EXERCISE 18

PRICING ALTERNATIVES FOR A MONOPOLY

In this exercise, profit maximisation is compared with alternative profit-objectives for a monopoly

The firm ***Power To The People*** (*PTTP*), the only electricity producer in the city, faces a demand given by : $P = -45/8\ Q + 2{,}750$ where P is the price, and Q the quantity.

Total costs function is : $TC = 1/30\ Q^3 - 15\ Q^2 + 2{,}500\ Q$

Question 1.

Find *PTTP* marginal revenue function.

Question 2.

PTTP actually sells 200 units, determine the selling price. Is this a profit-maximising situation ?

Question 3.

What is the profit per unit ?

Question 4.

Compute the own-price-elasticity of demand.

Question 5.

If *PTTP* wants to make a per unit profit equal to 10 % of average cost, what quantity must be sold and at what price ?

Question 6.

What is the maximum quantity that can be sold by *PTTP*, without suffering a loss, and at what price ?

Question 7.

What quantity and price would maximise *PTTP* cash-flow (or total revenue or **P·Q**) ?

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ANSWERS

Question 1.

The demand curve facing a monopolist is the same as the industry demand curve. Solving the demand equation for P yields the **average revenue curve** P or AR = -45/8 Q + 2,750. If the average revenue curve is a straight line then the marginal revenue curve will also be a straight line with the same intercept but with a slope twice steeper MR = -45/4 Q + 2,750.

Question 2.

For Q = 200, the price is equal to : P = -45/4 (200) + 2,750 = 1,625. In a profit-maximising situation, marginal revenue equals marginal cost, or MR = MC. For Q = 200 : MR = -5/4 (200) + 2,750 = 500 and

$$MC = 1/10\ Q^2 - 30\ Q + 2\ 500 = 1/10\ (200)^2 - 30\ (200) + 2{,}500 = \underline{500}$$

Since MR = MC, this is indeed a profit-maximising situation. [Point **A** on next figure]

Question 3.

The profit per unit is selling price P minus average cost (AC).

AC is given by : TC/Q = AC = 1/30 Q^2 - 15 Q + 2,500
for Q = 200, AC = 1/30 $(200)^2$ - 15 (200) + 2,500 = 833.33
Thus, profit per unit is π_u = 1,625 - 833,33 = 791.67.

Question 4.

The own-price elasticity of demand can easily be computed from the price and marginal revenue :

$$MR = P\left(1 + \frac{1}{E_{QP}}\right)$$

$$or\ \frac{MR}{P} - 1 = \frac{1}{E_{QP}}$$

$$or\ E_{QP} = \frac{P}{MR - P}$$

When P = 1,625 and MR = 500, then : $E_{QP} = \dfrac{1625}{500 - 1625} = -1.44$

The absolute value of the own-price elasticity of demand is greater than 1 as always when a monopolist charges a profit-maximising price. Why ? Because marginal revenue must be equal to marginal cost and the later cannot be negative. Hence a monopolist can NEVER maximise profit in the inelastic part of the linear demand, where MR is negative nor when total revenue is maximum, because there MR = 0 and marginal cost will never be zero.

Question 5.

Let ACP denotes average cost increased by 10 %,

$$\text{ACP} = 1.1 \ \text{AC} = \frac{1.1}{30} Q^2 - 16.5\,Q + 2750$$

PTTP will sell the maximum quantity of electricity, given that its cost (taking into consideration the profit margin) is equal to ACP. The maximum output is given by the intersection of the average revenue curve and the ACP curve.

$$\frac{1.1}{30} Q^2 - 16.5\,Q + 2750 = -\frac{45}{8} Q + 2750$$

$$or \frac{1.1}{30} Q^2 - \frac{33}{2} Q + \frac{45}{8} Q = 0$$

$$or \frac{1,1}{30} Q^2 - \frac{132}{8} Q + \frac{45}{8} Q = 0$$

$$or Q\left[\frac{1,1}{30} Q - \frac{87}{8}\right] = 0$$

Which implies Q = 0 or Q = 296.6. Given *PTTP'*s goal of maximising sales subject to the 10 % profit constraint, the quantity produced will be 296.6 Substituting Q = 296 into the demand function gives P = 1,085. [Point **B** on next figure]

Question 6.

The maximum quantity that can be sold without suffering a loss is found at the intersection of the average cost and the average revenue curve or by solving for Q :

$$-45/8\ Q + 2{,}750 = 1/30Q^2 - 15\ Q + 2{,}500$$
$$\text{or } Q^2/30 - 75/8\ Q - 250 = 0$$

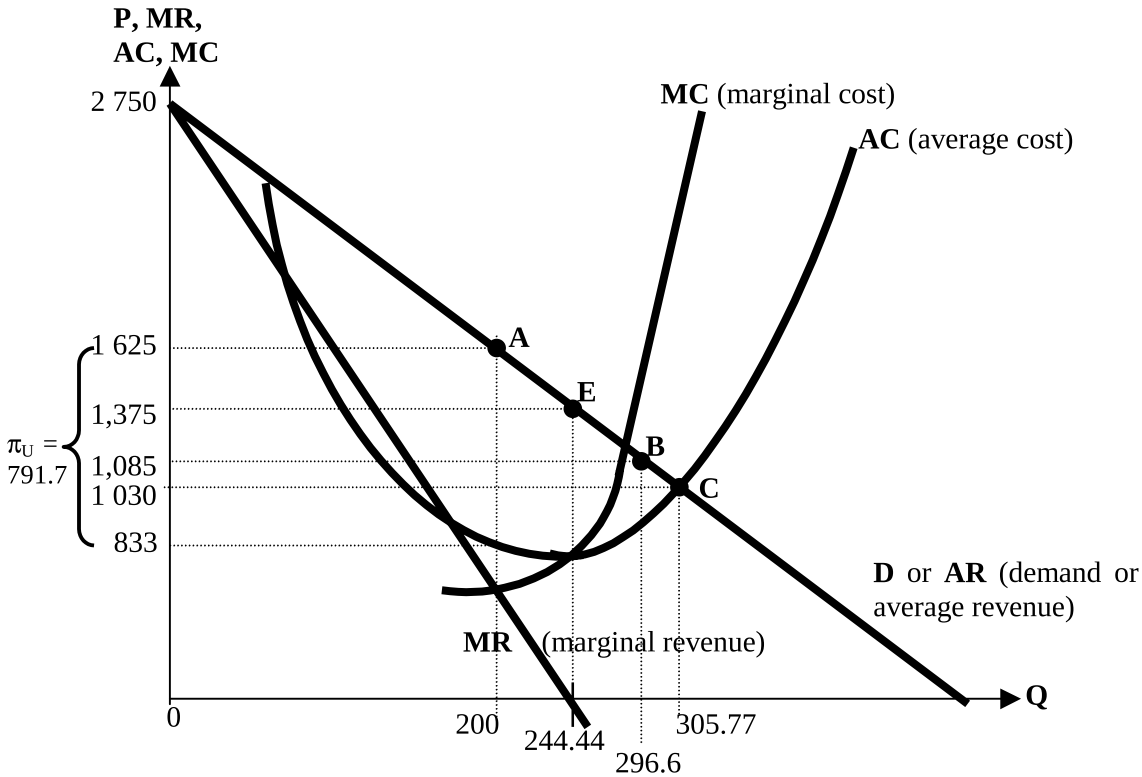
or Q = 305.775 (disregarding the meaningless negative root – 24.57) and P = 1,030.

[Point **C** on next figure]

Question 7.

PTTP maximises its cash-flow or its total revenue **P·Q** when its marginal revenue is zero : MR = -45/4 Q + 2,750 = 0 for Q = 244.44 and for P = 1,375 but as said above this can never be profit maximising. [Point **E** on next figure]

FIGURE **(approximate, not up to scale)**



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