending leads to a large and significant decline in private investment. The response of investment has an inverted hump with a trough occurring at around three quarters after the shock.

Overall, our empirical findings are similar to those reported by Fatás and Mihov (2001), Blanchard and Perotti (2002), Perotti (2004), Gál, López-Salido, and Vallés (2005), and to a

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6 This measure of government spending includes public investment but excludes transfers.
7 As pointed out by a referee, the double-sided nature of the standard H-P filter conflicts with the recursive identification approach adopted in this paper.
8 The ordering of the remaining elements of the vector $Z_t$ is irrelevant to the propagation of the government spending shock.
lesser extent, Mountford and Uhlig (2005). Interestingly, these authors find a positive effect of public spending on consumption albeit using different identification strategies: While, as stated above, Fatás and Mihov (2001) and Galí, López-Salido, and Vallés (2005) rely on a recursive identification scheme, where government spending is ordered first, Blanchard and Perotti (2002) and Perotti (2004) identify fiscal policy shocks by exploiting institutional information on the responsiveness of fiscal variables to economic activity. Mountford and Uhlig (2002), on the other hand, achieve identification by imposing sign restrictions on the impulse responses of fiscal variables as well as the orthogonality of fiscal policy shocks to business cycle and monetary policy shocks.

Not all identification schemes, however, lead to the prediction that consumption rises after a government spending shock. Ramey and Shapiro (1998) use a narrative approach to isolate three events that led to large military buildups in the U.S., commonly known as the Ramey-Shapiro episodes (the Korean War 1950:3, the Vietnam War 1965:1, and the Carter-Reagan defense buildup 1980:1), and identify fiscal shocks as innovations to a dummy variable that traces these episodes. Their empirical analysis shows that non-durable and service consumption slightly decreases following a government spending shock, although the effect is mostly statistically insignificant. Building on Ramey and Shapiro’s approach, Edelberg, Eichenbaum, and Fisher (1999) and Burnside, Eichenbaum, and Fisher (2004) also find a weak and statistically insignificant response of private consumption to the onset of a Ramey-Shapiro episode. The analysis based on this approach also yields different results regarding the response of non-residential investment and the real wage. An expansionary fiscal spending shock significantly raises the former and reduces the latter.

The main advantage of the narrative approach over alternative identification schemes is that the episodes identified by Ramey and Shapiro correspond to genuinely exogenous shifts in fiscal policy in the United States. Hence, no further arbitrary assumptions are needed to achieve identification. This approach, however, has some drawbacks. First, and most obviously, it only identifies three exogenous fiscal shocks. Second, it is often argued that the Ramey-Shapiro episodes, while exogenous, might not be fully unanticipated (see Fatás and Mihov 2001, for example). Third, as discussed by Perotti (2004), the identification of military buildup shocks may be polluted by the presence of other substantial fiscal shocks of different type or sign that might have occurred around the same time. Finally, a sensitivity analysis carried out by Fatás and Mihov (2001) reveals that when the Ramey-Shapiro episodes are treated asymmetrically, they lead to entirely different dynamics of private con-

\footnote{All these authors use U.S. data, except Perotti (2004), who uses data from five OECD countries: Australia, Canada, Germany, the United Kingdom, and the United States.}

\footnote{Durable consumption, in contrast, rises significantly on impact before declining a few quarters later.}
sumption, thus casting some doubts on the robustness of the results based on the narrative approach.\textsuperscript{11}

Based on these arguments, and given that the three alternative identification approaches—notwithstanding that they could be subject to criticisms of a different kind—find that public spending crowds-in private consumption, we will treat this result as an empirical fact that needs to be explained. This is the purpose of the model developed in the next section.

3. The Model

We extend the standard RBC model along two dimensions. First, we allow government spending to enter the utility function, and second, we assume that consumer preferences exhibit habit formation.

3.1 The representative household

The economy is populated by a single, infinitely lived, representative household that derives utility from effective consumption ($\tilde{C}$) and leisure ($1 - N$).\textsuperscript{12} Effective consumption is assumed to be a constant-elasticity-of-substitution (CES) index of private consumption ($C$) and government spending ($G$):

$$\tilde{C}_t = \left[ \phi C_t^{(\nu - 1)/\nu} + (1 - \phi) G_t^{(\nu - 1)/\nu} \right]^{\nu/(\nu - 1)}, \quad (2)$$

where $\phi$ is the weight of private consumption in the effective consumption index, and $\nu > 0$ is the elasticity of substitution between private consumption and government spending.\textsuperscript{13}

In the special case where $\nu = 0$, $C_t$ and $G_t$ become perfect complements. As $\nu \to \infty$, they become perfect substitutes. The CES specification captures the idea of diminishing marginal returns to public spending in order to achieve a given level of effective consumption \textit{ceteris paribus}.

We assume that the household’s preferences exhibit habit formation in effective consumption. More precisely, the household’s instantaneous utility function depends on the

\textsuperscript{11}Their results indicate that consumption increases for a few quarters following the Korean War, increases permanently after the Vietnam War, and falls permanently after the Carter-Reagan defense buildup.

\textsuperscript{12}It is assumed that, in each period, the representative household is endowed with one unit of time that is divided between labour and leisure.

current level of effective consumption relative to its previous level. The functional form of
the instantaneous utility function is the following:

\[
u(\tilde{C}_t, \tilde{C}_{t-1}, N_t) = \frac{1}{1-\epsilon} \left(\frac{\tilde{C}_t}{\tilde{C}_{t-1}}\right)^{1-\epsilon} + \psi \ln (1 - N_t), \tag{3}\]

where \(\epsilon, \psi\) are positive parameters and \(\gamma \in (0,1)\) measures the degree of habit forma-
tion.\(^{14}\) The utility function encompasses the standard case where preferences depend only
on the current level of private consumption (\(\phi = 1\) and \(\gamma = 0\)).

The representative household supplies labour and capital to firms, and pays a lump-sum
tax to the government. It allocates its disposable income to consumption and investment.
Investment increases the household’s stock of capital according to

\[
K_{t+1} = (1 - \delta)K_t + I_t, \tag{4}\]

where \(K_t\) is the stock of capital at the beginning of period \(t\), \(I_t\) is investment, and \(\delta \in (0,1)\)
is the depreciation rate of capital. Investment is subject to convex adjustment costs of the
following form:

\[
\varphi(I_t, K_t) = \frac{\kappa}{2} \left(\frac{I_t}{K_t} - \delta\right)^2 K_t, \tag{5}\]

where \(\kappa\) is a positive parameter. Therefore, the representative household’s budget constraint
in period \(t\) is

\[
C_t + I_t + \varphi(I_t, K_t) \leq w_t N_t + r_t K_t - T_t, \tag{6}\]

where \(w_t\) is the real wage, \(N_t\) is the number of hours worked, \(r_t\) is the real rental rate of
capital, and \(T_t\) is a lump-sum tax.

The representative household maximizes its lifetime utility function given by

\[
U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} u(\tilde{C}_s, \tilde{C}_{s-1}, N_s),
\]

subject to (2), (4), (5), and (6). The operator \(E_t\) denotes the mathematical expectation
conditional on the information available up to time \(t\), and the parameter \(\beta \in (0,1)\) is the
subjective discount factor. First-order conditions associated with the optimal choice of

\(^{14}\)An alternative specification of habit formation, also found in the literature, assumes that the argument
that enters the utility function is the difference between the current level of (effective) consumption and the
habit stock. A drawback of this specification is that the utility function is not defined when consumption
falls below the habit stock. The specification used in this paper does not have this shortcoming.
\( C_t, N_t, \) and \( K_{t+1} \) are

\[
\lambda_t = \phi(\tilde{C}_t/C_t)^{1/\nu} \left\{ (1/\tilde{C}_{t-1}^\gamma)(\tilde{C}_t/\tilde{C}_{t-1}^\gamma)^{-\epsilon} - \beta\gamma E_t \left[ (\tilde{C}_{t+1}/\tilde{C}_t^\gamma)(\tilde{C}_t/\tilde{C}_t^\gamma)^{-\epsilon} \right] \right\}, \quad (7)
\]

\[
\lambda_t = \psi/ \left[ w_t (1 - N_t) \right], \quad (8)
\]

\[
\lambda_t = \beta E_t \left\{ \lambda_{t+1} \left[ 1 + r_{t+1} - \delta + \kappa (I_{t+1}/K_{t+1} - \delta) + (\kappa/2) (I_{t+1}/K_{t+1} - \delta)^2 \right] \right\} / \left[ 1 + \kappa (I_t/K_t - \delta) \right], \quad (9)
\]

where \( \lambda_t \) is the Lagrange multiplier associated with the budget constraint at time \( t \). Equation (7) defines the marginal utility of consumption. Equation (8) equates the marginal rate of substitution between consumption and leisure to the real wage. Equation (9) determines the marginal value of capital.

### 3.2 Firms

Firms hire labour and rent capital to produce a homogeneous final good using the following Cobb-Douglas technology:

\[
Y_t = A_t K_t^\alpha N_t^{1-\alpha}, \quad (10)
\]

where \( A_t \) is a stochastic technology shock that follows a first-order autoregressive process given by

\[
\ln A_t = (1 - \rho_A) \ln A + \rho_A \ln A_{t-1} + \mu_{At}, \quad (11)
\]

where \( \rho_A \) is strictly bounded between \(-1\) and \(1\), \( A \) is the steady-state value of \( A_t \), and \( \mu_{At} \) is a normally distributed zero-mean disturbance with standard deviation \( \sigma_{\mu_A} \). Each firm chooses labour and capital inputs to maximize its profit. Profit maximization yields the following input-demand equations:

\[
w_t = (1 - \alpha)Y_t/N_t, \quad (12)
\]

\[
r_t = \alpha Y_t/K_t, \quad (13)
\]

which state that each factor must earn its marginal product.

### 3.3 The government

Government purchases are entirely financed by taxes. That is,

\[
G_t = T_t. \quad (14)
\]

Because Ricardian equivalence holds in this model, introducing public debt would be redundant. We assume that government spending is stochastic and follows an autoregressive process given by

\[
\ln G_t = (1 - \rho_G) \ln G + \rho_G \ln G_{t-1} + \mu_{Gt}, \quad (15)
\]
where $\rho$ is strictly bounded between $-1$ and $1$, $G$ is the steady-state level of government spending, and $\mu_{Gt}$ is a normally distributed zero-mean disturbance with standard deviation $\sigma_{\mu_G}$.

### 3.4 Market clearing and equilibrium

Substituting equations (12), (13), and (14) into the budget constraint (6) yields the following resource constraint

$$Y_t = C_t + I_t + G_t + \varphi(I_t, K_t),$$

which is the national accounting identity augmented with capital adjustment costs.

A competitive equilibrium for this economy is a collection of nine sequences $(\lambda_t, \tilde{C}_t, C_t, N_t, I_t, K_{t+1}, Y_t, w_t, r_t)_{t=0}^{\infty}$ that satisfy (i) the definition of effective consumption (2), (ii) the accumulation equation (4), (iii) the household’s maximization conditions (equations (7) to (9)), (iv) the production function (10), (v) the profit maximization conditions (equations (12) and (13)), and (vi) the market-clearing condition (16), given the initial stocks of habit and capital, and the exogenous stochastic processes $(A_t, G_t)$.

To solve the model, we log-linearize the equilibrium conditions around a deterministic steady state where all variables are constant. This yields a system of stochastic linear difference equations that can be solved using standard methods.

### 4. The Model’s Implications

In this section, we examine the model’s implications regarding the effects of a government spending shock. More precisely, we illustrate the extent to which the model’s predictions depart from those of a standard RBC framework, and the role of the key features of the model in accounting for this departure.

From the log-linearized version of the model (see Appendix A), it is easy to show that, for a given level of private consumption, the effect of a change in government spending on the marginal utility of consumption is given by

$$\frac{\partial \hat{\lambda}_t}{\partial G_t} = \frac{(1 - \phi)}{G/C_t^{(\nu-1)/\nu}} \left\{ \frac{1/\nu - \epsilon - \beta \gamma (\epsilon - 1) (1 + \gamma - \rho G)}{1 - \beta \gamma} \right\}.$$  

(17)

Recall that our model nests the standard RBC case, which can be obtained by imposing the restrictions $\phi = 1$ and $\gamma = 0$.\(^{15}\) In this case, the right-hand side of equation (17) collapses to zero, so that government spending affects consumption only through the wealth channel.

\(^{15}\)The parameter $\nu$ becomes irrelevant in this case.
Figure 2 depicts the impulse responses to a 1 per cent government spending shock, generated by this version of the model. The responses are obtained using a plausible parameterization of the model. Specifically, we set the preference parameters $\beta$ and $\epsilon$ to 0.99 and 2, respectively. The elasticity of output with respect to capital, $\alpha$, the depreciation rate, $\delta$, and the capital-adjustment-cost parameter, $\kappa$, are calibrated to 0.36, 0.025, and 0, respectively. We let the steady-state level of technology, $A$, and the scaling parameter $\psi$ adjust so that, given the calibration of the other parameters, the proportion of time allocated to work in the steady state, $N$, is equal to 0.31. The steady-state ratio of government spending to output, $G/Y$, is set to 0.2, and the autocorrelation coefficient of the government spending shock, $\rho_G$, is chosen to be 0.9.

Figure 2 shows that a positive government spending shock decreases consumption and investment and increases hours worked and output. Intuitively, an increase in government spending means a lower permanent income for the representative household, and thus a lower private consumption. To prevent a large drop in consumption, the household increases its labour supply. But this substitution effect is not strong enough to offset the negative wealth effect on consumption. The increase in labour supply translates into a higher output and a lower real wage. Owing to consumption smoothing, consumption decreases less, in absolute value, than disposable income. Thus, the representative household must dissave and, as a consequence, investment decreases. In summary, the standard RBC model is clearly unable to account for the documented increase in private consumption following a government spending shock.

Now consider a version of the model where we keep abstracting from habit formation ($\gamma = 0$), but where effective consumption depends on government spending ($\phi < 1$). In this case, the derivative $\partial \hat{\lambda}_t / \partial \hat{G}_t$ has the same sign as the term $1/\nu - \epsilon$. When the elasticity of substitution, $\nu$, is lower than $1/\epsilon$, government spending raises the marginal utility of consumption, ceteris paribus. Hence, an increase in government purchases has not only a negative wealth effect on consumption, but also a positive effect that stems from the Edgeworth complementarity between private and public spending. The latter effect is stronger the smaller the value of $\nu$ relative to $1/\epsilon$. For sufficiently low values of $\nu$, the complementarity effect may actually offset the wealth effect, causing consumption to increase in equilibrium.

Figure 3 illustrates the impact of complementarity between private and public spending on the economy’s response to a government spending shock.\footnote{As stated earlier, for government spending to play a role in the utility function, the weight of private consumption in the CES aggregator must be strictly less than 1. Hence, we set $\phi$ to 0.8.} We consider three different scenarios by setting the elasticity of substitution, $\nu$, to 1, 0.45, and 0.25, respectively. Fig-
Figure 3 shows that, when \( \nu \) is equal to 1, a government spending shock produces a larger crowding-out effect on consumption than that predicted by the standard RBC model. Intuitively, because \( \nu \) is higher than \( 1/\epsilon \), government spending decreases the marginal utility of consumption (that is, private and public spending are Edgeworth substitutes), which reinforces the negative wealth effect. When \( \nu \) is set to 0.45, private and public spending become Edgeworth complements as government spending now raises the marginal utility of consumption. Figure 3 clearly shows that the complementarity effect mitigates the wealth effect, but the overall effect of the shock on consumption is still negative. This suggests that the complementarity between private consumption and government spending is rather weak in this case. The last scenario corresponds to the case \( \nu = 0.25 \). Under this parameterization, the complementarity effect is strong enough to dominate the wealth effect, so that private consumption is crowded-in by government spending. Note that the increase in consumption implies that labour supply rises more than in the standard RBC model, which should amplify the decline in the real wage. Comparing Figures 2 and 3 confirms these predictions.

Hence, the model with strong complementarity between private and public spending seems to solve the puzzling increase in consumption in response to a government spending shock. However, the model's success is only partial, as it fails to reproduce the non-monotonic response of consumption and the U-shaped response of investment obtained from the VAR.

Finally, consider the model augmented with habit formation in effective consumption (\( 0 < \gamma \leq 1 \)). In this case, the derivative \( \partial \hat{\lambda}_t / \partial \hat{G}_t \) has the same sign as the expression in brackets in equation (17). Straightforward calculation shows that the derivative is unambiguously decreasing in \( \gamma \) (given our assumption that \( \epsilon = 2 > 1 \)). That is, habit formation reduces the effect of government spending on the marginal utility of consumption, which, in turn, dampens the complementarity effect. The intuition behind this result is as follows. Habit-forming households smooth both the absolute level of consumption and its rate of change. As a result, the consumption response to shocks is smaller on impact and more gradual under habit formation than under time-separable preferences. This suggests that habit formation could help the model replicate the non-monotonic consumption response predicted by the VAR.

Figure 4 illustrates the effect of habit formation on the impulse-response functions generated by the model. The figure depicts the responses obtained using three values of the parameter \( \gamma \), 0, 0.5, and 0.8, holding the intratemporal elasticity of substitution, \( \nu \), fixed at 0.25. A comparison of the impulse responses under time-separability (\( \gamma = 0 \)) and habit-forming preferences (\( \gamma = 0.5 \) and 0.8) shows that habit formation introduces a non-monotonicity
in the impulse response of consumption and investment. Note, however, that when habit formation is relatively important ($\gamma = 0.8$), private consumption decreases on impact. This occurs because large values of $\gamma$ attenuate the complementarity effect so severely that it fails to dominate the wealth effect. On the other hand, with a moderate amount of habit formation ($\gamma = 0.5$), the consumption response is positive on impact and reaches its peak with a delay, as predicted by the VAR.

In summary, to the extent that complementarity between private and public spending is strong, and habit formation is moderate, the model presented in Section 3 is capable of replicating the crowding-in effect of government spending on consumption as well as the non-monotonic responses of consumption and investment predicted by the VAR.

5. Estimation Methodology

The purpose of this section is to use U.S. data to obtain values for the model parameters. In particular, we are interested in measuring the extent of complementarity between public expenditures and private consumption and the degree of habit formation. We describe below our estimation strategy.

The model’s solution can be written in the following state-space form:

$$X_{t+1} = FX_t + \mu_{t+1}, \quad (18)$$
$$P_t = QX_t, \quad (19)$$

where $X_t = (A_t, G_t, K_t, \hat{C}_{t-1})'$ is a $4 \times 1$ vector that contains the state variables of the model; $P_t = (\lambda_t, \hat{C}_t, \hat{N}_t, \hat{\bar{I}}_t, \hat{\bar{Y}}_t, \hat{\bar{\ell}}_t)'$ is an $8 \times 1$ vector that contains the forward-looking variables; $\mu_t = (\mu_{A,t}, \mu_{G,t})'$ is a $2 \times 1$ vector that contains the innovations of the shocks; and $F$ and $Q$ are, respectively, $4 \times 4$ and $8 \times 4$ matrices, the elements of which are combinations of the model parameters.

The transition equation (18) and a measurement equation that collects a subset of the variables included in $P_t$ can be estimated by the ML method. The likelihood function can be evaluated recursively using the Kalman filter. This method has been used to estimate dynamic stochastic general-equilibrium models by (among others) McGrattan (1994), McGrattan, Rogerson, and Wright (1997), Kim (2000), Ireland (2001), Dib (2003), and Bouakez, Cardia, and Ruge-Murcia (2005). A crucial requirement of this method is that the number of observable variables used in the estimation does not exceed the number of shocks in the model; otherwise, the variance-covariance matrix of the residuals becomes singular, in which case the ML procedure fails. In our case, this would imply estimating the model parameters using no more than two series, since we have only two structural shocks.
One way to circumvent this problem is to add measurement errors to the variables in the measurement equation.\textsuperscript{17} This yields the following empirical model:

\begin{align*}
X_{t+1} &= FX_t + \mu_{t+1}, \\
Y_t &= HX_t + \eta_t,
\end{align*}

where $Y_t$ is the vector of observable variables, $\eta_t$ is the vector of measurement errors, and $H$ is a matrix of appropriate size, the elements of which are combinations of the model parameters. These elements are computed from the model's solution in each iteration of the optimization procedure. Thus, the estimation procedure takes into account the cross-equation restrictions implied by the theoretical model. In addition, we numerically restrict each estimated parameter to lie within its economically meaningful interval.

The series used in the estimation are consumption, hours worked, output and investment (see Section 2 for a description of the data). Adding the real wage to the list of observable variables yields almost identical results, while making convergence time-consuming.

Preliminary attempts to estimate all the model parameters were not successful. On the one hand, the parameters $\phi$ and $\epsilon$ turn out to be poorly identifiable, as one might suspect by inspecting the log-linearized versions of equations (2) and (7). On the other hand, in every attempt to estimate the parameters $\alpha$ and $\delta$, the algorithm keeps searching the parameter space either until it reaches the maximum number of iterations, or until it runs into regions where the model solution cannot be computed, and eventually crashes. To circumvent these issues, we set $\epsilon = 2$, $\phi = 0.8$, $\alpha = 0.36$, and $\delta = 0.025$ prior to estimation.\textsuperscript{18} These values are summarized in Table 1. The latter two values are widely accepted in the RBC literature. In contrast, there is less agreement about the value of the curvature parameter $\epsilon$, and very few empirical estimates of the parameter $\phi$ exist. For this reason, we performed an extensive sensitivity analysis to check the robustness of the results to different values of these two parameters. We found that the paper's main findings were robust to alternative plausible values of $\phi$ and $\epsilon$.\textsuperscript{19} The remaining parameters to be estimated by ML are $\nu$, $\gamma$, $\beta$, $\kappa$, $\rho_G$, $\rho_A$, $\sigma_{\mu_G}$, and $\sigma_{\mu_A}$.

\textsuperscript{17}The addition of measurement errors to get around the singularity problem has been done by McGrattan, Rogerson, and Wright (1997), Ireland (2004), and Bouakez (2005).

\textsuperscript{18}The steady-state quantities $G/Y$ and $N$ are also set to their respective values used in section 3.

\textsuperscript{19}Higher (lower) values of $\phi$ and $\epsilon$ result in a (higher) lower estimate of the parameter $\nu$. In all cases, however, the condition for Edgeworth complementarity between private and public spending holds, and the estimated model is able to generate a positive non-monotonic response of consumption to a government spending shock. To conserve space, these results are not reported, but are available from the authors upon request.
6. Results

6.1 Parameter estimates

Table 2 reports the estimation results.\textsuperscript{20} Figures between parentheses are standard errors, which are computed as the square root of the diagonal elements of the inverted Hessian of the (negative) log likelihood function evaluated at the maximum. At the estimated parameters, the condition for existence of a unique solution to the model is satisfied. That is, the number of explosive eigenvalues of the system of linear difference equations equals the number of non-predetermined variables.

The ML procedure yields a precise estimate of the elasticity of substitution, $\nu$, (0.332). Interestingly, this value is statistically lower than $1/\epsilon$. That is, the necessary condition for government spending to increase the marginal utility of consumption is satisfied, meaning that private and public spending are Edgeworth complements. This result contradicts earlier findings by Aschauer (1985), who finds that government spending and private consumption are Edgeworth substitutes in the United States. Amano and Wirjanto (1998), on the other hand, find that U.S. private and public expenditures are best described as unrelated. Karras (1994) examines evidence from 30 countries and concludes that the two aggregates are complements. A more recent contribution by Okubo (2003) confirms this result for Japan. These earlier studies are similar in that they use a partial-equilibrium approach based on Euler equations. However, they differ in many details that could explain the disparity of results. For example, some of these studies assume that private consumption and government purchases enter effective consumption linearly, whereas in other studies, the two arguments are non-separable. Ni (1995) finds private and public spending to be substitutes in the former case, and complements in the latter. His results, however, indicate that the non-separable specification is better supported by the data. Among the studies cited above that use U.S. data, only the one by Amano and Wirjanto (1998) assumes that effective consumption is a (non-linear) CES aggregator of private consumption and government spending. Although this paper also uses a CES specification, our estimate of $\nu$ cannot be directly compared to theirs for several reasons. First and most importantly, our estimate of the elasticity of substitution is conditional on the calibrated value of the curvature parameter $\epsilon$ (see footnote 19), while Amano and Wirjanto obtain an unconditional estimate by applying cointegration techniques to an intraperiod first-order condition. Second,

\textsuperscript{20}In the working-paper version of this article (Bouakez and Rebei 2003), we also estimated the model parameters by minimizing the distance between the impulse-response functions generated by the model and those obtained from a benchmark VAR. The results were very similar to those obtained using the ML method.
these authors assume that the utility function is time separable. In contrast, preferences in our model exhibit habit formation. This is an important distinction because, as shown by Ni (1995), time non-separability largely affects the estimates of substitutability between private and public spending. Third, unlike Amano and Wirjanto who use raw data, the empirical analysis in this paper is based on detrended data.

The estimate of the habit-formation parameter, $\gamma$, is lower than those reported by Heaton (1995), Fuhrer (2000), and Bouakez, Cardia, and Ruge-Murcia (2005), for example, and indicates a limited extent of habit-forming behaviour. The adjustment cost parameter, $\kappa$, is found to be small and statistically insignificant. Traditionally, very low values of the parameter $\kappa$ have been used to calibrate RBC models. Some models, such as that developed by Hansen (1985), do not even have capital adjustment costs. Finally, the estimated values of the autocorrelation coefficients $\rho_G$ and $\rho_A$ and the standard deviations $\sigma_{\mu_G}$ and $\sigma_{\mu_A}$ are precise and plausible.

6.2 Impulse-response functions

In this section, we investigate whether and to which extent the estimated model is able to account for the documented dynamic effects of government spending shocks and, in particular, the crowding-in effect on private consumption. Figure 5 compares the impulse-response functions implied by the estimated model with those obtained from the benchmark VAR. The size of the shock is normalized to 1 per cent.

Overall, the estimated model is successful in replicating the impulse responses obtained from the VAR. In particular, it generates a positive and non-monotonic response of consumption to an increase in government spending, with a peak at one quarter after the shock, exactly as predicted by the VAR. The response predicted by the model, however, is larger in magnitude at any given quarter than its VAR-based counterpart. The model also overpredicts the response of hours worked, and cannot account for the sign changes observed in the VAR. On the other hand, the estimated model performs remarkably well in matching the response of output and investment. In both cases, the model-based response is quantitatively similar to its VAR-based counterpart both in terms of magnitude and persistence.

The only major discrepancy between the model and the VAR is the response of the real wage. This outcome is to be expected given the simulation results discussed in Section 4. As explained above, neoclassical models generically imply that the real wage must decrease following a government spending shock because the resulting increase in labour supply drives the marginal product of labour down. The VAR, on the other hand, shows that the real
wage reacts weakly but positively to an increase in government spending.

6.3 Second moments

Having shown that the estimated model is, by and large, capable of replicating the conditional covariance of government spending and key economic variables, this section evaluates its ability to match the unconditional moments of the data. For this purpose, we report in Table 3 some selected historical moments and their counterparts predicted by the model. For a formal comparison between the model and the data, we estimate the historical moments using GMM and compute their standard deviations. The model’s performance is compared with that of the standard RBC model.

Table 3 shows that both the standard RBC model and the estimated model successfully account for the volatility of consumption, investment, the real wage, and government spending, but underpredict the volatility of hours worked. The two models replicate reasonably well the first-order autocorrelation of consumption, hours worked, investment, and government spending, as well as their correlation with output. However, both models generate too much persistence in the real wage, and predict that this variable is countercyclical, whereas it is essentially acyclical in the data. Note that although the estimated model improves upon the standard RBC model in almost any of these dimensions, the predictions generated by the two models remain quantitatively similar.

The two models diverge sharply, however, regarding the predicted correlation between consumption and government spending. This correlation is positive in the data, but the standard RBC model generates a negative value. On the other hand, although somewhat high, the correlation predicted by the estimated model has the correct sign.

The table also shows that both models counterfactually imply a negative comovement between government spending and the real wage, and that the estimated model does marginally worse than the standard RBC model in this dimension. The reason for this result is that the increase in consumption following a positive government spending shock requires that labour supply rises more in the estimated model than in the standard RBC model, which amplifies the decline in the real wage.

In view of these results, we may conclude that, overall, the model developed in this paper captures the (unconditional) covariance of consumption and government spending better than a standard RBC model, without worsening the dynamics of remaining variables. Admittedly, however, the model’s failure to account for the behaviour of the real wage is an important weakness that needs to be addressed. Perhaps for this reason, the proposed model should be regarded as a step towards a more elaborate theory of business-cycle fluctuations.
7. Conclusion

The purpose of this paper was to explain the puzzling crowding-in effect of government spending on private consumption. Departing from standard RBC models, we have assumed that public expenditures affect consumer preferences, and that those preferences exhibit habit formation in consumption. Rather than using calibration to assess the relevance of these two features, we estimated the model parameters using U.S. data. Estimation results reveal a strong complementarity between public and private spending, and a fairly limited extent of habit formation. The estimated model performs well in replicating the observed impulse responses of the key variables to a government spending shock. In particular, it is able to account for the documented crowding-in effect on consumption. Moreover, the consumption response generated by the model has similar dynamics to the response obtained from a benchmark VAR, both in terms of magnitude and persistence. Finally, the model outperforms the standard RBC model in matching the unconditional moments of the data.

Two important remarks about our results are worth noting. First, as emphasized by Karras (1994), the fact that private and public spending are found to be complements should not be considered valid for all types of publicly provided goods but only as holding in the aggregate.²¹

Second, throughout the paper, we have assumed that government spending is an exogenous variable. While this assumption is plausible from the households’ perspective, the fact that government spending affects the utility function implies that there is scope for optimal fiscal policy. Therefore, a natural extension of the model would be to allow for an optimizing government that chooses public spending endogenously to maximize the households’ welfare. This is left for future research.

²¹ Yet, one can think of a variety of specific cases where public spending can increase private consumption. Examples include education, transportation, and communication.
References


Appendix A: The Log-Linearized Model

In this appendix, variables without time subscripts denote steady-state values, and the circumflex denotes percentage deviation from steady state. Linearizing equations (2), (4), (7), (8), (9), (10), (12), (13), and (16) yields

\[
\hat{C}_t = \phi \left( \frac{C}{\hat{C}} \right)^{(u-1)/u} \hat{C}_t + (1 - \phi) \left( \frac{G}{\hat{C}} \right)^{(u-1)/u} \hat{G}_t,
\]

\[
\hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \delta \hat{I}_t,
\]

\[
\hat{\lambda}_t = \frac{\gamma (\epsilon - 1)}{1 - \beta \gamma} E_t \hat{C}_{t+1} - \frac{\beta \gamma (\gamma (\epsilon - 1) - 1) + \epsilon - (1 - \beta \gamma)/\nu}{1 - \beta \gamma} \hat{C}_t + \frac{\gamma (\epsilon - 1)}{1 - \beta \gamma} \hat{C}_{t-1} - \frac{1}{\nu} \hat{C}_t,
\]

\[
\hat{\lambda}_t = \frac{N}{1 - N} \hat{N}_t - \hat{w}_t,
\]

\[
\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \beta r E_t \hat{r}_{t+1} - \kappa (1 + \beta \delta) \hat{K}_{t+1} + \kappa \hat{K}_t + \beta \kappa \delta E_t \hat{I}_{t+1},
\]

\[
\hat{Y}_t = \alpha \hat{K}_t + (1 - \alpha) \hat{N}_t + \hat{A}_t,
\]

\[
\hat{w}_t = \hat{Y}_t - \hat{N}_t,
\]

\[
\hat{r}_t = \hat{Y}_t - \hat{K}_t,
\]

\[
\hat{Y}_t = \frac{C}{Y} \hat{C}_t + \frac{I}{Y} \hat{I}_t + \frac{G}{Y} \hat{G}_t.
\]

The stochastic processes of the shocks, (11) and (15), are already linear. Using the same notation as above, they are rewritten as

\[
\hat{A}_{t+1} = \rho_A \hat{A}_{t-1} + \mu_A,
\]

\[
\hat{G}_{t+1} = \rho_G \hat{G}_{t-1} + \mu_G.
\]
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural parameters</strong></td>
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<td></td>
</tr>
<tr>
<td>Weight of private spending in effective consumption</td>
<td>( \phi )</td>
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</tr>
<tr>
<td>Curvature parameter</td>
<td>( \epsilon )</td>
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</tr>
<tr>
<td>Elasticity of output with respect to capital</td>
<td>( \alpha )</td>
<td>0.36</td>
</tr>
<tr>
<td>Depreciation rate of capital</td>
<td>( \delta )</td>
<td>0.025</td>
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<tr>
<td><strong>Steady-state values</strong></td>
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</tr>
<tr>
<td>Fraction of time worked</td>
<td>( N )</td>
<td>0.31</td>
</tr>
<tr>
<td>Government-spending-to-output ratio</td>
<td>( G/Y )</td>
<td>0.2</td>
</tr>
<tr>
<td>Description</td>
<td>Parameter</td>
<td>Estimate</td>
</tr>
<tr>
<td>------------------------------</td>
<td>-----------</td>
<td>------------</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\nu$</td>
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</tr>
<tr>
<td>Habit-formation parameter</td>
<td>$\gamma$</td>
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<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
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<td>Adjustment-cost parameter</td>
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<td>Autocorrelation coefficient of $A_t$</td>
<td>$\rho_A$</td>
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<tr>
<td>Autocorrelation coefficient of $G_t$</td>
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<tr>
<td>Standard deviation of $\mu_{Gt}$</td>
<td>$\sigma_{\mu_G}$</td>
<td>0.0127</td>
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</table>

† Figures between parentheses are standard errors. The restrictions imposed on the parameters are $\gamma, \beta \in (0, 1), \rho^A, \rho^G \in (-1, 1)$, and $\nu, \kappa, \sigma_{\mu_A}, \sigma_{\mu_G} \in (0, \infty)$. 

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Table 3. Actual and Predicted Moments

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data†</th>
<th>Standard RBC model‡</th>
<th>Estimated model‡</th>
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<tr>
<td></td>
<td>Estimate</td>
<td>Std. error</td>
<td></td>
</tr>
<tr>
<td><strong>Std. Deviation Relative to Y</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.49 (0.02)</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>H</td>
<td>1.07 (0.05)</td>
<td>0.55</td>
<td>0.65</td>
</tr>
<tr>
<td>I</td>
<td>2.90 (0.30)</td>
<td>2.26</td>
<td>2.36</td>
</tr>
<tr>
<td>w</td>
<td>1.03 (0.10)</td>
<td>0.78</td>
<td>0.85</td>
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<tr>
<td>G</td>
<td>2.24 (0.57)</td>
<td>1.71</td>
<td>1.74</td>
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<tr>
<td><strong>Autocorrelation</strong></td>
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<tr>
<td>C</td>
<td>0.88 (0.15)</td>
<td>0.98</td>
<td>0.96</td>
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<tr>
<td>H</td>
<td>0.88 (0.11)</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td>I</td>
<td>0.90 (0.13)</td>
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<td>0.87</td>
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<tr>
<td>w</td>
<td>0.44 (0.12)</td>
<td>0.96</td>
<td>0.96</td>
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<tr>
<td>G</td>
<td>0.94 (0.62)</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>Correlation with Y</strong></td>
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</tr>
<tr>
<td>(Y,C)</td>
<td>0.81 (0.12)</td>
<td>0.63</td>
<td>0.62</td>
</tr>
<tr>
<td>(Y,H)</td>
<td>0.87 (0.12)</td>
<td>0.62</td>
<td>0.53</td>
</tr>
<tr>
<td>(Y,I)</td>
<td>0.73 (0.10)</td>
<td>0.94</td>
<td>0.87</td>
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<tr>
<td>(Y,w)</td>
<td>0.27 (0.09)</td>
<td>0.83</td>
<td>0.75</td>
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<tr>
<td>(Y,G)</td>
<td>0.35 (0.20)</td>
<td>0.10</td>
<td>0.17</td>
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<tr>
<td><strong>Correlation with G</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(G,C)</td>
<td>0.26 (0.14)</td>
<td>−0.27</td>
<td>0.62</td>
</tr>
<tr>
<td>(G,H)</td>
<td>0.22 (0.15)</td>
<td>0.41</td>
<td>0.58</td>
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<tr>
<td>(G,I)</td>
<td>−0.03 (0.07)</td>
<td>−0.21</td>
<td>−0.31</td>
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<tr>
<td>(G,w)</td>
<td>0.13 (0.14)</td>
<td>−0.16</td>
<td>−0.25</td>
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</table>

† The statistics are based on logged and H-P filtered quarterly data for the period 1948Q1–2005Q4. Figures between parentheses are standard errors.
‡ The standard deviation and the autocorrelation of the technology shock are chosen so that the standard deviation and the first-order autocorrelation of output are the same as in the data: 0.0177 and 0.88, respectively.
Figure 1: VAR-based impulse responses to a 1 per cent government spending shock. Solid lines: impulse responses, dotted lines: error bands.
Figure 2: Impulse responses to a 1 per cent government spending shock: Standard RBC model
Figure 3: Impulse responses to a 1 per cent government spending shock for different values of \( \nu \) (\( \gamma = 0 \)).
Solid lines: \( \nu = 1 \), dashed lines: \( \nu = 0.45 \), dotted lines: \( \nu = 0.25 \)
Figure 4: Impulse responses to a 1 per cent government spending shock for different values of $\gamma$ ($\nu = 0.25$).

Solid lines: $\gamma = 0$, dashed lines: $\gamma = 0.5$, dotted lines: $\gamma = 0.8$
Figure 5: Impulse responses to a 1 per cent government spending shock.
Solid lines: VAR-based responses, dotted lines: error bands, dashed lines: model-based responses