Terms of Trade and Current Account Fluctuations: The Harberger-Laursen-Metzler Effect Revisited[*]

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Abstract

This paper investigates whether extending the intertemporal model of the current account to allow for variations in the terms of trade improves its ability to fit the data. It derives a testable present-value representation of the current account that encompasses the Harberger-Laursen-Metzler (HLM) effect, according to which a temporary rise in the terms of trade improves the current account. The present-value model is tested using data from Australia, Canada, and the United Kingdom. The results show that terms-of-trade movements do not affect the current account in a significant way, and that, in two of the three cases, the model is strongly rejected by the data.

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1. Introduction

Since the seminal work of Sachs (1981), the intertemporal approach to the current account has become a popular tool to study the indebtedness of countries and the borrowing and lending behaviour in international financial markets. In a nutshell, the intertemporal approach explains current account fluctuations as the result of optimal saving and investment behaviour of rational, forward-looking economic agents in an open economy.[1] One of its testable implications is well known as the present-value model (PVM) of the current account. In its simplest form, the PVM implies that the current account is determined by the discounted sum of expected future changes in the economy's cash flow (or net output), and can be described as the small-open-economy version of Campbell's (1987) "rainy day" equation.[2]

The standard PVM of the current account has been evaluated by many studies, using time-series data from different countries, over various sample periods, and at different frequencies.[3] The common result of these studies is that the standard PVM fails to explain postwar current account fluctuations of typical small open economies such as Australia, Canada, and the United Kingdom. Indeed, the cross-equation restrictions that the standard PVM imposes on an unrestricted vector autoregression (VAR) are often jointly rejected in postwar data. Moreover, the current account series predicted by the standard PVM is often found to be too smooth compared with the actual series, a result that was characterized by Ghosh (1995) as the "excess smoothness puzzle" of the current account.

Though using different analytical frameworks, two recent papers by Bergin and Sheffrin (2000) and Nason and Rogers (2004) emphasize the importance of stochastic variations in relative prices as a potential explanation for current account fluctuations.[4] More precisely, Bergin and Sheffrin

[1]The small-open-economy, optimal-growth model of Hamada (1966) is an explicit precursor to the intertemporal approach to the current account. An excellent review of this approach can be found in Obstfeld and Rogoff (1995).

[2]We will refer to this version of the present-value model as the standard PVM.

[3]See, for example, Sheffrin and Woo (1990), Otto (1992), Ghosh (1995), and Bergin and Sheffrin (2000).

[4]Alternative explanations have been proposed by Normandin (1999), İşcan (2002) and Gruber (2004). Normandin (1999) shows that adopting an overlapping-generation framework, and introducing government bonds as part of household financial wealth help the standard model explain current account movements in Canada and the United States. İşcan (2002) shows that extending the standard PVM to allow for durable consumption can be helpful in explaining the Canadian current account. Gruber (2004) finds that habit formation in consumption improves the ability of the PVM to fit actual current account series in a number of small open economies such as Canada, Italy, the Netherlands, and the United Kingdom.

(2000) show that amending the standard intertemporal model of the current account to include variable interest rates and exchange rates improves its fit substantially. Nason and Rogers (2004), on the other hand, show that, among the features that are suspected to be responsible for the rejection of the standard PVM, world real interest rate shocks bring the "canonical" small-open economy, real-business-cycle model closest to the data.[5]

In this paper, we develop a small-open-economy model with tradable and non-tradable goods and a time-varying world real interest rate. Unlike Bergin and Sheffrin (2000), however, we distinguish between exportable and imported goods, thus introducing an additional relative price, i.e., the terms of trade. Terms-of-trade shocks are widely regarded as a major force driving business cycle fluctuations in small open economies.[6] This view has become even more popular after the oil-price shock in the early 1970s. Indeed, the subsequent two decades witnessed a secular decline in commodity prices along with an increase in their volatility.[7] In the same time, commodity-exporting countries, such as Australia and Canada, experienced an increase in their current account variability. This suggests that terms-of-trade shocks might be important in explaining current-account movements in these countries.

The effects of terms-of-trade movements on the current account have been initially studied by Harberger (1950) and Laursen and Metzler (1950), who show, using a Keynesian model, that an exogenous rise in the terms of trade of a small open economy leads to an improvement in its trade balance. The reason is obvious: an improvement in a country's terms of trade raises its current income, and given a marginal propensity to consume less than unity, current consumption increases less than current income, causing private saving to increase. This so-called Harberger-Laursen-Metzler (HLM) effect has subsequently been examined within deterministic intertemporal models by Sachs (1981), Obstfeld (1982), and Svensson and Razin (1983), among others. More recently, the HLM effect was recast within dynamic general-equilibrium models by Backus (1993) and Mendoza (1995), for example.

[5]No consensus has been reached in the literature regarding the importance of real interest rate variations for aggregate fluctuations in a small open economy. For example, Hercowitz (1986) and Blankenau, Kose, and Yi (2001) provide evidence that real interest rate variations matter for aggregate fluctuations, while Mendoza (1991) finds no significant role for the real interest rate.

[6]See Mendoza (1995), among others.

[7]See Reinhart and Wickham (1994).

This paper derives an approximate closed-form solution for the present-value representation of the current account which encompasses the HLM effect in addition to the usual consumption-smoothing motive and the effects of future changes in the interest rate and the exchange rate, highlighted in Bergin and Sheffrin (2000). Moreover, while Bergin and Sheffrin (2000) identify only the intertemporal substitution effect of expected future changes in the world real interest rate, our PVM allows us to study not only the intertemporal substitution effect but also the income and wealth effects associated with a change in the world real interest rate, as well as its instantaneous effect on net foreign interest payments.

The cross-equation restrictions implied by the extended PVM are tested using quarterly data from Australia, Canada, and the United Kingdom. The results show that for Australia and Canada, the model is strongly rejected by the data, while it cannot be rejected at conventional levels of significance for the United Kingdom. In all three cases, however, the extended model does not improve upon the fit of a benchmark PVM that allows for variable interest rates and exchange rates but excludes the terms of trade. This suggests that terms-of-trade shocks play little role, if any, in explaining current-account fluctuations in Australia, Canada, and the United Kingdom.

The rest of the paper is organized as follows. Section 2 discusses the effects of the terms of trade on the current account within a simple two-period deterministic model. Section 3 presents a stochastic infinite-horizon version of the model and derives a testable closed-form solution for the current account. Section 4 explains the testing procedure, describes the data, and discusses the results. Section 5 concludes.

2. A Two-Period Deterministic Model

The basic insights of the effects of the terms of trade on the current account can be better understood within a simple two-period deterministic model. Consider a small open economy where a representative household consumes a mix of tradable and non-tradable goods and has the following lifetime utility:

$$U_t = \frac{C_t^{1-1/\sigma}}{1-1/\sigma} + \beta \frac{C_{t+1}^{1-1/\sigma}}{1-1/\sigma}, \qquad 0 < \beta < 1, \quad \sigma > 0, \tag{1}$$

where β is the subjective discount factor and σ is the elasticity of intertemporal substitution. The consumption index, C_t, is a Cobb-Douglas aggregator given by

$$C_t = \omega_1 \left(C_t^T\right)^{\epsilon} \left(C_t^N\right)^{1-\epsilon}, \qquad 0 < \epsilon < 1, \tag{2}$$

where C_t^T is consumption of the tradable good, C_t^N is consumption of the non-tradable good, ϵ is weight of the tradable good in the consumption basket, and $\omega_1 \equiv \epsilon^{-\epsilon}(1-\epsilon)^{\epsilon-1}$ is a positive parameter. The Cobb-Douglas type aggregator (equation (2)) implies unit elasticity of intratemporal substitution between the two types of goods. We choose the the tradable good to be the numeraire, and we normalize its price to 1.[8] Denoting by Q_t the price of the non-tradable good, i.e., the real exchange rate, the demands for tradable and non-tradable goods are given by, respectively

$$C_t^T = \epsilon P_t^c C_t, \tag{3}$$

and

$$Q_t C_t^N = (1-\epsilon) P_t^c C_t, \tag{4}$$

where P_t^c is the consumption-based price index given by

$$P_t^c = Q_t^{1-\epsilon}. \tag{5}$$

As in Obstfeld (1996) and Obstfeld and Rogoff (1996, p. 266), we assume that the tradable good is itself a Cobb-Douglas aggregator of domestic exportable goods and imported goods,

$$C_t^T = \omega_2 X_t^{\gamma} M_t^{1-\gamma}, \qquad 0 < \gamma < 1, \tag{6}$$

where X_t is consumption of exportable goods, M_t is consumption of imported goods, γ is the weight of exportable goods in the traded-good basket, and ω_2 is a constant given by $\omega_2^{-1} = \gamma^{\gamma}(1-\gamma)^{1-\gamma}$. Denoting by P_t^x and P_t^m the prices of the exportable and imported goods, respectively, the demands for these goods are given by

$$P_t^x X_t = \gamma C_t^T \tag{7}$$

[8]This choice of the numeraire is not innocuous: it implies that the terms of trade do not affect the price of the tradable good. As will become clear further, this means that there are no substitution or income effects associated with changes in the consumption-based real interest rate that are induced by terms-of-trade variations (see Cashin and McDermott 2002 for a discussion of these effects). This will allow us to isolate the HLM effect as the only channel through which the terms of trade affect the current account.

and

$$P_t^m M_t = (1-\gamma) C_t^T. \tag{8}$$

In addition, the following condition must hold:

$$1 = (P_t^x)^{\gamma} (P_t^m)^{1-\gamma}. \tag{9}$$

Notice that from this equation, the terms of trade, defined as the relative price of exports in terms of imports, can be expressed as a function of the price of the exportable good

$$P_t \equiv P_t^x / P_t^m = (P_t^x)^{1/(1-\gamma)}. \tag{10}$$

The only tradable assets in this economy are one-period risk-free international bonds, which are indexed to the tradable consumption basket. International financial markets are perfect but incomplete.[9] That is, the representative household can borrow and lend freely at the world common real interest rate, r_{t+1}, to smooth consumption across the two periods, but cannot buy or sell state-contingent claims to diversify away idiosyncratic shocks. The assumption of a small open economy requires that this economy cannot affect the world real interest rate or the terms of trade.

At the beginning of period t, the representative household receives an endowment of exportable and non-tradable net outputs, denoted by NY_t^x and NY_t^n, respectively. It allocates its income to consumption of non-tradable, exportable, and imported goods. If total current consumption expenditure is less than current income, the households lends the difference to the rest of the world by acquiring international bonds. In period $t+1$, the household receives interest payments on its holdings of bonds. This additional income is used to finance consumption in period $t+1$.[10] Therefore, the household's budget constraints in periods t and $t+1$ (expressed in terms of the tradable consumption basket) are, respectively

$$B_{t+1} = P_t^x NY_t^x + Q_t NY_t^n - P_t^x X_t - P_t^m M_t - Q_t C_t^N$$

and

$$P_{t+1}^x X_{t+1} + P_{t+1}^m M_{t+1} + Q_{t+1} C_{t+1}^N = (1 + r_{t+1}) B_{t+1} + P_{t+1}^x NY_{t+1}^x + Q_{t+1} NY_{t+1}^n.$$

[9] See footnote 17.

[10] Since households do not live beyond period $t+1$, it must be the case that $B_{t+2} = 0$.

Combining these two equations and using the fact that $P_t^x X_t + P_t^m M_t + Q_t C_t^N = P_t^c C_t$, we obtain the following intertemporal budget constraint:

$$P_t^c C_t + \frac{1}{1+r_{t+1}} P_{t+1}^c C_{t+1} = P_t^x NY_t^x + Q_t NY_t^n + \frac{1}{1+r_{t+1}} \left[P_{t+1}^x NY_{t+1}^x + Q_{t+1} NY_{t+1}^n \right]. \qquad (11)$$

The representative household maximizes its lifetime utility (equation (14)) subject to its intertemporal budget constraint (equation (11)). The first-order condition for this problem yields the following Euler equation:

$$1 = \beta(1+r_{t+1}) \left(\frac{P_t^c}{P_{t+1}^c} \right) \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma}. \qquad (12)$$

This equation implies that the relevant interest rate for smoothing consumption between periods t and $t+1$ is the consumption-based world real interest rate $(1+r_{t+1}) \left(P_t^c / P_{t+1}^c \right)$. Other things being equal, a rise in the consumption-based price index in period $t+1$ lowers the one-period gross rate of return of foregoing one unit of consumption at time t.

In equilibrium, the condition $NY_t^n = C_t^N$ must hold. Thus, the budget constraint in period t can be written as

$$\begin{aligned} B_{t+1} &= P_t^x NY_t^x - P_t^x X_t - p_t^m M_t \\ &= P_t^x NY_t^x - C_t^T \\ &\equiv CA_t, \end{aligned} \qquad (13)$$

where the second equality follows from the fact that $C_t^T = P_t^x X_t + P_t^m M_t$, and where CA_t denotes the current account.

Substituting equation (12) into the intertemporal budget constraint (equation (11)), and using equations (3), (4), (5), and the market-clearing condition $C_t^N = NY_t^n$, we obtain

$$C_t^T = \frac{1}{1 + \beta^\sigma (1+r_{t+1})^{\sigma-1} \left(Q_t / Q_{t+1} \right)^{(\sigma-1)(1-\epsilon)}} \left[P_t^{1-\gamma} NY_t^x + \frac{1}{1+r_{t+1}} P_{t+1}^{1-\gamma} NY_{t+1}^x \right].$$

Substituting this equation into the current account identity (equation (13)) yields

$$CA_t = P_t^{1-\gamma} NY_t^x - \frac{1}{1 + \beta^\sigma (1+r_{t+1})^{\sigma-1} \left(Q_t / Q_{t+1} \right)^{(\sigma-1)(1-\epsilon)}} \left[P_t^{1-\gamma} NY_t^x + \frac{1}{1+r_{t+1}} P_{t+1}^{1-\gamma} NY_{t+1}^x \right].$$

This expression encompasses the effects of changes in the world real interest rate, the real exchange rate, and the terms of trade on the current account. A rise in the world real interest rate, r_{t+1}, has three distinct effects on the current account. First, a rise in r_{t+1} increases the consumption-based real interest rate, which raises the price of current consumption in terms of future consumption thus giving households an incentive to tilt their consumption towards the future. This intertemporal substitution effect improves the current account as households save more in the current period. Second, by lowering the price of future consumption, a rise in the consumption-based real interest rate expands the feasible consumption set for a given present value of lifetime resources. This income effect induces households to increase their current consumption and reduce their saving, which worsens the current account. Note that when $\sigma = 1$ (in which case the utility function is logarithmic), the substitution and income effects offset each other exactly. When σ is strictly higher (lower) than 1, the substitution (income) effect dominates. Finally, a rise in the world real interest rate lowers the market discount factor and therefore the present value of lifetime income. This negative wealth effect acts to reduce present consumption and to improve the current account.[11]

A rise in the real exchange rate, Q_t, also increases the consumption-based real interest rate. Hence, it has similar intertemporal substitution and income effects to those discussed above. The intertemporal substitution (income) effect reduces (raises) current consumption of tradable goods–and therefore total current consumption–and improves (worsens) the current account.[12]

Finally, an improvement in the terms of trade, that is, a rise in P_t, raises the present value of lifetime income and leads households to increase their current consumption. Because the fraction of lifetime income devoted to current consumption is less than 1, current consumption rises less than current income and the current account improves. This is the HLM effect. On the other hand, since the terms of trade do not affect the consumption-based real interest rate, they do not have any consumption-tilting effects.

[11] See Obstfeld and Rogoff (1996, chapter 1) for a further discussion of the effects of changes in the real interest rate on consumption and the current account.

[12] The positive effect of a rise in the relative price of non-traded goods, Q_t, on current consumption of traded goods can be interpreted as an *intratemporal* substitution effect.

3. A Stochastic Infinite-Horizon Model

To construct and empirically evaluate a present value representation of the current account with time-varying terms of trade, we need a stochastic infinite-horizon version of the small-open-economy model presented in the previous section. In the infinite-horizon model, the household's lifetime utility function is

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \frac{C_{t+i}^{1-1/\sigma}}{1-1/\sigma}, \qquad 0 < \beta < 1, \quad \sigma > 0, \tag{14}$$

where E_t is the mathematical expectation operator conditional on the information available at time t. In each period, the household faces the following budget constraint:

$$B_{t+1} = (1+r_t)B_t + P_t^x NY_t^x + Q_t NY_t^n - P_t^c C_t. \tag{15}$$

The problem of the representative household is to maximize lifetime utility (equation (14)) subject to a sequence of budget constraints (equation (15)) and to a non-Ponzi-game condition. The first-order conditions for this dynamic problem are the (stochastic) Euler equation

$$1 = \beta E_t (1+r_{t+1}) \left(\frac{P_t^c}{P_{t+1}^c} \right) \left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \tag{16}$$

and a transversality condition on bonds holdings $\lim_{i\to\infty} E_t R_{t,i} B_{t+1+i} = 0$, where $R_{t,i}$ is the market discount factor defined as

$$R_{t,i} \equiv \begin{cases} 1/\left(\prod_{j=t+1}^{t+i}(1+r_j)\right) & \text{if } i \geq 1, \\ 1 & \text{if } i = 0. \end{cases}$$

As in the two-period model, the budget constraint (equation (15)) can be expressed as

$$B_{t+1} = (1+r_t)B_t + P_t^x NY_t^x - C_t^T. \tag{17}$$

Iterating this equation forward and using the transversality condition yields the intertemporal budget constraint

$$\sum_{i=0}^{\infty} E_t R_{t,i} C_{t+i}^T = (1+r_t)B_t + \sum_{i=0}^{\infty} E_t R_{t,i} P_{t+i}^x NY_{t+i}^x.$$

Define Y_t as total real output in this economy. Dividing both sides of the equation above by $P_t^c Y_t$, and using the approximation $\ln(1+r_t) \approx r_t$, yields, after some algebra (see Appendix 1 for details),

$$\epsilon \tau_t \left[1 + \sum_{i=1}^{\infty} E_t \exp \left\{ \sum_{j=t+1}^{t+i} \left(\Delta c_j^T - r_j \right) \right\} \right] = \exp(r_t) b_t + \eta_t \left[1 + \sum_{i=1}^{\infty} E_t \exp \left\{ \sum_{j=t+1}^{t+i} \left(\Delta n y_j^x + \Delta p_j^x - r_j \right) \right\} \right],$$

where $\tau_t = C_t/Y_t$ is the consumption-output ratio, $\eta_t = P_t^x NY_t^x / P_t^c Y_t$ is the ratio of exportable net output to total output, $b_t = B_t / P_t^c Y_t$ is the ratio of foreign debt to total output, $\Delta c_t^T = \ln C_t^T - \ln C_{t-1}^T, \Delta n y_t^x = \ln NY_t^x - \ln NY_{t-1}^x$, and $\Delta p_t^x = \ln P_t^x - \ln P_{t-1}^x$. It is assumed that τ_t, η_t, b_t, Δc_t^T, $\Delta n y_t^x$, Δy_t, r_t, and Δp_t^x follow stationary processes with the unconditional means τ, η, b, g^c, g^{ny}, g^y, r, and g^p, respectively. The intertemporal budget constraint is then linearly approximated by taking a first-order Taylor expansion around the unconditional means. For any variable x_t, let $\tilde{x}_t$ denote the deviation from its unconditional mean. The linearly approximated intertemporal budget constraint then is (see Appendix 1 for details)

$$\epsilon \tilde{\tau}_t - \frac{1-\alpha}{1-\kappa} \tilde{\eta}_t \approx (1-\alpha) \exp(r) \tilde{b}_t + (1-\alpha) \exp(r) b \tilde{r}_t - \epsilon \tau \sum_{i=1}^{\infty} \alpha^i E_t \left[\widetilde{\Delta c_{t+i}^T} - \tilde{r}_{t+i} \right] + \eta \frac{1-\alpha}{1-\kappa} \sum_{i=1}^{\infty} \kappa^i E_t \left[\widetilde{\Delta n y_{t+i}^x} + \widetilde{\Delta p_{t+i}^x} - \tilde{r}_{t+i} \right], \quad (18)$$

where $\alpha = \exp(g^c - r)$ and $\kappa = \exp(g^p + g^{ny} - r)$ are assumed to be less than 1.[13]

To derive a log-linear approximation of the Euler equation for the consumption basket (equation (16)), we follow Campbell and Mankiw (1989), Campbell (1993), and Campbell, Lo, and MacKinlay (1997, p. 306) in assuming that the world real interest rate, r_t, the consumption price index P_t^c, and the consumption basket C_t are jointly conditionally homoscedastic and log-normally distributed. In this case, the Euler equation (16) can be log-linearized as

$$E_t \widetilde{\Delta c_{t+1}} = \sigma E_t \tilde{r}_{t+1} - \sigma(1-\epsilon) E_t \widetilde{\Delta q_{t+1}}, \quad (19)$$

[13] The conditions $\alpha < 1$ and $\kappa < 1$ are required to satisfy boundedness of the expected present value terms of equation (18).

where the unconditional mean of Δc_t is given by $E\Delta c_t = \sigma(\ln\beta + \delta - \mu) - \sigma(1-\epsilon)g^q$.[14] From this equation and the demand function for the tradable consumption basket (equation (3)), it is easy to show that the Euler equation for the tradable consumption basket is approximated by

$$E_t\widetilde{\Delta c^T_{t+1}} = \sigma E_t\tilde{r}_{t+1} + (1-\sigma)(1-\epsilon)E_t\widetilde{\Delta q_{t+1}}, \tag{20}$$

where the unconditional mean of Δc^T_t is given by $E\Delta c^T_t = \sigma r + (1-\sigma)(1-\epsilon)g^q$.

Finally, to derive an approximate solution for the current account-output ratio $ca_t \equiv (CA_t/P^c_t Y_t)$, we assume that the economy possesses a balanced growth path, where $\alpha = \kappa$. From the current account identity

$$CA_t \equiv r_t B_t + p^x_t N Y^x_t - C^T_t, \tag{21}$$

and equations (10), (18), (20), we can obtain the following present value representation of the current account-output ratio (see Appendix 2 for details):

$$\begin{aligned}\tilde{ca}_t = b\tilde{r}_t + [\eta + \epsilon\tau(\sigma-1)]\sum_{i=1}^{\infty}\kappa^i E_t\tilde{r}_{t+i} - \eta\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta ny^x_{t+i}} \\ + \epsilon\tau(1-\sigma)(1-\epsilon)\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta q_{t+i}} - \eta(1-\gamma)\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta p_{t+i}}.\end{aligned} \tag{22}$$

The first term in the RHS of equation (22), $b\tilde{r}_t$, shows the instantaneous effect of a change in the current world real interest rate on the current account. As can be seen from the current account identity (equation (21)), for example, if the economy is a net debtor (i.e. $B_t < 0$), a rise in r_t increases the net foreign interest payment instantaneously, and the current account moves into deficit. The second term, $[\eta + \epsilon\tau(\sigma-1)]\sum_{i=1}^{\infty}\kappa^i E_t\tilde{r}_{t+i}$, measures the impact of expected future changes in the world real interest rate on the current account. As discussed in Section 2, this impact can be decomposed into substitution ($\epsilon\tau\sigma$), income ($-\epsilon\tau$), and wealth (η) effects. The traditional consumption-smoothing motive is captured by the third term, $-\eta\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta ny^x_{t+i}}$,: the household adjusts the current account to smooth consumption to an income shock. The fourth term, $\epsilon\tau(1-\sigma)(1-\epsilon)\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta q_{t+i}}$, reflects the effect of expected future changes in the real exchange rate on the current account, which can also be decomposed into intertemporal substitution

[14] δ is a constant term including the constant variances of r_t, $\Delta\ln C_t$, and $\Delta\ln Q_t$ and the constant covariances across them.

$(-\epsilon\tau\sigma(1-\epsilon))$ and income $(\epsilon\tau(1-\epsilon))$ effects.[15] The last term, $-\eta(1-\gamma)\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta p_{t+i}}$, captures the effect of expected future changes in the terms of trade on the current account, or the HLM effect. Note that equation (22) implies that only transitory shocks to the terms of trade affect the current account. This is because households cannot smooth away a permanent shock, as the standard permanent-income hypothesis predicts.[16,17]

4. Econometric Analysis

4.1 Evaluating the PVM

To evaluate the PVM (equation (22)) empirically, we start by deriving the cross-equation restrictions that the PVM imposes on the unrestricted VAR. Let X_t denote a 5×1 column vector defined as $X_t = [\tilde{r}_t \quad \widetilde{\Delta ny_t^x} \quad \widetilde{\Delta q_t} \quad \widetilde{\Delta p_t} \quad \tilde{ca}_t]'$. It is assumed that the probability distribution of X_t is well approximated by a pth order unrestricted VAR: $X_t = A^1X_{t-1} + A^2X_{t-2} + \cdots + A^pX_{t-p} + \upsilon_t$, where A^i is the ith coefficient matrix with an $(m,n)th$ typical element $a(m,n)^i$ and υ_t is a 5×1 vector of unrestricted identically, independently distributed disturbances with variance-covariance matrix Ω.

Since any higher-order VAR has a first-order representation with a $5p\times 5p$ companion matrix $\mathcal{B}$, we can rewrite the pth order VAR as $\mathcal{Z}_t = \mathcal{B}\mathcal{Z}_{t-1} + \upsilon_t$ where $\mathcal{Z}_t = [X_t' \quad X_{t-1}' \quad \cdots \quad X_{t-p-1}']'$ and $\upsilon_t = [\upsilon_t' \; 0\cdots 0]'$, respectively. Let $\mathbf{e}_i$ denote the $1\times 5p$ row vector in which the ith element is 1 but the rest of elements are zeros. Then the PVM (equation (22)) implies the following cross-equation restrictions:

$$\mathbf{e}_5\mathcal{Z}_t = b\mathbf{e}_1\mathcal{Z}_t + \sum_{i=1}^{\infty}\kappa^i\left\{[\eta+\epsilon\tau(\sigma-1)]\mathbf{e}_1 - \eta\mathbf{e}_2 + \epsilon\tau(1-\sigma)(1-\epsilon)\mathbf{e}_3 - \eta(1-\gamma)\mathbf{e}_4\right\}\mathcal{B}^i\mathcal{Z}_t, \qquad (23)$$

[15] One advantage of the present-value representation derived in this paper is that it disentangles explicitly the effects of variable world real interest rates from the effects stemming from movements in the real exchange rate. In contrast, Bergin and Sheffrin (2000) embed these two effects in their model by constructing a measure of the consumption-based real interest rate. A possible drawback of this approach is that the series used in the estimation procedure depends on the structural parameters.

[16] See Svensson and Razin (1983) for the relationship between the HLM effect and the persistence of the terms of trade shocks.

[17] On the other hand, Backus (1993) shows that under the assumption of complete markets, the relationship between the terms of trade and the trade balance is independent of the properties of terms-of-trade shocks. This is because market completeness means that there exists a complete set of state-contingent securities that enable the representative household to insure against all idiosyncratic risks, thereby implying that there is no permanent-income effect arising from terms-of trade-shocks.

or, equivalently,

$$\mathbf{e}_5 = b\mathbf{e}_1 + \{[\eta + \epsilon\tau(\sigma - 1)]\mathbf{e}_1 - \eta\mathbf{e}_2 + \epsilon\tau(1-\sigma)(1-\epsilon)\mathbf{e}_3 - \eta(1-\gamma)\mathbf{e}_4\}\,\kappa\mathcal{B}[I_{5p} - \kappa\mathcal{B}]^{-1} \tag{24}$$

These cross-equation restrictions can be tested statistically using a Wald test. More precisely, let $\hat{\mathbf{e}}_5$ denote the estimated value of $\mathbf{e}_5$, $\frac{\partial \hat{\mathbf{e}}_5}{\partial \mathcal{B}}$ the matrix of derivatives of $\hat{\mathbf{e}}_5$ with respect to the elements $a(m,n)^i$ and V the variance-covariance matrix of those elements. Then

$$(\hat{\mathbf{e}}_5 - \mathbf{e}_5)' \, \mathbf{W}^{-1} \, (\hat{\mathbf{e}}_5 - \mathbf{e}_5) \sim \chi^2(5p),$$

where $\mathbf{W} = \left(\frac{\partial \hat{\mathbf{e}}_5}{\partial \mathcal{B}}\right)' V \left(\frac{\partial \hat{\mathbf{e}}_5}{\partial \mathcal{B}}\right)$. In addition, a predicted current account series, $\widehat{ca}_t$, can be constructed as

$$\widehat{ca}_t \equiv b\mathbf{e}_1\mathcal{Z}_t + \{[\eta + \epsilon\tau(\sigma - 1)]\mathbf{e}_1 - \eta\mathbf{e}_2 + \epsilon\tau(1-\sigma)(1-\epsilon)\mathbf{e}_3 - \eta(1-\gamma)\mathbf{e}_4\}\,\kappa\mathcal{B}[I_{5p} - \kappa\mathcal{B}]^{-1}\mathcal{Z}_t, \tag{25}$$

and compared with the actual series.

4.2 Data

The PVM is tested using quarterly data from three small open economies: Australia (1972Q1-2001Q4), Canada (1962Q2-2001Q2), and the United Kingdom (1971Q1-2001Q4). The data sources are Statistics Canada, International Financial Statistics, OECD, and National Statistics of the U.K. Government. All series are seasonally adjusted. Following Barro and Sala-i-Martin (1990), the world real interest rate series is constructed as a GDP-weighted average of real interest rates in the G-7 countries. For each country, the real interest rate is computed as the three–month Treasury bill rate, or an equivalent short-term rate, adjusted by expected CPI inflation, the latter being forecast using a sixth-order autoregression. To construct the net exportable output series, we proceed as follows. We measure gross exportable output by subtracting from total real GDP the (real) value of output in the service industry. Since we do not have disaggregated data on GDP components, we compute the net exportable output by assuming that the share of output in the service industry that is used for investment and government spending purposes is equal to the share of aggregate investment and public spending in total output. The series of net exportable output is expressed in per capita terms using the civilian population, age 16 and over. Following

Rogoff (1992) and Bergin and Sheffrin (2000), the real exchange rate is measured by the real effective exchange rate, which is obtained by multiplying the nominal effective exchange rate by the country's consumer price index and dividing it by the consumption price index for the G-7 countries. The terms of trade are computed as the ratio of export to import prices. In conformity with equation (22), the net exportable output, the real exchange rate, and the terms of trade series are logged and expressed in first differences. The current account series is constructed by adding the real trade balance and real foreign interest payments. The series is then divided by real output to obtain the current account-output ratio. Finally, all the series are demeaned.

Before testing the PVM (equation (22)), we need to check the stationarity of the series used in the unrestricted VAR. To do so, we conduct unit-root tests for the elements of the vector $X_t = [\tilde{r}_t \quad \widetilde{\Delta ny_t^x} \quad \widetilde{\Delta q_t} \quad \widetilde{\Delta p_t} \quad \tilde{ca}_t]'$, based on the augmented Dickey-Fuller (ADF) τ statistic. For each element x_t of X_t, we estimate the following ADF equation by ordinary least squares:

$$\Delta x_t = \rho x_{t-1} + \sum_{i=1}^{l} \alpha_i \Delta x_{t-i} + \varepsilon_t, \tag{26}$$

for different lag lengths, l. Because each element x_t is a demeaned series, we ignore the constant term in the above regression. The ADF τ statistic is calculated as the standard t statistic attached to the OLS estimate of ρ. Since this statistic is non-standard, Davidson and MacKinnon (1993) tabulate its asymptotic critical values under the unit-root null hypothesis. In particular, the critical values for the unit-root null at the 10, 5, and 1 percent significance levels are -1.62, -1.94, and -2.56, respectively.

Table 1 reports the unit-root test results for the three countries. In most cases, the null hypothesis of a unit root can be rejected (at least) at the 5 per cent significance level. In all remaining cases (except one), we can reject the presence of a unit root at the 10 per cent significant level. Therefore, we conclude that the elements of the vector X_t are stationary in the three countries.

4.3 Results

To test the PVM (equation (22)), we need to assign values to the parameters τ, η, b, κ, σ, ϵ, and γ.[18] The parameters τ, η, and b can be estimated directly using data on the consumption-output

[18] Table 2 summarizes the meaning of these parameters.

ratio, the ratio of exportable net output to total output, and the ratio of foreign debt to total output, respectively. Table 3 reports these ratios for Australia, Canada, and the United Kingdom. The remaining parameters are more problematic, either because they are preference parameters or because we do not have reliable data to estimate them directly. Our strategy, therefore, is to estimate these parameters by minimizing the distance between the prediction of the PVM (equation (25)) and the data. More precisely, we select the parameter values that minimize the root-mean-squared error (RMSE) associated with the predicted current account series.

Equation (22) states that the effect of the terms of trade on the current account is inversely proportional to the value of γ. In the limiting case where $\gamma = 1$, the current account does not depend on the terms of trade at all. Since our intention is to estimate the parameter γ (rather than calibrating it), our approach will allow us to determine whether or not the terms of trade are relevant in explaining current account fluctuations.

In evaluating the fit of the PVM (equation (22)), it is useful to compare its predictions with those generated by two "restricted" versions: the standard PVM, where there are no variations in the interest rate, the exchange rate, or the terms of trade, and a model that allows for time-varying real interest rates and real exchange rates but with constant terms of trade.[19]

In our context, the standard PVM yields

$$\tilde{ca}_t = -\eta \sum_{i=1}^{\infty} \kappa^i E_t \widetilde{\Delta ny_{t+i}},$$

where ny_t is the log of net output. Note that the parameters η and κ in the equation above have slightly different interpretations than in the extended model (equation (22)). In particular, since the standard PVM assumes that all goods are tradable, the parameter η must be calibrated to match the average ratio of net output to total output.[20] The model without variations in the terms of trade, labelled PVM without TOT, is obtained by imposing $\gamma = 1$ in the model discussed in section 2. This yields

$$\tilde{ca}_t = b\tilde{r}_t + [\eta + \epsilon\tau(\sigma - 1)] \sum_{i=1}^{\infty} \kappa^i E_t \tilde{r}_{t+i} - \eta \sum_{i=1}^{\infty} \kappa^i E_t \widetilde{\Delta ny^x_{t+i}} + \epsilon\tau(1-\sigma)(1-\epsilon) \sum_{i=1}^{\infty} \kappa^i E_t \widetilde{\Delta q_{t+i}}.$$

[19]The term "restricted" should not be interpreted in a statistical sense here, for two reasons. First, by construction, the PVM presented in this paper does not nest the standard PVM. Second, the VAR parameters used to compute the predicted current-account series are based on a different information set for each model.

[20]The discount factor, κ, is set to 0.99 in this case.

Results for Australia, Canada, and the United Kingdom are shown in Tables 4, 5, and 6, and Figures 1, 2, and 3, respectively. Each table reports estimates of the structural parameters, the Wald statistic with the corresponding p-value, the RMSE, and the relative standard deviation of the predicted current account series associated with each of the three PVMs considered. In the case of the standard PVM, the unrestricted VAR used to compute the Wald statistic and to generate a predicted current account series is bivariate and includes the variables $\widetilde{\Delta ny_t}$ and $\tilde{ca}_t$. In the case of the PVM without TOT, the results are based on a VAR that includes $\tilde{r}_t, \widetilde{\Delta ny^x_t}, \widetilde{\Delta q_t}$, and $\tilde{ca}_t$.[21] Each figure superposes on the actual current account series the one predicted by each of the three PVMs.

Australia

The top panel of Figure 1 shows that the standard PVM is quite successful in explaining the Australian current account. As can be seen from Table 4, the predicted series is more than 80% as volatile as the actual one. Yet, the cross-equation restrictions implied by this model are strongly rejected by the statistical test with a p-value of zero.

Next, consider the model with time-varying interest rate and exchange rate but with constant terms of trade. Our estimation strategy yields a point estimate of the intertemporal elasticity of substitution, σ, of 0.006. This low value is consistent with Bergin and Sheffrin's estimate of 0.087.[22] The share of tradable goods in total consumption, ϵ, is estimated to be 0.509, which is very close to the value of 0.5 reported by Stockman and Tesar (1995) and used by Bergin and Sheffrin. Finally, the estimated discount factor, κ, is 0.999.

Table 4 shows that allowing for stochastic variations in the interest rate and the exchange rate does not improve the fit of the standard PVM. Even though the extended model better matches the volatility of the Australian current account, its overall fit, as measured by the RMSE, is essentially the same as that of the standard PVM. In addition, the statistical test still rejects the cross-equations restrictions implied by the extended model at all conventional levels of significance. This

[21] The results discussed below are based on first-order VARs, as suggested by the Schwarz information criterion. However, we reach similar conclusions when we include more lags. To save space, results based on higher-order VARs are not reported, but are available upon request.

[22] Bergin and Sheffrin (2000) obtain this value by minimizing the Wald statistic.

result contrasts with Bergin and Sheffrin's conclusion that the PVM that includes variable interest rates and exchange rates is not statistically rejected by the Australian data.

Finally, consider the PVM augmented with terms-of-trade variations. The estimated values of σ, ϵ, and κ are roughly the same as before. For the parameter γ, we obtain a point estimate of 0.993, which implies that the terms of trade do not matter for the Australian current account. A comparison of the middle and bottom panels of Figure 1 shows that the current account series predicted by the PVMs with and without terms of trade are virtually identical. Table 4 shows that the PVM with variable terms of trade is also soundly rejected by the Wald test.

Canada

In the case of Canada, the top panel of Figure 2 shows that the current account series predicted by the standard PVM is much smoother than the actual one. That is, the standard model fails to generate movements in the current account of the same magnitude as those observed in the Canadian data. This failure, however, is not reflected in the result of the statistical test reported in Table 5. This table shows that the Wald statistic has a p-value of 0.075, meaning that the cross-equation restrictions implied by the standard PVM can be rejected at the 10% significance level, but not at the 5% level. This is somewhat surprising, given that earlier studies, such as those by Sheffrin and Woo (1990), Ghosh (1995), Bergin and Sheffrin (2000), and Nason and Rogers (2004), report strong rejections of the standard PVM in the case of Canada.[23]

The values of σ, ϵ, and κ that minimize the RMSE associated with the PVM augmented with variable interest rates and exchange rates are 0.008, 0.725, and 0.999, respectively. A low intertemporal elasticity of substitution of 0.039 is also reported by Bergin and Sheffrin (2000) for Canada. On the other hand, our estimate of the share of tradable goods in total consumption is relatively high but not implausible.

The middle panel of Figure 2 depicts the current account series predicted by the PVM with time-varying interest and exchange rates. The predicted series fits the data substantially better than that implied by the standard PVM, although it is only 60% as volatile as the actual series.

[23]When contrasting our findings with those of earlier papers, one should bear in mind an important distinction: the standard PVM model tested in this paper involves the current account output ratio, rather than the *level* of the current account, as is typically done in the literature.

This result corroborates Bergin and Sheffrin's findings that extending the standard PVM to allow for variations in the interest rate and the exchange rate improves its ability to fit the Canadian current account series. Despite this improvement, however, the cross-equation restrictions implied by the extended model are statistically rejected at all significance levels. Given that the standard PVM could not be rejected at the 5% significance level albeit its poor fit, this suggests that passing the statistical test does not necessarily guarantee a good fit and vice versa, which raises some doubt regarding the usefulness of the Wald test as an overall assessment of the matching performance of present-value models.

When we allow for terms-of-trade variations, our estimation procedure yields a point estimate of γ equal to 0.991. Thus, as is the case for Australia, the terms of trade appear to be irrelevant in explaining current-account movements in Canada. Table 5 shows that the predicted current account volatility and the RMSE associated with the PVM augmented with terms of trade are roughly the same as those associated with the PVM without terms of trade. This is illustrated in Figure 2, which shows that the current account series predicted by these two models are almost identical.

The United Kingdom

As is the case for Canada, the standard PVM performs very poorly in accounting for current-account fluctuations in the United Kingdom. The predicted current account series, depicted in the top Panel of Figure 3, is counterfactually smooth: the standard PVM explains less than 24% of current account variability. In this case, however, the Wald test strongly rejects the cross-equation restrictions implied by the standard PVM.

In preliminary attempts to jointly estimate the parameters σ, ϵ, and κ, we could not obtain convergence of the minimization procedure. To circumvent this problem, we calibrate the share of tradable goods in total consumption, ϵ, to 0.5. The estimated value of the intertemporal elasticity of substitution obtained in this case is relatively high compared with the values used in the real-business-cycle literature. As illustrated by the middle panel of Figure 3, amending the standard PVM to allow for variable interest rates and exchange rates improves its fit dramatically: the predicted current account series is, in this case, 70% as volatile as the actual one. Moreover, the p-value associated with the Wald statistic is 0.066, indicating that the extended PVM cannot be rejected at the 5% level.

To evaluate the PVM augmented with terms-of-trade variations, we continue to impose $\epsilon = 0.5$, and estimate the parameters σ, κ, and γ. We obtain a point estimate of γ of 0.497. Although imprecisely estimated, this value is much lower than 1, suggesting that the terms of trade are potentially important in explaining current-account movements in the United Kingdom. Figure 3 shows, however, that allowing for variable terms of trade only marginally improves the fit of the PVM: the predicted current account volatility is about 72% in this case. Nonetheless, the cross-equation restrictions implied by the model with variable terms of trade are not rejected by the statistical test.

Summary of the results

Our findings are summarized as follows. For Australia, Canada, and the United Kingdom, the standard PVM and the PVM augmented with variable interest rates and exchange rates are both statistically rejected by the data, at least at the 10 per cent significance level. In two of the three cases considered, namely, Australia and Canada, allowing for time-varying terms of trade does not improve the fit of the PVM. The extended model is still strongly rejected by the data, thus indicating that terms-of-trade shocks are not important in explaining current-account movements in these two countries. For the United Kingdom, once the terms of trade are included, the cross-equation restrictions implied by the extended model are no longer rejected by the data. The fit of this model, however, is only marginally better than that of a model that excludes terms-of-trade variations. Thus, by and large, our results suggest that terms-of-trade shocks are quantitatively unimportant in accounting for current-account fluctuations.

These findings partially contradict earlier results reported by Otto (2003), who uses a structural VAR approach to test for the presence of an HLM effect in a large sample of small open economies. He finds that the majority of countries exhibit a significant HLM effect, but his results indicate that Canada is an outlier in that terms-of-trade shocks were found to have no significant effect on the Canadian trade balance.

Our findings, however, are broadly consistent with earlier results by İşcan (2000) and Cashin and McDermott (2002). Using the methodology proposed by Glick and Rogoff (1995) and Panel data from the G-7 countries, İşcan (2000) finds little evidence that the terms of trade affect the current account. Cashin and McDermott (2002) perform a variance-decomposition exercise and find that terms-of-trade shocks are not important in explaining current account movements in Canada and the United Kingdom.[24]

5. Conclusion

This paper has extended the standard intertemporal model of the current account to allow for stochastic variations in three relative prices: the world real interest rate, the real exchange rate, and the terms of trade. Previous research by Bergin and Sheffrin (2000) has shown that variations in the world real interest rate and the real exchange rate substantially improve the fit of the intertemporal model. For Canada and the United Kingdom, however, a significant portion of current-account fluctuations remains unexplained even when movements in the world real interest rate and the real exchange rate are taken into account. The purpose of this paper, therefore, was to investigate whether including the terms of trade improves the ability of the intertemporal model to explain current account movements.

To do this, the paper has derived a closed-form solution for the present-value representation of the current account that highlights, in addition to the usual consumption-smoothing motive, three different channels through which movements in the real interest rate, the real exchange rate and the terms of trade affect the current account. In particular, the model encompasses the HLM effect according to which an increase in a country's terms of trade leads to an improvement in its current account. The restrictions implied by the extended model were subjected to present-value tests using data from Australia, Canada, and the United Kingdom. Our results show that terms-of-trade movements do not affect the current account in a significant way, and that, in two of the three countries, the model augmented with variables terms of trade is firmly rejected by the data.

Beyond the results reported in this paper, two methodological aspects of our research need to be further investigated. First, our approach does not allow us to distinguish between transitory

[24] For Australia, however, Cashin and McDermott (2002) find that terms-of-trade shocks account for a large fraction of current account variability.

and permanent terms-of-trade shocks. Since the theory predicts that only temporary shocks affect the current account, decomposing the terms of trade into transitory and permanent components is an obvious extension of our framework, and might lead to different conclusions. Second, this paper has raised some doubts on the usefulness of the Wald test as an appropriate assessment of the empirical validity of the PVM. Therefore, future research should focus on developing more powerful statistical tests for this class of models.

Appendices

Appendix 1: Derivation of the Log-Linearized Intertemporal Budget Constraint (equation (18))

Our log-linearizatrion of the intertemporal budget constraint follows Kano (2003). Dividing both sides of equation (17) by $P_t^c Y_t$ gives

$$\frac{C_t^T}{P_t^c Y_t}\sum_{i=0}^{\infty} E_t R_{t,i}\frac{C_{t+i}^T}{C_t^T} = (1+r_t)\frac{B_t}{P_t^c Y_t} + \frac{P_t^x NY_t^x}{P_t^c Y_t}\sum_{i=0}^{\infty} E_t R_{t,i}\frac{P_{t+i}^x NY_{t+i}^x}{P_t^x NY_t^x}. \qquad (A1.1)$$

Notice that for any variable x_t, the relation $\frac{x_{t+i}}{x_t} = \frac{x_{t+1}}{x_t}\frac{x_{t+2}}{x_{t+1}}\cdots\frac{x_{t+i}}{x_{t+i-1}}$ holds. Also, from equation(3), $C_t^T/P_t^c Y_t = \epsilon C_t/Y_t = \epsilon\tau_t$. Therefore, the intertemporal budget constraint can be rewritten as

$$\begin{aligned}\epsilon\tau_t\left[1+\sum_{i=1}^{\infty} E_t R_{t,i}\prod_{j=t+1}^{t+i}\left(\frac{C_j^T}{C_{j-1}^T}\right)\right] &= (1+r_t)\frac{B_t}{P_t^c Y_t}\\ &\quad + \eta_t\left[1+\sum_{i=1}^{\infty} E_t R_{t,i}\prod_{j=t+1}^{t+i}\left(\frac{P_j^x NY_j^x}{P_{j-1}^x NY_{j-1}^x}\right)\right]. \qquad (A1.2)\end{aligned}$$

Notice that for any variable X_t, the relation $\prod_{j=t+1}^{t+i} X_j = \exp\{\sum_{j=t+1}^{t+i}\ln(X_j)\}$ holds. From this relation and the definition of $R_{t,i}$, equation $(A1.2)$ can be further rearranged as

$$\begin{aligned}\epsilon\tau_t\left[1+\sum_{i=1}^{\infty} E_t \exp\left\{\sum_{j=t+1}^{t+i}\left(\Delta c_j^T - r_j\right)\right\}\right] &= \exp(r_t)b_t\\ &\quad + \eta_t\left[1+\sum_{i=1}^{\infty} E_t \exp\left\{\sum_{j=t+1}^{t+i}\left(\Delta\ln p_j^x + \Delta ny_j^x - r_j\right)\right\}\right], \qquad (A1.3)\end{aligned}$$

where $(1+r_t) = \exp\{\ln(1+r_t)\} \approx \exp(r_t)$ and $b_t \equiv B_t/P_t^c Y_t$.

Taking a first-order Taylor expansion of the LHS of equation $(A1.3)$ around the mean values gives

$$\widetilde{\text{the LHS}} \approx \frac{\epsilon}{1-\alpha}\tilde{\tau}_t + \frac{\epsilon\tau}{1-\alpha}\sum_{i=1}^{\infty}\alpha^i E_t\left\{\widetilde{\Delta c_{t+i}^T} - \tilde{r}_{t+i}\right\}, \qquad (A1.4)$$

where $\alpha = \exp(g^c - r)$ is assumed to be less than 1. The RHS of equation $(A1.3)$ is also approximated as

$$\widetilde{\text{the RHS}} \approx \exp(r)\tilde{b}_t + \exp(r)b\tilde{r}_t + \frac{\eta_t}{1-\kappa}\sum_{i=1}^{\infty}\kappa^i E_t\left\{\widetilde{\Delta p_{t+i}^x} + \widetilde{\Delta ny_{t+i}^x} - \tilde{r}_{t+i}\right\} + \frac{1}{1-\kappa}\tilde{\eta}_t, \qquad (A1.5)$$

where $\kappa = \exp(g^p + g^{ny} - r)$ is assumed to be less than 1. From $(A1.4)$ and $(A1.5)$, we have

$$\epsilon\tilde{\tau}_t - \frac{1-\alpha}{1-\kappa}\tilde{\eta}_t \approx (1-\alpha)\exp(r)\tilde{b}_t + (1-\alpha)\exp(r)b\tilde{r}_t$$
$$- \epsilon\tau\sum_{i=1}^{\infty}\alpha^i E_t\left[\widetilde{\Delta c^T_{t+i}} - \tilde{r}_{t+i}\right] + \eta\frac{1-\alpha}{1-\kappa}\sum_{i=1}^{\infty}\kappa^i E_t\left[\widetilde{\Delta ny^x_{t+i}} - \tilde{r}_{t+i} + \widetilde{\Delta p^x_{t+i}}\right],$$

which is equation (18) in the main text.

Appendix 2: Derivation of the Present-Value Model (equation (22))

For simplicity, assume that the economy is on a balanced growth path: the growth rate of the tradable consumption basket is the same as that of the exportable net output. That is, $\alpha = \kappa$. Then, substituting equation (20) into the intertemporal budget constraint (equation (18)) yields

$$\epsilon\tilde{\tau}_t - \tilde{\eta}_t = (1-\kappa)\exp(r)\tilde{b}_t + (1-\kappa)\exp(r)b\tilde{r}_t - [\eta + \epsilon\tau(\sigma-1)]\sum_{i=1}^{\infty}\kappa^i E_t\tilde{r}_{t+i}$$
$$+ \eta\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta ny^x_{t+i}} - \epsilon\tau(1-\sigma)(1-\epsilon)\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta q_{t+i}} + \eta\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta p^x_{t+i}}. \quad (A2.1)$$

To derive the optimal current account ratio equation, divide both sides of the current account identity (equation (21)) by $P^c_t Y_t$; this yields

$$ca_t \equiv \frac{CA_t}{P^c_t Y_t} = \{\exp[\ln(1+r_t)] - 1\}b_t + \eta_t - \epsilon\tau_t \approx [\exp(r_t) - 1]b_t + \eta_t - \epsilon\tau_t.$$

Taking a first-order Taylor expansion of this equation gives

$$\tilde{ca}_t \approx [\exp(r) - 1]\tilde{b}_t + \exp(r)b\tilde{r}_t + \tilde{\eta}_t - \epsilon\tilde{\tau}_t. \quad (A2.2)$$

Substituting $(A2.1)$ into $(A2.2)$ and using equation (10), yields the optimal current account ratio,

$$\tilde{ca}_t = [\kappa\exp(r) - 1]\tilde{b}_t + \kappa\exp(r)b\tilde{r}_t + [\eta + \epsilon\tau(\sigma-1)]\sum_{i=1}^{\infty}\kappa^i E_t\tilde{r}_{t+i}$$
$$- \eta\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta ny^x_{t+i}} + \epsilon\tau(1-\sigma)(1-\epsilon)\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta q_{t+i}} - \eta(1-\gamma)\sum_{i=1}^{\infty}\kappa^i E_t\widetilde{\Delta p_{t+i}}.$$

Since $\kappa\exp(r) = \exp(g^{ny} + g^p)$ takes a value close to one, in particular, in quarterly data, it is a reasonable approximation to set the coefficients on the first and second terms in the RHS to zero and b, respectively.

Thus,

$$\tilde{ca}_t = b\tilde{r}_t + [\eta + \epsilon\tau(\sigma - 1)] \sum_{i=1}^{\infty} \kappa^i E_t \tilde{r}_{t+i} - \eta \sum_{i=1}^{\infty} \kappa^i E_t \widetilde{\Delta n y^x_{t+i}} + \epsilon\tau(1-\sigma)(1-\epsilon) \sum_{i=1}^{\infty} \kappa^i E_t \widetilde{\Delta q_{t+i}} - \eta(1-\gamma) \sum_{i=1}^{\infty} \kappa^i E_t \widetilde{\Delta p_{t+i}},$$

which is equation (22) in the main text.

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Table 1. Unit-Root Test Results

Variable	Lag length 1	3	5	7
	Australia			
$\tilde{r}_t$	$-2.813^{\ddagger}$	$-3.512^{\ddagger}$	$-3.582^{\ddagger}$	$-2.411^{\dagger}$
$\widetilde{\Delta ny_t^x}$	$-5.203^{\ddagger}$	$-6.302^{\ddagger}$	$-7.032^{\ddagger}$	$-4.775^{\ddagger}$
$\widetilde{\Delta q_t}$	$-7.631^{\ddagger}$	$-5.849^{\ddagger}$	$-4.175^{\ddagger}$	$-4.597^{\ddagger}$
$\widetilde{\Delta p_t}$	$-7.215^{\ddagger}$	$-6.519^{\ddagger}$	$-4.503^{\ddagger}$	$-4.721^{\ddagger}$
$\tilde{ca}_t$	$-2.049^{\dagger}$	-1.822^{*}	-1.814^{*}	-1.803^{*}
	Canada			
$\tilde{r}_t$	$-2.103^{\dagger}$	$-2.105^{\dagger}$	$-2.048^{\dagger}$	-1.805^{*}
$\widetilde{\Delta ny_t^x}$	$-8.206^{\ddagger}$	$-6.036^{\ddagger}$	$-5.125^{\ddagger}$	$-5.408^{\ddagger}$
$\widetilde{\Delta q_t}$	$-9.148^{\ddagger}$	$-5.234^{\ddagger}$	$-4.113^{\ddagger}$	$-3.248^{\ddagger}$
$\widetilde{\Delta p_t}$	$-7.109^{\ddagger}$	$-6.970^{\ddagger}$	$-4.782^{\ddagger}$	$-4.670^{\ddagger}$
$\tilde{ca}_t$	$-2.103^{\dagger}$	$-2.273^{\dagger}$	-1.731^{*}	$-2.094^{\dagger}$
	United Kingdom			
$\tilde{r}_t$	$-3.065^{\ddagger}$	$-2.677^{\ddagger}$	$-2.515^{\dagger}$	$-2.498^{\dagger}$
$\widetilde{\Delta ny_t^x}$	$-5.389^{\ddagger}$	$-4.593^{\ddagger}$	$-5.058^{\ddagger}$	$-3.912^{\ddagger}$
$\widetilde{\Delta q_t}$	$-7.771^{\ddagger}$	$-5.162^{\ddagger}$	$-4.397^{\ddagger}$	$-3.697^{\ddagger}$
$\widetilde{\Delta p_t}$	$-7.870^{\ddagger}$	$-3.978^{\ddagger}$	$-4.248^{\ddagger}$	$-3.943^{\ddagger}$
$\tilde{ca}_t$	$-2.116^{\dagger}$	-1.920^{*}	-1.812^{*}	-1.566

Note: The superscripts $*, \dagger$, and $\ddagger$ denote rejections of the null hypothesis of a unit root at the 10, 5, and 1 per cent significance levels, respectively.

Table 2. Description of Relevant Parameters

Parameter	Description
τ	Unconditional mean of the consumption-output ratio
η	Unconditional mean of the ratio of exportable net output to total output
b	Unconditional mean of is the ratio of foreign debt to total output
κ	Discount factor
σ	Elasticity of intertemporal substitution
ϵ	Share of tradable goods in the consumption basket
γ	Share of exportable goods in the traded-good basket

Table 3. Calibrated Parameters

	Standard PVM			Extended PVM		
Parameter	Australia	Canada	United Kingdom	Australia	Canada	United Kingdom
τ	0.583	0.469	0.603	0.583	0.469	0.603
η	0.567	0.489	0.611	0.210	0.179	0.237
b	–	–	–	−0.320	−0.269	−0.610

Table 4. Tests of Different Versions of the Present-Value Model of the Current Account for Australia

	Model		
Parameter	Standard PVM	PVM without TOT	PVM with TOT
κ	0.9900	0.9991	0.9999
		(0.2172)	(0.0194)
σ	–	0.0063	0.0071
		(2.4333)	(2.6949)
ϵ	–	0.5091	0.5058
		(3.6538)	(3.6368)
γ	–	–	0.9935
			(1.9672)
Statistic			
RMSE	0.0059	0.0059	0.0059
Wald statistic	45.599	31.725	32.502
p-value	0.0000	0.0000	0.0000
$\sigma_{\widehat{\tilde{ca}}}/\sigma_{\tilde{ca}}$	0.8204	0.9798	0.9797

Notes: Standard errors are between parentheses. RMSE is the root-mean-squared error.

Table 5. Tests of Different Versions of the Present-Value Model of the Current Account for Canada

	Model		
Parameter	Standard PVM	PVM without TOT	PVM with TOT
κ	0.9900	0.9993	0.9999
		(0.0823)	(0.0225)
σ	–	0.0085	0.0001
		(1.1559)	(0.1414)
ϵ	–	0.7246	0.7164
		(4.5193)	(4.4208)
γ	–	–	0.9911
			(1.4580)
Statistic			
RMSE	0.0096	0.0071	0.0070
Wald statistic	5.189	31.725	32.502
p-value	0.0747	0.0000	0.0000
$\sigma_{\widehat{\tilde{ca}}}/\sigma_{\tilde{ca}}$	0.3859	0.6047	0.6105

Notes: Standard errors are between parentheses. RMSE is the root-mean-squared error.

Table 6. Tests of Different Versions of the Present-Value Model of the Current Account for the United Kingdom

	Model		
Parameter	Standard PVM	PVM without TOT	PVM with TOT
κ	0.9900	0.9998	0.9999
		(0.0829)	(0.0485)
σ	–	1.8251	1.7212
		(10.454)	(10.363)
ϵ	–	0.5000	0.5000
γ	–	–	0.4971
			(9.4654)
Statistic			
RMSE	0.0142	0.0126	0.0114
Wald statistic	21.234	8.8257	7.4142
p-value	0.0000	0.0656	0.1916
$\sigma_{\widehat{\tilde{ca}}}/\sigma_{\tilde{ca}}$	0.2371	0.7033	0.7233

Notes: Standard errors are between parentheses. RMSE is the root-mean-squared error. For the PVMs with and without terms of trade, the parameter ϵ is calibrated to 0.5.

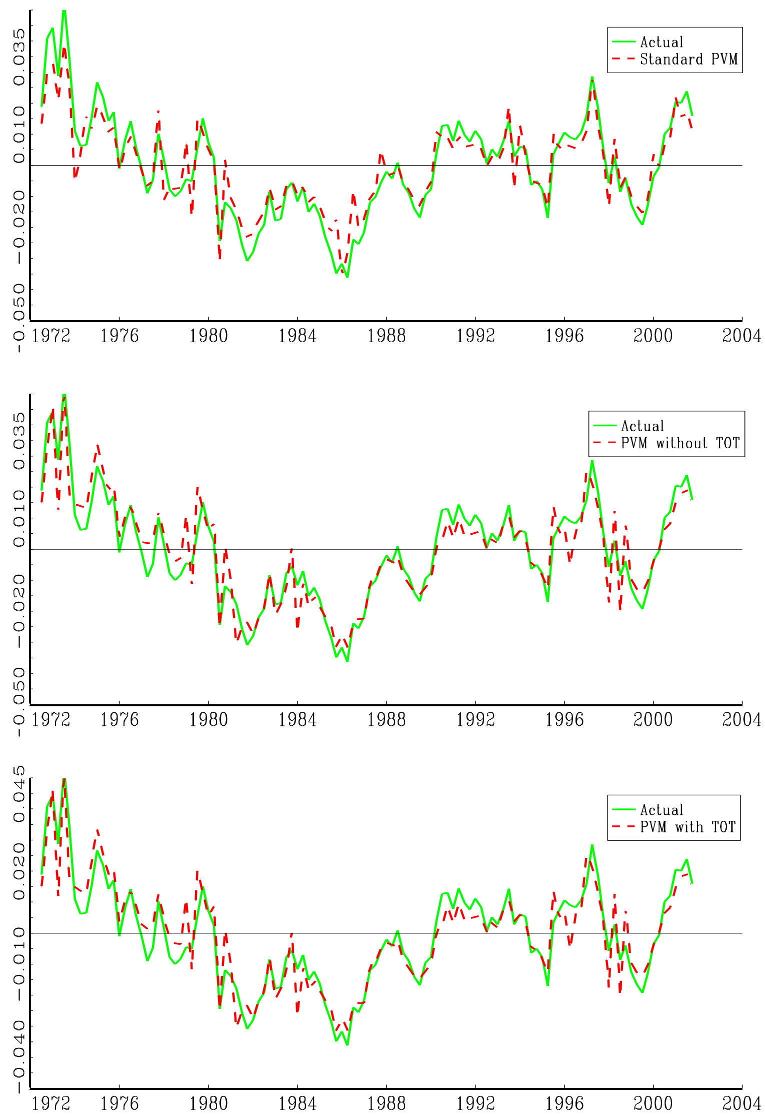

Figure 1: Actual and predicted current account for Australia

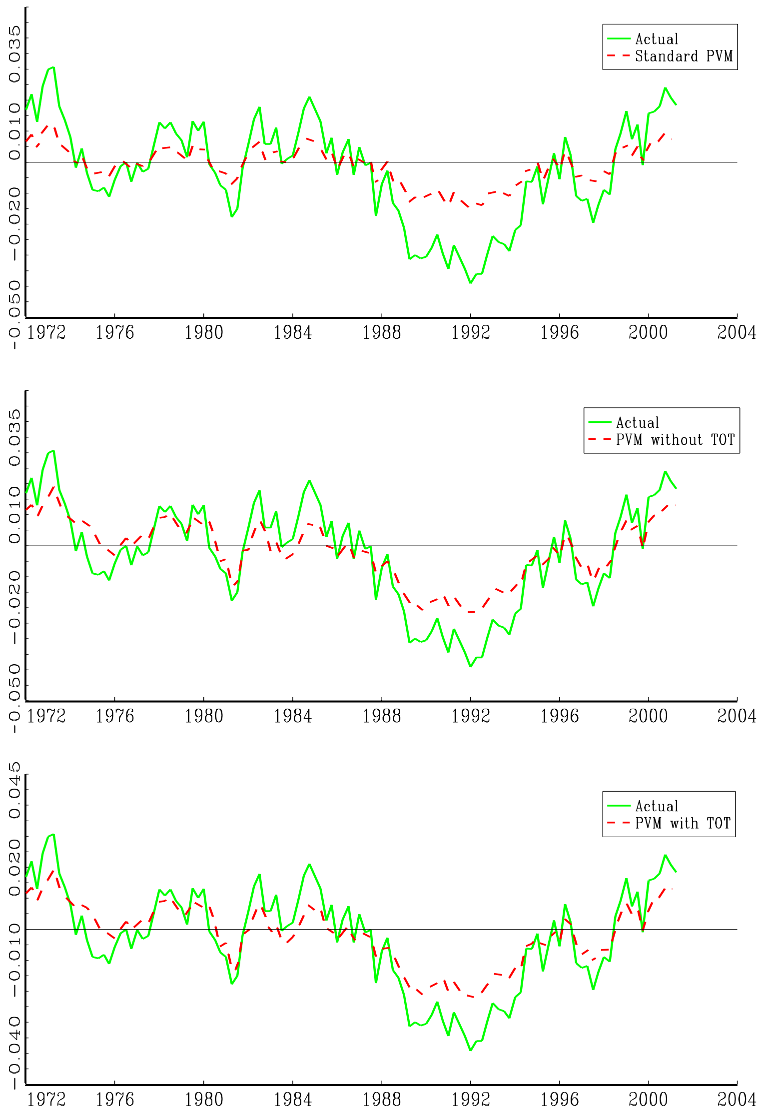

Figure 2: Actual and predicted current account for Canada

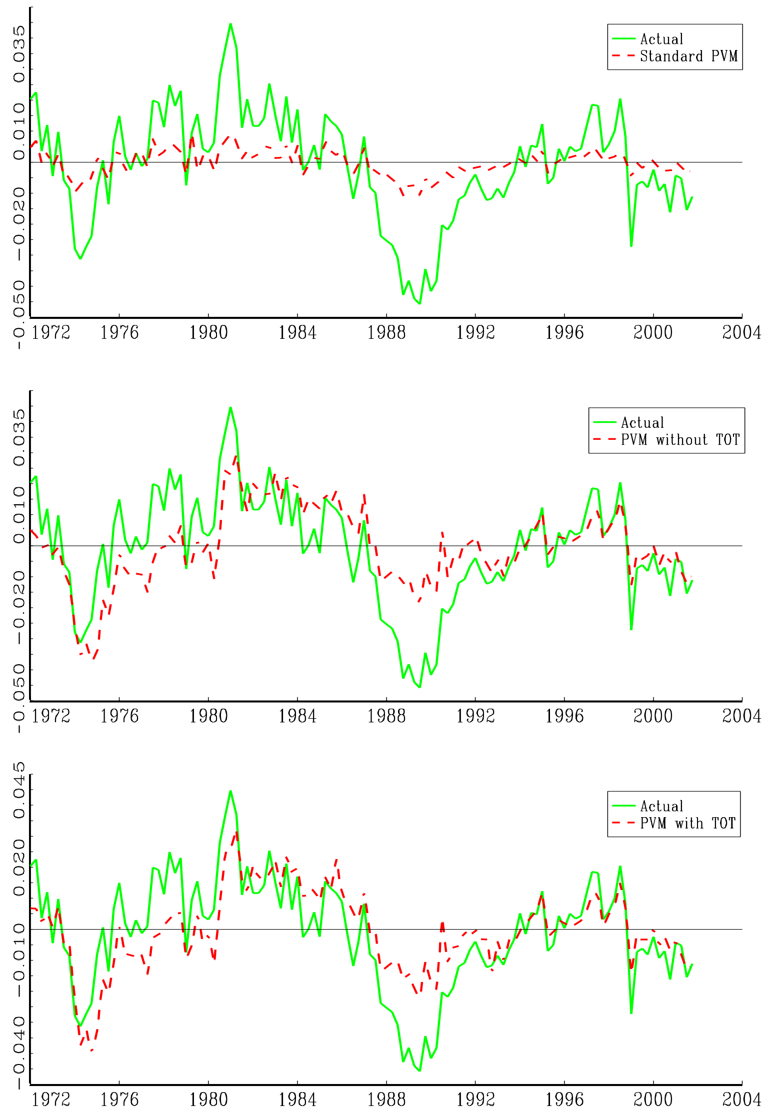

Figure 3: Actual and predicted current account for the United Kingdom