Malthus to Solow: A New Classical Approach to Induced Innovation

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Abstract

An important question in economic development is how European countries made the transition from a Malthusian economy, in which living standards were largely determined by land-labor ratios, to a modern Solow economy, in which living standards were delinked from land scarcity and linked to capital accumulation. In this paper, we set up a new classical in which population pressure leads to an endogenous change from a Malthusian to a Solowian production technique through vertical division of labor and the creation of land-saving intermediate goods. This approach provides new microfoundations for the transition from Malthus to Solow that are based on the concept of induced innovation.

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Keywords: division of labor, induced technical change, structural change, Boserupian transition

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Introduction

One of the fundamental questions in economic development is how European and Asian countries made the structural transition from a Malthusian economy in which living standards were largely determined by land-labor ratios, to a modern Solow economy in which living standards were unlinked from land scarcity and linked to capital accumulation. This question arises from the observation that since 1700 the long-run trend in the ratio of wages to land rents took on a u-shape in Europe despite continued population growth (O'Rourke and Williamson, 2002). As shown in figure 1, prior to the 19th century there has been a long period during which the wage-land rent ratio declined, a trend consistent with Malthus' theory of population pressure on a quasi-fixed land endowment (Malthus, 1928). During the first half of the 19th century, however, this pattern reversed and wages started to rise relative to land rent despite continued population growth. This trend is consistent with the Solow theory that in the aftermath of the industrial revolution capital and labor were the two primary inputs into aggregate production (Solow, 1956).

[figure 1 and 2 about here]

The reversal of the wage-rent ratio has often been attributed to the structural transformation from agriculture to manufacturing and services at the advent of the industrial revolution. A number of growth studies have recently attempted to capture this transformation by incorporating non-homothetic preferences, sector-specific technical change or a combination of both into standard neoclassical growth models.[1] Hansen and Prescott (2002) build a one-good Diamond growth model with two asymmetric production technologies that are perfectly substitutable and with sector-specific technical change. The Malthus technology, which represents agricultural production on family farms, uses land, labor and reproducible capital as inputs. The Solow technology, which represents factory production, only uses labor and capital to produce the same good. In this setting, exogenous technological progress in the Solow sector is found to speed up the transition while exogenous technological progress in the Malthus sector delays the transition.[2] This last

[1] We use the terms technique and technology as in Atkinson and Stigliz (1969). A technique is a blue print describing how inputs can be combined to produce a certain amount of output. Technology is the set of available techniques.

[2] See also Goodfriend and McDermott, 1995; Lucas, 2002; and Ngai, 2000.

implication is in sharp contrast with historical experience, since several economic historians have documented that the Industrial Revolution in England was either preceded or accompanied by an agricultural revolution (e.g. Allen, 2000; Overton, 1996).[3] Structural transformations in many Asian countries also started with a period of rapid productivity growth in agriculture.

A number of dual economy studies have attempted to salvage the role of an agricultural revolution on the structural transformation by stepping away from a one-good model and assuming non-homothetic preferences over two consumer goods, an agricultural good and a non-agricultural good. Jorgenson (1961) illustrated that a low income elasticity for the agricultural good combined with exogenous technological progress in the agricultural sector can induce a transition. Technological progress in the agricultural sector increases income, thus raising demand for the nonagricultural good more than demand for the agricultural good and therefore leading to a transfer of labor from the agricultural sector to the non-agricultural sector. Similarly, Gollin, Parente and Rogerson (2002) illustrate that a combination of Stone-Geary preferences and technological progress in the agricultural sector can replicate this result in a two-sector neoclassical growth setting. Once per capita output in the agricultural sector reaches a critical level, all remaining labor will flow out of agriculture regardless of the state of the nonagricultural sector. Yang and Zhu (2004) demonstrate that a combination of Stone-Geary preferences and productivity growth in the nonagricultural sector can induce both an agricultural revolution and a structural transformation through feedback effects of nonagricultural growth on agriculture. In their model, they distinguish between two types of agricultural technology: a traditional technology that solely uses land and labor, and a modern technology that uses land, labor and intermediate goods produced in the non-agricultural sector. Nonagricultural productivity growth induces both an agricultural revolution through the reduction in the price of intermediate goods and a transfer of labor from agriculture to industry due to the income effect.

We also take as a starting point to this paper that an agricultural revolution preceded the industrial revolution. However, distinct to the existing literature, our paper uses a single-sector agricultural model to explain the transition from Malthus to Solow. In particular, we set up a model in which exogenous population pressure can induce technical and organizational change in

[3] Clark (2002) questions that there was an agricultural revolution either alongside or before the Industrial Revolution.

the agricultural sector that leads to both an increase in the share of capital in aggregate production and a decrease in the share of land in aggregate production. This setup thus allows us to focus on the type of organizational and technological changes in agricultural production required to instigate an industrial revolution.

In setting up our model, we draw from two separate streams of literature that have provided insights into the mechanisms through which population pressure might induce technical and organizational change in agricultural production. A first set of studies is based on Boserup's (1981) and Lee's (1986) intuition that increased population density can have a positive impact on economic development through improved economic and social infrastructure. Boserup asserts that before the Industrial Revolution, the main advantage of a dense population was "the better possibilities to create infrastructure" (Boserup, 1981 p. 129). She argued that the irrigation technology for agriculture (p. 66), the building and maintenance of roads (p. 67), the canalization of a river (pp. 68, 97), and the laying of the railroad system (p. 132) were all possible only with the support of a large population. Krautkraemer (1994) formalized Boserup's hypothesis by identifying irrigation infrastructure as an important source of nonconvexity in the agricultural production function. In his model, the population is initially too small to exploit the economies of scale inherent to an irrigation project, but once the population reaches a certain critical value, the more labor intensive irrigated agriculture becomes the more efficient mode of production. Lee (1986) emphasized the role of division of labor in the Boserupian transition: "the larger the population engaged in non-food producing activities, the greater the possible division of labor and the greater the possibilities of technological advance." Chu (1997) and Chu and Tsai (1998) have built new classical general equilibrium models in the spirit of Yang Xiaokai to formalize this idea.[4] In their models, there is a fixed cost to infrastructure investment much in the same way as modeled by Krautkraemer. As population reaches a threshold where investment is wortwhile, division of labor occurs and some people move out of the agricultural sector into the professional infrastructure sector. In turn, the increased labor force in the infrastructure sector results into an increase in transaction efficiency in society, thus improving labor productivity and further inducing economic development.

[4]Yang Xiaokai (2001, 2003) defines New Classical Economics as general equilibrium models with endogenous specialization and division of labor

The theory of induced innovation, on the other hand, focuses on the effect of factor endowments on the direction of technical change (Hicks, 1932; Kennedy, 1964; Hayami and Ruttan, 1970; Acemoglu, 2002). In this literature, a change in factor endowments induces a change in the relative prices of the factors of production, thus spurring firms and households to invent and implement technologies that are directed at economizing the use of a factor which has become relatively expensive.[5]

In this paper, we combine the ideas of induced innovation and vertical division of labor to explain the transition from Malthus to Solow. We consider an agricultural society with many *ex ante* identical consumer-producers who derive utility from a single consumption good, food. Each individual's food production function uses labor, land and land-saving capital (fertilizer) as inputs. Each individual's production function of land-saving capital (fertilizer) uses labor as its sole input. Initially, the economy resides in autarky and no capital is being produced. Each individual uses its endowment of labor and land to produce agricultural output solely for own consumption. As the population exogenously expands, the wage-rent ratio falls and, much as in the theory of induced innovation, this induces division of labor with a number of people moving out of food production into the production of previously unused land-saving capital. As population pressure continues, more people move out of food production and into capital production, thus increasing the share of capital in aggregate production and reducing the share of land. This paper thus contributes to our understanding of the transition from a Malthusian economy to a Solow economy, by emphasizing the intricate importance of Smith's vertical division of labor and the theory of induced innovation in the process.

The Model

Consider an agricultural society with a continuum of N *ex ante* identical individuals who are both consumers and producers. The consumer-producers derive utility from a single consumption good, food y, which uses as inputs labor l, land h and capital k. Capital can be produced out of labor.

[5] As noted by Olmstead and Rhode (1993), the theory of induced technological change is ambiguous regarding whether it is the change in factor prices or their levels that matters. Clearly, either is compatible with the theory. If one is explaining the evolution, it is the changes that are more germaine. in either case, however, the theory would predict that the factor that is relatively more expensive (than that in the other country or that which previously prevailed) is the one that is "saved".

Each individual in the society is endowed with a given quantity of labor which we normalize to one, and an equal fraction of the primary resource land $\frac{1}{N}$. For simplicity, we assume that each individual at all times completely specializes in a single task.

Individual utility is given by:

$$u = y + ty^d \tag{1}$$

where y is the self-provided quantity and y^d is the quantity of the good purchased from the market. Transactions efficiency is represented by the parameter t that shows the fraction of goods received from purchasing one unit of final goods. The transaction costs per unit of final good purchased are thus given by $(1 - t)$. This is sometimes referred to as ice-berg transaction costs.

Individuals face the following nested production function for food:

$$y^p = y + y^s = Y(A(k + tk^d, h + th^d), l_y) \tag{2}$$

y^p stands for the amount of food produced, y stands for the self-provided quantity of food and y^s stands for the amount of food that the individual sells to the market. Food production is a function of three inputs: the amount of labor that an individual allocates to food production l_y; the amount of self-provided land h and the amount of land bought from the market h^d; and the amount of self-provided capital k and the amount of capital bought from the market k^d. We assume that the composite input A exhibits constant returns to scale and diminishing marginal returns, and that the traditional Inada conditions apply.

Individuals face the following production function for capital:

$$k^p = k + tk^s = X(l_k) \tag{3}$$

In other words, total capital production by an individual k^p is a function of the amount of labor l_K allocated to its production.

All *ex ante* identical consumer-producers have the same initial endowments of labor and land:

$$l = l_k + l_y = 1 \tag{4}$$

$$h = \frac{1}{N} \tag{5}$$

We identify two possible economic structures for this agricultural society: autarky and division of labor. In *autarky*, each individual produces agricultural output solely for own consumption. As illustrated in figure 3, in this case no capital is produced and no trade between individuals occurs. With *division of labor*, the *ex ante* identical consumers divide themselves into two groups. A portion of the population produces food y, while the other portion produces capital k. As illustrated in figure 1, there is an emergence of trade under this economic structure. In particular, three markets emerge: a market for food, land and capital. This is because food producers will sell food for capital and land, while capital producers will sell capital and land for food.

[figure 3 about here]

The equilibrium industry structure is determined by a two-step procedure. In step one, individuals independently choose their professions. In step two, the individuals select the utility maximizing levels of output given the profession chosen. As usual, the problem is solved through backward induction.

1 Autarky

In autarky each individual produces agricultural output only for own consumption. In that case, there is no division of labor and trade between individuals. Our assumption that each individual at all times completely specializes in a single task implies that there is no capital production. As a result, the optimization problem that we set up above reduces to the following problem:

Maximize

$$u^A = y \tag{6}$$

subject to

$$y = Y\left(A\left(0, \frac{1}{N}\right), 1\right) = F\left(0, \frac{1}{N}\right) \tag{7}$$

As in autarky all resources are devoted to the production of the final good. As a result $l_y = 1$ and

$z = \frac{1}{N}$. This implies that the autarky level of utility is:

$$u^{A*} = F\left(0, \frac{1}{N}\right) \tag{8}$$

Note that the equation above complies to the standard Malthusian result that living standard for each person u^{A*} declines as the population expands because each individual is now left with a smaller endowment of land.

2 Division of Labor

Under division of labor, the *ex ante* identical consumer-producers divide themselves into two sectors. A portion of the population specializes in the production of food y, while the other portion specializes in the production of capital k. Under this structure, three markets emerge in the society that did not exist under autarky: a market for food, land and capital. This is because food producers now use their income from selling a portion of their food output y^s to buy capital k^d and land h^d. Capital producers, on the other hand, use their income from selling capital k^s and land h^s for food y^d.

Individuals maximize their utility with respect to the quantities of the goods produced, traded and consumed given the relative prices. The market clearing conditions in the three markets and the utility equalization condition between food producers and capital producers ultimately allow us to determine the equilibrium. Note that the utility equalization condition allows us to endogenously determine the division of labor. If capital producers gain a higher (lower) level of utility than food producers, then food producers will try to arbitrage away this discrepancy by moving into the capital (food) sector. This arbitrage ultimately determines the share of individuals in each sector.

2.1 Food Producers

In each profession, individuals choose their levels of production and trade to maximize utility. We start off with the individuals in the food sector.

Maximize

$$u^Y = y \tag{9}$$

subject to

$$y^p = y + y^s = F\left(tk^d, \frac{1}{N} + th^d\right) \tag{10}$$

and

$$p_k k^d + p_h h^d = y^s \tag{11}$$

Food producers do not produce capital. They use all of their initial endowments to produce food. As a result, $k^s = 0$, $h = \frac{1}{N}$, $l_y = 1$, $p_y = 1$

If we maximize utility with respect to the demand for capital, the demand for land and the supply of food, we gain the following first-order conditions:

$$tF_k - p_k = 0 \tag{12}$$

$$tF_h - p_h = 0 \tag{13}$$

$$p_k k^d + p_h h^d = y^s \tag{14}$$

If we combine the first two first-order conditions:

$$\frac{F_k}{F_h} = \frac{p_k}{p_h} \tag{15}$$

If we plug the equation above into the first two first-order conditions:

$$p_k = tF_k \tag{16}$$

$$p_h = tF_h \tag{17}$$

Plug into budget constraint:

$$y_s = t(F_k k^d + F_h h^d) \tag{18}$$

This gives us utility of a food producer under specialization.

$$u^{Y*} = F\left(tk^d, \frac{1}{N} + th^d\right) - t(F_k k^d + F_h h^d) \tag{19}$$

2.2 Capital Producers

Capital producers specialize in the production of capital. They use the income of selling their capital k and their share of the land $\frac{1}{N}$ to buy food y. Capital producers face the following optimization problem:

Maximize

$$u^F = ty^d \tag{20}$$

subject to the capital production function

$$K = X(1) = X \tag{21}$$

and the budget constraint

$$Y^d = p_k k^s + p_h h^s \tag{22}$$

As capital producers use all their endowment of labor to produce capital, and sell all of their land to the food producers, the optimization problem yields to the following results:

$$K^{s*} = X \tag{23}$$

$$H^{s*} = \frac{1}{N} \tag{24}$$

$$Y^{d*} = p_k X + \frac{p_h}{N} = tF_k X + \frac{tF_h}{N} \tag{25}$$

$$u^{F*} = t^2(F_k X + \frac{F_h}{N}) \tag{26}$$

2.3 Market Clearing Conditions

In order to determine the equilibrium prices and the share of people in each sector, we need to solve for the market clearing conditions and the utility equalization condition. We have three market clearing conditions:

$$\alpha N k^{d*} = (1-\alpha) N k^{s*} \tag{27}$$

$$\alpha N h^{d*} = (1-\alpha) N h^{s*} \tag{28}$$

$$\alpha N y^{s*} = (1-\alpha) N y^{d*} \tag{29}$$

Note that α is the share of the population in the food sector and $1-\alpha$ is the share of the population in the capital sector.

The three market clearing conditions give us the following equalities:

$$k^{d*} = \frac{1-\alpha}{\alpha} X \tag{30}$$

$$h^{d*} = \frac{(1-\alpha)}{\alpha N} \tag{31}$$

2.4 Utility Equalization

As all individuals are *ex ante* identical and have the freedom to move between professions, all individuals at all times have the same level of utility. This leads to the final condition that closes the model:

$$F\left(tX, \frac{1}{N}\left(\frac{\alpha}{1-\alpha} + t\right)\right) = t\left(F_k X + \frac{F_h}{N}\right)\left(\frac{\alpha}{1-\alpha} t + 1\right) \tag{32}$$

2.5 Population Pressure and the Extent of Division of Labor

The main interest of this paper is the impact of population pressure on the organization of agricultural production. In our model, we treat population size as exogenous and simply assume that population increases due to improvements in public health.

We can use comparative statics on equation (32) to analyze the impact of population pressure on the share of people working in the capital sector α. An increase in population will increase the share of the total population working in the capital sector if the following condition holds:

$$\frac{d\alpha}{dN} = \frac{-F_h \alpha (1-\alpha)(1-t^2)}{N(t^2 F_k X N - (1-t^2) F_h)} \tag{33}$$

This will be the negative if:

$$\frac{F_K}{F_H} > \frac{1-t^2}{t^2} \frac{1}{N} \frac{1}{X}$$

This condition indicates that for population pressure to lead to an increase in the share of people working in the capital sector, (1) population is large; (2) transaction efficiency is high; (4)

productivity in the capital sector is high; (3) relative marginal product of capital to land is large.

A related question is whether population growth increases the share of capital in aggregate production Y^T. We define the share of capital in aggregate production as:

$$\frac{K^T}{Y^T} = \frac{(1-\alpha)NK^p}{\alpha NY^p} = \frac{X}{F(tX, \frac{1}{N}(\frac{\alpha}{1-\alpha}+t))} \tag{34}$$

$$\frac{d\left(\frac{K}{Y}\right)}{d\alpha} = \frac{-F_h X}{(1-\alpha^2)NF(tX, \frac{1}{N}(\frac{\alpha}{1-\alpha}+t))} < 0 \tag{35}$$

This implies that, as population increases, the share of land-saving capital in aggregate production increases and the share of land decreases in all circumstances. This is consistent with the idea that European countries made the transition from a Malthusian economy, in which livings standards were largely determined by land-labor ratios, to a modern Solow economy, in which living standards were delinked from land scarcity and linked to capital accumulation.

3 Comparing Autarky and Division of Labor

In stage 1, individuals need to decide which production structure they want to use to produce the agricultural product.

The general equilibrium of the economy is either autarky or specialization and division of labor. Given the various parameter values, individuals will choose the configuration that maximizes their utility, i.e.

$$U_D^* > U_A^*$$

.

We can use equations (8) and (26) to determine this condition:

$$t^2\left(F_k X + \frac{F_h}{N}\right) > F\left(0, \frac{1}{N}\right) \tag{36}$$

From this condition, we can determine whether or not population pressure can induce division of labor. In order to do so, we need to differentiate the left-hand side and the right-hand side with respect to N. If utility under division of labor reduces less than under autarky, then there is a possibility of transition.

An increase in population will eventually induce a change in economic structure from autarky to division of labor if transaction costs are low and if the marginal product of land is sufficiently reduced by switching to division of labor.

4 Conclusion

An important question in economic development is how European and Asian countries made the transition from a Malthusian economy, in which living standards were largely determined by land-labor ratios, to a modern Solow economy, in which living standards were delinked from land scarcity and linked to capital accumulation. In this paper we have addressed this issue by considering the linkage between changes in factor prices, division of labor and land-saving technical change. We find that as population increases more people move from the food production into the previously nonexistent land-saving capital production, thus increasing the share of capital in aggregate production and reducing the share of land. This approach provides new microfoundations to why population pressure might induce economic growth through division of labor.

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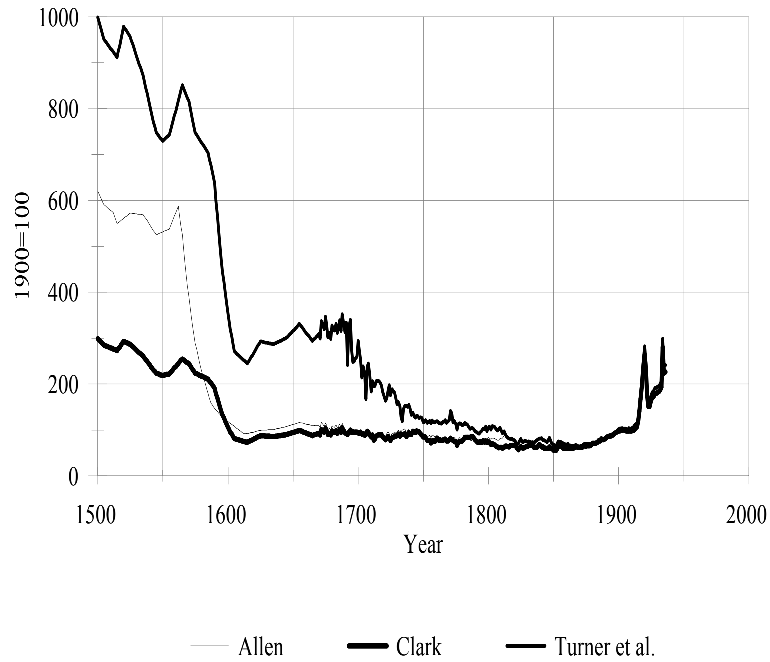

Figure 1: Long-run wage-rent ratio: England 1500-1936, 1900=100 (source: O'Rourke and Williamson, 2002)

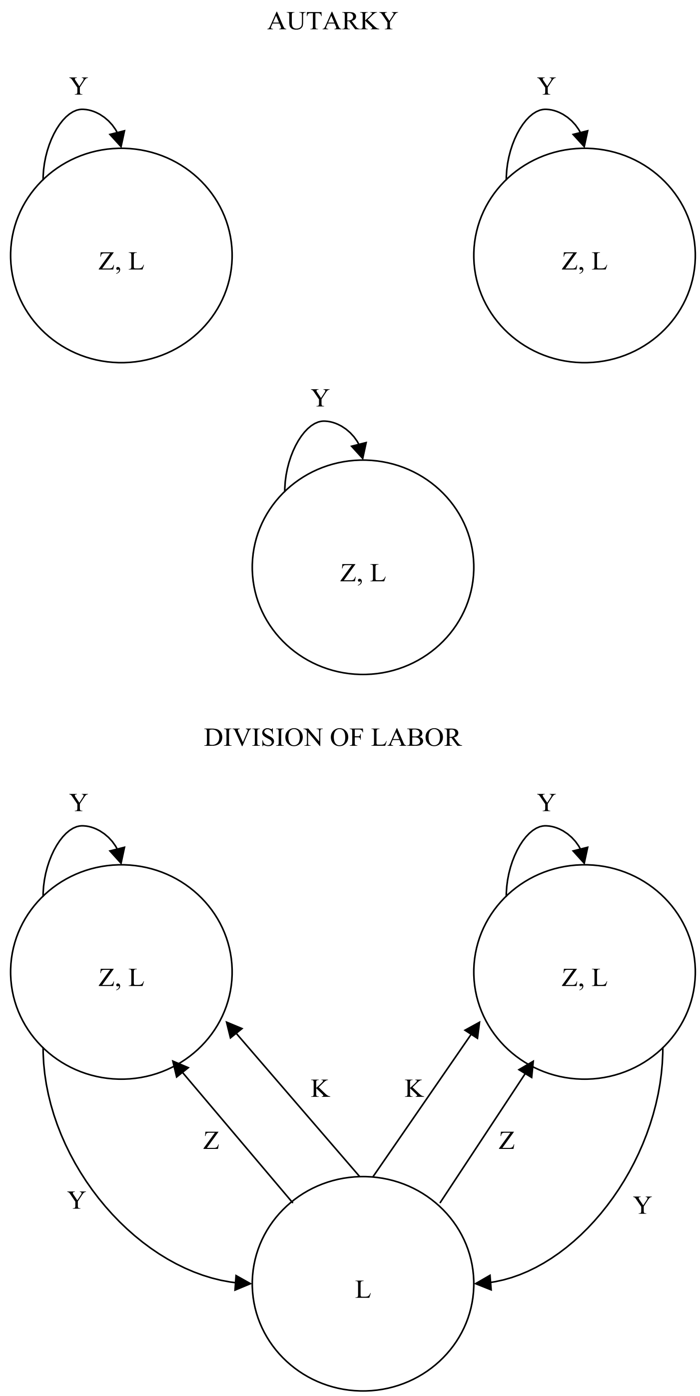

Figure 2: Autarky vs. Division of Labor