A Model of Informal Sector Labor Markets

Carol Ann Rogers
Department of Economics
Georgetown University
37th and O Sts., NW
Washington, DC 20057

Kenneth A. Swinnerton
U.S. Department of Labor
Bureau of International Labor Affairs
200 Constitution Ave., NW
Room S-5317
Washington, DC 20210

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Abstract: When labor standards signal where desirable jobs are, they alter the job-matching process, with the result that both aggregate output and employment can rise. Our model distinguishes formal-sector from informal-sector firms by their compliance with labor standards. Formal-sector firms comply. Informal-sector firms do not. Workers prefer formal-sector jobs, and take informal-sector jobs only to tide themselves over while searching in the formal sector. To gain access to more workers, more productive employers join the formal sector.

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I. Introduction

In policy discussions about enforcement of labor standards, proponents of employer interests typically make the case that excessive government regulation drives firms into the informal sector, and should therefore be scaled back. This view of the informal sector as being an undesirable side effect of distortionary government policies is echoed in much of the literature on informal sector labor markets. Worker representatives paint a different picture. They insist that workers are exploited in the absence of labor standards, and that standards can increase both output and employment.[1]

We develop a model in which both these claims can be true. Our starting point is the search model in Swinnerton (1996), from which we adopt the critical features that there are diminishing returns to labor in production and heterogeneous production technologies. In our model, labor standards signal job quality. Compliance with labor standards provides a mechanism through which high-productivity firms can attract more workers. Lower productivity firms, which do not wish to attract more workers, choose to operate in the informal sector, and do not comply with labor standards. If labor standards are not too strict, they have the net effect of causing output and employment to rise, as more workers move to higher-productivity opportunities.

There is a strand of recent research that shares with our model the feature that the regulations that demarcate the formal and informal sectors can have beneficial effects.[2] Douglas Marcouiller and Leslie Young (1995), Sylvain Dessy and Stephane Pallage

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[1] Both of these views are evident in International Labor Office (ILO: 2002), which contains "Conclusions Concerning Decent Work and the Informal Economy," reached at the International Labor Conference by the Committee on the Informal Sector. The Committee was made up of representatives of governments, worker and employer organizations.

[2] A comprehensive survey of the shadow economy appears in Friedrich Schneider and Dominik H. Enste (2000).

(2003), and Yoshiaki Azuma and Herschel Grossman (2002), develop models in which formal and informal sectors arise when the government provides public goods to producers in the (tax-paying) formal sector. In Marcouiller and Young, the government can maximize tax revenues net of expenditure on "social order" by increasing tax rates on formal producers and tolerating flight to the informal sector. In Dessy and Pallage, both formal and informal producers benefit from an output-enhancing public good, but the benefit is greater in the formal sector, where the public good facilitates adoption of an advanced technology. In Azuma and Grossman, the public good is a direct input into the formal production function. Informal producers can produce a similar input themselves, and thereby avoid paying taxes. In their model, a uniform tax that keeps all producers in the formal sector could be dominated by a high tax that drives poorly endowed producers into the informal sector, if the government cannot observe producers' endowments. The important difference between these and our work is that they do not model labor markets or analyze how labor market regulation can affect firms' incentives to operate informally.

Another set of papers (Tito Boeri and Pietro Garibaldi (2002), Pinelopi Goldberg and Nina Pavcnik (2003), and James Rauch (1991)) does focus on modeling the labor market, but unlike our work presumes that the regulations that create the formal and informal sectors necessarily interfere with economic efficiency. These models cannot account for the stated views of worker representatives in discussions of labor standards.[3]

Boeri and Garibaldi develop a matching model of the labor market with on-the-job search, in which non-negative incomes taxes always drive less-productive firms into the informal sector. Taxes must be non-negative in order to raise government revenue.

[3] In fact, they do not address explicitly from where the support for regulation derives. Presumably, in the case of labor standards, it would be from rent-seeking behavior on the part of some "insider" group of workers.

Boeri and Garibaldi characterize optimal tax and enforcement policies, and discuss how these policies create a trade-off between unemployment and informal-sector employment.

Goldberg and Pavcnik offer an efficiency wage model of the informal labor market. In their model, regulation protecting formal workers ensures they cannot be monitored, and these workers must therefore receive above-market wages in order to discourage shirking. Since the focus of their paper is on testing empirically for the effect trade policies might have on the share of informal to total employment, Goldberg and Pavcnik do not take up the question of why formal employment might be so highly regulated. The informal sector in their model could be eliminated to good effect by dispensing with regulation of the formal labor market.

Of the work in this vein, our model is closest in spirit to Rauch (1991). He posits a continuum of agents possessing different levels of managerial ability. Agents sort themselves endogenously as workers (low managerial ability) or managers (high ability). If they become workers, they are employed instantaneously, i.e., there is no job search. In Rauch's model, an informal sector arises if the government imposes a minimum wage on firms that are above a certain size. The key difference with our paper is that Rauch's model of frictionless labor markets necessarily dooms regulation to interfere with economic efficiency. Because we allow for search frictions, regulation and the creation of the formal-informal distinction can have beneficial effects on employment and output.

In the next section we describe the behavior of workers in our model. Section III discusses labor standards and their impact on the search process. Section IV addresses firms' choice whether or not to comply with labor standards and operate in the formal sector. Section V evaluates the effects of labor standard on aggregate output and employment. Section VI concludes. Proofs of all formally stated propositions are in the appendix.

II. Workers

There are k homogeneous workers in the economy, each of whom faces a constant probability of death, $\tau \in (0,1]$, in every period. New workers enter the labor force at rate τ, so that flows out of the labor force due to death are exactly matched by new flows into the labor force. In every period, there will be τk new entrants searching for work for the very first time.

Every worker has a utility function of the form

$$u = x + (m - \ell)v, \tag{1}$$

where m is an endowment of units of time and ℓ is time supplied to a firm as labor. We assume that by performing a home-based activity, any worker can reach a subsistence level of utility per unit of time, which we denote by v. x is a consumption good that is produced outside a worker's home. Workers pay for x from the income they earn working for firms. We normalize the price of x to 1. An implication of equation (1) is that any worker who has a firm-based job that pays a wage greater than v (the marginal product of time spent in home-based production), will want to devote all time to working for the firm. We assume that indifference between home-based and firm-based work is resolved in favor of firm-based work, if workers can find such work.

III. Labor Standards and the Search Process

Remuneration to a worker in the formal sector is administratively regulated by a set of labor standards so that each unit of time spent in formal-sector employment yields the worker a utility value of *at least* $w^* > v$. w^* could, for example, result from a minimum wage policy that says that all workers must be paid at least a wage (denominated in terms of the consumption good *x*) of w^* for each unit of time devoted to formal-sector work. An equivalent outcome could result from a set of health and safety standards that ensures that each unit of time spent in formal-sector work is less onerous (more enjoyable) than any unit of time spent in home-based production, i.e., each unit of time rather than having a base value to a worker of *v* has a base value of at least w^*. Since many combinations of the two types of standards can be conceived to have equivalent utility values, we can think of w^* as the outcome of a set of standards. In what follows, w^* is taken as a useful shorthand for some set of labor standards. A formal sector firm is defined as one that adopts labor standards, i.e, that "pays" $w_F \geq w^*$. There is no regulation imposed on the informal sector, so informal sector firms pay $w_I < w^*$.

Workers searching for jobs know which firms are formal.[4] The mechanisms for conveying this information can be many. We give three examples. First, formal-sector firms may register with the government or some other entity that publicizes who they are, perhaps through a public employment service. Second, informal-sector firms may not wish to draw the attention of regulators, so they do not advertise job openings in newspapers or other outlets that regulators as well as searchers can easily view. Thus, only formal-sector firms advertise. Finally, trade unions or employer organizations may exist either as signals of formality or at least in part to raise awareness about where the "good jobs" are.

Even though searchers distinguish the set of formal-sector firms from the set of informal ones, they do not know--without searching--which firms will offer them jobs or

[4] The sorting of firms into the formal and informal sector, and the job offer probabilities associated with each firm type, are equilibrium outcomes determined in Section V. For now, we simply assume that both types of firms exist.

on what precise terms. Since $w_F > w_I$, there are two sequential episodes of random search at the start of every period. In the first, new entrants to the labor market and workers who have not yet found employment in the formal sector randomly apply for employment to a firm in the formal sector. Any worker who receives an offer accepts it. Formal-sector workers never search again, and remain with the same firm until they die.

In the second episode of search, formal-sector applicants who do not receive a job offer interpret the rejection as a signal that there are no immediate job opportunities in the formal sector, and turn to the informal sector to support themselves while they await the next opportunity to apply to the formal sector for employment.   Applicants to informal-sector firms who do not receive offers spend the period "unemployed" in the sense that they do not work for a firm. At the beginning of the next period, workers who spent the previous one unemployed or at informal sector firms join new labor market entrants in search, which repeats the cycle just explained.

IV. Firms

Individual firms are atomistic in the sense that they cannot affect marginally the flow of searchers to their doors (after they identify themselves as formal or informal). From the point of view of searchers, a formal-sector firm can distinguish itself from all informal-sector firms by adopting labor standards, but it has no way to distinguish itself from other formal-sector firms. Since it must pay at least w^* per unit of labor hired to signal that it is formal, and since beyond that it can do nothing more to affect the flow of applicants to its door, a formal firm pays no more, i.e., $w_F = w^*$. Any firm that chooses the informal sector, i.e., chooses not to comply with labor standards, knows that any searcher it meets will turn down any offer of less than v, but that offering more than v

will not affect its marginal flow of searchers. So informal-sector firms offer $w_I = v$. We now discuss how and why firms sort into the formal and informal sectors.

Normalize to unity the number of firms in the economy. The production function for a firm is

$$\lambda f(\ell), \tag{2}$$

with $f(0)=0$, $f'(\ell)>0$, $f''(\ell)<0$, $\lim_{\ell\to\infty} f'(\ell)=0$, and $\lim_{\ell\to 0} f'(\ell)=\infty$. λ is an index of firm productivity, and ℓ is labor input. Firms are heterogeneous in λ, which is distributed on $[0,\bar{\lambda}]$ according to the distribution function $A(\lambda)$ with associated density $a(\lambda)$. We assume $a(\lambda)$ is continuous. A firm may operate only in one sector, and chooses the sector that brings it higher profits.

The quantity of labor demanded by a formal sector firm is implicitly defined by $\lambda f'(\ell)=w^*$. We use the notation $\ell^d(w^*/\lambda)$ to stand for this demand for labor. In the informal sector, each firm has labor demand $\ell^d(v/\lambda)$. Note that $\partial \ell^d(w/\lambda)/\partial\lambda > 0$: higher-productivity firms demand more labor, at any given wage rate.

Since searchers within sectors are allocated randomly to firms, a firm can be *labor-supply constrained* within a sector if its demand for labor at the going wage rate is greater than the per-firm supply in that sector. If this constraint could be relaxed, the firm's employment level and profits would both be higher. Given the assumptions of the model, the constraint cannot be relaxed within a sector. But the search process sends all searchers first to formal-sector firms, which means that formal-sector firms have "first dibs" on workers and thus a larger labor supply. So a firm that would be labor supply constrained in the informal sector may find it more profitable to operate in the formal sector, in spite of the higher unit labor costs, because its formal-sector status brings more workers. Letting ℓ^j *(j = I, F)* denote per-firm labor supply in each sector, we have Proposition 1.

Proposition 1: If a formal sector exists, then $\ell^F > \ell^I$.

Corollary 1: A firm never enters the formal sector if $\ell^I \geq \ell^d(v/\lambda)$.

Corollary 2: Formal sector firms are larger than informal sector firms.

From Corollary 1 we see that all firms with values of λ *above* some cut-off level, which we will call λ_1, are supply constrained in the informal sector. λ_1 is the productivity index of the firm where labor demand at the informal sector wage just equals informal-sector labor supply:

$$\lambda_1 = v / f'(\ell^I) . \tag{3}$$

Firms in the formal sector will have $\lambda > \lambda_1$. Let us denote by $\lambda_2 > \lambda_1$ the highest productivity index for any firm in the informal sector. The aggregate measure of informal sector firms is $A(\lambda_2)$; $1 - A(\lambda_2)$ firms are formal.

It may happen that in equilibrium, all firms will prefer informality (i.e., that $\lambda_2 = \overline{\lambda}$). In order to accommodate this possibility, we define λ_{IF} as the productivity level that would be needed for a firm to be *indifferent* between two sectors (i.e., to earn the same profit in either sector). This level of productivity could be outside the support of the distribution of λ. Thus,

$$\lambda_2 = \min(\lambda_{IF}, \overline{\lambda}) \tag{4}$$

To write out the equal-profits condition that defines λ_{IF}, we note that a firm having $\lambda = \lambda_{IF}$ would *not* be supply constrained in the formal sector. If it were, then all formal firms would be supply constrained. Formal firms would hire every searcher, leaving no labor for the informal sector. Consequently, all firms would prefer the formal sector, where profits would be positive, to the informal sector, where profits would equal zero, and λ_{IF} would have to equal zero. But this is not possible. So long as labor supply is positive in the formal sector and $a(\lambda)$ is continuous, there will be firms with very low productivity indices (very low labor demand) that will not be supply constrained.

We conclude that a firm that would be indifferent between the two sectors would be supply constrained in the informal sector and on its demand curve in the formal sector. λ_{IF} is therefore defined by

$$\lambda_{IF}\, f(\ell^I) - v\ell^I = \lambda_{IF}\, f(\ell^d(w^*/\lambda_{IF})) - w^*\ell^d(w^*/\lambda_{IF})\,. \qquad (5)$$

We next derive ℓ^F and ℓ^I, using the logic from Albrecht and Axell (1984). Let q be the probability that a formal-sector job applicant receives a job offer. q is determined in equilibrium, but for now we take it as a parameter. The flow of workers to a formal-sector firm at any search date consists of its share of new entrants into the labor force at that date, $\tau k/[1 - A(\lambda_2)]$; its share of the new-entrants from the previous period who did not get formal-sector jobs and did not die, $(1-q)(1-\tau)\tau k/[1 - A(\lambda_2)]$; its share of the still-living searchers who first entered two periods in the past, $(1-q)^2(1-\tau)^2\tau k/[1 - A(\lambda_2)]$; and so forth. The total flow equals $\dfrac{\sum_{j=0}^{\infty}(1-\tau)^j(1-q)^j\tau k}{1 - A(\lambda_2)} = \dfrac{\tau k}{[1 - A(\lambda_2)][1-(1-q)(1-\tau)]}$. If a firm were to offer jobs to all of the workers who applied, then its potential labor supply would be equal to this flow plus survivors from the total flows from previous periods. Adding these up gives us the potential labor supply to a firm in the formal sector:

$$\ell^F(q,\lambda_2) = \frac{k}{[1-A(\lambda_2)][1-(1-q)(1-\tau)]}. \tag{6}$$

The informal sector provides employment to workers who are unable to secure jobs in the formal sector. Since workers in the informal sector do not wish to work there forever, we do not aggregate all surviving workers who failed to secure employment in the formal sector. Potential labor supply to a firm in the informal sector thus equals its period flow of applicants:

$$\ell^I(q,\lambda_2) = \frac{(1-q)\tau k}{A(\lambda_2)[1-(1-q)(1-\tau])}. \tag{7}$$

V. Equilibrium

We close the model by determining determine *q*, the probability that a searcher receives a formal-sector job offer, and *p*, the probability of an informal-sector offer.

We begin with *q*. Denote by λ_3 the highest productivity level of a formal firm that is able to satisfy its demand for labor in the formal sector. Firms having $\lambda > \lambda_3$ have such high demand for labor that they are supply-constrained (even in the formal sector), while firms having $\lambda \le \lambda_3$ satisfy their labor demand in the formal sector. For given λ_2 and *q*, λ_3 is defined by

$$\lambda_3 = \begin{cases} \bar{\lambda}, & w^*/f'(\ell^F) \ge \bar{\lambda} \\ w^*/f'(\ell^F) & \text{if } \lambda_2 < w^*/f'(\ell^F) < \bar{\lambda}, \\ \lambda_2, & w^*/f'(\ell^F) \le \lambda_2 \end{cases} \tag{8}$$

The first line of equation (8) describes a situation where, for given λ_2 and q, no formal firms are supply constrained: $w^* > \bar{\lambda} f'(\ell^F)$. The second line describes a situation in which the formal sector includes both supply-constrained firms and firms that satisfy their labor demand. The third line describes a situation in which, for given λ_2 and q, all formal firms are supply constrained: $w^* < \bar{\lambda} f'(\ell^F)$.

Because of each worker's constant death risk, τ, a formal-sector firm has, in every period, $\tau\ell^F$ job-openings if it is labor-supply constrained, and $\tau\ell^d(w^*/\lambda)$ if it is not. The per-period flow of searchers to each formal sector firm is $\tau\ell^F$. Thus, a searcher who contacts a supply-constrained firm (having $\lambda > \lambda_3$) receives an offer with probability $\tau\ell^F/\tau\ell^F = 1$. If a searcher contacts an unconstrained firm (having $\lambda_2 < \lambda < \lambda_3$), the conditional offer probability is only $\tau\ell^d(w^*/\lambda)/\tau\ell^F < 1$, as the firm's flow of job openings, $\tau\ell^d$, is less than its flow of applicants, $\tau\ell^F$. The unconditional probability of receiving an offer from *some* formal-sector firm is just the weighted average of the formal-sector firms' offer probabilities:

$$q = \frac{\int_{\lambda_2}^{\lambda_3} \frac{\ell^d(w^*/\lambda)}{\ell^F(q,\lambda_2)} a(\lambda)d\lambda}{1 - A(\lambda_2)} + \frac{\int_{\lambda_3}^{\bar{\lambda}} a(\lambda)d\lambda}{1 - A(\lambda_2)}. \tag{9}$$

Note that if $\lambda_3 = \bar{\lambda}$, then the last term in equation (9) vanishes, since all formal firms are then on their labor demand curves. If all formal-sector firms are supply constrained, i.e., $\lambda_3 = \lambda_2$, then the first term on the right-hand side of equation (9) vanishes, and $q = 1$.

To complete the model, we follow the derivation of the equation for q to define the probability of a job offer in the informal sector, p:

$$p = \frac{\int_0^{\lambda_1} \frac{\ell^d(v,\lambda)}{\ell^I(q,\lambda_2)} a(\lambda) d\lambda}{A(\lambda_2)} + \frac{A(\lambda_2) - A(\lambda_1)}{A(\lambda_2)} \qquad (10)$$

Since turnover occurs in every period in the informal sector (as workers quit to search for "better" formal jobs), the probability of a job offer from an informal firm (*p*) has no effect on the probability of a formal offer (*q*) or on the productivity level of the marginal entrant into the formal sector (λ_2). Equation (10) therefore determines *p* recursively, for given values of the model's other endogenous variables.

Equilibrium values of the other endogenous variables in the model, $\{\ell^I, \ell^F, q, \lambda_1, \lambda_{IF}, \lambda_2, \lambda_3\}$, are determined by equations (3) – (9). To characterize equilibrium further, we simplify by first noting from equations (6) and (7) that ℓ^F and ℓ^I are functions of *q* and λ_2 but not of any other endogenous variable. Substitute $\ell^I(q,\lambda_2)$ into equation (3) and $\ell^F(q,\lambda_2)$ into equation (8), so that both λ_1 and λ_3 are also functions of only *q* and λ_2 . We denote these functions by $\lambda_1(q,\lambda_2)$ and $\lambda_3(q,\lambda_2)$. Finally, we substitute $\ell^I(q,\lambda_{IF})$ for ℓ^I in equation (5).

We are left with three equations in the unknowns, $\{q, \lambda_{IF}, \lambda_2\}$:

$$\lambda_{IF} f(\ell^I(q,\lambda_{IF})) - v\ell^I(q,\lambda_{IF}) = \lambda_{IF} f(\ell^d(w^*/\lambda_{IF})) - w^*\ell^d(w^*/\lambda_{IF}) \qquad (11A)$$

$$\lambda_2 = \min(\lambda_{IF}, \bar{\lambda}) \qquad (11B)$$

$$q = \frac{\int_{\lambda_2}^{\lambda_3(q,\lambda_2)} \frac{\ell^d(w^*/\lambda)}{\ell^F(q,\lambda_2)} a(\lambda) d\lambda}{1 - A(\lambda_2)} + \frac{\int_{\lambda_3(q,\lambda_2)}^{\bar{\lambda}} a(\lambda) d\lambda}{1 - A(\lambda_2)} \qquad (12)$$

We can represent equilibrium solutions for *q* and λ_2 graphically, and we provide three examples in Figures 1 through 3. We will discuss the distinctive features of each Figure shortly.   First, we identify the common features of each case, and explain why at least one stable equilibrium always exists.

Equations (11A) and (11B) define λ_2, the highest productivity index in the informal sector, for any $q \in [0,1]$. If q = 1, then all searchers receive offers in the formal sector: none are left over to apply to the informal sector ($\ell^I = 0$). Every firm with $\lambda > 0$ will therefore locate in the formal sector, making $\lambda_2 = \lambda_{IF} = 0$. Decreases in q increase informal labor supply, and raise λ_2. As q becomes progressively smaller, either of two things may happen. One, which is illustrated in Figures 1 and 3, is that λ_2 rises quickly enough to cause all firms become informal ($\lambda_2 = \bar{\lambda}$) at some q > 0. In this case, equations (11) give rise to a negatively sloped curve in (q, λ_2)-space, which ranges from the point (1,0) on the horizontal axes, moves up and to the left, and eventually becomes horizontal, at $\lambda_2 = \bar{\lambda}$. The second possibility, illustrated in Figure 2, is that for the entire range of q, $\lambda_2 \le \bar{\lambda}$. In this case, equations (11) imply a negatively sloped curve with no horizontal portion.

Equation (12) defines, for any $\lambda_2 \in [0, \bar{\lambda}]$, the probability, q, of a job offer from a formal firm.   When $\lambda_2 = 0$, then q must be less than 1: if all firms are formal, some will necessarily be on their labor demand curves and will not hire every applicant. If $\lambda_2 = \bar{\lambda}$, then all firms are informal, which implies q = 0. q is not monotonically related to λ_2: an increase in λ_2 has two opposing effects.[5] The first effect is to decrease the measure of firms in the formal sector, which reduces the probability of a formal-sector job offer. The second effect is to increase the weight - - in determining q - - given to formal-sector firms

[5] A formal derivation of the slope of the graph of equation (12) may be found in the proof of Proposition 3.

with comparatively high offer probabilities (the firms that remain in the formal sector), thereby increasing *q*. If λ_2 is close to zero (there are a lot of formal sector firms), then the variance in offer probabilities across formal-sector firms is relatively large and there is a large positive effect of removing the firms with small offer probabilities on the average offer probability (*q)*: the graph of equation (12) has a positive slope near the horizontal intercept. On the other hand, if there are very few formal sector firms, i.e., λ_2 is large, there is comparatively little variance in job offer probabilities across formal-sector firms, so the effect of firms dropping out of the formal sector dominates: as the graph of equation (12) approaches its vertical intercept, its slope is negative.

An equilibrium always exists. Equations (11) and (12) define continuous relationships between *q* and λ_2. The horizontal intercept of the graph of equations (11) always is to the right of that of the graph of equation (12). As $q \to 0$, there are two possibilities. The first possibility, which is illustrated in Figures 1 and 3, is that the curves have the same vertical intercept. In this case, there is always at least one equilibrium, i.e., one in which all firms are informal (i.e., the point $(0, \bar{\lambda})$). The second possibility, illustrated in Figure 2, is that the vertical intercept of equation (11) is below that for equation (12). In this case, continuity ensures that the two curves must cross.

In Figure 1, there is no formal sector in equilibrium: the equilibrium has $q = 0$ and $\lambda_2 = \bar{\lambda}$. Such an outcome is clearly possible if complying with labor standards is very costly; or, if in the absence of labor standards, labor supply constraints are not binding. Here are three, not-mutually-exclusive, ways the equilibrium in Figure 1 could occur. First, $a(\lambda)$ could have heavy density near zero and very little near $\bar{\lambda}$. In this case, the economy is heavily populated with unproductive firms that have low labor demand. Second, the population of workers relative to firms could be so large that binding labor-supply constraints on informal sector firms are never much of an issue. Finally, if labor standards are set too high (w^* much larger than v), then all firms will find operating in the informal sector (with lower wages) to be more profitable, and none will comply with labor standards.

In Figure 2, there is a single equilibrium with both a formal and an informal sector. This occurs when: k, the measure of workers, is not too large, so that in the absence of labor standards there are supply constrained firms; and when labor standards are not very onerous (that is, when w^* is "close" to v).

Finally, in Figure 3, there are multiple equilibria, labeled A, B, and C. "A" and "C" are stable. In "A" the equilibrium has only an informal sector, while in "C" both formal and informal sectors exist. The intuition for the possible existence of two stable equilibria is straightforward. If a large enough formal sector exists (equilibrium C), the formal sector soaks up many workers making it more likely that the informal-sector labor-supply constraint binds on any individual firm should it choose informality; therefore, it is more likely to be most profitable to go formal. In equilibrium A, the formal sector soaks up no workers and so the labor-supply constraint from remaining informal is not as severe as in equilibrium A. Thus in the economy depicted in Figure 3, an individual firm's choice to go formal or not tends to be reinforced by heavy incidence of other firms making exactly the same choice. Formality and informality feed on themselves.

VI. The Effects of Labor Market Regulation

In our model, regulation comprises two interconnected features: one is the set of labor standards ($w^* > v$), which impose a cost on formality. The second is that regulation directs workers towards higher-productivity formal firms. In tandem, these two features can have beneficial effects on labor markets and aggregate output. We now explore this possibility.

In the absence of labor market regulation, the wage would equal v for all workers. There is no reason for workers to prefer work at any firm over another. As a result, per-firm labor supply would be equal to $k/[1-(1-\hat{p})(1-\tau)]$, where $\hat{p}$ is the equilibrium probability of a job in the absence of labor regulations.

Let us suppose there *would* be some supply-constrained firms in the absence of regulation.[6] Lower-productivity firms would operate on their labor demand curves, and hire fewer workers than apply for work, while supply-constrained higher-productivity firms would hire every worker who applies. If the government knew individual firm productivities, it might be able to channel the excess supply from the low-productivity firms to the high-productivity firms, thereby raising both aggregate output and employment. Unfortunately, the government is unlikely to observe all firm productivities, and workers have no incentive to present themselves in larger numbers to high-productivity firms, since all firms pay the same wage.

Labor standards, viewed from this perspective, serve as an allocation device. By raising remuneration for workers, they make search at the (known) formal sector more desirable. Workers search there first. Since not all firms will join the formal sector, this

[6] Otherwise, all informal firms will satisfy their labor demands and there will be no formal sector.

aggregate supply of labor is spread out over fewer firms than in the no-regulation case. As a result, firms that join the formal sector face a greater per-firm labor supply than they would in the absence of regulation.

Proposition 2: In equilibrium, $\ell^F = \dfrac{k/[1-A(\lambda_2)]}{1-(1-q)(1-\tau)} > \dfrac{k}{1-(1-\hat{p})(1-\tau)}$.

In turn, since only high-productivity firms will benefit from a higher labor supply, the labor standard encourages high-productivity firms to sort themselves into the formal sector.

When both of these features of labor market regulation are taken into account, it becomes clear that *some* labor market regulation is *always* desirable.

Proposition 3: If some firm faces labor shortages in an unregulated labor market, then a set of labor standards exists that raises aggregate output and aggregate employment.

Proposition 3 does not imply that *any* labor standard increases output and employment. Our model associates stricter labor standards (higher values of w^*) with a larger informal sector. As we noted above, w^* could be made so high as to drive all firms to the informal sector.

Such a high level of w^* is benign in our model: it only returns the economy to the no-regulation equilibrium. Very costly labor standards that fall just a little short of driving away the formal sector are not so benign. Labor standards initiate high turnover in the informal sector, since all workers search first at formal firms. This turnover makes it impossible for informal-sector firms to build their workforces by retaining workers over time. As w^* increases, more and more informal-sector production is directed to high-turnover, low-output firms. Before w^* gets so high as to eliminate the formal sector completely, there is a point at which increasing w^* causes aggregate output and employment to fall below non-regulatory levels.

VII. Conclusions

Our model distinguishes between the formal and informal sectors by compliance with labor regulation. Formal-sector firms' compliance with labor standards signals where the good jobs are. More productive employers join the formal sector to offer these jobs. Accordingly, labor standards improve the job-matching process, with the result that both aggregate output and employment can rise.

In this paper, we assumed that there was no penalty for non-compliance with labor standards. We did not pursue the issues of whether some firms will announce compliance and then not comply, or whether there are net benefits to forcing compliance among all firms. It would be worthwhile to augment the model of this paper with an explicit model of what it takes to enforce labor standards, exploring the effects of a cost to non-compliance and the optimal structure of enforcement. We think a careful treatment of these issues would lead to interesting insights on economically beneficial regulation-and-enforcement regimes.

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Appendix: Proofs of Propositions

Proposition 1: If a formal sector exists, then $\ell^F > \ell^I$.

Proof: First, suppose for some firm in the formal sector with productivity level $\hat{\lambda}$, that $\ell^d(w^*/\hat{\lambda}) > \ell^F$. Then, it has to be the case that $\ell^F > \ell^I$, or else profits for this firm will be greater in the informal sector, and the firm would not be in the formal sector.

Next suppose that for some firm in the formal sector with productivity level $\hat{\lambda}$, that $\ell^d(w^*/\hat{\lambda}) \le \ell^F$. Then, since $\ell^d(v/\hat{\lambda}) > \ell^d(w^*/\hat{\lambda})$, it has to be the case that $\ell^d(v/\hat{\lambda}) > \ell^I$ and $\ell^I < \ell^F$, or else profits will be greater in the informal than in the formal sector, and the firm would not be in the formal sector.

Proposition 2: In equilibrium, $\ell^F = \dfrac{k/[1-A(\lambda_2)]}{1-(1-q)(1-\tau)} > \dfrac{k}{1-(1-\hat{p})(1-\tau)}$

Proof: For convenience, define $\hat{\ell} = \dfrac{k}{1-(1-\hat{p})(1-\tau)}$. We show that for *any* (q,λ_2) pair that satisfies equation (12), $\ell^F \ge \hat{\ell}$. After defining two functions useful to the proof, we do this in two steps. (i) We show that for the smallest possible $\lambda_2 (\lambda_2 = 0)$, $\ell^F \ge \hat{\ell}$. (ii) We show that increasing λ_2 while satisfying equation (12) always leads to increases in ℓ^F .

Define,

$$G(q,\lambda_2) = q - \{\frac{\int_{\lambda_2}^{\lambda_3(q,\lambda_2)} \frac{\ell^d(w^*/\lambda)}{\ell^F(q,\lambda_2)} a(\lambda)d\lambda}{1-A(\lambda_2)} + \frac{1-A(\lambda_3(q,\lambda_2))}{1-A(\lambda_2)}\}. \tag{A1}$$

Note that $G(q,\lambda_2) = 0$ is equivalent to equation (12).

Next define,

$$H(\hat{p}) = \hat{p} - \{ \int_0^{v/f'(\hat{\ell})} \frac{\ell^d(v/\lambda)}{\hat{\ell}} a(\lambda)d\lambda + 1 - A(\frac{v}{f'(\hat{\ell})})\} = 0 \tag{A2}$$

Equation (A2) gives the probability of receiving a job offer ($\hat{p}$) when there is no formal sector and no quitting of jobs.

(i) We now show that for the smallest possible $\lambda_2 (\lambda_2 = 0)$, $\ell^F \ge \hat{\ell}$. If $w^* = v$, the point $(q,\lambda_2) = (\hat{p},0)$ is a solution to $G(q,\lambda_2) = 0$, because if we plug these values into equation (A1), then (A1) is identical to equation (A2). In this case, $\ell^F = \hat{\ell}$. If $w^* > v$, then we see from equation (A1) that $G(\hat{p},0) > 0$, because $\lambda_3(\hat{p},0)$ is no smaller for $w^* > v$ than for $w^* = v$; $\ell^d(w^*/\lambda) < \ell^d(v/\lambda)$; and, $\ell^F(\hat{p},0) = \hat{\ell}$. If we hold λ_2 =0 for any $w^* > v$, it must be he case that the q that solves $G(q,0) = 0$ is less than $\hat{p}$. We know this because

$$\frac{\partial G}{\partial q} = 1 + \frac{1}{1-A(\lambda_2)} \int_{\lambda_2}^{\lambda_3(q,\lambda_2)} \frac{\ell^d(w^*/\lambda)}{[\ell^F(q,\lambda_2)]^2} \frac{\partial \ell^F(q,\lambda_2)}{\partial q} a(\lambda) d\lambda$$
$$= 1 - (1-\tau) \int_{\lambda_2}^{\lambda_3(q,\lambda_2)} \frac{\ell^d(w^*/\lambda)}{k} a(\lambda) d\lambda > 0. \qquad \text{(A3)}$$

The sign of this expression follows from noticing that the integrand in the second line is less than one because labor demand at any firm that is not labor-supply constrained cannot be large enough to absorb the entire population of workers. Finally, since ℓ^F is decreasing in q (see equation (A5) below) it follows that for any $q < \hat{p}$, $\ell^F(q,0) > \ell^F(\hat{p},0) = \hat{\ell}$.

(ii) We now show that increasing λ_2 while satisfying equation (12) always leads to increases in ℓ^F. To do this, we establish that the derivative

$$\frac{d\ell^F}{d\lambda_2} = \frac{\partial \ell^F}{\partial q}\frac{\partial q}{\partial \lambda_2} + \frac{\partial \ell^F}{\partial \lambda_2} \qquad \text{(A4)}$$

is positive.

From equation (7) we have,

$$\frac{\partial \ell^F}{\partial q} = \frac{-(1-\tau)\ell^F}{1-(1-q)(1-\tau)} \qquad \text{(A5)}$$

$$\frac{\partial \ell^F}{\partial \lambda_2} = \frac{a(\lambda_2)}{1-A(\lambda_2)} \ell^F \qquad \text{(A6)}$$

We derive $\frac{\partial q}{\partial \lambda_2}$ by noting that equation (A1) may be viewed as defining q implicitly as a function of λ_2. From equation (A1) we have,

$$\frac{\partial G}{\partial \lambda_2} = -\frac{a(\lambda_2)}{1-A(\lambda_2)}\left[-\frac{1-A(\lambda_3)}{1-A(\lambda_2)} + \frac{\ell^d(w^*/\lambda_2)}{\ell^F}\right] \qquad \text{(A7)}$$

By the implicit function theorem: we have $\frac{\partial q}{\partial \lambda_2} = -\frac{\partial G}{\partial \lambda_2} \Big/ \frac{\partial G}{\partial q}$. Substituting from equations (A3) and (A5)-(A7) into equation (A4) yields, after some manipulation:

$$\frac{d\ell^F}{d\lambda_2} = \frac{\partial \ell^F}{\partial \lambda_2}\left[\frac{k-(1-\tau)E^F + (1-\tau)(1-A(\lambda_2))\ell^d(w^*/\lambda_2)}{k\frac{\partial G}{\partial q}}\right] > 0. \quad \text{(A8)}$$

In equation (A8), E^F is total formal sector employment, i.e., $E^F = \int_{\lambda_2}^{\lambda_3} \ell^d(w^*/\lambda)a(\lambda)d\lambda + [1-A(\lambda_3)]\ell^F(q,\lambda_2)$. The term $k-(1-\tau)E^F$ is equal to the flow of searchers to the formal-sector in the aggregate at the beginning of each period. Increasing λ_2 decreases the stock of formal-sector firms, and the workers released from the marginal firm switching to the informal sector—i.e., $(1-\tau)(1-A(\lambda_2))\ell^d(w^*/\lambda_2)$ -- increases the flow of searchers to each remaining formal-sector firm. In the steady state, each remaining formal sector firm's potential labor supply increases.

Proposition 3: If some firm faces labor-shortages in an unregulated labor market, then a set of labor standards exists that raises aggregate output and aggregate employment.
Proof: We derive expressions for the change in aggregate output ($\Delta Y(w^*)$) and aggregate employment ($\Delta E(w^*)$), when an economy goes from having no labor market regulation to having *some* labor market regulation. We then show that $\lim_{w^*\to v} \Delta Y(w^*) = \Delta Y(v)$ and $\lim_{w^*\to v} \Delta E(w^*) = \Delta E(v)$ are strictly positive. Existence of these limits establishes that $\Delta Y(w^*)$ and $\Delta E(w^*)$ are continuous at *w** = *v*, so that we know that at least at values of *w** that are slightly greater than *v*, $\Delta Y(w^*)$ and $\Delta E(w^*)$ are positive.

<u>Aggregate output:</u>

In the absence of any labor market regulation, aggregate output equals

$$\int_0^{v/f'(\hat{\ell})} \lambda f(\ell^d(v/\lambda))a(\lambda)d\lambda + f(\hat{\ell})\int_{v/f'(\hat{\ell})}^{\bar{\lambda}} a(\lambda)d\lambda\,, \quad \text{(A9)}$$

this is the sum of output at firms that satisfy their labor demand and of firms that are labor-supply constrained.

With labor market regulations, aggregate output becomes

$$\int_0^{v/f'(\ell^I)} \lambda f(\ell^d(v/\lambda))a(\lambda)d\lambda + f(\ell^I)\int_{vf'(\ell^I)}^{\lambda_2} \lambda a(\lambda)d\lambda + \int_{\lambda_2}^{w^*/f'(\ell^F)} \lambda f(\ell^d(w^*/\lambda))a(\lambda)d\lambda +$$

$$f(\ell^F)\int_{w^*/f'(\ell^F)}^{\bar{\lambda}} \lambda a(\lambda)d\lambda\,. \quad \text{(A10)}$$

The *change* in aggregate output (ΔY) equals

$$\Delta Y(w^*) = \int_0^{v/f'(\ell^I)} \lambda f(\ell^d(v/\lambda))a(\lambda)d\lambda + f(\ell^I)\int_{vf'(\ell^I)}^{\lambda_2} \lambda a(\lambda)d\lambda + \int_{\lambda_2}^{w^*/f'(\ell^F)} \lambda f(\ell^d(w^*/\lambda))a(\lambda)d\lambda +$$

$$f(\ell^F)\int_{w^*/f'(\ell^F)}^{\bar{\lambda}} \lambda a(\lambda)d\lambda - \left[\int_0^{v/f'(\hat{\ell})} \lambda f(\ell^d(v/\lambda))dA(\lambda) + f(\hat{\ell})\int_{v/f'(\hat{\ell})}^{\bar{\lambda}} \lambda a(\lambda)d\lambda\right]. \qquad \text{(A11)}$$

As w^* approaches v, in the limit, $\lambda_2 \to \lambda_1 = v/f'(\ell^I)$, and $\ell^I = \ell^d(v/\lambda_2)$. Making these changes and also setting w^*'s equal to v everywhere gives us:

$$\lim_{w^*\to v} \Delta Y = \int_0^{v/f'(\ell^F)} \lambda f(\ell^d(v/\lambda))a(\lambda)d\lambda + f(\ell^F)\int_{v/f'(\ell^F)}^{\bar{\lambda}} \lambda a(\lambda)d\lambda -$$
$$\left[\int_0^{v/f'(\hat{\ell})} \lambda f(\ell^d(v/\lambda))a(\lambda)d\lambda + f(\hat{\ell})\int_{v/f'(\hat{\ell})}^{\bar{\lambda}} \lambda a(\lambda)d\lambda\right] \qquad \text{(A12)}$$

From proposition 2, we know that $\ell^F > \hat{\ell}$ which in turn implies that $v/f'(\ell^F) > v/f'(\hat{\ell})$; so, the limit above can be rewritten as

$$\lim_{w^*\to v} \Delta Y = \int_{v/f'(\hat{\ell})}^{v/f'(\ell^F)} \lambda\left[f(\ell^d(v/\lambda)) - f(\hat{\ell})\right]a(\lambda)d\lambda + \left[f(\ell^F) - f(\hat{\ell})\right]\int_{v/f'(\ell^F)}^{\bar{\lambda}} \lambda a(\lambda)d\lambda > 0,$$

since both terms in the sum on the r.h.s are strictly positive.

<u>Aggregate Employment.</u>

With no regulation, aggregate employment is

$$\int_0^{v/f'(\hat{\ell})} \ell^d(v/\lambda)a(\lambda)d\lambda + \hat{\ell}\int_{v/f'(\hat{\ell})}^{\bar{\lambda}} a(\lambda)d\lambda. \qquad \text{(A13)}$$

With the regulation in place it equals:

$$\int_0^{v/f'(\ell^I)} \ell^d(v/\lambda)a(\lambda)d\lambda + \ell^I\int_{vf'(\ell^I)}^{\lambda_2} a(\lambda)d\lambda$$
$$+\int_{\lambda_2}^{w^*/f'(\ell^F)} \ell^d(w^*/\lambda)a(\lambda)d\lambda + \ell^F\int_{w^*/f'(\ell^F)}^{\bar{\lambda}} a(\lambda)d\lambda \qquad \text{(A14)}$$

The change in employment (due to having regulation) equals (A14) minus (A13). The same reasoning as before establishes that:

$$\lim_{w^*\to v} \Delta E = \int_{v/f'(\hat{\ell})}^{v/f'(\ell^F)} \left[\ell^d(v/\lambda) - \hat{\ell}\right]a(\lambda)d\lambda + \left[\ell^F - \hat{\ell}\right]\int_{v/f'(\ell^F)}^{\bar{\lambda}} a(\lambda)d\lambda > 0.$$

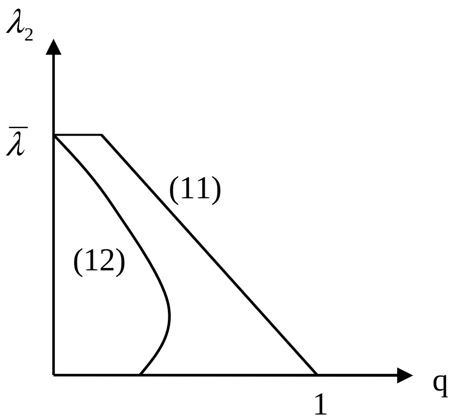

Figure 1: An Equilibrium with No Formal Sector

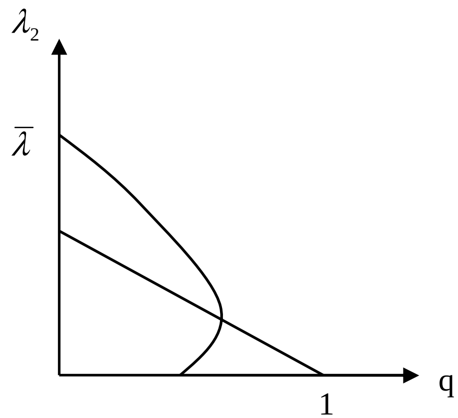

Figure 2: A Unique Equilibrium with a Formal and an Informal Sector

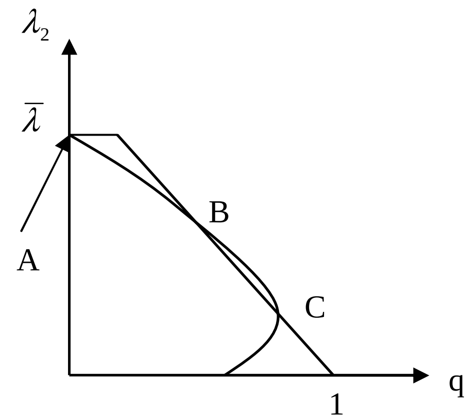

Figure 3: Multiple Equilibria