
EXERCISE 24

SWEEZY AND CARTEL OLIGOPOLY

The Sweezy model, or the kinked demand model, shows that price stability can exist without collusion in an oligopoly.

Two firms "squabble" over a market. Observers have noticed that whenever the price of one firm was increased, the price of the other firm remained constant. On the other hand, whenever the price of one firm fell, its rival would reduce its own price too to maintain its market share. It is believed this behaviour will be maintained and right now each firm holds half the market each selling 1,500 tons per day.

It has been established that the demand facing each producer depends on its volume of sales and also on the sales of its competitor. The following functions are a good approximation :

$$\begin{array}{ll} \text{for firm A :} & P_A = 240 - 4 Q_A - 2 Q_B \\ \text{and for firm B :} & P_B = 210 - 2 Q_A - 4 Q_B \end{array}$$

where Q_A and Q_B are, respectively, the amounts of sales by firm A and by B, expressed in hundreds of units.

Nor A nor B have any fixed costs, their marginal and average cost functions are :

$$\begin{array}{ll} MC_A = 60 + 2 Q_A & \text{for } Q_A > 2.5 \text{ (in hundreds of units)} \\ AVC_A = 60 + Q_A & \\ MC_B = 50 + Q_B & \text{for } Q_B > 3 \text{ (in hundreds of units)} \\ AVC_B = 50 + 0.5 Q_B & \end{array}$$

Currently, both firm's managers are considering co-ordinating their operations like a cartel, believing that in this case the demand for the cartel product would be : $P = 225 - 3 Q$.

Question 1.

Compute each firm price.

Question 2.

What is the average revenue function of each product ?

Question 3.

What is the marginal revenue function ?

Question 4.

Is the market presently in equilibrium ?

Question 5.

Compute the cartel global revenue.

Question 6.

Compute the cartel marginal cost.

Question 7.

What price should the profit-maximising cartel charge ?

Question 8.

How many units will be produced by each firm under the cartel ?

Question 9.

Draw a graph to illustrate the cartel.

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ANSWERS

Question 1.

A and B sales are given ; therefore prices could be obtained by substituting Q_A and Q_B by their respective values into the demand equations :

$$\begin{aligned} \text{thus} \quad & P_A = -4(15) - 2(15) + 240 = 150 \\ \text{and} \quad & P_B = -2(15) - 4(15) + 210 = 120. \end{aligned}$$

The product are slightly differentiated and command slightly different prices.

Question 2.

The demand equations show that each competitor chosen price is directly influenced by the quantity its rival sells on the market.

Some modifications to the demand functions are necessary to plot the graphs on a traditional two dimensions axis chart. That can be done keeping in mind the underlying behaviour assumption that each oligopolist is expected to follow any price reduction (to protect its market share) but not a price increase (which presents no threat to its market share). Each rival expects his competitor to have the same reaction. The resulting demand curve will be kinked ; algebraically it follows that each average revenue curve is described by two equations, one for each segment.

Average revenue of A :

- **If $P_A \geq 150$** (or $Q_A \leq 15$) P_B remains constant ($P_B = 120$) and we may write :
 $P_B = 120 = -2 Q_A - 4 Q_B + 210$ or $Q_B = -0.5 Q_A + 22.5$.
 Substituting this value of Q_B into the demand for A, we obtain :
 $P_A = AR_A = -4 Q_A - 2 [-0.5 Q_A + 22.5] + 240 = -3 Q_A + 195$
- **If $P_A \leq 150$** (or $Q_A \geq 15$)
 As B follows the price reduction to preserve its actual market share, we have $Q_A = Q_B$
 Thus $P_A = AR_A = -4 Q_A - 2 Q_A + 240 = -6 Q_A + 240$

Average revenue of B : The same method is applied to B as for A.

- **If $P_B \geq 120$** (or $Q_B \leq 15$)
 As $P_A = 150$, we have : $P_A = 150 = -4 Q_A - 2 Q_B + 150$ or $Q_A = -0.5 Q_B + 22.5$.
 Therefore, $AR_B = -2 [-0.5 Q_B + 22.5] - 4 Q_B + 210 = -3 Q_B + 165$.
- **If $P_B \leq 120$** (or $Q_B \geq 15$), we have $Q_A = Q_B$ and $AR_B = -6 Q_B + 210$

The two average revenue functions reflect the asymmetrical behaviour in reaction to price change.

Question 3.

Since the kinked demand curve has two segments, so has the corresponding marginal revenue curve and two equations are necessary to describe the marginal revenue for each firm.

The marginal revenue curve of A is discontinuous for a level of output of 15 at a price of $P_A = 150$. The marginal revenue curve of B is also discontinuous for a level output of 15 at a price of $P_B = 120$.

On the following figures 1, marginal revenue curve lies directly below its corresponding average revenue curve price at each level of output. Note that the length of the gap in the marginal revenue curve depends upon the slope of the average revenue curves and hence upon the value of the price elasticities at the kink.

As each segment of average revenue curves is linear, the corresponding marginal revenue segments are linear too. Algebraically we just have to double the slope and keep the same vertical intercept.

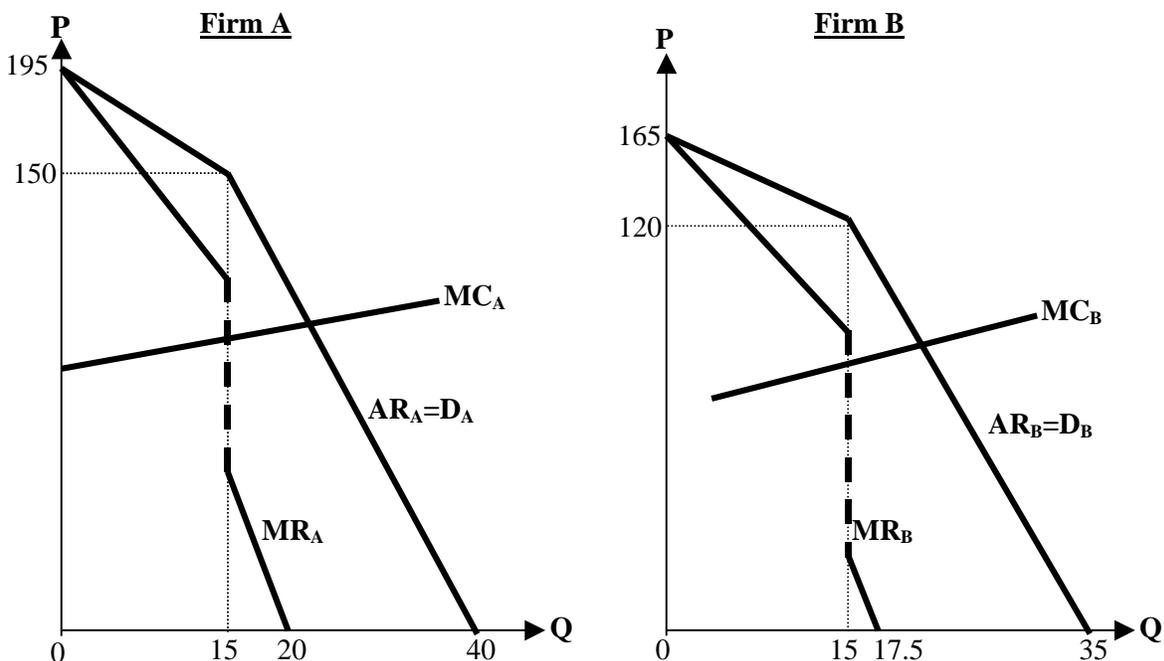
→ **Marginal revenue of A**

- If $Q_A \leq 15$, as $AR_A = -3 Q_A + 195$ we have $MR_A = -6 Q_A + 195$
- If $Q_A \geq 15$, as $AR_A = -6 Q_A + 240$ we have $MR_A = -12 Q_A + 240$

→ **Marginal revenue of B**

- If $Q_B \leq 15$, as $AR_B = -3 Q_B + 165$ we have $MR_B = -6 Q_B + 165$
- If $Q_B \geq 15$, as $AR_B = -6 Q_B + 240$ we have $MR_B = -12 Q_B + 240$

FIGURE 1 : Duopoly à la Sweezy



Question 4.

Sweezy model was developed to explain why (nominal or relative) prices in oligopolistic markets tend to be relatively stable or rigid, without collusion among the competitors.

In the present case, both A and B maximise their profit, as the marginal cost curve pass through the discontinuity of their respective marginal revenue curve, as shown on the previous figure. The market is in equilibrium. There is no reason for A or B to initiate a price change.

Note that moderate variations in cost conditions would not affect the present equilibrium. Profit maximisation decisions ($Q_A = 15$, $P_A = 150$ and $Q_B = 15$, $P_B = 120$) remain as long as the marginal cost stays in a range of values compatible with the discontinuity of the marginal revenue curve at the kink.

Question 5.

The demand for the cartel is the sum of the demand for each product :

$$Q_A = 240/6 - P_C / 6$$

$$Q_B = 210/6 - P_C / 6$$

$$\overline{Q_C} = Q_A + Q_B = 550/6 - 2P_C / 6 = 75 - P_C / 3$$

$$\text{or } P_C = 225 - 3 Q_C$$

Total revenue of the cartel is : $TR = P_C \cdot Q_C = - 3 Q_C^2 + 225 Q_C$.

Question 6.

To maximise profit, A and B should enter in an agreement to allocate the production such as to minimise production cost.

Optimal allocation of production among A and B requires that $MC_A = MC_B = MC_{\text{CARTEL}}$ for any level of output where MC_{CARTEL} is the cartel marginal cost. Thus the cartel marginal cost curve (MC_{CARTEL}) is the horizontal sum of the marginal cost curves of A and B.

Comparing MC_A and MC_B we notice that MC_{CARTEL} is identical to MC_B , for $50 \leq MC_{\text{CARTEL}} \leq 60$, the first ten units of output will be produced by B, to take advantage of its lower marginal cost.

Thus,

- **If $Q < 10$** , $MC_{\text{CARTEL}} = Q_C + 50$
- **If $Q > 10$** , we have to aggregate the two marginal cost functions :

$$\begin{aligned} Q_A &= MC_A / 2 - 30 \\ + Q_B &= MC_B - 50 \\ \hline Q_C &= MC_A / 2 + MC_B - 80 \end{aligned}$$

Since $MC_A = MC_B$, $Q_C = 3/2 MC_{\text{CARTEL}} - 80$ thus $MC_{\text{CARTEL}} = 2/3 Q_C + 160/3$

Question 7.

To maximize its joint-profit, the new cartel will operate at the level of output where its marginal revenue equals its combined marginal cost of production.

Thus

$$-6 Q_C + 225 = 2/3 Q_C + 160/3 \quad \text{and} \quad Q_C^* = \mathbf{25.75}. \quad \text{The cartel price is : } P_C^* = \mathbf{147.75}$$

Question 8.

The value of MC_{CARTEL} for $Q_C = 25.75$ is 70.50.

Therefore $MC_A = MC_B = 70.50$.

Substituting this value in the MC_A and MC_B equations gives $Q_A^* = \mathbf{5.25}$ and $Q_B^* = \mathbf{20.50}$

Previously both duopolists produced 30 units (15 each) sold at an average price of $(150+120)/2=135$.

Their profit (remember none had fixed costs) amounted then to :

$$\begin{aligned} \pi_A &= 15 (P - AVC_A) = 15 [150 - (60 + Q_A)] = 15(150 - 60 - 15) = 1,125 \\ \pi_B &= 15 (P - AVC_B) = 15 [120 - (50 + 0.5 Q_B)] = 15(120 - 50 - 7.5) = 937.5 \\ \text{and } \pi_A + \pi_B &= \text{amounted to } 1125. + 937.5 = 2,062.5 \quad \text{and } \pi_A / \pi_B = 1.2 \end{aligned}$$

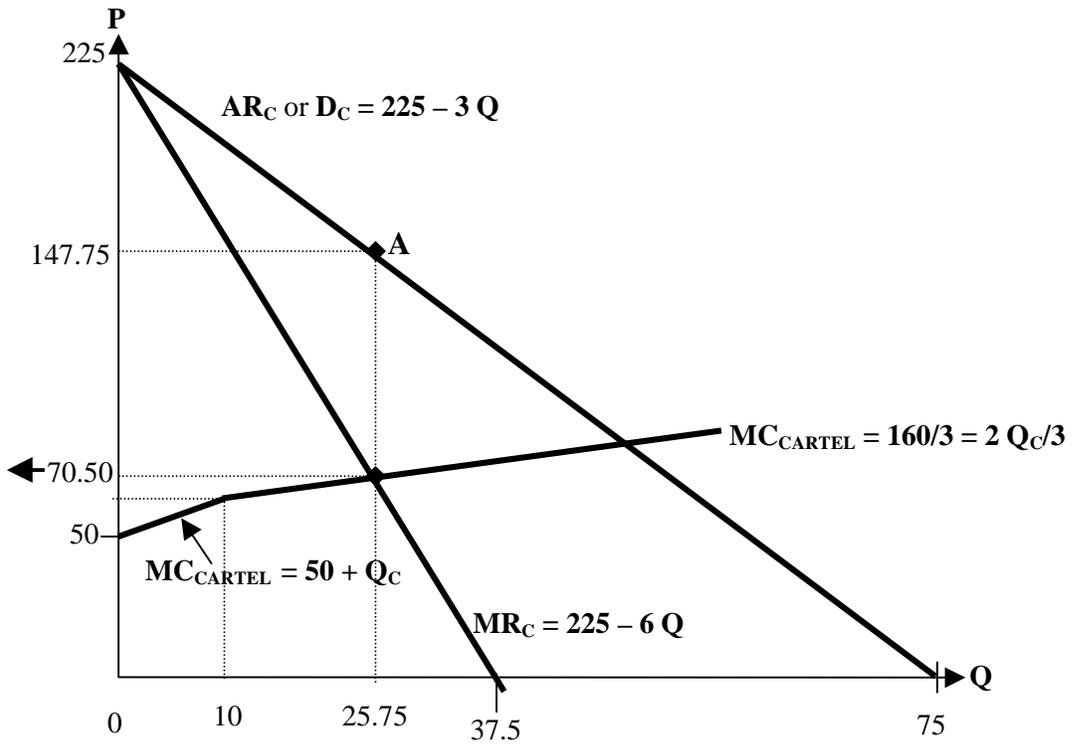
By co-ordinating their production forces and maximising their joint profit, they restrict their sales to 25.75 units and raise their price to 147.50.

The cartel profit is

$$\begin{aligned} \pi_{\text{CARTEL}} &= 5.25[147.75 - (60 + 5.25)] + 20.50 [147.75 - (50 + 20.50/2)] \\ &= 5.25 * 82.50 + 20.50 * 87.50 = 433.125 + 1,793.95 = 2,226.875 \\ \text{and } \pi_A / \pi_B &= 0.24. \end{aligned}$$

Once the market sharing (15/15) agreement is disregarded, the lower cost producer B is advantaged ; some agreement of profit sharing must then replace the output agreement for the cartel to be stable.

FIGURE 2 : The Two Firms Create a Cartel



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