The Optimal Composition of Public Spending in a Deep Recession*

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Abstract

We study optimal fiscal policy in an economy where monetary policy is constrained by the zero lower bound on the nominal interest rate, and where the government can allocate spending to public consumption and public investment. We show that the optimal response to an adverse shock that precipitates the economy into a liquidity trap entails a small and short-lived increase in public consumption but a large and persistent increase in public investment, which lasts well after the natural rate of interest has ceased to be negative. During this period, the optimal composition of public spending is therefore heavily skewed towards public investment. Contrary to the literature that abstracts from public investment, we find that the optimal increase in total public spending in a deep recession is sizable. To the extent that monetary policy is set optimally, this fiscal expansion has little to do with a stabilization motive and is instead mainly warranted by the intertemporal allocation of resources that efficiency dictates. In contrast, when monetary policy is sub-optimal, there is an important scope for using public spending in general and public investment in particular as a stabilization tool.

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1 Introduction

What is the optimal composition of a fiscal expansion in a depressed economy? Despite the widespread interest in the stimulative effects of fiscal policy generated by the unprecedentedly large stimulus plans enacted in most industrialized economies at the onset of the Great Recession, this question has remained largely overlooked by the literature. Existing studies indeed mostly focus on the size of the spending multiplier when the economy is plunged in a liquidity trap, that is, when nominal interest rates are at their zero lower bound (ZLB), and on the desirability of public spending from a welfare standpoint in such circumstances.

Two main conclusions emerge from that literature. First, the spending multiplier can be substantially large when the ZLB binds.[1] Second, it is optimal to temporarily increase public spending (at least relative to its flexible-price level) while the economy is in a liquidity trap.[2] The intuition for why public spending improves welfare is the following. When monetary policy is unconstrained — so that it can replicate the flexible-price allocation — and to the extent that government spending provides utility to households, optimality requires that the marginal utilities of private and public goods be equated, a condition commonly referred to as the Samuelson rule. The latter thus implies that government spending co-moves with consumption: if an adverse shock causes a fall in consumption, it will also lead to a fall in public spending. When the ZLB binds, however, nominal interest rates cannot be used to fully stabilize the economy, and an undesired (negative) output gap emerges, creating another motive for varying public spending. In response to an adverse shock, public spending rises above its flexible-price level to help close the output gap.[3]

Bilbiie et al. (2014), however, argue that the optimal increase in public spending in response to a typical recession is tiny — less than 0.5 percent of steady-state output. Essentially, this result reflects the fact that optimal public spending needs to strike a balance between stabilizing the output gap and meeting the Samuelson condition. Under empirically plausible scenarios about the size of the adverse shock, optimal public spending rises but only by a small amount in order not to deviate too far from the Samuelson condition. Bilbiie et al. (2014) show that to obtain large optimal levels of public spending, one needs to assume implausibly severe recessions that take the economy close to the starvation point (the point where private consumption is at its minimum level). Furthermore, Sims & Wolff (2013) argue that the welfare multiplier of public spending, defined as the change in aggregate welfare for a one-unit change in government spending, albeit positive, is procyclical. That is, it tends to be low during recessions and high during expansions. Together,

[1] See Christiano et al. (2011), Eggertsson (2011), and Woodford (2011), among many others.

[2] See, for instance, Woodford (2011), Werning (2011), Mankiw & Weinzierl (2011), Schmidt (2013), Nakata (2013), Bilbiie et al. (2014), and Nakata (2015) in the context of a closed economy, and Bhattarai & Egorov (2016) in the context of a small open economy.

[3] A similar motive for increasing public spending is shown by Michaillat & Saez (2015) in the context of a model with search and matching frictions in the goods and labor markets.

these findings cast doubt on the usefulness of public spending from a normative perspective.

A common characteristic of all of these studies is the assumption that public spending consists exclusively of purchases of consumption goods, so that there is no scope for public investment. This assumption is unlikely to be innocuous when it comes to determining the optimal level of public spending and its welfare consequences. But perhaps more importantly, it precludes the analysis of the optimal composition of a fiscal expansion, an issue that was at the center of policy debates during the Great Recession. The various fiscal plans that have been implemented worldwide in 2008-2009 assigned a significant fraction of the additional spending to public investment in infrastructure,[4] but to our knowledge, there has not been any formal attempt to determine whether this allocation scheme was warranted from a welfare standpoint.

The objective of this paper is to study optimal fiscal policy in an economy where monetary policy is constrained by the ZLB on the nominal interest rate, and where the government can allocate spending to public consumption and public investment. More specifically, we study the optimal policy response to an adverse shock that precipitates the economy into a deep recession characterized by a liquidity trap. As is common in the literature cited above, we assume that the ZLB binds as a result of a large preference shock that raises agents' discount factor. The main novelty of our model with respect to that literature is the possibility for the government to accumulate public capital, which is an external input in the firms' production technology.[5] As in Leeper et al. (2010), Leduc & Wilson (2013), and Bouakez et al. (2017), we assume that the accumulation of public capital is subject to lengthy time-to-build delays, a distinctive feature of public infrastructure projects.[6]

We study optimal fiscal policy under two different scenarios about the conduct of monetary policy: one in which the latter is set optimally under the assumption that the policy maker can pre-commit to a future path of the nominal interest rate even if the ZLB is currently binding, and one in which monetary policy is set sub-optimally according to a Taylor-type rule. As a useful benchmark, we compute the first-best (efficient) allocation, i.e., the welfare-maximizing allocation chosen by a benevolent central planner. In our model, the first best coincides with the flexible-price allocation under optimal fiscal policy.

We find that the optimal fiscal policy response to an adverse shock that makes the ZLB bind is to initially raise public consumption and public investment above their steady-state levels. The increase in public consumption is negligible and short-lived, followed by a prolonged cut that persists even after the natural rate of interest has ceased to be negative. In contrast, the increase

[4] For instance, the American Recovery and Reinvestment Act and the European Economic Recovery Plan allocated, respectively, 40 and 71 percent of the additional public spending to investment in infrastructure.

[5] Earlier papers that also study optimal public consumption and investment include those by Lansing (1998), Ambler & Paquet (1996), and Ambler & Cardia (1997). Unlike our paper, however, these authors consider a real-business-cycle framework and abstract from monetary policy and the ZLB.

[6] None of the three papers studies optimal policy.

in public investment is relatively large and persistent, lasting well after the natural rate of interest has become positive again. These paths are subsequently reversed, as the optimal plan eventually entails raising public consumption and decreasing public investment relative to their steady-state levels. Qualitatively, these patterns are observed regardless of whether monetary policy is set optimally or not.

These findings have two salient implications. First, the optimal fiscal plan features a change in the composition of public spending in a way that assigns a larger weight to public investment relative to public consumption for a prolonged period of time after the shock. Second, the optimal fiscal expansion is sizable; under our baseline calibration, the cumulative increase in total public spending exceeds 10 percent of steady-state output. This result stands in sharp contrast with the conclusion based on optimal fiscal plans in which only public consumption is adjusted, as is the case in existing studies.

We then ask: how much of this fiscal expansion is due to the presence of a negative output gap, or, equivalently, is warranted by a stabilization motive? We refer to this spending component as *stimulus spending* and we compute it as the difference between the spending level obtained under the optimal plan and that obtained under fully flexible prices. The latter, which we label *neoclassical spending*, would occur even if monetary policy were able to fully eliminate any output gap. We show that the answer to the question above crucially depends on the stance of monetary policy.

When monetary policy is chosen optimally, we find that the stimulus component of public consumption and public investment is front-loaded but negligible, summing to less than 0.05 percent of steady-state output at the time of the shock and cumulating to approximately -0.05 percent of steady-state output over time. In contrast, when monetary policy is set according to a Taylor rule, stimulus spending is both non-monotonic and considerable. It culminates at about 0.7 and 2.4 percent of steady-state output for, respectively, public consumption and public investment, and remains positive for roughly 3 years after the shock. The reason for these results is the following: when monetary policy is set optimally, it goes a long way towards closing the output gap and replicating the flexible-price allocation even when the ZLB is binding. In this case, the desirability of a fiscal expansion and the larger weight assigned to public investment in response to an adverse shock have very little to do with a stabilization motive; instead, they essentially reflect the role of public investment in enabling the intertemporal allocation of resources that efficiency dictates even in the absence of an output gap. In contrast, when monetary policy is sub-optimal, an adverse shock that causes the ZLB to bind gives rise to a large output gap, thus creating a stabilization role for public spending in general and public investment in particular. The latter increases significantly more than its flexible-price level in order to absorb some of the idle resources in the economy and increase aggregate demand.

These results conflict with the conclusion reached by Mankiw & Weinzierl (2011), who argue that public investment is not particularly useful for putting idle resources to work. In their model, public investment is always chosen based on "classical principles" — by equating its marginal product to that of private capital — regardless of whether the output gap is zero or negative. In other words, the stimulus component of public investment is always nil. We show that this conclusion is specific to their model, which assumes a production technology in which private and public capital enter in an additively separable manner, thus implying that public capital does not affect the marginal productivity of private inputs. Once this assumption is relaxed, our analysis shows that there is an important scope for using public investment as a stabilization tool.

We show how our main results are affected by perturbations to key structural parameters, namely the elasticity of output with respect to public capital and the length of time to build. We also study the robustness of our results along three dimensions. First, we consider the possibility that the planner relies on distortionary taxes to finance public spending; second, we extend the model to allow for the accumulation of private capital; and finally, we assume that the ZLB binds as a result of a liquidity premium shock. We find our conclusions to be robust to these three alternative modelling assumptions.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 characterizes the first-best response to the preference shock. Section 4 studies optimal monetary and fiscal policy. Section 5 studies optimal fiscal policy when monetary policy is sub-optimal. Section 6 performs a sensitivity and robustness analysis. Section 7 discusses the case in which fiscal authorities lack the ability to pre-commit to future announcements. Section 8 concludes.

2 A New Keynesian Model with Public Capital

We consider a simple new-Keynesian economy without private capital as in Bouakez et al. (2017). The economy is composed of infinitely lived households, firms, a government, and a monetary authority. The key feature of the model is that a fraction of government spending can be invested in public capital subject to a time-to-build requirement. The remaining fraction, i.e., government consumption, directly affects households' utility. The breakdown of public spending into investment and consumption expenditures is chosen optimally by the government. The stock of public capital enters as an external input in the production of intermediate goods, which are used to produce an homogenous final good. The latter is used for consumption and investment purposes. There is a continuum of monopolistically competitive intermediate-good producers, indexed by $z \in (0, 1)$, which set prices subject to a Rotemberg (1982)-type adjustment cost. Final-good producers are perfectly competitive. The nominal interest rate is subject to a non-negativity constraint.

2.1 Households

The economy is populated by a large number of identical households who have the following lifetime utility function:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \xi_{t+s} \left[U\left(C_{t+s}, N_{t+s}\right) + V\left(G_{t+s}^c\right) \right], \tag{1}$$

where β is the discount factor, C_t is consumption, N_t denotes hours worked, G_t^c denotes government consumption. We assume that $U(.)$ is increasing and concave in C_t and decreasing and concave in N_t; and $V(.)$ is increasing and concave in G_t^c. The total time endowment of the representative household is normalized to 1, so that leisure time is equal to $1 - N_t$. Both consumption and leisure are assumed to be normal goods. The term ξ_t is a preference shock that evolves according to the following process:

$$\log(\xi_t) = \rho \log(\xi_{t-1}) + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0,1),$$

where $\rho \in (0,1)$. A negative shock to ξ_t thus raises the effective discount factor in the subsequent periods, which increases households' incentive to save, *ceteris paribus*.

The representative household enters period t with $\mathcal{B}_{t-1}$ units of one-period riskless nominal bonds. During the period, it receives a nominal wage payment, $W_t N_t$, and dividends, $\mathcal{D}_t = \int_0^1 \mathcal{D}_t(z)\, dz$, from monopolistically competitive firms. This income is used to pay a lump-sum tax, $\mathcal{T}_t$, to the government, to consumption, and to the purchase of new bonds. The household's budget constraint is therefore

$$P_t C_t + \mathcal{T}_t + \frac{\mathcal{B}_t}{1+R_t} \leq W_t N_t + \mathcal{D}_t + \mathcal{B}_{t-1}, \tag{2}$$

where P_t is the price of the final good, W_t is nominal wage rate, and $\frac{1}{1+R_t}$ is the price of a nominal bond purchased at time t, R_t being the nominal interest rate. The household maximizes (1) subject to (2) and to a no-Ponzi-game condition. The first-order conditions for this problem are given by

$$w_t = -\frac{U_{N,t}}{U_{C,t}}, \tag{3}$$

$$\frac{1}{1+R_t} = \beta \mathbb{E}_t \left(\frac{\xi_{t+1}}{\xi_t} \frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right), \tag{4}$$

where $w_t = \frac{W_t}{P_t}$ is the real wage rate and $U_{X,t} = \partial U\left(C_t, N_t\right) / \partial X_t$.

2.2 Firms

The final good is produced by perfectly competitive firms using the following constant-elasticity-of-substitution technology:

$$Y_t = \left(\int_0^1 Y_t(z)^{1-1/\theta}\, dz \right)^{\frac{\theta}{\theta-1}}, \tag{5}$$

where $Y_t(z)$ is the quantity of intermediate good z and $\theta \geq 1$ is the elasticity of substitution between intermediate goods. Denoting by $P_t(z)$ the price of intermediate good z, demand for z is given by

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\theta} Y_t. \tag{6}$$

Firms in the intermediate-good sector are monopolistically competitive, each producing a differentiated good using labor as a direct input and public capital as an external input

$$Y_t(z) \leq F(N_t(z), K_{G,t}), \tag{7}$$

where $F(.)$ is increasing and concave in both of its arguments. This specification implies that public capital acts as a positive externality that improves the marginal productivity of private inputs. This assumption is consistent with the empirical evidence reported by Aschauer (1989), Fernald (1999), Leduc & Wilson (2013), and Bouakez et al. (2017), among many others.[7] Aschauer (1989) estimates an aggregate production function for the U.S. economy and finds that non-military infrastructure has a strong positive effect on total factor productivity. Fernald (1999) finds a causal effects of growth in roads (the largest component of U.S. public infrastructure) on the productivity of U.S. vehicle-intensive industries. Leduc & Wilson (2013) find evidence that federal highway spending works like an anticipated productivity shock, raising output several years in the future. Finally, using cointegration techniques and post-War U.S. data, Bouakez et al. (2017) find a positive long-term relationship between the stock of public capital and (a purified measure of) total factor productivity.

We assume that firms set prices subject to a Rotemberg (1982)-type adjustment cost. That is, in each period, a given firm pays a quadratic adjustment cost to reset its nominal price, $P_t(z)$, measured in terms of the final good and given by

$$\Xi_t(z) = \frac{\psi}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 Y_t, \tag{8}$$

where $\psi \geq 0$ governs the magnitude of the price-adjustment cost.

Dividends paid by firm z are given by

$$\mathcal{D}_t(z) = (1+\tau) P_t(z) Y_t(z) - W_t N_t(z) - P_t \Xi_t(z), \tag{9}$$

where $\tau = 1/(\theta - 1)$ is a subsidy that corrects the steady-state distortion stemming from monopolistic competition in the goods market. Firm z chooses $P_t(z)$ for all t to maximize its total real market value

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\xi_{t+s}}{\xi_t} \frac{U_{C,t+s}}{U_{C,t}} \frac{\mathcal{D}_{t+s}(z)}{P_{t+s}}, \tag{10}$$

[7] For recent surveys of the empirical literature on the productivity of public capital see Romp & de Haan (2007) and Bom & Ligthart (2014).

subject to the production technology (7) and the Hicksian demand function (6).

Since all the firms face an identical problem, the optimal price will satisfy the following condition:

$$0 = \theta \left(mc_t - 1 \right) - \psi \left[\left(1 + \pi_t \right) \pi_t - \beta \mathbb{E}_t \frac{\xi_{t+1}}{\xi_t} \frac{U_{C,t+1}}{U_{C,t}} \frac{Y_{t+1}}{Y_t} \left(1 + \pi_{t+1} \right) \pi_{t+1} \right], \tag{11}$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation rate and mc_t is the real marginal cost of production, defined by

$$mc_t = \frac{w_t}{F_N \left(N_t, K_{G,t} \right)}. \tag{12}$$

2.3 Fiscal and monetary authorities

The government levies lump-sum taxes to finance its expenditures and the subsidy given to firms in the intermediate-good sector. Its budget constraint is given by

$$G_t + \tau Y_t = \frac{\mathcal{T}_t}{P_t}, \tag{13}$$

where G_t is total government spending, which is composed of two parts, public consumption and public investment

$$G_t = G_t^c + G_t^i. \tag{14}$$

Public investment increases the stock of public capital according to the following accumulation equation:

$$K_{G,t+T} = \left(1 - \delta \right) K_{G,t+T-1} + \left(1 - S \left(\frac{G_t^i}{G_{t-1}^i} \right) \right) G_t^i, \tag{15}$$

where $T \geq 0$ and the function $S(\cdot)$ satisfies $S'(\cdot) \geq 0$, $S''(\cdot) \geq 0$, $S(1) = 0$, and $S'(1) = 0$. Equation (15) allows for the possibility that several periods may be required to build new productive capital, i.e., time to build (see Kydland & Prescott (1982)). This feature reflects the implementation delays typically associated with the different stages of public investment projects (planning, bidding, contracting, construction, etc.). The function $S(\cdot)$ captures adjustment costs of public investment, as an additional unit of investment at time t increases the stock of capital at time $t + T$ by less than one unit.

2.4 Market clearing and private-sector equilibrium

Since households are identical and there is no private capital, the net supply of bonds must be zero in equilibrium ($\mathcal{B}_t = 0$). Substituting the definition of dividends in the representative household's budget constraint and using (14), one obtains the resource constraint of this economy

$$\Delta_t F(N_t, K_{G,t}) = \Delta_t Y_t = C_t + G_t^c + G_t^i, \tag{16}$$

where $\Delta_t = \left(1 - \frac{\psi}{2} \pi_t^2 \right)$.

A private-sector equilibrium for this economy is a sequence of quantities and prices $\{N_t, C_t, K_{G,t}, \pi_t, w_t, mc_t\}_{t=0}^{\infty}$ such that, for a given sequence of policy variables $\{R_t, G_t^c, G_{t-T}^i\}_{t=0}^{\infty}$ and exogenous variables $\{\xi_t\}_{t=0}^{\infty}$, and given an initial stock of public capital, $K_{G,-T}$, and the definitions of the utility, production and adjustment-cost functions, equations (3), (4), (11), (12), (15), and (16) hold.

A Ramsey planner maximizes utility by choosing (and committing to) a path for the policy variables subject to the constraints implied by the optimizing behavior of households and firms. In other words, the Ramsey planner selects among all the implementable private-sector equilibria described above the one that maximizes social welfare.

3 The First Best Allocation

Before studying the Ramsey (second-best) equilibrium, it is useful to have as a benchmark the efficient (or first-best) allocation of resources, i.e., the allocation that would be chosen by a benevolent central planner.

3.1 Maximization program and solution

The planner's problem is to choose the sequence of allocations that maximize households' lifetime utility given the sequence of the economy's resource constraints and accumulation equations for capital. Formally,

$$\begin{aligned}
&\max_{Z_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \xi_t \left[U\left(C_t, N_t\right) + V\left(G_t^c\right) \right] \\
&+ \lambda_{1,t} \left[F(N_t, K_{G,t}) - C_t - G_t^c - G_t^i \right] \\
&+ \lambda_{2,t} \left[\left(1-\delta\right) K_{G,t+T-1} + \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right) G_t^i - K_{G,t+T} \right] \Big\},
\end{aligned}$$

where $Z_t = \left[C_t, N_t, G_t^c, G_t^i, K_{G,t+T}\right]$ is the vector of choice variables and the λ's are Lagrange multipliers. Note that, due to the time-to-build delay, the planner does not have any control over K_{t+i} for $i < T$ at time t, so that the true choice variables are K_{t+T+i} for $i \geq 0$. The efficient allocation is the solution to the following set of equations:

$$0 = \xi_t U_{C,t} - \lambda_{1,t}, \tag{17}$$

$$0 = \frac{U_{N,t}}{U_{C,t}} + F_N(N_t, K_{G,t}), \tag{18}$$

$$0 = \xi_t V_{G,t} - \lambda_{1,t}, \tag{19}$$

$$0 = \lambda_{1,t} - \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right) - \frac{G_t^i}{G_{t-1}^i} S'\left(\frac{G_t^i}{G_{t-1}^i}\right)\right]\lambda_{2,t} - \beta \mathbb{E}_t \lambda_{2,t+1}\left(\frac{G_{t+1}^i}{G_t^i}\right)^2 S'\left(\frac{G_{t+1}^i}{G_t^i}\right), \tag{20}$$

$$0 = \lambda_{2,t} - \beta(1-\delta)\mathbb{E}_t\lambda_{2,t+1} - \beta^T \mathbb{E}_t \lambda_{1,t+T} F_{K_G}\left(N_{t+T}, K_{G,t+T}\right), \tag{21}$$

$$0 = F(N_t, K_{G,t}) - C_t + G_t^C + G_t^i, \tag{22}$$

$$0 = K_{G,t+T} - (1-\delta)\, K_{G,t+T-1} - \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right] G_t^i. \tag{23}$$

Equation (17) defines the Lagrangian multiplier $\lambda_{1,t}$ as the marginal utility of consumption. Equation (18) equates the marginal rate of substitution between consumption and labor to their marginal rate of transformation, which is equal to the marginal product of labor. Equations (17) and (19) imply the so-called Samuelson condition:

$$V_{G,t} = U_{C,t}, \tag{24}$$

which equates the marginal utilities of private and public consumption. This condition states that since final output can be transformed into public as well as private consumption goods, the marginal rate of substitution between C_t and G_t^c must be equal to their marginal rate of transformation, which is 1. Equation (20) is the first-order condition with respect to public investment, where $\lambda_{2,t}$ is the shadow value of capital installed in time t. Without investment adjustment cost, we have $\lambda_{1,t} = \lambda_{2,t}$. The Euler equation (21) equates the costs and benefits of an additional unit of capital in period t. The cost is equal to the shadow value of capital in utils ($\lambda_{2,t}$). The two additional terms in the right-hand side of the equation are interpreted as follows. At time $t+1$, the invested unit of capital is worth $\beta(1-\delta)\mathbb{E}_t\lambda_{2,t+1}$ in utils as of time t. Furthermore, investing in public capital today will generate higher output T periods in the future by a factor of $F_{K_G,t+T}$, which is valued at $\beta^T \mathbb{E}_t \lambda_{1,t+T}$ in utils as of time t. Finally, equations (22) and (23) are, respectively, the resource constraint and the accumulation equation for public capital.

3.2 Functional forms and calibration

In order to study the way in which the efficient allocation changes in response to a negative shock to ξ_t, we need to specify functional forms for the utility and production functions, and to assign values to the model parameters. We assume that preferences take the form

$$\begin{aligned} U\left(C_t, N_t\right)+V\left(G_t^c\right) &= \frac{\left(C_t^\gamma (1-N_t)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}+\chi \frac{\left(G_t^c\right)^{1-\sigma}}{1-\sigma} && \text{if } \sigma \neq 1 \\ &= \gamma \ln C_t+(1-\gamma) \ln (1-N_t)+\chi \ln \left(G_t^c\right) && \text{if } \sigma=1, \end{aligned} \tag{25}$$

where $\sigma > 0$ and $0 < \gamma \leq 1$, and that the production function is given by

$$F\left(N_t, K_{G,t}\right)=N_t^a K_{G,t}^b, \tag{26}$$

where $0 \leq a, b \leq 1$. This specification nests the linear technology assumed by Christiano et al. (2011) and Woodford (2011) as a special case in which $a = 1$ and $b = 0$. We also assume that the adjustment-cost function, S, is given by

$$S\left(\frac{G_t^i}{G_{t-1}^i}\right)=\frac{\varpi}{2}\left(1-\frac{G_t^i}{G_{t-1}^i}\right)^2,$$

where $\varpi > 0$.

Our calibration closely follows Bouakez et al. (2017), and is summarized in Table 1. The values of β and σ are based on Christiano et al. (2011). The value of b is based on the meta-regression results of Bom & Ligthart (2013) and is very close to the values considered by Baxter & King (1993) and Leeper et al. (2010). The elasticity of substitution between domestic goods, θ, is chosen so as to yield a steady-state markup of 20 percent. The price-cost-adjustment parameter, ψ, is set such that, conditional on the chosen value of θ, it implies a slope of the (linearized Phillips curve) equal to 0.03. Consistent with the evidence discussed in Leeper et al. (2010), Leduc & Wilson (2013), and Bouakez et al. (2017) regarding the delays associated with the completion of public investment projects, we set $T = 16$. We also follow Leeper et al. (2010) and set the depreciation rate, δ, to 0.02. The investment-adjustment-cost parameter, ϖ, is more difficult to pin down, as empirical estimates are only available for private investment. We set $\varpi = 2.5$, which is very close to the macro estimate of 2.48 obtained by Christiano et al. (2005) and to the micro estimate of 1.86 obtained by Eberly et al. (2012) for private investment. Finally, we calibrate γ and χ such that, given the values of the remaining parameters, the fraction of time devoted to work in steady state, N, is equal to 1/3, and the steady-state ratio of total government spending to output, $g \equiv \frac{G^c+G^i}{Y}$, is equal to 0.2.[8] Given our calibration, the implied share of public investment in total public spending, $\alpha \equiv \frac{G^i}{G^c+G^i}$, is equal to 0.2286, which is very close to the historical average of 0.23 that we observe in U.S. data.

[8]See Appendix A.3 for a detailed description of the steady state.

Table 1: Parameter values.

Discount factor	$\beta = 0.99$
Preference parameter	$\sigma = 2$
Preference parameter	$\gamma = 0.29$
Preference parameter	$\chi = 0.054$
Elasticity of output w.r.t public capital	$b = 0.08$
Elasticity of output w.r.t hours worked	$a = 1$
Elasticity of substitution between intermediate goods	$\theta = 6$
Time-to-build delay	$T = 16$
Price-adjustment-cost parameter	$\psi = 200$
Depreciation rate of public capital	$\delta = 0.02$
Investment-adjustment-cost parameter	$\varpi = 2.5$
Steady-state ratio of public spending to output	$g = 0.2$
Autocorrelation of the preference shock	$\rho = 0.9$

3.3 The efficient response to a negative preference shock

Assume that the economy is initially at the steady state when a negative preference shock hits. The shock is assumed to be persistent, with an autocorrelation coefficient of 0.9.[9] Using the equilibrium conditions (17)–(21), we compute the economy's response to the shock. The results are depicted in Figure 1. All the responses, except that of hours worked, are expressed as percentage deviations from steady-state output; the response of hours is expressed in percentage deviation from their steady-state level.

The negative preference shock increases households' desire to save. Because the accumulation of physical (public) capital allows the intertemporal substitution of consumption while raising future production capacities, current consumption falls and public investment rises in response to the shock. Due to the time-to-build delays, public investment increases the stock of capital — and thus the marginal productivity of labor — 16 quarters later. Eventually, public investment falls below its steady-state level, before converging to it from below. Note that due to investment-adjustment costs, the response of public investment is hump shaped. In the absence of these costs, the maximum increase in investment would take place at the time of the shock, and the response would be less persistent. The figure also shows that the optimal path of government consumption follows closely that of private consumption, in accordance with the Samuelson condition.

Hours worked respond in an opposite way to consumption during the first 15 quarters after the shock. This follows from (18): since public capital is predetermined for $t < 16$, and given our assumptions that the production function is concave in labor, that the utility is concave in consumption and leisure, and that the latter are assumed to be normal goods, an increase in the marginal utility of consumption ($U_{C,t}$) requires an increase in the marginal disutility of labor

[9] The size of the shock will be discussed in section 4.2.

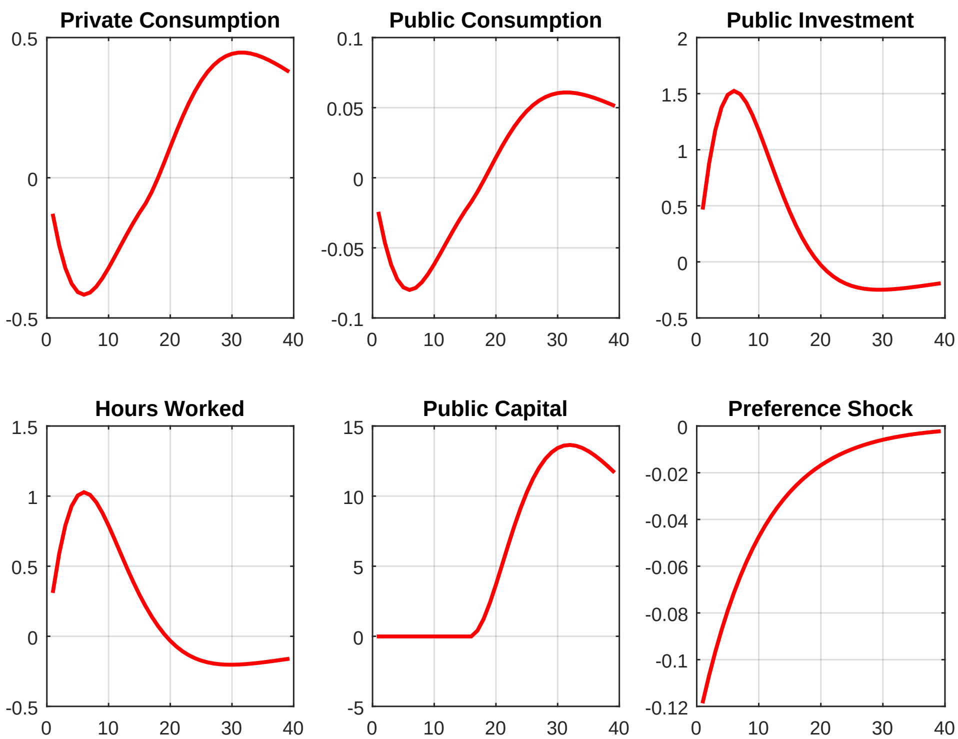

Figure 1: First best allocation after a negative preference shock.

Notes: The figure shows the economy's efficient response to a negative preference shock. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output.

$(-U_{N,t})$ to restore the optimality condition. This in turn implies that hours worked must increase following the shock.[10]

[10] To see this, log-linearize equation (18) around the steady state. This yields

$$\Upsilon \hat{C}_t + \Theta \hat{N}_t = \frac{NF_{NN}}{F_N}\hat{N}_t + \frac{K_G F_{NK_G}}{F_N}\hat{K}_{G,t}, \tag{27}$$

where $\hat{X}_t \equiv \frac{X_t - X}{X}$, variables without a time subscript denote steady-state values, and

$$\begin{aligned} \Upsilon &\equiv \frac{CU_{CN}}{U_N} - \frac{CU_{CC}}{U_C}, \\ \Theta &\equiv \frac{NU_{NN}}{U_N} - \frac{NU_{CN}}{U_C}. \end{aligned}$$

Because public capital is predetermined for $t < 16$, the last term in the right-hand side of equation (27) is equal to zero. The condition for consumption and leisure to be both normal goods is

$$\frac{\Upsilon}{\Theta} > 0.$$

Concavity of the utility function with respect to consumption and leisure in turn implies that $\Upsilon, \Theta > 0$. Rearranging equation (27) yields

$$\left(\frac{NF_{NN}}{F_N} - \Theta\right)\hat{N}_t = \Upsilon \hat{C}_t.$$

The main take-away from these results is that the efficient response of public investment to a (negative) preference shock differs drastically from that of public consumption. At least during the first quarters following the shock, the optimal allocation of resources calls for a simultaneous increase in public investment and a cut in public consumption. In other words, the efficient response to the shock requires a substantial change in the composition of public spending, assigning a substantially larger weight on public investment compared with the steady-state allocation.

4 Optimal Monetary and Fiscal Policy

Consider now the case in which monetary and fiscal policy are chosen optimally under full commitment by a Ramsey planner. The planner has three tools : *i*) the nominal interest rate, R_t, *ii*) government consumption, G_t^c, and *iii*) government investment, G_t^i. Assuming that the planner can commit to future paths of these variables, the Ramsey problem consists in maximizing household's lifetime utility subject to the private-sector equilibrium defined in Section 2.4 and the non-negativity constraint on the nominal interest rate. The Lagrangian of this problem is given by

$$\begin{aligned}
&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \left[U\left(C_t, N_t\right) + V\left(G_t^c\right) \right] \\
&+ \phi_{1,t} \left[1 - \Lambda_{t,t+1} \left(\frac{1+R_t}{1+\pi_{t+1}} \right) \right] \\
&+ \phi_{2,t} \left[\theta\left(mc_t - 1\right) - \psi \left(d\left(\pi_t\right) - \Lambda_{t,t+1} \frac{F\left(N_{t+1}, K_{G,t+1}\right)}{F\left(N_t, K_{G,t}\right)} d\left(\pi_{t+1}\right) \right) \right] \\
&+ \phi_{3,t} \left[F(N_t, K_{G,t}) \left(1 - \frac{\psi}{2} \pi_t^2 \right) - C_t - G_t^c - G_t^i \right] \\
&+ \phi_{4,t} \left[(1-\delta) K_{G,t+T-1} + \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) \right) G_t^i - K_{G,t+T} \right] \\
&+ \phi_{5,t} \left[R_t - 0 \right] \Bigg\},
\end{aligned}$$

where $mc_t = -\frac{U_{N,t}}{U_{C,t} F_{N,t}}$, and where we have defined $d(\pi_t) \equiv \pi_t(1+\pi_t)$ and $\Lambda_{t,t+1} \equiv \beta \frac{\xi_{t+1}}{\xi_t} \frac{U_{C,t+1}}{U_{C,t}}$.

The first-order conditions for this problem are listed in Appendix B. In what follows, we study optimal monetary and fiscal policy under two distinct scenarios: one in which the preference shock is relatively small, such that the economy never hits the ZLB, and one in which the preference shock is large enough to make the ZLB bind, sending the economy into a liquidity trap. In solving the system of (non-linear) first-order conditions, we assume that agents have perfect foresight over the simulation horizon.[11] In what follows, we will refer to the level of public spending that would

Finally, noting that $\frac{NF_{NN}}{F_N} \leq 0$, one can easily see that hours and consumption move in opposite directions.

[11] This approach has also been followed by Christiano et al. (2011), Werning (2011), and Bhattarai & Egorov (2016), among others.

be optimal under fully flexible prices as *neoclassical spending*, and label as *stimulus spending* the difference between the spending level obtained under the Ramsey plan and neoclassical spending. Stimulus spending would therefore be solely due to the distortions stemming from price stickiness (or, equivalently, the presence of a non-zero output gap) that the monetary authority cannot fully eliminate.[12]

4.1 Non-binding ZLB

Consider first the case in which monetary policy is not constrained by the ZLB. Since the only distortion in this economy stems from price rigidity in the goods market,[13] monetary policy can replicate the flexible-price allocation by equating the nominal interest rate to the (efficient) natural rate of interest. This policy fully stabilizes inflation and the output gap in every period, a result that has come to be known as the divine coincidence (Blanchard & Galí (2007)). The optimal levels of government consumption and investment are therefore obvious: they must coincide with those obtained under the efficient allocation, discussed in Section 3. In other words, to the extent that monetary policy can maneuver freely without hitting the ZLB, the Ramsey allocation replicates the first best. In this case, stimulus spending is nil by construction.

4.2 Binding ZLB

We now consider the scenario in which the preference shock is large enough to precipitate the economy into a liquidity trap. To calibrate the size of this shock in a realistic manner, we proceed as follows. Consider a version of our economy in which monetary policy is set according to the following Taylor rule

$$R_t = \max\left[0, \beta^{-1}(1+\pi_t)^{\phi_\pi} - 1\right], \tag{28}$$

where $\phi_\pi > 1$. We select the size of the shock such that, with $\phi_\pi = 1.5$, the resulting decline in output in the absence of any fiscal-policy response matches the observed decline in U.S. GDP from peak to trough during the Great Recession, which amounted to 5.65%. In this environment, the shock leads to a deflation and causes the nominal interest rate to bind for 8 quarters. This outcome will be referred to as the baseline scenario and is represented with the black solid line in Figure 2.

4.2.1 The economy's response under optimal monetary and fiscal policy

The economy's response to the preference shock under optimal monetary and fiscal policy is shown with the dotted green line in Figure 2. The figure also reproduces the economy's response under

[12]Werning (2011) proposes an alternative decomposition whereby *stimulus* spending is defined as deviations from *opportunistic* spending; the latter being defined as "the level of government purchases that is optimal from a static, cost-benefit standpoint, taking into account that, due to slack resources, shadow costs may be lower during a slump."

[13]Recall that the monopolistic competition distortion is corrected using per-unit subsidy given to monopolistically competitive producers.

fully flexible prices, i.e., the first best allocation, reported in Figure 1 (dashed red line). Under the Ramsey plan, the nominal interest rate falls until it reaches the ZLB floor where it remains for 5 quarters before gradually reverting to its steady-state level. In line with the results obtained by Eggertsson & Woodford (2003), Jung et al. (2005), and Adam & Billi (2006), the nominal interest rate remains equal to zero even after the natural rate has become positive, reflecting the planner's commitment to generating a boom in consumption at some point in the future, which helps mitigate the fall in current consumption. The initial decline in consumption is larger than that occurring under the efficient allocation of resources, but the gap is very small — barely 0.5 percent of steady-state output. Public consumption rises by 0.02 percent of steady-state output at the time of the shock, before falling below its steady-state level for a prolonged period of time that extends well after the natural rate has ceased to be negative. While the ZLB is binding, public consumption temporarily exceeds, before eventually falling below, its first-best counterpart. Public investment tracks its efficient level very closely at any given horizon: it initially increases by 0.5 percent of steady-state output and reaches its maximum response of 1.5 percent of steady-state output 5 quarters after the shock. It remains above average for roughly 5 years — i.e., well after the natural interest rate has become positive again — before eventually falling below its steady-state level. Together, the response of public consumption and public investment indicate the optimal fiscal plan is front-loaded, and eventually entails a spending cut.

4.2.2 Implications

Three important observations emerge from the results above. First, the Ramsey fiscal plan entails a change in the composition of public spending in a way that assigns a larger weight to public investment relative to public consumption for a prolonged period of time. Figure 3 depicts the (time-varying) optimal share of public investment in total public spending. At the steady state, this fraction is equal to 22.8 percent. It surges to 24.7 percent at the time of the shock, reaches a peak of 28.5 percent, and remains above its steady-state level for about 5 years.

Second, the fiscal expansion triggered by the shock is sizable, exceeding 10 percent of steady-state output in cumulative terms. Only a small fraction of this expansion (about 1 percent of steady-state output) is accounted for by the cumulative increase in public consumption; an observation that echoes Bilbiie et al. (2014)'s result that the optimal increase of public consumption in response to a shock that makes the ZLB bind is modest. This means that 90 percent of the total increase in public spending comes from the increase in public investment. In this regard, our findings highlight the misleading inference one can make about optimal fiscal policy in a liquidity trap when public investment is neglected.

Third, the increase in public spending is mostly neoclassical in nature. In other words, the stimulus component of the fiscal expansion is negligible. The latter observation is illustrated in

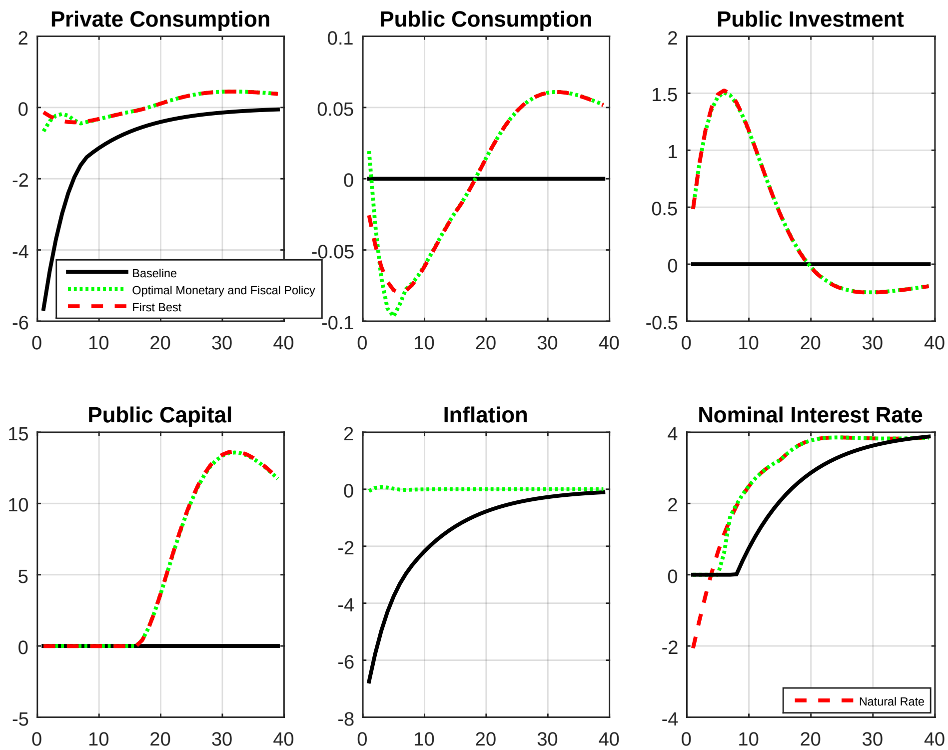

Figure 2: The economy's response to a negative preference shock: optimal monetary and fiscal policy vs alternative scenarios.

Notes: The green dotted line refers to the economy's response under optimal monetary and fiscal policy. The black solid line (baseline) refers to the economy's response under constant public spending and a Taylor rule for the nominal interest rate. The red dashed line refers to the first-best (flexible-price) allocation. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output. The response of inflation and the interest rate are expressed in annualized percentage terms.

Figure 4, which depicts the stimulus component of public consumption and public investment in response to the shock. In both cases, stimulus spending — albeit positive at the time of the shock — is tiny, reaching a maximum of roughly 0.045 percent of steady-state output for public consumption and 0.005 percent of steady-state output for public investment. In cumulative terms, total stimulus spending amounts to -0.05 percent of steady-state output.

In order to understand why the initial response of public consumption exceeds its flexible-price counterpart (or, equivalently, why the stimulus component of public consumption is positive at the time of the shock), differentiate the households' lifetime utility with respect to G_t^c and evaluate it at the maximum. This yields

$$\xi_t\left(U_{C,t}\frac{dC_t}{dG_t^c}+U_{N,t}\frac{dN_t}{dG_t^c}+V_{G,t}\right)+\mathbb{E}_t\sum_{s=t+1}^{\infty}\beta^{s-t}\xi_s\left(\frac{dU(C_s,N_s)}{dG_t^c}+\frac{dV\left(G_s^c\right)}{dG_t^c}\right)=0.$$

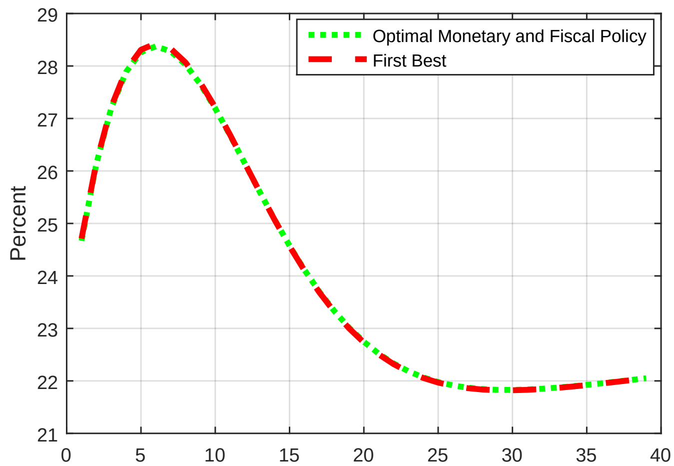

Figure 3: Share of public investment in total public spending (in percent): optimal monetary and fiscal policy vs the first best.

For simplicity, let us abstract from public investment and from the effects of G_t^c on future variables. Since $\frac{dC_t}{dG_t^c} = \Delta_t \left(\frac{dY_t}{dG_t^c} + \frac{Y_t}{\Delta_t} \frac{d\Delta_t}{dG_t^c} \right) - 1$ and $\frac{dN_t}{dG_t^c} = \frac{dY_t}{dG_t^c} F_{N,t}^{-1}$, the condition above can be rewritten as

$$\begin{aligned} \frac{V_{G,t}}{U_{C,t}} &= 1 - \left(\Delta_t + \frac{U_{N,t}}{U_{C,t} F_{N,t}} \right) \frac{dY_t}{dG_t^c} - Y_t \frac{d\Delta_t}{dG_t^c} \\ &= 1 - (1 - mc_t) \frac{dY_t}{dG_t^c} + \frac{\psi}{2} \pi_t^2 \frac{dY_t}{dG_t^c} - Y_t \frac{d\Delta_t}{dG_t^c}. \end{aligned} \tag{29}$$

Under flexible prices, $\Delta_t = 1$, $d\Delta_t = 0$ and $mc_t = 1$, so that the Samuelson condition holds: $V_{G,t} = U_{C,t}$. This condition would also hold when prices are sticky but the monetary authority can fully stabilize inflation and real marginal cost (thus eliminating the output gap). In contrast, when monetary policy is constrained by the ZLB — so that full stabilization is unattainable — the optimal choice of G_t^c deviates from the Samuelson condition. In particular, when the economy is hit by a preference shock that makes the ZLB bind, real marginal cost falls, which implies that the term $(1 - mc_t) \frac{dY_t}{dG_t^c}$ is positive.[14] Equation (29) therefore implies that the marginal rate of substitution (MRS) between public and private consumption, $\frac{V_{G,t}}{U_{C,t}}$, must be smaller than 1.[15] Intuitively, this condition reflects the trade-off that the Ramsey planner faces in the presence of a negative output gap: the Samuelson condition calls for lowering G_t^c but a lower G_t^c further widens the output gap. In this case, the response of G_t^c will be larger than under flexible prices, but whether G_t^c exceeds

[14] Following the shock, mc_t falls below its steady-state of 1. Thus, $1 - mc_t$ is positive. In addition, a well established result is that the output multiplier associated with public consumption, $\frac{dY_t}{dG_t^c}$, is positive and even exceeds 1 when the ZLB binds. The inflationary effect of higher public spending lowers the real interest rate (since the nominal rate is constant) and raises consumption.

[15] The last two terms in the right-hand side of this equation are of second order.

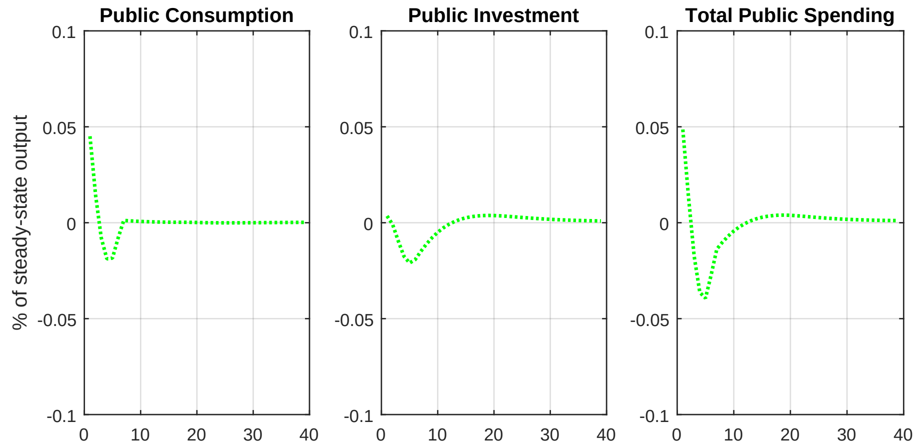

Figure 4: Stimulus spending under optimal monetary and fiscal policy.

Notes: Stimulus spending is defined as the difference between the spending level obtained under the optimal plan and that obtained under the flexible-price allocation. Stimulus spending is therefore only due to the presence of an output gap.

its pre-shock level will depend on how negative is the output gap.

To understand why the stimulus component of public investment is initially positive, it is instructive to examine the condition determining the optimal choice of $K_{G,t+T}$. Assume for simplicity that there are no investment adjustment costs (which implies that $\hat{\phi}_{3,t} = \hat{\phi}_{4,t}$) and let a circumflex denote the percentage deviation of a variable from its steady-state value. Log-linearizing the first-order condition with respect to $K_{G,t+T}$ and rearranging yields

$$\hat{K}_{G,t+T} = \frac{F_{K_G}}{K_G F_{K_G K_G}} \left[\frac{\hat{\phi}_{3,t} - \beta(1-\delta)\mathbb{E}_t \hat{\phi}_{3,t+1}}{1-\beta(1-\delta)} - \mathbb{E}_t \left(\hat{\phi}_{3,t+T} + \frac{N F_{K_G N}}{F_{K_G}} \hat{N}_{t+T} - \frac{\theta F_{K_G N}}{U_C F_N F_{K_G}} \phi_{2,t+T} \right) \right]. \tag{30}$$

This equation implies that, for given expected values of the lagrange multipliers and hours worked, $\hat{K}_{G,t+T}$ — and thus $\hat{G}^i_t$ — will be larger the smaller is the shadow cost of installed capital, $\hat{\phi}_{3,t}$ (recall that $F_{K_G K_G} < 0$). To see how $\hat{\phi}_{3,t}$ behaves, log-linearize the first-order condition with respect to private consumption. This yields

$$\hat{\phi}_{3,t} = \frac{C U_{CC}}{U_C} \hat{C}_t + \frac{N U_{CN}}{U_C} \hat{N}_t + \hat{\xi}_t - \theta \frac{1}{U_C F_N} \left(\frac{U_{CN} U_C - U_{CC} U_N}{U_C^2} \right) \phi_{2,t} + \frac{U_{CC}}{U_C} \phi_{1,t} - \frac{U_{CC}}{\beta U_C} \phi_{1,t-1}. \tag{31}$$

Under the efficient allocation of resources, the consumption Euler equation and the Phillips curve do not bind, which implies that $\phi_{1,t} = \phi_{2,t} = 0$. When prices are rigid and marginal cost is not completely stabilized, however, $\phi_{1,t}, \phi_{2,t} > 0$. From (31), one can easily see that, at the time of the shock, $\hat{\phi}_{3,t}$ is smaller under the Ramsey plan than under the efficient allocation, *ceteris* *paribus*.[16] This in turn implies that public investment will be temporarily larger than its flexible-price level. By raising public investment above the level implied by classical principles, the Ramsey planner "loosens" the constraints implied by nominal rigidities and helps close the output gap. The observation that public investment exceeds its flexible-price counterpart by a negligible amount, however, reflects the fact that the output gap is inconsequential.

In sum, we find that even when monetary policy is constrained by the ZLB, it can a go a long way towards closing the output gap and thus replicating the efficient allocation of resources if it can optimally commit to being expansionary once the economy has exited the liquidity trap. In this case, little scope is left for fiscal policy to act as a stabilization tool and consequently, changes in public spending will be almost exclusively warranted by efficiency motives. A similar conclusion is reached by Mankiw & Weinzierl (2011) in the context of a simple two-period model.

5 Optimal Fiscal Policy when Monetary Policy is Sub-Optimal

Next, we consider the case in which monetary policy is set sub-optimally by the monetary authority. More specifically, we assume that monetary policy is set according to the Taylor rule (28). Fiscal policy, on the other hand, continues to be chosen optimally under full commitment by a "fiscal Ramsey planner". The latter solves a similar program to that described in Section 4, with the constraint $R_t \geq 0$ being replaced with equation (28). The first-order conditions for this problem are listed in Appendix C.

5.1 The economy's response under optimal fiscal policy and sub-optimal monetary policy

Figure 5 shows the economy's response to the preference shock in the case where fiscal policy is optimal but monetary policy is set according to (28). The shock is calibrated as in Section 4. The figure also reproduces the baseline scenario, the efficient allocation, as well as the economy's response under optimal monetary and fiscal policy. Under optimal fiscal policy, public consumption rises for 10 quarters before falling below its steady-state level during the subsequent 15 quarters. At the peak, the increase in public consumption reaches roughly 0.6 percent of steady-state output. Public investment rises by 1 percent of steady-state output on impact and by 3.9 percent at the peak, and remains above its steady-state level for 15 quarters. Note that both categories of public spending remain higher than their flexible-price counterparts for 11 and 14 quarters, respectively. By committing to temporarily raising public spending above its flexible-price level and subsequently reducing it, the planner commits to generating a consumption boom in the future, which raises inflation expectations at the time of the shock. This explains the consumption and inflation responses shown in Figure 5. Private consumption falls initially by roughly 5 percent of steady-state

[16] Note that $\frac{U_{CC}}{U_C} \leq 0$, $\frac{U_{CN}U_C - U_{CC}U_N}{U_C^2} > 0$, and $\phi_{1,t-1} = 0$.

output, that is, less than under the baseline scenario (of no fiscal-policy response), and eventually rises above its steady-state level for a prolonged period of time. Inflation also falls less than under the baseline scenario and converges to its steady-state value of 0 from below. Importantly, the optimal fiscal plan endogenously shortens the duration of the ZLB episode, which lasts for a single period.

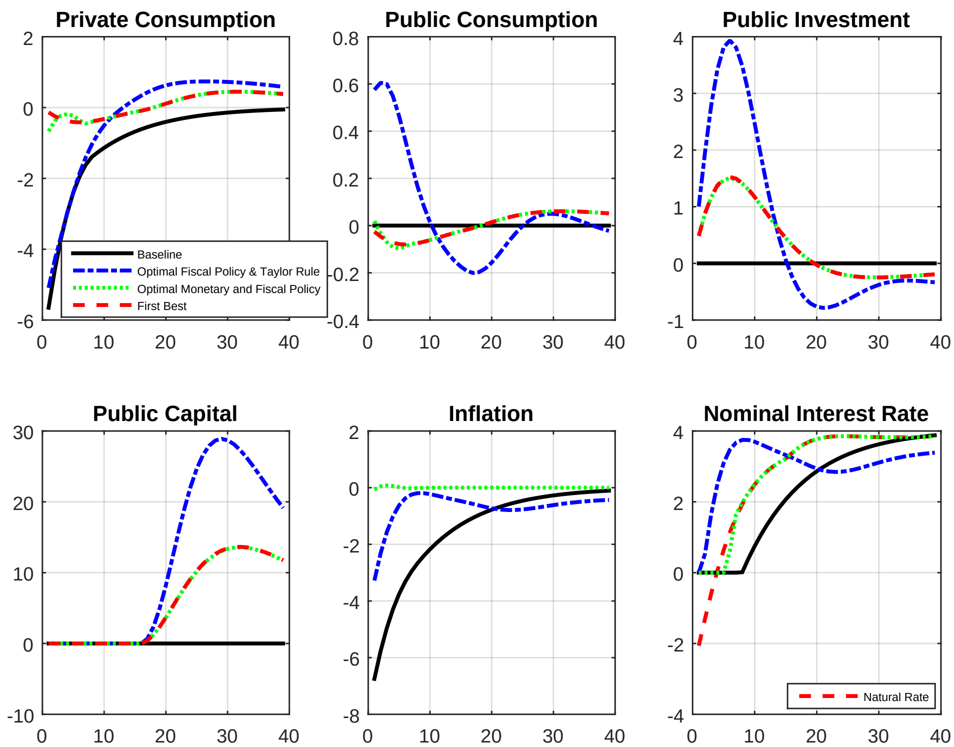

Figure 5: The economy's response to a negative preference shock: optimal fiscal policy and sub-optimal monetary policy vs alternative scenarios.

Notes: The blue dotted-dashed line refers to the economy's response under optimal fiscal policy and a Taylor rule for the nominal interest rate. The black solid line (baseline) refers to the economy's response under constant public spending and a Taylor rule for the nominal interest rate. The red dashed line refers to the first-best (flexible-price) allocation. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output. The response of inflation and the interest rate are expressed in annualized percentage terms.

5.2 Implications

We discuss the implications of the Ramsey fiscal plan along the same three dimensions studied in Section 2.4, namely, the shift in the composition of public spending, the size of the fiscal expansion, and the size of stimulus spending.

First, as illustrated by Figure 6, the shift in the composition of public spending implied by the optimal fiscal plan is much more pronounced than that obtained under the flexible-price allocation. The fraction of public investment in total public spending rises to 25.8 percent at the time of the shock and reaches a peak of 35 percent a few quarters later. Again, these two observations emphasize the importance of allowing for public investment when studying the optimal provision of public goods in depressed economies.

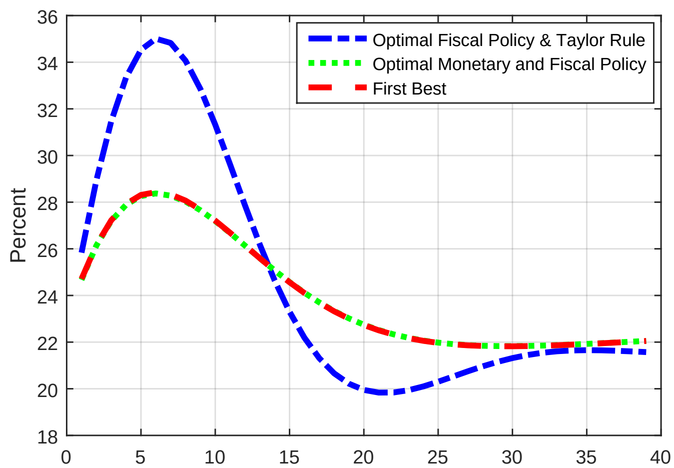

Figure 6: Share of public investment in total public spending (in percent): optimal fiscal policy and sub-optimal monetary policy vs alternative scenarios.

Second, while the initial increase in public consumption is relatively large compared with the case in which both monetary and fiscal policy are chosen optimally, in cumulative terms, this spending category rises only by 0.1 percent of steady-state output. Therefore, public investment continues to account almost entirely for the fiscal expansion engendered by the shock: additional public investment cumulates to roughly 15 percent of steady-state output.

Third, a significant fraction of the increase in public spending serves a stimulus purpose. This fraction is an order of magnitude larger than that obtained in the case where both monetary and fiscal policy are set optimally. Figure 7 shows that in the case of public consumption, the stimulus component is positive during the first 10 quarters after the shock — with a peak of roughly 0.7 percent of steady-state output — and negative afterwards. As for stimulus investment, it is hump shaped — with a peak of 2.4 percent of steady-state output — and positive during the first 3 years after the shock. In cumulative terms, the stimulus components of public consumption and investment amount to, respectively, -1 and 6 percent of steady-state output, thus implying that total stimulus spending cumulates to 5 percent of steady-state output.

This result means that when monetary policy cannot be set optimally in response to an adverse shock, a large negative gap emerges and calls for an increase in public expenditures above the

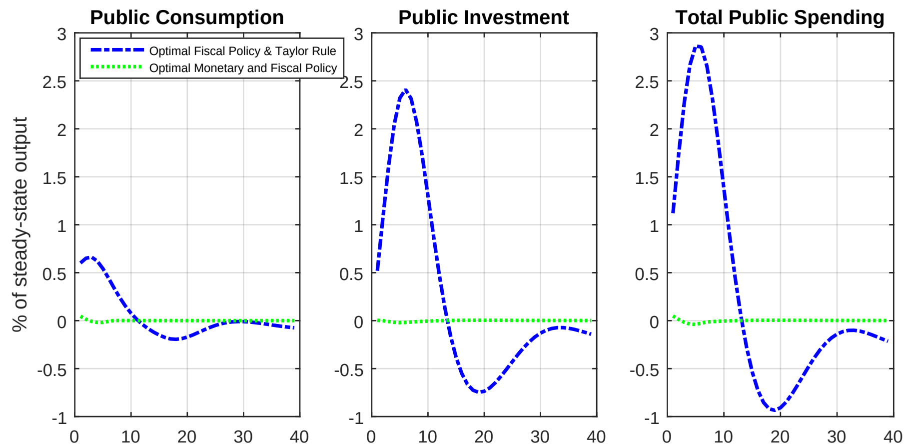

Figure 7: Stimulus spending: optimal fiscal policy and sub-optimal monetary policy vs alternative scenarios.

Notes: Stimulus spending is defined as the difference between the spending level obtained under the optimal plan and that obtained under the flexible-price allocation. Stimulus spending is therefore only due to the presence of an output gap.

level that would be observed under fully flexible prices. Public investment, in particular, rises substantially and remains above it flexible-price level several quarters after the natural rate has ceased to be negative. This result sharply contrasts with the conclusion reached by Mankiw & Weinzierl (2011) who find no stabilization motive for public investment even when monetary policy is restricted both by the ZLB and by the monetary authority's inability to optimally commit to future actions. The main reason for this discrepancy lies in the non-standard functional form of the production technology that Mankiw & Weinzierl (2011) assume, in which public capital enters in an additively separable manner, thereby implying that it does not affect the marginal productivity of private inputs. By relaxing the assumption of additive separability, our analysis shows that there is an important scope for using public investment as a stabilization tool.

5.3 Welfare results

The fact that the planner finds it optimal to temporarily increase public spending in response to the adverse shock necessarily implies that there are welfare gains to this policy relative to the scenario in which public spending is kept constant. How large are these welfare gains? As is typically done in the literature, we answer this question by computing the compensating variation in private consumption; that is, the perpetual percentage increase in consumption that would make households as well off under constant public spending as under optimal fiscal policy. We describe how we compute this object in Appendix D.

We find that the welfare gain associated with optimal public spending amounts to 0.14%. This number is one order of magnitude larger than those obtained in models that focus exclusively on public consumption (e.g., Nakata (2015)), thereby highlighting the importance of public investment in attenuating the welfare loss associated with liquidity traps, and rationalizing the compositional shift in public expenditures entailed in the optimal fiscal plan in such circumstances.

6 Sensitivity and Robustness Analysis

In the simple model studied so far, which we henceforth refer to as the benchmark model, the optimal policy response to the adverse shock involved raising public spending and shifting its composition towards public investment. The fraction of this fiscal expansion intended to fill the negative output gap — stimulus spending — is large only to the extent that monetary policy is chosen sub-optimally; otherwise, the stimulus component of public spending is negligible. In this section, we perform a sensitivity analysis with respect to two key parameters of the model, namely, the elasticity of output to the stock of public capital and the length of time to build. We also study the robustness of our results along three different dimensions: the reliance on distortionary taxes to finance public spending, the inclusion of private capital, and the assumption that a shock of a different nature drives the economy into a liquidity trap.

6.1 Sensitivity analysis

The role of public investment in the model crucially depends on the elasticity of output with respect to public capital, b, and the time-to-build delay, T. To see how our results are affected by alternative values of the parameter b, we follow Leeper et al. (2010) and consider the values of 0.05 and 0.1 (recall that b is set to 0.08 in our benchmark calibration). For each of these values, Table 2 reports the cumulative increase in public consumption and public investment both when monetary policy is optimal and sub-optimal. As b increases, optimal public spending becomes more heavily skewed towards public investment and the stimulus component of investment spending rises as a percentage of steady-state output. The intuition for these results resides in the fact that larger values of b imply a larger marginal productivity of labor.

Next, we consider two alternative scenarios about the length of time to build: the standard one-quarter delay ($T = 1$) commonly assumed in the literature, and an intermediate delay of four quarters ($T = 4$). The results are shown in the bottom two rows of Table 2. The main observation that emerges from the table is that when monetary policy is sub-optimal, cumulative stimulus investment spending decreases as the time to build becomes longer — from 10.8% of steady state output when $T = 1$ to 6% when $T = 16$. The intuition is the following. Irrespective of the length of time to build, public investment rises markedly above its socially optimal level to help dampen the deflationary pressure caused by the preference shock. Because monetary policy is constrained to follow a Taylor rule, the policy maker cannot promise to keep the nominal interest rate below the natural rate in the future so as to stimulate current consumption. Instead, a lower future nominal rate is achieved via lower future inflation. With $T = 1$, lower inflation comes naturally with the increase in the stock of public capital and the resulting fall in real marginal cost. With $T = 16$, this increase comes too late and the fall in inflation requires a fall in public investment. This explains why the cumulative stimulus component of public investment tends to be smaller as T increases. By choosing $T = 16$ in our benchmark calibration, we have thus made a rather conservative assumption that diminishes the importance of public investment as a stabilization tool.[17]

Table 2: Cumulative increase in public spending.

	Optimal Monetary Policy		Sub-Optimal Monetary Policy	
	Public Consumption	Public Investment	Public Consumption	Public Investment
Benchmark	1.118 -0.004	9.052 -0.045	0.106 -1.017	15.064 5.967
$b = 0.05$	0.778 -0.002	5.754 -0.022	-0.444 -1.224	10.749 4.973
$b = 0.1$	1.296 -0.006	11.240 -0.064	0.424 -0.879	17.830 6.526
$T = 1$	0.863 -0.003	9.112 -0.036	0.059 -0.807	19.929 10.781
$T = 4$	0.926 -0.004	9.251 -0.048	0.079 -0.851	19.013 9.715

Notes: In each cell, the top entry refers to the cumulative increase in public spending (consumption or investment), while the bottom entry refers to cumulative stimulus spending. Numbers are percentages of steady-state output.

6.2 Distortionary taxation

In the benchmark model, public spending is assumed to be entirely financed through lump-sum taxes, thus implying that there is no "cost" of using fiscal policy to soak up idle resources in the economy. But what if the planner cannot fully rely on lump-sum taxes to finance public expenditures? To keep things simple and for ease of comparison with the benchmark model, we consider an economy in which only public spending in excess of its steady-state level is financed through distortionary labor-income taxes. That is,

$$G_t - G = \tau_{N,t} w_t N_t. \tag{32}$$

[17] In Bouakez et al. (2017), we show that the spending multiplier associated with public investment tends to increase with the elasticity of output with respect to public capital and with the length of time to build when the ZLB binds. This result, however, holds in the context of a model in which public investment is assumed to follow an exogenous AR(1) process.

where $\tau_{N,t}$ is the tax rate.[18] This assumption ensures that the steady state is identical to that of the benchmark economy. The presence of the labor-income tax distorts the labor supply condition, which becomes

$$(1-\tau_{N,t})w_t = -\frac{U_{N,t}}{U_{C,t}}.$$

Figure 8 shows the Ramsey allocations when monetary policy is set optimally and following the interest-rate rule (28). Note that the flexible-price allocation (i.e., the relevant reference scenario to compute stimulus spending) no longer coincides with the first best in this case. Two key results come out of Figure 8. First, the initial increase in public consumption is larger than that obtained under lump-sum taxes, regardless of whether monetary policy is set optimally or sub-optimally. The intuition for this result can be easily understood by noticing that under distortionary income taxes, equation (29) becomes

$$\frac{V_{G,t}}{U_{C,t}} = 1 - \left[1-(1-\tau_{N,t})mc_t\right]\frac{dY_t}{dG_t^c} + \frac{\psi}{2}\pi_t^2\frac{dY_t}{dG_t^c} - Y_t\frac{d\Delta_t}{dG_t^c},$$

which implies that the MRS between public and private consumption is a decreasing function of $\tau_{N,t}$.[19] Abstracting again from public investment, an increase in public consumption relative to steady state implies — *ceteris paribus* — an increase in the income-tax rate, which means that the MRS is smaller than under lump-sum taxation. This in turn implies that the increase in public consumption must be larger than that occurring under lump-sum taxation. When monetary policy is set optimally, however, the difference in the response of G_t^c due to distortionary taxation is negligible. In contrast, under the Taylor rule (28), the initial increase in G_t^c is more than 15 percent larger than in the case with lump-sum taxes.

Second, regardless of how monetary policy is set, public investment rises less under distortionary taxes than under lump-sum taxes. This result can be understood in light of equation (30). Because the decline in private consumption is larger under distortionary taxes, the shadow cost of installed capital, $\phi_{3,t}$, falls by less in response to the preference shock, leading to a milder increase in public investment. Again, the difference in results relative to the benchmark case is quantitatively important only when monetary policy is sub-optimal. In this case, public investment rises by roughly 0.9 percent of steady-state output on impact and by 3 percent at the peak; the corresponding numbers

[18] This amounts to assuming that in each period the government levies just enough lump-sum taxes to pay the subsidy given to monopolistically competitive firms and to finance steady-state government spending:

$$G + \tau Y_t = \frac{\mathcal{T}_t}{P_t}.$$

Combining this equation with the government budget constraint:

$$G_t + \tau Y_t = \frac{\mathcal{T}_t}{P_t} + \tau_{N,t} w_t N_t,$$

yields equation (32).

[19] This is of course the case as long as $\frac{dY_t}{dG_t^c}$ is positive, which is always true in this model.

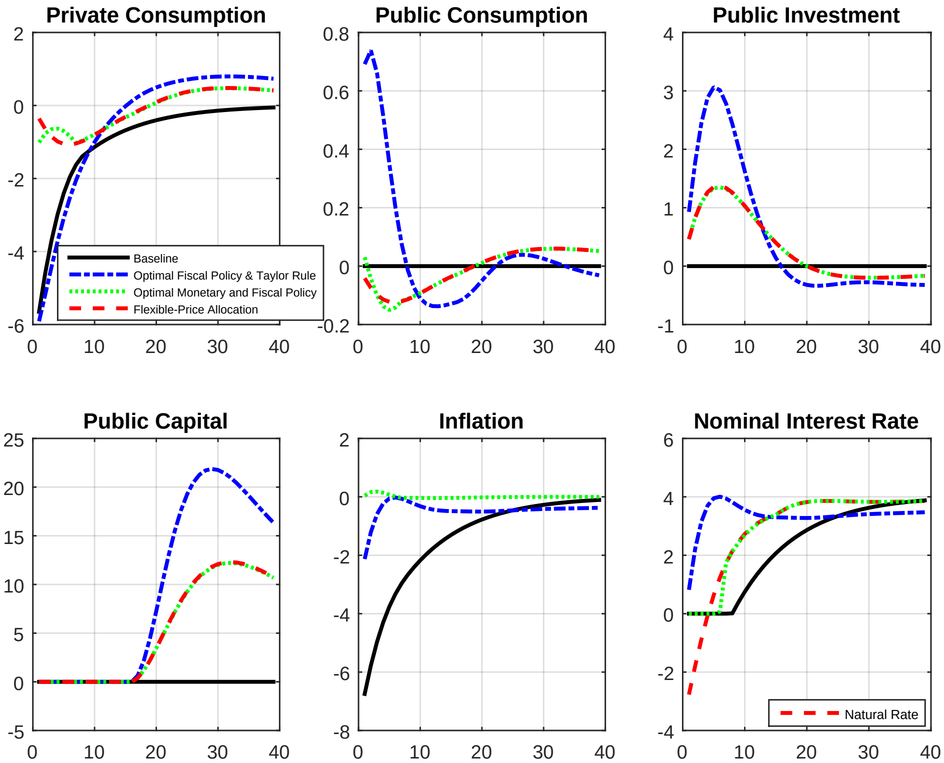

Figure 8: The economy's response to a negative preference shock in the model with distortionary taxes under alternative scenarios.

Notes: The blue dotted-dashed line refers to the economy's response under optimal fiscal policy and a Taylor rule for the nominal interest rate. The black solid line (baseline) refers to the economy's response under constant public spending and a Taylor rule for the nominal interest rate. The red dashed line refers to the flexible-price allocation. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output. The response of inflation and the interest rate are expressed in annualized percentage terms.

are 1 and 3.9 percent in the benchmark economy.

Together, these results suggest that distortionary taxation mitigates the shift in the composition of fiscal spending towards public investment. Nonetheless, the latter still receives a significantly larger share than public consumption in the fiscal expansion, and this compositional shift becomes more salient when monetary policy is set sub-optimally. Figure 9 shows that, under the Taylor rule (28), the share of public investment in total public spending rises to 25.5 percent on impact and 32.7 percent at the peak.

Finally, Figure 8 suggests that, while stimulus spending continues to be negligible under distortionary taxation when monetary policy is set optimally, there is still room for using public expenditures in general and public investment in particular for stabilization purposes when monetary policy is sub-optimal. As shown in Figure 10, under the Taylor rule (28), stimulus consumption

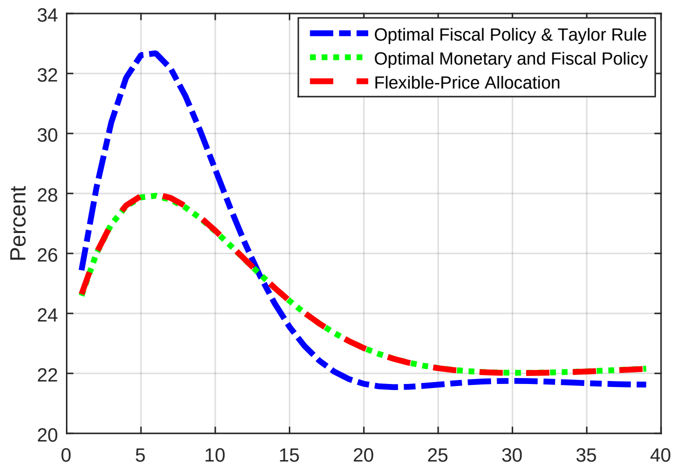

Figure 9: Share of public investment in total public spending (in percent) in the model with distortionary taxes under alternative scenarios.

is positive during the first 8 quarters after the shock, peaking at around 0.8 percent of steady-state output. On the other hand, stimulus investment, albeit smaller in magnitude than under lump-sum taxes, is still sizable, reaching 1.7 percent of steady-state output at the peak and cumulating to roughly 2.5 percent of steady-state output.

6.3 Model with private capital

A natural question one might ask is whether there would be still scope for using public investment as a stabilization tool if the private sector can also accumulate capital? To investigate this question, we augment the baseline model with private capital, which is owned by households and rented out to firms. As is the case with public capital, the accumulation of private capital is also subject to investment-adjustment costs. For simplicity, however, we abstract from time-to-build delays.[20] In this extended version of the model, the production function takes the following form:

$$F(N_t, K_t, K_{G,t}) = N_t^a K_t^{1-a} K_{G,t}^b,$$

where $a, b \in [0, 1]$. To conserve space, we leave out the full description of the model and the equilibrium conditions.[21] In our simulations, we set a to $2/3$. We also assume that private and public capital have the same depreciation rate and adjustment-cost parameter. The remaining parameters are assigned identical values to those in the baseline simulations. The preference shock is again calibrated such that the resulting fall in output obtained under the interest-rate rule (28)

[20]This assumption is consistent with the observation that public investment projects typically entail longer time-to-build delays than private investment projects.

[21]The full description of the model with private capital is available upon request.

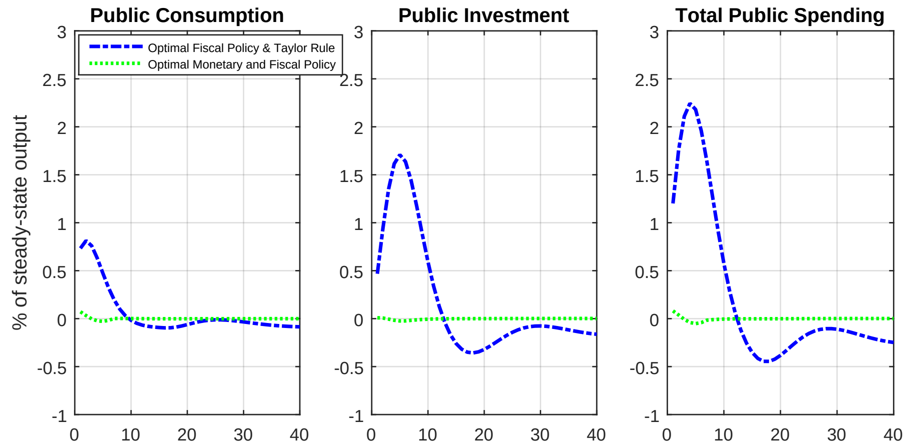

Figure 10: Stimulus spending in the model with distortionary taxes under alternative scenarios.

Notes: Stimulus spending is defined as the difference between the spending level obtained under the optimal plan and that obtained under the flexible-price allocation. Stimulus spending is therefore only due to the presence of an output gap.

and in the absence of any fiscal response matches the observed decline in U.S. GDP from peak to trough during the Great Recession. This scenario is represented by the black solid line in Figure 11. The figure also shows the efficient allocation (red dashed line) as well as the economy's response to the shock under optimal fiscal policy (blue dashed-dotted line) and optimal monetary and fiscal policy (green dotted line).

Consider first the efficient allocation. As in the benchmark model, the preference shock initially triggers a fall in private and public consumption, which persists for 15 quarters. Notice that the effect on C_t and G_t^c is now larger because the shock is larger. Nonetheless, public investment rises less than in the benchmark model, increasing by roughly 0.9 percent of steady-state output at the peak — compared to 1.5 percent in the model without private capital. Intuitively, because the latter can also be used to reallocate resources intertemporally, public investment need not increase by as much as in the benchmark model. Private investment surges by 2.1 percent of steady-state output on impact and by 5 percent at the peak.

Next, consider the scenario in which monetary and fiscal policy are chosen optimally by a Ramsey planner. Under this scenario, the nominal interest rate remains at the ZLB for 7 quarters. Although the deviations of the equilibrium allocations from their efficient levels are slightly more pronounced than in the benchmark model, they remain negligible. As a consequence, the shift in the composition of public spending remains very similar to that implied by the efficient allocation (see Figure 12), and the stimulus component of public spending remains small (see Figure 13). The latter never exceeds 0.5 percent of steady-state output for public consumption and 0.15 percent for

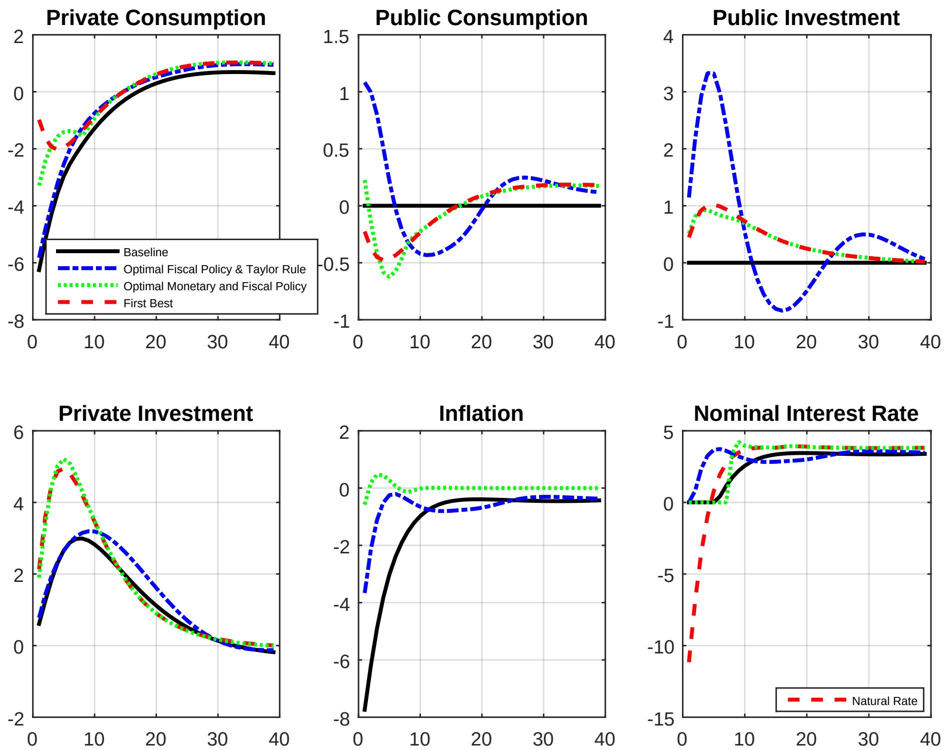

Figure 11: The economy's response to a negative preference shock in the model with private capital under alternative scenarios.

Notes: The blue dotted-dashed line refers to the economy's response under optimal fiscal policy and a Taylor rule for the nominal interest rate. The black solid line (baseline) refers to the economy's response under constant public spending and a Taylor rule for the nominal interest rate. The red dashed line refers to the first-best (flexible-price) allocation. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output. The response of inflation and the interest rate are expressed in annualized percentage terms.

public investment at any given horizon.

Finally, consider the case in which fiscal policy is set optimally but monetary policy follows the Taylor rule (28). The paths of public consumption and public investment are very similar to those obtained in the model without private capital. Both fiscal instruments rise temporarily before eventually falling below their steady-state levels. This fiscal response helps attenuate the private consumption and investment gaps relative to their efficient levels. Quantitatively, the increase in public investment is somewhat smaller in magnitude than under the benchmark model, but it is still significantly larger than the increase in public consumption. As a result the optimal (fiscal) policy still entails a much more pronounced shift in the composition of public spending — towards public investment — than that obtained under the efficient allocation of resources (see Figure 12).

From Figure 11, one can also easily infer that the fraction of government spending that is carried

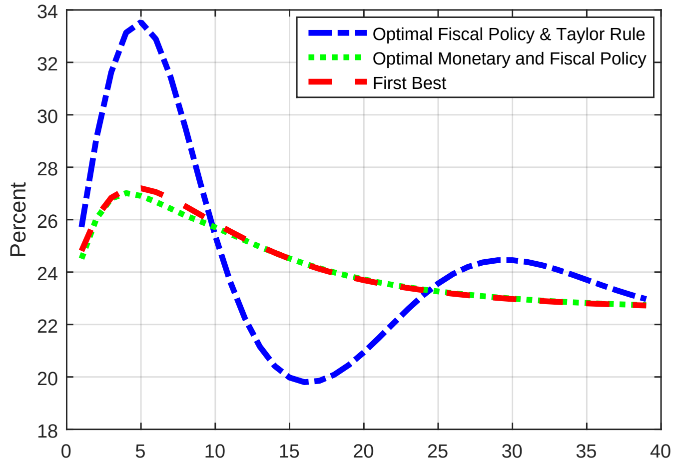

Figure 12: Share of public investment in total public spending (in percent) in the model with private capital under alternative scenarios.

out for stimulus purposes is also significant in this version of the model when monetary policy is sub-optimal. This is confirmed by Figure 13. Stimulus consumption spending represents 1.4 percent of steady-state output at the peak, and cumulates to -0.7 percent of steady-state output. The corresponding numbers for the stimulus component of public investment are 2.3 and 3.7 percent of steady-state output. Total stimulus spending thus cumulates to 3 percent of steady-state output.

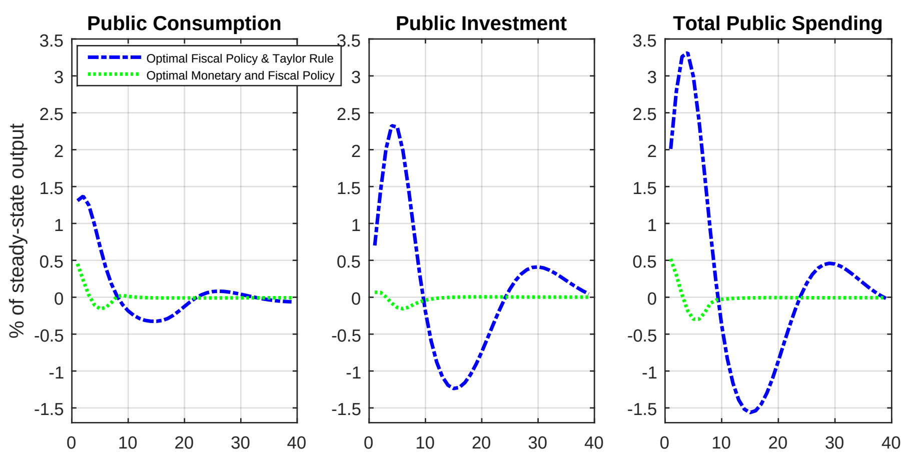

Figure 13: Stimulus spending in the model with private capital under alternative scenarios.

Notes: Stimulus spending is defined as the difference between the spending level obtained under the optimal plan and that obtained under the flexible-price allocation. Stimulus spending is therefore only due to the presence of an output gap.

6.4 A shock of a different nature

In the exercise considered above, the preference shock leads to a negative comovement of private consumption and investment during an extended period of time: while consumption falls, investment actually rises in response to a fall in ξ_t. This negative comovement contrasts strikingly with the simultaneous decline in private consumption and investment spending typically witnessed during deep recessions, as was prominently the case during the Great Recession. In this section, we consider an alternative shock that can deliver such a positive comovement, namely a *liquidity premium* shock (ζ_t). This shock appends the consumption Euler equation as follows

$$U_{C,t} - \zeta_t = \beta \left(1 + R_t\right) \mathbb{E}_t \left(\frac{U_{C,t+1}}{1 + \pi_{t+1}}\right), \tag{33}$$

which implies that an increase in ζ_t lowers the marginal cost of saving in the risk-free bond thus increasing the incentive to save, but only through this vehicle rather than via capital accumulation.[22] As a result, in a decentralized economy with sticky prices, a positive liquidity premium shock should lead to a simultaneous fall in private consumption and investment, which is exactly what Figure 14 shows (solid black line). When the shock is calibrated to generate the same trough in output as in the benchmark model, it gives rise to a liquidity trap of 8 quarters. On the other hand, since this shock only affects the pricing condition of the risk-free bond, it does not affect the first best. Figure 14 also shows that the allocation obtained under optimal monetary and fiscal policy is virtually identical to the first best, thus suggesting that monetary policy is very powerful in this case, which in turn leaves no room for fiscal policy as a stabilizing tool.

When monetary policy is set (sub-optimally) according to a Taylor rule but fiscal policy is chosen optimally, both public consumption and public investment follow the same pattern as in the case with the preference shock, rising temporarily then falling below their steady-state levels. While the size of the fiscal expansion is slightly smaller in magnitude in this case, it still entails a shift in the composition of public spending towards public investment during several quarters. Figure 15 shows that the share of public investment in total public spending reaches a peak that exceeds 30 percent and remains above its steady-state level for about three years after the shock.

Because public spending remains constant at its steady-state level under flexible prices, the fiscal expansion obtained in this case is exclusively comprised of stimulus spending. The latter is depicted in Figure 16. At the peak, stimulus consumption and investment reach roughly 0.9 and 2.5 percent of steady-state output, respectively.

In sum, the three robustness exercises discussed above confirm the main conclusion drawn from

[22] This specification follows Fisher (2015) and is derived by assuming that the utility function depends positively (and linearly) on the real stock of risk-free assets, scaled by an exogenous term ζ_t. Fisher (2015) shows that ζ_t can be viewed as an alternative (structural) interpretation of the risk-premium shock introduced by Smets & Wouters (2007). Under certain conditions, equation 33 is isomorphic — up to a first-order approximation — to the consumption euler equation in Smets & Wouters (2007).

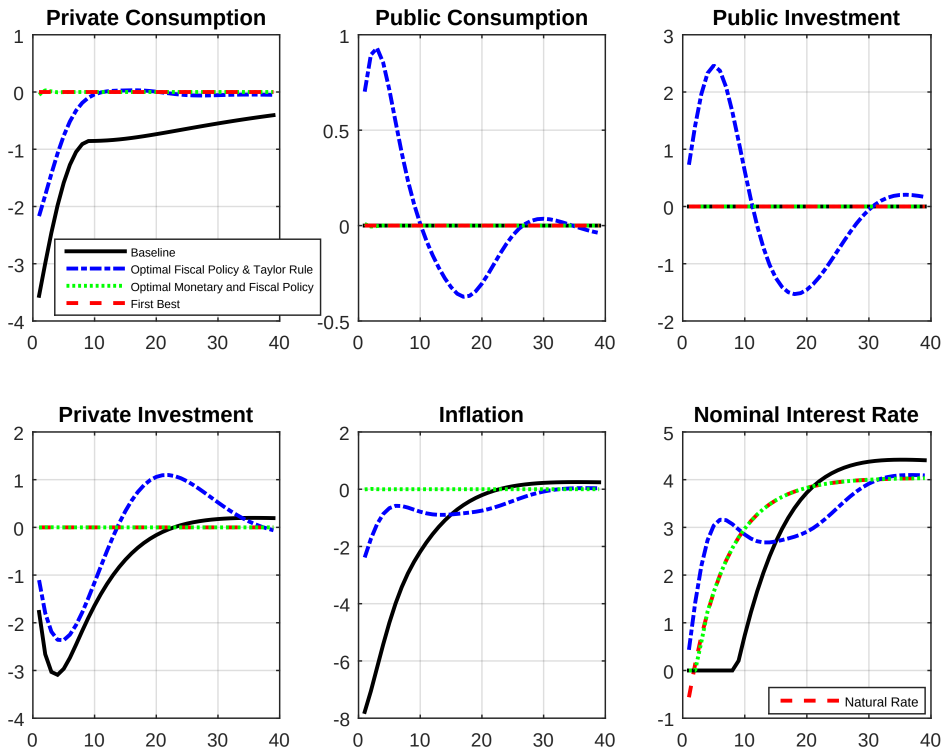

Figure 14: The economy's response to a positive liquidity premium shock under alternative scenarios.

Notes: The blue dotted-dashed line refers to the economy's response under optimal fiscal policy and a Taylor rule for the nominal interest rate. The black solid line (baseline) refers to the economy's response under constant public spending and a Taylor rule for the nominal interest rate. The red dashed line refers to the first-best (flexible-price) allocation. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output. The response of inflation and the interest rate are expressed in annualized percentage terms.

the benchmark model. In the face of shocks that drive the nominal interest rate down to its lower bound of zero, and to the extent that monetary policy is not chosen optimally, it is optimal to raise public spending beyond its flexible-price level and to shift its composition towards public investment.

7 Discussion: How Important is Commitment?

As is well known, Ramsey policies are generally not time consistent. One then might ask whether the government's inability to commit to future actions undermines our main conclusions about the design of fiscal policy and its stabilizing role. Put differently, when the government is constrained to act in a discretionary fashion and monetary policy is set sub-optimally, would it still be optimal to

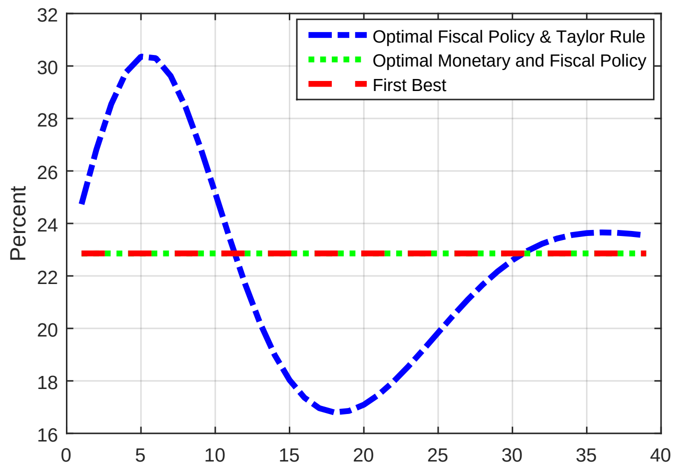

Figure 15: Share of public investment in total public spending (in percent) in response to a positive liquidity premium shock under alternative scenarios.

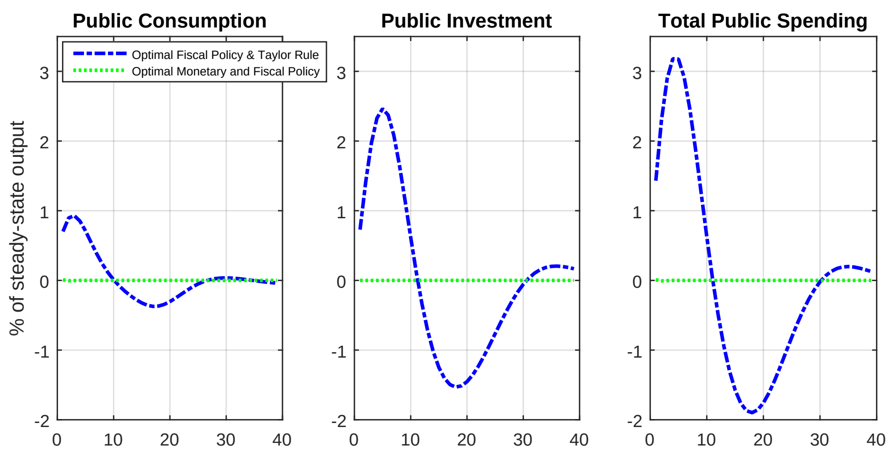

Figure 16: Stimulus spending in response to a positive liquidity premium shock under alternative scenarios.

Notes: Stimulus spending is defined as the difference between the spending level obtained under the optimal plan and that obtained under the flexible-price allocation. Stimulus spending is therefore only due to the presence of an output gap.

temporarily raise public consumption and investment beyond their flexible-price levels in response to an adverse demand shock and if so, would this stimulus spending be still large? While we do not solve explicitly for optimal fiscal policy under discretion, we argue that the answer to both questions is affirmative.[23] In fact, we explain below why stimulus spending should be larger under discretion than under commitment.

When a negative output gap occurs, reflecting a sub-optimal level of private spending, a rise in public spending can — under our maintained assumption about the production technology — help absorb idle resources as long as the negative gap persists. Since liquidity traps are typically characterized by a long lasting fall in output below its natural level, a fiscal expansion would be warranted even if the government lacked the ability to commit to future actions. Furthermore, the time-to-build delays associated with public investment projects lend some form of commitment to government's decisions even if these are taken on a discretionary basis. For these reasons, stimulus spending should continue to be positive even in the absence of commitment. Note that the trade-off between the stabilization motive and meeting the Samuelson condition continues to hold under discretionary fiscal policy, which means that government consumption cannot deviate too much from its flexible-price level, thus implying that the optimal plan will continue to assign a significantly larger weight to public investment in this case.

A noticeable feature about the optimal fiscal plan under commitment is that it eventually entails a spending cut, which actually helps mitigate the output loss while the economy is still in the liquidity trap. This promised fiscal consolidation is unlikely to be credible and would therefore not occur (or at least would be milder) under discretion, a result that has been shown by, for instance, Schmidt (2013) and Nakata (2013) in the context of models with only public consumption. This in turn implies that public spending has to rise even more in the aftermath of the shock to help stabilize the economy. In other words, stimulus spending ought to be larger under discretionary fiscal policy than under commitment. Based on this argument, one can view our results about the magnitude of stimulus spending under full commitment as a lower bound and that, consequently, our conclusions would remain valid under loose commitment (as in Debortoli et al. (2014)) or limited-time commitment (as in Clymo & Lanteri (2016)).

8 Conclusion

This paper has shown that to the extent that monetary policy is set sub-optimally, fiscal policy can play a potent role in stabilizing an economy plunged in a liquidity trap. In the presence of nominal rigidities, an adverse shock causes employment to fall below its socially optimal level, creating a wedge between the marginal productivity of labor and its marginal disutility and driving

[23] Solving the non-linear model numerically under discretion is significantly complicated by the large number of state variables arising from the long time-to-build delays. The dynamic programming problem associated with the optimal choice of fiscal instruments under discretion involves terms that capture the effect of those instruments on agents' expectations. Those terms, however, do not have closed-form solutions and must therefore be approximated as polynomials of the state variables, a procedure that becomes increasingly burdensome as the number of state variables increases.

the real marginal cost of production below unity. When monetary policy is unable to eliminate the distortions associated with nominal rigidities, an increase in public spending can stimulate employment and help close the gap relative to the efficient allocation. The main question we have attempted to address is: how should this fiscal expansion be divided between public consumption and public investment?

We have shown that the optimal fiscal plan entails a shift in the composition of public spending in a manner that assigns a significantly larger weight to public investment for a prolonged period of time that extends beyond the length of the liquidity trap. This result lends support to the IMF's recent recommendations to both advanced and developing economies to promote public investment in infrastructure,[24] as well as to the efforts currently made in the U.S. and Europe in that regard.[25] An equally important issue in these economies, however, is the sustainability of public debt; an issue from which our model abstracts as public spending is assumed to be financed only through taxes. A natural extension of this work would therefore be to study optimal fiscal policy in a non-Ricardian framework with public debt. The existence of a fiscal limit, i.e., a maximum level of debt that can be financed through taxes, would lead to interesting trade-offs in determining the optimal level of stimulus spending. We leave this extension for future research.

[24]See the 2014, 2015, and 2016 editions of the IMF World Economic Outlook.

[25]During his presidential campaign, Donald Trump promised a $1 trillion infrastructure program. The Juncker Plan announced by the European commission in November 2014 aims at increasing private and public investment by € 315 billion over the period 2015 – 2017.

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A First Best

A.1 Equilibrium conditions

The Lagrangian is given by

$$\begin{aligned}
&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \left[U\left(C_t, N_t\right) + V\left(G_t^c\right) \right] \\
&+ \lambda_{1,t} \left[F(N_t, K_{G,t}) - C_t - G_t^c - G_t^i \right] \\
&+ \lambda_{2,t} \left[\left(1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) \right) G_t^i + (1-\delta) K_{G,t+T-1} - K_{G,t+T} \right] \Bigg\}.
\end{aligned}$$

The efficient allocation is the solution to the following set of equations:

$$0 = \xi_t U_{C,t} - \lambda_{1,t}, \tag{34}$$

$$0 = \xi_t U_{N,t} + \lambda_{1,t} F_N(N_t, K_{G,t}), \tag{35}$$

$$0 = \xi_t V_{G,t} - \lambda_{1,t}, \tag{36}$$

$$0 = \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) - \left(\frac{G_t^i}{G_{t-1}^i} \right) S'\left(\frac{G_t^i}{G_{t-1}^i} \right) \right] \lambda_{2,t} + \beta \mathbb{E}_t \lambda_{2,t+1} \left(\frac{G_{t+1}^i}{G_t^i} \right)^2 S'\left(\frac{G_{t+1}^i}{G_t^i} \right) - \lambda_{1,t}, \tag{37}$$

$$0 = \lambda_{2,t} - \beta(1-\delta)\mathbb{E}_t \lambda_{2,t+1} - \beta^T \mathbb{E}_t \lambda_{1,t+T} F_{K_G}\left(N_{t+T}, K_{G,t+T} \right), \tag{38}$$

$$0 = F(N_t, K_{G,t}) - C_t + G_t^C + G_t^i, \tag{39}$$

$$0 = \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) \right) G_t^i + (1-\delta) K_{G,t+T-1} - K_{G,t+T}. \tag{40}$$

A.2 Functional forms

We consider the following functional forms:

Utility function

$$\begin{aligned}
U\left(C_t, N_t\right) + V\left(G_t^c\right) &= \frac{\left(C_t^{\gamma} (1-N_t)^{1-\gamma} \right)^{1-\sigma}}{1-\sigma} + \chi \frac{\left(G_t^c\right)^{1-\sigma}}{1-\sigma} && \text{if } \sigma \neq 1 \\
&= \gamma \ln C_t + (1-\gamma) \ln(1-N_t) + \chi \ln\left(G_t^c\right) && \text{if } \sigma = 1,
\end{aligned}$$

which implies the following first and second derivatives:

$$\begin{aligned}
&U_C = \gamma \frac{\left(C^{\gamma} (1-N)^{1-\gamma} \right)^{1-\sigma}}{C}, && U_{CC} = -\left(1 + \gamma(\sigma - 1)\right) \tfrac{U_C}{C}, \\
&U_N = -(1-\gamma) \frac{\left(C^{\gamma} (1-N)^{1-\gamma} \right)^{1-\sigma}}{1-N}, && U_{NN} = \left(1 + (1-\gamma)(\sigma-1)\right) \tfrac{U_N}{1-N}, \qquad U_{CN} = (1-\gamma)(\sigma-1) \tfrac{U_C}{1-N}, \\
&V_G = \chi \left(G^c\right)^{-\sigma}, && V_{GG} = -\sigma \chi \left(G^c\right)^{-(1+\sigma)}.
\end{aligned}$$

Production function

$$F(N_t, K_{G,t}) = N_t^a K_{G,t}^b, \qquad 0 \le a, b \le 1,$$

which implies the following first and second derivatives:

$$\begin{aligned} F_N &= a\tfrac{N^a K_G^b}{N}, \qquad F_{NN} = -(1-a)\tfrac{F_N}{N}, \\ F_{K_G} &= b\tfrac{N^a K_G^b}{K_G^b}, \qquad F_{K_G K_G} = -(1-b)\tfrac{F_{K_G}}{K_G}, \qquad F_{NK_G} = b\tfrac{F_N}{K_G}. \end{aligned}$$

Investment-adjustment-cost function

$$S\left(\frac{G_t^i}{G_{t-1}^i}\right) = \frac{\varpi}{2}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right)^2, \qquad \varpi \ge 0,$$

which implies the following first and second derivatives:

$$\begin{aligned} S^{'}\left(\frac{G_t^i}{G_{t-1}^i}\right) &= -\frac{\varpi}{G_{t-1}^i}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right), \\ S\text{"}\left(\frac{G_t^i}{G_{t-1}^i}\right) &= \frac{\varpi}{\left(G_{t-1}^i\right)^2}. \end{aligned}$$

A.3 Steady state

We focus on a deterministic steady state in which the preference shock takes a value of 1. In what follows, variables without time-subscript denote steady-state values. Evaluating system (34)–(40) at steady state and using $Y = F(N, K_G)$ and the specifications of preferences and technology, one obtains:

$$U_C = V_G \iff \gamma\frac{\left(C^\gamma(1-N)^{1-\gamma}\right)^{1-\sigma}}{C} = \chi\left(G^c\right)^{-\sigma}, \tag{41}$$

$$-\frac{U_N}{U_C} = F_N \iff \frac{1-\gamma}{\gamma}\frac{C}{1-N} = a\frac{Y,}{N}, \tag{42}$$

$$F_{K_G} = \frac{1-\beta(1-\delta)}{\beta^T} \iff b\frac{Y}{K_G} = \frac{1-(1-\delta)\beta}{\beta^T}, \tag{43}$$

and

$$Y = N^a K_G^b, \tag{44}$$

$$Y = C + G^C + G^i, \tag{45}$$

$$G^i = \delta K_G. \tag{46}$$

Theoretically, this system of equations allows one to find C, N, Y, G^c, G^i, and K_G for a given parameter set: $\{\chi, \gamma, \sigma, a, b, \beta, \delta, T\}$. In practice, and in order to discipline our calibration, we choose χ and γ in order to match the fraction of time worked in steady state, N, as well as the steady-state ratio of total public spending to GDP, given by

$$g \equiv \frac{G^c + G^i}{Y}. \tag{47}$$

In other words, our calibration strategy consists in assigning values to $\{\sigma, a, b, \beta, \delta, T, g, N\}$ in order to find the equilibrium values of $\left\{C, Y, G^c, G^i, K_G, \chi, \gamma\right\}$ from equations (41) to (47). In particular, using (41), (45), and (47), one finds

$$\gamma = \frac{(1-g)\,N}{(1-g)\,N + a\,(1-N)},$$

and from (43) and (46), we obtain

$$g^i \equiv \frac{G^i}{Y} = \frac{b\delta\beta^T}{1-(1-\delta)\beta}.$$

The resource constraint (45) can be re-written as

$$Y = N^a \left(\frac{K_G}{Y}\right)^b Y^b = N^{\frac{a}{1-b}} \left(\frac{g^i}{\delta}\right)^{\frac{b}{1-b}},$$

where we have used (46) again. The remaining endogenous variables K_G, G^i, G^c, C, and χ can be found recursively (and in this order) from equations (44), (46), (47), (45), and (41).

B Optimal Monetary and Fiscal Policy

B.1 Equilibrium conditions

The Lagrangian can be expressed as

$$\begin{aligned}
&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \left[U\left(C_t, N_t\right) + V\left(G_t^c\right)\right] \\
&+ \phi_{1,t} \left[1 - \Lambda_{t,t+1}\left(\frac{1+R_t}{1+\pi_{t+1}}\right)\right] \\
&+ \phi_{2,t} \left[\theta\left(mc_t - 1\right) - \psi\left(d\left(\pi_t\right) - \Lambda_{t,t+1}\frac{F\left(N_{t+1}, K_{G,t+1}\right)}{F\left(N_t, K_{G,t}\right)} d\left(\pi_{t+1}\right)\right)\right] \\
&+ \phi_{3,t} \left[\left(1 - \frac{\psi}{2}\pi_t^2\right) F\left(N_t, K_{G,t}\right) - C_t - G_t^c - G_t^i\right] \\
&+ \phi_{4,t} \left[\left(1-\delta\right) K_{G,t+T-1} + \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right) G_t^i - K_{G,t+T}\right] \\
&+ \phi_{5,t} \left[R_t - 0\right] \Bigg\},
\end{aligned}$$

where $mc_t = -\frac{U_{N,t}}{U_{C,t}F_{N,t}}$, and where we have defined $d(\pi_t) \equiv \pi_t(1+\pi_t)$ and $\Lambda_{t,t+1} \equiv \beta \frac{\xi_{t+1}}{\xi_t} \frac{U_{C,t+1}}{U_{C,t}}$.

The equilibrium conditions are

$$
\begin{aligned}
0 &= \xi_t U_{C,t} + \left(\phi_{1,t} - \beta^{-1}\phi_{1,t-1}\right)\frac{U_{CC,t}}{U_{C,t}} + \theta\phi_{2,t}\left(\frac{U_{CN,t}}{U_{N,t}} - \frac{U_{CC,t}}{U_{C,t}}\right) mc_t - \psi\left(\phi_{2,t}\mathbb{E}_t\Omega_{t,t+1} - \beta^{-1}\phi_{2,t-1}\Omega_{t-1,t}\right)\frac{U_{CC,t}}{U_{C,t}} - \phi_{3,t}, \\
0 &= \xi_t U_{N,t} + \left(\phi_{1,t} - \beta^{-1}\phi_{1,t-1}\right)\frac{U_{CN,t}}{U_{C,t}} + \theta\phi_{2,t}\left(\frac{U_{NN,t}}{U_{N,t}} - \frac{U_{CN,t}}{U_{C,t}} - \frac{F_{NN,t}}{F_{N,t}}\right) mc_t \\
&\quad -\psi\left(\phi_{2,t}\mathbb{E}_t\Omega_{t,t+1} - \beta^{-1}\phi_{2,t-1}\Omega_{t-1,t}\right)\left(\frac{U_{CN,t}}{U_{C,t}} + \frac{F_{N,t}}{F_t}\right) + \phi_{3,t}F_{N,t}\left(1 - \psi\frac{\pi_t^2}{2}\right), \\
0 &= \xi_t V_{G,t} - \phi_{3,t}, \\
0 &= \phi_{3,t} - \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right) - \frac{G_t^i}{G_{t-1}^i}S'\left(\frac{G_t^i}{G_{t-1}^i}\right)\right]\phi_{4,t} - \beta\mathbb{E}_t\phi_{4,t+1}\left(\frac{G_{t+1}^i}{G_t^i}\right)^2 S'\left(\frac{G_{t+1}^i}{G_t^i}\right), \\
0 &= \phi_{4,t} - \beta(1-\delta)\mathbb{E}_t\phi_{4,t+1} + \theta\beta^T\mathbb{E}_t\phi_{2,t+T}\left(\frac{F_{NK_G,t+T}}{F_{N,t+T}}\right) mc_{t+T} \\
&\quad +\psi\beta^T\mathbb{E}_t\left(\phi_{2,t+T}\Omega_{t+T,t+T+1} - \beta^{-1}\phi_{2,t+T-1}\Omega_{t+T-1,t+T}\right)\frac{F_{K_G,t+T}}{F\left(N_{t+T},K_{G,t+T}\right)} - \beta^T\mathbb{E}_t\phi_{3,t+T}\left(1 - \psi\frac{\pi_{t+T}^2}{2}\right)F_{K_G,t+T}, \\
0 &= -\beta^{-1}\phi_{1,t-1} + \psi\left[\phi_{2,t} - \beta^{-1}\phi_{2,t-1}\Lambda_{t-1,t}\frac{F\left(N_t,K_{G,t}\right)}{F\left(N_{t-1},K_{G,t-1}\right)}\right]d'\left(\pi_t\right) + \phi_{3,t}\psi F\left(N_t,K_{G,t}\right)\pi_t, \\
0 &= \phi_{5,t} - \phi_{1,t}\mathbb{E}_t\left(\frac{\Lambda_{t+1}}{1+\pi_{t+1}}\right), \\
0 &= \Lambda_{t,t+1}\left(\frac{1+R_t}{1+\pi_{t+1}}\right) - 1, \\
0 &= \psi\mathbb{E}_t\Omega_{t+1} - \psi d\left(\pi_t\right) + \theta\left(mc_t - 1\right), \\
0 &= \left(1 - \frac{\psi}{2}\pi_t^2\right)F(N_t,K_{G,t}) - C_t - G_t^c - G_t^i, \\
0 &= (1-\delta)K_{G,t+T-1} + \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right]G_t^i - K_{G,t+T}, \\
0 &= \min(R_t, \phi_{5,t}),
\end{aligned}
$$

where we have defined $\Omega_{t-1,t} = \Lambda_{t-1,t}\frac{F(N_t,K_{G,t})}{F(N_{t-1},K_{G,t-1})}d\left(\pi_t\right).$

B.2 Steady state

In zero-inflation steady state, the equilibrium allocation is identical to the first best. This implies that

$$\phi_1 = \phi_2 = \phi_5 = 0,$$

$$\phi_3 = \phi_4 = U_C,$$

and

$$
\begin{aligned}
\pi &= 0, \\
mc &= 1.
\end{aligned}
$$

C Optimal Fiscal Policy and Sub-Optimal Monetary Policy

C.1 Equilibrium conditions

The Lagrangian can be expressed as

$$
\begin{aligned}
&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \left[U\left(C_t, N_t\right) + V\left(G_t^c\right)\right] \\
&+ \phi_{1,t} \left[1 - \Lambda_{t,t+1} \left(\frac{1+R_t}{1+\pi_{t+1}}\right)\right] \\
&+ \phi_{2,t} \left[\theta\left(mc_t - 1\right) - \psi\left(d\left(\pi_t\right) - \Lambda_{t,t+1} \frac{F\left(N_{t+1}, K_{G,t+1}\right)}{F\left(N_t, K_{G,t}\right)} d\left(\pi_{t+1}\right)\right)\right] \\
&+ \phi_{3,t} \left[\left(1 - \frac{\psi}{2}\pi_t^2\right) F\left(N_t, K_{G,t}\right) - C_t - G_t^c - G_t^i\right] \\
&+ \phi_{4,t} \left[\left(1-\delta\right) K_{G,t+T-1} + \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right) G_t^i - K_{G,t+T}\right] \\
&+ \phi_{5,t} \left[R_t - \max\left(0, \beta^{-1}(1+\pi_t)^{\phi_\pi} - 1\right)\right] \Bigg\}.
\end{aligned}
$$

The first-order conditions for this problem are

$$
\begin{aligned}
0 &= \xi_t U_{C,t} + \left(\phi_{1,t} - \beta^{-1}\phi_{1,t-1}\right) \frac{U_{CC,t}}{U_{C,t}} + \theta\phi_{2,t}\left(\frac{U_{CN,t}}{U_{N,t}} - \frac{U_{CC,t}}{U_{C,t}}\right) mc_t - \psi\left(\phi_{2,t}\mathbb{E}_t\Omega_{t,t+1} - \beta^{-1}\phi_{2,t-1}\Omega_{t-1,t}\right)\frac{U_{CC,t}}{U_{C,t}} - \phi_{3,t}, \\
0 &= \xi_t U_{N,t} + \left(\phi_{1,t} - \beta^{-1}\phi_{1,t-1}\right) \frac{U_{CN,t}}{U_{C,t}} + \theta\phi_{2,t}\left(\frac{U_{NN,t}}{U_{N,t}} - \frac{U_{CN,t}}{U_{C,t}} - \frac{F_{NN,t}}{F_{N,t}}\right) mc_t \\
&\quad -\psi\left(\phi_{2,t}\mathbb{E}_t\Omega_{t,t+1} - \beta^{-1}\phi_{2,t-1}\Omega_{t-1,t}\right)\left(\frac{U_{CN,t}}{U_{C,t}} + \frac{F_{N,t}}{F_t}\right) + \phi_{3,t}F_{N,t}\left(1 - \psi\frac{\pi_t^2}{2}\right), \\
0 &= \xi_t V_{G,t} - \phi_{3,t}, \\
0 &= \phi_{3,t} - \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right) - \frac{G_t^i}{G_{t-1}^i} S'\left(\frac{G_t^i}{G_{t-1}^i}\right)\right]\phi_{4,t} - \beta\mathbb{E}_t\phi_{4,t+1}\left(\frac{G_{t+1}^i}{G_t^i}\right)^2 S'\left(\frac{G_{t+1}^i}{G_t^i}\right), \\
0 &= \phi_{4,t} - \beta(1-\delta)\mathbb{E}_t\phi_{4,t+1} + \theta\beta^T\mathbb{E}_t\phi_{2,t+T}\left(\frac{F_{NK_G,t+T}}{F_{N,t+T}}\right) mc_{t+T} \\
&\quad +\psi\beta^T\mathbb{E}_t\left(\phi_{2,t+T}\Omega_{t+T,t+T+1} - \beta^{-1}\phi_{2,t+T-1}\Omega_{t+T-1,t+T}\right)\frac{F_{K_G,t+T}}{F\left(N_{t+T}, K_{G,t+T}\right)} - \beta^T\mathbb{E}_t\phi_{3,t+T}\left(1 - \psi\frac{\pi_{t+T}^2}{2}\right) F_{K_G,t+T}, \\
0 &= -\beta^{-1}\phi_{1,t-1} + \psi\left[\phi_{2,t} - \beta^{-1}\phi_{2,t-1}\Lambda_{t-1,t}\frac{F\left(N_t, K_{G,t}\right)}{F\left(N_{t-1}, K_{G,t-1}\right)}\right] d'\left(\pi_t\right) + \phi_{3,t}\psi F\left(N_t, K_{G,t}\right)\pi_t + \phi_{5,t}R_t\frac{\beta^{-1}\phi_\pi(1+\pi_t)^{\phi_\pi-1}}{\beta^{-1}(1+\pi_t)^{\phi_\pi} - 1} \\
0 &= \phi_{5,t} - \phi_{1,t}\mathbb{E}_t\left(\frac{\Lambda_{t,t+1}}{1+\pi_{t+1}}\right), \\
0 &= \Lambda_{t,t+1}\left(\frac{1+R_t}{1+\pi_{t+1}}\right) - 1, \\
0 &= \psi\mathbb{E}_t\Omega_{t+1} - \psi d\left(\pi_t\right) + \theta\left(mc_t - 1\right), \\
0 &= \left(1 - \frac{\psi}{2}\pi_t^2\right) F(N_t, K_{G,t}) - C_t - G_t^c - G_t^i, \\
0 &= \left(1-\delta\right) K_{G,t+T-1} + \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right] G_t^i - K_{G,t+T}, \\
0 &= R_t - \max\left(0, \beta^{-1}(1+\pi_t)^{\phi_\pi} - 1\right),
\end{aligned}
$$

C.2 Steady state

The steady state is identical to that derived in Section B.2.

D Welfare Criterion

The welfare gain associated with optimal fiscal policy relative to the constant-spending case is measured by the compensating variation in consumption; that is, the perpetual percentage increase in consumption that would make the representative household as well off under constant spending as under optimal fiscal policy. In both scenarios, monetary policy is set according to the Taylor rule (28). Let us define

$$\mathcal{W}_0^{cons} \equiv \mathbb{E}_0 \sum_{t\geq 0} \beta^t \xi_t \Big\{ U\left(C_t^{cons}, N_t^{cons}\right) + V\left(G^c\right) \Big\}, \tag{48}$$

and

$$\mathcal{W}_0^{opt} \equiv \mathbb{E}_0 \sum_{t\geq 0} \beta^t \xi_t \Big\{ U\left(C_t^{opt}, N_t^{opt}\right) + V\left(G_t^{c,opt}\right) \Big\}, \tag{49}$$

as the representative household's lifetime utility under constant and optimal public spending, respectively. The compensating variation in consumption, which we denote by Ξ, solves the following equation:

$$\mathcal{W}_0^{opt} = \mathbb{E}_0 \sum_{t\geq 0} \beta^t \xi_t \Big\{ U\left(C_t^{cons}(1+\Xi), N_t^{cons}\right) + V\left(G^c\right) \Big\}. \tag{50}$$

Defining $\mathcal{V}_0 \equiv \mathbb{E}_0 \sum_{t\geq 0} \beta^t \xi_t V\left(G^c\right)$, and given our assumed functional form for preferences, it follows that

$$\Xi = \left[\frac{\mathcal{W}_0^{opt} - \mathcal{V}_0}{\mathcal{W}_0^{cons} - \mathcal{V}_0} \right] - 1. \tag{51}$$