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Currency Options and Central Bank Intervention: The Case of Colombia

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Abstract

In November 1999, the Colombian monetary authority implemented an intervention scheme in which it would intervene through the selling of call and put currency options, instead of undertaking direct operations in the foreign exchange spot market. Although it has been suggested that intervention with currency options should help to stabilize the exchange rate, it has also been claimed that its success depends to a great extent on the particular specification of the option contract. In this work, we study the stabilizing potential of the option contracts introduced by the Colombian monetary authority to intervene in the foreign exchange market. For this purpose, we evaluate their impact on the volatility and the short-term equilibrium level of the exchange rate COP/USD. Our results indicate that, even though option contracts introduced by the Colombian Central Bank affect the exchange rate in the manner suggested in the literature, they do not significantly reduce its volatility during the intervention period.

Sommaire

En novembre 1999, la Banque centrale de Colombie a implanté un système d'intervention dans lequel elle entreprendrait la vente des options d'achat et de vente sur devises, au lieu de faire des opérations au comptant sur le marché des changes. Bien qu'il ait été suggéré qu'une intervention avec des options sur devises puisse aider à stabiliser le taux de change, il a été également signalé que le succès d'un tel schéma dépend fortement de la spécification du contrat d'optionalité. Dans ce travail, nous étudions le potentiel de stabilisation des contrats qui ont été introduits par l'autorité monétaire colombienne pour intervenir sur le marché des changes. À cette fin, nous évaluons leur impact sur la volatilité et le niveau d'équilibre à court terme du taux de change COP/USD. Nos résultats indiquent que, même si les contrats introduits par la Banque centrale de Colombie ont une influence sur le niveau d'équilibre du taux de change tel qu'il est suggéré dans la littérature, ils ne réduisent pas sa volatilité de manière significative pendant la période d'intervention.

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1 Introduction

Intervention in currency markets has always been one of the main options of central banks to control the evolution of the exchange rate and to achieve their monetary goals. In addition to common interventions derived from the engagement of monetary authorities with a specific exchange rate level, a central bank might also wish to intervene, even under free-floating regimes, in order to constrain extreme exchange rate volatility. Recently, the increasing importance of derivatives in financial markets has allowed the use of forward and option contracts as intervention instruments, motivating certain central banks to explore the feasibility of these new schemes.

It has been frequently argued that central bank intervention in the foreign exchange market via option contracts is a convenient alternative to traditional direct interventions in the spot market since it should induce, at least theoretically, a stabilizing effect on the exchange rate, reduce the pressure of agents' dynamic hedging strategies on the spot market, provide additional hedging instruments in incomplete markets, and allow the defense of an exchange rate target with a lower spending of foreign reserves. However, despite these supposed advantages, systematic and publicly announced interventions with currency options have remained mostly a theoretical possibility and, so far, only Mexico and Colombia have formally implemented schemes of this type. Thus, after the turbulences that followed the financial crisis of 1995, the Mexican Central Bank introduced an option's sale scheme to increase its foreign exchange reserves, which allowed it to accumulate roughly 16000 millions USD between August 1996 and June 2001. Thereafter, Colombia introduced in November 1999, following the elimination of the target-zone system that had been in place since 1994, a similar scheme designed to provide an effective mechanism to manage the foreign exchange reserves without sending specific signals to the market and to permit the control of extreme exchange rate volatility. As in the Mexican instance, during the time this scheme has been in place the Colombian central bank has accumulated foreign exchange reserves amounting to roughly 2500 millions USD. However, contrary to its Mexican counterpart, the Colombian monetary authority has auctioned currency options in moments of great market instability in order to constrain the excessive volatility of the exchange rate COP/USD.

Although several studies have been made to test the performance of the Mexican intervention scheme, in the Colombian instance the only research on the subject has been that conducted by Mandeng [33]. Specifically, he used an econometric approach to assess the effect on the volatility of the exchange rate COP/USD of three interventions with currency options made by the Central Bank in July, August and October, 2002. However, and despite its broad use, the stand-alone implementation of such econometric methods have at least two drawbacks: in the first place, since real intervention data is required, these techniques can be used only to evaluate *ex post* the performance of the intervention scheme, so that knowledge about its stabilizing potential is available only after it has come into play. Secondly, econometric methods do not permit analysis of the effects of particular option contracts on the exchange rate process, so that it remains impossible to know which changes should be made to the contract in order to increase its effectiveness.

An alternative approach that avoids such shortcomings was proposed in 2003 by Zapatero and Reverter [49]. Specifically, they suggested the use of a Monte Carlo simulation method to estimate the effects that the intervention strategy with currency options would have in a hypothetical economy. Despite the fact that in this case conclusions have to be drawn from a very simplified model of the economy, this method still has the advantage of permitting the analysis of the effects of changes in the financial conditions of the option. Moreover, it can facilitate comparisons of its performance with other types of contract, which can be used as convenient benchmarks, and, contrary to econometric approaches, it provides *ex ante* extremely useful information about the impact of the option on the exchange rate process.

In this work we study the stabilizing potential of the option contracts introduced by the Colombian Central Bank to manage the foreign exchange reserves and to control the extreme exchange rate volatility. With this purpose in mind, we propose a partial equilibrium model of the foreign exchange market, inspired by that developed by Schönbucher and Wilmott [40], to analyze the feedback effect of dynamic hedging strategies introduced by a large market maker. In order to price the option contracts we propose the use of the methodology developed by Longstaff and Schwartz [32] to value Bermudan-style claims. We justify its utilization based on the results obtained by Bilger [4], as well as on a comparison exercise we carry on to test its precision against a binomial model we present in chapter 6. Likewise, we compare the results obtained from our implementation of the Longstaff-Schwartz model with those obtained from the pricing models developed by Fernández, Galán and Saavedra [18] and Fernández and Saavedra [20] to price the similar kind of options introduced by the Mexican Central Bank. Finally, we carry out several simulation exercises in which we estimate the impact on the exchange rate process of the trading strategies induced by the introduction of the option contracts written by the Colombian Central Bank. Furthermore, we study the stabilizing potential of a plain-vanilla American-style option contract we have used as a benchmark, and we perform a comparative analysis for different values of the parameters of the model that describes the functioning of the foreign exchange market.

This paper is organized as follows: Chapter 2 gives an account of the literature related to the topic of central bank intervention with currency options. Chapter 3 describes the context in which the Colombian Central Bank implemented its intervention strategy and presents the main features of the option contracts. Chapter 4 introduces the market model used to test their performance and the theoretical foundations of the different methods used to price them. Chapter 5 summarizes the relevant details concerning the implementation of the Monte Carlo simulation procedure. Chapter 6 presents the results of our simulation experiments. Finally, chapter 7 concludes the work with some final remarks.

2 Central bank intervention with currency options

It has been suggested that the use of currency options to intervene in the foreign exchange market, as distinct from the use of traditional spot operations, introduces an additional dimension to the mechanics of central bank intervention related to the trading strategies adopted by investors to maintain the risk level of their portfolios. Indeed, theoretically such trading strategies affect the demand for currencies and the short-run equilibrium exchange rate, so that, when carefully designed, the intervention scheme should induce an "engagement" in the agents toward exchange rate stability (Breuer [9], Wiseman [47], and Zapatero and Reverter [49]).

In an initial attempt to develop a strategy in which the central bank would profit from the supposed advantages of intervention through currency options, Taylor [41] proposed, in 1995, a mechanism in which the central bank would purchase out-of-the-money call options on foreign currencies as an insurance against sharp depreciations in the local currency. Specifically, he suggested that this kind of strategy would make it possible for the central bank to buy cheap foreign currencies that could be sold in the foreign exchange market to defend the exchange rate. Moreover, the profits derived from this intervention scheme would partially counteract the eventual losses suffered by the central bank as the result of interventions in the foreign exchange spot market, thereby creating additional room for further defense manoeuvres. However, as pointed out by Breuer [9], Taylor's proposal turned out to be flawed since its implementation could also generate destabilizing effects in the exchange rate and expose the monetary authority to both risks and policy dilemmas. In particular, since it is reasonable to expect market makers to hedge a short position in such options by buying currencies when the exchange rate goes up and by selling them when it drops, then trading strategies derived from the introduction of this purchase strategy could amplify any variation in the exchange rate. Moreover, in addition to the obvious credit risk the central bank would incur under this scheme, which should not be considered a key drawback given the usual lender-of-last-resort role played by most monetary authorities, a conflict of interests can result since the central bank obtains profits from the evolution of the price of an asset it appears to control. Indeed, the implementation of such a scheme could induce the central bank to intervene in the foreign exchange market hoping to increase the economic benefits derived from the exercise of the options by influencing the exchange rate evolution. Finally, Taylor's purchase intervention strategy could lead to a scenario in which the exercise of the options by the central bank could exacerbate a financial crisis through the increase of losses on the part of banking institutions, contributing to the weakening of the entire financial system.

As a response to these drawbacks, Wiseman [48] proposed a public intervention scheme in which the central bank would carry out auctions of short-term currency options with different strike prices, instead of intervening directly through spot operations in the foreign exchange market. Moreover, since in periods of financial turbulence market participants could interpret any change in the intervention mechanism as the acceptance of a critical situation, which could induce an increase in the exchange rate volatility, he suggests the permanent implementation of this intervention scheme rather than relegating its use to moments of extreme market instability. Wiseman [48] argued that the implementation of such an intervention mechanism would help the central bank to stabilize the exchange rate and to reduce the exchange rate volatility. Additionally, as pointed out by Breuer [9], such a sale scheme could avoid, at least under certain conditions, the problem of conflicting interests introduced by Taylor's purchasing strategy. Indeed, if the central bank were actually able to specify the financial conditions of the contracts in such a way that its potential losses would be limited, then under this scheme the monetary authority would not have strong incentives to misuse its prominent position to influence the evolution of the exchange rate beyond the effect produced by the original intervention.

Nevertheless, the selling of options by the central bank is not a risk-free proposition. In particular, as noticed by both authors, it requires a commitment that makes it eventually impossible for the monetary authority to change the strategy before the maturity of the options and could even make it difficult to anticipate the evolution of the level of foreign exchange reserves. Furthermore, as pointed out by Blejer and Schumacher [6], the selling of currency options introduces a contingent liability in the portfolio of the central bank that makes it complicated to assess its financial position. Fortunately, as Werner and Milo [46] remarked concerning the Mexican example, the first of these problems can be easily solved by selling short-maturity claims, while the latter can be mitigated through the specification of a variable strike that limits the magnitude of the losses - and therefore, the value of the liability - faced by the central bank.

In addition to these theoretical justifications proclaiming its advantages, there have been also attempts to assess the benefits of an intervention scheme with currency options. Particularly interesting in this respect is the work of Zapatero and Reverter [49], in which they compare the effects on the exchange rate and the risk-free interest rate of a traditional intervention strategy in the spot market with a scheme in which the monetary authority intervenes through the purchase and sale of options on foreign exchange. In order to carry out their exercise, the authors introduced equilibrium models of the foreign exchange market and the local bonds market that explicitly acknowledge the effects of the trading strategies resulting from the introduction of the claims. Despite the wide range of admissible intervention strategies, Zapatero and Reverter [49] focused on a "zero cost" intervention strategy in which the central bank, interested in reducing the exchange rate volatility, writes call options on foreign exchange and buys put currency options from private agents, and fixes the strike price of both contracts at a value that minimizes the initial rebalancing that should be introduced by private agents in order to keep unchanged the risk level of their portfolios. The authors estimated the dynamic hedging strategies derived from such intervention scheme from delta values for the currency options obtained using the Garman and Kohlhagen [25] option model, and assessed their effect on the price of the currency by computing the corresponding new equilibrium exchange rate from their proposed partial equilibrium framework. In accordance with the prior suggestions of Breuer [9] and Wiseman [48], they found that in general an intervention strategy that makes use of currency options does indeed allow the central bank to reduce the exchange rate volatility and to maintain a superior level of foreign exchange reserves than that observed under the spot intervention strategy. However, they also concluded that, ultimately, the effectiveness of the intervention scheme with currency options still depends critically on the characteristics of the economy in which it is implemented.

Given its supposed advantages, during the last decade some central banks have attempted the implementation of intervention mechanisms with currency options. Specifically, option sale schemes have been introduced in Mexico and Colombia, and their use has been suggested in Hong Kong (Cheng et al., [11], Hong Kong Monetary Authority, [29]) and Guatemala (Edwards and Vergara, [15]). With regard to the Mexican example, in 1996 the central bank considered that it would be advisable to increase the level of foreign exchange reserves in order to reduce the vulnerability of the country to external shocks, and adopted an intervention scheme in which it would auction put options on foreign exchange instead of buying foreign currencies directly on the spot market. Distinctively, this put option contract - called the Banxico option - had a floating strike price and its exercise was allowed only when the official exchange rate was below its 20-day moving average the day before the exercise date. Moreover, the Mexican intervention scheme contemplated a public commitment on the part of the central bank to sterilize all purchases derived from the exercise of the options. Interestingly, the econometric tests carried out by Werner [45] showed that this commitment on the part of the monetary authority allowed the accumulation of a significant amount of foreign exchange reserves - 5095 millions dollars during the period August 1996 - November 1997 - without generating a significant impact on the long-run equilibrium exchange rate.

In the wake of the Mexican experience, in 1999, the Colombian central bank introduced a more wide-ranging intervention scheme in which two different sets of put and call option contracts were used to accumulate foreign exchange reserves and to control the excessive exchange rate volatility. Specifically, the strategy that was put into place to manage the foreign exchange reserves entailed regular auctions of put currency options that allowed the Colombian Central Bank to increase the country's reserves by 2560 millions USD between November 1999 - August 2004. Moreover, the Colombian monetary authority has intervened occasionally, selling options with the explicit purpose of constraining the volatility of the exchange rate in moments of extreme market instability. Despite the lack of data related to this kind of interventions, Mandeng [33] tested econometrically the short-term impact of this set of options on the volatility of the exchange rate

COP/USD and informally analyzed the implications of the particular option contract specification. He found evidence of a moderate effect that vanishes 10 days after the intervention date. As a result of this, he claimed that the ineffectiveness of the scheme could be correlated to the low delta and gamma values of the option contracts and accordingly suggested the modification of their financial conditions in order to increase the stabilizing effects of these interventions.

3 The Colombian experience

3.1 Background

During the nineties, the Colombian exchange regime suffered important modifications in response to the requirements imposed by the opening of the economy to international markets. This adjustment process began in June 1991 when the board of directors of Colombia's monetary authority introduced changes that allowed financial intermediaries to exchange foreign currencies directly among themselves. As a result, from that moment on the exchange rate was determined by the foreign exchange market, instead of being administratively fixed as in the former crawling-peg regime that had been in place since 1967. Additionally, the intervention mechanism was also modified and, thus, after June 1991 the Central Bank intervened in the foreign exchange market purchasing, rather than foreign currencies, a new class of dollar denominated bonds: the exchange certificates. These changes permitted the creation of a hybrid free-floating-target-zone regime in which the central bank influenced the exchange rate through the modification of the maturity of the exchange certificates or the official rate at which these dollar denominated bonds could be redeemed. However, in practice the use of exchange certificates as intervention instruments turned out to be problematic because it tended to postpone any increase in the monetary base derived from an inflow of foreign currencies, so that by the end of 1992 exchange certificates accounted for almost 50% of the monetary base and their redemption created policy problems throughout 1993 (Villar and Rincón, [44], p. 30).

In this context, and "with the purpose of reestablishing some degree of monetary control" (Urrutia, [43], p. 12), an explicit target-zone regime was introduced to replace the former implicit target-zone system. It was argued that under this scheme marginal spot interventions would increase exchange rate stability and improve the degree of control over the domestic money stock. Nevertheless, the benefits derived from this explicit target-zone regime proved to depend critically on the market consensus concerning the ability of the central bank to defend its publicly established limits. Indeed, during the life of this system the explicit bounds were shifted several times in accordance with changes in the international financial environment and finally in September 1999, after a period of international turbulence, the target-zone was eliminated and a system close to a free-floating regime "was successfully adopted following the intervention scheme introduced in Mexico in 1996" (Villar and Rincón, [44], p. 34).

3.2 The intervention scheme

Since November 1999 the Colombian Central Bank has implemented a publicly announced intervention scheme based on the selling of contingent claims, rather than on direct operations on the foreign exchange spot market. Under this new regime, Colombia's monetary authority intervene using two publicly announced mechanisms: the first aimed to allow the Central Bank to manage the foreign currency reserves via the auction of call and put currency options. The second intended to control the extreme volatility of the exchange rate through the auction of call or put option contracts in which stronger exercise restrictions are imposed. To guarantee the transparency of this scheme, it seemed desirable for the Central Bank to intervene in the foreign exchange market without sending specific signals to economic agents about its exchange rate target. Moreover, it seemed advisable to specify the financial conditions of the options in such a way that the potential losses faced by the monetary authority and the anticipated commitment of foreign reserves would be limited. In order to meet these requirements, all claims sold by the Colombian Central Bank under this intervention scheme share the following features:

1. All currency options auctioned have a fixed maturity equal to one month.

2. All options can be exercised every trading day before the expiration of the option.

3. The strike price of all options is fixed so as to be equal to the official exchange rate observed the day before the exercise date[1].

4. When the option is exercisable its payoff is equal to the difference between two consecutive official exchange rates. Thus, the losses faced by the Central Bank are roughly limited to the value of the exchange rate variation over one trading day.

However, given their dissimilar purposes, some differences were established in the financial conditions of both types of option contract. In particular, as shown in table 3.1., the Colombian Central Bank established that call (put) options for the management of foreign currency reserves can be exercised only if the official exchange rate COP/USD observed the day before the exercise date is above (below) its 20-day moving average. On the other hand, in the case of options for the control of the extreme exchange rate volatility this exercise restriction would be strengthened, so that call (put) contracts would be exercisable only if the official rate observed the day before the exercise date were to be at least 4% above (below) its 20-day moving average.

[1]The official exchange rate is provided on a daily basis by the central bank. For a given trading day it is computed as the arithmetic average of the buy and sale rates quoted by all currency dealers during the preceding trading day (Banco de la República [3])

Contract specifications	Type of contract	
	Options for the management of the foreign exchange reserves	Options to control the extreme exchange rate volatility
Currency of quotation	Colombian peso (COP)	Colombian peso (COP)
Maturity	One month	One month
Option style	American	American
Strike price	Official exchange rate the day before the exercise date	Official exchange rate the day before the exercise date
Exercise condition	Call/put options can be exercised only if the official exchange rate for the day before the exercise date is above/below its 20-day moving average	Call/put options can be exercised only if the official exchange rate for the day before the exercise date is at least 4% above/below its 20-day moving average

Table 3.1 Contract specification of the currency options auctioned by the Colombian Central Bank.

With regard to the intervention's frequency, the scheme introduced by the Colombian Central Bank to manage the foreign exchange reserves considered monthly auctions of put options and occasional auctions of call options. Thus, as shown in table A.1.1., during the period November 1999 - August 2004, the monetary authority made 46 auctions of this kind of put option for an average notional value of 102 millions USD each time, while call options for the management of foreign exchange reserves were auctioned as recently as February 2003, March 2003 and April 2003, for a notional value of 200 millions USD each time. On the other hand, since the introduction of the scheme the Central Bank made public that it would intervene using options for the control of extreme exchange rate volatility only in moments of great market instability. Indeed, as established by its board of directors, a fixed notional value of 180 millions USD of this type of call (put) option would be auctioned each time the official exchange rate were to be 4% above (below) its 20-day moving average. Consequently, Colombia's monetary authority intervened using call options for the control of extreme exchange rate volatility in July 2002, August 2002 and October 2002, while this kind of put currency option was auctioned only in December 2004.

3.3 Foreign exchange market activity

The foreign exchange market is one of the most dynamic markets in the Colombian economy, as is corroborated by its daily spot turnover, which averaged 348 millions of US dollars during the period January 2001 - March 2005 and it represented approximately 14% of the overall monthly volume of Colombia's international trade. As a result of this dynamism, the official exchange rate has exhibited a highly variable monthly volatility. In particular, the standard deviation of the annualized log-returns of the official exchange rate was on average 6.99% during the period January 2001 - March 2005, with a minimum of 0.82% in March 2003 and a maximum of 17.84% in August 2002. Furthermore, economic and political instability has also been reflected by a highly variable annualized log-return. Indeed, during the same period it has been equal on average to 0.63%, ranging between a minimum of -48.39% in October 2002 and a maximum of 104.93% in July 2002. Interestingly, such instability has not been reflected in the differential between the Colombian interbank rate and the LIBOR overnight rate, which during the same period has remained relatively stable around 5.50%, with a low standard deviation of 2.33%.

Trading in the Colombian foreign exchange market takes place among the so called authorized currency dealers, composed mostly of credit intermediaries and currency brokers. All of these currency dealers are subject to foreign exchange position limits and their activity is constantly monitored by the central bank, the regulatory authorities, and the Colombian Stock Exchange. In particular, each currency intermediary is authorized to hold foreign exchange denominated assets up to a value representing 50% of the firm's capital and can take individual short positions on currencies on condition that the overall value of its portfolio in foreign denominated assets must not become negative. Indeed, as shown in figure 3.1., during the period January 2003 - December 2004 the cash position in USD held by all currency dealers was on average 318 millions USD, with a minimum of 119 millions USD in January 2004 and a maximum of 574 millions USD in June 2003.

Finally, transactions in the Colombian foreign exchange market are made mainly through a screen-based centralized electronic dealing system called SET FX, although other telephonic brokering systems are also available. This dealing interface, established in 2003 to replace the former DATATEC transactional system, permits the trade of currencies and currency derivatives among authorized users on both spot and next-day settlement modalities and was introduced to promote the adequate formation of market prices, to encourage the wide and opportune diffusion of relevant information, and to bring into existence a dealing system that guarantees equal access conditions to similar market participants.

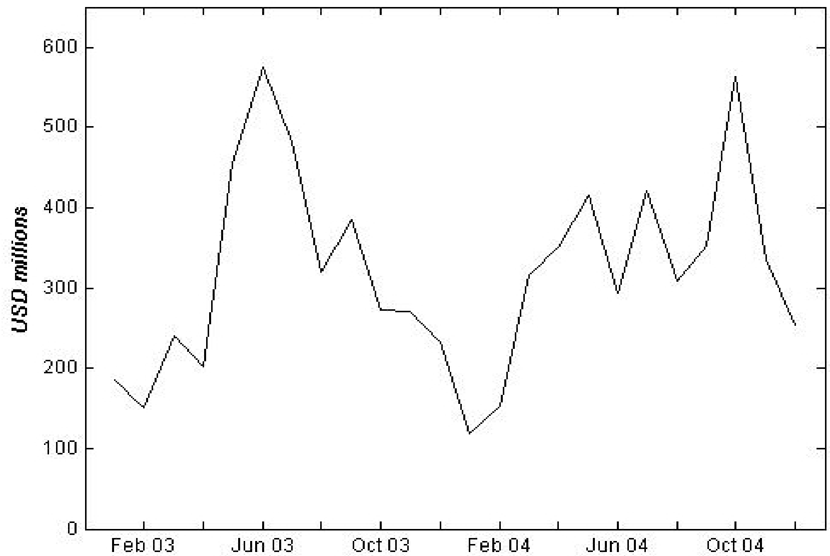


Figure 3.1. Cash position in USD (end of month) of all currency dealers (Source: Informe mensual de operaciones de derivados, Banco de la República, www.banrep.gov.co).

4 Theoretical framework

As mentioned in chapter 2, the stabilizing potential of a scheme in which the central bank intervenes with currency options depends crucially on its ability to induce the adoption of adequate trading strategies on the part of the market participants. As a result of this, the analysis of the effects of such intervention strategies has lead to a sharper focus on the influence of the trading process on the short-run equilibrium exchange rate, so that models of the microstructure of the foreign exchange market that view the market price as the outcome of individual decisions arising from underlying optimization problems seem at first sight to be well suited to address this kind of problem. However, as pointed out by O'Hara [35], Frankel and Froot [22], [23], [24], and further argued by Sarno and Taylor [37], implementation of such micro-based models is often hindered by the limited knowledge we have concerning the investment process, and by the fact that the degree of detail and the amount of information they demand makes them computationally intensive and difficult to put into practice.

To overcome these problems, the analysis of the effects of central bank intervention with currency options has focused on the interaction between the supply and demand of currencies, without taking into account the process from which they arise. Moreover, trading strategies introduced by market participants are usually assumed to be the result of optimization problems simpler than those we find in real-life situations, so that their effects can be readily evaluated using a specific partial equilibrium model. As a result of these simplifications, this approach has proved capable of providing not only a simpler alternative to fully micro-based models, but it is also well adapted to Monte Carlo simulation methods that can be implemented with little difficulty.

This chapter introduces the framework through which the theoretical impact of option contracts introduced by the Colombian Central Bank will be analyzed. Section 4.1 presents a simple partial equilibrium model inspired by that proposed by Schönbucher and Wilmott [40]. Section 4.2 introduces the basic principles of the traditional American-style option pricing paradigm. Section 4.3 presents the particular optimization problem from which we assume market participants derive their decisions about their optimal trading strategies and, finally, section 4.4 formalizes the financial conditions of the option contract introduced by the Colombian monetary authority and details the main features of the three alternative pricing models we present.

4.1 Market model

In 2000 Schönbucher and Wilmott [40] developed a market model they used to analyze the influence of trading strategies on the short-run equilibrium prices of financial assets. They studied the effect of a hedging strategy introduced by a large trader in an illiquid market using as the point of departure for their model the so-called excess demand function, instead of individual supply and demand relationships. One advantage of this approach arises from the fact that the functional form of the excess demand can be chosen in such a way that the price process of the underlying asset follows a discrete version of a geometric Brownian motion, which is specially well adapted to the traditional option pricing paradigm. Furthermore, although they considered the case of a perfectly informed large trader, their framework is easy to adapt to the more real situation in which agents' expectations about market equilibrium do not coincide with the actual equilibrium that is attained.

4.1.1 General setting

The economy. We assume the existence of an open small economy in which the arrival of new exogenous information - such as political events or changes in macro-economic fundamentals - is explicitly modeled by a Brownian motion W defined on a probability space $(\Omega, \mathcal{F}, P)$ equipped with a discrete filtration $\mathbb{F} = (\mathcal{F}_t)_{t \in \mathbb{T}}$. We denote by $\mathbb{T} = \{0, 1, ..., T\}$ the time set of the economy and we call the time interval $[0, T]$ the trading interval. We take for granted the existence of a foreign exchange market with limited liquidity in which the equilibrium exchange rate is determined.

Trading dates and information update. Trades in both markets take place at equally spaced discrete dates $t \in \mathbb{T}$ and all relevant exogenous information is assumed to arrive just before each trading date t.

Market participants. We take for granted the existence of a central bank that seeks to smooth the evolution of the exchange rate. We also assume that there is a risk-averse market maker who do not manipulate the foreign exchange market and a large population of similar small traders. We suppose that the market maker and the large population of small traders do not know the true market structure, have a limited ability to process the available information, and hold a portfolio comprising positions in the money market account and in the risky currency.

Assets. There are two assets in this economy: a money market account and a risky currency. For the sake of simplicity we assume that during each subinterval $(t, t+1]$, $t \in \{0, 1, ..., T-1\}$, the money market account and the risky currency yield a constant return equal to the local interest rate r and the foreign interest rate r^*, respectively. The price processes of the currency - also called the exchange rate process - and the money market account are denoted by R and B, respectively.

Contingent claim. The central bank writes at date $t = 0$ a currency option with maturity T, exercisable at dates $\mathbb{E} = \{1, 2, ..., T\}$ and with a payoff depending on the exchange rate process R.

Trading strategies. We assume that the market maker holds the currency option with hedging purposes, so that its possession induces the introduction of a trading strategy at time $t \in \mathbb{T}$. Formally, we represent this trading strategy by a predictable stochastic process generically denoted by η_t, which stands for the amount of currencies the market maker decides to hold during the period $(t, t+1]$ as the result of the possession of one contingent claim written on one unit of foreign currency.

Price process of the money market account. The price of the money market account is given at each date $t \in \mathbb{T}$ by

$$B_t = \exp\{rt\},$$

where B_0 is normalized to 1.

Supply and demand functions for the risky currency. We assume that the supply and demand for the risky currency at each date $t \in \mathbb{T}$ arise as exogenous outcomes of investment problems solved by the market maker and the large population of small traders. As usual, the aggregate supply of currencies at time t represents the relationship between the current exchange rate R_t and the overall stock of each asset available in the economy, and is denoted by a function $\mathcal{S}_t$, twice continuously differentiable in R_t and satisfying

$$\frac{\partial \mathcal{S}_t}{\partial R_t} > 0.$$

On the other hand, the aggregate demand for currencies is the sum of the individual long positions in the risky currency that all market participants are willing to hold for each exchange rate R_t. We represent it as a function $\mathcal{D}_t$, again twice continuously differentiable in R_t, such that

$$\frac{\partial \mathcal{D}_t}{\partial R_t} < 0.$$

Excess demand function. We define the excess demand in the foreign exchange market as the difference between the aggregate demand and the aggregate supply of currencies. The excess demand is represented by a function ψ_t such that

$$\psi_t = \mathcal{D}_t - \mathcal{S}_t$$

and satisfying

$$\frac{\partial \psi_t}{\partial R_t} < 0.$$

For the sake of simplicity, we assume the following specification, suggested by Schönbucher and Wilmott [40], for the excess demand function in the foreign exchange market[2]

$$\psi_t\left(R_0, W_t, x, \mu, \sigma\right) = x\left(R_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right\} - R_t\right) \tag{1}$$

where $x \in \mathbb{R}^+$ is a scaling parameter that represents the change in the excess demand for currencies that results from a marginal change in the value of R_t.

[2]Indeed, as it can be easily noted, the assumption of an excess demand function with specification such as in expression (1) results in an equilibrium exchange rate process that follows a discrete version of a geometric Brownian motion. This simplifies considerably not only the pricing of the contingent claims we consider, but also the simulation exercises we carry on to determine the impact of their introduction on the exchange rate process.

Likewise, $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$ are parameters that relate the available information up to time t with a corresponding value for the equilibrium exchange rate.

Market mechanism. We assume there is a market-clearing mechanism such that differences between the aggregate demand and the aggregate supply are instantaneously adjusted.

Equilibrium price function. At each date $t \in \mathbb{T}$ the equilibrium of the risky currency is represented by a price function R_t that equals the aggregate supply and demand in every state of nature. In the absence of trading strategies - i.e. when $\eta_t = 0 \ \forall t \in \mathbb{T}$ - the equilibrium exchange rate, called in the following the undisturbed exchange rate, is given by that price function R_t that satisfies

$$\psi_t = 0. \tag{2}$$

Specifically, from (2) and (1) the undisturbed exchange rate function R_t turns out to be equal to

$$R_t = R_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right\}. \tag{3}$$

Market maker's expectations. We assume that the imperfect information accessible to the market maker forces him to represent the functioning of the economy in a simplified way. In particular, we suppose that at time $t \in \mathbb{T}$ the market maker believes that the equilibrium price function that will hold at any date $s \in \mathbb{E}_t$, $\mathbb{E}_t = \mathbb{E} \cap (t, T]$, is given by

$$R_s^e = R_t \exp\left\{\left(\alpha\mu - \frac{(\alpha\sigma)^2}{2}\right)(s-t) + \alpha\sigma\left(W_s - W_t\right)\right\} \tag{4}$$

for some $\alpha \in \mathbb{R}^+$.

It should be noted that when a perfectly informed market maker is assumed to exist, as Schönbucher and Wilmott [40] do, then his expectation about the equilibrium price must coincide with the price that actually arises once all market participants have submitted their orders. However, an imperfectly informed market maker, such as the one we assume to exist, cannot know what will be the result of the trades submitted by all market participants and can only make a fair guess of the equilibrium price function. We support our assumption that the market maker adjusts the parameters μ and σ in the same proportion α based only on the fact that the introduction of an additional adjustment parameter will make the final analysis more complex - i.e. we will have another parameter to vary - without necessarily making it richer. Moreover, we justify the particular specification of the market maker's expected equilibrium price function by noticing that it is particularly well suited to the traditional contingent claim pricing paradigm and that it is broadly used in practice, even if evidence against the faithfulness with which it represents the behavior of the underlying asset's price is readily available.

Pricing model. We take for granted that the market maker prices the contingent claim written by the central bank and determines his optimal trading strategy using a valuation model in which the foreign exchange market is assumed to be perfectly liquid. We suppose also that the market maker obtains the inputs required by the model from his expectations function as in expression (4) and tries to offset the limitations of the pricing model by adjusting the parameter α. In particular, we justify this assumption based on the fact that in real-life situations agents price contingent claims by subscribing to simplified option pricing methods based on simple unrealistic hypotheses that are not usually completely met, such as the Black-Merton-Scholes model (Black and Scholes [5], Merton [34]).

4.1.2 Market equilibrium in the presence of trading strategies

As noticed by Schönbucher and Wilmott [40], the introduction of contingent claims in illiquid markets is likely to produce modifications in the price process of the asset on which the option's payoff depends, so that it is reasonable to expect the writing of the currency option by the central bank to produce changes in the equilibrium price function.

Let θ be the notional value of all currency options written by the central bank in $t = 0$. The market maker's hedging-originated demand at time t is equal to the product of the trading strategy implemented by him as the result of the possession of contingent claim written on one unit of foreign currency, η_t, and the notional amount of options auctioned by the central bank θ. Consequently, since in equilibrium the demand and supply of currencies must be both equal, in the presence of such a trading strategy condition (2) becomes

$$\psi_t + \theta\eta_t = 0. \tag{5}$$

For the sake of simplicity and computational purposes, we assume that the introduction of the contingent claim by the central bank induces a flux of information such that the market works at each time $t \in \mathbb{T}$ following the sequence:

1. At time $t-$ exogenous information represented by W_t arrives.

2. Immediately thereafter all small traders determine their investment strategy and submit their trades. At the same time the market maker computes the expected equilibrium price using the price function (4) and submits his trade, including that generated by the possession of the contingent claim.

3. At time t, once all submitted trades have been executed, a price R_t^d, called hereafter the disturbed exchange rate, is obtained from the disturbed equilibrium condition (5). In particular, if we replace ψ_t by (1) in (5), then this disturbed equilibrium condition becomes

$$x\left(R_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right\} - R_t^d\right) + \theta\eta_t = 0,$$

so that at each date $t \in \mathbb{T}$ the disturbed exchange rate function is given by

$$R_t^d = R_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right\} + \frac{\theta}{x}\eta_t. \tag{6}$$

4. At $t+$ the disturbed exchange rate is announced.

4.2 American-style option pricing

As it is well known, the traditional contingent claim pricing paradigm is based on the assumption of absence of riskless arbitrage opportunities. Moreover, stronger suppositions such as perfect liquidity, so that the trading strategies derived from the possession of the contingent claim do not affect the price of the underlying asset, and equal access to the available information, in order to avoid the existence of manipulation strategies based on the strategic disclosure of privileged information, are necessary. Given these premises, Black and Scholes [5] and Merton [34] found that under certain conditions a continuously rebalanced self-financing portfolio could replicate the behavior of the option contract, so that the pricing of the claim could be reduced to the solution of a partial differential equation subject to boundary conditions given by the payoff of the claim. As pointed out by Cox and Ross [13], under such non-arbitrage condition agents' preferences should not affect the value of the option and an interesting approach in which risk-neutral investors are assumed to prevail could be used to price it. This valuable insight was subsequently formalized by Harrison and Kreps [27] and by Harrison and Pliska [28], who found that a competitive economy is arbitrage-free only if there is at least one martingale measure Q equivalent to the objective probability measure P, and they showed that a "rational" price for a contingent claim would be the expected value of its discounted payoff computed under Q, which originated the so-called martingale pricing approach.

4.2.1 Formulation of the American-style option pricing problem

Unlike the European option valuation problem, in which the exercise date is fixed, contingent claims that involve an early exercise feature are harder to price. Indeed, since it is in the interest of the option's holder to obtain the maximum profit derived from the possession of the claim, this early exercise feature introduces the need for a strategy that specifies the optimal conditions under which the option should be exercised.

In order to consider the American-style option pricing problem, let us assume again, as in section 4.1.1, that at time $t \in \mathbb{T}$ each agent believes that the equilibrium exchange rate at any date $s > t$, $s \in \mathbb{E}_t$, is given by expression (4), so that there is only one price for the contingent claim corresponding to the unique martingale measure Q. Likewise, let's denote by X_s the $(\mathbb{F}, Q)-$adapted process that represents the payoff derived from the exercise of the contingent claim at any date s and let $\widetilde{X}$ be the discounted option's payoff process, so that $\widetilde{X}_s = \frac{B_t}{B_s} X_s$. Since it is reasonable to expect the holder of the option to be unable to perfectly foresee future states of the nature, then the exercise decision at date s must be based only on the information available up to that moment. In particular, let us represent the strategy that underlies the exercise decision by a random variable $\tau : \Omega \to \mathbb{E}_t$, called an admissible stopping time, that satisfies

$$\{\omega \in \Omega : \tau = s\} \in \mathcal{F}_s.$$

On the other hand, let's denote by $\widetilde{X}^{\tau}$ the stopped payoff process resulting from the introduction of the stopping rule τ and define

$$E^Q\left[\widetilde{X}^{\tau} \mid \mathcal{F}_t\right] = E^Q\left[\widetilde{X}_{\tau} \mid \mathcal{F}_t\right].$$

Formally, the holder's option pricing problem, also known as the "primal" valuation problem, can be stated as

$$E^Q\left[\widetilde{X}^{\tau^*} \mid \mathcal{F}_t\right] = \sup_{\tau \in \mathcal{T}_t} E^Q\left[\widetilde{X}_{\tau} \mid \mathcal{F}_t\right], \tag{7}$$

where $\mathcal{T}_t = \{\tau \in \mathcal{T} : \tau \geq t\}$ denotes the set of all admissible stopping times and τ^* is the so-called optimal stopping time.

4.2.2 Recursive solution of the primal valuation problem

As is well known, the "primal" valuation problem (7) can be solved at time $t \in \mathbb{T}$ by defining for $s \geq t$ the adapted process $\widetilde{Z}_s$

$$\begin{gathered} \widetilde{Z}_T = \widetilde{X}_T \\ \widetilde{Z}_s = \max\left[\widetilde{X}_s, E^Q\left[\widetilde{Z}_{s+1} \mid \mathcal{F}_s\right]\right] \end{gathered} \tag{8}$$

called the "Snell envelope" of the sequence $\widetilde{X}$.

In particular, for the optimal stopping time

$$\tau^* = \min\left\{s \in \mathbb{E}_t : \widetilde{Z}_s = \widetilde{X}_s\right\}, \tag{9}$$

we have

$$\widetilde{Z}_t = E^Q\left[\widetilde{X}_{\tau^*} \mid \mathcal{F}_t\right] = \sup_{\tau \in \mathcal{T}} E^Q\left[\widetilde{X}_\tau \mid \mathcal{F}_t\right],$$

which justifies calling $\widetilde{Z}_t$ the solution to the "primal" valuation problem and (9) the optimal stopping time[3].

4.3 Trading strategy

Again, let η_t denote the trading strategy introduced by the market maker at date $t \in \mathbb{T}$ as the result of the possession of the contingent claim written by the central bank. At date $t < \tau^*$ the market maker values the portfolio composed by a long position in one contingent claim and a short position in η_t currencies as

$$\pi_t = C_t - \eta_t R_t,$$

where $C_t = E^Q\left[\widetilde{X}_{\tau^*} \mid \mathcal{F}_t\right]$ represents the continuation value of the option and R_t stands for the currency equilibrium price the market maker expects to hold at t[4].

[3] For a detailed proof of this result see section 5.4. in Elliot and Kopp [16].

[4] One detail about the way the market maker values his strategy remains to be considered. Indeed, as noticed by Schonbucher and Wilmott [40], in illiquid markets the paper value of any strategy differs from its real value due to the fact that it cannot be liquidated at the current equilibrium price. Although a perfectly informed market maker must know the "real value" of his strategy and act accordingly, as mentioned in section 4.1.1, an imperfectly informed

In order to characterize the market maker optimization problem from which his trading strategy is assumed to arise, lets define the process

$$\varepsilon_{t+1} = \Delta\widetilde{V}_{t+1} - \eta_t \Delta\widetilde{R}_{t+1},$$

where the discounted variations $\Delta\widetilde{V}_{t+1}$ and $\Delta\widetilde{R}_{t+1}$ are given by

$$\Delta\widetilde{V}_{t+1} = \left(\frac{B_t}{B_{t+1}} \max\left[X_{t+1}, C_{t+1}\right] - C_t \right),$$

$$\Delta\widetilde{R}_{t+1} = \left(\frac{B_t}{B_{t+1}} R_{t+1} - R_t \right).$$

As noted by Schäl [39], since in the discrete-time case a perfect hedging is no longer possible, then it is necessary for agents to select a suitable criterion to choice the adequate trading strategy. In particular, as he remarks, criteria such as the minimization of the local conditional risk, the conditional remaining risk or the total risk of the trading strategy are often found in the literature. Therefore, even though we assumed that the market maker holds the currency option with hedging purposes, there are still innumerable admissible trading strategies that vary depending on the particular risk measure and the time horizon he considers, which force us to make additional assumptions about his behavior. For the sake of simplicity, we assume that at time $t < \tau^*$ the market maker is interested in minimizing the quadratic local hedging error of his portfolio $\mathcal{R}_t$, defined as

market maker cannot know the equilibrium price that will result from the trades of all market participants. Consequently, it is reasonable to suppose that he uses his expectation of the equilibrium price to value his trading strategy and to take what he thinks is the appropriate decision according to the information available to him.

$$\mathcal{R}_t = E^P \left[\varepsilon_{t+1}^2 \mid \mathcal{F}_t \right],$$

so that the optimization problem he faces turns out to be

$$\eta_t^* = \underset{\eta_t}{\text{argmin}} \;\; E^P \left[\left(\Delta \widetilde{V}_{t+1} - \eta_t \Delta \widetilde{R}_{t+1} \right)^2 \mid \mathcal{F}_t \right], \tag{10}$$

where it is easy to remark that η_t^* is equivalent to the best linear estimator of $\frac{B_t}{B_{t+1}} \max \left[X_{t+1}, C_{t+1} \right]$, given C_t and R_t.

In order to solve such problem, note that since ε_{t+1} is continuous in η_t, then the first order condition of this optimization problem turns out to be

$$\frac{\partial \mathcal{R}_t}{\partial \eta_t} = \frac{\partial E^P \left[\varepsilon_{t+1}^2 \mid \mathcal{F}_t \right]}{\partial \eta_t} = E^P \left[\frac{\partial \varepsilon_{t+1}^2}{\partial \eta_t} \mid \mathcal{F}_t \right] = 0.$$

Replacing ε_{t+1}^2 by $\left(\Delta \widetilde{V}_{t+1} - \eta_t \Delta \widetilde{R}_{t+1} \right)^2$ we obtain

$$\frac{\partial \mathcal{R}_t}{\partial \eta_t} = E^P \left[-2 \left(\Delta \widetilde{V}_{t+1} - \eta_t \Delta \widetilde{R}_{t+1} \right) \Delta \widetilde{R}_{t+1} \mid \mathcal{F}_t \right] = 0,$$

$$E^P \left[\Delta \widetilde{V}_{t+1} \Delta \widetilde{R}_{t+1} \mid \mathcal{F}_t \right] = E^P \left[\eta_t \left(\Delta \widetilde{R}_{t+1} \right)^2 \mid \mathcal{F}_t \right],$$

and, since η_t is predictable, then

$$\eta_t^* = \frac{E^P \left[\Delta \widetilde{V}_{t+1} \Delta \widetilde{R}_{t+1} \mid \mathcal{F}_t \right]}{E^P \left[\left(\Delta \widetilde{R}_{t+1} \right)^2 \mid \mathcal{F}_t \right]}, \tag{11}$$

which coincides with the well known solution to (10).

On the other hand, since the second order condition turns out to be equal to

$$\frac{\partial^2 \mathcal{R}_t}{\partial \eta_t^2} = E^P \left[\left(\Delta \widetilde{R}_{t+1} \right)^2 \mid \mathcal{F}_t \right] \geq 0,$$

then the quadratic local hedging error attains its minimum when the market maker chooses the trading strategy $\eta_t = \eta_t^*$.

4.4 Pricing models

Several attempts have been made to address the issue of the valuation of currency options written by the Mexican and Colombian central banks. In particular, it is worthwhile to mention two specific efforts: the heuristic exercise rule developed by Fernández, Galán and Saavedra [18] and the binomial pricing model proposed by Fernández and Saavedra [20]. Despite the interest these models have aroused, they are not adapted to the pricing of the type of options introduced by the Colombian Central Bank to control excessive exchange rate volatility and, therefore, an alternative pricing model needs to be proposed. In particular, and despite the non-Markovian character of the average-based payoff process of the currency options used by the Colombian monetary authority to intervene in the foreign exchange market, we propose, supported by the recent work of Bilger [4], to use the regression-based algorithm proposed by Longstaff and Schwartz [32] to price the option contracts sold by the Colombian Central Bank.

This section is divided into four sub-sections as follows: sub-section 4.4.1 formalizes the financial conditions of the option contracts. Sub-sections 4.4.2 and 4.4.3 present briefly the Fernández-Galán-Saavedra heuristic exercise rule and the

Fernández-Saavedra binomial model. Finally, sub-section 4.4.4 presents the main features of the proposed Longstaff-Schwartz pricing algorithm.

4.4.1 Option contract description

Again, let $\mathbb{T} = \{0, 1, ..., T\}$ represents the time set of the economy, where T is set to be equal to the maturity (in days) of the currency option written by the central bank. Let's define the D-day simple moving average of the equilibrium exchange rate as

$$M_{D,t-1} = \frac{1}{D}\sum_{i=1}^{D} R_{t-i}.$$

As mentioned in chapter 3, the exercise of all options used by the Colombian Central Bank to intervene in the foreign exchange market depends on the fulfillment of a condition that varies according to the type of option considered. Formally, while call and put options used to manage the level of foreign exchange reserves can be exercised at time $t \in \mathbb{E}$ only if $R_{t-1} \geq M_{20,t-1}$ and $R_{t-1} \leq M_{20,t-1}$, respectively, options for the control of extreme exchange rate volatility can be exercised at t only if $R_{t-1} \geq 1.04M_{20,t-1}$ in the case of the call option, and if $R_{t-1} \leq \frac{1}{1.04}M_{20,t-1}$ in the case of the put option. Clearly, all exercise conditions share a similar structure, differing only by a factor κ that rescales the moving average depending on the type of option. Let $\mathbf{1}_t$ denote the indicator function that represents the fulfilment of the exercise restriction at time $t \in \mathbb{E}$. The payoff at time $t \in \mathbb{E}$ of any of the call options used by the Colombian Central Bank can be written as

$$X_t^{(c)}\left(R_t, R_{t-1}, M_{D,t-1}, \kappa\right) = \max\left[R_t - R_{t-1}, 0\right] \; \mathbf{1}_{t\left\{R_{t-1} \geq \kappa M_{D,t-1}\right\}}, \qquad \kappa \in (1, \infty],$$

whereas the payoff of any of the put options at time $t \in \mathbb{E}$ can be expressed as

$$X_t^{(p)}\left(R_t, R_{t-1}, M_{D,t-1}, \kappa\right) = \max\left[R_{t-1} - R_t, 0\right] \; \mathbf{1}_{t\left\{R_{t-1} \leq \kappa M_{D,t-1}\right\}}, \qquad \kappa \in (0,1].$$

4.4.2 The Fernández-Galán-Saavedra heuristic stopping criteria

In 2003, and as an attempt to obtain a pricing model for the option used by the Mexican Central Bank to manage its foreign exchange reserves, Fernández, Galán and Saavedra [18] developed a heuristic exercise rule in a theoretical framework in which the official exchange rate process under the risk-neutral measure Q was supposed to follow a discrete version of a geometric Brownian motion. To develop their stopping criteria, they initially focused their attention on a hypothetical option contract that did not take the exercise restriction into account and they found that it could be optimally exercised only when the observed daily return of the exchange rate exceeded a threshold for which a closed-form solution is available.

Let's first consider the pricing at time $t \in \mathbb{T}$ of the options that does not take the exercise restriction into account. In their article, the authors proved that this hypothetical option should be exercised in $s \in \mathbb{E}_t$ when the ratio $\frac{\Delta W_s^Q}{\sigma\sqrt{\Delta s}}$ goes beyond an exercise frontier denoted Y_s, so that the optimal stopping times of the call and put options are

$$\tau^{(c),w} = \min\left\{ s \in \mathbb{E}_t : \frac{\Delta W_s^Q}{\sigma\sqrt{\Delta s}} > Y_s^{(c)} \right\}, \tag{12}$$

and

$$\tau^{(p),w} = \min\left\{s \in \mathbb{E}_t : \frac{\Delta W_s^Q}{\sigma\sqrt{\Delta s}} < Y_s^{(p)} \right\}, \tag{13}$$

where $Y_t^{(\cdot)}$ is obtained recursively in each case as

$$\begin{aligned} Y_T^{(c)} &= K \\ Y_s^{(c)} &= \frac{\sigma\sqrt{\Delta s}}{2} - \frac{1}{\sigma\sqrt{\Delta s}} \ln\left(e^{(r-r^*)\Delta s} - e^{-r^*\Delta s} A_{s+1}^{(c)}\right) \end{aligned},$$

$$\begin{aligned} Y_T^{(p)} &= K \\ Y_s^{(p)} &= \frac{\sigma\sqrt{\Delta s}}{2} - \frac{1}{\sigma\sqrt{\Delta s}} \ln\left(e^{(r-r^*)\Delta s} - e^{-r^*\Delta s} A_{s+1}^{(p)}\right) \end{aligned},$$

with $A_t^{(\cdot)}$ and K given by

$$\begin{aligned} A_T^{(c)} &= e^{(r-r^*)\Delta s}\Phi\left(\sigma\sqrt{\Delta s} - K\right) - \Phi\left(-K\right) \\ A_s^{(c)} &= -\Phi\left(-Y_s^{(c)}\right) - \Phi\left(\sigma\sqrt{\Delta s} - Y_s^{(c)}\right)\left(e^{(r-r^*)\Delta s} - e^{-r^*\Delta s} A_{s+1}^{(c)}\right) + e^{-r^*\Delta s} A_{s+1}^{(c)} \end{aligned},$$

$$\begin{aligned} A_T^{(p)} &= \Phi\left(K\right) - e^{(r-r^*)\Delta s}\Phi\left(K - \sigma\sqrt{\Delta s}\right) \\ A_s^{(p)} &= \Phi\left(Y_s^{(p)}\right) - \Phi\left(Y_s^{(p)} - \sigma\sqrt{\Delta s}\right)\left(e^{(r-r^*)\Delta s} + e^{-r^*\Delta s} A_{s+1}^{(p)}\right) + e^{-r^*\Delta s} A_{s+1}^{(p)} \end{aligned},$$

$$K = \frac{1}{2}\left(\sigma\sqrt{\Delta s} - \frac{2\left(r - r^*\right)\Delta s}{\sigma\sqrt{\Delta s}}\right).$$

Although the authors were unable to develop an optimal stopping rule for the more complex options that consider the exercise restriction, they suggested the use of (12) and (13) as sub-optimal stopping criteria. In particular, Fernández, Galán and Saavedra [18] proposed that the option with the exercise restriction should be exercised when a positive payoff is observed and $\frac{\Delta W_s^Q}{\sigma\sqrt{\Delta s}}$ goes beyond the exercise threshold Y_s, so that the (sub-optimal) stopping times of the call and put contracts would in this case be

$$\tau^{(c)} = \min\left\{s \in \mathbb{E}_t : X_s^{(c)}\left(\frac{\Delta W_s^Q}{\sigma\sqrt{\Delta s}} - Y_s^{(c)}\right) > 0\right\},$$

$$\tau^{(p)} = \min\left\{s \in \mathbb{E}_t : X_s^{(p)}\left(\frac{\Delta W_s^Q}{\sigma\sqrt{\Delta s}} - Y_s^{(p)}\right) < 0\right\},$$

where once again $X_s^{(c)}$ and $X_s^{(p)}$ represent the payoff of the call and put options at time s, respectively.

To support the validity of their (sub-optimal) rule, Fernández, Galán and Saavedra [18] compared the profits derived from the exercise of the option obtained by all Mexican financial institutions during the period August 1996 - June 2001 with those they could have obtained from the use of the proposed stopping criteria. Surprisingly, they found that their heuristic rule systematically outperformed the stopping criterion implicit in the practitioners' exercise decisions. However, they presented neither convergence results for the option that takes account of the exercise restriction, nor simple numerical exercises carried out to test the accuracy of their model. Consequently, the degree of sub-optimality of their exercise rule, and therefore the downward bias of the option prices obtained through it, remains unknown.

4.4.3 The Fernández-Saavedra binomial model

As an alternative to the heuristic model proposed by Fernández, Galán and Saavedra [18], in 2003 Fernández and Saavedra [20] suggested the pricing of the Banxico option using a standard binomial tree in which the time-step Δs is set as equal to one trading day. Under this framework the official exchange rate would take two possible values at every date $s \in \mathbb{E}_t$

$$R_s^{(\cdot)} = R_{s-1}\zeta^{(\cdot)},$$

$$\zeta^{(\cdot)} = \begin{cases} \zeta^{(u)} = e^{\sigma\sqrt{\Delta s}} \\ \zeta^{(d)} = e^{-\sigma\sqrt{\Delta s}} \end{cases},$$

with risk-neutral transition probabilities

$$Q^{(u)} = \frac{e^{(r-r^*)\Delta s} - e^{-\sigma\sqrt{\Delta s}}}{e^{\sigma\sqrt{\Delta s}} - e^{-\sigma\sqrt{\Delta s}}},$$

$$Q^{(d)} = \frac{e^{\sigma\sqrt{\Delta s}} - e^{(r-r^*)\Delta s}}{e^{\sigma\sqrt{\Delta s}} - e^{-\sigma\sqrt{\Delta s}}}.$$

Fernández and Saavedra [20] proved that in this model the optimal stopping time for the call and put options for the management of foreign exchange reserves is given by the first moment a positive payoff is observed. This is,

$$\tau^* = \begin{cases} \min\{s \in \mathbb{E}_t : X_s^{(\cdot)} > 0\} & \text{if } \; X_s^{(\cdot)} > 0 \\ T & \text{otherwise} \end{cases},$$

so that at each date $s \in \mathbb{E}_t$ the discounted value of the option could be computed as

$$\widetilde{Z}_s = \begin{cases} \widetilde{X}_s^{(\cdot)} & \text{if } \widetilde{X}_s^{(\cdot)} > 0 \\ E^Q\left[\widetilde{Z}_{s+1}^{(\cdot)} \mid \mathcal{F}_s\right] & \text{otherwise} \end{cases} .$$

Despite its simplicity, this exercise rule permits a considerable reduction in the computational burden imposed by the path dependent character of the claim by avoiding the use of any recursive procedure to compute the option's price. However, it should be noted that the assumption that in each trading day the official exchange rate increases or decreases in the same proportion is not realistic and then it is reasonable to expect this model to produce highly biased estimates of the option value.

4.4.4 The Longstaff-Schwartz pricing algorithm

In 2001, Longstaff and Schwartz [32] suggested a regression-based algorithm to price American-style options through the use of a Monte Carlo simulation technique. As has been frequently noted, the advantage of this pricing technique lies in the fact that it allows the estimation of continuation values with a relatively low degree of computational effort. Moreover, this simulation based method has been proven to be very flexible and it is applicable to a wide range of exotic options, although the accuracy of the method depends to a great extent on the particular choice of the basis functions used to estimate the continuation value of the claim - the "approximation architecture", as Tsitsiklis and Van Roy [42] term it -.

Longstaff and Schwartz [32] suggested the use of a linear function that depends on a vector of free parameters and on a set of $\mathcal{F}_s-$measurable functions to obtain at each date $s \in \mathbb{E}_t$, $\mathbb{E}_t = \mathbb{E} \cap (t, T]$, a first approximation to the continuation value of the option. Specifically, they proposed that for a given set of simulated paths of the state variable starting at R_t, $\left\{\overrightarrow{R}_n\right\}$, $\overrightarrow{R}_n = (R_t, R_{t+1,n}, ..., R_{T,n})$,, $n \in \{1, 2, ..., N\}$, and a vector composed of M basis functions $\gamma_{s,n} = \left(\gamma_{s,n}^{(1)}, ..., \gamma_{s,n}^{(M)}\right)$, this continuation value could be approximated as

$$\widetilde{C}_{s-1,n}^{(1)} = \gamma_{s-1,n}\beta_{s-1}, \quad \text{for } s > t+1, \tag{14}$$

where β_{s-1} is estimated as

$$\widehat{\beta}_{s-1} = \left(\gamma_{s-1}^{\mathsf{T}}\gamma_{s-1}\right)^{-1}\left(\gamma_{s-1}^{\mathsf{T}}\widetilde{C}_{s}^{(2)}\right), \tag{15}$$

$$\gamma_{s-1} = \begin{bmatrix} \gamma_{s-1,1} \\ \vdots \\ \gamma_{s-1,n} \\ \vdots \\ \gamma_{s-1,N} \end{bmatrix} \qquad \widetilde{C}_{s}^{(2)} = \begin{bmatrix} \widetilde{C}_{s,1}^{(2)} \\ \vdots \\ \widetilde{C}_{s,n}^{(2)} \\ \vdots \\ \widetilde{C}_{s,N}^{(2)} \end{bmatrix}.$$

In contrast to pure regression-based algorithms which use only this first estimate of the continuation value obtained through regression to implement the recursive pricing procedure (8), the Longstaff-Schwartz method makes use of an additional approximation to the continuation value of the option. In particular, while the first approximation to this continuation value is obtained from (14) and provides the stopping criteria that defines the exercise rule, the second approximation, obtained from the application of such an exercise rule to each one of the simulated paths, replaces the continuation value in the recursive procedure (8) introduced in page 28 to solve the "primal" valuation problem. As noted by Glasserman [26], this feature results in a greater degree of precision than would be possible with pure regression-based methods, whose estimated prices can show a considerable upward bias.

Formally, Longstaff and Schwartz [32] proposed to compute the second approximation to the continuation value $\widetilde{C}^{(2)}_{n,s}$ for each trajectory as

$$\widetilde{C}^{(2)}_{T,n} = \widetilde{X}_{T,n}$$

$$\widetilde{C}^{(2)}_{s,n} = \begin{cases} \widetilde{X}_{s,n} & \text{if } \widetilde{X}_{s,n} \geq \widetilde{C}^{(1)}_{s,n} \\ \widetilde{C}^{(2)}_{s+1,n} & \text{if } \widetilde{X}_{s,n} < \widetilde{C}^{(1)}_{s,n} \end{cases}, \quad \text{for } t < s < T$$

so that the continuation value of the option at date $t \in \mathbb{T}$ would be estimated as

$$\widetilde{C}_{t,N} = \frac{1}{N} \sum_{n=1}^{N} \widetilde{C}^{(2)}_{t+1,n}. \tag{16}$$

In their article, Longstaff and Schwartz [32] provide convergence results for their algorithm for an American-style option with payoff depending on a single variable. Additionally, they present several numerical examples through which they test the performance of their methodology by comparing their results with the exact option prices obtained using finite difference methods. Subsequently, Clément, Lamberton and Lapeyre [12] formally justified the use of this algorithm to price American-style options and presented a study of its convergence rate. In particular, they showed that if the set of basis functions $\gamma_{s,n} = \left(\gamma^{(1)}_{s,n}, ..., \gamma^{(M)}_{s,n}\right)$ is total in L^2, then the option price obtained using the Longstaff-Schwartz algorithm converges almost surely to the true option price as $N \to \infty$ (Clément, Lamberton and Lapeyre [12], Theorem 3.2., p. 456), although for $M < \infty$ and $N < \infty$

$$\sup_{\tau \in \mathcal{T} \cap (t,T]} E^Q \left[\widetilde{X}_\tau \mid \mathcal{F}_t \right] > \widetilde{C}_{t,N} \qquad \text{a.s.}$$

so that in practice it tends to underestimate the true option price. However, as has been pointed out by Longstaff and Schwartz [32] and by Glasserman [26], for a careful choice of the set of basis functions this algorithm can provide accurate option prices with very reduced downward bias.

Unfortunately, such convergence results hold only when the option's payoff follows a Markov process. Although in the case of the average-based options written by the Colombian central bank the evolution of the discounted payoff can be represented by a Markov process that at time t depends on the $D+1$ variables $(R_t, R_{t-1}, ..., R_{t-D-1})$, this increases the dimension of the problem significantly and makes the pricing of the option very time-consuming. On the other hand, when we use as state variables R_t and $M_{D,t-1}$ the payoff function becomes non-Markovian and the convergence of the Longstaff-Schwartz algorithm is not guaranteed. Although this might be a reason to avoid the use of this algorithm with basis functions depending only on R_t and $M_{D,t-1}$, Bilger [4] suggests in his masters' thesis that the use of the Longstaff-Schwartz method to price contingent claims with non-Markovian payoff processes could be justified. Numerical exercises carried out by the author show that this algorithm provides accurate option prices in the case of a non-Markovian American-style Asian option with a rolling time window when he include basis functions depending on the rolling average from which the option's payoff is computed. Nevertheless, despite this encouraging evidence, the accuracy of this algorithm for the option contracts we have considered remains to be determined. Section 6.1 presents numerical exercises carried out to test the performance of the Longstaff-Schwartz pricing method in comparison with a standard binomial model we present in Appendix 3. Our results suggest that for the choice of basis functions presented in section 5.2 this regression-based method performs well, providing acceptably accurate option prices for all option contracts used by the Colombian Central Bank to intervene in the foreign exchange market.

5 Monte Carlo implementation

The growing complexity of financial derivatives and the availability of powerful computers have generated an increasing interest in the application of the Monte Carlo simulation method to the pricing of exotic contingent claims. As has been frequently noted, in addition to its flexibility and simplicity, the Monte Carlo simulation technique allows to obtain option price estimates with good statistical properties that rest on weak assumptions, usually satisfied by the way in which the sample of paths is generated.

This chapter presents the details of the Monte Carlo implementation of the theoretical model introduced in chapter 4. Section 5.1 outlines some basic concepts that underlie the application of this simulation method, identifies the main features of the variance reduction technique that will be used to estimate the option prices, and introduces the error measures we will compute to test the performance of the proposed pricing algorithm. Finally, section 5.2 presents implementation details concerning the market and the option pricing models we proposed in the previous chapter.

5.1 Basic concepts

The utility of the Monte Carlo simulation method for the valuation of contingent claims lies in the fact that the prices of derivatives can be expressed as mathematical expectations, so that, when analytic expressions are not available, it is logical to try to estimate such prices from finite sets of paths of the state variables on which the option's payoff depends.

However, in order to guarantee the convergence of the Monte Carlo estimator to the value of the corresponding mathematical expectation, the simulation of N independent paths of the state variable must be feasible. Indeed, let's assume we can express the payoff of the option obtained from the implementation of the exercise rule τ^0 as a function of a uniform random vector $\overrightarrow{u}_n = (u_{t+1,n}, ..., u_{T,n})$, $u_{i,j} \sim U[0,1]$. Formally, the convergence of the Monte Carlo estimator requires each realization of the option's payoff $\widetilde{x}_{\tau^0,n}(\overrightarrow{u}_n)$ to be independent from every other payoff $\widetilde{x}_{\tau^0,j}(\overrightarrow{u}_j)$, $j \neq n$. Fortunately, this requisite is ordinarily met by the generation of a set of N independent random vectors $\{\overrightarrow{u}_n\}$, so that, by the strong law of large numbers, the mean of the random sample $\{\widetilde{x}_{\tau^0,1}, ..., \widetilde{x}_{\tau^0,N}\}$ converges in probability to the expected value of the random variable $\widetilde{X}_{\tau^0}$ as $N \to \infty$. This makes it possible to estimate a lower bound for the price of an American-style claim using the stopping rule τ^0 as

$$\widetilde{C}_{t,N} = \frac{1}{N}\sum_{n=1}^{N} \widetilde{x}_{\tau^0,n},$$

so that

$$E^Q\left[\widetilde{X}_{\tau^*} \mid \mathcal{F}_t\right] \geq \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} \widetilde{x}_{\tau^0,n}, \qquad \text{a.s.}$$

Moreover, since the central limit theorem guarantees that the distribution of the standarized mean of any square-integrable random variable converges to a standard normal distribution as $N \to \infty$, then

$$\sqrt{N}\left(\widetilde{C}_{t,N} - E^Q\left[\widetilde{X}_{\tau^0} \mid \mathcal{F}_t\right]\right) \sim N\left(0, \mathrm{Var}^Q\left[\widetilde{X}_{\tau^0} \mid \mathcal{F}_t\right]\right). \tag{17}$$

Although expression (17) includes the (generally) unknown variance of the random variable $\widetilde{X}_{\tau^0}$, fortunately, it still holds if $\mathrm{Var}^Q\left[\widetilde{X}_{\tau^0} \mid \mathcal{F}_t\right]$ is replaced by the unbiased estimator

$$\widehat{s}_{t,N}^2 = \frac{1}{N-1} \sum_{n=1}^{N} \left(\widetilde{x}_{\tau^0,n} - \widetilde{C}_{t,N} \right)^2 ,$$

and

$$\sqrt{N} \left(\widetilde{C}_{t,N} - E^Q \left[\widetilde{X}_{\tau^0} \mid \mathcal{F}_t \right] \right) \sim N \left(0, \widehat{s}_{t,N}^2 \right) , \tag{18}$$

from which confidence intervals for the estimated price $\widetilde{C}_{t,N}$ can be easily constructed.

Despite its benefits, the main disadvantage of the Monte Carlo simulation method is its slow convergence speed. In particular, from expression (18) it is apparent that in order to cut by half the standard deviation of the estimator - i.e. to reduce the length of the confidence interval by a factor of two - it is necessary to multiply N by four. Since in terms of computational effort it is likely that this will imply an increase in the processing time of roughly the same proportion, then it is clear that improvements in the efficiency of the crude Monte Carlo estimator should be attempted using alternative variance-reduction techniques, rather than by just increasing the sample size.

5.1.1 Variance reduction technique: antithetic variates

To increase the convergence speed of the Monte Carlo method several variance-reduction techniques have been suggested, one of the simplest being the so called antithetic variates method. Again, let $\widetilde{x}_{\tau^0,n} (\overrightarrow{u}_n)$ denote the value of the random variable $\widetilde{X}_{\tau^0}$ obtained from a realization of the uniformly distributed random vector $\overrightarrow{u}$. Additionally, let's set $\widetilde{x}^{a}_{\tau^0,n}\left(\overrightarrow{u}_n\right) = \widetilde{x}_{\tau^0,n}\left(\overrightarrow{1} - \overrightarrow{u}_n\right)$ and define the random variable

$$\widetilde{X}^{*}_{\tau^0} = \frac{\widetilde{X}_{\tau^0,n} + \widetilde{X}^{a}_{\tau^0,n}}{2},$$

for which it is easy to check that

$$E^Q\left[\widetilde{X}^{*}_{\tau^0} \mid \mathcal{F}_t\right] = E^Q\left[\widetilde{X}_{\tau^0} \mid \mathcal{F}_t\right].$$

On the other hand, the variance of the random variable $\widetilde{X}^{*}_{\tau^0}$ is given by

$$\mathrm{Var}^Q\left[\widetilde{X}^{*}_{\tau^0} \mid \mathcal{F}_t\right] = \frac{1}{2}\mathrm{Var}^Q\left[\widetilde{X}_{\tau^0} \mid \mathcal{F}_t\right] + \frac{1}{2}\mathrm{Cov}^Q\left[\widetilde{X}_{\tau^0}, \widetilde{X}^{a}_{\tau^0} \mid \mathcal{F}_t\right], \qquad (19)$$

where it is clear that if $\widetilde{x}_{\tau^0,n}$ depends monotonically on $\overrightarrow{u}_n$, which is the case for most options' payoffs, then by construction it is assured that $\mathrm{Cov}^Q\left[\widetilde{X}_{\tau^0}, \widetilde{X}^{a}_{\tau^0} \mid \mathcal{F}_t\right] < 0$ and antithetic variates help to increase the efficiency of the Monte Carlo estimation procedure.

The main advantage of the use of this variance-reduction technique is principally derived from two sources: in the first place, since the generation of each pseudo-random vector $\overrightarrow{u}_n$ implies a certain computational burden, it is reasonable to expect the set of realizations of the random variables $\widetilde{X}_{\tau^0}$ and $\widetilde{X}^{a}_{\tau^0}$ to require less time to be generated than an ordinary set of realizations of size $2N$. In the second place, as noted by Boyle, Broadie and Glasserman [8], random pairs $\left\{\overrightarrow{u}_n, \overrightarrow{1} - \overrightarrow{u}_n\right\}$, $n \in \{1, ..., N\}$, are more regularly distributed than a collection of random realizations $\{\overrightarrow{u}_n\}$, $n \in \{1, ..., 2N\}$. This results in better "representability" of the sample and leads the mean over all antithetic pairs $\left\{\overrightarrow{u}_n, \overrightarrow{1} - \overrightarrow{u}_n\right\}$ to be always equal to the theoretical mean of the uniform distribution, contrary to the mean of the collection $\{\overrightarrow{u}_n\}$, which almost invariably differs from that value.

5.1.2 Error measures for biased estimates of the option price

In the case in which $\widetilde{C}_{t,N}$ is not guaranteed to converge to $E^Q\left[\widetilde{X}_{\tau^*} \mid \mathcal{F}_t\right]$, as is the situation when it is not assured that the stopping rule τ^0 obtained from our pricing model is close to the optimal exercise rule, it is necessary to develop a measure of precision that incorporates not only the variance of the estimator - i.e. the dispersion of the Monte Carlo estimation -, but also the deviation from the true solution - i.e. the estimation bias -. A precision indicator that considers both sources of error is the so-called root mean square error (RMSE) defined as in Glasserman [26] as

$$RMSE\left(\widetilde{C}_{t,N}\right) = \sqrt{Bias^2\left(\widetilde{C}_{t,N}\right) + \widehat{s}^2_{t,N}},$$

where $Bias^2\left(\widetilde{C}_{t,N}\right) = \left(\widetilde{C}_{t,N} - E^Q\left[\widetilde{X}_{\tau^*} \mid \mathcal{F}_t\right]\right)^2$.

5.2 Monte Carlo implementation

5.2.1 Monte Carlo implementation of the market model

Simulation of the undisturbed exchange rate process. We simulate all paths of the undisturbed exchange rate process from expression (3) in page 23 and do not implement any variance reduction technique.

Computation of the trading strategy. We compute the trading strategy introduced by the market maker at $t \in \mathbb{T}$ from (11) in page 31.

Computation of the disturbed exchange rate processes. In order to obtain the disturbed exchange rate at each date $t \in \mathbb{T}$, we affect the undisturbed exchange rate by the market maker's trading strategy, as in (6) in page 26.

5.2.2 Implementation of the Longstaff-Schwartz pricing model

Path simulation. We generate all paths used to compute the Longstaff-Schwartz option prices from the risk-neutral version of the process (4) in page 23 through the technique of antithetic variates.

Choice of basis functions. As mentioned in section 4.4.4, the implementation of the Longstaff-Schwartz pricing algorithm requires the specification of the particular set of basis functions that will be used to obtain the first approximation to the continuation value of the option. Since the payoff of the option at time $s \in \mathbb{E}_t$ can be represented in all cases as a function of R_s, R_{s-1}, $M_{D,s-1}$ and κ, we consider as basis functions simple powers of the difference between the state variables R_{s-1} and $\kappa M_{D,s-1}$ that determine the value of the indicator function that represents the fulfilment of the exercise restriction presented in sub-section 4.4.1, so that the first M basis functions are given by

$$\gamma_{s-1}^{(1)} = 1$$
$$\gamma_{s-1}^{(2)} = (R_{s-1} - \kappa M_{D,s-1})$$
$$\vdots$$
$$\gamma_{s-1}^{(M)} = (R_{s-1} - \kappa M_{D,s-1})^{M-1}$$

We compute the first approximation to the continuation value of the option for each simulated path as

$$\widetilde{C}^{(1)}_{s-1,n} = \beta^{(1)}_{s-1} + (R_{s-1} - M_{D,s-1})\,\beta^{(2)}_{s-1} + ... + (R_{s-1} - M_{D,s-1})^{M-1}\,\beta^{(M)}_{s-1},$$

where the parameters' vector β_{s-1} is estimated from expression (15) in page 39, using, as in Longstaff and Schwartz [32], only in-the-money paths[5].

On the other hand, we obtain the second approximation to the continuation value for each trajectory from

$$\widetilde{C}^{(2)}_{s,n} = \sum_{i=t+1}^{T} \widetilde{X}_{i,n}\mathbf{1}_i,$$

where

$$\mathbf{1}_i = \begin{cases} 1 & \text{if } i = \min\left\{s \in \mathbb{E}_t : \widetilde{X}_{s,n} \geq \widetilde{C}^{(1)}_{s,n}\right\} \\ 0 & \text{otherwise} \end{cases}.$$

Least squares problem solution. We solve the least squares minimization problem applying the ordinary least squares (OLS) method, which is implemented using the *regress* function provided by Matlab®.

Computation of the Longstaff-Schwartz option price. As suggested by Glasserman [26], we obtain the continuation value of the option at time $t \in \mathbb{T}$ from the application of the exercise rule obtained through the Longstaff-Schwartz algorithm to a new set of paths of the exchange rate generated using once again

[5] Longstaff and Schwartz [32] support the use of only in the money paths by noting that this limits "the region over which the conditional expectation must be estimated, (so that) ... fewer basis functions are needed to obtain an accurate approximation to the conditional expectation function" (Longstaff and Schwartz, [32], p. 123).

the technique of antithetic variates. We then compute the variance of the price estimate from expression (19) in page 46.

6 Numerical results

This chapter presents the numerical results obtained from the implementation of the theoretical model introduced in chapter 4. In particular, section 6.1 presents numerical exercises carried out to verify the precision of the Longstaff-Schwartz algorithm when applied to the type of options used by the Colombian Central Bank - called hereafter the Banrep option contract - to intervene in the foreign exchange market. For this purpose, we build a standard binomial model and compare its results with those obtained from the Longstaff-Schwartz method. We carry out additional exercises to study the performance of the Fernández-Galán-Saavedra heuristic rule and the Fernández-Saavedra binomial model presented in section 4.4. On the other hand, section 6.3 presents a brief study of the behavior of the trading strategy proposed in section 4.3. In particular, we compare the results of the trading strategy that arise from the introduction of the Banrep option contracts with those obtained for the plain-vanilla American-style option contract we use as a benchmark. Finally, section 6.4 presents the results of the simulation exercises carried out to assess the theoretical short-term effects of the option contracts introduced by the Colombian Central Bank on the volatility of the exchange rate and on the exchange rate equilibrium level.

6.1 Performance of the Longstaff-Schwartz algorithm

As pointed out in section 4.4.4, although the payoff process option can be reformulated as a Markovian process, this significantly increases the dimension of the pricing problem. On the other hand, when we use basis functions that depend just on R_t and $M_{D,t}$ the convergence of the Longstaff-Schwartz algorithm is not guaranteed, which calls for numerical exercises that compare the results obtained from this method to those computed using models for which convergence is guaranteed. One of the simplest of these models is the well-known binomial model developed by Cox, Ross and Rubinstein [14], which converges to a log-normal diffusion as the number of time steps in which the time interval is divided rises to infinity. However, as pointed out by Hull [30], one problem with tree approaches is the difficulty encountered when they are applied to options whose payoff depends on the past history of the underlying asset. Indeed, in such cases the branches of the binomial tree do not recombine anymore - i.e. all state variables are never identical on two different nodes -, so that the resulting tree can have up to 2^{IT} different nodes at T. Consequently, the pricing of the option can become very time-consuming as the number of time-steps increases and can turn out to be unfeasible even for low values of I and T due to memory or processing time restrictions.

Fortunately, in the case of the Banrep option contract the memory requirements of the traditional binomial approach are significantly reduced by the fact that the moving average that affects the exercise restriction and the option's payoff are computed only at daily-spaced dates $t \in \mathbb{E}$. Indeed, in this case the number of final nodes of the tree is reduced to $(I+1)^T$ and it is no longer necessary to keep a record of the values of R_t and $M_{D,t}$ between two consecutive exercise dates. Furthermore, when the exercise restriction is satisfied at date $t = T-1$ the option becomes a standard European option with strike price equal to R_{T-1}, so that its value can be computed using the version of the standard Black-Scholes pricing model developed by Garman and Kohlhagen [25] to price currency options, which allows to eliminate all the nodes in the tree at time T. This modification to the original approach proposed by Broadie and Detemple [10] permits an additional reduction in the number of final nodes of the proposed binomial structure - from $(I+1)^T$ to $(I+1)^{T-1}$ -, and at the same time improves the convergence of the model by providing exact continuation values at $t = T - 1$[6]. Nevertheless these improvements to the binomial model, it is easy to note that its application to more realistic cases - such as the one in which we are interested - remains unfeasible. Indeed, in the case in which we consider a number of exercise dates T equal to 20, the partition of each time interval between two successive exercise dates in a number of time-steps as low as $I = 2$ results in a tree composed of roughly 1.162×10^9 different branches, which turns computationally unfeasible the pricing of the option. This, in addition to the fact that the pricing of the option using the binomial model in the simplest case in which $I = 1$ takes a mean processing time of 9.34 seconds - versus 2.18 seconds in the case of the Longstaff-Schwartz model -, makes unpractical - or even impossible - its application to the simulation exercises we carry on hereafter and leads us to implement the Longstaff-Schwartz methodology in order to assess the effects of the option contracts on the exchange rate process, making use of the binomial model only for benchmarking purposes.

Tables A.1.2. to A.1.4. present a comparison between the prices obtained using the Longstaff-Schwartz algorithm and those provided by the proposed binomial model for Banrep contracts with 5 exercise dates. Longstaff-Schwartz prices were computed from 20000 simulated paths of the exchange rate (10000 originally generated and 10000 antithetic) and using 2, 3, 4 and 5 basis functions to approximate the continuation value of the option. On the other hand, binomial prices were obtained from a tree in which each sub-interval $(t-1, t]$ between two successive exercise dates was divided in $I = 19$ time-steps[7]. In all cases the maturity of the option T, the local interest rate r, the foreign interest rate r^* and the initial

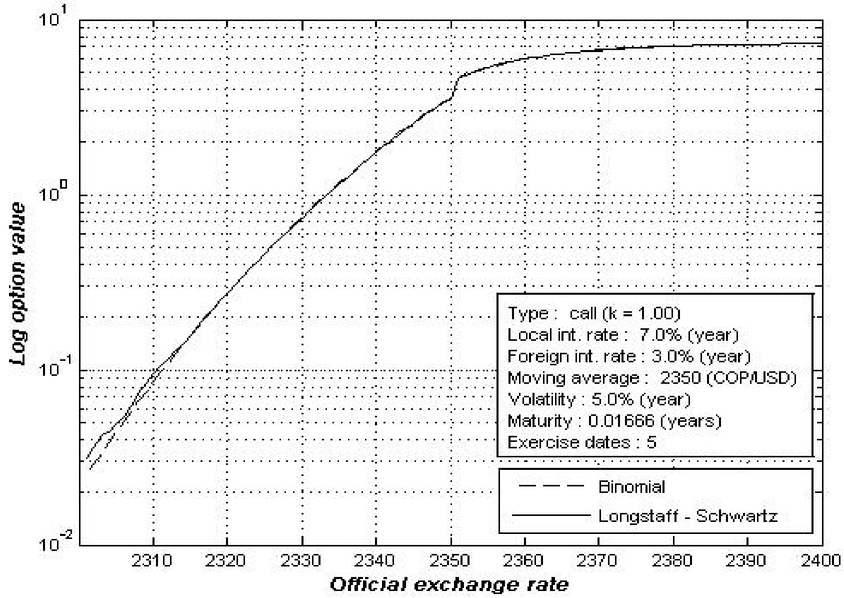
[6] In Appendix 2 we provide a justification for the use of the recursive procedure introduced in section 4.2 when the number of exercise dates differs from the number of time-steps into which the trading interval is divided. Likewise, Appendix 3 presents the implementation details of the binomial approach in the case of the Banrep option contract.

[7] The number of time-steps was chosen so that every value of the exchange rate at t is followed by 20 possible values of the same variable at $t+1$. This results in time steps equal to 0.00087 years - or likewise, 0.05263 trading days - and a tree composed of 130321 different branches.

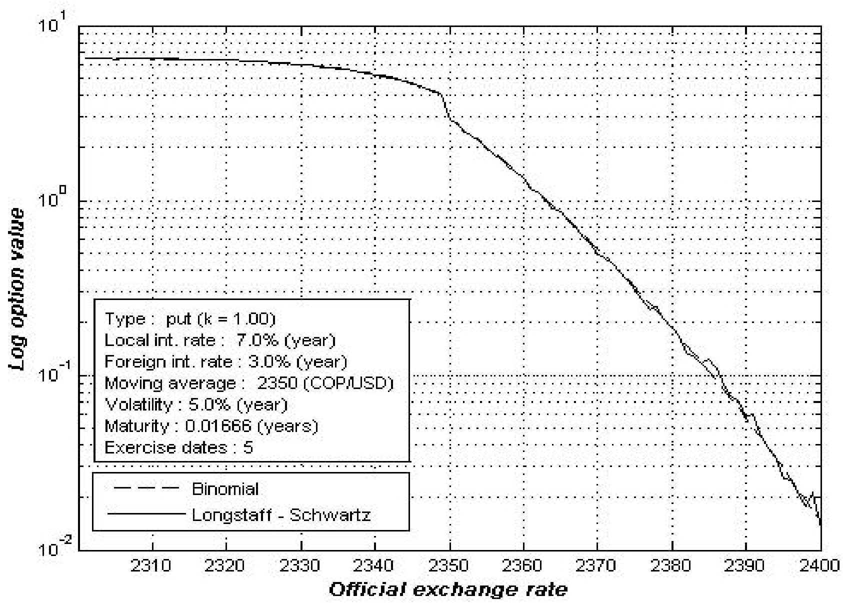
value of the $5-$day moving average of the official exchange rate were set to be equal to 0.01666 years, 0.07, 0.03 and 2350 COP/USD, respectively. Since the implementation of the Longstaff-Schwartz model provides biased estimates of the option values, in addition to average prices, we present average relative errors and relative root mean square errors (RMSE) for the call and put options for the management of foreign exchange reserves and the call and put options for the control of extreme exchange rate volatility. In the case of the call and put options for the management of foreign exchange reserves, all averages were computed from the results obtained for a set of exchange rates equal to $\{2301, 2302, ..., 2400\}$. On the other hand, average prices and errors for call and put options for the control of extreme exchange rate volatility were computed from results obtained for sets of exchange rates equal to $\{2401, 2402, ..., 2500\}$ and $\{2201, 2202, ..., 2300\}$, respectively.

As it can be noted, average prices obtained through the implementation of the Longstaff-Schwartz algorithm do not differ significantly from prices obtained from the proposed binomial approach. In general, the Longstaff-Schwartz method seems to perform better in cases of low exchange rate volatility. However, even though it seems to systematically loose its precision as the volatility of the exchange rate increases, we observe that the average relative RMSE never goes far beyond 1.70% in the case of the reserves' management options and 3.00% in the case of the volatility control options. Moreover, the absolute value of the average relative error - a proxy measure of the estimation bias - for the Longstaff-Schwartz estimated prices remains low in all cases, never exceeding 0.4% in the case of options for the management of foreign exchange reserves and 1.9% in the case of options for the control of extreme exchange rate volatility. Consequently, even though on average the Longstaff-Schwartz estimated prices are persistently downward biased, we infer from our results that the magnitude of the departure from the true option price tends to remain small. On the other hand, it appears that in this case there is no significant dependence in regard to the accuracy of the Longstaff-Schwartz algorithm on the number of basis functions used to obtain the first approximation to the continuation value and even for a number of basis functions as low as 2 it seems to provide acceptably precise option prices. Indeed, it is clear that for all of the sets of basis functions and for all of the values of volatility we have considered, the Longstaff-Schwartz method seems to behave well without significant improvements in the model's performance as the number of basis functions increases.

In order to evaluate the performance of the proposed pricing method for punctual values of the exchange rate, figures 6.1. (a) and (b) present the behavior of the continuation value function of the options for the management of foreign exchange reserves obtained using the Longstaff-Schwartz algorithm and the proposed binomial methodology. To observe clearly the differences between the results of both methods, we present in each case the logarithm of the continuation value of the option. As can be seen, for all of the initial exchange rate values considered the two prices turn out to be almost indistinguishable. Particularly, the Longstaff-Schwartz algorithm seems to provide accurate option prices for out-of-the-money, at-the-money and in-the-money options. We can explain the precision of the Longstaff-Schwartz algorithm by noticing that at any exercise date the chosen set of basis functions incorporates information about the past official rates observed in the last D exercise dates through the variable $M_{D,t}$, so that it is reasonable to expect our choice of basis functions to provide enough information to estimate with acceptable precision the first continuation value of the option required by the Longstaff-Schwartz algorithm. On the other hand, we observe that the continuation price of the option jumps when $R_t = M_{5,t}$. This can be explained by noting that when $R_t < M_{5,t}$ ($R_t > M_{5,t}$) the call (put) option cannot be exercised at $t+1$, so that the value of the option at that date is always equal to its continuation value. On the contrary, when R_t is above (below) the moving average the call (put) option becomes exercisable at $t + 1$, which increases the continuation value of the option at time t given that the price of the option at $t + 1$ becomes the maximum between the continuation value and the observed payoff.



(a)

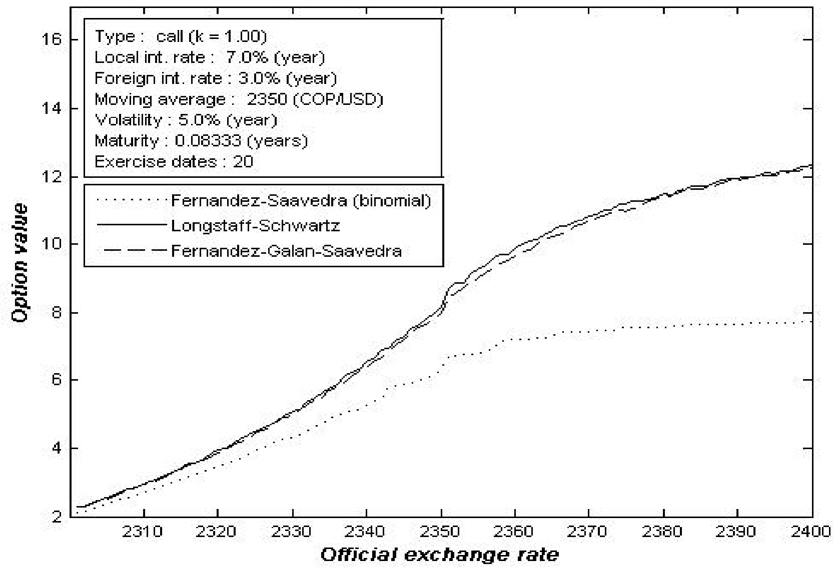


(b)

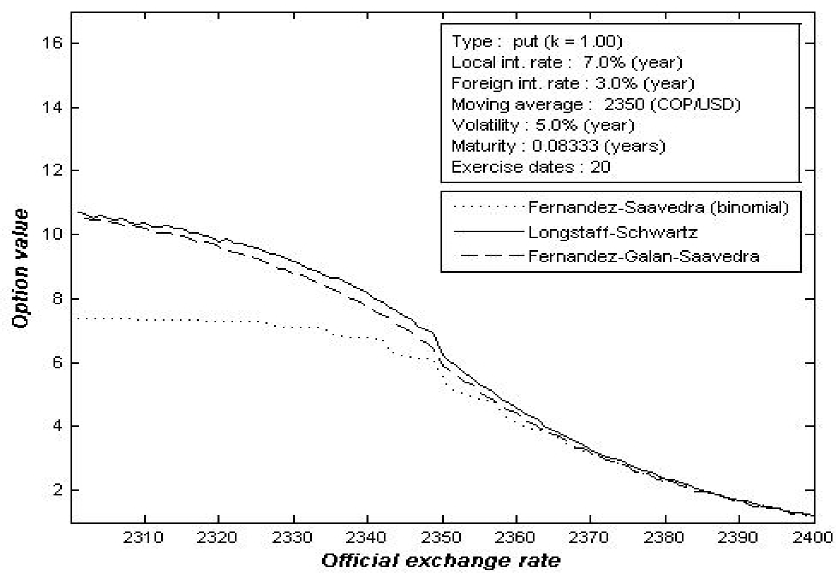
Figure 6.1. Continuation value of the option obtained using the binomial model and the Longstaff-Schwartz algorithm. (a) Call option for the management of foreign exchange reserves and, (b) Put option for the management of foreign exchange reserves.

6.2 Comparison between the performance of the Longstaff-Schwartz method and the Fernández-Saavedra and Fernández-Galán-Saavedra models

Figures 6.2. and 6.3. present a comparison between the call and put reserves' management option prices provided by the Longstaff-Schwartz method, and those obtained using the Fernández-Saavedra and the Fernández-Galán-Saavedra models introduced in sub-sections 4.4.2 and 4.4.3. As can be seen, our results suggest that the use of the Fernández-Saavedra binomial model leads to the underestimation of call and put prices for in-the-money, at-the-money and out-of-the-money options, which, in addition to its considerable computational burden - reflected in a mean processing time of 9.34 sec. -, constitutes an irremediable handicap that forbids its implementation. On the other hand, the option prices obtained using the Longstaff-Schwartz method and the Fernández-Galán-Saavedra heuristic exercise rule turn out to be - surprisingly - close, with the difference between both methods increasing slightly as the volatility of the exchange rate rises. However, despite the fact that the Fernández-Galán-Saavedra model requires on average only 0.66 seconds to obtain a price estimation - compared to 2.18 seconds in the case of the Longstaff-Schwartz algorithm -, one disadvantage of this heuristic exercise rule lies in the fact that it ignores the magnitude of the current difference between the official exchange rate and its moving average, merely taking account of its sign. Indeed, given that the continuation value of the call (put) option augments (diminishes) as the difference $R_{t_i} - \kappa M_{D,t_i}$ increases, then it is reasonable to expect the heuristic criteria to suggest the premature exercise of the claim, leading to the underestimation of the continuation value of the option. Furthermore, while the proposed heuristic stopping rule is applicable only to call and put options for the management of foreign exchange reserves, the Longstaff-Schwartz method can be easily adapted to the pricing of options for the control of extreme exchange rate volatility with good results. In the end, the usual trade-off between the flexibility and the computational efficiency of the pricing model appears again.

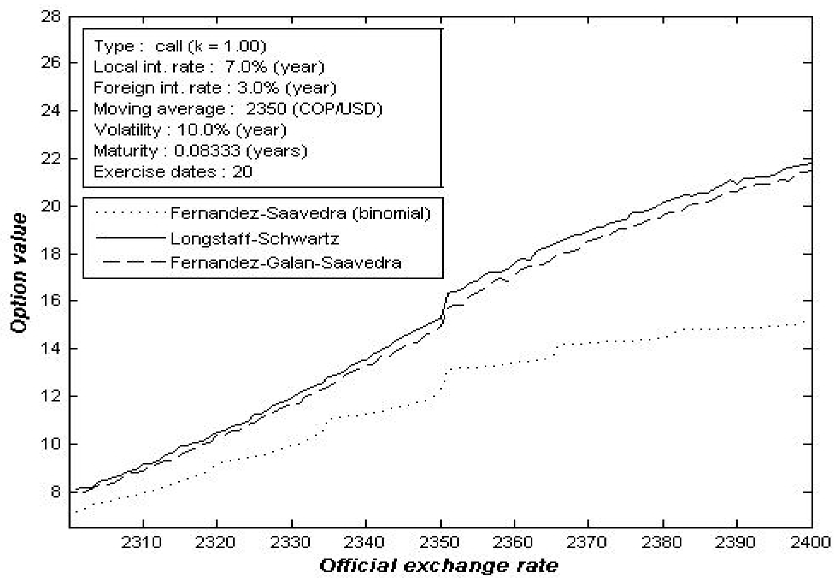


(a)

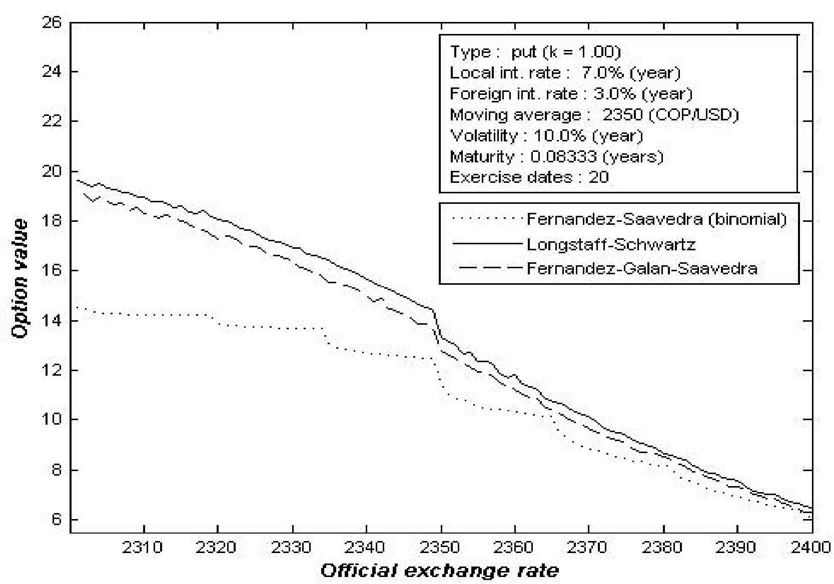


(b)

Figure 6.2. Continuation values of the option obtained using the Longstaff-Schwartz algorithm, the Fernández-Saavedra binomial model and the Fernández-Galán-Saavedra exercise rule. The volatility of the exchange rate is set to be equal to 0.05. (a) Call option for the management of foreign exchange reserves and, (b) Put option for the management of foreign exchange reserves.



(a)



(b)

Figure 6.3. Continuation values of the option obtained using the Longstaff-Schwartz algorithm, the Fernández-Saavedra binomial model and the Fernández-Galán-Saavedra exercise rule. The volatility of the exchange rate is set to be equal to 0.10. (a) Call option for the management of foreign exchange reserves and, (b) Put option for the management of foreign exchange reserves.

6.3 Behavior of the trading strategy

Figure 6.4. presents the behavior of the trading strategy computed from expression (11) for the call and put options for the management of foreign exchange reserves, as well as that corresponding to the plain-vanilla American-style option contract[8]. In all cases the annual local interest rate r, the annual foreign interest rate r^* and $M_{20,t-1}$ are set to be equal to 0.07, 0.03 and 2350 COP/USD, respectively.

As can be seen, while in the case of the plain-vanilla contract the trading strategy turns out to be close - as expected - to the delta of the option and varies smoothly as R_t changes, in the case of the Banrep contract it changes abruptly when R_t equals $M_{20,t}$. In particular, this jump we observe in the trading strategy can be explained by noting that, as mentioned in the preceding section, the continuation value of the option changes significantly when $R_t = M_{20,t}$. As a result, the trading strategy induced by the Banrep contract jumps exactly at the value of the barrier $M_{20,t}$. On the other hand, we can observe that the trading strategy in the case of the Banrep contract seems to remain stable for values of R_t different from $M_{20,t}$. However, another jump can be observed at the exercise frontier - i.e. the value of the exchange rate for which the payoff of the option becomes larger than its continuation value -. In this case, this abrupt change in the trading strategy can be intuitively explained by noting that even in the case in which the exercise restriction is satisfied in t, there is always a considerable probability that the option will not be exercised in $t+1$, so that the absolute magnitude of the trading strategy is always much less than one. Moreover, the almost linear behavior of the continuation value of the option we observe from figures 6.3. and 6.4. results

[8] Since option contracts used for the management of foreign exchange reserves are essentially equal to those utilized to control extreme exchange rate volatility, for the sake of exposition we focus our analysis only on the first.

in a situation in which changes in R_t induce modifications in it whose magnitudes do not increase as R_t approaches the exercise frontier. This, in addition to the fact that the slope of the continuation value function invariably turns out to be of small magnitude, causes a jump in the trading strategy to 1 in the case of the call option and -1 in the case of the put option when R_t reaches the exercise frontier. Finally, it is interesting to note that while the trading strategy of the plain-vanilla contract changes significantly as the remaining maturity of the option diminishes - especially for out-of-the-money options -, in the case of the Banrep contracts it remains stable with only slight changes in the location of the exercise frontier, so that as time goes on it slowly approaches $M_{20,t}$.

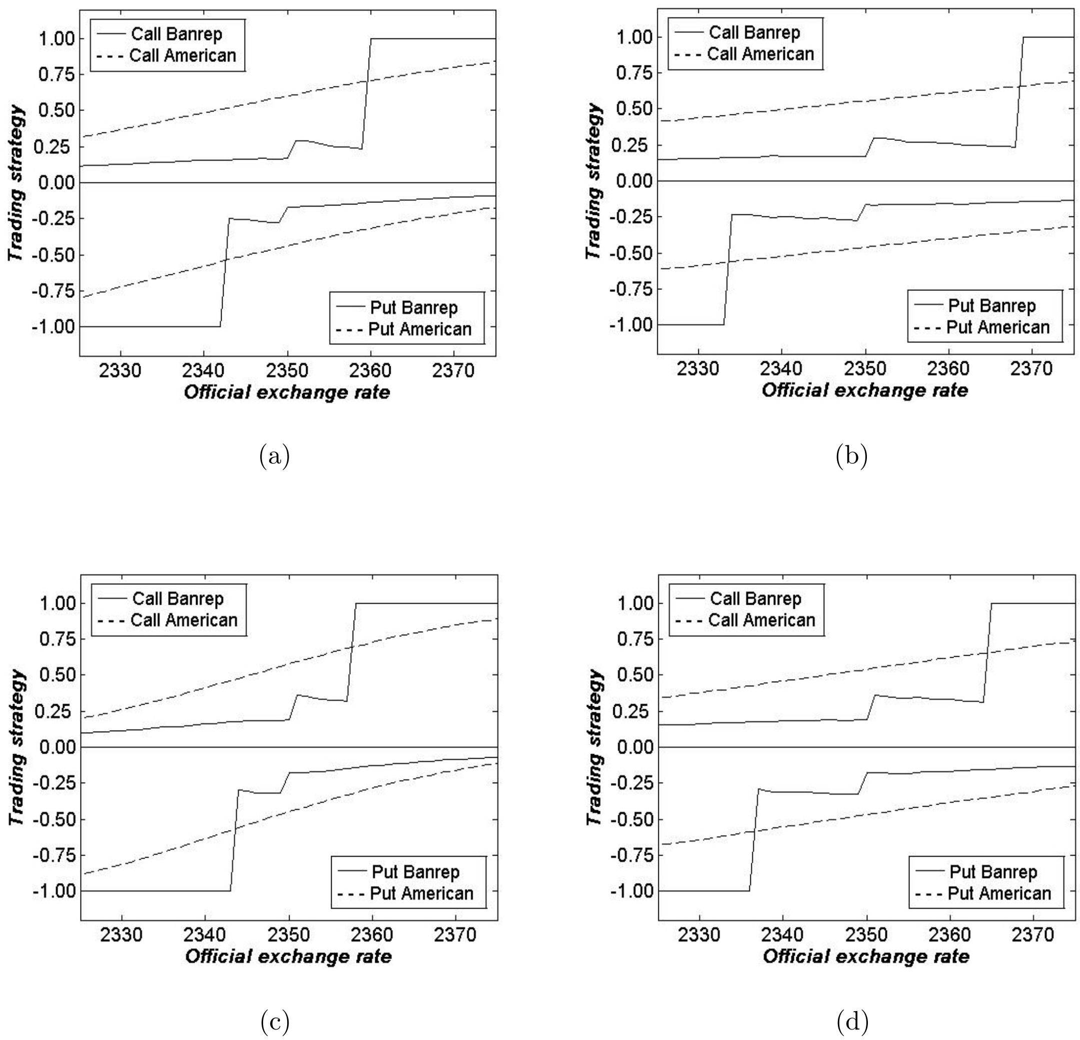


Figure 6.4. Behavior of the trading strategy for the call and put options for the management of foreign exchange reserves. (a) and (b) remaining maturity equal to 19 trading days and volatility equal to 0.05 and 0.10, respectively and, (c) and (d) remaining maturity equal to 10 trading days and volatility equal to 0.05 and 0.10, respectively.

6.4 Performance of the option contracts

This section presents the estimated effects of intervention with the currency option contracts introduced by the Colombian Central Bank on the standard deviation of the exchange rate's log-returns, the average exchange rate, and the final exchange rate. All of these results were obtained from simulations carried out for options for the management of foreign exchange reserves, as well as for options for the control of exchange rate volatility. In order to provide a benchmark, we made additional simulations to compute the effect on the same variables of call and put plain-vanilla American-style option contracts with a strike price equal to $\kappa M_{20,0}$. The market maker's expectations, as in expression (4), were affected by a factor α equal to 0.8 and 1.0 in order to analyze the consequences of changes in the agents' expectations on the effects of the intervention strategy with currency options. Specifically, a value of $\alpha = 0.8$ represents a situation in which the market maker anticipates the drift and the volatility of the exchange rate process to be reduced by 20% once the intervention strategy has been introduced. On the contrary, a value of $\alpha = 1.0$ represents the situation in which the market maker expects the exchange rate process to remain the same, as in the absence of central bank intervention. On the other hand, three values of the ratio θ/x equal to 10, 20 and 30 were also considered. Since x represents the change in the excess demand resulting from a marginal change in the exchange rate, we interpret the ratio θ/x as the modification in the equilibrium exchange rate that would follow a direct intervention in the foreign exchange market of size θ. In particular, this allowed us to focus our analysis on the effect of changes in the pair intervention size-excess demand sensitivity, rather than on the effect of individual changes to those parameters.

Each simulation exercise consisted in the generation of 100 paths of the undisturbed exchange rate and the subsequent computation of the disturbed exchange rate resulting from the introduction of the market maker's trading strategy. Such trading strategies for both the Banrep and plain-vanilla option contracts were obtained at each exercise date through the simulation of 100 subsequent values of the official exchange rate. While the plain-vanilla prices used to compute the corresponding trading strategy were obtained using a standard binomial tree in which the trading interval was divided into 100 time-steps, for computational purposes the Longstaff-Schwartz prices from which we computed the Banrep trading strategy were obtained from 100 paths (50 simulated and 50 antithetic) and using 2 basis functions to estimate the first approximation to the continuation value of the option. In all cases the number of paths was selected with the aim of providing acceptable accuracy in the estimation of the effects of the intervention strategy on the undisturbed exchange rate process.

Tables A.1.5. to A.1.10. present the estimated effects of the intervention strategy for all of the simulation exercises carried out. Tables A.1.5. and A.1.6. present the effect on the standard deviation of the log-returns of the disturbed exchange rate process. As predicted in the literature, in all cases the plain-vanilla and the Banrep option contracts were found to reduce the volatility of the exchange rate, with this effect being greater insofar as the ratio θ/x increases. Since the variance of the disturbed exchange rate is affected by the covariance between the undisturbed exchange rate and the trading strategy introduced by agents, which is always negative due to the fact that the market maker is long in the option, then it is reasonable to expect the introduction of such contingent claims to reduce the volatility of the exchange rate. Moreover, given that trading strategies introduced by agents have a larger effect on the equilibrium exchange rate as θ/x increases, then it is natural to observe more important effects on the exchange rate volatility for larger values of the ratio θ/x. However, we note that for all values of θ/x the effect of the plain-vanilla contract on the volatility of the disturbed exchange rate process is in general larger than that for the option contract introduced by the Colombian Central Bank. In particular, we find that while the plain-vanilla option allows an average relative reduction of the volatility of the exchange rate of roughly 21%, the introduction of the Banrep contracts permits an average reduction of approximately only 7.5%. Moreover, we observe that differences in the reduction of the exchange rate volatility provided by both types of claims are statistically different from zero with a confidence level of 5% in around 99% of the scenarios we considered, so that the contract specification does indeed seem to have an effect on the reduction of the volatility we can expect from the introduction of the option. This result is possibly due to the fact that plain-vanilla claims tend to be exercised later than Banrep contracts, so that the stabilizing effect tends to last longer in the first case than in the second. In fact, as can be noted from expression (6) in page 26, the variance of the exchange rate is affected only while $\eta_t \neq 0$, so that once the option is exercised its effects on the exchange rate process disappear. Furthermore, as can be seen from figure 6.5., the plain-vanilla contract induces a trading strategy that changes more uniformly than that induced by the Banrep contract. Given that the trading strategy induced by the second contract changes abruptly at $R_t = M_{D,t}$ and at the exercise frontier, then it is likely that the Banrep contract will induce some instability in the exchange rate process at those points, with a moderate effect on the exchange rate volatility during the days when the option remains in-the-money or out-of-the-money. We also observe that for small values of the volatility of the undisturbed exchange rate process both contracts induce ambiguous effects, even producing increments in the exchange rate volatility as the ratio θ/x increases. Interestingly, our results suggest that despite their original purpose the Banrep options used to control extreme volatility do not have a larger stabilizing effect on the exchange rate than that observed for the contracts introduced to manage the foreign exchange reserves.

Tables A.1.7. and A.1.8. present the effect of the plain-vanilla contracts and the Banrep contracts on the average exchange rate observed during the intervention period. In the same way, Tables A.1.9. and A.1.10. present the effect of both types of contract on the final exchange rate. In accordance with what has usually been suggested, call option contracts induce a reduction in the average and final exchange rate, while the introduction of put option contracts tend to produce an increase in these variables. In contrast to the effect on the exchange rate volatility, we cannot conclude that the impact of plain-vanilla contracts and Banrep contracts on the average and final exchange rates is significantly different. Indeed, the differences we observe in the impact of both types of contracts on such variables are statistically different from zero with a confidence level of 5% only in around 33% and 46% of the scenarios, respectively. On the other hand, our results again suggest that the option contracts used to control extreme exchange rate volatility do not seem to produce larger effects on the average and final exchange rates than those induced by option contracts for the management of foreign exchange reserves.

In order to analyze the impact of changes in the expectations' parameter α, the initial degree of moneyness of the options, or the volatility of the undisturbed process, we have summarized the results obtained in tables 6.1. to 6.3. according to several values of α, R_0 and σ. In particular, table 6.1. presents the effects for the two considered values of the expectations parameter α. As can be seen, the impact of a change in the expectations' parameter from 0.8 to 1.0 on the volatility of the disturbed exchange rate process turns out to be of a small magnitude in all cases. Moreover, its effect on the average exchange rate and on the final exchange rate is not statistically different from zero - with a confidence level of 5% - for all of the types of option contract we have considered.

	SD log-returns		Average exch. rate		Final exch. rate	
α	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep
			Call option reserves' management			
0.8	-0.01004	-0.00339	-11.17	-14.38	-10.47	-10.24
	(0.00068)	(0.00038)	(1.00)	(0.92)	(0.45)	(0.55)
1.0	-0.00975	-0.00339	-11.21	-13.47	-10.55	-9.59
	(0.00053)	(0.00036)	(1.01)	(0.97)	(0.43)	(0.57)
Difference	0.00029	0.00000	-0.04	0.92	-0.09	0.66
	(0.00061)	(0.00037)	(1.00)	(0.94)	(0.44)	(0.56)
			Call option volatility control			
0.8	-0.01228	-0.00521	-11.49	-10.04	-10.99	-8.57
	(0.00064)	(0.00049)	(1.05)	(1.02)	(0.50)	(0.77)
1.0	-0.01065	-0.00527	-11.19	-9.50	-10.69	-8.10
	(0.00052)	(0.00044)	(1.05)	(0.99)	(0.47)	(0.72)
Difference	0.00163	-0.00006	0.30	0.54	0.29	0.47
	(0.00058)	(0.00047)	(1.05)	(1.01)	(0.49)	(0.75)
			Put option reserves' management			
0.8	-0.00895	-0.00430	6.44	9.88	6.84	7.83
	(0.00064)	(0.00033)	(0.94)	(0.97)	(0.46)	(0.61)
1.0	-0.00932	-0.00440	5.84	9.41	6.85	7.41
	(0.00054)	(0.00033)	(0.91)	(0.99)	(0.41)	(0.60)
Difference	-0.00037	-0.00010	-0.60	-0.47	0.00	-0.42
	(0.00059)	(0.00033)	(0.93)	(0.98)	(0.44)	(0.60)
			Put option volatility control			
0.8	-0.01406	-0.00572	9.01	9.13	9.11	7.99
	(0.00077)	(0.00046)	(1.04)	(0.98)	(0.52)	(0.76)
1.0	-0.01274	-0.00617	8.21	8.23	8.82	7.16
	(0.00058)	(0.00046)	(1.05)	(1.00)	(0.48)	(0.75)
Difference	0.00132	-0.00044	-0.80	-0.91	-0.29	-0.83
	(0.00068)	(0.00046)	(1.05)	(0.99)	(0.50)	(0.76)

Table 6.1. Average effect of the plain-vanilla and Banrep option contracts on the standard deviation of the log-returns of the exchange rate process, the average exchange rate and the final exchange rate for different values of the expectations parameter α. Standard errors in parentheses.

Table 6.2. shows the effects of the introduction of the option contracts on the standard deviation of the log-returns and on the average and final disturbed exchange rates for two initial values of R_0. As can be seen, the impact of changes in the initial degree of moneyness of the options on the volatility of the disturbed exchange rate process is of a small magnitude. In this case this is due to the fact that the curvature of the continuation value function does not depend substantially on the initial degree of moneyness of the options, so that changes in the initial value of the exchange rate do not significantly affect the behavior of the trading strategy. On the contrary, it appears that changes in R_0 substantially alter the effect of the option contracts on the average and final exchange rates. We explain this result by noting that the effect of the option contracts on the average and final exchange rates depends on the magnitude of the trading strategy during the intervention period and on the probability of exercise of the option, respectively, which are larger for in-the-money options than for out-of-the-money options.

	SD log-returns		Average exch. rate		Final exch. rate	
R_0	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep
			Call option reserves' management			
2,330	-0.01047	-0.00402	-7.34	-6.92	-7.98	-11.22
	(0.00060)	(0.00038)	(0.45)	(0.51)	(1.04)	(1.07)
2,370	-0.00932	-0.00275	-13.68	-12.91	-14.39	-16.63
	(0.00061)	(0.00035)	(0.42)	(0.61)	(0.97)	(0.80)
Difference	0.00115	0.00127	-6.34	-5.99	-6.41	-5.42
	(0.00061)	(0.00037)	(0.44)	(0.56)	(1.00)	(0.94)
			Call option volatility control			
2,430	-0.01169	-0.00525	-9.26	-5.22	-9.58	-6.67
	(0.00057)	(0.00046)	(0.50)	(0.65)	(1.07)	(0.98)
2,470	-0.01124	-0.00523	-12.42	-11.44	-13.11	-12.88
	(0.00060)	(0.00047)	(0.47)	(0.84)	(1.03)	(1.03)
Difference	0.00044	0.00002	-3.17	-6.22	-3.53	-6.21
	(0.00058)	(0.00047)	(0.49)	(0.75)	(1.05)	(1.01)
			Put option reserves' management			
2,330	-0.01030	-0.00410	9.91	11.12	9.08	13.46
	(0.00063)	(0.00034)	(0.52)	(0.73)	(1.07)	(1.02)
2,370	-0.00798	-0.00460	3.78	4.12	3.21	5.83
	(0.00056)	(0.00032)	(0.34)	(0.45)	(0.76)	(0.94)
Difference	0.00232	-0.00050	-6.14	-7.00	-5.87	-7.63
	(0.00059)	(0.00033)	(0.44)	(0.60)	(0.93)	(0.98)
			Put option volatility control			
2,230	-0.01411	-0.00642	10.78	10.75	10.18	11.78
	(0.00069)	(0.00043)	(0.52)	(0.86)	(1.07)	(1.05)
2,270	-0.01270	-0.00547	7.15	4.40	7.04	5.58
	(0.00068)	(0.00049)	(0.48)	(0.63)	(1.02)	(0.92)
Difference	0.00141	0.00095	-3.62	-6.35	-3.13	-6.21
	(0.00068)	(0.00046)	(0.50)	(0.76)	(1.05)	(0.99)

Table 6.2. Average effect of the plain-vanilla and Banrep option contracts on the standard deviation of the log-returns of the exchange rate process, the average exchange rate and the final exchange rate for different values of the initial exchange rate. Standard errors in parentheses.

Table 6.3. summarizes the results obtained for the two different values of volatility of the undisturbed exchange rate process we considered in each case. Contrary to what was observed for changes in the expectations parameter α or in the initial exchange rate, the effects on the variability of the log-returns of the exchange

rate process seem to differ significantly depending on the value of the volatility of the undisturbed exchange rate process we consider. In particular, since the effect of the plain-vanilla and banrep options contracts on the volatility of the disturbed exchange rate is ambiguous for small values of the undisturbed volatility, we observe significant changes in this variable when σ goes from 0.05 to 0.10. However, for larger values of σ the effect on the volatility of the disturbed exchange rate process tends to disappear, so that the impact of a change in σ from 0.10 to 0.20 seems to be of much smaller magnitude, specially for plain-vanilla contracts. In general, it appears that changes in σ do indeed affect the stabilizing effect of the considered option contracts, although the impact of such changes tends to disappear as the volatility of the undisturbed exchange rate increases. Finally, our results suggest that the effect of changes in σ on the average and final exchange rates is of a small magnitude and often not statistically different from zero.

σ	SD log-returns Plain-Vanilla	SD log-returns Banrep	Average exch. rate Plain-Vanilla	Average exch. rate Banrep	Final exch. rate Plain-Vanilla	Final exch. rate Banrep
			Call option reserves' management			
0.05	-0.00778	-0.00173	-10.61	-9.83	-11.52	-14.52
	(0.00063)	(0.00040)	(0.40)	(0.50)	(0.96)	(0.90)
0.10	-0.01200	-0.00504	-10.41	-10.00	-10.86	-13.33
	(0.00058)	(0.00033)	(0.48)	(0.61)	(1.05)	(0.99)
Difference	-0.00423	-0.00331	0.20	-0.17	0.66	1.18
	(0.00061)	(0.00037)	(0.44)	(0.56)	(1.00)	(0.94)
			Call option volatility control			
0.10	-0.01145	-0.00434	-10.86	-7.35	-11.36	-8.33
	(0.00056)	(0.00043)	(0.48)	(0.74)	(1.05)	(0.98)
0.20	-0.01148	-0.00613	-10.82	-9.31	-11.33	-11.21
	(0.00061)	(0.00050)	(0.49)	(0.75)	(1.05)	(1.04)
Difference	-0.00003	-0.00179	0.05	-1.96	0.03	-2.88
	(0.00058)	(0.00047)	(0.49)	(0.75)	(1.05)	(1.01)
			Put option reserves' management			
0.05	-0.00651	-0.00304	6.17	6.81	5.28	8.34
	(0.00055)	(0.00031)	(0.42)	(0.58)	(0.86)	(0.93)
0.10	-0.01176	-0.00565	7.52	8.43	7.00	10.94
	(0.00063)	(0.00035)	(0.46)	(0.63)	(0.99)	(1.03)
Difference	-0.00525	-0.00261	1.36	1.62	1.72	2.60
	(0.00059)	(0.00033)	(0.44)	(0.60)	(0.93)	(0.98)
			Put option volatility control			
0.10	-0.01297	-0.00412	8.58	6.69	8.08	7.25
	(0.00069)	(0.00039)	(0.49)	(0.74)	(1.03)	(0.92)
0.20	-0.01383	-0.00777	9.35	8.46	9.14	10.11
	(0.00068)	(0.00052)	(0.51)	(0.77)	(1.06)	(1.05)
Difference	-0.00087	-0.00365	0.77	1.77	1.07	2.86
	(0.00068)	(0.00046)	(0.50)	(0.76)	(1.05)	(0.99)

Table 6.3. Average effect of the plain-vanilla and Banrep option contracts on the standard deviation of the log-returns of the exchange rate process, the average exchange rate and the final exchange rate for different values of volatility of the undisturbed exchange rate process. Standard errors in parentheses.

7 Conclusion

In this work we studied the stabilizing potential of the option contracts introduced in 1999 by the Colombian Central Bank to manage the foreign exchange reserves and to control the extreme exchange rate volatility. In order to assess the effect of these contracts, we developed a simple model of the foreign exchange market from which we estimated the effect of the trading strategies introduced by agents on the volatility and on the short-run equilibrium level of the exchange rate.

In accordance with the findings presented in the literature, our results indicate that the option contracts used by the Colombian Central Bank to intervene in the foreign exchange market indeed help to stabilize the exchange rate, although the magnitude of their effect on the exchange rate volatility is moderate in all of the situations we studied. Interestingly, we found that while their impact on the volatility of the exchange rate depends on the size of the intervention, on the parameters of the demand and supply of currencies, and on the volatility that prevailed before the introduction of the scheme, changes in the agents' expectations or in the initial degree of moneyness of the options do not seem to substantially alter their effect on such variable. On the contrary, the initial degree of moneyness seems to significantly affect the impact of all option contracts on the average and final exchange rates. In general, our results suggest that the performance of all options introduced by the Colombian Central Bank is similar in all of the scenarios we considered, so that contracts introduced to counteract the excessive volatility of the exchange rate do not appear to offer any real advantage over those put into place to manage the foreign exchange reserves. Moreover, the effect of all Banrep contracts seems to be important only in the case of the average and final exchange rates, so that it is not so clear why it would be advisable to use them instead of implementing a direct intervention scheme.

In each of the cases we studied the effect of the Banrep option contracts on the exchange rate volatility turned out to be of an inferior magnitude to that of the plain-vanilla contracts we used as a benchmark, so that, in accordance with what has been suggested by Mandeng [33] and Wiseman [48], the option contract specification indeed seems to have a significant impact on the stabilizing potential of the option used by the central bank to intervene in the foreign exchange market. On the other hand, the results of our exercises appear to indicate that option contracts help to reduce the exchange rate volatility while they remain unexercised, so that contracts with larger continuation values could have a greater stabilizing potential than those contracts for which this value is smaller. Specifically, insofar as the continuation value of the option contract diminishes, the intervention scheme with currency options becomes much more like a direct intervention strategy and consequently loses its advantages. Therefore, since the value of the option and the potential losses faced by the central bank are directly related, there seems to be a trade-off between the risks the monetary authority takes under a particular intervention scheme with currency options and its stabilizing potential.

Finally, although strictly outside the original goals of the work, we found that the Longstaff-Schwartz algorithm seems to provide accurate prices in the case of the non-Markovian options introduced by the Colombian Central Bank. The numerical exercises we carried out suggest that in this instance this pricing procedure performs well even for approximations of the continuation value made from few basis functions. This surprising result, in addition to those obtained by Bilger [4] for American-Asian options with rolling time-window, should deepen the interest in conducting further research concerning the potential of application of this algorithm to other non-Markovian contingent claims.

Appendix 1. Tables

Auction date	Approved amount	Minimum bid	Maximum bid	Final price	Exercised amount
Put option reserves' management					
Nov 1999	200.0	0.00	8.00	4.00	200.0
Dec 1999	80.0	0.21	8.10	4.50	0.0
Jan 2000	80.0	0.01	3.02	0.21	12.0
Feb 2000	80.0	0.13	4.10	1.60	0.0
Mar 2000	100.0	0.10	4.80	3.00	74.0
Apr 2000	100.0	0.23	5.00	0.60	0.0
May 2000	100.0	0.05	4.50	2.25	0.0
Jun 2000	100.0	0.05	5.10	3.00	15.5
Jul 2000	100.0	0.20	6.10	3.65	0.0
Aug 2000	99.9	0.22	6.96	3.50	17.1
Sep 2000	100.0	0.22	7.00	4.51	100.0
Oct 2000	100.0	0.01	7.54	6.35	100.0
Nov 2000	100.0	0.21	5.16	3.56	0.0
Dec 2000	100.0	0.32	4.11	1.86	80.0
Jan 2001	75.0	0.21	7.10	5.56	69.3
Feb 2001	50.0	0.50	7.57	3.80	0.0
Mar 2001	50.0	0.31	5.57	3.00	0.0
Apr 2001	30.0	1.29	5.56	3.51	30.0
May 2001	30.0	3.96	10.00	9.55	30.0
Jun 2001	30.0	1.50	11.56	10.20	30.0
Jul 2001	30.0	1.00	12.50	6.00	30.0
Aug 2001	80.0	0.10	10.57	7.18	0.0
Sep 2001	100.0	0.10	11.00	5.78	100.0
Oct 2001	140.0	0.10	8.21	5.21	140.0
Nov 2001	119.9	0.21	10.20	6.01	119.9
Dec 2001	50.0	0.00	10.51	8.00	50.0
Jan 2002	49.9	3.60	10.57	10.00	1.5
Feb 2002	50.0	3.00	9.01	8.56	50.0
Mar 2002	100.0	4.00	12.00	8.30	100.0
Apr 2002	100.0	1.50	8.22	3.57	0.0
May 2002	100.0	2.01	9.22	6.01	0.0
Jun 2002	100.0	2.01	6.80	3.51	0.0
Jul 2002	50.0	0.51	2.02	0.65	0.0
Aug 2002	50.0	1.00	5.50	4.01	0.0
Sep 2002	50.0	1.51	6.00	2.59	50.0
Dec 2002	50.0	0.90	12.00	4.00	0.0
Jul 2003	49.8	0.01	13.00	5.00	6.2
Dec 2003 - 1st	100.0	1.00	8.00	5.68	100.0
Dec 2003 - 2nd	200.0	2.00	7.10	5.56	200.0
Jan 2004	200.0	1.05	7.10	4.26	200.0
Mar 2004	200.0	1.73	6.20	5.05	200.0
Apr 2004	250.0	1.50	6.23	4.16	0.0
May 2004	200.0	2.00	7.60	5.10	200.0
Jun 2004	199.9	2.50	8.25	6.30	199.9
Jul 2004	199.8	2.70	8.33	7.00	199.8
Aug 2004	200.0	3.20	10.20	7.23	200.0
Total	4724.2				2905.2

Table A.1.1. Results of the auctions of currency options carried out by the Colombian Central Bank.

Auction date	Approved amount	Minimum bid	Maximum bid	Final price	Exercised amount
Call option reserves' management					
Feb 2003	200.0	0.00	10.25	6.20	144.6
Mar 2003	200.0	1.00	6.50	4.90	0.0
Apr 2003	199.9	0.01	4.20	2.10	199.9
Total	599.9				344.5
Call option volatility control					
Jul 2002	180.0	1.80	10.01	3.80	180.0
Aug 2002	180.0	1.00	8.01	4.22	109.5
Oct 2002	180.0	1.80	9.01	5.16	124.5
Total	540.0				414.4
Put option volatility control					
Dec 2004	179.9	1.00	10.11	4.00	179.9
Total	179.9				179.9

Table A.1.1. (Cont.) Results of the auctions of currency options carried out by the Colombian Central Bank.

Number of basis functions	Volatility					
	0.05	0.10	0.15	0.20	0.25	0.30
Call option reserves' management ($\kappa = 1.00$)						
Binomial	3.76548	7.44474	11.13305	14.82854	18.52311	22.21273
2	3.76141	7.43358	11.13489	14.8137	18.5191	22.22178
3	3.75573	7.43012	11.11244	14.81666	18.49649	22.21153
4	3.75799	7.42722	11.10509	14.82469	18.51536	22.2154
5	3.75395	7.43115	11.11601	14.80349	18.5188	22.22431
Put option reserves' management ($\kappa = 1.00$)						
Binomial	3.22813	6.72270	10.30473	13.91196	17.53215	21.16283
2	3.21929	6.70428	10.27897	13.91052	17.51864	21.14706
3	3.2163	6.71164	10.28521	13.89994	17.50895	21.1237
4	3.21535	6.71202	10.28147	13.91128	17.52941	21.14057
5	3.21908	6.70828	10.28322	13.89727	17.51145	21.12637
Call option volatility control ($\kappa = 1.04$)						
Binomial	2.18111	4.12909	6.20737	8.71919	11.59801	14.86968
2	2.1505	4.07827	6.14892	8.63213	11.45316	14.62134
3	2.15232	4.06897	6.13062	8.60483	11.47377	14.6126
4	2.15296	4.06603	6.12728	8.59474	11.47981	14.66039
5	2.15708	4.07015	6.12313	8.59926	11.46121	14.67042
Put option volatility control ($\kappa = 1.04^{-1}$)						
Binomial	1.96162	3.91009	5.94085	8.34786	11.00912	14.04884
2	1.93864	3.85861	5.89014	8.2463	10.88436	13.79518
3	1.93888	3.84806	5.87152	8.21862	10.8845	13.79539
4	1.93571	3.83965	5.86557	8.21321	10.88825	13.82018
5	1.94122	3.84695	5.87171	8.21185	10.89086	13.81866

Table A.1.2. Average option values obtained using the Longstaff-Schwartz algorithm.

Number of basis functions	Volatility					
	0.05	0.10	0.15	0.20	0.25	0.30
	Call option reserves' management ($\kappa = 1.00$)					
2	0.108%	0.15%	-0.016%	0.1%	0.022%	-0.041%
3	0.26%	0.197%	0.185%	0.08%	0.144%	0.005%
4	0.199%	0.236%	0.252%	0.026%	0.042%	-0.012%
5	0.307%	0.183%	0.153%	0.169%	0.023%	-0.052%
	Put option reserves' management ($\kappa = 1.00$)					
2	0.274%	0.275%	0.251%	0.01%	0.077%	0.075%
3	0.368%	0.165%	0.19%	0.086%	0.132%	0.185%
4	0.398%	0.159%	0.226%	0.005%	0.016%	0.105%
5	0.281%	0.215%	0.209%	0.106%	0.118%	0.173%
	Call option volatility control ($\kappa = 1.04$)					
2	1.424%	1.246%	0.951%	1.009%	1.265%	1.698%
3	1.338%	1.478%	1.252%	1.329%	1.083%	1.759%
4	1.308%	1.551%	1.307%	1.448%	1.03%	1.428%
5	1.114%	1.448%	1.376%	1.395%	1.194%	1.358%
	Put option volatility control ($\kappa = 1.04^{-1}$)					
2	1.185%	1.334%	0.861%	1.232%	1.146%	1.839%
3	1.173%	1.612%	1.181%	1.573%	1.145%	1.837%
4	1.338%	1.835%	1.283%	1.64%	1.11%	1.655%
5	1.051%	1.641%	1.178%	1.656%	1.086%	1.666%

Table A.1.3. Average relative error of the option values obtained using the Longstaff-Schwartz algorithm.

Number of basis functions	Volatility					
	0.05	0.10	0.15	0.20	0.25	0.30
Call option reserves' management ($\kappa = 1.00$)						
2	0.832%	1.111%	1.25%	1.374%	1.469%	1.463%
3	0.986%	1.228%	1.278%	1.432%	1.49%	1.549%
4	0.926%	1.143%	1.317%	1.351%	1.47%	1.503%
5	0.991%	1.201%	1.283%	1.453%	1.541%	1.465%
Put option reserves' management ($\kappa = 1.00$)						
2	1.045%	1.224%	1.353%	1.51%	1.537%	1.607%
3	1.057%	1.184%	1.427%	1.484%	1.499%	1.603%
4	1.081%	1.286%	1.372%	1.537%	1.448%	1.621%
5	1.052%	1.218%	1.332%	1.481%	1.548%	1.708%
Call option volatility control ($\kappa = 1.04$)						
2	2.044%	2.427%	2.927%	2.94%	2.997%	2.955%
3	1.94%	2.518%	2.84%	2.96%	2.964%	2.975%
4	1.939%	2.566%	2.942%	3.005%	2.951%	3.007%
5	1.816%	2.454%	2.94%	3.094%	2.953%	2.883%
Put option volatility control ($\kappa = 1.04^{-1}$)						
2	1.893%	2.35%	2.561%	2.869%	2.783%	2.904%
3	1.888%	2.508%	2.529%	3.075%	2.72%	2.868%
4	1.98%	2.687%	2.581%	2.93%	2.709%	2.748%
5	1.8%	2.511%	2.707%	3.014%	2.704%	2.878%

Table A.1.4. Average relative RMSE of the option values obtained using the Longstaff-Schwartz algorithm.

R_0	σ	$\theta/x = 10$ Plain-Vanilla	$\theta/x = 10$ Banrep	$\theta/x = 20$ Plain-Vanilla	$\theta/x = 20$ Banrep	$\theta/x = 30$ Plain-Vanilla	$\theta/x = 30$ Banrep
				Call option reserves' management ($\alpha = 0.80$)			
2330	0.05	-0.00627	-0.00291	-0.01133	-0.00334	-0.00711	-0.00132
		(0.00032)	(0.00016)	(0.00051)	(0.00035)	(0.0011)	(0.00066)
2330	0.10	-0.00686	-0.00316	-0.01338	-0.00588	-0.0191	-0.00763
		(0.00033)	(0.00018)	(0.00059)	(0.00035)	(0.00082)	(0.0005)
2370	0.05	-0.00453	-0.00227	-0.00843	-0.00166	-0.00634	0.00081
		(0.00035)	(0.00014)	(0.00051)	(0.00032)	(0.00107)	(0.00046)
2370	0.10	-0.00613	-0.00284	-0.01233	-0.00502	-0.01864	-0.00545
		(0.00031)	(0.00014)	(0.00064)	(0.00034)	(0.00087)	(0.00048)
				Call option reserves' management ($\alpha = 1.00$)			
2330	0.05	-0.00581	-0.00255	-0.01093	-0.00373	-0.01035	-0.00156
		(0.00026)	(0.00015)	(0.00045)	(0.00033)	(0.00077)	(0.00061)
2330	0.10	-0.00567	-0.00306	-0.01164	-0.00644	-0.01714	-0.00669
		(0.00026)	(0.00013)	(0.00047)	(0.00031)	(0.00072)	(0.00041)
2370	0.05	-0.00392	-0.00247	-0.00812	-0.00165	-0.01021	0.00184
		(0.00027)	(0.00016)	(0.0005)	(0.00031)	(0.00071)	(0.00059)
2370	0.10	-0.0052	-0.00311	-0.01055	-0.00464	-0.01741	-0.00659
		(0.00027)	(0.00014)	(0.00051)	(0.00027)	(0.00071)	(0.00044)
				Call option volatility control ($\alpha = 0.80$)			
2430	0.10	-0.00644	-0.00193	-0.01268	-0.00458	-0.01892	-0.00557
		(0.0003)	(0.00021)	(0.0006)	(0.00049)	(0.00077)	(0.00053)
2430	0.20	-0.00589	-0.00317	-0.01193	-0.00687	-0.0187	-0.00961
		(0.00031)	(0.00022)	(0.00058)	(0.00054)	(0.00089)	(0.00071)
2470	0.10	-0.00521	-0.00257	-0.01198	-0.00411	-0.01807	-0.007
		(0.00031)	(0.0002)	(0.00062)	(0.00036)	(0.00083)	(0.00072)
2470	0.20	-0.00663	-0.00287	-0.01099	-0.00523	-0.01989	-0.00895
		(0.00032)	(0.00023)	(0.00059)	(0.00041)	(0.00102)	(0.00074)
				Call option volatility control ($\alpha = 1.00$)			
2430	0.10	-0.00543	-0.00273	-0.01109	-0.00477	-0.01685	-0.00484
		(0.00026)	(0.00027)	(0.00043)	(0.00045)	(0.00069)	(0.00042)
2430	0.20	-0.00548	-0.00341	-0.01084	-0.00633	-0.01597	-0.00913
		(0.00027)	(0.0002)	(0.00046)	(0.00041)	(0.00078)	(0.00069)
2470	0.10	-0.00493	-0.00269	-0.0099	-0.00493	-0.0159	-0.00639
		(0.00027)	(0.00021)	(0.00054)	(0.00047)	(0.0007)	(0.00052)
2470	0.20	-0.00509	-0.00338	-0.01071	-0.00621	-0.01563	-0.00841
		(0.00024)	(0.00019)	(0.00054)	(0.00044)	(0.00066)	(0.00068)

Table A.1.5. Estimated change in the standard deviation of the log-returns of the exchange rate process in the case of call options. Standard errors in parentheses.

R_0	σ	$\theta/x=10$ Plain-Vanilla	$\theta/x=10$ Banrep	$\theta/x=20$ Plain-Vanilla	$\theta/x=20$ Banrep	$\theta/x=30$ Plain-Vanilla	$\theta/x=30$ Banrep
		Put option reserves' management ($\alpha=0.80$)					
2330	0.05	-0.00573	-0.00257	-0.01096	-0.00322	-0.00166	-0.00143
		(0.00036)	(0.00015)	(0.00063)	(0.00031)	(0.00101)	(0.00049)
2330	0.10	-0.00601	-0.00338	-0.01383	-0.0061	-0.02103	-0.00757
		(0.00033)	(0.00015)	(0.00064)	(0.00033)	(0.00094)	(0.00049)
2370	0.05	-0.00368	-0.00238	-0.00661	-0.00388	-0.00578	-0.00395
		(0.00034)	(0.00017)	(0.00056)	(0.0003)	(0.00049)	(0.00036)
2370	0.10	-0.00571	-0.00339	-0.00995	-0.00565	-0.01648	-0.00804
		(0.00033)	(0.00018)	(0.00064)	(0.00031)	(0.00089)	(0.00047)
		Put option reserves' management ($\alpha=1.00$)					
2330	0.05	-0.00599	-0.00237	-0.01164	-0.00387	-0.00933	-0.00177
		(0.0003)	(0.00015)	(0.00045)	(0.00028)	(0.00076)	(0.00048)
2330	0.10	-0.00645	-0.00365	-0.01227	-0.0062	-0.01866	-0.00702
		(0.00029)	(0.00016)	(0.00049)	(0.00035)	(0.00078)	(0.00041)
2370	0.05	-0.00357	-0.00229	-0.00568	-0.00394	-0.0075	-0.00481
		(0.00029)	(0.00016)	(0.00042)	(0.00024)	(0.00057)	(0.00036)
2370	0.10	-0.00497	-0.00323	-0.01111	-0.00572	-0.01467	-0.00787
		(0.0003)	(0.00019)	(0.00061)	(0.00031)	(0.00084)	(0.00053)
		Put option volatility control ($\alpha=0.80$)					
2230	0.10	-0.00655	-0.00281	-0.01565	-0.00493	-0.0219	-0.0064
		(0.00037)	(0.00019)	(0.00075)	(0.00038)	(0.00107)	(0.0006)
2230	0.20	-0.00715	-0.00446	-0.01522	-0.00785	-0.02283	-0.01175
		(0.00033)	(0.00023)	(0.00071)	(0.0004)	(0.0011)	(0.00074)
2270	0.10	-0.00664	-0.00177	-0.01251	-0.00346	-0.01787	-0.00346
		(0.00037)	(0.00022)	(0.00066)	(0.0004)	(0.00108)	(0.00039)
2270	0.20	-0.00718	-0.0041	-0.01451	-0.00749	-0.02069	-0.0102
		(0.00035)	(0.00029)	(0.00074)	(0.00054)	(0.00102)	(0.00072)
		Put option volatility control ($\alpha=1.00$)					
2230	0.10	-0.00686	-0.00278	-0.01333	-0.00525	-0.02063	-0.00748
		(0.00029)	(0.00015)	(0.00058)	(0.00036)	(0.00079)	(0.00048)
2230	0.20	-0.00636	-0.00394	-0.0132	-0.00834	-0.01959	-0.01108
		(0.00029)	(0.00022)	(0.00053)	(0.00041)	(0.00076)	(0.00058)
2270	0.10	-0.00586	-0.00202	-0.01098	-0.00379	-0.01682	-0.00532
		(0.00028)	(0.00022)	(0.00059)	(0.00043)	(0.00079)	(0.00058)
2270	0.20	-0.00648	-0.00385	-0.01297	-0.00807	-0.01983	-0.01209
		(0.00026)	(0.00022)	(0.00054)	(0.00057)	(0.0008)	(0.0008)

Table A.1.6. Estimated change in the standard deviation of the log-returns of the exchange rate process in the case of put options. Standard errors in parentheses.

		$\theta/x = 10$		$\theta/x = 20$		$\theta/x = 30$	
R_0	σ	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep
		Call option reserves' management ($\alpha = 0.80$)					
2330	0.05	-3.49	-3.40	-6.54	-6.66	-8.46	-9.05
		(0.23)	(0.23)	(0.41)	(0.42)	(0.55)	(0.66)
2330	0.10	-4.43	-4.18	-8.42	-7.64	-10.92	-10.74
		(0.24)	(0.28)	(0.49)	(0.53)	(0.69)	(0.80)
2370	0.05	-8.24	-7.13	-15.84	-14.56	-21.27	-21.33
		(0.15)	(0.23)	(0.32)	(0.45)	(0.49)	(0.70)
2370	0.10	-6.67	-6.52	-12.65	-12.48	-18.67	-19.24
		(0.22)	(0.30)	(0.47)	(0.61)	(0.67)	(0.90)
		Call option reserves' management ($\alpha = 1.00$)					
2330	0.05	-4.14	-3.44	-7.38	-7.09	-9.58	-8.52
		(0.21)	(0.22)	(0.41)	(0.46)	(0.53)	(0.51)
2330	0.10	-4.43	-4.04	-8.34	-7.63	-11.95	-10.67
		(0.23)	(0.27)	(0.45)	(0.54)	(0.64)	(0.75)
2370	0.05	-7.81	-5.92	-15.31	-13.87	-19.25	-16.97
		(0.18)	(0.26)	(0.32)	(0.50)	(0.60)	(0.90)
2370	0.10	-6.86	-6.70	-12.85	-11.64	-18.73	-18.53
		(0.18)	(0.25)	(0.44)	(0.61)	(0.58)	(0.92)
		Call option volatility control ($\alpha = 0.80$)					
2430	0.10	-4.51	-1.58	-9.52	-4.10	-13.30	-5.63
		(0.24)	(0.27)	(0.48)	(0.61)	(0.72)	(0.85)
2430	0.20	-5.15	-4.04	-9.41	-6.83	-15.29	-11.22
		(0.25)	(0.34)	(0.50)	(0.67)	(0.69)	(1.02)
2470	0.10	-7.00	-6.48	-13.19	-11.97	-18.59	-14.87
		(0.22)	(0.39)	(0.41)	(0.84)	(0.67)	(1.24)
2470	0.20	-5.52	-5.01	-12.07	-12.50	-18.28	-18.55
		(0.23)	(0.39)	(0.50)	(0.76)	(0.68)	(1.09)
		Call option volatility control ($\alpha = 1.00$)					
2430	0.10	-5.27	-2.61	-9.07	-3.23	-11.68	-3.70
		(0.24)	(0.33)	(0.43)	(0.51)	(0.64)	(0.63)
2430	0.20	-4.86	-3.39	-8.74	-5.66	-14.28	-10.69
		(0.23)	(0.32)	(0.44)	(0.61)	(0.70)	(0.99)
2470	0.10	-6.76	-5.68	-13.44	-12.64	-18.04	-15.73
		(0.21)	(0.41)	(0.40)	(0.77)	(0.65)	(1.22)
2470	0.20	-6.33	-5.86	-11.90	-10.60	-17.96	-17.38
		(0.21)	(0.35)	(0.44)	(0.74)	(0.62)	(1.08)

Table A.1.7. Estimated change in the average exchange rate in the case of call options. Standard errors in parentheses.

R_0	σ	$\theta/x = 10$ Plain-Vanilla	$\theta/x = 10$ Banrep	$\theta/x = 20$ Plain-Vanilla	$\theta/x = 20$ Banrep	$\theta/x = 30$ Plain-Vanilla	$\theta/x = 30$ Banrep
		Put option reserves' management ($\alpha = 0.80$)					
2330	0.05	6.06	5.65	11.10	12.41	13.73	16.13
		(0.30)	(0.33)	(0.54)	(0.67)	(0.76)	(1.05)
2330	0.10	5.40	5.92	10.25	12.85	13.80	16.83
		(0.28)	(0.34)	(0.51)	(0.67)	(0.71)	(1.01)
2370	0.05	1.48	1.69	2.40	3.12	2.79	3.28
		(0.17)	(0.20)	(0.27)	(0.40)	(0.37)	(0.44)
2370	0.10	2.67	2.76	4.74	4.86	7.69	8.48
		(0.21)	(0.25)	(0.42)	(0.48)	(0.56)	(0.73)
		Put option reserves' management ($\alpha = 1.00$)					
2330	0.05	5.42	5.06	9.64	10.34	13.33	14.89
		(0.25)	(0.33)	(0.43)	(0.67)	(0.66)	(0.99)
2330	0.10	5.33	5.67	10.31	11.77	14.57	15.95
		(0.23)	(0.33)	(0.45)	(0.67)	(0.70)	(0.99)
2370	0.05	1.43	1.51	2.51	2.75	4.09	4.92
		(0.13)	(0.16)	(0.20)	(0.29)	(0.37)	(0.54)
2370	0.10	2.74	2.83	6.08	6.18	6.69	7.08
		(0.20)	(0.26)	(0.41)	(0.51)	(0.44)	(0.68)
		Put option volatility control ($\alpha = 0.80$)					
2230	0.10	6.02	5.98	11.31	12.28	16.73	16.33
		(0.26)	(0.41)	(0.49)	(0.82)	(0.74)	(1.25)
2230	0.20	5.86	5.91	10.72	11.32	16.86	18.33
		(0.25)	(0.38)	(0.51)	(0.80)	(0.74)	(1.14)
2270	0.10	3.22	1.04	6.83	2.99	8.24	3.33
		(0.22)	(0.22)	(0.47)	(0.56)	(0.65)	(0.74)
2270	0.20	4.21	3.47	8.00	6.03	11.33	8.88
		(0.24)	(0.34)	(0.49)	(0.67)	(0.72)	(1.01)
		Put option volatility control ($\alpha = 1.00$)					
2230	0.10	5.51	4.79	11.19	11.32	15.08	14.57
		(0.22)	(0.42)	(0.48)	(0.83)	(0.68)	(1.27)
2230	0.20	5.47	5.38	10.21	9.73	14.37	13.07
		(0.25)	(0.38)	(0.46)	(0.74)	(0.69)	(1.14)
2270	0.10	3.47	1.29	5.95	2.00	9.43	4.34
		(0.22)	(0.25)	(0.39)	(0.40)	(0.63)	(0.79)
2270	0.20	4.11	3.39	8.57	6.68	12.49	9.36
		(0.22)	(0.34)	(0.47)	(0.65)	(0.66)	(0.95)

Table A.1.8. Estimated change in the average exchange rate in the case of put options. Standard errors in parentheses.

		$\theta/x=10$		$\theta/x=20$		$\theta/x=30$	
R_0	σ	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep
				Call option reserves' management ($\alpha = 0.80$)			
2330	0.05	-4.20	-6.10	-8.80	-13.20	-7.80	-16.20
		(0.49)	(0.49)	(0.99)	(0.95)	(1.32)	(1.50)
2330	0.10	-4.70	-6.00	-10.20	-12.00	-11.70	-15.90
		(0.50)	(0.49)	(1.00)	(0.98)	(1.47)	(1.50)
2370	0.05	-8.80	-9.30	-16.80	-18.60	-22.80	-27.90
		(0.32)	(0.25)	(0.73)	(0.51)	(1.28)	(0.76)
2370	0.10	-6.10	-7.90	-13.20	-15.80	-18.90	-23.70
		(0.49)	(0.40)	(0.95)	(0.81)	(1.45)	(1.22)
				Call option reserves' management ($\alpha = 1.00$)			
2330	0.05	-4.70	-5.60	-8.60	-11.80	-9.90	-15.60
		(0.50)	(0.49)	(0.99)	(0.98)	(1.41)	(1.50)
2330	0.10	-4.70	-6.00	-8.80	-11.20	-11.70	-15.00
		(0.50)	(0.49)	(0.99)	(0.99)	(1.47)	(1.50)
2370	0.05	-8.20	-8.10	-16.60	-18.40	-21.00	-23.40
		(0.38)	(0.39)	(0.75)	(0.54)	(1.38)	(1.24)
2370	0.10	-7.10	-8.80	-13.40	-14.60	-19.80	-23.10
		(0.45)	(0.32)	(0.94)	(0.89)	(1.42)	(1.26)
				Call option volatility control ($\alpha = 0.80$)			
2430	0.10	-4.50	-2.00	-10.20	-5.40	-14.70	-7.80
		(0.5)	(0.40)	(1.00)	(0.89)	(1.50)	(1.32)
2430	0.20	-5.30	-5.30	-9.60	-9.00	-15.00	-13.80
		(0.50)	(0.50)	(1.00)	(1)	(1.50)	(1.50)
2470	0.10	-6.90	-7.20	-14.80	-12.60	-18.90	-16.20
		(0.46)	(0.45)	(0.88)	(0.97)	(1.45)	(1.50)
2470	0.20	-5.60	-5.70	-12.60	-14.20	-19.80	-21.30
		(0.49)	(0.49)	(0.97)	(0.91)	(1.42)	(1.36)
				Call option volatility control ($\alpha = 1.00$)			
2430	0.10	-5.20	-3.30	-10.40	-3.80	-11.10	-4.80
		(0.50)	(0.47)	(1.00)	(0.78)	(1.45)	(1.10)
2430	0.20	-4.40	-4.20	-8.60	-6.80	-15.90	-13.80
		(0.49)	(0.49)	(0.99)	(0.95)	(1.50)	(1.50)
2470	0.10	-7.00	-5.90	-14.60	-14.20	-18.00	-16.80
		(0.46)	(0.49)	(0.89)	(0.91)	(1.47)	(1.49)
2470	0.20	-6.70	-7.00	-12.60	-12.40	-19.80	-21.00
		(0.47)	(0.46)	(0.97)	(0.97)	(1.42)	(1.38)

Table A.1.9. Estimated change in the final exchange rate in the case of call options. Standard errors in parentheses.

		$\theta/x = 10$		$\theta/x = 20$		$\theta/x = 30$	
R_0	σ	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep	Plain-Vanilla	Banrep
		Put option reserves' management ($\alpha = 0.80$)					
2330	0.05	5.80	6.90	10.80	14.60	12.90	18.90
		(0.49)	(0.46)	(1.00)	(0.89)	(1.49)	(1.45)
2330	0.10	5.40	7.40	8.20	15.00	13.20	20.40
		(0.50)	(0.44)	(0.98)	(0.87)	(1.49)	(1.40)
2370	0.05	1.70	2.60	1.40	3.60	2.40	3.90
		(0.37)	(0.44)	(0.51)	(0.77)	(0.81)	(1.01)
2370	0.10	2.40	3.70	5.00	6.80	8.10	14.70
		(0.42)	(0.48)	(0.87)	(0.95)	(1.33)	(1.50)
		Put option reserves' management ($\alpha = 1.00$)					
2330	0.05	4.30	6.20	7.60	12.40	11.10	19.20
		(0.49)	(0.48)	(0.97)	(0.97)	(1.45)	(1.44)
2330	0.10	5.20	7.00	10.00	14.00	14.40	19.50
		(0.50)	(0.46)	(1.00)	(0.92)	(1.50)	(1.43)
2370	0.05	1.10	1.90	1.60	3.60	2.70	6.30
		(0.31)	(0.39)	(0.54)	(0.77)	(0.86)	(1.22)
2370	0.10	1.80	3.70	5.80	8.60	4.50	10.50
		(0.38)	(0.48)	(0.91)	(0.99)	(1.07)	(1.43)
		Put option volatility control ($\alpha = 0.80$)					
2230	0.10	5.50	6.40	11.60	13.20	16.80	18.00
		(0.5)	(0.48)	(0.99)	(0.95)	(1.49)	(1.47)
2230	0.20	5.90	6.90	10.40	12.00	16.20	20.40
		(0.49)	(0.46)	(1.00)	(0.98)	(1.50)	(1.40)
2270	0.10	2.60	1.20	7.20	3.60	8.70	3.90
		(0.44)	(0.32)	(0.96)	(0.77)	(1.36)	(1.01)
2270	0.20	3.80	4.80	9.80	8.40	9.60	10.80
		(0.48)	(0.50)	(1.00)	(0.99)	(1.40)	(1.44)
		Put option volatility control ($\alpha = 1.00$)					
2230	0.10	4.90	4.80	10.00	11.80	11.70	15.60
		(0.50)	(0.50)	(1.00)	(0.98)	(1.47)	(1.50)
2230	0.20	5.30	6.20	9.40	11.40	14.40	14.70
		(0.50)	(0.48)	(1.00)	(0.99)	(1.50)	(1.50)
2270	0.10	3.50	1.20	4.80	2.20	9.60	5.10
		(0.47)	(0.32)	(0.85)	(0.62)	(1.40)	(1.13)
2270	0.20	3.60	4.30	10.20	9.40	11.10	12.00
		(0.48)	(0.49)	(1.00)	(1.00)	(1.45)	(1.47)

Table A.1.10. Estimated change in the final exchange rate in the case of put options. Standard errors in parentheses.

Appendix 2 (Sub-section 6.1, page 53). Solution of the Snell problem when the number of exercise dates differs from the number of time-steps in which the trading interval is divided

Again, let $\widetilde{X}_t$ denote the discounted payoff of the option at time $t \in \mathbb{E}$, where $\mathbb{E}$ represents the set of dates in which the option can be exercised. Likewise, let the time-interval $(t-1, t]$ be divided between two consecutive exercise dates in I equal sub-intervals and define the adapted process $\widetilde{Z}$ as

$$\widetilde{Z}_T = \widetilde{X}_T$$

$$\widetilde{Z}_{t+\frac{i}{I}} = \max\left[\widetilde{X}_{t+\frac{i}{I}}, E^Q\left[\widetilde{Z}_{t+\frac{i+1}{I}} \mid \mathcal{F}_{t+\frac{i}{I}}\right]\right], \quad 0 \leq i < I$$

where $\widetilde{X}_{t+\frac{i}{I}} = 0$ for $i \neq 0$.

On the other hand, let $\widetilde{X}^\tau$ be the payoff process stopped using the admissible exercise rule τ defined on the set of exercise dates $\mathbb{E}$. Since $\widetilde{Z}$ dominates $\widetilde{X}$ for all $t \in \mathbb{E}$, then

$$E^Q\left[\widetilde{Z}^\tau \mid \mathcal{F}_0\right] \geq E^Q\left[\widetilde{X}^\tau \mid \mathcal{F}_0\right].$$

Assuming positive interest rates, $E^Q\left[\widetilde{X}^\tau \mid \mathcal{F}_0\right]$ equals $E^Q\left[\widetilde{Z}^\tau \mid \mathcal{F}_0\right]$ when τ is chosen so as to be equal to

$$\tau^* = \min\left\{t \in \mathbb{E} : \widetilde{Z}_t = \widetilde{X}_t\right\},$$

so that for τ^* we have

$$E^Q\left[\widetilde{Z}^{\tau^*} \mid \mathcal{F}_0\right] = E^Q\left[\widetilde{X}^{\tau^*} \mid \mathcal{F}_0\right] = \sup_{\tau\in\mathcal{T}} E^Q\left[\widetilde{X}^{\tau} \mid \mathcal{F}_0\right].$$

On the other hand, as in Elliot and Kopp [16], chapter 5, let's define $\phi_{t+\frac{i}{I}} = \mathbf{1}_{\left\{\tau^*\geq t+\frac{i}{I}\right\}}$. $\widetilde{Z}^{\tau^*}_{t+\frac{i}{I}}$ can be expressed as

$$\widetilde{Z}^{\tau^*}_{t+\frac{i}{I}} = Z_0 + \sum_{u=1}^{t}\sum_{j=1}^{i} \phi_{u+\frac{j}{I}} \Delta \widetilde{Z}_{u+\frac{j}{I}},$$

so that $\widetilde{Z}^{\tau^*}_{t+\frac{i+1}{I}} - \widetilde{Z}^{\tau^*}_{t+\frac{i}{I}} = \phi_{t+\frac{i+1}{I}}\left(\widetilde{Z}_{t+\frac{i+1}{I}} - \widetilde{Z}_{t+\frac{i}{I}}\right)$.

Since $\phi_{(\cdot)}$ is predictable and

$$\widetilde{Z}_{t+\frac{i}{I}} = E^Q\left[\widetilde{Z}_{t+\frac{i+1}{I}} \mid \mathcal{F}_{t+\frac{i}{I}}\right], \qquad \text{for all } \tau^* \geq t + \frac{i+1}{I},$$

then

$$E^Q\left[\widetilde{Z}^{\tau^*}_{t+\frac{i+1}{I}} - \widetilde{Z}^{\tau^*}_{t+\frac{i}{I}} \mid \mathcal{F}_{t+\frac{i}{I}}\right] = \phi_{t+\frac{i+1}{I}} E^Q\left[\widetilde{Z}_{t+\frac{i+1}{I}} - E^Q\left[\widetilde{Z}_{t+\frac{i+1}{I}} \mid \mathcal{F}_{t+\frac{i}{I}}\right] \mid \mathcal{F}_{t+\frac{i}{I}}\right] = 0,$$

and the stopped process $\widetilde{Z}^{\tau^*}$ is a martingale for all $t < T$.

Consequently,

$$\widetilde{Z}_0 = E^Q\left[\widetilde{Z}_{\tau^*} \mid \mathcal{F}_0\right],$$

which justifies the use of the proposed recursive procedure to compute the option price when the number of exercise dates differs from the number of time-steps in which the trading interval is divided.

Appendix 3 (Sub-section 6.1, page 53). Implementation of the binomial model used to test the performance of the Longstaff-Schwartz algorithm

Let's assume that at each time $t+\frac{i+1}{I}$, $t\in\mathbb{E}$ and $i\in\{0,...,I-1\}$, the official exchange rate is given by a binomial model such that $R_{t+\frac{i}{I}}$ is followed by two possible values

$$\begin{aligned} R^{(u)}_{t+\frac{i+1}{I}} &= R_{t+\frac{i}{I}}\zeta^{(u)}, \qquad \zeta^{(u)}=e^{\sigma\sqrt{\frac{1}{I}}}, \\ R^{(d)}_{t+\frac{i+1}{I}} &= R_{t+\frac{i}{I}}\zeta^{(d)}, \qquad \zeta^{(d)}=e^{-\sigma\sqrt{\frac{1}{I}}}, \end{aligned}$$

with risk-neutral transition probabilities

$$Q^{(u)}=\frac{e^{(r-r^{*})\frac{1}{I}}-e^{-\sigma\sqrt{\frac{1}{I}}}}{e^{\sigma\sqrt{\frac{1}{I}}}-e^{-\sigma\sqrt{\frac{1}{I}}}},$$

$$Q^{(d)}=\frac{e^{\sigma\sqrt{\frac{1}{I}}}-e^{(r-r^{*})\frac{1}{I}}}{e^{\sigma\sqrt{\frac{1}{I}}}-e^{-\sigma\sqrt{\frac{1}{I}}}}.$$

Again, let's define the $D-$day simple moving average as

$$M_{D,t}=\frac{1}{D}\sum_{j=1}^{D}R_{t-j+1}.$$

At every exercise date $t\in\mathbb{E}$ the option's payoff in the case of the call option is given by

$$X^{(c),(\cdot)}_{t+1}=\max\left[R^{(\cdot)}_{t+1}-R_t,0\right]\ \mathbf{1}_{t\left\{R_{t-1}\geq\kappa M_{D,t-1}\right\}},$$

whereas the payoff of any of the put options can be expressed as

$$X_{t+1}^{(p),(\cdot)} = \max\left[R_t - R_{t+1}^{(\cdot)}, 0\right] \; \mathbf{1}_{t\left\{R_{t-1} \leq \kappa M_{D,t-1}\right\}},$$

where $\mathbf{1}_t$ denotes the indicator function that represents the fulfilment of the exercise restriction.

Following the approach used in Appendix 2, let's define the adapted process

$$Z_{T-1} = \max\left[X_{T-1}, GK\left(r, r^*, R_{T-1}, R_{T-1}, \sigma, \tfrac{1}{T}\right)\right],$$

$$Z_{t+\frac{i}{I}} = \max\left[X_{t+\frac{i}{I}}, \frac{B_{t+\frac{i}{I}}}{B_{t+\frac{i+1}{I}}}\left(Z^{(u)}_{t+\frac{i+1}{I}} Q^{(u)} + Z^{(d)}_{t+\frac{i+1}{I}} Q^{(d)}\right)\right], \quad 0 \leq i < I,$$

where the continuation value of the option in the penultimate node is replaced by the option price computed using the Garman and Kohlhagen [25] model, denoted by $GK\left(r, r^*, R_{T-1}, R_{T-1}, \sigma, \frac{1}{T}\right)$, and once again $X_{t+\frac{i}{I}}$ is set to be equal to zero for $i \neq 0$.

The price of the option at $t = 0$ is given by $Z_0 = \left(B_{\frac{1}{I}}\right)^{-1}\left(Z^{(u)}_{\frac{1}{I}} Q^{(u)} + Z^{(d)}_{\frac{1}{I}} Q^{(d)}\right)$.

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