A Test of Collective Rationality Within Bigamous Households in Burkina Faso

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Abstract

This paper provides different tests to ascertain efficiency of consumption decisions in households with many decision-makers based on the effect of distribution factors . It also presents a method of determining the number of these decision-makers. The tests are used to investigate collective rationality within monogamous and bigamous households living in rural Burkina Faso. The data are found to be consistent with collective rationality in both setting. Furthermore, it is found that the all the spouses take part in the internal decision process.

KEYWORDS: Intra-household allocation, collective model, polygamy, extended family, Pareto optimality, African households.
JEL numbers: D11, D70

PRELIMINARY AND INCOMPLETE.
NOT TO BE QUOTED WITHOUT PERMISSION.

1 Introduction

The dynamic of household decision-making has been the object of intense debate over the last twenty years. The conventional unitary model which postulates that the members of an

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household behave as if they were maximising an unique utility function under an household budget constraint is now very seriously challenged. One of the reason is that its theoretical foundations are weak: the type of behavior postulated by the unitary model is legetimate only in very special conditions.[1] Another reason is that the testable restrictions imposed by the unitary model, which concern the nature and the structure of the price effects (that is the negative semidefinite Slutsky matrix) and the pooling of household income, have been rejected by a substantial number of studies carried out throughout the world.[2]

Probably the most serious alternative proposed to the unitary model is the collective model initially developped by Chiappori (1988, 1992) In its most general form, the collective model makes only two hypothesis: each household member has his own preferences, which may differ from one member to an other, and the decision process yield pareto-efficient outcomes for those members who were able to influence the outcomes.[3] The type of rationality provided by the collective model is thus qualified of "collective rationality" (CR) instead of "individual rationality" like the one provided by the unitary model. This makes its theoretical foundations much more robust than those of the unitary model, since less restrictive, and closer to a methodological individualism framework on which microeconomic theory is grounded. To validate empirically the collective model, many have sought to identify its testable restrictions in different contexts (*e.g.*, Chiappori (1988), Bourguignon, Browning and Chiappori (1995), Udry (1996) and Browning and Chiappori (1998)). Two kinds of testables restrictions applicable to the consumption decisions have been developed. The first kind relates to the price effects, while the second kind relates to the effects of the so-called distribution factors. A distribution factor is a variable that influences the decision process within the household, but which doesn't influence preferences or the household budget set.

The test pertaining to the first category are due to Browning and Chiappori (1998) and have been generalised by Chiappori and Ekeland (2002). They have shown that CR generates testable restrictions on consumption price effects even in very general settings that allow for public and private commodities and externalities. When there are two members in the household, they have shown that the Pseudo-Slutsky matrix is the sum of a symmetric negative semi-definite matrix and a matrix that has, at most, rank one. They have also shown how this condition can be generalized to households with more than two members. This extension is important since it is likely that in many households adult children who live with their parents influence the family decision process. Likewise, polygamous or extended families are quite

[1]Either when all the household members have the same preferences or when all the household members agree on the preferences to maximise or else when the household is in fact governed by a dictator.

[2]See for example, Thomas (1990), Bourguignon, Browing *et al.* (1993), Hoddinott and Kanbur (1992), Lachaud (1998), Quisumbing and Maluccio (2000), Fortin and Lacroix (1997).

[3]It is important to note that the collective model doesn't assume that all the household members participate in the decision process and influence the outcomes, but it doesn't exclude this possibility either. Said differently, the collective model only assumes that each household member may influence the decision outcomes.

common in many developing countries. It is thus likely that the decision process may involve more than two decision-makers in such households. As a by-product of their analysis, Browning and Chiappori (1998) have also provided a simple test which allows the number of decision-makers in a multi-person household to be determined.

These tests face two limitations, however. First, they cannot be used with cross-sectional data that have no variability in regional prices. Yet, it is often very difficult to find panel data or cross-sectional data with reliable regional price variations in developping countries. Second, these tests cannot be implemented when the number of observed commodities is less than twice the number of household members. Otherwise, the symmetry plus rank restrictions is always satisfied. This implies, for instance, that the tests do not apply in the standard labor supply model with one Hicksian consumption good, two leisure commodities and two members.

The test pertaining to the second category, that is applying to the effects of distribution factors on consumption, are due to Bourguignon et al. (1995) and are less prone to these limitations. They can be implemented with cross-sectional data and, as shown below, only require the number of observed commodities to be greater than the number of household members. In households with only two members, Bourguignon et al. (1995) have shown that the restrictions imposed by distribution factors stem from the fact that only affect the point chosen by the household on the Pareto frontier of consumption possibilities, and not the frontier itself. Chiappori and Ekeland (2002) have generalised this result to household with more than two members. This paper also proposes such a generalization, but which is different than the one of Chiappori and Ekeland and which produce a test easier to implement. Furthermore a new result is proposed. These two resultses are then tested on bigamous and monogamous households from Burkina Faso based on a survey that we have carried out in 1999. We find that our survey data is consistent with collective rationality in both type of households . Furthermore, the data indicates that all the spousess influence to some extent the household expenditures in both setting. To our knowledge, this is the paper to test the collective rationality with household including more than two members.

The next section presents the theoretical framework and the different testable results. It is followed by a section presenting the data and the context in Burkina Faso. The last section presents the econometric approach and the empirical results.

2 The Theoretical Framework

Consider a household with $I+1$ members (with $I \geqslant 1$). Each member i, with $i = 1, ..., I+1$, draws his/her well-being from the consumption of N market commodities, which we represent by the N-vector $\mathbf{x}_i$ for private consumption and by $\mathbf{X}$ for public consumption.[4]. Each commodities may thus serve private and public uses simultanously. All prices are normalized to one. The household consumption is given by $\sum_{i=1}^{I+1} \mathbf{x}_i + \mathbf{X} \equiv \mathbf{x}$ and the household budget constraint is therefore given by: $\boldsymbol{\iota}' \left(\sum_{i=1}^{I+1} \mathbf{x}_i + \mathbf{X} \right) \equiv \boldsymbol{\iota}'\mathbf{x} = m$, where $\boldsymbol{\iota}$ is a unit vector of dimension N and m represents the level of total household expenditures.[5]

Axiom 1 *Each member i has preferences given by a strongly concave and twice continuously differentiable utility function $U_i(\mathbf{x}_i, \mathbf{X})$, which differs from those of the other members.*

Axiom 2 *The outcomes of the decision process are (weakly) Pareto-efficient.*

Axiom 3 *The household's decision process depends on a set of K variables, $\mathbf{y} \equiv [y_1, y_2, ..., y_K]'$, called distribution factors, that are independent of individual preferences and which do not affect the overall household budget constraint.*

A discussion on distribution factors, a crucial concept for our model, is necessary before going any further. The influence of distribution factors on decision-making can be interpreted as the result of their effect on the bargaining power of the household members. The bargaining power of a member is generally conceived as following from his threat point, that is the member's vulnerability in the event of a disagreement over the resources allocation. The greater a member's vulnerability, the more he will need to reach an agreement, and consequently, the more he will make concessions. He will thus have a lower bargaining power and as a result, the outcomes of the decision process will correspond less to his or her own preferences. The possibility that a distribution factor influence the decision process other than through its effect on the bargaining power is however not excluded.

The threat point of an individual can vary from one person to another and from a culture to another. It can also differ with the importance of the disagreement. For example, it could consist of adopting a non-cooperative behavior in the case of minor disagreements (Woolley 1988) and to separate when the disagreements are major (Manser and Brown, 1980, McElroy and Horney, 1981). Many distribution factors have been proposed in the literature when the threat point corresponds to a separation. Becker (1981) has suggested the state of the marriage market, caracterised for example by the sex-ratio (Chiappori, Fortin and Lacroix, 2002) and

[4]Following convention, we will denote vectors and matrices in boldface characters. Further, the expression $D_{\mathbf{z}}\mathbf{f}(\mathbf{z})$ will denote the partial derivatives matrix of any vector-valued differentiable function $\mathbf{f}(\mathbf{z})$ with respect to $\mathbf{z}$, whose mn^{th} entry is $\partial f_m(\mathbf{z})/\partial z_n$.

[5]This assumes that the household does not produce any of these N goods, or equivalently, that market goods are perfect substitutes for household production.

the divorce laws (Gray 1988, Chiappori, Fortin and Lacroix, 2002). McElroy (1990) also proposes as potential distribution factors the "parameters that characterize government taxes and government or private transfers that are conditioned on marital or family status". Haddad and Kanbur (1991) are adding to this list, the economic possibilities of the individuals external to the household, such as the access to communes, the laws regarding food pensions and the care of children, the capacity of women to return in their native family and the discrimination against women in the market place. The next section will proposes potential distribution factors specific to the Burkina Faso context.

Let's now go back to our model. Technically, the three axioms postulated above are equivalent to saying that there exists I scalar functions $0 \leq \mu_i(m, \mathbf{y}) \leq 1 \; \forall \; i$, which we will call Pareto weights, and such that $\mathbf{x}$ is the solution to the following program:

$$\underset{\{\mathbf{x}_1,...,\mathbf{x}_{I+1},\mathbf{X}\} \in \mathbb{R}_+^N}{\text{Max}} \boldsymbol{\mu}_{I+1}(m, \mathbf{y})' \; [U_1(\mathbf{x}_1, \mathbf{X}), ..., U_{I+1}(\mathbf{x}_{I+1}, \mathbf{X})]$$
$$\text{subject to } \boldsymbol{\iota}' \left(\sum\nolimits_{i=1}^{I+1} \mathbf{x}_i + \mathbf{X} \right) = m,$$

where $\boldsymbol{\mu}_{I+1}(m, \mathbf{y}) \equiv [\mu_1(m, \mathbf{y}), ..., \mu_I(m, \mathbf{y}), 1]$. Thus the household pseudo-utility function[6] to be maximized in this program is a weighted sum of the members utility functions, with the vector $\boldsymbol{\mu}_{I+1}(m, \mathbf{y})$ holding for the relative utility weights of the I first members with respect to the $I + 1$th member. The vector $\boldsymbol{\mu}_{I+1}(m, \mathbf{y})$ can be interpretated as representing the bargaining power of the I first members relatively to the $I + 1$th member. One important characteristic of this collective approach is that the I relative utility weights are not constant in general, but instead are functions of the overall household expenditures and of the distribution factors $\mathbf{y}$.

The demand system under collective rationality obtained from solving the program (P) for $\mathbf{x}$ can be written as: $\mathbf{x} = \hat{\mathbf{x}}(m, \; \boldsymbol{\mu}_{I+1}(m, \mathbf{y}))$, with $\boldsymbol{\iota}'\hat{\mathbf{x}}(m, \; \boldsymbol{\mu}_{I+1}(m, \mathbf{y})) = m$ from the adding-up restriction. This system shows that the distribution factors influence household consumption choices only through the I relative utility weights entering the household pseudo-utility function. This is a consequence of the fact that the distribution factors do not affect the Paretian frontier of the household consumption possibilities (which depends only on preferences and the household budget constraint), but only the location of the point chosen by the household on this frontier. The basic issue therefore is to find a way to test whether the household demand system can be written as $\hat{\mathbf{x}}(m, \; \boldsymbol{\mu}_{I+1}(m, \mathbf{y}))$. Yet, even if the I Pareto weights were existing, it would be possible to observe them directly. Only the following reduced form $\widetilde{\mathbf{x}}(m, \mathbf{y})$ is observable. We must therefore find a way to test whether the demand system $\widetilde{\mathbf{x}}(m, \mathbf{y})$ satisfy:

$$\widetilde{\mathbf{x}}(m, \mathbf{y}) \equiv \hat{\mathbf{x}}(m, \; \boldsymbol{\mu}_{I+1}(m, \mathbf{y})). \tag{1}$$

[6] Strictly speaking, this is not a utility function, since it depends on the total expenditures of the household.

In order to keep the presentation as simple as possible, we shall drop m from all functions for the remainder of the paper. Thus (1) becomes:

$$\widetilde{\mathbf{x}}(\mathbf{y}) \equiv \hat{\mathbf{x}}(\boldsymbol{\mu}_{I+1}(\mathbf{y})). \tag{2}$$

Now, based on a particular type of conditional demand system generalizing the approach suggested by Bourguignon et al. (1995), we will show that it is indeed possible to derive two local tests of collective rationality. For this, we shall consider a partition $\mathbf{x} = [\mathbf{x}_1', \mathbf{x}_2']'$ of the demand system and a partition $\mathbf{y} = [\mathbf{y}_1', \mathbf{y}_2']'$ of the distribution factors, with $\mathbf{x}_1$and $\mathbf{y}_1$having the same dimension J. Given such a partition, (2) can be written as:

$$\begin{aligned} \mathbf{x}_1 &= \widetilde{\mathbf{x}}_1(\mathbf{y}_1, \mathbf{y}_2) \equiv \hat{\mathbf{x}}_1(\boldsymbol{\mu}_{I+1}(\mathbf{y}_1, \mathbf{y}_2)), & (3) \\ \mathbf{x}_2 &= \widetilde{\mathbf{x}}_2(\mathbf{y}_1, \mathbf{y}_2) \equiv \hat{\mathbf{x}}_2(\boldsymbol{\mu}_{I+1}(\mathbf{y}_1, \mathbf{y}_2)). & (4) \end{aligned}$$

Lemma 1 *Let $\mathbf{y}^* \in \mathbb{R}^K$ be a point at which $\widetilde{\mathbf{x}}_1(\mathbf{y})$ is differentiable and such that $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*)$ is non-singular. Then, conditional on $\mathbf{x}_1^* = \widetilde{\mathbf{x}}_1(\mathbf{y}_1^*, \mathbf{y}_2^*)$, there exists a unique and continuously differentiable function $\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2)$ that solves (3) for $\mathbf{y}_1$ in some neighborhood of $(\mathbf{x}_1^*, \mathbf{y}_2^*)$ and such that:*

$$\mathbf{x}_1^* = \overline{\mathbf{x}}_1(\mathbf{x}_1^*, \mathbf{y}_2) \equiv \widetilde{\mathbf{x}}_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2), \mathbf{y}_2) \equiv \hat{\mathbf{x}}_1(\boldsymbol{\mu}_{I+1}(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2), \mathbf{y}_2)). \tag{5}$$

See the annex for the proofs. Under the conditions of Lemma 1, one can define the function $\overline{\mathbf{x}}_2 : \mathbb{R}^K \rightarrow \mathbb{R}^{N-J}$:

$$\overline{\mathbf{x}}_2(\mathbf{x}_1^*, \mathbf{y}_2) \equiv \widetilde{\mathbf{x}}_2(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2), \mathbf{y}_2) \equiv \hat{\mathbf{x}}_2(\boldsymbol{\mu}_{I+1}(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2), \mathbf{y}_2)). \tag{6}$$

The right-hand side of (6) yields a (local) demand sub-system for $\mathbf{x}_2$ conditional on the J-vector $\mathbf{x}_1^*$ and the $K-J$-vector $\mathbf{y}_2$.[7] One should note, that a demand function that is insensitive to a distribution factor may respond to it once it is conditioned on $\mathbf{x}_1$ through the function $\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2)$.

This sub-system will be used to state two theorems which will provide the tests of collective rationality. Prior to stating the theorems, though, we need to define additional notation. Let the n^{th} demand function of the sub-system defined by $\mathbf{x}_2$ be denoted by x_{2n} with $n = 1, \ldots, N-J$. Likewise, let the k^{th} distribution factor in $\mathbf{y}_2$ be denoted as y_{2k}, with $k = 1, \ldots, K-J$.

[7] It should be noted that the ordering of the demands and the distribution factors is not important.

Theorem 1 *Assume that $\boldsymbol{\mu}_{I+1}(\mathbf{y})$ and $\widehat{\mathbf{x}}(\boldsymbol{\mu}_{I+1}(\mathbf{y}))$ are differentiable at respectively $\mathbf{y}^*$ and $\boldsymbol{\mu}_{I+1}(\mathbf{y}^*)$. Assume also that $K \geqslant I$, $N \geqslant I$. Then, when $J = I-1$ and the conditions of the Lemme 1 are satisfied, we have that:*

$$\begin{aligned}
\forall\, D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) = \;& \mathbf{0}: \qquad (7)\\
& D_{\mathbf{y}_2}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0} \;\; ou \;\; D_{\mathbf{y}_{21}}\widetilde{x}_{2n}(\mathbf{y}^*) \neq\neq \mathbf{0} \;\; et \;\; D_{\mathbf{y}_{22}}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0},\\
\forall\, D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) \neq \;& \mathbf{0}: \qquad (8)\\
& D_{\mathbf{y}_2}\overline{x}_{2n}(\mathbf{x}_1^*, \mathbf{y}_2^*) = \mathbf{0} \, ou \, D_{\mathbf{y}_{23}}\overline{x}_{2n}(\mathbf{x}_1^*, \mathbf{y}_2^*) \neq\neq \mathbf{0} \, et \, D_{\mathbf{y}_{24}}\overline{x}_{2n}(\mathbf{x}_1^*, \mathbf{y}_2^*) = \mathbf{0},
\end{aligned}$$

where $\mathbf{y}_{21}, \mathbf{y}_{22}, \mathbf{y}_{23}$ and $\mathbf{y}_{24}$ represent sub-vectors of $\mathbf{y}_2$ having dimensions that can vary from zero to $K-J$, and such that $\mathbf{y}_2 \equiv [\mathbf{y}_{21}\ \mathbf{y}_{22}] \equiv [\mathbf{y}_{23}\ \mathbf{y}_{24}]$.

This theorem says that all the demands belonging to $\mathbf{x}_2$ that are not influenced by $\mathbf{y}_1$ are, either not influenced at all by the distribution factors contained in $\mathbf{y}_2$, or, are only influenced by a common sub-group of distribution factors $\mathbf{y}_{21}$ contained in $\mathbf{y}_2$. All the other demands, that is those that are influenced by at least one of the distribution factors contained in $\mathbf{y}_1$, turned out to be, once they are conditionned on $\mathbf{x}_1$, again either not influenced at all by the distribution factors contained in $\mathbf{y}_2$, or, are only influenced by a common sub-group of distribution factors $\mathbf{y}_{23}$ contained in $\mathbf{y}_2$, which can be different from $\mathbf{y}_{21}$.

The intuition behind these results is fairly straightforward. If a demand x_{2n} is not influenced by $\mathbf{y}_1$, it must be because it is not influenced by the Pareto weights through which $\mathbf{y}_1$ exerts its effect. Since it is a condition of the Lemme 1 that $\mathbf{y}_1$ is at least influencing $I-1$ Pareto weights (otherwise $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*)$ would be singular), this can happen under two circonstances. The first one is when x_{2n} is not function of any of the Pareto weights in which case, it will necessary be independent of $\mathbf{y}_2$. The second one is when x_{2n} is only function of the one remaining Pareto weight which is not itself function of $\mathbf{y}_1$, and in this case, will only be influenced by the distribution factors entering in this specific weight. Now, if x_{2n} is influenced by $\mathbf{y}_1$, it must be because it is influenced by at least one of the Pareto weights through which $\mathbf{y}_1$ exerts its effect. It will then be possible to condition it on $\mathbf{x}_1$, which will have the effect of maintaining $\mathbf{x}_1$ constant. For the $I-1$ demands contained in $\mathbf{x}_1$ to remain constant, which are themselves function of at least $I-1$ Pareto weights, it is necessary that $\mathbf{y}_1$ compensate the variations of $\mathbf{y}_2$ in such a way tgat the variations in the $I-1$ weights either are equal to zero or cancelled out. Therefore, $\overline{x}_{2n}$ will either be influenced by the one remaining weight, and thus by the distribution factors influencing this specific weight, or will not be influenced by it, and thus not influenced at all by the distribution factors contained in $\mathbf{y}_2$.

While the results of the Theorem 1 hold when $K \geqslant I$, $N \geqslant I$.and the conditions of the Lemme 1 are satisfied for $J = I-1$, they will provide a test of (local) collective rationality only when $K \geqslant I+1$ and $N > I+1$. Indeed, if $K = I$, $\mathbf{y}_2$ will only contain one distribution factor, and thus the results of Theorem 1 will always be satisfied. Same thing if $N = I$, since a single demand equation would be contained in $\mathbf{x}_2$. When $N = I + 1$, $\mathbf{x}_2$ will contain two demands, but the problem will still arise if one of the demand is not influenced by $\mathbf{y}_1$, but the second one is. Even if both were influenced by $\mathbf{y}_1$, the budgetary constraint would imply that $\overline{x}_{21}(\mathbf{x}_1^*, \mathbf{y}_2^*) + \overline{x}_{22}(\mathbf{x}_1^*, \mathbf{y}_2^*) = m - \iota'\mathbf{x}_1^*$. Therefore $\mathbf{y}_2$ would automatically either influenced the two conditional demands or none at all, irrespective of collective rationality. Thus the results of the Theorem 1 provide a test of collective rationality only when $N > I + 1$ and $K \geqslant I + 1$.

To our knowledge the Theorem 1 set out new results. We will see further how theses results differ from the ones of Bourguignon et al. (1995), which we will now generalise to the case where there are $I + 1$ members in the household.[8]

Theorem 2 *Assume that $\boldsymbol{\mu}_{I+1}(\mathbf{y})$ and $\widehat{\mathbf{x}}(\boldsymbol{\mu}_{I+1}(\mathbf{y}))$ are differentiable at respectively $\mathbf{y}^*$ and $\boldsymbol{\mu}_{I+1}(\mathbf{y}^*)$. Assume also that $K \geqslant I + 1$, $N \geqslant I + 1$. Then, when $J = I$ and the conditions of the Lemme 1 are satisfied, we have that:*

$$
\begin{aligned}
\forall\, D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) \quad &= \quad \mathbf{0}: \quad D_{\mathbf{y}_2}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0}, \qquad (9) \\
\forall\, D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) \quad &\neq \quad \mathbf{0}: \quad D_{\mathbf{y}_2}\overline{x}_{2n}(\mathbf{x}_1^*, \mathbf{y}_2^*) = \mathbf{0}. \qquad (10)
\end{aligned}
$$

The intuition behind these result is the following. If a demand x_{2n} is not influenced by $\mathbf{y}_1$, it must be because it is not influenced by the Pareto weights through which $\mathbf{y}_1$ exerts its effect. Since it is a condition of the Lemme 1 that $\mathbf{y}_1$ is influencing the I Pareto weights (otherwise $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*)$ would be singular), this can happen under only one circonstance: when x_{2n} is not influenced by any of the Pareto weights. It can not therefore be influenced by $\mathbf{y}_2$. Now, if x_{2n} is influenced by $\mathbf{y}_1$, it must be because it is influenced by at least one of the Pareto weights through which $\mathbf{y}_1$ exerts its effect. It will then be possible to condition it on $\mathbf{x}_1$, which will have the effect of maintaining $\mathbf{x}_1$ constant. For the I demands contained in $\mathbf{x}_1$ to remain constant, which are themselves function of the I Pareto weights, it is necessary that $\mathbf{y}_1$ compensate the variations of $\mathbf{y}_2$ in such a way the variations in the I weights either are equal to zero or cancelled out. In brief, it is like we were maintaining the I Pareto weights constant.

Like for the Theorem 1, the results of the Theorem 3 will only provide a (local) test of collective rationality when $N > I + 1$. When $N = I + 1$, one has $\overline{x}_2(\mathbf{x}_1^*, \mathbf{y}_2^*) = m - \boldsymbol{\iota}'\mathbf{x}_1^*$ from the adding-up restriction. Therefore, the second result is always satisfied in this case.[9] Note that in practice however, only K^o $(\leqslant K)$ distribution factors and N^o $(\leqslant N)$ commodities are observed. It is nevertheless still possible to test the restrictions imposed by collective

[8]More precisely, we are generalising the third result of the their proposition 1 which applies to two-person household. Note that this generalization has already been published in Dauphin and Fortin 2001.

[9]Note that when $N = I + 1$, the only remaining demand x_2 is necessarily influenced by $\mathbf{y}_1$. Otherwise, the lemme 1 would not be satisfied for$J = I$.

rationality in the two theorems if $K^o \geqslant I+1$ and $N^o \geqslant I+1$ for $N^o < N$ or $N^o > I+1$ for $N^o = N$.

These last results provide a way of determining the actual number of household members having a Pareto weight different from zero, that is the number of members influencing the decision process.

Corollary 1 *Assume that the household decisions are collectively rational. Assume also that* $rank(D_{\boldsymbol{\mu}_I}\widehat{\mathbf{x}}_2(\boldsymbol{\mu}_{I+1}(\mathbf{y}^*)) = I \quad \forall\, J < I$*. Then, under the conditions of Theorem 2, the number of members in the household influencing the decision process is given by the smallest number of goods upon which the demand functions must be conditioned in order to satisfy restrictions (10, plus one.*

The two tests provided by the results of the theorems 1 and 2 (thereafter to be referred as test 1 and test 2) differ in some ways. First, while they both require to have $K \geqslant I+1$ and $N > I+1$, the test 1 require that $I-1$ demands satisfied the Lemme 1, compared to I demands for the test 2. Therefore the conditions under which the test 1 can be performed are slighty less restrictive than those of the test 2. It comes at a cost however. The results of the two theorems are both necessary conditions. Under collective rationality, they will thus all by satisfied, but in the absence of collective rationality, they will not necessarily all be rejected. In fact, as it is shown in the annex, the satisfaction of the results (7) and (8) is a necessary condition for the satisfaction of the results (9) and (10). Therefore, in the absence of collective rationality, it could be possible to reject the results of the Theorem 2, but to not reject those of the Theorem 1. In this sense, the second test is more reliable or powerful than the first one.[10]

Before moving to the next section presenting the Burkina Faso context and the data, we will compare the results of our two theorems with the one of the Chiappori and Ekeland (2002)[11]:

Theorem 3 *Assume that* $\boldsymbol{\mu}_{I+1}(\mathbf{y})$ *and* $\widehat{\mathbf{x}}(\boldsymbol{\mu}_{I+1}(\mathbf{y}))$ *are differentiable at respectively* $\mathbf{y}^*$ *and* $\boldsymbol{\mu}_{I+1}(\mathbf{y}^*)$*. Assume also that* $K \geqslant I$ *and* $N \geqslant I$*. Then we have that:*

$$rank\left[D_{\mathbf{y}}\widetilde{\mathbf{x}}(\mathbf{y}^*)\right] \leq I. \tag{11}$$

As for the results of our theorems, this result will provide a test of collective rationality only when $K \geqslant I+1$ and $N > I+1$. On the other hand, Chiappori and Ekeland theorem doesn't require that a certain a number of demands satisfy the Lemme 1. It can thus

[10] It should be noted that the choice of the elements in $\mathbf{x}_1$ and $\mathbf{y}_1$ doesn't influence the outcome of the tests. If collective rationality is not rejected for a certain choice of $\mathbf{x}_1$ and $\mathbf{y}_1$, it will not be rejected with another combinaison of $\mathbf{x}_1$ and $\mathbf{y}_1$, as long as the Lemme 1 and the regularity conditions are respected. The same thing applies if collective rationality is rejected.

[11] More precisely, the result in question is the first result of their proposition 10.

be performed under more general circonstances, but again, this comes at a cost. By not imposing that $I-1$ or I demands satisfy the Lemme 1, it makes it possible to not reject this result even when there is less then I members influencing the decisions. For example, if $rank\left[D_{\mathbf{y}}\widetilde{\mathbf{x}}(\mathbf{y}^{*})\right]=I-2$ was found, it would be consistent with collective rationality of $I+1$, I and $I-1$ members influencing the decision process. By imposing that $I-1$ or I demands satisfy the Lemme 1,we are eliminating the possibility that respectively less than I or $I+1$ members influence the decision. When the Lemme 1 is satisfied for $J=I$, then Chiappori and Ekeland' results, which becomes $rank\left[D_{\mathbf{y}}\widetilde{\mathbf{x}}(\mathbf{y}^{*})\right]=I$, provide a restriction that is equivalent to those provided by our Theorem 2. That is if (11) is rejected (not rejected), then (9) and (10) are rejected (not rejected) and vice versa. In these circumstances, the only disavantage of the test provided by the (11) compared to the one provided by (9) and (10) is that tests of rank are more difficult to implement.

The above theoretical results will be used to test collective rationality within monogamous and bigamous households in Burkina Faso. But before presenting the data, the family context in Burkina Faso will be discussed.

3 The Context in Burkina Faso

Burkina Faso is one of poorest country of the world. In 2001 the country was classified as the 147th over 162 in terms of life expectency, the 161th in terms of literacy and the 142th in terms of GDP per capita.[12] The proportion of the population living in rural area is 83% and the economy is still mainly dominated by the agricultural sector, which occupies as the principal activity 90% of the active population.[13] The agricultural techniques are still traditionnal, that is mainly manual, without animals or tractors. Burkina's population was evaluated at 11.2 millions in 1999 and is composed of around 60 ethnic groups of different sizes.[14] The most important one is the Mossi, which count for more than half of the total population. The animist religion, which was once the dominant one, is now abandon for muslim and cathologic religions.

3.1 Family Context

The family context of the different ethnic groups, while similar on certain aspects, shows some differences. Since the survey which will be used to test the theoritical results was done in a

[12] Human Development Report, 2001, UNDP.
[13] Burkina Faso Atlas, 1998, Jeune Afrique Atlases, Les éditions J.A., Paris, p.35.
[14] UNDP, loc. cit.

region with a verty strong Mossi predominance, we will limit our presentation to this ethnic group.[15]

In the Mossi society, as it is the case in many african societies, the living unit is the concession. It is formed by a collection of construction generally surrounded by a fence. At the minimum, the concession give shelter to a household composed of a man with his wife(s) and his children. Sometimes, the man's brothers and sons also live there with their wifes and children. Traditionnaly, the concession is the economic unit and is runned by the chief of the concession, who is usually the oldest man of the concession.

Most concessions farm many plots at the same time. Some of these plots are family plots, while others are personal plots. Each member of the concession must work on the family plots since its crops are intented for family consumption and obligations. The farm operations and the disposal of the crops are however under the autority of the concession's chief. After having fulfilled their tasks, adult women and men (with the exception of the concession's chief) go farm their own personal plots. The crops of theses plots belong to their cultivator. It should be noted that the plots belong to men. Women only have usufruct rights. The women living in concession carry out all the domestic work. Early on, around 5 years old, little girls help their mother with domestic and farming work.

Marriage is considered, above all, as an agreement between two families. Not only must the marriage receive the benediction of two families, but the couple will not be allowed to divorce without their consent. Girls have a limited influence on the decision took by their family. Besides, mariage are sometimes arranged when girls are very young. Girls marry around 16 and 18 years old, while boys marry latter, around 25 and 30 years old. Three kind of marriage exist: customary marriage, religious marriage and civil marriage. Each of these marriages is accompanied with differents norms. Customary marriage, which are by far the most common type, doesn't restrict the number of wives than a man can marry. Muslim marriage restrict the number of wives to four, while catholic and civil marriages autorize only one wife. In practice however, it is frequent to encounter a catholic man married to more than one wife. Polygamy is indeed quite common in the Mossi society, especially in rural areas, where some old men have up to six wives It is also very common to see couples cumulating more than one type of marriage.

[15]The anthropological litterature on Mossi families, on which this section is based, date back to the seventies and heigthies and seems outdated on some aspects. The main references are Lallemand (1977), Rookhuizen (1986) and Rohatynskyj (1988).

3.1.1 Distribution Factors in the Burkina Context

As it was mentionned earlier, the influence of distribution factors on decision-making can be interpreted as the result of their effect on the bargaining power of the household members. The bargaining power of a member is generally conceived as ensuing from his threat point, that is the situation in which the member would end-up in the event of a disagreement over the resources allocation. The lower is the member's well-being associated to the disagreement situation, the more he will want to reach an agreement, and thus the more he will make concessions. He will thus have a lower bargaining power. One possibility to identify distribution factors relevant to Burkina Faso is therefore to look for factors that will influence the members' well-being in the disagreement situation, without however influencing their preferences and the overall budget.

In the Mossi society, but also generally in Burkina Faso, it appears that a non-cooperative behavior is adopted by the household members in case of disagreement, at least as a first step. The husband will typically cut his material aid to his wife and in return she will reduce the quantity or the quality of the services that she is producing for him. When there is a conflict, the husband "refuses to give cereals, money and gifts to his wife and will prefer another wife. The wife at her turn refuses to carry out her domestic duties and her conjugal duties. [...] The wife can thus refuse to fetch water from the well for him, to heat it for him, to wash his clothes and give him food that she has herself produced or bought".[16] Following this logic, the more a man is dependant on his wife to produce these services for him, the worse will be his threat point, and the more a women will be independant financially, the best will be her threat point. The number of wives a man has should thus decrease his dependance over one or another of his wives, but it might as well be influenced by the husband's preferences and could also modify the household budget constraint, which disqualifies it as a distribution factor. On the wives side, the wife contribution to the household income could be a distribution factor. The income of an individual should be understood here as including the value of the crops from family plots for the chief of the concession, and the value of the crops from their personnel crops for the other household members.

When the situation becomes unbearable for the spouses, either the husband will chase out her wife, or she will leave him. The circumstances under which the wife and the husband are allowed to divorce and the sharing of the family wealth will depend on the kind of marriage they opted for. The civil mariage is more advantagous than the customary marriage for the wife.[17] Firstly, the circumstances under which a wife is allowed to ask for divorce are the same than those allowed to her husband, while in the customary marriage, they are much more re-

[16] Rookhuizen (1986), p. 59, free translation.

[17] Whether the religous catholic and muslim marriages offers greather protection to the wife than the civil mariage is not clear.

stricted for the wife. Secondly, the civil matrimonial regime is the community of goods, while it is the separtion of goods under the customary marriage.[18] The type of marriage could therefore influence the bargaining power of the spouses, but it could be endogenous. Furthermore, since it is a discrete variable, it would not allow us to perform the tests of collective rationality presented in the first section. The options available to a divorced woman are to go back live with her parents, or to find another man. The woman financial independance, the fact that her parents are still alive and the sex ratio, as a proxy of the marriage market, should thus all influence her well-being in case of a separation.

In addition to these potential distribution factors, the anthropological litterature suggests a few variables that could influence the status and the power of Mossi wifes, without however intervening through their threat point. One of these is the number of years since the marriage and the rank of the spouse in the case of polygamy. The custom in the Mossi society, and in Burkina Faso more generally, is that new wifes be initially under the authority of the oldest woman of the concession, who is usually the mother's husband. With time passing, the wife will acquire more automony. Furthermore, "When a wife is married to a man who already has many wives, in addition of being the mother-in-law helper, she also becomes the helper of the other wives".[19] The co-wives "must submit to an internal hierarchy conditionned by age and the length of the marriage: negligable when less than a decade has separated either their birth or their union, it is perceptible beyond. [...] the first wife has authority on the other wifes".[20]

In summary, some potential distribution factors for the Mossi society that could be used to test our collective rationality results are: the wives financial independance, proxied by their contribution to the household income, the marriage market, proxied by the sex-ratio, the number of years since marriage of the different wifes and the numbers of years separating their marriage or their birth.[21]

4 Data and Institutional Setting

4.1 The Survey

The data we use are taken from a field survey in Burkina Faso that we conducted between January and March 1999. The survey was conducted under the auspices of the Centre canadien d'étude et de coopération internationale (CECI) and its primary purpose was to collect

[18] Boye (1987), Lallemand (1977) and Rookhuizen (1985).

[19] Rookhuizen, loc.cit. p. 58.

[20] Lallemand, loc. cit., p. 263.

[21] The fact the wives' parent are alive and the rank of the wife are not continuous variables, and thus are dropped.

information easy to measure, but determining for household decision-making on consumption spending, time allocation and fertility. All the potential (continues) distribution factors identified above were collected excepted the sex-ratio, which would have been too complicated and costly.[22] That is the contribution of the different spouses to the household income, the number of years since marriage of the different wifes and their age. The information on the income of the different spouses was collected indirectly. Since most households lived out of agriculture and since agricultural production survey are very complex, we have favored an indicator of their permanent income: their expenditures. More precisely, we have collected the expenditures and the auto-consumption made by each spouse from his own income and agricultural production.on food and non-food products. Expenditures on certain assignable goods were also collected: the household expenditures on clothes and hairdressing for the husband, the wife and their respective children.

The survey was conducted in the Province of Passoré, which has a population of approximately 300 000[23], primarily because the CECI has been involved in the region for a long time and has established close links with the local institutions. The administrative regions chosen in this province by the Enquête Prioritaire 1994-1995 were chosen to form the base de sondage of the primary units of this survey.. Of the nine regions available, five were selected in a way to represent the economic and social fabric of the province.and to minimize transportation costs. These are: Dakiégré, Pelegtanga, Rallo, and sectors 1 and 5 of the City of Yako.

To be included in the sample a household had to meet the following two conditions: (1) The (male) household head as well as his spouse(s) had to be less than 70 years of age and; (2) his spouse(s) had to live permanently on his compound. Prior to sampling, a census was conducted in each of the five regions to identify married households and to determine eligibility. Over 125 married households were then randomly selected among the eligibles, except for the village of Dakiégré where all 111 households were selected.

Table 1 indicates the number of potential households as well as the number of households who were present at the time of the survey and agreed to answer the detailed questionnaire. Overall, as many as 552 households out of 611 were interviewed (response rate = 90.3%). The questionnaires were pre-tested during a period of two weeks by local trained investigators. For each household, the head and each of his spouses were interviewed individually and separately using a “female” and “male” version of the questionnaire. Heads were interviewed by a male investigator and each spouse was interviewed by a female investigator.

[22] First, we would have had to determine what was the right perimeter to consider for the sex-ratio of each of the localities surveyed. Second, we would have had to survey these perimeters, since the last census dates back to 1991.

[23] According to the 1991 Demographic Survey.

4.2 Sample Characteristics

Polygamy is more prevalent in rural areas. Indeed, Table 2 shows that the number of spouses per head is lower in Yako(1) and Yako(5), the only two urban areas in our sample, and is highest in the village of Dakiégré. The table also shows that nearly 72% of households are monogamous (393 out of 552).

As mentioned earlier, collective rationality will be investigated within bigamous households only. This restricts our sample to fewer than 117 observations. The main characteristics of this sample are presented in Table 3 which is divided into three separate panels. The first of these shows that husbands are on average relatively older than both spouses and have somewhat less schooling.[24] Younger spouses, not surprisingly, have fewer children than older spouses.

The second panel concerns distribution factors. Recall that distribution factors are variables that are thought to influence the decision process within the household, but to have no impact either on preferences or on the household budget set. A natural candidate for bigamous households is to consider years of marriage. Indeed, it seems likely that seniority within the household may confer some privileges with respect to expenditures decisions. We will thus consider the difference in years of marriage between the two female spouses as a potential candidate.

In line with much of the recent empirical literature on collective models, we also consider income shares as two additional distribution factors. In our data, both female spouses earn on average the same proportion of total household income. Indeed, the table indicates that each one earns approximately 17% of family income.

The potential to assess the impact of distribution factors on expenditures is greatly enhanced if we focus on so-called "assignable" or "exclusive" goods. Assignable goods may be consumed by more than one household member but individual consumption is observable in the data. Exclusive goods, on the other hand, are consumed entirely by a single individual and its consumption is thought not to provide any (positive/negative) externality to other family members. *A priori*, distribution factors that favour a particular household member should have a noticeable impact on household expenditures on his/her exclusive goods.

The field survey was designed to collect information on the main exclusive goods consumed by each spouse in the household. Survey pre-testing indicated that clothing and hairdressing were the best two items that could qualify as exclusive goods. Each spouse in the household was thus questioned about the expenditures made on these goods both for his/her

[24] The schooling variable is categorized from 1 (no schooling) to 18 (high-school completed). The literacy rate is as low as 4.25% for men and 11.25% for women. Younger spouses have a literacy rate of 12.2% compared to 7.7% for older spouses.

own purposes, and for those of the other spouses. The third panel of the table reports the average household expenditure on male and female clothing and hairdressing.

5 Estimation Results

The following Working-Leser system of non-conditional demands is estimated using our data.

$$x_i = Z_i\delta_i + \rho_i \mathrm{m} + \theta_i \mathrm{m} \log \mathrm{m} + \alpha_i(\mathrm{m}_1/\mathrm{m}) + \beta_i(\mathrm{m}_2/\mathrm{m}) + \gamma_i \ \Delta(\text{Years of Marriage}), \quad (12)$$

where m_i/m is the income share of female spouse i, $i = 1, 2$, and Δ(Years of Marriage) is the difference in years of marriage between the older and the younger spouses. Likewise, Z_i is a vector of taste shifters and δ_i is an appropriately dimensioned vector of parameters.

In practice, only K^o $(\leqslant K)$ distribution factors and N^o $(\leqslant N)$ commodities are observed. It is nevertheless still possible to test the restrictions imposed by collective rationality if $K^o \geqslant I + 1$ and $N^o \geqslant I + 1$ for $N^o < N$ or $N^o > I + 1$ for $N^o = N$. At the very least, each of the K^o variables should significantly affect at least one of the N unconditional demands, which is required for these variables to be distribution factors.

With regards to the test itself, the choice of the elements of $\mathbf{x}_1$ on which the demand sub-system is conditioned should not influence the result.[25] Furthermore, note that the estimation of this conditional sub-system raises an identification issue even when $\mathbf{y}$ and $\mathbf{m}$ are exogenous, since the $\mathbf{x}_1$ variables are endogenous. However, since the number of exogenous variables excluded (that is, the number of elements in vector $\mathbf{y}_1$) is equal to the number of right-hand side endogenous variables included ($= I$), the order criterion for exact identification in a linear model is satisfied.

5.1 Estimation Results for Lemma 1

The results reported in Table (4) is based on the following specification:

$$\begin{aligned}
\text{Husb Cloth} &= Z_1\delta_1 + \rho_1 \mathrm{m} + \theta_1 \mathrm{m} \log \mathrm{m} + \alpha_1(\mathrm{m}_1/\mathrm{m} + \beta_1(\mathrm{m}_2/\mathrm{m}) + \gamma_1 \ \Delta(\text{YM}) \quad (13)\\
\text{Wife Cloth} &= Z_2\delta_2 + \rho_2 \mathrm{m} + \theta_2 \mathrm{m} \log \mathrm{m} + \alpha_2(\mathrm{m}_1/\mathrm{m} + \beta_2(\mathrm{m}_2/\mathrm{m}) + \gamma_2 \ \Delta(\text{YM}) \quad (14)\\
\text{Wife Hair} &= Z_3\delta_3 + \rho_3 \mathrm{m} + \theta_3 \mathrm{m} \log \mathrm{m} + \alpha_3(\mathrm{m}_1/\mathrm{m} + \beta_3(\mathrm{m}_2/\mathrm{m}) + \gamma_3 \ \Delta(\text{YM}) \quad (15)
\end{aligned}$$

where YM stands for “Years of Marriage”. In both equations (14) and (15), the expenditures pertain to the youngest spouse. Only these three demand functions depicted any sensitivity

[25] The reason for this is that, regardless of whether Pareto-efficiency holds or not, if (2) holds for a given order of elements of $\mathbf{x}$ and $\mathbf{y}$, it will also hold for any other order for which the regularity conditions are satisfied.

to the distribution factors considered in the analysis. The village dummy variables must be interpreted with respect to Yako (5), one of the two urban areas represented in our data. The parameter estimates indicate that men's expenditures are lowest in Yako (5). Otherwise no clear pattern emerges from the data. The religion dummy variables have no particular impact on any of the expenditures considered in the table. The presence of young children, on the other hand, has no impact of men's clothing but definitely decreases expenditures on both the second wife's clothing and hairdressing. Finally, men's expenditures on clothing increases rapidly with income, while the second spouse's expenditures decreases slowly as household income increases.

The second panel of the table reports the parameter estimates of the distribution factors for the three demand functions. Most are statistically significant and have the expected sign. For instance, household expenditures on men's clothing decreases as the income shares of both spouses increase. Likewise, as the difference in years of marriage between the two spouses increases, household expenditures on men's clothing decreases as well. Household expenditures on the younger spouse's clothing and hairdressing decrease as the income share of the older spouse increases, but increase as her own share of income increases. Finally, expenditures on the younger spouse's clothing and hairdressing decrease with the difference in years of marriage.

5.2 Estimation Results for Theorems 1 and 2

The only distribution factor that has a statistically significant impact on each unconditional demand function is the difference in years of marriage. This suggests three different ways to test the results of Theorems 1 and **??** assuming there are three decision-makers in the household. Indeed, each demand function can in turn be inverted with respect to this distribution factor and the remaining demand functions will be conditioned accordingly. The test conditions specified in (**??**) and (**??**) can be verified parametrically.

The results for the conditional demands are reported in Table 5. The three columns report the estimation results of two conditional demand functions. The demand upon which they are conditioned is reported in the last panel of the table. In all three cases, the conditioning demand is inverted with respect to the difference in years of marriage as mentioned earlier. This is why all three specifications are expressed in terms of the two income shares.

The first specification reports the parameter estimates of the expenditures functions for husband's clothing and (second) wife's clothing. Both demand functions are conditioned on expenditures on the second wife's hairdressing. Notice first that the parameter estimates of most demographic variables change little relative to those reported in the unconditional demand functions (Tables 4), except perhaps for the fact that some parameter in the husband's equations are no longer statistically significant. Furthermore, the parameter estimates of both Y and $Y \log(Y)$ are also no longer statistically significant in the husband's equation.

The parameter estimates of the income shares are highly statistically significant in both equations. These parameter estimates need to be interpreted with caution. Indeed, since the demand functions are conditioned on hairdressing expenditures, the two remaining distribution factors have to adjust so that any change in the omitted distribution factor leave the conditioning demand constant. According to Theorem 1, the remaining distribution factors should either have no impact on the conditional demands, or a subset of those should have an impact on each conditional demands. Thus this specification satisfies the conditions of Theorem 1.

Recall that Theorem **??** requires the ratio of the marginal impacts of the distribution factors to be the same across the conditional demands. The P-Value of the χ^2 statistic is reported in the last line of the table. According to the P-Value, this restriction is not satisfied by the data. Since Theorem **??** constitutes a necessary condition for CR, this specification leads one to reject it.

The second specification conditions husband's clothing expenditures and (second) wife's hairdressing expenditures on the (second) wife's clothing expenditures. The parameter estimates of husband's expenditures are similar to those of the unconditional demand function (Table 4). On the other hand, a number of parameter estimates of wife's hairdressing equation lose their statistical significance, namely those related m and $m \log(m)$. As was the case with the first specification, the conditions required by Theorem 1 are also satisfied in this specification. The main difference with the previous case is that the income share of the first wife is not statistically significant in either conditional demands. Hence, the parametric test of Theorem **??** can not be computed.

The last specification focuses on (second) wife's clothing and hairdressing expenditures, conditional on household expenditures on husband's clothing expenditures. The parameter estimates associated with the demographic variables are very similar to those of the unconditional demand functions. As with the first specification, both distribution factors are statistically significant. Thus the conditions of Theorem 1 are satisfied once again. Interestingly, the parametric test of Theorem **??** is also satisfied (P-Value = 0.400). Hence, on the basis of this specification CR can not be rejected.

5.3 Estimation Results for Theorem 3

As stressed by Bourguignon et al. (1995), CR tests based on parametric restrictions across demand functions are less robust than those based on single equation restrictions. Although the restrictions in (**??**) and (2) are formally equivalent, their empirical counterpart may not agree or may not be consistent for many reasons. Fortunately, our framework allows us to investigate the validity of CR using both avenues.

The manner in which the parametric restriction of Theorem **??** can be carried out is dependent upon the conditions of Lemma 1. Indeed, it is required that $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*)$ be non-singular (and that $\mathbf{x}_1^* = \widetilde{\mathbf{x}}_1(\mathbf{y}_1^*, \mathbf{y}_2^*)$). The non-singularity of $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*)$ is tested parametrically for each pairwise combination of distribution factors across the three demand equations. The P-Value of the χ^2 statistics are reported for each combination in Table 6. The first two columns of the table indicates which demand functions and corresponding distributions factors are used in computing the tests. The third column states the tests formally in terms of equations (13)–(15).

Using only those combinations for which the P-Value is less than 5% allows to test Theorem **??** in six different ways. Indeed, each $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*)$ that is non-singular allows inverting the two unconditional demand functions with respect to the two distribution factors prior to substituting them into the remaining demand function. In principles, this conditional demand equation should be insensitive to the remaining distribution factor under collective rationality and assuming there are three decision-makers in the household.

Table 7 reports the parameter estimates of the six conditional demand functions. The second row of the table indicates which demand function is estimated. The second panel of the table indicates which distribution factor is left once the other two are substituted out. Finally, the last panel indicates the conditioning demands. The first noteworthy feature of the table is that the parameter estimates appear to be relatively similar to those of the unconditional demand functions (see Table 4). The second important feature is that most parameter estimates associated with the conditioning demands are statistically significant. This is to be expected since they summarize the influence of the distribution factors, which were found to affect significantly the unconditional demand functions. Lastly, and most importantly, in all six specifications the remaining distribution factor is found not to have any impact on the conditional demands. This is consistent with Theorem **??**.

Recall from Theorem **??** that whenever the number of conditioning demands is equal to the number of decision-makers minus 1, then collective rationality insures that the last $K - I$ distribution factors should have no impact on the conditional demands. Likewise, from Corollary 1 we know that the number of decision-makers is given by the smallest number of conditioning demands plus one. The results presented in Table 5 show that conditioning upon one distribution factor does not suffice to have the marginal impacts of the remaining distribution factors vanish. One must thus conclude that there are more than two decision-makers in the bigamous households of our sample. The results reported in Table 7, when interpreted along with the results of Table 5 and Corollary 1, indicate that there are three decision-makers in the bigamous households of rural Burkina Faso, and that the decision process is Pareto-efficient.

6 Conclusion

Household collective rationality (CR) has become a very important topic of research over the past few years. Many have sought to devise ways of testing CR in different contexts (*e.g.*, Chiappori (1992), Browning, Bourguignon, Chiappori and Lechene (1994), Udry (1996) and Fortin and Lacroix (1997)). The interest in this topic stems from the fact that efficiency has always been thought to be an innocuous and natural assumption to make when modeling household decisions. Surprisingly, though, Browning and Chiappori (1998) have shown that efficiency generates testable restrictions on consumption even in very general settings that allow for public and private commodities and externalities. households. As a by-product of their analysis, Browning and Chiappori (1998) have also provided a simple test which allows the number of decision-makers in a multi-person household to be determined.

Unfortunately, these tests face two limitations. First, they cannot be used with cross-sectional data that have no variability in regional prices. Second, it is easy to show that they cannot be implemented when the number of observed commodities is less than twice the number of intra-household decision-makers.

Fortunately, a complementary approach based on so-called distribution factors (see Browning et al. (1994)) provides tests that are less prone to these limitations. Bourguignon et al. (1995) have shown which restrictions distribution factors imply in the context of households with two decision-makers. Unfortunately, their results do not extend trivially to the case of multi-person households.

In this paper we generalize the distribution factors tests to households in which there are potentially more than two individuals in the decision process. We also provide a simple method of determining the number of decision-makers when the intra-household consumption decision process is efficient. The tests are used to investigate whether the decision process within bigamous households in rural Burkina Faso is efficient. Our survey data is consistent with collective rationality. Furthermore, the data indicates that all three spouses influence to household expenditures.

Proof of Lemme 1: See the theorem of implicit functions.

Proof of Theorem 1: First note that for the matrix $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*)$ of dimension $I-1$ to be non-singular as requested by the Lemme 1, it is necessary that $D_{\boldsymbol{\mu}_I}\hat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))$ and $D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*)$ be itself of rank $I-1$, since $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*) = D_{\boldsymbol{\mu}_I}\hat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*)$.by (3). Now let's start with the case where $D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0}$. By taking the derivative of the nth element of (4) with respect to $\mathbf{y}_1$ we get that:

$$D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) = D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*) = 0, \tag{16}$$

where $D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))$ is a vector of dimension I and $D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*)$ is matrix $I \times I-1$. Let's define the following partitions:

$$\begin{aligned} D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*)) &\equiv \left[D_{\mu_1}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))\ \ D_{\boldsymbol{\mu}_{I-1}}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))\right], \\ D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*) &\equiv \left[D_{\mathbf{y}_1}\mu_1(\mathbf{y}^*)'\ \ D_{\mathbf{y}_1}\boldsymbol{\mu}_{I-1}(\mathbf{y}^*)'\right]', \end{aligned} \tag{17}$$

where $D_{\mu_1}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))$ is a scalor, $D_{\boldsymbol{\mu}_{I-1}}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))$ and $D_{\mathbf{y}_1}\mu_1(\mathbf{y}^*)'$ are vectors of dimension $I-1$ and $D_{\mathbf{y}_1}\boldsymbol{\mu}_{I-1}(\mathbf{y}^*)'$ is a matrix of dimension $I-1$. Since $D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*)$ is of rank $I-1$, it necessarily containts a square sub-matrix of rank $I-1$. Assume thus, in all generalities, that the Pareto weight are classifiy in such a way that $D_{\mathbf{y}_1}\boldsymbol{\mu}_{I-1}(\mathbf{y}^*)'$ be of full rank, which allow us to rewrite (16) as:

$$D_{\boldsymbol{\mu}_{I-1}}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*)) = -D_{\mu_1}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{\mathbf{y}_1}\mu_1(\mathbf{y}^*)[D_{\mathbf{y}_1}\boldsymbol{\mu}_{I-1}(\mathbf{y})]^{-1}. \tag{18}$$

By taking now the derivative of the nth element of (4) with respect to y_{2k} :

$$\begin{aligned} D_{y_{2k}}\widetilde{x}_{2n}(\mathbf{y}^*) &= D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{y_{2k}}\boldsymbol{\mu}_I(\mathbf{y}^*) \\ &= [D_{\mu_1}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))\ \ D_{\boldsymbol{\mu}_{I-1}}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))]D_{y_{2k}}\boldsymbol{\mu}_I(\mathbf{y}^*) \qquad (19) \\ &= D_{\mu_1}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))\left[1\ \ -D_{\mathbf{y}_1}\mu_1(\mathbf{y}^*)[D_{\mathbf{y}_1}\boldsymbol{\mu}_{I-1}(\mathbf{y}^*)]^{-1}\right]D_{y_{2k}}\boldsymbol{\mu}_I(\mathbf{y}^*), \qquad (20) \end{aligned}$$

where we have used (17) et we have substitued the result of (18) in (19). If $D_{y_{2k}}\widetilde{x}_{2n}(\mathbf{y}^*) \neq 0$, it must be because $D_{\mu_1}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*)) \neq 0$ and $\left[1\ \ -D_{\mathbf{y}_1}\mu_1(\mathbf{y}^*)[D_{\mathbf{y}_1}\boldsymbol{\mu}_{I-1}(\mathbf{y}^*)]^{-1}\right]D_{y_{2k}}\boldsymbol{\mu}_I(\mathbf{y}^*) \neq 0$. However, if $D_{y_{2k}}\widetilde{x}_{2n}(\mathbf{y}^*) \neq 0$, then $D_{y_{2k}}\widetilde{x}_{2m}(\mathbf{y}^*) \neq 0$ for all the $\widetilde{x}_{2m}(\mathbf{y}^*)$ which don't react to $\mathbf{y}_1$, but which react to at least one of distribution factor contained in $\mathbf{y}_2$. In effect, if $\widetilde{x}_{2m}(\mathbf{y}^*)$ was depending on any distribtion factor, it would necessarily be because $D_{\mu_1}\widehat{x}_{2m}(\boldsymbol{\mu}_I(\mathbf{y}^*)) \neq 0$. But if $D_{\mu_1}\widehat{x}_{2m}(\boldsymbol{\mu}_I(\mathbf{y}^*)) \neq 0$, then $D_{y_{2k}}\widetilde{x}_{2m}(\mathbf{y}^*) \neq 0$ since $D_{y_{2k}}\widetilde{x}_{2m}(\mathbf{y}^*) = D_{\mu_1}\widehat{x}_{2m}(\boldsymbol{\mu}_I(\mathbf{y}^*))\left[1\ \ -D_{\mathbf{y}_1}\mu_1(\mathbf{y}^*)[D_{\mathbf{y}_1}\boldsymbol{\mu}_{I-1}(\mathbf{y}^*)]^{-1}\right]D_{y_{2k}}\boldsymbol{\mu}_I(\mathbf{y}^*)$. Moreover, $\widetilde{x}_{2n}(\mathbf{y}^*)$ will not react necessarily to all the elements in $\mathbf{y}_2$ since the expression $\left[1\ \ -D_{\mathbf{y}_1}\mu_1(\mathbf{y}^*)[D_{\mathbf{y}_1}\boldsymbol{\mu}_{I-1}(\mathbf{y}^*)]^{-1}\right]D_{y_{2j}}\boldsymbol{\mu}_I(\mathbf{y}^*)$ could be zero for certain y_{2j}. Therefore:

$$\forall\, D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0}: \quad D_{\mathbf{y}_2}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0}\ \mathit{or}\ D_{\mathbf{y}_{21}}\widetilde{x}_{2n}(\mathbf{y}^*) \neq\neq \mathbf{0}\, \mathit{and}\, D_{\mathbf{y}_{22}}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0},$$

with $\mathbf{y}_2 \equiv [\mathbf{y}_{21}\ \mathbf{y}_{22}]$.

Let's now take the other case where $D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) \neq \mathbf{0}$. In these circonstances, x_{2n} can be conditionned on $\mathbf{x}_1$. We can thus derivate the equations (**??**) and (**??**) with respect to y_{2k} at the point $\mathbf{y}_2^*$, which gives:

$$\begin{aligned} \mathbf{0} &= D_{\boldsymbol{\mu}_I}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{y_{2k}}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*), \qquad (21) \\ D_{y_{2k}}\overline{x}_{2n}(\mathbf{x}_1^*, \mathbf{y}_2^*) &= D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{y_{2k}}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*). \qquad (22) \end{aligned}$$

The matrix $D_{\boldsymbol{\mu}_I}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))$ is of dimension $I-1 \times I$, while the vectors $D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))$ et $D_{y_{2k}}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*)'$ are both of dimension $1 \times I$. Let's use again the partition (17) et define in addition the next two:

$$\begin{aligned} D_{\boldsymbol{\mu}_I}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*)) &\equiv \left[D_{\mu_1}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))\ \ D_{\boldsymbol{\mu}_{I-1}}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))\right] \equiv [\mathbf{A}_{11}\ \ \mathbf{A}_{12}], \\ D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*)) &\equiv \left[D_{\mu_1}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))\ \ D_{\boldsymbol{\mu}_{I-1}}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))\right] \equiv [B_{11}\ \ \mathbf{B}_{12}], \\ D_{y_{2k}}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*)' &\equiv \left[D_{y_{2k}}\mu_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*)\ \ D_{y_{2k}}\boldsymbol{\mu}_{I-1}(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*)'\right]', \end{aligned}$$

where $D_{\mu_1}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))$ and $D_{y_{2k}}\mu_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*)$ are both scalars, $D_{\boldsymbol{\mu}_{I-1}}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))$ and $D_{y_{2k}}\boldsymbol{\mu}_{I-1}(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*)'$ are both of dimension $1 \times I-1$, and $D_{\mu_1}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))$ and $D_{\boldsymbol{\mu}_{I-1}}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))$ are respectively of dimension $I-1 \times 1$ and $I-1 \times I-1$. The equations (21) and (**??**) can then be rewritten as:

$$\begin{aligned} \mathbf{0} &= \mathbf{A}_{11}D_{y_{2k}}\mu_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*) + \mathbf{A}_{12}D_{y_{2k}}\boldsymbol{\mu}_{I-1}(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*) \qquad (23) \\ D_{y_{2k}}\overline{x}_{2n}(\mathbf{x}_1^*, \mathbf{y}_2^*) &= B_{11}D_{y_{2k}}\mu_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*) + \mathbf{B}_{12}D_{y_{2k}}\boldsymbol{\mu}_{I-1}(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*, \mathbf{y}_2^*), \mathbf{y}_2^*) \qquad (24) \end{aligned}$$

Since $D_{\boldsymbol{\mu}_I}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))$ is of rank $I-1$, it necessarily contains a square sub-matrix of rank $I-1$. So let's suppose, in all generalities, that the Pareto weights are classified in such a way that $\mathbf{A}_{12}$ be of full rank, which allow us to obtain the next result: $D_{y_{2k}}\boldsymbol{\mu}_{I-1}(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*) = -[\mathbf{A}_{12}]^{-1}\mathbf{A}_{11}D_{y_{2k}}\mu_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*)$. By substituting it the equation (**??**), we obtain:

$$D_{y_{2k}}\overline{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*) = (B_{11} - \mathbf{B}_{12}[\mathbf{A}_{12}]^{-1}\mathbf{A}_{11})D_{y_{2k}}\mu_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*). \tag{25}$$

If $D_{y_{2k}}\overline{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*) \neq 0$, it must thus be because $(B_{11} - \mathbf{B}_{12}[\mathbf{A}_{12}]^{-1}\mathbf{A}_{11}) \neq 0$ and $D_{y_{2k}}\mu_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*) \neq 0$. However, if $D_{y_{2k}}\overline{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*) \neq 0$, then $D_{y_{2k}}\overline{x}_{2m}(\mathbf{x}_1^*,\mathbf{y}_2^*) \neq 0$ for all the $\widetilde{x}_{2m}(\mathbf{y}^*)$ reactomg to $\mathbf{y}_1$, and which once conditionned on $\mathbf{x}_1$, also depend on at least one of the distribution factor contained in $\mathbf{y}_2$. In effect, if $\overline{x}_{2m}(\mathbf{x}_1^*,\mathbf{y}_2^*)$ was depênding on any distribution factor, it would necessarily because $(D_{\mu_1}\widehat{x}_{2m}(\boldsymbol{\mu}_I(\mathbf{y}^*)) - D_{\boldsymbol{\mu}_{I-1}}\widehat{x}_{2m}(\boldsymbol{\mu}_I(\mathbf{y}^*))[\mathbf{A}_{12}]^{-1}\mathbf{A}_{11}) \neq 0$. But in this case $D_{y_{2k}}\widetilde{x}_{2m}(\mathbf{y}^*) \neq 0$ since $D_{y_{2k}}\widetilde{x}_{2m}(\mathbf{y}^*) =$
$(D_{\mu_1}\widehat{x}_{2m}(\boldsymbol{\mu}_I(\mathbf{y}^*)) - D_{\boldsymbol{\mu}_{I-1}}\widehat{x}_{2m}(\boldsymbol{\mu}_I(\mathbf{y}^*))[\mathbf{A}_{12}]^{-1}\mathbf{A}_{11})D_{y_{2k}}\mu_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*)$. Morever, $\overline{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*)$ will not react necessarily to all the elements contained in $\mathbf{y}_2$ since the expression $(B_{11} - \mathbf{B}_{12}[\mathbf{A}_{12}]^{-1}\mathbf{A}_{11})D_{y_{2k}}\mu_1(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*)$ could be zero for certain y_{2j}. Therefore:

$$\forall\, D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) \neq \mathbf{0}: \quad D_{\mathbf{y}_2}\overline{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*) = \mathbf{0} \; or \; D_{\mathbf{y}_{23}}\overline{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*) \neq\neq \mathbf{0} \, and \, D_{\mathbf{y}_{24}}\overline{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*) = \mathbf{0},$$

with $\mathbf{y}_2 \equiv [\mathbf{y}_{23}\ \mathbf{y}_{24}]$. By comparing the equations (20) and (17), we also that $\mathbf{y}_{21}$ is not necessarily identical to $\mathbf{y}_{23}$.■

Proof of Theorem 3: First note that for the matrix $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*)$ of dimension I to be non-singular as requested by the Lemme 1, it is necessary that $D_{\boldsymbol{\mu}_I}\hat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))$ and $D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*)$ be itself of rank I, since $D_{\mathbf{y}_1}\widetilde{\mathbf{x}}_1(\mathbf{y}^*) = D_{\boldsymbol{\mu}_I}\hat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*)$.by (3). Now let's start with the case where $D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0}$. By taking the derivative of the nth element of (4) with respect to $\mathbf{y}_1$ we get that: $D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) = D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*) = 0$. Since $D_{\mathbf{y}_1}\boldsymbol{\mu}_I(\mathbf{y}^*)$ is non-singular as we just noted it, $D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0}$ only if $D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*)) = \mathbf{0}$. It follows that $D_{\mathbf{y}_2}\widetilde{x}_{2n}(\mathbf{y}^*) = \mathbf{0}$. Now let's take the other case where $D_{\mathbf{y}_1}\widetilde{x}_{2n}(\mathbf{y}^*) \neq \mathbf{0}$. In theses circumstances, x_{2n} can be conditionned on $\mathbf{x}_1$. We can thus derive the equations (5) and (6) with respect to $\mathbf{y}_2$ at the point $\mathbf{y}_2^*$, which gives us:

$$\begin{aligned} \mathbf{0} &= D_{\boldsymbol{\mu}_I}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{\mathbf{y}_2}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*), &(26)\\ D_{\mathbf{y}_2}\overline{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*) &= D_{\boldsymbol{\mu}_I}\widehat{x}_{2n}(\boldsymbol{\mu}_I(\mathbf{y}^*))D_{\mathbf{y}_2}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*). &(27)\end{aligned}$$

Now consider a sub-system (26) which contains I equations with I variables. Since $D_{\boldsymbol{\mu}_I}\hat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))$ is non-singular, the only solution to this system is $D_{\mathbf{y}_2}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*) = \mathbf{0}$, which implies that $D_{\mathbf{y}_2}\overline{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*) = 0$.■

Proof of Corollary 1: We know from the Theorem 3 that $D_{\mathbf{y}_2}\overline{\mathbf{x}}_2(\mathbf{x}_1^*,\mathbf{y}_2^*) = \mathbf{0}$ when $J = I$. When $J < I$ ($I > 1$), there are an infinity of non-trivial solutions for $D_{\mathbf{y}_2}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*)$ that are consistent with the system of J equations in I variables $D_{\boldsymbol{\mu}_I}\widehat{\mathbf{x}}_1(\boldsymbol{\mu}_I(\mathbf{y}^*))\,D_{\mathbf{y}_2}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*) = \mathbf{0}$. Therefore, under the assumption that $(\mathbf{x}_1^*,\mathbf{y}_2^*)$ does not correspond to the trivial solution, we will have $D_{\mathbf{y}_2}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*) \neq \mathbf{0}$. Now, since $rank(D_{\boldsymbol{\mu}_I}\hat{\mathbf{x}}_2(\boldsymbol{\mu}_I(\mathbf{y}^*)) = I$ (which implies that $N \geqslant I + k$), the only solution for the system of $N-k$ equations in I variables $D_{\boldsymbol{\mu}_I}\hat{\mathbf{x}}_2(\boldsymbol{\mu}_I(\mathbf{y}^*)D_{\mathbf{y}_2}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*)= \mathbf{0}$ is $D_{\mathbf{y}_2}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*)= \mathbf{0}$. Thus one must have $D_{\mathbf{y}_2}\overline{\mathbf{x}}_2(\mathbf{x}_1^*,\mathbf{y}_2^*) \neq \mathbf{0}$ since $D_{\mathbf{y}_2}\boldsymbol{\mu}_I(\widetilde{\mathbf{y}}_1(\mathbf{x}_1^*,\mathbf{y}_2^*),\mathbf{y}_2^*) \neq \mathbf{0}$.■

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Table 1: Sample Selection

Village	Population*	Total Number of Married Households	Number of Eligible Households	Number of Surveyed Households	Number of Households
Dakiégré	1141	133	111	111	103
Pelegtanga	551	201	170	125	122
Rallo	1053	207	162	125	108
Yako (1)	856	221	128	125	117
Yako (5)	1 311	246	236	125	102
Total	4 912	1 008	800	600	552

According to the 1991 Demographic Survey.

Table 2: Household types

	Dakié-gré	Peleg-tange	Rallo	Yako(1)	Yako(5)	Urban	Rural	Total
Head	103	122	108	117	102	219	333	552
Spouses	171	169	154	144	129	273	494	767
Spouses/Head	1.7	1.4	1.4	1.2	1.3	1.2	1.5	1.4
Number of spouses								
1	58	84	73	94	84	215	178	393
2	27	32	28	10	20	87	30	117
3	14	4	5	5	4	23	9	32
4	3	1	1	2	0	5	2	7
5	1	1	0	0	0	2	0	2
6	0	0	1	0	0	1	0	1
Total	103	122	108	117	102	219	333	552

Table 3: Sample Descriptive Characteristics

Household Characteristics		
	Mean	Std.-Err.
Age		
Husband	48.77	11.36
Wife 1	41.48	11.12
Wife 2	30.75	8.66
Schooling		
Husband	1.29	1.64
Wife 1	1.44	1.71
Wife 2	1.56	1.61
Religion (%)		
Muslim	43.59	
Animism	41.88	
Christian	14.53	
No. Children, Wife 1	4.80	2.34
No. Children, Wife 2	3.09	2.15
Distribution Factors		
Years of Marriage		
Wife 1	22.43	10.10
Wife 2	11.39	8.56
Income Share (%)		
Wife 1	17.10	9.70
Wife 2	17.49	10.47
Endogenous Variables		
Clothing Expenditures (FCFA)		
Husband	5473.50	11667.52
Wife 1	8511.97	8885.89
Wife 2	9762.72	9021.20
Hairdressing Expenditures (FCFA)		
Husband	314.79	527.46
Wife 1	241.67	602.40
Wife 2	464.22	1204.36

Table 4: GMM Parameter Estimates –Unconditional Demands

Variables	Men's Clothing	Women's Clothing (2)	Women's Hair Dressing (2)
Dakiégré	6.776	3.002	-0.144
	(1.617)	(1.159)	(0.147)
Pelegtanga	7.246	3.412	0.275
	(1.577)	(1.030)	(0.146)
Rallo	2.606	-2.259	0.018
	(1.568)	(1.208)	(0.164)
Yako (1)	13.418	-2.260	0.300
	(2.736)	(1.313)	(0.152)
Muslim	-0.378	1.056	-0.124
	(0.908)	(1.049)	(0.115)
Catholic	0.938	-0.432	0.244
	(1.042)	(1.320)	(0.162)
Age (Husband)	0.533		
	(0.447)		
Age (Wife 2)		-0.161	-0.034
		(0.053)	(0.006)
Young Children (1)	0.623		
	(0.201)		
Older Children (1)	-0.262		
	(0.280)		
Young Children (2)	-0.030	-0.557	-0.053
	(0.288)	(0.222)	(0.028)
Older Children (2)	-0.566	-1.626	0.025
	(0.475)	(0.388)	(0.041)
m	-1.583	10.879	1.350
	(2.382)	(1.440)	(0.211)
m$\log$(m)	2.770	-3.521	-0.525
	(1.343)	(0.659)	(0.096)
Distribution Factors			
m_1/m	-5.724	-24.821	-1.171
	(9.900)	(5.782)	(0.676)
m_2/m	-21.252	15.358	0.073
	(7.911)	(6.221)	(0.639)
Δ(Years of Marriage)	-0.153	-0.108	-0.024
	(0.068)	(0.045)	(0.005)
Function Value		59.795	

Table 5: GMM Parameter Estimates –Conditional Demands (Theorem 1)[†]

Variable	Specification 1		Specification 2		Specification 3	
	Husband clothing	Wife clothing	Husband clothing	Wife hair-dressing	Wife clothing	Wife hair-dressing
Dakiégré	0.608	3.390	8.061	0.490	1.120	-0.332
	(1.076)	(0.977)	(2.642)	(0.274)	(1.330)	(0.151)
Pelegtanga	-1.492	2.788	6.753	0.535	0.856	-0.054
	(1.247)	(1.071)	(2.378)	(0.225)	(1.250)	(0.141)
Rallo	-2.232	-4.020	5.750	0.831	-2.383	0.015
	(1.147)	(1.058)	(2.055)	(0.211)	(1.069)	(0.136)
Yako (1)	2.434	-1.440	13.887	0.757	-7.036	-0.347
	(1.363)	(1.011)	(2.905)	(0.179)	(1.682)	(0.212)
Muslim	-2.657	3.647	0.278	0.217	2.218	0.031
	(0.935)	(0.777)	(1.247)	(0.158)	(0.692)	(0.084)
Catholic	-0.506	0.588	1.376	0.282	0.333	0.221
	(0.722)	(1.028)	(1.083)	(0.136)	(1.041)	(0.120)
Age (Husband)	0.540		0.399			
	(0.384)		(0.525)			
Age (Wife 2)		-0.295		-0.016	-0.294	-0.024
		(0.057)		(0.007)	(0.054)	(0.006)
Young Children (1)	0.952		0.260			
	(0.282)		(0.227)			
Older Children (1)	-0.260		-0.212			
	(0.150)		(0.296)			
Young Children (2)	0.215	0.057	0.153	-0.042	-0.396	-0.063
	(0.183)	(0.189)	(0.274)	(0.036)	(0.197)	(0.022)
Older Children (2)	-0.094	-0.574	-0.083	0.101	-0.591	0.034
	(0.231)	(0.348)	(0.400)	(0.041)	(0.367)	(0.033)
m	0.724	6.525	-5.961	0.090	9.027	0.863
	(2.061)	(1.155)	(3.221)	(0.188)	(1.185)	(0.152)
mlog(m)	-0.276	-2.272	3.631	-0.087	-3.546	-0.387
	(0.942)	(0.490)	(1.622)	(0.078)	(0.585)	(0.076)
Distribution Factors						
m_1/m	-5.395	-11.352	-5.673	-0.593	-8.809	-0.685
	(2.888)	(4.622)	(5.229)	(0.694)	(3.376)	(0.393)
m_2/m	-10.848	36.972	-13.761	-1.224	30.183	0.735
	(3.440)	(4.037)	(5.529)	(0.542)	(3.993)	(0.393)
Conditioning Demand						
Clothing Expenditures						
Husband					0.348	0.036
					(0.036)	(0.004)
Wife 2			0.543	0.066		
			(0.106)	(0.009)		
Hairdressing Expenditures						
Wife 2	5.966	2.512				
	(1.052)	(0.515)				
Function Value	42.613		40.465		48.136	
P-Value $\left(\frac{D_{\mathbf{y}_{2k}}\bar{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*)}{D_{\mathbf{y}_{2l}}\bar{x}_{2n}(\mathbf{x}_1^*,\mathbf{y}_2^*)} = \frac{D_{\mathbf{y}_{2k}}\bar{x}_{2m}(\mathbf{x}_1^*,\mathbf{y}_2^*)}{D_{\mathbf{y}_{2l}}\bar{x}_{2m}(\mathbf{x}_1^*,\mathbf{y}_2^*)}\right)$	0.0012		N.A.		0.400	

[†] "Wife" refers to the younger spouse in all three specifications.

Table 6: Non-Singularity Tests – Unconditional Demands[†]

Likelihood-Ratio Tests				
Demand Equations	Distribution Factors	Test	χ^2	P-Value
Husband clothing, Wife clothing	m_1/m, m_2/m	$\alpha_1\beta_2 - \beta_1\alpha_2 = 0$	7.32	0.007
Husband clothing, Wife hairdressing	m_1/m, m_2/m	$\alpha_1\beta_3 - \beta_1\alpha_3 = 0$	5.26	0.022
Husband clothing, Wife clothing	m_1/m, Δ(YM)	$\alpha_1\gamma_2 - \gamma_1\alpha_2 = 0$	3.03	0.082
Husband clothing, Wife hairdressing	m_1/m, Δ(YM)	$\alpha_1\gamma_3 - \gamma_1\alpha_3 = 0$	0.04	0.842
Wife clothing, Wife hairdressing	m_1/m, m_2/m	$\alpha_2\beta_3 - \beta_2\alpha_3 = 0$	7.07	0.008
Wife clothing, Wife clothing	m_1/m, Δ(YM)	$\alpha_2\gamma_3 - \gamma_2\alpha_3 = 0$	7.19	0.007
Husband clothing, Wife clothing	m_2/m, Δ(YM)	$\beta_1\gamma_2 - \gamma_1\beta_2 = 0$	0.87	0.350
Wife clothing, Wife hairdressing	m_2/m, Δ(YM)	$\beta_2\gamma_3 - \gamma_2\beta_3 = 0$	4.96	0.026
Husband clothing, Wife hairdressing	m_2/m, Δ(YM)	$\beta_1\gamma_3 - \gamma_1\beta_3 = 0$	6.89	0.009

[†] "Wife" refers to the younger spouse.

Table 7: GMM Parameter Estimates – Conditional Demands (Theorem 3)†

Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
	Husband clothing	Husband clothing	Wife clothing	Wife hair-dressing	Wife hair-dressing	Wife hair-dressing
Dakiégré	4.563	6.249	2.064	-0.359	-0.378	-0.388
	(2.267)	(2.240)	(1.938)	(0.171)	(0.185)	(0.177)
Pelegtanga	3.397	4.252	1.024	-0.012	-0.044	-0.008
	(2.253)	(2.353)	(1.959)	(0.189)	(0.202)	(0.192)
Rallo	2.927	2.554	-2.274	0.234	0.236	0.235
	(1.926)	(1.984)	(1.831)	(0.192)	(0.197)	(0.191)
Yako (1)	8.031	6.841	-4.288	0.124	0.080	0.166
	(3.549)	(3.639)	(2.609)	(0.231)	(0.245)	(0.244)
Muslim	-0.135	0.543	0.408	-0.102	-0.079	-0.168
	(1.577)	(1.499)	(1.443)	(0.150)	(0.148)	(0.168)
Catholic	1.673	0.809	-0.831	0.033	0.069	-0.020
	(1.615)	(1.484)	(1.627)	(0.149)	(0.129)	(0.152)
Age (Husband)	0.277	-0.244				
	(0.783)	(0.727)				
Age (Wife 2)			-0.203	-0.018	-0.016	-0.022
			(0.101)	(0.009)	(0.010)	(0.009)
Young Children (1)	0.189	0.053				
	(0.428)	(0.413)				
Older Children (1)	-0.139	-0.021				
	(0.377)	(0.388)				
Young Children (2)	0.143	-0.086	0.283	-0.063	-0.071	-0.065
	(0.351)	(0.365)	(0.358)	(0.031)	(0.037)	(0.032)
Older Children (2)	0.336	-0.313	-0.388	0.073	0.071	0.072
	(0.493)	(0.567)	(0.552)	(0.046)	(0.049)	(0.048)
m	-2.577	-1.028	8.022	0.546	0.506	0.691
	(3.741)	(4.227)	(2.171)	(0.222)	(0.204)	(0.238)
mlog(m)	1.820	1.399	-3.113	-0.244	-0.225	-0.288
	(1.913)	(2.107)	(1.037)	(0.103)	(0.096)	(0.106)
Distribution Factors						
m_1/m		-11.107	3.217		-0.235	
		(10.088)	(10.286)		(0.928)	
m_2/m	-21.508			-0.188		
	(12.623)			(0.942)		
Δ(Years of Marriage)						-0.011
						(0.008)
Conditioning Demand						
Clothing Expenditures						
Husband			0.246	0.019	0.018	0.017
			0.077)	(0.008)	(0.008)	(0.008)
Wife 2	0.499	0.321		0.037	0.040	0.036
	(0.215)	(0.183)		(0.016)	(0.017)	(0.015)
Hairdressing Expenditures						
Wife 2	1.115	1.186	2.857			
	(1.885)	(1.833)	(1.461)			
Function Value	18.079	17.795	27.231	14.702	15.054	14.154

† "Wife" refers to the younger spouse in all six specifications.