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**EXERCISE 26**

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**PRICE DISCRIMINATION**

*The Celestial Megawatt Company*, the only electricity producer in the city, sells electricity to industrial users and to households, two groups with different sensibility to price change as shown in the two demand functions :

$$\begin{array}{ll} P_I & = -4 Q_I + 48 & \text{for the industrial sector} \\ \text{and } P_R & = -20/3 Q_R + 80 & \text{for households or the residential market.} \end{array}$$

The production cost function is  $C(Q) = 100 + 4 Q + Q^2$ .

**Question 1**

What is the highest profit attainable if *Celestial Megawatt Co.* sells at the same price on both markets ?

**Question 2**

At what conditions can *Celestial Megawatt Co.* practice third-degree price discrimination among buyers ?

**Question 3**

Which rule should guide *Celestial Megawatt Co.* to allocate its production among markets ?

**Question 4**

Compute each market profit-maximising price and quantity with third-degree price discrimination. Compare total profit with your answer to question 1.

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**ANSWERS**


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**Question 1.**

Without discrimination we have a classical case of a monopoly serving two markets. The maximum attainable profit level is determined by the  $MC = MR$  rule. The total average revenue curve ( $AR_T$ ) is the summation of the industrial and household demands.

Thus, for a given level of selling price we have :

$$\begin{aligned}
 Q_I &= -(5/20) P_I + 12 \\
 + Q_R &= -(3/20) P_R + 12 \\
 \hline
 Q_T = Q_R + Q_I &= -5/20 P_I - 3/20 P_R + 24 \\
 \text{or } Q_T &= -8/20 P + 24
 \end{aligned}$$

or the average revenue function is  $AR$  or  $P_T = -5/2 Q_T + 60$  where  $P_T$  is the common price charged on both markets. However the  $AR_T$  curve is kinked as shown on the first figure because for a price higher than 48, *Celestial Megawatt Co.* can sell only in the residential market ; at that high price, the industrial users presumably produce their own power. So, for a price higher than 48,  $AR_T$  is identical to the residential demand.

Note that if  $P = 48$ , the firm sells 4.8 units on the residential market.

Thus :

- If  $0 < Q \leq 4.8$                        $AR_T = -20/3 Q_T + 80$
- If  $Q > 4.8$                                $AR_T = -(5/2) Q_T + 60$

Both segments of  $AR_T$  being linear, the corresponding marginal revenue curves are linear, with slopes twice steeper than the corresponding  $AR_T$  curves.

Thus :

- If  $0 < Q \leq 4.8$                        $MR_T = -(40/3) Q_T + 80$
- If  $Q > 4.8$                                $MR_T = -5 Q_T + 60$

To maximise profit  $MR_T$  and  $MC$  must be equal :  $-5 Q_T + 60 = 2 Q + 4$  or  $Q^* = 8$ .

*Celestial Megawatt Co.* sells to both the industrial and the residential markets since  $Q > 4.8$ .

From the  $AR_T$  equation, the common price for  $Q^* = 8$  is :  $P^* = -(5/2) \cdot 8 + 60 = 40$ .

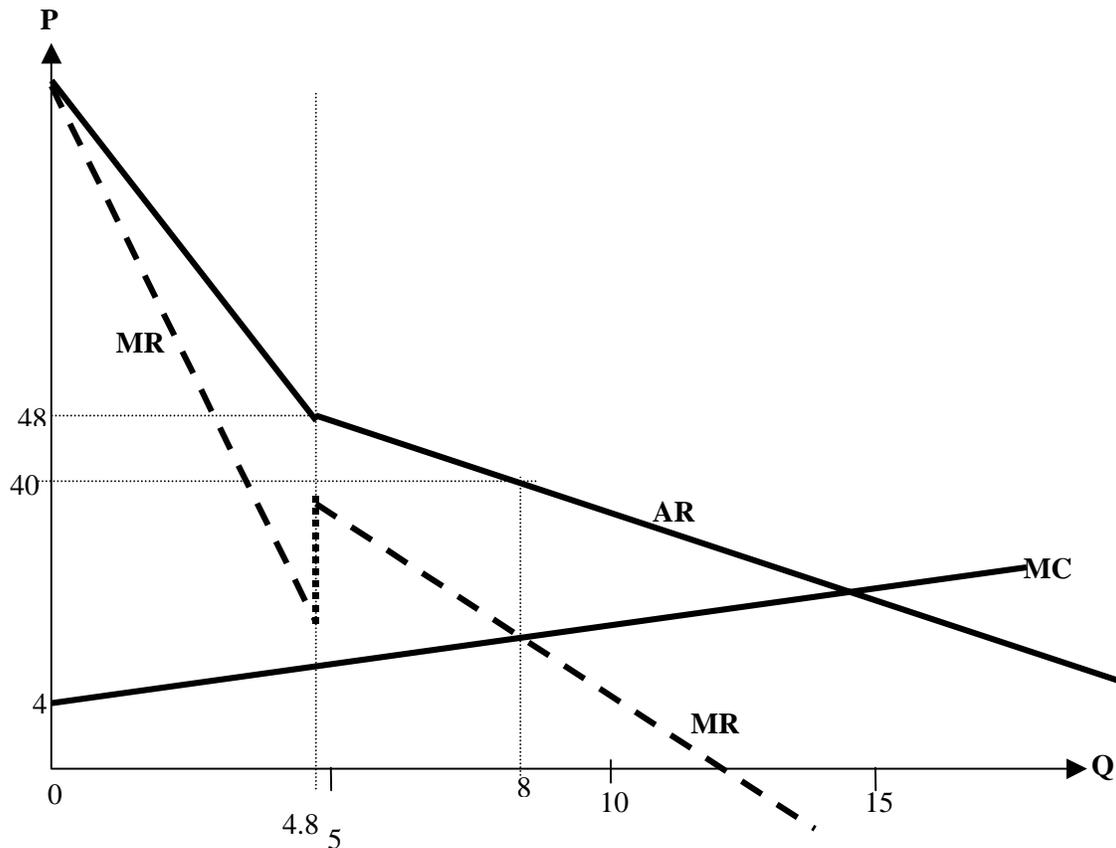
Subtracting total cost from total revenue gives *Celestial Megawatt Co.* total profit.

$$\pi_T = TR - TC = 40(8) - [(8^2 + 4(8) + 100)] = 124.$$

Given a price of 40, the quantity demanded by each market is easily obtained :

- $Q_I = -(5/20) (40) + 12 = 2$  for the industrial market
- $Q_R = -(3/20) (40) + 12 = 6$  for the residential market ; and  $Q_T = Q_I + Q_R = 8$

Figure 1. No Price Discrimination.

**Question 2.**

- A) *Celestial Megawatt Co.* must have some market power (to fix price).
- B) Markets must be kept separated or well insulated, no household should be able to buy electricity at the lower industrial price.
- C) Own-price elasticities must be different on both market, otherwise prices must be the same.

**Question 3.**

The total quantity produced should be allocated between markets as to equalise the marginal revenue brought in by the last unit sold on each one. As for the multiplant producer who would take each additional unit from the lower cost plant, here each additional unit produced will be supplied to the market providing the highest additional revenue.

**Question 4.**

If  $MR_T$  is the combined marginal revenue, profit-maximisation requires  $MR_T = MC$ .

The  $MR_T$  curve is obtained by summing horizontally the MR curves of the industrial market and the residential market. The resulting curve is kinked since the marginal revenue can never higher than 48 in the industrial market. Consequently the best allocation of output is to sell on the residential market alone as long as  $MR_R > 48$ .

- if  $MR_R \leq 48$  or  $Q \geq 2.4$ , we sum the marginal revenue functions :

$$\begin{aligned} Q_I &= -(1/8)MR_I + 6 \\ Q_R &= -(3/40)MR_R + 6 \\ \hline Q_I + Q_R &= -1/8 MR_I - 3/40 MR_R + 12 \end{aligned}$$

$$\text{or } Q_T = 12 - 8/40 MR_T \text{ or } MR_T = -5 Q_T + 60$$

- and if  $MR_T > 48$  or  $Q < 2.4$  :  $MR_T = -40/3 Q_T + 80$

The  $MR = MC$  rule implies that :  $-5 Q + 60 = 2 Q + 4$  and  $Q^* = 8$ .

Note that whether *Celestial Megawatt Co.* uses discriminatory pricing or not, the most profitable level is 8 units in this case.

The level of marginal revenue of the last unit supplied is :  $MR_T = -5 (8) + 60 = 20$ .

Profit maximisation requires that :  $MR_T = MR_I = MR_R = 20$ .

Substituting this value in the  $MR_I$  and  $MR_R$  equations we have :  $20 = -8 Q_I + 48$

or  $Q_I^* = 3.5$

and  $20 = -(40/3) Q_R + 80$  or  $Q_R^* = 4.5$

Prices are determined by the two demand equations :

- for the industrial market :  $P_I = -4 (Q_I) + 48 = 34$
- and for the residential market :  $P_R = -(20/3) Q_R + 80 = 50$

Note that the price charged in the residential market is higher than the price charged in the industrial market. With third-degree price discrimination one must charge a higher price in the market whose demand is relatively inelastic and a lower price in the market with a relatively elastic demand. Let  $E_I$  and  $E_R$  be the price elasticities in the markets for the profit maximizing outputs.

$$E_I = \frac{dQ_I}{dP_I} \frac{P_I}{Q_I} = -\frac{1}{4} \frac{34}{3.5} = -2.43 \quad \text{and} \quad E_R = \frac{dQ_R}{dP_R} \frac{P_R}{Q_R} = -\frac{3}{20} \frac{50}{4.5} = -1.67$$

$$\text{We know that } P_I/P_R = \frac{1+E_R}{E_R} / \frac{1+E_I}{E_I}$$

$$\text{or } P_I/P_R = (-0.67/-1.67)/(-1.43/-2.43) = 0.4012/0.5885 = 0.68 \text{ equal indeed to } = 34/50.$$

The revenue from the industrial market is  $34 * 3.5 = 119$  and  $4.5 * 50 = 225$  from the residential market. The profit maximizing output remains 8 as in question 1, thus the total cost remains 196.

Total profit is  $\pi_T = 119 + 225 - 196 = 148$ . Third-degree price discrimination increased profit by 24.

Figure 2. Case of Price Discrimination

