Habit Formation and the Persistence of Monetary Shocks

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Abstract

The dynamic effects and relative importance of monetary shocks in the U.S. business cycle are studied using a sticky-price Dynamic Stochastic General Equilibrium model with habit formation and capital adjustment costs. The model is estimated via Maximum Likelihood using data on output, real money balances, and the nominal interest rate. Econometric results indicate that the model has a strong internal propagation mechanism that can explain the persistent and hump-shaped response of U.S. output and consumption to monetary shocks.

Keywords: Habit formation, endogenous persistence, monetary policy

JEL classification: E3, E4, E5

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1. Introduction

This paper studies the dynamic effects and relative importance of monetary shocks in the U.S. business cycle using a sticky-price Dynamic Stochastic General Equilibrium (DSGE) model with habit formation and capital adjustment costs. The model is estimated by Maximum Likelihood (ML) using quarterly data on output, real balances and the nominal interest rate. At the ML estimates, impulse response analysis indicates that money supply shocks induce a persistent and hump-shaped response in output, consumption and hours worked. Variance decomposition attributes to money supply shocks more than 50 per cent of the output variance at horizons of less than a year and 27 per cent in the long run. Technology shocks explain 71 per cent of the output variance in the long run.

The empirical analysis of two restricted versions of the model indicates that habit formation interacts with costly capital adjustment to magnify the propagation of monetary shocks in the model. Adjustment costs to capital spread out investment and output responses to shocks, while habit formation increases output persistence through its effect on labor supply. In particular, habit formation induces agents to adjust their labor supply more gradually to attain a smoother and more persistent consumption profile than under time-separable preferences.

The model also generates predictions regarding data series that were not used in the estimation procedure. The predicted consumption and investment dynamics are similar to those of U.S. data. However, the model does poorly in explaining the behavior of the real marginal cost and inflation. This result reflects a drawback of inflation models based on forward-looking pricing rules. Since inflation inherits the dynamic properties of the real marginal cost and current inflation is not helpful in predicting future inflation, forward-looking pricing rules imply that inflation is less persistent than usually found in the data.

Recent literature that studies the persistent effects of monetary shocks include Bergin and Feenstra (2000); Dotsey and King (2001); Dib and Phaneuf (2001); Ambler, Guay and Phaneuf (2003); and Christiano, Eichenbaum and Evans (2005). Bergin and Feenstra construct a model where the interaction of materials inputs and translog preferences leads to endogenous output persistence. Translog preferences dissuade firms from charging higher prices by making the elasticity of demand facing a given firm depend on the firm's relative price. Dotsey and King construct a model that incorporates variable capital utilization, materials input, and labor flexibility. Results indicate that these three features are mutually reinforcing and magnify output persistence. Dib and Phaneuf, and Ambler, Guay and Phaneuf construct DSGE models with sticky prices and costly adjustment to labor. Their results show that adding adjustment costs to the labor input generates endogenous output persistence to monetary shocks. Christiano, Eichenbaum and Evans (2005) use a limited participation model that incorporates price and wage rigidities, optimizing and non-optimizing price and wage setting, habit formation, adjustment costs in investment and variable capital utilization. Their results suggest that wage rigidity and variable capital utilization are also important to explain output persistence in response to monetary shocks. Although apparently distinct, the crucial features of
these models work through the same channel to increase output persistence. They prevent a rapid change in the real marginal cost after a monetary shock and lead to stronger nominal rigidity.

The rest of the paper is organized as follows: Section 2 presents the theoretical model, Section 3 describes the estimation procedure and data, Section 4 reports empirical results and Section 5 concludes.

2. The Model

The economy consists of an infinitely-lived representative household, a representative final good producer, a continuum of intermediate good producers indexed by \( i \in [0, 1] \) and a government. There is no population growth and the population size is normalized to one.

The representative household maximizes lifetime utility

\[
E_s \sum_{t=s}^{\infty} \beta^{t-s} \left( \frac{(c_t/c_{t-1})^{1-\eta_1}}{1-\eta_1} + \frac{b_t(m_t)^{1-\eta_2}}{1-\eta_2} + \frac{\psi(t_t)^{1-\eta_3}}{1-\eta_3} \right),
\]

where \( \beta \in (0, 1) \) is the subjective discount factor, \( \eta_1, \eta_2 \) and \( \eta_3 \) are positive parameters, \( c_t \) is consumption of the final good, \( m_t \) is real money balances, \( t_t \) is leisure, \( b_t \) is a preference shock and \( \psi > 0 \) is the utility weight of leisure. The time endowment is normalized to one. The household’s preferences exhibit internal habit formation. That is, utility depends on current consumption relative to a habit stock determined by the household’s own past consumption. Then, consumption levels in adjacent periods are complements.\(^1\) In the special case where \( \gamma = 0 \), there is no habit formation and households care only about the absolute level of current consumption.

The household’s budget constraint (expressed in real terms) is:

\[
c_t + a_t + m_t + x_t = (R_t - 1/\pi_t)a_{t-1} + (m_{t-1}/\pi_t) + w_t n_t + q_t k_t + d_t + \tau_t,
\]

where \( a_t \) is the real value of nominal bond holdings, \( x_t \) is investment, \( R_t \) is the gross nominal interest rate, \( \pi_t \) is the gross inflation rate, \( w_t \) is the real wage, \( n_t \) is hours worked, \( q_t \) is the real rental rate of capital, \( k_t \) is the capital stock, \( d_t \) are dividends and \( \tau_t \) are lump-sum transfers or taxes.

Investment increases the household’s capital stock according to:

\[
k_{t+1} = (1 - \delta)k_t + \Gamma(x_t/k_t)k_t,
\]

where \( \delta \in (0, 1) \) is the depreciation rate and \( \Gamma(\cdot) \) is a strictly positive and concave function. The assumption that \( \Gamma(\cdot) > 0 \) means that investment unambiguously increases the capital stock. The assumption that \( \Gamma(\cdot) \) is concave means that large proportional changes in the capital stock are marginally more costly than smaller

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\(^1\)In principle, the habit stock could also include consumption levels prior to time \( t-1 \). Fuhrer (2000) estimates a model where the stock of habit is a weighted average of past consumption and finds that the habit-formation reference level is essentially the previous period’s consumption level.
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In the special case where $\Gamma(\cdot) = x_t/k_t$, there are no adjustment costs and one unit invested becomes one unit of capital.

Final good producers are perfectly competitive and aggregate the intermediate goods into a single perishable commodity using the CES (Constant Elasticity of Substitution) technology

$$y_t = \left( \int_0^1 y_t(i)^{(\theta-1)/\theta} \, di \right)^{\theta/(\theta-1)}, \quad (4)$$

where $y(i)$ is the input of intermediate good $i$ and $\theta > 1$ is the elasticity of substitution between different goods. As $\theta \to \infty$, goods become perfect substitutes in production. Cost minimization by the final good producer delivers the input demand of good $i$

$$y_t(i) = \left( P_t(i)/P_t \right)^{-\theta} y_t, \quad (5)$$

where $P_t(i)$ is the price of good $i$ and $P_t$ is the aggregate price level.

Intermediate good producers are monopolistically competitive. Firm $i$ produces its differentiated good using the Cobb-Douglas technology

$$y_t(i) = z_t k_t(i)\alpha n_t(i)^{1-\alpha}, \quad (6)$$

where $\alpha \in (0,1)$ and $z_t$ is an aggregate technology shock. The firm chooses its nominal price taking as given the demand function (5), aggregate demand and the price level. Nominal prices are assumed to be sticky à la Calvo (1983).2 The probability that a firm reoptimizes its price in every period is $1 - \varphi$.3 It is easy to show that the optimal price is

$$P_t^* = \frac{\theta}{(\theta - 1)} \left( \frac{E_t \left( \sum_{s=1}^{\infty} (\beta \varphi)^{s-t} \Lambda_{t,s} y_s^* \Phi_s \right)}{E_t \left( \sum_{s=1}^{\infty} (\beta \varphi)^{s-t} \Lambda_{t,s} y_s^* \right)} \right). \quad (7)$$

where $\Lambda_{t,s} = (\lambda_s/P_s)/(\lambda_t/P_t)$, $\lambda_t$ is the household’s marginal utility of wealth, $\Phi_s$ is the nominal marginal cost and $y_s^* = (P_t^*/P_s)^{-\theta} y_s$. Thus, the optimal price depends on current and expected future demands and nominal marginal costs. Since labor and capital are perfectly mobile, wages and rental rates are the same for all firms. The optimal demand for labor and capital inputs equates their marginal products times the real marginal cost to the real wage and real rental rate, respectively.

2Alternatively, one could assume explicit costs of changing prices or Taylor’s staggered price setting. Quadratic price-adjustment costs yield an aggregate pricing equation similar to the one obtained using Calvo’s model. Aggregation is somewhat easier using Calvo-type than Taylor-type price rigidity because it is not necessary to keep track of heterogeneous price cohorts. From the point of view of estimating the average length of price contracts, Calvo’s model is also easier to implement because the log likelihood function is continuous in the probability that a firm reoptimizes its price. In contrast, the contract length in Taylor’s model is an integer number and, consequently, the log likelihood function is discontinuous in this parameter.

3As in Yun (1996), firms that do not reoptimize change their prices in proportion to the steady-state inflation rate.
The government comprises both fiscal and monetary authorities. The government makes lump-sum transfers to households each period. Transfers are financed by printing additional money in each period. Thus, the government budget constraint is:

\[ \tau_t = m_t - m_{t-1}/\pi_t, \]  

where the term on the right-hand side is seigniorage revenue at time \( t \). Money is supplied exogenously by the government according to

\[ M_t = \mu_t M_{t-1}, \]

where \( \mu_t \) is the stochastic gross rate of money growth.

The technology, money supply and money demand shocks follow the processes

\[
\begin{align*}
\ln(z_t) &= (1 - \rho_z) \ln(z_{ss}) + \rho_z \ln(z_{t-1}) + \epsilon_{z,t}, \\
\ln(\mu_t) &= (1 - \rho_\mu) \ln(\mu_{ss}) + \rho_\mu \ln(\mu_{t-1}) + \epsilon_{\mu,t}, \\
\ln(b_t) &= (1 - \rho_b) \ln(b_{ss}) + \rho_b \ln(b_{t-1}) + \epsilon_{b,t},
\end{align*}
\]

where \( \rho_z, \rho_\mu, \rho_b \in (-1, 1) \), \( \ln(z_{ss}), \ln(\mu_{ss}) \) and \( \ln(b_{ss}) \) are the unconditional means of their respective shocks, and the innovations \( \epsilon_{z,t}, \epsilon_{\mu,t} \) and \( \epsilon_{b,t} \) are serially uncorrelated and normally distributed with mean zero and variances \( \sigma^2_z, \sigma^2_\mu \) and \( \sigma^2_b \), respectively.

Since the model cannot be solved analytically, its equations were log-linearized around the deterministic steady state. The resulting system of linear difference equations was solved using the method in Blanchard and Kahn (1980).

3. Estimation Method and Data

The model is estimated by the method of Maximum Likelihood (ML) using the Kalman filter to evaluate the likelihood function. The Kalman filter allows us to deal with unobserved or poorly measured predetermined variables (like the stock of capital) and yields the optimal solution to the problem of predicting and updating state-space models. Hansen and Sargent (1998) show that the ML estimator obtained by applying the Kalman filter to the state-space representation of DSGE models is consistent and asymptotically efficient.

For the Kalman filter, the transition equation is the joint process of the predetermined and exogenous variables, \( H_{t+1} = QH_t + e_t, \) where \( H_t = (k_t, m_{t-1}, c_{t-1}, z_t, \mu_t, b_t)' \) and \( e_t = (0, 0, 0, \epsilon_{z,t}, \epsilon_{\mu,t}, \epsilon_{b,t})' \). The measurement equation is the joint process of output, real money balances and the nominal interest rate expressed as a function of the state variables, \( \xi_t = WH_t, \) where \( \xi_t = (m_t, y_t, R_t)' \).4 The elements of \( Q \) and \( W \) are nonlinear functions of the structural parameters of the

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4 As is well known, the Maximum Likelihood estimation of DSGE models using more observable variables than structural shocks leads to a singular variance-covariance matrix of the residuals. One strategy to address this issue is to add measurement errors to the observable variables. A possible drawback to this approach is that measurement errors lack a structural interpretation and essentially capture specification error. Still, in preliminary work, we considered this approach. When we added measurement errors to all observable variables, we found that not all variances were identified or that some of them converged to zero. When we added only as many errors as needed to make the system nonsingular, we found that results were very sensitive to the variable that was assumed to be measured with noise.
model. These elements are computed from the Blanchard-Kahn solution of the DSGE model in each iteration of the optimization procedure that maximizes the log likelihood function. Note that the estimation procedure imposes all restrictions implied by the theoretical model. Standard errors were computed as the square root of the diagonal elements of the inverted Hessian of the log likelihood function evaluated at the maximum. At the estimated ML parameters, the condition for existence of a unique model solution is satisfied. That is, the number of explosive characteristic roots of the system of linear difference equations equals to the number of non-predetermined variables.

The model is estimated using quarterly U.S. data on output, real money balances and the rate of nominal interest. The series were taken from the database of Federal Reserve Bank of St. Louis. The sample is 1960:Q1 to 2001:Q2. Output is measured by real GDP per capita. The stock of nominal money is measured by M2 per capita. By measuring these two series in per capita terms, we aim to make the data compatible with our model, where there is no population growth. Population is measured by the civilian, noninstitutional population, 16 years old and over. The gross nominal interest rate is measured by the 3-month U.S. Treasury bill rate. Since the variables in the model are expressed in percentage deviations from the steady state, the output and real money series were logged and detrended linearly. The nominal interest rate series was logged and demeaned. The model was also estimated using HP filtered data and results were very similar to the ones reported below.

4. Empirical Results

4.1. Maximum Likelihood Estimates

The estimated structural parameters are the preference parameters $\eta_2$ and $\eta_3$, the habit persistence parameter ($\gamma$), the probability that an intermediate-good producer does not reoptimize its price in a given quarter ($\varphi$), the parameters of the shock processes ($\rho^z, \rho^\mu, \rho^b, \sigma^z, \sigma^\mu$ and $\sigma^b$) and the elasticity of investment with respect to the price of installed capital at the steady state (denoted below by $\chi$).

The rest of the parameters were either poorly identified or additional evidence about their magnitude is available. Data on national income accounts suggest that a plausible value for the share of capital in production is 0.36. The subjective discount factor is fixed to 0.99, meaning that the steady-state gross real interest rate is approximately 1.01. The rate of depreciation is fixed to 0.025. The gross rate of money growth (and inflation) at the steady state is fixed to 1.017. This value corresponds to the average gross rate of money growth during the sample period. Two important structural parameters that are poorly identified are the elasticity of demand ($\theta$) and the curvature parameter of the consumption component in the utility function ($\eta_1$). Markup estimates reported by Basu and Fernald (1994) for U.S. data indicate that $\theta$ is approximately 10. Estimates of the curvature of the utility function with respect to consumption range from 0.5 to 5. We assume $\eta_1 = 2$, but sensitivity analysis indicates that the results do not depend crucially
on the magnitudes of $\theta$ and $\eta_1$.\footnote{We also performed single and joint Lagrange Multiplier tests of the null hypothesis that the true values of $\beta, \delta, \eta_1, \alpha,$ and $\theta$ are the ones assumed during estimation. In all cases, one cannot reject the null hypothesis. However, these results should be interpreted with caution because they might also reflect low test power.} Finally, fixing the proportion of time worked in steady state ($n_{ss}$) amounts to fixing either the mean of the technology shock ($z$) or the weight of leisure in the utility function ($\psi$). The values of these two parameters are set so that, along with the ML estimate of $\eta_3$, the proportion of time worked in steady state is 0.31.

Maximum Likelihood (ML) estimates of the parameters and their standard errors are reported in Column 1 of Table 1. The ML estimate of the habit-formation parameter ($\gamma$) is 0.98 (0.016). The term in parenthesis is the standard error. This estimate is significantly different from zero, but is not significantly different from one at standard levels. Its magnitude is larger than, but still consistent with, the values of 0.80 (0.19) and 0.90 (1.83), reported by Fuhrer (2000); 0.63 (0.14), reported by Christiano, Eichenbaum and Evans (2005); 0.73, reported by Boldrin, Christiano and Fisher (2001); and 0.938 (1.775), reported by Heaton (1995).

The estimated elasticity of investment with respect to the price of installed capital is 0.47 (0.11). This value is higher than the point estimates of 0.34 and 0.28 reported by Kim (2000) and Christiano, Eichenbaum and Evans (2005), respectively, but it is considerably lower than the typical value used to calibrate standard Real Business Cycle (RBC) models (see, for example, Baxter and Crucini, 1993).

The estimated probability that an intermediate good producer does not reoptimize its price is $\varphi = 0.847$ (0.034) per quarter. This implies that the average length of price contracts is $1/(1 - 0.847) = 6.56$ (1.44) quarters. Previous estimates on the average time between price adjustments vary substantially. Gali and Gertler (1999) find that $\varphi$ is approximately 0.83. Their estimate implies that prices are fixed between five and six quarters. Cecchetti (1986) reports that the average number of years since the last price adjustment for U.S. magazines ranges from 1.8 to 14. Kashyap (1995) finds that the average time between price changes in 12 mail-order catalog goods is approximately 4.9 quarters. Taylor (1999) surveys empirical studies on price setting and finds that the average duration of price contracts is about 4 quarters in the United States. Bils and Klenow (2004) document substantial heterogeneity in the frequency of price adjustments among the goods surveyed by the U.S. Bureau of Labor Statistics and report a median price duration of only 1.66 quarters. Christiano, Eichenbaum and Evans (2005) find that the average length of price contracts is about 2 quarters and that of wage contracts is roughly 3.3 quarters.

The parameter estimates for the curvature parameters of leisure and real balances in the utility function are 1.59 (2.88) and 3.9 (0.83), respectively. These results imply an elasticity of labor supply with respect to the real wage (for a given marginal utility of consumption) of $(1 - n)/(\eta_3 n) = (1 - 0.31)/(1.591 \cdot 0.31) = 1.4$ (2.99), and an interest elasticity of money demand of $-1/\eta_2 = -1/3.09 = -0.32$ (0.09). The latter estimate is very close to the one of 0.39 reported by Chari, Kehoe and McGrattan (2000), but larger than the estimates of 0.10 and 0.11 found...
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by Christiano, Eichenbaum and Evans (2005) and Dib and Phaneuf (2001), respectively. Finally, estimates of the autoregressive coefficients of the shock processes indicate that all shocks are very persistent. Very persistent technology and money demand shocks are also reported by Kim (2000) and Ireland (2001).

4.2. Fit of the Model

This section evaluates the goodness of fit of the model. Consider Figure 1 that plots the observed and fitted series of U.S. output, real money balances and the nominal interest rate. The Figure suggests that the model tracks well the dynamics of the three variables. This impression is confirmed quantitatively by the $r^2$ statistic, that measures the proportion of the total variation in the dependent variable that is explained by the model. The $r^2$s for output, real money balances and the nominal interest rate are 0.948, 0.945 and 0.893, respectively. Thus, roughly 95 per cent of the total variation of real money balances and output can be explained by a sticky-price DSGE model with habit formation and costly capital adjustment. The model does not explain as well the nominal interest rate, but still can account for more than 89 per cent of the total variation of this series.6

The model also generates predictions regarding data series that were not used in the estimation procedure. Figure 2 plots the actual and predicted series of U.S. consumption, investment, hours worked, real wage, real marginal cost and inflation. The real marginal cost is not directly observable, but under certain conditions, it can be proxied by the labor share in national income (for a detailed discussion, see Gali and Gertler, 1999). The model generates consumption and investment dynamics that are similar to the ones of their detrended U.S. counterparts. However, predicted investment is much smoother than the data.

The model is less successful in explaining the behavior of the other variables. The model predicts much more volatility in hours worked and real wages than in the data. The reason for these results is that, while the model assumes capital adjustment costs and price stickiness, hours worked can be adjusted without cost and nominal wages are flexible. Predicted inflation is more volatile and less persistent than in the data. This result reflects a drawback of inflation models based on forward-looking pricing rules. It is easy to show that, under Calvo-type pricing, the inflation deviation from steady state equals the present discounted value of current and future expected real marginal cost deviations from steady state. This means that inflation inherits the dynamic properties of the real marginal cost and that current inflation is not helpful in predicting future inflation. Because lagged inflation is absent from the inflation equation, forward-looking pricing rules imply that inflation is less persistent than usually found in the data. To address this shortcoming of the model, some authors (for example, Gali and Gertler, 1999) assume the existence of rule-of-thumb firms that fix their prices as a function of past

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6 The working paper version of this article (Bouakez, Cardia and Ruge-Murcia, 2003) reports tests for serial correlation of the residuals and neglected Autoregressive Conditional Heteroskedasticity (ARCH). The hypotheses of no autocorrelation and no conditional heteroskedasticity cannot be rejected for real money balances and output, but they are rejected for the nominal interest rate.
inflation.

Finally, the real marginal cost in the model is much more volatile than the labor share in national income would suggest. One possibility is that the labor share in national income is a poor empirical proxy for the real marginal cost. More likely, the real marginal cost in our model is excessively volatile because it abstracts from supply-side features like variable capital utilization and adjustment costs to labor input.

4.3. Impulse-Response Analysis

This section examines the response of the model economy to a money supply shock. Figure 4 reports the responses of output, consumption, investment, hours worked, inflation, the nominal interest rate, real money balances, the real wage and the real marginal cost to a temporary shock that makes the growth rate of the money supply 1 percent higher than in steady state. The model responses are compared with those generated by a Vector Autoregression (VAR) of order 2.

Following the shock, there is an increase in aggregate demand that causes output and consumption to increase. The consumption response is hump-shaped because, under habit formation, agents smooth both the level and the change of consumption. Both the VAR and the model predict hump-shaped responses, but the peak of the output (consumption) response in the model takes place after two (five) quarters, rather than the four (four) quarters in the VAR.

Investment and hours worked increase following a positive monetary shock. This result is due to the fact that aggregate demand is expected to increase in subsequent periods because prices adjust slowly. Note, however, that the investment response does not exhibit the hump-shaped dynamics found in the VAR. Also, the hours response is hump-shaped as in the VAR but its magnitude is much larger. The predicted large effect of the monetary shock on hours worked is most likely due to the fact that employment changes are costless in the model. Following the shock, the increase in output is accommodated initially with an increase in hours worked because the capital stock is predetermined. Without frictions, this initial effect can be large. Moreover, since capital is costly to adjust in subsequent periods, the hours response is persistent.

The model predicts an increase in the rates of nominal interest and inflation after a monetary shock. In contrast, the VAR shows that the initial response by both variables is negative. Thus, the model does not generate a liquidity effect, nor does it explain the price puzzle. The reason is that the estimated money growth process is highly autocorrelated. Thus, after a positive money supply shock, expected inflation increases by a magnitude that is larger in absolute value than the decrease in the real interest rate. As a result, the net effect of the money supply shock on the nominal interest rate is positive. Since the model generates a very large inflation response, real balances in the model decline, rather than increase as they do in the VAR. Finally, the model predicts strong effects of the money supply shock on the real wage and real marginal cost, while the VAR responses are considerably muted. The large response by the real wage is attributable to the model’s assumption that
nominal wages are flexible.

4.4. Variance Decomposition

This section examines the relative importance of monetary shocks for the fluctuations of output, investment, consumption, hours worked, inflation and the nominal interest rate. To that effect, we compute the fraction of the conditional variance of the forecasts at different horizons that is attributed to each of the shocks. Recall that the money supply shock is a shock to the growth rate of the money supply and the money demand shock is a shock to the preference parameter of money in the utility function. As the horizon increases, the conditional variance of the forecast error of a given variable converges to the unconditional variance of that variable. The decompositions of the variances at the one-year, three-year and infinite horizons are reported in Table 2.

Several results are apparent from this Table. First, money supply shocks account for the largest part of the conditional variance in forecasting output in the short run (i.e., less than a year). At higher horizons, most of the conditional variance is due to technology shocks. In the long-run, 27 per cent of the unconditional variance of output is attributed to money supply shocks, 2 per cent to money demand shocks and 71 per cent to technology shocks. Second, money supply shocks and technology shocks are both important in explaining the conditional variance of consumption in the very short run. However, as the horizon increases the contribution of technology shocks increases and that of money supply shocks decreases. In the long-run, 77.3 per cent of the variance of consumption is explained by technology shocks and only 21.6 per cent by money supply shocks. Third, money supply shocks account for the largest part of the conditional variance in forecasting investment in the short-run. As the horizon increases, the contribution of technology shocks increases and the one of money supply shocks decreases but even in the long-run, money supply shocks explain half of the variance of investment. Fourth, money supply shocks explain most of the fluctuations of the rate of inflation at all horizons. Fifth, technology shocks explain most of the variation in hours worked at all horizons. Finally, money demand shocks play an important role in explaining the fluctuations of the nominal interest rate. At horizons of less than six quarters, money demand shocks explain more than 50 per cent of the conditional variance of the nominal interest rate. In the long-run, money demand shocks explain roughly 45 per cent of the conditional variance of the nominal interest rate.

4.5. The Role of Habit Formation and Capital Adjustment Costs

This section examines the empirical contribution of habit formation and capital adjustment costs to the propagation mechanism of the model. To that end, two restricted versions of the model are solved and estimated via Maximum Likelihood. One version is a sticky-price model with neither habit formation, nor frictional costs in capital adjustment. This model is similar to the one calibrated by Yun (1996).
and King and Watson (1996) and is obtained by restricting $\gamma = 0$ and $\Gamma(\cdot) = x_t/k_t$ in the model in Section 2. The other version allows capital adjustment costs but keeps the restriction that $\gamma = 0$.

First, consider the version of the DSGE model with sticky prices and capital adjustment costs but without habit formation. Comparing this model and the one in Section 2, will allow us to assess the role of habit formation in the propagation of money supply shocks. Maximum Likelihood estimates of the structural parameters of this restricted model are reported in Column 2 of Table 1. Note that estimates are not very different from the ones reported in Column 1 for the model with both habit formation and capital adjustment costs. The exception is the elasticity of investment with respect to the price of installed capital evaluated at the steady state ($\chi$). Regardless of the starting values in the optimization routine, $\chi$ would converge to zero. Zero is the minimum value of $\chi$ in the parameter space, and implies that the capital stock is infinitely costly to adjust. Thus, investment is zero. Since the estimate of $\chi$ is on the boundary of the parameter space, regularity conditions fail and standard errors cannot be computed as usual. To address this issue, we imposed the restriction $\chi = 0$ at the maximum (or, more precisely, $\chi = 1 \times 10^{-5}$) and then computed standard errors as the square root of the diagonal elements of the inverted Hessian.

Figure 4 plots the responses predicted by this restricted model to a money supply shock (semi-continuous lines). The responses predicted by the full model are plotted in continuous lines. Output and consumption jump immediately after the shock and return fairly quickly to their steady-state levels. Without habit formation, the dynamics are monotone and lack the hump-shaped pattern predicted by the VAR and the model in Section 2. The nominal interest and inflation rates respond to a money supply shock in a qualitatively similar manner as in the model with habit formation. These results indicate that habit formation magnifies output persistence in response to a monetary shock. The reason is that habit-forming agents allocate resources to obtain a smoother and more persistent consumption profile than agents with time-separable preferences. This implies a more persistent path of labor supply and output following a shock.

Our results are broadly consistent with Fuhrer (2002). Fuhrer examines the effect of habit formation in a monetary policy model. His model is a structural Vector Autoregression with restrictions that arise from the Euler equation for consumption, a Taylor-type monetary policy rule and a price contracting model similar to that in Fuhrer and Moore (1995). Fuhrer shows that habit formation improves the output dynamics without worsening the other dynamic interactions of the model. However, our results regarding the inflation dynamics are different from his. Comparing the inflation responses in Figure 4 indicates that habit formation has little effect on the inflation dynamics in our DSGE model. The reason for this discrepancy is the assumed pricing behavior of firms. Calvo-type pricing is forward looking and implies a lower inflation persistence than in the data. On the other hand, Fuhrer and Moore’s formulation is equivalent to a two-sided inflation specification where inflation depends on both its leads and lags. This formulation appears to capture better the observed inflation persistence.
Finally, consider a DSGE model with sticky prices alone. This model is obtained by imposing the restrictions $\gamma = 0$ and $\Gamma(\cdot) = x_t/k_t$ on the model in Section 2. These restrictions mean that there is neither habit formation, nor adjustment costs to the capital stock. Because it is costless to adjust the capital stock, the elasticity of investment with respect to the price of installed capital is infinite.

Maximum Likelihood estimates of the parameters are reported in the Column 3 of Table 1. The estimate of the probability that an intermediate good producer does not reoptimize its price in a given quarter is $\hat{\varphi} = 0.346 (0.457)$. This estimate is very imprecise and one cannot reject the hypothesis that the true value is either zero or one. Estimates of the preference parameters $\eta_2$ and $\eta_3$ are quantitatively very large, imprecisely estimated, and in the latter case not statistically different from zero. Finally, estimates of $\rho_2$, $\rho_3$ and $\rho_6$ indicate that structural shocks are persistent. Because the parameters are imprecisely estimated, the results for this model should be interpreted with caution. Figure 4 plots the responses predicted by this model to a money supply shock (dotted lines). Monetary shocks have a small effect on output and consumption. These variables (and inflation) adjust rapidly in response to monetary shocks and return near their steady state values after only one period. The reason is that, without capital adjustment costs, firms can pull forward in time their response to a monetary shock.\footnote{In some unreported counterfactual experiments, we set the parameter that determines price rigidity ($\varphi$) to different values from its Maximum Likelihood estimate and obtained similar impulse responses to the ones reported here. Only the magnitude of the effects varies depending on the degree of price stickiness.}

Earlier research by Jermann (Jermann, 1998) finds that habit formation alone is insufficient to explain the equity premium puzzle and salient business cycles facts in a general equilibrium framework. The reason is that, in general equilibrium (with production), households can modify the intertemporal allocation of resources. Hence, the only difference between RBC models with and without habit formation is that consumption is smoother in the former than in the latter. With no adjustment costs, consumers can reallocate resources freely to decrease the volatility of the marginal rate of substitution and obtain a smoother consumption profile than that predicted by the Permanent Income Hypothesis. Thus, capital adjustment costs in Jermann (1998) and factor allocation frictions across sectors in Boldrin, Christiano and Fisher (2001) limit the scope of resource allocation and, along with habit formation, help to explain the equity premium puzzle in a general equilibrium setup. In this paper, price rigidities are necessary for money to be non-neutral, and the interaction of some factor-market inflexibility (capital adjustment costs) and habit formation is necessary to explain the persistent output response to monetary shocks.

5. Conclusion

This paper constructs a DSGE model with sticky prices, habit formation and costly capital adjustment that accounts for the persistent and hump-shaped response of output to monetary shocks. The model is estimated by the method of Maximum Likelihood using U.S. data on output, real money balances and the nomi-
nal interest rate. Econometric results indicate that U.S. prices are fixed on average for six and a half quarters. The peak of the output response takes place after two quarters, that is less than the four quarters found in a benchmark VAR model. Also, the size of the hump is smaller in the DSGE model than in the VAR. Variance decomposition indicates that money growth explains more than 50 per cent of the (conditional) output variability at horizons of less than a year. In the long run, money growth explains only 27 per cent of the unconditional output variability while 71 per cent is explained by technology shocks. The empirical analysis of two restricted versions of the model suggests that habit formation interacts with costly capital adjustment to strengthen the internal propagation mechanism of the model. Although the model tracks well the behavior of consumption and investment, it is less successful in explaining hours worked and the inflation rate. In the former case, this is due to the assumption that the labor input can be adjusted without cost. In the latter case, this is due to the assumption of a purely forward-looking pricing rule that implies a lower inflation persistence than in the data.
### Table 1. Maximum Likelihood Estimates

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habit parameter</td>
<td>$\gamma$</td>
<td>0.982*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment elasticity</td>
<td>$\chi$</td>
<td>0.469*</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.106)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of no price change</td>
<td>$\varphi$</td>
<td>0.847*</td>
<td>0.808*</td>
<td>0.346</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.034)</td>
<td>(0.188)</td>
<td>(0.457)</td>
</tr>
<tr>
<td>Preference parameter</td>
<td>$\eta_2$</td>
<td>3.089*</td>
<td>2.564*</td>
<td>33.51*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.826)</td>
<td>(0.813)</td>
<td>(13.50)</td>
</tr>
<tr>
<td>Preference parameter</td>
<td>$\eta_3$</td>
<td>1.591</td>
<td>2.243</td>
<td>23.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.879)</td>
<td>(22.020)</td>
<td>(57.56)</td>
</tr>
<tr>
<td>Coefficient technology shock</td>
<td>$\rho_z$</td>
<td>0.867*</td>
<td>0.930*</td>
<td>0.981*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.051)</td>
<td>(0.031)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>Coefficient money supply shock</td>
<td>$\rho_\mu$</td>
<td>0.880*</td>
<td>0.937*</td>
<td>0.957*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.033)</td>
<td>(0.027)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Coefficient money demand shock</td>
<td>$\rho_b$</td>
<td>0.924*</td>
<td>0.882*</td>
<td>0.995*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.019)</td>
<td>(0.021)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>S.D. of technology innovation</td>
<td>$\sigma_{e_z}$</td>
<td>0.040</td>
<td>0.021</td>
<td>0.009*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.023)</td>
<td>(0.035)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>S.D. of money supply innovation</td>
<td>$\sigma_{e_\mu}$</td>
<td>0.007*</td>
<td>0.003*</td>
<td>0.002*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>S.D. of money demand innovation</td>
<td>$\sigma_{e_b}$</td>
<td>0.077*</td>
<td>0.072*</td>
<td>0.312*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.005)</td>
<td>(0.004)</td>
<td>(0.127)</td>
</tr>
</tbody>
</table>

| Value of log-likelihood function     | 2408.30   | 2384.33   | 2345.51   |

**Notes:** S.D. is standard deviation. The figures in parenthesis are standard errors. The superscript * denotes the rejection of the null hypothesis that the parameter is zero at the 5 per cent significance level. The restrictions imposed on the parameters were $\gamma, \varphi \in (0, 1)$, $\rho_z, \rho_\mu, \rho_b \in (-1, 1)$, and $\eta_2, \eta_3, \chi, \sigma_{e_z}, \sigma_{e_\mu}, \sigma_{e_b} \in (0, \infty)$. For the purpose of computing standard errors for the second model, $\chi$ is fixed to $1 \times 10^{-5}$. 
Table 2. Variance Decomposition
Model with Habit Formation and Capital Adjustment Costs

<table>
<thead>
<tr>
<th>Variable</th>
<th>Technology Shocks</th>
<th>Money Supply Shocks</th>
<th>Money Demand Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One-Year Horizon</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.423</td>
<td>0.529</td>
<td>0.048</td>
</tr>
<tr>
<td>Investment</td>
<td>0.194</td>
<td>0.735</td>
<td>0.071</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.579</td>
<td>0.388</td>
<td>0.033</td>
</tr>
<tr>
<td>Hours</td>
<td>0.929</td>
<td>0.065</td>
<td>0.006</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.247</td>
<td>0.727</td>
<td>0.026</td>
</tr>
<tr>
<td>Nominal interest</td>
<td>0.071</td>
<td>0.372</td>
<td>0.557</td>
</tr>
<tr>
<td><strong>Three-Year Horizon</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.652</td>
<td>0.326</td>
<td>0.022</td>
</tr>
<tr>
<td>Investment</td>
<td>0.401</td>
<td>0.553</td>
<td>0.046</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.722</td>
<td>0.262</td>
<td>0.016</td>
</tr>
<tr>
<td>Hours</td>
<td>0.880</td>
<td>0.112</td>
<td>0.008</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.214</td>
<td>0.765</td>
<td>0.021</td>
</tr>
<tr>
<td>Nominal interest</td>
<td>0.154</td>
<td>0.387</td>
<td>0.460</td>
</tr>
<tr>
<td><strong>Infinite Horizon</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.714</td>
<td>0.271</td>
<td>0.015</td>
</tr>
<tr>
<td>Investment</td>
<td>0.469</td>
<td>0.493</td>
<td>0.038</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.773</td>
<td>0.216</td>
<td>0.011</td>
</tr>
<tr>
<td>Hours</td>
<td>0.872</td>
<td>0.120</td>
<td>0.008</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.221</td>
<td>0.756</td>
<td>0.023</td>
</tr>
<tr>
<td>Nominal interest</td>
<td>0.163</td>
<td>0.389</td>
<td>0.448</td>
</tr>
</tbody>
</table>

*Notes:* The money supply shock is a shock to the growth rate of the money supply. The money demand shock is a shock to the preference parameter of money in the utility function.
References


Bouakez, H., E. Cardia, and F. J. Ruge-Murcia, 2003, Habit Formation and the Persistence of Monetary Shocks, Université de Montréal, Mimeo.


Figure 1: Actual (continuous lines) and predicted (dashed lines) values of variables in measurement equation.
Figure 2: Actual (continuous lines) and predicted (dashed lines) values of other model variables
Figure 3: Responses to a 1 per cent money supply shock in the model (continuous lines) and the VAR (dashed lines)
Figure 4: Sensitivity Analysis. Responses to a 1 percent money supply shock in the models with habit formation and adjustment costs (continuous line), no habit formation and adjustment costs (semi-continuous line), and neither habit formation nor adjustment costs (dashed line).