Abstract

This paper identifies a new industry-equilibrium channel through which a firm’s productivity affects its organizational choice. In a two-country model with firm heterogeneity and incomplete contracts, we show that the degree of input specificity and the hold-up friction in an outsourcing relation become a function of the final good firm’s productivity when inputs are not completely specific. We examine the implications for the equilibrium international sorting pattern of firms.

**JEL Codes:** F23, F12.

**Key words:** firm heterogeneity; input specificity; hold-up problem; incomplete contracts.
1 Introduction

The role of firm-specific productivity on a firm’s international organization of production has become the focus of an emerging research area in international trade (Helpman, 2006). Recent empirical studies have used firm-level data to identify a systematic relationship between a firm’s productivity and its internationalization strategy. Tomiura (2005) and Kurz (2006), for example, have used data from Japan and the United States to show that firms tend to offshore more of their production activities when their productivity is higher. Furthermore, Nunn and Trefler (2008) have used U.S. data and found that, in capital-intensive industries, firms with higher productivity tend to rely more on intra-firm imports than on arm’s length imports.

To explain the relationship between a firm’s productivity and its organizational form, existing empirical studies have primarily relied on the theoretical contribution of Antràs and Helpman (2004). Antràs and Helpman map the property rights theory of Grossman and Hart (1986) and Hart and Moore (1990) into a two-country industry-equilibrium trade model with heterogeneous firms.¹ In this model, an intermediate and final good firm are both required to provide non-contractible, relationship-specific inputs to produce joint output. Since contracts are incomplete, there is a classic two-sided holdup problem. Antràs and Helpman’s framework then predicts that the optimal ownership structure depends on the relative importance of the parties’ inputs. If the final good firm provides the bulk of the inputs (headquarter-intensive industries), underinvestment is reduced by incentivizing the final good firm. Vertical integration thus emerges as the optimal ownership structure because it gives the final good firm the residual rights over the intermediate good firm’s inputs. Conversely, when the intermediate good firm provides the bulk of the inputs (component-intensive industries), underinvestment is reduced by incentivizing the intermediate good firm. Outsourcing is therefore optimal since it gives the intermediate good firm residual rights over its inputs. In Antràs and Helpman (2004), productivity affects a firm’s organizational choice due to the critical assumption that fixed costs differ across organizational forms. The assumption that fixed costs are higher when components are sourced from foreign countries implies that only the most productive firms in component-intensive indus-

¹Antràs and Helpman (2008) generalize this model to accommodate for varying degrees of contractual frictions. Both papers build on a vast literature that has analyzed the international organization of production by incorporating elements of incomplete contracts theory into industry-equilibrium trade models. See Spencer (2005), Helpman (2006) and Antràs and Rossi-Hansberg (2009) for comprehensive reviews of this literature.
tries will outsource input production internationally. In addition, the assumption that integration entails larger fixed costs than outsourcing yields that only the most productive firms in headquarter-intensive industries will choose to conduct foreign direct investment.\textsuperscript{2}

In this paper, we do not question the existence of fixed cost differences across organizational forms. Rather, we analyze whether there are other industry-equilibrium mechanisms through which a firm’s productivity affects its organizational form.\textsuperscript{3} Our starting point is a simplified version of Antràs-Helpman’s (2004) model where only intermediate good firms provide non-contractible inputs, and fixed costs are identical across organizational forms. We modify this model by assuming that inputs can be used for the production of another final good variety, rather than being completely specific as in the original Antràs and Helpman framework. We take on this assumption because, as Grossman and Helpman (2002) acknowledge, complete specificity may not be a reasonable hypothesis in many industries. Our assumption implies that, under outsourcing, an intermediate good firm may have a positive ex post outside option. Specifically, should an outsourcing relationship with a final good firm break down after the intermediate good firm has produced its inputs (i.e. ex post), the intermediate good firm can form a new relationship with another non-committed final good firm in the market, and offer its inputs to this latter firm to produce joint revenue. The presence of this positive ex post outside option strengthens the intermediate good firm’s bargaining power and therefore improves its incentives to contribute inputs.

Within-industry firm heterogeneity in our model implies that the ex post outside option affects an intermediate good firm’s bargaining power differently depending on its final good partner’s productivity. If the final good firm’s productivity is low, we will show that the value of the intermediate good firm’s inputs in the outside relationship is not much lower than within the original relationship. The ex post outside option therefore significantly increases the intermediate good firm’s bargaining power. Conversely, if the final good firm’s productivity is high, the value of inputs in the outside relationship is significantly lower than in the original relationship. The ex

\textsuperscript{2} This assumption has recently been questioned by a number of studies. Grossman et al. (2005) and Defever and Toubal (2007) assume that the fixed cost of outsourcing is larger than that of vertical integration. The latter study finds that, for French firms, empirical evidence favors this alternative fixed cost ranking.

\textsuperscript{3} In a similar vein, Naghavi and Ottaviano (2009) set up an endogenous growth model of North-South offshoring with heterogeneous firms in which the organizational choice is not driven by fixed cost differences across organizational forms.
post outside option thus leads to a small increase in the intermediate good firm’s bargaining power. This heterogeneous effect of productivity on the bargaining outcome within an industry implies that an increase in the final good firm’s productivity corresponds to a rise in the hold-up friction within an outsourcing relationship.

Our model is constructed in a two-country setting where final good firms are located in the North and intermediate good firms can be located in both the North and South. The countries differ along two dimensions. On the one hand, wages are lower in the South. On the other hand, it is more costly for a Southern intermediate good firm to ex post search for and coordinate with an outside partner. These extra cross-border costs imply that, all else equal, a Southern intermediate good firm’s ex post outside option is lower than that of its Northern counterpart. As a result, outsourcing internationally entails a larger hold-up friction than outsourcing domestically. Interestingly, our model also shows that this extra hold-up friction associated with outsourcing internationally diminishes as the final good firm’s productivity rises. This leads to the key prediction of our model that only the most productive firms are willing to outsource internationally to take advantage of the lower Southern wages, while less productive firms outsource domestically. This prediction is not based on fixed cost differences across organizational forms, and our paper therefore provides an alternative theory to Antrás and Helpman (2004) why only the most productive firms outsource internationally in component-intensive industries.

We can link the central results of our model to the concept of input specificity. In the transaction-cost literature, an investment is considered specific to a relationship if its value is higher inside that particular relationship than outside of it (Klein et al., 1978; Williamson, 1979). Traditionally, the degree of input specificity has been considered as purely technology driven, i.e. as linked to the technological characteristics of a product. However, recent studies demonstrate that when inputs can be put to an alternative use, input specificity also depends on the organization of the industry (Erkal, 2007) and on firm heterogeneity (Andrabi et al., 2006). In line with these insights, our model with within-industry firm heterogeneity finds that technologically identical inputs are more specific when produced for a high productivity firm than for a low productivity firm. Our prediction that a rise in productivity increases the hold-up friction in an outsourcing relationship thus reflects that input specificity rises with productivity. Similarly, our finding that the extra hold-up friction related to outsourcing internationally relative to outsourcing domestically diminishes with productivity reflects parallel trends in input specificity.
Our paper is related to two other recent studies that have constructed two-country industry-equilibrium models with incomplete contracts and partially specialized inputs. First, McLaren (2000) analyzes how international openness affects the organization of production by assuming that an independent supplier’s probability of finding an attractive outside buyer is increasing in the “thickness of the market” as determined by the number of unintegrated final-good firms. Since countries’ openness to trade increases the number of available unintegrated buyers, trade liberalization serves to increase independent suppliers’ ex post outside options, thus reducing input specificity and making outsourcing more attractive. Second, Grossman and Helpman (2002) build an industry-equilibrium model where intermediate good firms can choose the degree of technological specificity of their inputs, thus affecting their ability of ex post selling their inputs on a secondary input market. In this setting, the existence of the secondary market reduces input specificity and therefore makes outsourcing more attractive. Our approach differs from these papers in an important way. In McLaren (2000) and Grossman and Helpman (2002), firms are considered identical within an industry so that input specificity can only differ across industries, but not within a sector. Conversely, in our model with firm heterogeneity, input specificity varies by firm within an industry. This allows us to contribute to the literature by studying the impact of a final good firm’s productivity on its organizational choice through its effect on input specificity.

2 Model

Our model is a simplified version of Antràs and Helpman (2004) where only intermediate good firms provide non-contractible inputs and fixed costs are identical across organizational forms. We extend this model by assuming that inputs are not completely specific, so that they can ex post be put to an alternative use.

Consider a world with two countries, North and South, and a single monopolistically competitive industry that produces differentiated consumer goods \( y(i) \). Preferences across varieties have the standard CES form, with an elasticity of substitution \( \sigma = 1/(1 - \alpha) > 1.4 \) These preferences generate the inverse demand function

\[
p(i) = \lambda^{1-\alpha} y(i)^{-\alpha},
\]

4The utility function is \( U = \left( \int_{i \in \Omega} y(i)^\alpha di \right)^{1/\alpha} \), where \( \Omega \) is the set of available varieties.
where $y(i)$ is the quantity demanded of variety $i$, $p(i)$ is its price and $\lambda$ is the aggregate consumption index. Since there is a continuum of firms, we assume that firms treat $\lambda$ as given.\(^5\)

To enter the industry, any final good firm located in the North needs to incur an irreversible fixed cost of entry equal to $F_e$ units of Northern labor.\(^6\) The entrant then randomly draws a productivity $\theta(i)$ from a common and known cumulative distribution function $G(\theta)$ with support $[0, \infty)$.\(^7\) After observing its productivity, it decides whether to start producing a variety or to remain idle in the market. Initiating production requires a final good firm to spend an additional fixed cost equal to $F$ units of Northern labor, which contrary to other studies, we consider to be identical across organizational forms. The extra fixed cost $F$ entails that final good firms with a productivity $\theta$ above threshold level $\theta$ become active in the market, while firms with $\theta \leq \theta$ remain idle. After entry, the active final good firms engage in monopolistic competition.

To initiate production, a final good firm needs to contract with one of a perfectly elastic supply of potential intermediate good firms in North or South that can provide the required inputs $x(i)$. An intermediate good firm produces a unit of input with a unit of labor. To reflect higher labor costs in North, we assume that Northern wages $\omega^N$ are strictly higher than Southern wages $\omega^S$ and normalize the latter to 1: $\omega^N = \omega > \omega^S = 1$. Once the final good firm receives inputs $x(i)$ from the intermediate good firm, it can use the following production function to produce output:\(^8\)

$$y(i) = \theta(i)x(i). \quad (2)$$

By combining equations (1) and (2), the potential revenue from the sale of $y(i)$ is given by

$$R(x(i)) = \lambda^{1-\alpha}(\theta(i)x(i))^\alpha. \quad (3)$$

In the spirit of Grossman and Hart (1986), contracts are incomplete in the sense that the only items that the two parties can contract upon ex ante (i.e., before inputs have been produced) are the allocation of residual rights

\(^5\)The utility function implies that $\lambda = \frac{E}{\int_0^n p(i) \, \, \, d\theta}$, where $E$ is the aggregate expenditure on the industry and $n$ is the number (measure) of varieties available.

\(^6\)We assume that only firms in North have the know-how (i.e. technology and distributional network) to produce final goods.

\(^7\)This approach to introducing within-industry firm heterogeneity was first developed by Melitz (2003).

\(^8\)This production function corresponds to the special case in Antràs and Helpman (2004) of a component-intensive industry where the degree of headquarter-intensity $\eta = 0$. 

6
and a lump-sum transfer $t$ between parties. As a result, the two contracting parties need to ex post bargain over the surplus of the relation. We model this ex post bargaining as a symmetric Nash bargaining game in which the parties share equally the ex post surplus.

Following the property rights approach to the theory of the firm, we assume that final good firms form incomplete contracts with intermediate good firms under both outsourcing ($O$) and integration ($I$). The choice of ownership structure is relevant since it affects parties’ ex post outside options, thus impacting the division of revenue in the ex post bargaining stage. Under outsourcing, the intermediate good firm has the residual rights over its inputs, thus providing it with a positive ex post outside option if it finds an alternative use for its inputs. This strengthens the intermediate good firm’s ex post bargaining power, and therefore increases its revenue share. Conversely, under integration, the final good firm has the residual rights over inputs. It therefore has the right to ex post fire its partner and seize at least a fraction of the inputs $x(i)$ produced by the intermediate good firm. This provides the final good firm with a positive ex post outside option, which lowers the intermediate good firm’s bargaining power and therefore its revenue share. In appendix A, we show that, in our model, integration is always dominated by outsourcing. In the main text, we therefore solely focus on outsourcing.

Our key departure from Antràs and Helpman (2004) is that inputs are not specific: inputs produced for one final good firm can ex post be used for the production of another final good variety. This implies that an intermediate good firm under outsourcing may have a positive ex post outside option. Specifically, if the original outsourcing relation breaks down, it can form a new outsourcing relation with a non-committed (idle) final good firm in the market and provide its inputs $x(i)$ to create joint revenue. The intermediate good firm’s ex post outside option is then half of this outside revenue.

Ex post searching for and coordinating with a non-committed final good firm is generally more costly across borders than within borders. To capture such cross-border coordination costs, we assume that intermediate good firms lose a fraction of their inputs when ex post searching and coordinating with a new final good firm. As a result, Southern intermediate good firms can only offer fraction $\tau \in [0,1]$ of the inputs it has produced to create

\[9\] In a previous version, we considered the case where, due to technological specificity, the intermediate good firm can ex post only use a fraction of its inputs to produce another final good variety. Introducing technological specificity does not affect the results of the model and has been dropped in this version. Calculations can be obtained upon request.
outside revenue, whereas Northern intermediate good firms can offer all its inputs.

The model can be summarized by the following sequences of moves: in period 0, each Northern final good firm $i$ decides whether it enters the market. If it enters, it incurs a fixed entry cost $F_e$ and draws its productivity $\theta(i)$. In period 1, the final good firm decides if it wants to produce output or remain idle. If it decides to produce output, it chooses the location to outsource its input production $l \in L = \{N, S\}$. In period 2, the final good firm signs an outsourcing contract with an intermediate good firm and there is a lump-sum transfer between both parties. In period 3, the intermediate good firm produces its inputs. In period 4, there is symmetric Nash bargaining between the intermediate good firm and the final good firm. The final goods are then produced and sold, after which the revenue is equally divided between the parties. In our analysis below, we will solve for the optimal organizational form through backward induction. We will denote periods 0, 1 and 2 as ex ante to reflect that they take place prior to input production, and period 4 as ex post to reflect that it takes place after input production.

2.1 Ex Post Revenue Distribution

We start by calculating each party’s revenue share that results from the symmetric Nash bargaining in period 4. To simplify notation, we from now on will drop the i’s and refer to a firm’s “ex post outside option” and “ex post bargaining power” as its “outside option” and “bargaining power”.

A standard result of symmetric Nash bargaining is that each party receives its outside option plus half of the ex post surplus. Let $v_l$ and $V_l$ denote the intermediate and final good firm’s outside options respectively. The intermediate good firm then obtains

$$s_l R_l = v_l + \frac{1}{2} \left( R_l - v_l - V_l \right),$$

where $s_l$ denotes the intermediate good firm’s revenue share and $v_l$, $V_l$, and $R_l$ are functions of $x_l$:

$$v_l = v_l(x_l), V_l = V_l(x_l), \text{ and } R_l = R_l(x_l).$$

The final good firm obtains the remaining portion of the revenue $(1 - s_l) R_l$.

To derive the intermediate good firm’s revenue share $s_l$, we will proceed by determining $v_l$ and $V_l$ for outsourcing in North $(O, N)$ and outsourcing in South $(O, S)$ separately.
Outsourcing in North. Under outsourcing in North \((O, N)\), the intermediate good firm has the residual rights over its inputs. These residual rights may provide the intermediate good firm with a positive outside option. Specifically, if both parties fail to agree in the bargaining, the intermediate good firm can use its inputs \(x^N\) in a new outsourcing relation with any idle final good firm in the market with \(\theta \leq \bar{\theta}\).\(^{10}\) If both parties agree with this new relation, they sign a new outsourcing contract that provides the intermediate good firm with the residual rights over its inputs and specifies a lump-sum transfer between both parties. In Appendix B, we demonstrate that any active Northern intermediate good firm (regardless of its original partner’s productivity) is ex post better off forming an outside relation with the threshold firm with productivity \(\bar{\theta}\) than remaining idle.

It is intuitive that an intermediate good firm prefers to form an outside relation with the threshold firm with productivity \(\bar{\theta}\). By definition, the threshold firm is the most productive final good firm that is idle in the market. Equation (3) then suggests that forming an outside relation with the threshold firm maximizes the outside revenue \(R\) that can be generated with inputs \(x^N\), where

\[
R^N = \lambda^{1-\alpha} (\bar{\theta} x^N)^\alpha.
\]  
(5)

Since the intermediate good firm receives half of the outside revenue \(R\), forming a relation with the threshold firm thus maximizes the intermediate good firm’s outside option.\(^{11}\)

By combining equations (3) and (5), the outside revenue can be expressed as a constant fraction of the inside revenue that could have been created if the original parties had agreed in the bargaining:

\[
R^N = \left( \frac{\bar{\theta}}{\theta} \right)^\alpha R^N.
\]  
(6)

Notice that \(R^N\) is smaller than inside revenue \(R^N\). Since the intermediate good firm’s inside partner has a higher productivity than the threshold firm

\(^{10}\)We define a party’s outside option as the deviation payoff when a relation breaks down, taking as given the continuance of all other relationships. Since all the active firms with \(\theta > \bar{\theta}\) are already committed, an intermediate good firm in our model can ex post only form an outside relation with an idle firm with \(\theta \leq \bar{\theta}\).

\(^{11}\)The assumption that the intermediate good firm receives half of total revenue in the outside relation is equivalent to assuming that there is no further separation possible in the outside option. In principle there might be further separations, but for simplicity we take the outside options after the second stage as nil. See Grossman and Helpman (2002) for a similar assumption.
(θ ≥ θ), less outside revenue than inside revenue can be generated with the same amount of inputs.

The intermediate good firm in its outside option receives half of outside revenue R, while the threshold final good firm receives the other half. Using equation (6), an intermediate good firm’s outside option thus equals:

\[ v^N = \frac{1}{2} \left( \frac{\theta}{\bar{\theta}} \right)^\alpha R^N. \]  

(7)

We assume that the final good firm ex post does not have the possibility of purchasing inputs on a spot market. Furthermore, it takes too long for it to sign a contract with a new intermediate good firm and await the production of new inputs. Given the continuance of all other relationships, this implies that the final good firm does not have an outside option under \((O, N)\).\(^{12}\) Thus, the final good firm’s outside option is

\[ V^N = 0. \]  

(8)

We can insert (7) and (8) into (4) to derive the intermediate good firm’s revenue share under \((O, N)\):

\[ s^N = \frac{1}{2} + \frac{1}{4} \left( \frac{\theta}{\bar{\theta}} \right)^\alpha. \]  

(9)

The intermediate good firm’s revenue share exceeds 1/2 since the positive outside option \(v^N\) tilts the intermediate good firm’s bargaining power in its favor. The amount that its bargaining power increases is negatively related to its original final good partner’s productivity θ. If the final good firm’s productivity θ is larger, the outside option becomes relatively less attractive than the value that can be generated inside the original relationship so that the intermediate good firm’s bargaining power is smaller. In Section 2.2, we further explore this link between productivity and bargaining power.

**Outsourcing in South.** Under outsourcing in South \((O, S)\), the Southern intermediate good firm also has the residual rights over its inputs. Due to the extra effort of searching for and coordinating with the threshold firm across borders, however, it can only use fraction τ of inputs \(x^S\) to generate outside revenue. The extra coordination costs implies that some Southern

\(^{12}\)In appendix F, we show that the key results of our model are largely unaffected if we assume that final good firms in their outside option can purchase inputs from a spot market.
intermediate good firms may ex post be better off remaining idle than forming an outside relation with the threshold firm. In appendix C, we derive the industry-equilibrium condition under which any active Southern intermediate good firm has a strictly positive outside option:

**Assumption 1** $\omega \geq \frac{3}{2+\tau^a} \left(\frac{4-3\alpha}{4+\tau^a}\right)^{\frac{1+\alpha}{4}}$.

This condition is more likely to hold if $\omega$ and $\tau$ are large. To simplify the exposition of the model, we adopt Assumption 1 throughout the paper.\(^{13}\)

Given that the outside option of any active Southern intermediate good firm is to sign a new outsourcing contract with the threshold final good firm, the derivation of $v^S$ and $V^S$ is similar to that of $v^N$ and $V^N$. The Southern intermediate good firm then provides fraction $\tau$ of its inputs $x^S$ to generate the following outside revenue:

$$R^S = \left(\frac{\tau \bar{\theta}}{\bar{\theta}}\right)^\alpha R^S.$$  \hfill (10)

The intermediate good firm’s outside option then equals half of this outside revenue:

$$v^S = \frac{1}{2} \left(\frac{\tau \theta}{\bar{\theta}}\right)^\alpha R^S.$$  \hfill (11)

Similar to $(O,N)$, the final good firm does not have an outside option:

$$V^S = 0.$$  \hfill (12)

Finally, we can insert (11) and (12) into (4) to determine the intermediate good firm’s revenue share:

$$s^S = \frac{1}{2} + \frac{1}{4} \left(\frac{\tau \theta}{\bar{\theta}}\right)^\alpha.$$  \hfill (13)

Equation (13) shows that, similar to $s^N$, the intermediate good firm’s revenue share $s^S$ is negatively related to the final good firm’s productivity. Furthermore, a comparison of equations (9) and (13) suggests that, for a given productivity $\theta$, $s^N \geq s^S$ due to the cross-border coordination costs under $(O,S)$. Finally, it is easy to show that the intermediate good firm’s extra revenue share under $(O,N)$ relative to $(O,S)$ decreases with productivity $\theta$. To understand the logic behind these results, it is instructive to calculate the degree of input specificity under both organizational forms. We proceed to do so in the next section.

\(^{13}\)In appendix E, we formally demonstrate that the key results of our paper are largely unaffected if Assumption 1 is relaxed.
2.2 Productivity and Input Specificity

Investments are considered specific to a relationship if the value of investments are higher within a relation than outside the relation (Klein et al., 1978; Williamson, 1979). In a similar spirit, we define the degree of input specificity \( d^l \) in an outsourcing relation as the difference between the inside revenue \( R^l \) and outside revenue \( \bar{R}^l \) that can be created with inputs \( x^l \) normalized by inside revenue:

\[
d^l = \frac{R^l - \bar{R}^l}{R^l}.
\]  

(14)

A higher value of \( d^l \) implies a higher degree of input specificity. If \( d = 0 \), there is no input specificity since the same inside and outside revenue can be created with inputs \( x^l \). If \( d = 1 \), there is complete input specificity since inputs are worthless in the outside relation. By inserting (6) and (10) into (14), input specificity in our model equals

\[
d^N = 1 - \left( \frac{\theta}{\bar{\theta}} \right)^\alpha,
\]

(15)

\[
d^S = 1 - \left( \frac{\tau \theta}{\bar{\theta}} \right)^\alpha.
\]

(16)

for the organizational forms \((O,N)\) and \((O,S)\), respectively. In Figure 1, we use equations (15) and (16) to graph input specificity under \((O,N)\) and \((O,S)\) by depicting the final good firm’s productivity relative to that of the threshold firm \( \theta / \bar{\theta} \) on the horizontal axis and input specificity \( d^l \) on the vertical axis.

[Figure 1 about here]

Figure 1 illustrates that \( d^N \) and \( d^S \) depend on productivity. Under both \((O,N)\) and \((O,S)\), input specificity is greater for high productivity firms than for low productivity firms. We state this in the following proposition:

**Proposition 1** Ceteris paribus, input specificity under both \((O,N)\) and \((O,S)\) is an increasing and concave function of the final good firm’s productivity \( \theta \).
The logic is the following. If the final good firm’s productivity is low, the value that can be created with the intermediate good firm’s inputs within the original relationship only marginally exceeds the value that can be created in the outside relationship. Conversely, if the final good firm’s productivity is high, the value that can be created with inputs is significantly higher within the original relationship than in the outside relation. As a result, inputs are more specific if produced for high productivity firms than if produced for low productivity firms. Note that input specificity in this model is not driven by the technological characteristics of the inputs. Rather, it is determined by intra-industry firm heterogeneity and cross-border coordination costs.

Productivity not only affects the degree of input specificity under \((O,N)\) and \((O,S)\), but also the difference in input specificity between organizational forms:

\[
d^S - d^N = (1 - \tau^\alpha) \left( \frac{\theta}{\bar{\theta}} \right)^{\alpha}.
\]

(17)

We can infer the next proposition from equation (17):

**Proposition 2** *Ceteris paribus, the difference in input specificity under \((O,S)\) relative to \((O,N)\) declines with the final good firm’s productivity \(\theta*.*

Propositions 1 and 2 can be used to explain the effect of productivity on the intermediate good firm’s revenue share (see equations (9) and (13)). From Proposition 1, the intermediate good firm’s revenue share decreases with productivity since inputs become more specific, thus reducing the intermediate good firm’s bargaining power. From Proposition 2, the difference in the intermediate good firm’s revenue share between \((O,N)\) and \((O,S)\) decreases with productivity since the difference in input specificity between both organizational forms is smaller for high productivity firms than for low productivity firms.

### 2.3 Productivity and Hold-Up

We can now role the clock back to period 3 in which the intermediate good firm decides how many inputs to produce. Since the delivery of inputs \(x^l\) is not contractible ex ante, the intermediate good firm non-cooperatively chooses the amount of inputs that maximizes its profits \(\pi^l\):

\[
\max_x \pi^l = s^l R(x^l) - \omega^l x^l + t,
\]

(18)

where \(s^l\) is defined by equations (9) and (13), and \(R(x^l)\) by equation (3). Solving for the maximization problem yields the amount of inputs produced
by the intermediate good firm:

\[ x^{l*} = \lambda \left( \frac{s^l \theta^\alpha}{\omega^l} \right)^{\frac{1}{1-\alpha}}. \]  

(19)

The amount of inputs that are produced by the intermediate good firm is strictly lower than if parties could write complete contracts. It is straightforward to show that if contracts were complete, the final good firm would set \( s = 1 \). The suboptimal amount of inputs reflects the distortion arising from incomplete contracting. Intuitively, the intermediate good firm underinvests in \( x^l \) because it fails to capture the full marginal return to its investment in the ex post bargaining.

Compared to the complete contract scenario (where \( s = 1 \)), the amount of underinvestment or “hold-up” decreases with the intermediate good firm’s revenue share \( s^l \). We can thus measure the degree of hold-up friction in a relation with the term \( 1 - s^l \). In Figure 2, we graph the hold-up friction for the various organizational forms. We depict the final good firm’s productivity relative to that of the threshold firm \( \theta/\bar{\theta} \) on the horizontal axis and the hold-up friction \( 1 - s^l \) on the vertical axis.

[Figure 2 about here]

It is straightforward to interpret the results displayed in Figure 2. First, for a given \( \theta/\bar{\theta} \), the hold-up friction is larger under \((O,S)\) than \((O,N)\), because a Southern outsourcee has relatively less bargaining power than a Northern outsourcee. Second, the hold-up friction is a function of productivity under \((O,N)\) and \((O,S)\). Specifically, we can infer the following corollaries:

**Corollary 1** *Ceteris paribus, the hold-up friction under \((O,N)\) and \((O,S)\) is an increasing and concave function of the final good firm’s productivity \( \theta \).*

The logic behind this corollary directly follows from Proposition 1. As the final good firm’s productivity rises under \((O,N)\) and \((O,S)\), inputs become more specific, thus eroding the intermediate good firm’s bargaining power and therefore its revenue share. The lower revenue share reduces the intermediate good firm’s incentives to produce inputs, therefore raising the hold-up friction.

14Under complete contracts, the final good firm sets \( s = 1 \) and uses the lump-sum transfer \( t \) to extract the intermediate good firm’s profits.
Corollary 2  
*Ceteris paribus, the extra hold-up friction under (O, S) relative to (O, N) declines with the final good firm’s productivity \( \theta \).*

We can use Proposition 2 to explain this corollary. Since the difference in input specificity between (O, S) and (O, N) is higher for low productivity firms, the difference in bargaining power is larger and therefore the difference in hold-up friction is also higher.

2.4 Optimal Organizational Form

In period 2, the final good firm offers a take-it-or-leave-it contract to the intermediate good firm that specifies lump-sum transfer \( t \) and the allocation of residual rights. Since the intermediate good sector in both North and South is highly competitive, the final good firm chooses the lump-sum transfer that guarantees that the intermediate good firm breaks even. Specifically, it solves:

\[
\max_t \Pi^l = (1 - s^l)R(\omega^l x^l) - F + t
\]

subject to

\[
\pi^l = s^l R(\omega^l x^l) - \omega^l x^l - t \geq 0,
\]

where \( \Pi^l \) is the final good firm’s profits. By inserting equation (3) and (19) into the solution of maximization problem (20), the final good firm’s profits can be expressed as:

\[
\Pi^l(\theta) = \lambda \theta^\frac{\alpha}{1-\alpha} \left(1 - \alpha s^l(\theta)\right) \left(\frac{\alpha s^l(\theta)}{\omega^l}\right)^{\frac{\alpha}{1-\alpha}} - F,
\]

where \( s^l \) is given by equations (9) and (13), and \( \omega^N = \omega > \omega^S = 1 \). From equation (21), it is straightforward to derive that \( \Pi^l \) increases with \( s^l \) and decreases with \( \omega^l \).

In period 1, the final good firm chooses the production location \( l \) that maximizes its profits: \( \Pi^* = \arg\max_{l \in L} \Pi^l \). From equation (21), the firm’s choice depends on both the wage difference \( \omega^l \) between North and South and on the difference in hold-up friction between the organizational forms measured by \( 1 - s^l \). The lower wages in South act as an incentive for final good firms to source their inputs from South. Conversely, the lower hold-up friction under (O, N) than under (O, S) acts as a deterrent against outsourcing to a Southern intermediate good firm.
From a comparison of the profit functions of \((O, N)\) and \((O, S)\) in equation (21), it follows that \(\Pi^N \geq \Pi^S\) if and only if \(A(\theta) \leq \omega\), where

\[
A(\theta) = \frac{s^N(\theta)}{s^S(\theta)} \left( \frac{1 - \alpha s^N(\theta)}{1 - \alpha s^S(\theta)} \right)^{\frac{1 - \alpha}{\alpha}}. \tag{22}
\]

Otherwise, \((O, S)\) is the optimal organizational form.

In Appendix D, we derive the properties of curve \(A(\theta)\). We show that \(A(\theta) > 1\) for all \(\theta \in [\bar{\theta}, +\infty)\) and that \(\lim_{\theta \to +\infty} A(\theta) = 1\). Furthermore, we demonstrate that the shape of \(A(\theta)\) can take two forms. If \(\tau\) is sufficiently small and \(\alpha\) sufficiently large, \(A'(\theta) \leq 0\) for all \(\theta \in [\bar{\theta}, +\infty)\). Otherwise, \(A(\theta)\) first rises in \(\theta\) and then declines with \(\theta\), with a unique maximum at \(\theta_{\text{max}}\). These properties suggest that \((O, S)\) is necessarily the optimal organizational form when the final good firm’s productivity is sufficiently high; and that \((O, N)\) can only be the optimal organizational form if the wages in the North \(\omega\) are not too high. Taken together, these properties imply that \(A(\bar{\theta}) \geq \omega\) is a sufficient condition for both the organizational forms \((O, N)\) and \((O, S)\) to coexist in industry-equilibrium. This will be the case if:

\[
A(\theta) = \frac{3}{2 + \tau^\alpha} \left( \frac{4 - 3\alpha}{4 - \alpha(2 + \tau^\alpha)} \right)^{\frac{1 - \alpha}{\alpha}} \geq \omega. \tag{23}
\]

In that case, there exists a unique cutoff productivity \(\theta_1 \geq \bar{\theta}\) such that \(A(\theta) \geq \omega\) for \(\theta \in [\bar{\theta}, \theta_1]\) and \(A(\theta) < \omega\) for \(\theta \in [\theta_1, +\infty)\). We state this in the following proposition:

**Proposition 3** If \(A(\bar{\theta}) \geq \omega\), there exists a unique cutoff productivity \(\theta_1 \geq \bar{\theta}\) such that final good firms with \(\theta \in [\bar{\theta}, \theta_1]\) choose \((O, N)\), while final good firms with \(\theta \in [\theta_1, +\infty)\) choose \((O, S)\).

In Figure 3, we provide a graphical description of the industry-equilibrium sorting pattern for \(A(\theta) \geq \omega\) by depicting \(\theta\) on the horizontal axis and \(A(\theta)\) on the vertical axis. Consistent with Proposition 3, we find that \((O, N)\) is optimal for \(\theta \leq \theta_1\) and \((O, S)\) is optimal for \(\theta > \theta_1\). The intuition follows from Corollary 2: the benefits of lower wages under \((O, S)\) are able to offset the extra hold-up problem only when the final good firm’s productivity is sufficiently large.

[Figure 3 about here]
When $A(\theta) < \omega$, it is generally the case that $(O, S)$ is the optimal organizational firm for all firms (see Appendix D for the derivation and further details). The intuition behind this is that when $\omega$ is too large, the advantage of having a lower hold-up friction under $(O, N)$ is insufficient to overcome the higher wages in the North. We also show, however, that for a limited parameter range it might be the case that $(O, N)$ becomes optimal for a middle range of productivity.

Finally, we investigate how robust our results are to changes in the model’s assumptions. In Appendix E, we demonstrate that the key results of our model are robust to relaxing Assumption 1. Suppose that only Southern intermediate good firms which collaborate with firms with productivity $\theta_0 > \theta$ have a strictly positive outside option, while the other Southern intermediate good firms prefer to ex post remain idle. In that case, we show that for low productivity levels $\theta \leq \theta < \theta_0$, the extra hold-up friction under $(O, S)$ relative to $(O, N)$ is even larger than under Assumption 1. As a result, the equilibrium sorting pattern presented in Proposition 3 generally remains.\(^{15}\)

In Appendix F, we show that the key results of the model are largely unaffected if final good firms have the outside option of purchasing inputs from a spot market. While this reduces the intermediate good firm’s bargaining power, it does not affect our results that the hold-up friction under $(O, N)$ and $(O, S)$ is an increasing function of the final good firm’s productivity. Furthermore, it remains the case that the difference in hold-up friction between $(O, N)$ and $(O, S)$ decreases with productivity. As a result, Proposition 3 remains valid.

\section{Conclusion}

In this paper, we have identified a new industry-equilibrium mechanism through which a firm’s productivity affects its organizational choice. To unveil this mechanism, we have modified the Antràs-Helpman model by assuming that only intermediate good firms provide non-contractible inputs, but that these inputs can ex post be put to an alternative use. In such a setting, any intermediate good firm under outsourcing has the possibility of ex post forming an outside relationship with the idle threshold firm in the market. The presence of this ex post outside option, however, differentially affects the hold-up friction in the relationship depending on the final good

\(^{15}\)In Appendix E, we show that under a limited parameter range the sorting pattern is more complex than presented in Proposition 3.
firm’s productivity. All else equal, final good firms with low productivity face a smaller hold-up friction than final good firms with high productivity.

We derive two additional results in a two-country setting where wages are higher in the North, but Southern intermediate good firms ex post face extra costs of coordinating with the threshold firm. First, we show that a final good firm faces a higher hold-up friction if it outsources in the South than in the North. Second, we show that the extra hold-up friction of outsourcing in the South diminishes with a final good firm’s productivity. This latter result allows us to predict that only the highest productivity firms in an industry will choose to outsource in the South, while less productive firms will outsource in the North. Contrary to Antràs and Helpman (2004), this result is not linked to fixed cost differences across organizational forms, and therefore illustrates a new mechanism through which only the most productive firms in an industry decide to outsource internationally. Our results thus stress the need for empirical research to further investigate the link between a firm’s productivity and its organizational choice.

The driving force behind our results is that input specificity in the model is not driven by technology alone, but also depends on the final good firm’s productivity and on cross-border coordination costs. In future research, we plan to further explore this point in the following ways. First, the existing literature on within-industry firm heterogeneity (including this paper) assumes that final good firms differ in their productivity, but intermediate good firms are symmetric. An important extension of our model should also allow for heterogeneity in intermediate good firms’ productivity. This approach will likely unveil more sophisticated predictions of the determinants of input specificity and a firm’s organizational choice. Second, it would also be interesting to explore the effects of other sources of heterogeneity such as the degree of technological specificity of suppliers. A third promising extension is to link relationship-specificity to the notion of ‘power’ developed by Rajan and Zingales (1998). Rajan and Zingales show that the distribution of power between a firm and its suppliers critically depends on the firm’s control of access to its critical resource and on the suppliers’ outside options. Introducing Rajan and Zingales’ concept of power in our model will allow us to gain insights into the within-industry distribution of power between final good firms and their domestic and foreign suppliers.

To conclude, we view our analysis as part of a larger effort to better understand the role of within-industry firm heterogeneity on transaction costs in an industry-equilibrium setting.
4 Appendix

A. Vertical integration

Here we demonstrate that integration \((I, l)\) is always dominated by outsourcing \((O, l)\) in our model. Under \((I, l)\), the final good firm has the residual rights over the inputs. We follow Antràs and Helpman (2004) by assuming that, if a relation breaks down, the final good firm has the power to fire the intermediate good firm and seize a fraction \(\delta \in [0, 1]\) of inputs. Using equations (1) and (2), it is straightforward to derive that this provides the final good firm with an outside option of \(V_I^l = \delta^\alpha R_I^l\). The intermediate good firm does not have an outside option since it has no residual rights over the inputs it produces. As a result, \(v_I^l = 0\). By inserting these outside options into (4), the intermediate good firm’s revenue share under \((I, l)\) amounts to \(s_I^l = \frac{1}{2}(1 - \delta^\alpha)\). By comparing with equations (9) and (13), it is straightforward to derive that the intermediate good firm’s revenue share is strictly smaller under integration than under outsourcing: \(s_I^l < s^l\). From equation (21), we can then conclude that integration \((I, l)\) is dominated by outsourcing \((O, l)\).

B. Proof of existence of outside option under \((O, N)\)

Here we provide a formal proof that, under \((O, N)\), any intermediate good firm (regardless of its original partner’s productivity) has a strictly positive outside option. In the text, we have derived that, if the intermediate good firm and threshold firm with productivity \(\theta^\bar{\phantom{\overline{\phantom{\bar{\phantom{\bar{}}}}}}}_\) agree to ex post form an outside relation, they each receive half of outside revenue: \(\frac{1}{2}R^N = \frac{1}{2} \left( \frac{\theta}{\bar{\theta}} \right)^\alpha R^N\). To complete our proof, we need to show that both parties are willing to form such an outside relation.

When forming an outside relation with the threshold firm, the intermediate good firm offers lump-sum transfer \(T\) that guarantees the threshold firm’s participation:

\[
\max_T \pi^N = \frac{1}{2} \left( \frac{\theta}{\bar{\theta}} \right)^\alpha R^N - T
\]

subject to

\[
\frac{1}{2} \left( \frac{\theta}{\bar{\theta}} \right)^\alpha R^N - F + T \geq 0.
\]

The solution of this maximization problem suggests that a Northern intermediate good firm is willing to participate in an outside relation if and only
if:
\[
\left( \frac{\theta}{\tilde{\theta}} \right)^\alpha R^N \geq F.
\]

Using equations (3), (9), and (19), it is straightforward to derive that the left-hand side of the condition decreases with \( \theta \). The condition will therefore hold for any Northern intermediate good firm if and only if:

\[
R^N(\theta) \geq F. \tag{B-1}
\]

We can use the threshold firm’s zero-profit condition to derive \( F \). Proposition 3 suggests that, in industry equilibrium, the threshold firm is indifferent between \((O,N)\) and remaining idle. As a result,

\[
F = (1 - \alpha s^N(\theta)) R^N(\theta). \tag{B-2}
\]

Inserting (B-2) into (B-1) and rearranging suggests that any Northern intermediate good firm has a strictly positive outside option if and only if \( s^N(\theta) \geq 0 \). This is always the case and thus completes the proof.

### C. Derivation of Assumption 1

We derive here the condition under which, under \((O,S)\), any Southern intermediate good firm (regardless of its original partner’s productivity) has a strictly positive outside option. By taking the same steps as in the previous proof, it is straightforward to derive that this will be the case under the following condition:

\[
\tau^\alpha R^S(\theta) \geq F. \tag{C-1}
\]

By inserting equation (B-2) into (C-1) and using equations (3) and (19), we can rearrange the condition to the following form:

\[
\omega \geq \frac{s^N(\theta)}{s^S(\theta)} \left( \frac{1 - \alpha s^N(\theta)}{\tau^\alpha} \right)^{\frac{1-\alpha}{\alpha}}. \tag{C-2}
\]

From equation (9) and (13), we can calculate that \( s^N(\theta) = \frac{3}{4} \) and \( s^S(\theta) = \frac{1}{2} + \frac{\tau^\alpha}{4} \). Inserting these shares into (C-2) then allows us to derive that any Southern intermediate good firm will in industry equilibrium have a positive outside option as long as:

\[
\omega \geq \frac{3}{2 + \tau^\alpha} \left( \frac{4 - 3\alpha}{4\tau^\alpha} \right)^{\frac{1-\alpha}{\alpha}}.
\]
D. Derivation of Proposition 3

We start off by deriving key properties of curve \( A(\theta) \) provided in equation (22). Straightforward algebra delivers that \( A(\theta) > 1 \) for all \( \theta \geq \theta \bar{\theta} \) and that \( \lim_{\theta \to +\infty} A(\theta) = 1 \). Furthermore, \( A'(\theta) \leq 0 \) for all \( \theta \geq \theta \bar{\theta} \) if and only if

\[
(1 - \alpha s^N)(1 - s^S)s^N \tau^\alpha - (1 - \alpha s^S)(1 - s^N)s^N \geq 0.
\]

Inserting (9) and (13) into the preceding condition entails that \( A'(\theta) \leq 0 \) if and only if:

\[
4(2 - \alpha) \left( \frac{\theta}{\theta \bar{\theta}} \right)^{2\alpha} - 2(2 - \alpha)(1 + \tau^\alpha) \left( \frac{\theta}{\theta \bar{\theta}} \right)^\alpha - (2 - 3\alpha)\tau^\alpha \geq 0 \tag{D-1}
\]

This condition will hold for all \( \theta \geq \theta \bar{\theta} \) as long as:

\[
\tau^\alpha \leq \frac{2(2 - \alpha)}{6 - 5\alpha}.
\]

Conversely, if condition (D-1) does not hold, \( A(\theta) \) will first increase with \( \theta \) and then decrease with \( \theta \), with a unique maximum at \( \theta^{\text{max}} \), which solves:

\[
\left( \frac{\theta^{\text{max}}}{\theta} \right)^\alpha = \frac{1}{4} \left( 1 + \tau^\alpha + \sqrt{(1 + \tau^\alpha)^2 + (2 - 3\alpha)\tau^\alpha} \right) \tag{D-2}
\]

Notice that the value of \( \theta^{\text{max}} \) cannot exceed \( \theta \) by much. Specifically, the maximum value that \( \theta^{\text{max}} / \theta \) can take is 1.06. This will be the case when \( \alpha \) approaches zero and \( \tau \) approaches 1.

We can use these properties to derive Proposition 3. Consider first the scenario where condition (D-1) holds such that \( A'(\theta) \leq 0 \) for all \( \theta \geq \theta \bar{\theta} \). The assumption that \( A(\theta) \geq \omega \) and the property of the \( A(\theta) \) curve that \( \lim_{\theta \to +\infty} A(\theta) = 1 \) then implies that there exists a unique cutoff productivity \( \theta_1 \) for which \( A(\theta_1) = \omega \).

Next, consider the case where condition (D-1) does not hold so that \( A(\theta) \) first slopes upward in \( \theta \) and then slopes downward. The assumption that \( A(\theta) \geq \omega \) and the properties that \( A(\theta) \) has a unique maximum and that \( \lim_{\theta \to +\infty} A(\theta) = 1 \) then confirms that there exists a unique cutoff productivity \( \theta_1 \) for which \( A(\theta_1) = \omega \). This completes the proof of Proposition 3.

To finalize our analysis, we need to derive the equilibrium sorting pattern when \( A(\theta) < \omega \). To start off, it is easy to show the equilibrium sorting
pattern of organizational forms when condition (D-1) holds. In that case, \( A(\theta) < \omega \) and \( A'(\theta) \leq 0 \) for all \( \theta \geq \theta \) so that \( A(\theta) < \omega \) for all \( \theta \geq \theta \). As a result, \((O,S)\) is the optimal organizational form for all final good firms.

We get a similar result when condition (D-1) does not hold, but \( A(\theta_{\text{max}}) < \omega \). In that case, the \( A(\theta) \) curve rises with \( \theta \) for \( \theta \leq \theta_{\text{max}} \), but the unique maximum at \( A(\theta_{\text{max}}) \) does not exceed \( \omega \). As a result, \( A(\theta) < \omega \) for all \( \theta \geq \theta \) so that \((O,S)\) is the optimal organizational form for all final good firms.

The only exception occurs when condition (D-1) does not hold and \( A(\theta_{\text{max}}) \geq \omega > A(\theta) \). In that case, \( A(\theta) \) equals \( \omega \) for two values of \( \theta \) so that the model features a more complex sorting pattern. Denote these two thresholds by \( \theta'_1 \) and \( \theta_1 \), where \( \theta < \theta'_1 < \theta_1 \). As in Proposition 3, firms will choose \((O,N)\) when \( \theta < \theta_1 \) and \((O,S)\) when \( \theta \geq \theta_1 \). The only difference is that for the lowest productivity firms, \( \theta \in [\theta, \theta'_1] \), \((O,S)\) emerges as the optimal organizational form.

### E. Relaxing Assumption 1

We prove here that the key results of our model do not depend on assumption 1. Suppose that assumption 1 is relaxed so that it is not the case that all Southern intermediate good firms have a positive outside option. Denote \( \theta_0 > \theta \) such that

\[
\tau^\alpha R^S(\theta_0) = F.
\]

Our analysis in Appendix C then suggests that the Southern intermediate good firm that collaborates with the firm with \( \theta_0 \) is ex post indifferent between forming an outside relation or remaining idle. Furthermore, Southern intermediate good firms that collaborate with firms with \( \theta \geq \theta_0 \) ex post have an outside option, while those that work for firms with \( \theta \leq \theta < \theta_0 \) have no outside option. Specifically,

\[
v^S = \begin{cases} 
0 & \text{if } \theta < \theta_0 \\
\frac{1}{2} (\frac{\tau \theta}{\tau})^\alpha R^S & \text{if } \theta \geq \theta_0.
\end{cases}
\]

By inserting (12) and (E-1) into (4) and rearranging, the Southern intermediate good firm’s revenue share then is:

\[
s^S = \begin{cases} 
\frac{1}{2} + \frac{1}{4} (\frac{\tau \theta}{\tau})^\alpha & \text{if } \theta < \theta_0 \\
\frac{1}{2} + \frac{1}{4} (\frac{\tau \theta}{\tau})^\alpha & \text{if } \theta \geq \theta_0.
\end{cases}
\]

The intermediate good firm’s revenue share is strictly lower when it does not have an outside option, since an outside option tilts its bargaining power in
its favor. Furthermore, by taking into account that outside revenue $R^S = 0$ for $\theta \leq \theta_0$, equation (14) can be used to show that input specificity under $(O, S)$ equals:

$$d^S = \begin{cases} 
1 & \text{if } \theta < \theta_0 \\
1 - \left(\frac{r\theta}{\bar{\theta}}\right)^\alpha & \text{if } \theta \geq \theta_0
\end{cases}$$

(E-3)

For $\theta \leq \theta < \theta_1$, inputs are completely specific since the Southern intermediate good firm does not have sufficient incentives to put its inputs to an outside use. This implies that the input specificity for these intermediate good firms is strictly higher than for the firms with an outside option.

While Proposition 1 and Corollary 1 no longer hold, equations (15) and (E-3) can be used to illustrate that Proposition 2 and Corollary 2 still hold. The difference in input specificity under $(O, S)$ relative to $(O, N)$ declines with the final good firm’s productivity. Furthermore, the extra hold-up friction under $(O, S)$ relative to $(O, N)$ declines with the final good firm’s productivity.

Finally, the equilibrium sorting pattern in Proposition 3 remains largely robust to relaxing Assumption 1. To see this, we start off by discussing the impact on the properties of $A(\theta)$. A number of key properties of curve $A(\theta)$ remain unchanged. Using equations (9), (22) and (E-2), it is easy to show that $A(\theta) > 1$ for all $\theta \geq \theta$ and that $\lim_{\theta \to +\infty} A(\theta) = 1$. Furthermore, under condition (D-1), $A'(\theta) \leq 0$ for all $\theta \geq \theta$. To see this latter property, it is straightforward to derive that

$$A'(\theta) \leq 0 \text{ for all } \theta \in [\bar{\theta}, \theta_0].$$

(E-4)

Since curve $A(\theta)$ discontinuously shifts down at $\theta_0$, this implies that there remains a unique cutoff productivity $\theta_1$ such that final good firms with $\theta \in [\bar{\theta}, \theta_1]$ choose $(O, N)$, while final good firms with $\theta \in [\theta_1, +\infty)$ choose $(O, S)$.

If condition (D-1) does not hold, the equilibrium sorting pattern is also generally in line with Proposition 3. As we have seen in Appendix D, if all intermediate good firms have a strictly positive outside option, the curve $A(\theta)$ first rises with $\theta$ and then declines with a unique maximum at $\theta_{max}$. If $\theta_0 \geq \theta_{max}$, however, the upward sloping portion of the slope is replaced by equation (E-4). As a result, there remains a unique cutoff productivity $\theta_1$ such that final good firms with $\theta \in [\bar{\theta}, \theta_1]$ choose $(O, N)$, while final good firms with $\theta \in [\theta_1, +\infty)$ choose $(O, S)$. The only scenario where Proposition 3 might not hold is if $\theta_0 < \theta_{max}$. In that case, there is a very small parameter range where with rising productivity, the following sorting pattern might occur: $(O, N), (O, S), (O, N), (O, S)$. 

23
Finally, when $A(\theta) < \omega$, we get the simple prediction that $(O, S)$ is always the optimal organizational form. This is because the downward shift of the $A(\theta)$ curve at $\theta_0$ is always larger than $A(\theta_{max}) - A(\theta)$.

**F. Introduction of a spot market for inputs.**

We demonstrate here that the key results of the model are largely unaffected if we follow Spencer and Qiu (2001), Qiu and Spencer (2002) Head et al. (2004) and Feenstra and Spencer (2006) by assuming that the outside option for the final good firm is to purchase inputs from a spot market at a higher unit price $\rho > \omega$. We impose a number of restrictions on this extension of the model. First, we assume that $\rho$ is sufficiently high so that any final good firm prefers to form an outsourcing relation over purchasing inputs on the spot market. Second, the intermediate good firm ex post favors forming an outside relation with the threshold firm over selling its inputs on the spot market. Finally, the final good firm ex post purchases the same amount of inputs from the spot market as the intermediate good firm had originally produced. In that case, the final good firm’s outside option equals the operating profits of purchasing inputs at price $\rho$ rather than $\omega^l$:

$$V^l = R^l(1 - \alpha \zeta^l) \left( \frac{\omega^l}{\rho} \right)^{\frac{\alpha}{1-\alpha}} \quad \text{(F-1)}$$

where $\zeta^l$ equals the intermediate good firm’s revenue share in the inside relation. Inserting (F-1), (7) and (11) into (4) and rearranging,

$$\zeta^l = \frac{s^l - \frac{1}{2} \left( \frac{\omega^l}{\rho} \right)^{\frac{\alpha}{1-\alpha}}}{1 - \frac{\alpha}{2} \left( \frac{\omega^l}{\rho} \right)^{\frac{\alpha}{1-\alpha}}} \quad \text{(F-2)}$$

where $s^l$ equals the revenue share depicted in equations (9) and (13). From equation (F-2), it is easy to derive that $\zeta^l < s^l$. The intuition behind this is that the positive outside option for the final good firm tilts the bargaining power away from the intermediate good firm, thus reducing the intermediate good firm’s revenue share.

A number of key results of our model remain unaffected. First, the intermediate good firm’s revenue share under both $(O, N)$ and $(O, S)$ decreases with productivity. In line with Corollary 1, this implies that the hold-up

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16The fixed cost $F$ has already been sunk and therefore does not feature in the final good firm’s outside option.
friction under \((O, N)\) and \((O, S)\) is an increasing and concave function of the final good firm’s productivity. Second, the difference in the intermediate good firm’s revenue share between \((O, N)\) and \((O, S)\) decreases with productivity. In line with Corollary 2, this suggests that the extra hold-up friction of \((O, S)\) declines with the final good firm’s productivity \(\theta\).

A difference from before, however, is that the intermediate good firm’s revenue share is not necessarily larger under \((O, N)\) than \((O, S)\) for all \(\theta \in [\theta, \theta]\). It is straightforward to show from (F-2) that \(\zeta^N(\theta) \geq \zeta^S(\theta)\) if and only if the following condition holds:

\[
\tau^\alpha \leq 1 - \frac{(\omega - 1)(4 - 3\alpha)}{2\rho - \alpha\omega}.
\]

Furthermore, as \(\theta\) approaches infinity, \(\zeta^N < \zeta^S\). The intuition comes from equation (F-1). Since the final good firm’s outside option \(V^l\) is a larger share of original revenue \(R^l\) under \((O, N)\) than under \((O, S)\), it tilts the bargaining power away from the intermediate good firm more under \((O, N)\) than under \((O, S)\). This further reinforces the results that for firms with a sufficiently high productivity level, \((O, S)\) always is the optimal organizational form.

Finally, straightforward algebra delivers that \(A(\theta) > 1\) for all \(\theta \in [\theta, +\infty]\) and that \(\lim_{\theta \to +\infty} A(\theta) = 1\). Furthermore, \(A'(\theta) \leq 0\) for all \(\theta \in [\theta, +\infty]\) if and only if

\[
(1 - \alpha\zeta^N)(1 - \zeta^S)\zeta^N \tau^\alpha - (1 - \alpha\zeta^S)(1 - \zeta^N)\zeta^S \geq 0.
\]

If this condition is not upheld, curve \(A(\theta)\) will first rise in \(\theta\) and then decline with a unique maximum. In line with Proposition 3, as long as \(A(\theta) \geq \omega\), there then exists a unique cutoff productivity \(\theta_1\) such that final good firms with \(\theta \in [\theta, \theta_1]\) choose \((O, N)\), while final good firms with \(\theta \in [\theta_1, \infty]\) choose \((O, S)\).
References


Figure 1: Degree of input specificity
Figure 2: Hold-up friction
Figure 3: Equilibrium sorting pattern when $A(\theta) \geq \omega$. 