Unpleasant Implications of Insecure Property*

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Abstract: We embed a model of imperfect property rights into an endogenous growth model. This is used to analyze the impact that government fiscal policy could have in tempering the inefficiencies associated with insecure property rights. Looking at optimal fiscal policy in this context gives insight into the problems involved with incomplete property rights and points to the limitations that governments must face in dealing with these problems. The main finding of the paper is that poor enforcement of property rights places an optimal ceiling on growth below the first-best rate. Our analysis aims to illustrate that the nature of this result is rather subtle and that it has somewhat unpleasant implications. The main lesson that can be drawn from the analysis is that pro-growth policies may well be undesirable in societies that lack the full rule of law.

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1 Introduction

The critical importance for economic activity of a social structure of political and judicial institutions that provide a foundation for individuals in the enforcement of their property claims is widely recognized (e.g., Hall and Jones (1999) and Acemoglu et al. (2001)). Without such social institutions homo economicus will, in North’s phrase, maximize at every possible margin of activity, producing where profitable, but equally devoting resources to cheating, theft and plunder where profitable. In such a world rights to property must rely heavily on private enforcement; how much of one’s own product and how much of others’ product that an individual secures for consumption depends on the technology of security and on how much individuals invest in protecting their own output and in diverting the production of others. Productive investment and growth are harmed because economic agents do not receive the full benefit of their marginal product and because they devote otherwise productive resources to diversionary activity, both predatory and defensive.

Social control of diversion brings substantial economic benefits. However, the development of the necessary institutions – political system, courts, police, social attitudes etc., nation-building in short – may require a considerable expenditure of time and resources, and a radical change in public attitudes and ideology. An important policy question is whether less fundamental instruments could help to ameliorate the problems that lack of social control implies for the rate of economic growth and the level of consumption in a society. This paper analyzes the impact that government fiscal policy could have in tempering the inefficiencies associated with insecure property rights. We do not suggest that benevolent fiscal planners are a key part of economies that are characterized by conflict and ineffective property rights systems. Rather, looking at optimal fiscal policy in this context gives insight into the problems involved with incomplete property rights and points to the limitations that governments must face in dealing with these problems. The main finding of this paper is that poor enforcement of property rights places an optimal ceiling on growth below the first-best rate. Our analysis aims to illustrate that the nature of this result is rather subtle and that it has somewhat unpleasant implications. The main lesson that can be drawn from the analysis is that pro-growth policies may well be undesirable in societies that lack the full rule of law.

This outcome is superficially counter-intuitive. Both the negative effect of insecure property on the incentive to invest and the diversion of otherwise productive resources associated with

\[1\text{See North (1987).}\]
conflict over economic distribution, when taken separately, suggest that pro-growth government policies are the appropriate ones. Thus, most models which emphasize the importance of rent-seeking or imperfect property rights for growth, though relying on different mechanisms, have the common implication that increases in productive investment and growth and reductions in diversion must go together (e.g. Murphy et al. 1991, 1993, Grossman and Kim 1996, and Tornell and Lane 1999). This is not so in a fully articulated model of endogenous property rights and growth, as shown by González (2004), because circumstances that would increase the marginal return on productive investment, leading to growth, will simultaneously lead to and increase in diversionary investments, as individuals allocate resources to maintain equality of marginal returns. Therefore, a reduction in productive investment might in fact be welfare-improving. It reduces growth, which reduces welfare, but by complementarity of investment activities, it also reduces diversion and so increases current consumption. Put another way, a tradeoff between current consumption and growth exists because faster growth exacerbates the problem of diversion. Building on this insight, our analysis will show why the lack of the full rule of law can greatly influence optimal, incentive-compatible, fiscal policy.

The kind of growth model with imperfect property rights studied here is likely to be most relevant to the study of less developed economies, where the rule of law is not pervasive. Most recent theories of development and underdevelopment are well aware of the importance of imperfect institutions in explanations of the failure of economies to develop. This paper contributes directly to this literature. There is a key difference in emphasis between the present paper and most of the recent literature however. The latter tends to see imperfect institutions as a constraint or barrier to growth, and if the constraints could be relaxed somewhat then further desirable growth could take place. The institutional constraints thus place a ceiling on growth, which is to be pushed against. This paper emphasizes the somewhat different idea that imperfect institutions, rather than placing a ceiling on growth, may make growth itself undesirable. The development of institutions – nation building – is not to release constraints on growth, but rather to create a situation in which growth itself becomes desirable. This kind of idea is found in the work of Bates (2001), for example, who sees the fundamental problem of development as resulting from the fact that, absent institutions that guarantee property rights, prosperity breeds conflict or diversion; underdevelopment may then be chosen as a rational conflict-reducing solution in societies with insufficient protection of property – poverty is the price of peace. While Bates’ arguments are based on exten-

sive observation and fieldwork, the present paper provides an explicit economic modeling of how such a result might emerge: growth with imperfect institutions brings with it a deadweight loss from self-protection that may be so burdensome that a benevolent government would choose to use fiscal instruments to reduce growth rather than increase it, to favor current consumption over wealth creation, in the interest of its citizens.

It is important to be clear on the normative role ascribed to government here. The literature on low growth and insecure property rights often fingers government itself as one of the main predators on economic activity. In a related literature, government actors are considered as part of the social game, and their behavior is derived as part of the equilibrium outcome (e.g. Bates et al. (2002), Grossman (2001)). These important aspects of government are not modeled here. Rather, first, property insecurity arises only from the covetousness of individuals in a situation where the state is either not willing or not powerful enough to support fully – with a police, court and punishment system – the defensive efforts of individual property holders. An extreme version of this abstraction is Hobbes’ state of nature, the “war of all against all”. A less abstract version is found in Bates’ discussion of stateless, but nonetheless highly structured societies, where property claims are mediated through complex clan and tribal systems. Similar issues are often emphasized in the context of failed states such as Bosnia, Afghanistan, etc. Even in highly organized societies private enforcement of property rights is essential; the state does not replace this, rather it provides institutions that augment and magnify individual self-enforcement expenditures, which remain the basis for effective property rights. This is a dimension receiving increasing attention in the context of the East European transition to capitalism (e.g. Johnson et al. (2002), Roland and Verdier (2003), Shleifer (1997)).

And second, government here is specified by the model to be a benevolent agent. In this framework, Barro (1990) and Barro and Sala-i-Martin (1992) have discussed the potential for fiscal policy to generate Pareto-improving growth in the presence of production externalities, public goods, etc. We contribute to this normative line of research by extending a standard model of endogenous growth to include a specific model of imperfect and endogenous property rights, and by using this model to evaluate the Pareto-improving potential of fiscal policy. A useful feature of the present model is that it includes the model studied by Barro (1990) as the special case in which property rights are perfectly and costlessly enforced. When this is not so, the present analysis highlights the interaction of two effects arising from externalities associated with imperfect property rights. First, there is an investment effect, because individuals do not internalize the fact that
some of their own output accrues to others. On this count, optimal fiscal policy ought to promote
investment and growth. Second, there is a conflict or diversion effect, because individuals do not
internalize the fact that the social cost of productive investment in terms of consumption is too
high, as growth breeds conflict. On this count, the private cost of investment and growth tends to
be too high, and optimal fiscal policy ought to limit investment and growth.

Within this paradigm we restrict the types of fiscal policy studied. A fully empowered planner
would deal with the investment and the conflict effects simultaneously by simply commanding
agents to set both predatory and defensive investments to zero. Likewise, with enough fiscal
instruments and with perfect and costless enforcement of the government’s ability to tax and
subsidize, the economy could be brought arbitrarily close to the first best outcome. In view of
this we suppose that the planner has access only to specific fiscal instruments: lump-sum taxes
(equivalent here to consumption taxes), income taxes (equivalent here to output taxes), and a
productive-investment tax. We introduce these taxes sequentially since they can be used to control
the two externalities separately, and this allows us to disentangle their individual impacts on
equilibrium.

The plan of the paper is as follows. Section 2 presents the basic model of growth and property
rights to be used, and derives some useful equilibrium properties. Section 3 analyzes a particular
second-best planning problem that arises naturally from considering the impact of a combination
of lump-sum and income taxes on the model. In effect these taxes allow a second-best solution
in which the investment effect is fully accounted for, but in which the conflict effect is completely
untouched. This second best is useful in allowing policy interpretations to be drawn. Section 4
considers the addition of a tax on productive-investment to the model. We show that there is a
tax/subsidy package which would implement the first-best (perfect property rights) welfare level.
This is not a possibility that we seek to emphasize however. Achievement of a first-best in this way
is possible only if the economy is effectively centrally controlled. And governments of societies in
which property rights are weak typically do not have the power to remove diversion through the
tax system. Section 5 provides some additional remarks on the results obtained.
2 Model

Consider an economy with a unit measure of household-producers in which each individual agent, at each point in time, can produce output

\[ q_i(t) = k_i(t) \psi \left( \frac{G(t)}{k_i(t)} \right), \]

where \( \psi \) is an increasing concave function, \( k_i(t) \) is the individual’s capital stock, and \( G(t) \) is the level of government-provided infrastructure capital. We will occasionally refer to a Cobb-Douglas version of this technology, given by \( \psi(G/k) = (G/k)^\alpha \), and a CES version given by \( \psi(G/k) = (\alpha(G/k)^{-\varsigma} + 1 - \alpha)^{-1/\varsigma} \), where \( \varsigma \in [-1, \infty) \). To focus attention on the externalities associated with imperfect property rights we assume that the infrastructure capital is a non-rivalrous and non-excludable public good.

Output produced by an individual is insecure in that \( i \)’s claim to his own output \( q_i \) can be contested by other individuals, and \( i \) in turn may contest the claims of others to their production. For simplicity we model the possibilities here by imagining that the members of society are paired randomly in each period, and that the outputs produced by each pair are reallocated between them according to sharing rules that depend on how much each individual has invested in defending his own output \( q_i \), and in seeking to appropriate the other’s output, \( q_j \).

Let \( x_i(t) \) be \( i \)’s stock of defensive capital – weapons, walls, ditches, secured storage etc. – and let \( z_j(t) \) be \( j \)’s stock of offensive capital – weapons, driving or hauling capacity etc. Then the share of his own output that \( i \) can hold onto is given by the function

\[ p_{ii}^m(t) = \frac{\pi x_i^m(t)}{\pi x_i^m(t) + z_j^m(t)} \]

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3 Random matching of this sort implies generalized diversion in the sense of a war of all against all, though one opponent at a time. There are other modeling possibilities. Each individual might be matched against the average of the other individuals in the economy, implying a different kind of generalized diversion. Alternatively, society might be divided into groups, with a member of each group being matched against a member of another group in each period. This would accord with the kind of differences in within-group and between-group interaction described in Bates outline of clan-based and tribe-based property rights enforcement; and with the empirical work of Easterly and Levine (1997) on ethnic conflict and economic growth.

4 For example, Fraser (1986) gives a vivid account of the extensive system of cattle raiding in the England-Scotland border region in the 13-16th centuries. Raiding was multi-lateral, fully integrated into the culture and way of life, and required substantial stocks of both predatory and defensive capital. In a modern society the aggregate of \( x \) and \( z \) is approximated loosely by the share of GNP that is spent in the non-government legal sector and the security sector in the economy.
while $j$ secures the complementary fraction $p^{ji}(t) = 1 - p^{ii}$.

This general form of sharing rule or ‘contest success function’ has been widely used in the rent-seeking literature Dixit (1987), Rowley et al. (1988) etc., and in the conflict literature Hirshleifer (1988, 1989), Skaperdas (1992) etc. Axiomatizations appropriate to these contexts can be found in Skaperdas (1996) and Clark and Riis (1998). We model property rights through this type of function because it allows a clear specification of the investment and diversion externalities that we are interested in analyzing, and it emphasizes some fundamental points about actual property rights, once we have abandoned the abstraction of ‘perfect’ property rights. In particular property rights are both costly and endogenously determined. The cost is borne by the agents, who must invest real resources in the project to secure their own property and to commandeer that of others. This is inescapable in any society. The contest success function allows both the defensive and predatory aspects of this project to be realized in a simple way. The government’s role in ‘providing’ property rights is to provide resources – legal framework, police, court system, prison system – that leverage the defender’s defensive resources. This is a compact abstraction of how a property rights system actually works. The government provides support for the efforts of individuals to defend their property; the impact of this support is summarized by the value of the parameter $\pi$, which we refer to as the ‘state of the law’ or ‘property rights’ parameter. This parameter introduces an asymmetry which gives a differentiated effect to defensive capital in the success function. For any capital stocks $\{x_i, z_j\}$, the larger is $\pi$ the larger is the share in output, $p^{ii}$, that the defender receives. At one extreme, when $\pi$ is one there is no government support for the defender’s investment, no differentiated effect, and so the model gives equal access to both predator and defender. As the value of $\pi$ rises access becomes differentiated, favoring the defender. Ceteris paribus, an increase in $\pi$ allows the defender to reduce defensive capital and still maintain the previous claim on output; alternatively, holding defensive capital fixed makes it more costly for the predator to maintain the previous claim. The ideal of perfectness of property rights emerges as $\pi$ approaches infinity. Then the defender receives a 100% share of his own output, irrespective of the values of offensive and defensive capital. Equilibrium in this limit will involve no externality since private defensive and offensive investments are irrelevant and approach zero. In a very natural way then the model allows property rights to be seen not as a dichotomy between perfect and imperfect, but as a continuum of imperfectness as $\pi$ varies between 1 and infinity. In practical terms the state of the system will involve a higher or lower value of $\pi$ with associated higher or lower values of diversionary capital. Effective property rights are the consequence of the interaction between $\pi$ and
the levels of offensive and defensive capital. Finally, we have chosen a model that is symmetric
across agents in the interests of simplicity, and to highlight imperfect property rights and the in-
teraction of the resulting externalities in abstraction from the particularities of a specific case (see
e.g. Tornell and Lane (1999) for an institutionally specific example of imperfect property rights
and growth).

The other key parameter of the success functions is \( m \). It is a measure of the “effectiveness of
conflict”. Write the relative shares of \( i \)'s output (the “odds” in probability terms) as
\[
\frac{p_{ij}^{ii}}{1 - p_{ij}^{ii}} = \pi \left( \frac{x_i}{z_j} \right)^m.
\]
Then \( m \) is the elasticity of the relative share of output with respect to a change in the capital ratio
\( x_i/z_j \). In the base case when \( \pi = 1 = m \) an individual’s share of output equals his share of the
diversionary capital stocks. The larger is \( m \) the greater is the impact on one’s relative share of
output from an increase in one’s relative capital stock. The incentive to hold capital stocks for
diversion is therefore larger the larger is \( m \).

To complete the modeling of the property rights system note that, by analogy with the above,
individual \( i \) claims a share of the output of individual \( j \) according to the function
\[
p_{ij}(t) = \frac{z_i^m(t)}{\pi x_j^m(t) + z_i^m(t)} = 1 - p_{ij}(t).
\]
The gross income that is fully secured to an individual at any time \( t \) is given by
\[
y_i(t) = p_{ii}(t)q_i(t) + (1 - p_{ij}(t))q_j(t). \tag{1}
\]
Net income in turn is allocated to current consumption \( c_i(t) \), and to investment in the three capital
stocks, \( k_i, x_i \) and \( z_i \).

For later convenience we include right away a simple income tax and lump-sum tax scheme
that allows us to consider the effects of these taxes on agents’ equilibrium behavior. Taxes are a
combination of a linear income tax and a lump-sum tax
\[
T(y_i(t)) = \tau(t)y_i(t) + \ell(t).
\]
We will see in the next section that a linear tax structure with time-invariant slope is sufficient to
achieve constrained optimality.\footnote{Linear output taxes can be shown to give the same outcomes
in our model.} Gross infrastructure investment is denoted by \( I^G \). We assume (i) that the government
budget is balanced at every time \( t \):
\[
I^G(t) = \int_0^1 T(y_i(t)) \, di. \tag{2}
\]
(ii) that the government acts as a benevolent planner, seeking to maximize the sum of individual utilities; and (iii) the government commits to all future infrastructure investment and taxes.

We assume for simplicity that the capital stocks, including infrastructure capital, are subject to the same depreciation rate $\delta$. The individual’s budget constraint at $t$ is then

$$
(1 - \tau)y_i(t) = c_i(t) + I_k^i(t) + I_x^i(t) + I_z^i(t) + \ell(t),
$$

(3)

where the $I_i$’s are gross investment levels.

Now consider the investment externality that is involved in imperfect property rights. This arises because individuals receive less than one hundred percent of the marginal output of their capital, and is seen by differentiating gross income $[1]$ with respect to the capital stock $k_i$:

$$
\frac{\partial y_i}{\partial k_i} = p^{ii} \frac{\partial q_i}{\partial k_i}.
$$

The individual’s marginal income is a fraction, $p^{ii}$, of the marginal output generated. This effect is well known and leads to under-investment and lower growth. The appropriate fiscal remedy involves an output subsidy (or income subsidy – here they amount to the same thing) that recognizes the positive externality provided by $i$’s capital stock for $j$’s income. That is, the marginal impact of $i$’s capital stock on total income is

$$
\frac{\partial (y_i + y_j)}{\partial k_i} = p^{ii} \frac{\partial q_i}{\partial k_i} + (1 - p^{ii}) \frac{\partial q_i}{\partial k_i} = \frac{\partial q_i}{\partial k_i}.
$$

An output subsidy at rate $-\tau = (1 - p^{ii}) / p^{ii}$ would induce $i$ to internalize this external effect. In sum, this aspect of the incomplete property rights problem suggests that investment and growth will be lower than desirable because individual’s fail to internalize the investment externality, and that a subsidy is appropriate to raise investment and growth.

2.1 Private equilibrium

To complete the model’s specification, each individual maximizes the present value of utility from his consumption stream, where $\rho$ is the rate of time preference. Utility in each period is given by $u_i(t) = \log(c_i(t))$. The individual thus wishes to choose a plan $\{c_i(t), k_i(t), x_i(t), z_i(t)\}_{t=0}^\infty$ to solve the problem

$$
\max \int_0^\infty e^{-\rho t} \log(c_i(t)) dt
$$

subject to the feasibility constraints [1] and (3), the equations of motion

$$
\dot{k}_i(t) = I_k^i(t) - \delta k_i(t);
\dot{x}_i(t) = I_x^i(t) - \delta x_i(t);
\dot{z}_i(t) = I_z^i(t) - \delta z_i(t),
$$

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and taking as given the government’s commitment to \( \{ I^G(t), \tau(t), \ell(t) \} \). Because each individual is small and is randomly matched in each period there are no strategic effects in decision-making. We restrict attention to symmetric equilibria, in which all agents choose a common plan \( \{ c(t), k(t), x(t), z(t) \} \). A necessary condition for this plan to be an equilibrium is that it is a best response by the \( i \)th agent, given that all others have chosen that allocation.

We begin by characterizing some aspects of this equilibrium allocation of the individuals, taking government policies as given. The individual’s Hamiltonian is

\[
H_i = e^{-\rho t} \log(c_i) + \mu_i \left( (1 - \tau)y_i - c_i - I^k_i - I^\ell_i - \ell \right) + \mu^k_i (I^k_i - \delta k_i) + \mu^\ell_i (I^\ell_i - \delta \ell_i)
\]  

Maximization conditions include

\[
\frac{\partial H_i}{\partial c_i} = e^{-\rho t} \frac{1}{c_i} - \mu_i = 0 \quad \text{which implies} \quad -\frac{\dot{\mu}_i}{\mu_i} = \frac{\dot{c}_i}{c_i} + \rho; 
\]  

\[
\frac{\partial H_i}{\partial I^k_i} = \mu^k_i - \mu_i = 0; \quad \frac{\partial H_i}{\partial I^\ell_i} = \mu^\ell_i - \mu_i = 0; \quad \frac{\partial H_i}{\partial \ell_i} = \mu^\ell_i - \mu_i = 0; 
\]

and

\[
\frac{\partial H_i}{\partial k_i} = -\dot{\mu}^k_i \quad \implies \quad \mu_i(1 - \tau)\frac{\partial y_i}{\partial k_i} - \mu^k_i \delta = -\dot{\mu}^k_i
\]  

\[
\frac{\partial H_i}{\partial x_i} = -\dot{\mu}^\ell_i \quad \implies \quad \mu_i(1 - \tau)\frac{\partial y_i}{\partial x_i} - \mu^\ell_i \delta = -\dot{\mu}^\ell_i
\]  

\[
\frac{\partial H_i}{\partial z_i} = -\dot{\mu}^\ell_i \quad \implies \quad \mu_i(1 - \tau)\frac{\partial y_i}{\partial z_i} - \mu^\ell_i \delta = -\dot{\mu}^\ell_i.
\]

Use (5) and (6) to substitute the multipliers in (7)-(9) and get

\[
\frac{\partial y_i}{\partial k_i} = \frac{\partial y_i}{\partial x_i} = \frac{\partial y_i}{\partial z_i} = \frac{1}{1 - \tau} \left( \frac{\dot{c}_i}{c_i} + \rho + \delta \right). 
\]

This is the key set of relationships in the model. Individuals optimize by equating at the margin the impacts on their income of each type of capital, whether productive or diversionary. This general principle is independent of the particular specification of property rights we are using. What it implies within the current model is a simple set of equilibrium complementarities between the three capital stocks.

First, differentiating in (11) gives

\[
\frac{\partial y_i}{\partial x_i} = \frac{\partial y_i}{\partial z_i} \iff q_i \frac{m}{x_i} p_i^{ii}(1 - p_i^{ii}) = q_i \frac{m}{z_i} p_i^{ii}(1 - p_i^{ii}) \iff \frac{x_i}{z_i} = \frac{q_i}{q_j} \frac{p_i^{ii}(1 - p_j^{ii})}{p_j^{ii}(1 - p_i^{ii})},
\]
Again differentiating in (1) gives
\[ \frac{\partial y_i}{\partial x_i} = \frac{\partial y_i}{\partial k_i} \iff k_i \psi \frac{m}{x_i} p^{ii} (1 - p^{ii}) = p^{ii} \left( \psi_i - \frac{G}{k_i} \psi'_i \right) \]
\[ \iff \frac{x_i}{k_i} = m (1 - p^{ii}) \frac{\psi_i}{\psi_i - \frac{G}{k_i} \psi'_i}. \] (12)

It is immediate from (11) and (12) that the three capital stocks are proportional to each other in the individual’s optimization. In a symmetric equilibrium, where \((k_i, x_i, z_i) = (k, x, z)\) for each \(i\), then
\[ p^{ii} = p(\pi) = \frac{\pi}{\pi + 1} \quad \text{and} \quad q_i = k \psi, \] (13)
which together imply
\[ z = x = \phi k \quad \text{for} \quad \phi = m (1 - p(\pi)) \frac{\psi}{\psi - \frac{G}{k} \psi'}. \] (14)

Equations (13) and (14) clearly show the impact of the two externalities. In symmetric equilibrium each individual receives a share \(\bar{p}(\pi) < 1\) of own output, which generates the investment externality. The quantitative impact of this externality in turn depends directly on the magnitude of \(\pi\) which measures the degree to which society helps individuals protect their property claims by leveraging their defensive investments through a legal system.

And the diversion externality appears because the diversionary stocks \(x = z\) are positive, being a multiple of \(k\). For every unit of productive capital held in equilibrium, \(2\phi\) units of diversionary capital are held as well; reducing the growth rate of productive capital may be socially desirable since the growth rate of diversionary capital will be correspondingly reduced. The extreme simplicity of these complementarities depends on homogeneity and symmetry of the conflict technologies, symmetry of the depreciation rates, and symmetry of the individuals in equilibrium. Similar complementarities, albeit more complex, can be expected in less symmetric scenarios.

The magnitude of the diversion coefficient \(\phi\) is larger the larger is the effectiveness of conflict, \(m\), which increases diversionary incentives\(^6\). It is larger the smaller is the equilibrium output share, \(p(\pi)\), since the return to productive investment is lower in that case. Finally, the multiplier depends on the production technology through \(\psi/(\psi - (G/k) \psi')\). In the Cobb-Douglas case, \(\psi = (G/k)^a\), this expression reduces to \(1/(1 - \alpha)\) so that \(\phi\) becomes entirely determined by
\[ \phi = \frac{m (1 - p(\pi))}{1 - \alpha}. \]

\(^6\) As \(m\) goes to zero \(\phi\) also goes to zero, eliminating the diversionary externality, but leaving the investment externality in place.
φ increases as α, the elasticity of q with respect to G, increases; the more important is G relative to k in production, the larger will be the ratio of diversionary capital to productive capital. Finally, note that the diversion coefficient is independent of the parameters of the tax system, the marginal tax rate τ in particular. This tax rate therefore has no impact on the diversion externality. However, it is clear from (10) that τ creates a wedge between the marginal products of capital and the growth rate of consumption. In this way τ can be used to control the level of output in the model, addressing the investment externality.

In equilibrium the share of output that each individual receives is independent of his diversionary capital stocks. A grand coalition that agreed to this share ex ante, with no diversionary expenditures, would make everyone better off since then the diversionary resources could be consumed or invested productively. However, given such an agreement, each individual playing non-cooperatively against the others would have an incentive to cheat and increase his share of output by building a predatory capital stock zi. The share scheme p(π) is self-enforcing only when individuals hold positive diversionary stocks (x, z).

The diversion externality is associated with the socially wasteful, but privately profitable, individual investments in the stocks of diversionary capital xi and zi. This cost of incomplete property rights is emphasized in the rent-seeking and conflict literatures, in a typically static context. In these static models investment in appropriation and investment in production are natural substitutes, since they are modeled as alternative uses of a given resource stock. Any policy that encourages investment in production would automatically reduce investment in diversion. Intuition here suggests that fiscal policy be designed to increase productive investment. However, this intuition is not supported in a dynamic model. Equality of the marginal returns in (10) ensures that the levels of all three capital stocks are positively related. Current consumption and total investment are substitutes in decision-making (see (1) and (10)), but the different types of investment are strict complements to each other. This complementarity is the specific instance in our model of Bates’ observation that, with non-neoclassical property rights, prosperity breeds conflict. Any policy that increases the equilibrium level of productive capital will result in a complementary increase in the levels of diversionary capital. Endogeneity of the defense of property rights thus imposes a “drag effect” on productive investment, so much so that, starting from a benchmark no-tax equilibrium, optimal policy might well call for the government to impose a tax on output thus reducing the productive capital stock, and the rate of growth, in order to reduce simultaneously the burden placed on the economy by excessive diversionary expenditures. Lower growth and
capital stocks in this case would result in higher utility because of increased current consumption.

3 Primal Problem

The full solution to the agents’ equilibrium cannot be completed without a specification of the government’s taxation and infrastructure program. We could solve directly for the optimal tax rates and infrastructure levels, taking as given the solution to the agents’ equilibrium as outlined in (5)-(9). However it is more convenient to work in the context of a corresponding second-best primal problem where the government chooses the plan \[ \{\{c_i(t), I^k_i(t)\}\forall i, I^G(t)\}_{t=0}^\infty \] to maximize aggregate utility of the agents, subject to incentive compatibility of the agents’ diversionary behavior, and to an aggregate resources constraint.

Equation (14) describes the relationship between diversionary capital and productive capital for any agent in symmetric private equilibrium. To convert this into an incentive compatible restriction on the investment choices of the reference agent note that

\[
\frac{\partial z_i}{\partial t} = \frac{\partial x_i}{\partial t} \quad \text{and} \quad \frac{\partial x_i}{\partial t} = \frac{\partial k_i}{\partial t} \phi_i + k_i \frac{\partial \phi_i}{\partial t}.
\]

With a common depreciation rate on all stocks this implies that in a symmetric equilibrium each agent’s diversionary investments must be determined in relation to \( I^k_i \) by the following incentive-compatibility constraints:

\[
I^x_i = I^z_i = \phi_i I^k_i + \frac{\partial \phi_i}{\partial t} k_i \quad \forall i
\]

where

\[
\phi_i \equiv m (1 - p(\pi)) \left( \frac{\psi_i}{\psi_i - \frac{G}{k_i} \psi'} \right); \quad p(\pi) = \frac{\pi}{\pi + 1}; \quad \frac{\partial \phi_i}{\partial t} = \frac{\partial \phi_i}{\partial G} \frac{\partial G}{\partial t} + \frac{\partial \phi_i}{\partial k_i} \frac{\partial k_i}{\partial t}.
\]

Thus, while the planner optimizes over \( \{c_i(t), I^k_i(t)\} \) and \( I^G(t) \), he must take into account the fact that equilibrium diversionary investments will be made by the agents according to (15). Given (15) the appropriate aggregate resources constraint is

\[
\int_0^1 \left( y_i - (1 + 2\phi_i) I^k_i - 2 \frac{\partial \phi_i}{\partial t} k_i - c_i \right) di - I^G = 0. \quad (16)
\]

Finally, the equation of motion of the infrastructure stock is

\[
\frac{\partial G}{\partial t} = I^G - \delta G. \quad (17)
\]

In sum, the primal problem is a planning problem that seeks to maximize the sum of agents’ discounted utilities, subject to the diversionary investment constraints in (15), the aggregate resources constraint (16), and the equations of motion of the stocks.
The Hamiltonian for this problem is

\[
H = \int_0^1 e^{-\rho t} \log(c_j) dj + \int_0^1 \lambda_k (I_j^k - \delta k_j) dj + \lambda_G (I^G - \delta G) + \lambda \left[ \int_0^1 \left( y_j - (1 + 2\phi_j) I_j^k - 2k_j \left( \frac{\partial \phi_i}{\partial G} (I^G - \delta G) + \frac{\partial \phi_i}{\partial k_j} (I_j^k - \delta k_j) \right) - c_j \right) dj - I^G \right]
\]  

(18)

The first-order conditions for agent \( i \)'s plan include

\[
\frac{\partial H}{\partial c_i} = 0 : \quad e^{-\rho t} = \frac{\lambda}{\lambda} \Rightarrow \gamma = \frac{\dot{c}_i}{c_i} = -\frac{\dot{\lambda}}{\lambda} - \rho
\]

(19)

\[
\frac{\partial H}{\partial I_j^k} = 0 : \quad \lambda_k - \lambda \left( 1 + 2\phi_i + 2k_j \frac{\partial \phi_i}{\partial k_j} \right) = 0
\]

(20)

\[
\frac{\partial H}{\partial I^G} = 0 : \quad \lambda_G - \lambda \int_0^1 \left( 1 + 2k_j \frac{\partial \phi_i}{\partial G} \right) dj = 0
\]

(21)

and, after some manipulation,

\[
\frac{\partial H}{\partial G} = -\dot{\lambda}_G : \quad -\delta \lambda_G + \lambda \left[ \int_0^1 \left( \frac{\partial y_j}{\partial G} - 2k_j \frac{\partial^2 \phi_i}{\partial G^2} \left( \frac{\partial G}{\partial t} - \frac{\partial k_j G}{\partial t} \frac{k_j}{G} \right) \right) dj \right] = -\dot{\lambda}_G
\]

(22)

\[
\frac{\partial H}{\partial k_j} = -\dot{\lambda}_k_j : \quad -\delta \lambda_{k_j} + \lambda \left[ \int_0^1 \left( \frac{\partial y_j}{\partial k_j} \right) dj + 2G \frac{\partial^2 \phi_i}{\partial G^2} \left( \frac{\partial G}{\partial t} - \frac{\partial k_j G}{\partial t} \frac{k_j}{G} \right) \right] = -\dot{\lambda}_{k_j}
\]

(23)

We look for a balanced growth solution to the primal, where \( G/k_i = g \), a constant, for all \( i \)

Then (22) and (23) reduce to

\[
-\delta \lambda_G + \lambda \int_0^1 \left( \frac{\partial y_j}{\partial G} \right) dj = -\dot{\lambda}_G
\]

(24)

\[
-\delta \lambda_{k_j} + \lambda \int_0^1 \left( \frac{\partial y_j}{\partial k_j} \right) dj = -\dot{\lambda}_{k_j}
\]

(25)

Note that in (25) the planner is fully valuing the contribution of \( k_j \) to social welfare, since all incomes that depend on \( k_j \) are included in the evaluation of marginal product: that is

\[
\int_0^1 \left( \frac{\partial y_j}{\partial k_j} \right) dj = \frac{\partial q_i}{\partial k_j} = \psi - g \psi' \quad \forall i
\]

In this sense the planner fully internalizes the investment externality. However, this does not mean that private capital is chosen at an efficient level, since the diversionary externality, linking diversionary capital to productive capital, remains.

Constancy of \( G/k_j \) also implies that \( \phi_i \) is constant with respect to \( t \), since \( g \) is; and that

\[
k_i \left( \frac{\partial \phi_i}{\partial k_j} \right) = -g \frac{\partial \phi}{\partial G} \quad \text{and} \quad k_i \frac{\partial \phi_i}{\partial G} = \frac{\partial \phi}{\partial G} \quad \forall i
\]

This will require that \( G(0) = gk(0) \) if the initial conditions of the problem are to be consistent with the balanced growth solution that we are interested in characterizing.
are also constant. It follows from conditions (20) and (21) that, with symmetry of the agents,

\[ \lambda_k = \lambda (1 + 2(\phi - g\phi_g)) \quad \text{and} \quad \dot{\lambda}_k = \dot{\lambda} (1 + 2(\phi - g\phi_g)) \]  
\[ \lambda_G = \lambda (1 + 2\phi_g) \quad \text{and} \quad \dot{\lambda}_G = \dot{\lambda} (1 + 2\phi_g), \]  

(26)  

(27)

where \( \phi_g \equiv \partial \phi / \partial g \). The term \((1 + 2(\phi - g\phi_g)) > 1\) here is the relative price of capital good in terms of consumption good. It is greater than 1 because of the fact that 1 unit of capital good costs more than one unit of consumption good as it induces an accompanying \(2\phi\) units of diversionary expenditure. This effect is complicated somewhat by the indirect effect of a change in \(k\) on the value of \(\phi\). A one unit increase in capital stock, at given \(G\), reduces \(g\) and hence changes the coefficient \(\phi\) (thus \(k\phi_k = -g\phi_g\)).

Taking both effects into account therefore, an extra unit of capital investment reduces current consumption by the amount \(1 + 2(\phi - g\phi_g)\).

Likewise, the term \(1 + 2\phi_g\) is the relative price of government infrastructure in terms of current consumption. An increase in infrastructure capital does not induce a direct increase in the stocks of diversionary capital, since infrastructure capital is not part of the individual decision-maker’s process. However, it has an indirect effect on the level of diversionary capital by changing the value of the diversion coefficient \(\phi\) that characterizes the agents’ equilibrium behaviour (thus \(k\phi_G = \phi_g\)). An extra unit of government investment therefore changes current consumption by the amount \(1 + 2\phi_g\).

Neither of these indirect effects is present when the production function is Cobb-Douglas (that is, \(\psi(g) = g^\alpha\)), since then the diversion coefficient is independent of infrastructure: \(\phi_g \equiv 0\). This is the pure second-best equilibrium in which the impact of the diversion externality is unaffected by any government policy. More generally \(\phi_g \neq 0\) and hence the diversion multiplier can be manipulated somewhat by the planner’s implicit choice of \(g\). This manipulability is limited however. We will see below that a subsidy on productive investment provides a much more powerful useful tool for manipulating the diversionary investment parameter.

Finally, the relative price of agents’ productive capital in terms of government infrastructure is

\[ \frac{\lambda_k}{\lambda_G} = \frac{1 + 2(\phi - g\phi_g)}{1 + 2\phi_g}. \]

---

8The term \(\phi - g\phi_g = m(1 - p(\pi)) (1 - ((g^2\phi''(\psi))/(\psi - g\psi)^2))\) is positive since \(\phi'' \leq 0\).

9\(\phi_g = m(1 - p(\pi)) \partial(\psi / (\psi - g\psi)) / \partial g\). If \(F(k, G)\) is CES of the form \((aG^{-\zeta} + (1 - a)k^{-\zeta})^{-1/\zeta}\) for \(-1 \leq \zeta < \infty\) then \(\phi_g\) has the sign of \(-a\zeta / (1 - a)\zeta^{1+\zeta}\). This has sign opposite to \(\zeta\). Hence the sign of \(\phi_g\) depends on the elasticity of substitution between \(G\) and \(k\) in production. It is zero in the Cobb-Douglas case, given by \(\zeta \to 0\).
With perfect property rights $\phi = \phi_g = 0$ and the relative price of the two capitals is unity. This price ratio will be distorted with imperfect property rights.

### 3.1 Infrastructure and private capital

Substituting from (26)–(27) into (24)–(25) gives a pair of equations that can be jointly solved for the growth rate $\gamma$ and the capital ratio $g$.

$$\gamma + \rho + \delta = \frac{\psi_g}{1 + 2\phi_g} \quad (28)$$

$$\gamma + \rho + \delta = \frac{\psi - g\psi_g}{1 + 2(\phi - g\phi_g)}. \quad (29)$$

In particular $g$ can be found by solving

$$\frac{\psi - g\psi_g}{\psi_g} = \frac{1 + 2(\phi - g\phi_g)}{1 + 2\phi_g} \left( = \frac{\lambda_k}{\lambda_G} \right). \quad (30)$$

This states that the marginal rate of substitution between government infrastructure and private capital is equated to the relative price of the two capitals. With perfect property rights the relative price is 1, so $G$ and $k$ are chosen such that their respective marginal values are equal; the optimal value of $g$ solves $\psi - g\psi_g = 1$.

In the Cobb-Douglas case this first-best solution is

$$g^{fb} = \frac{\alpha}{1 - \alpha}. \quad (31)$$

With imperfect property rights the relative price in (30) is distorted away from 1; in the Cobb-Douglas case $\phi_g \equiv 0$ and the equation simplifies to

$$\frac{1 - \alpha}{\alpha} g = 1 + 2\phi > 1$$

with solution

$$g^* = (1 + 2\phi) \frac{\alpha}{1 - \alpha} = (1 + 2\phi)g^{fb} > g^{fb}.$$  

This states that the planner, in a second-best solution constrained by the diversionary externality, will choose an inefficiently large stock of infrastructure relative to productive capital. This is true in particular for the case of Cobb-Douglas technology. We will see later that it is true more

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10. The derivative $\partial((\psi - g\psi_g)/\psi_g)\partial\psi_g = -\psi\psi''/\psi'' > 0$, so the solution is unique.

11. More generally, in the CES case it is $g^{fb}(\xi) = (\alpha/1 - \alpha)^{1/1+\xi}$.  

---
generally, for CES technologies. The reason that infrastructure capital is inefficiently large, in first-best terms, relative to private capital, is contained in the previous discussion of the price ratio of the two capitals. To produce any given level of output the planner will substitute away from private capital to infrastructure capital because the private capital induces $2\phi$ times its own stock of diversionary capital, while the latter induces none.

Note that, since $\phi$ is decreasing in $p(\pi)$, countries with stronger legal systems would have less distorted values of $g$ than would countries with weaker legal systems. In this way relatively larger governments sectors and relatively weaker property rights systems are likely to be positively associated. The causality is not between larger, more predatory government and consequently weakened property rights however; quite to the contrary, relatively larger government is a second-best response to the problem of weak property rights. We do not advocate larger government in this sense as an unfailing positive remedy to property rights problems. However, it points to one avenue of response to weak property rights that involves reducing indirectly the capacity of agents to engage in diversionary investment.

The second-best crowding-out of private investment by government investment foreshadows a theme that will emerge more fully below. We have assumed a benevolent planner and malfeasant agents. Insofar as resources are being controlled and allocated by the planner then the agents have lesser opportunities to implement their bad behavior, as in this case where $g$ is inefficiently high. We do not allow the planner to control diversionary investments directly. However, an investment subsidy together with a linear income tax system will enable the planner to crowd out diversionary investment, at the cost of having the planner effectively control all quantities in the system. We examine this issue in the next section.

### 3.2 Growth and consumption

Turning now to growth, given the capital ratio $g^*$ the optimal rate of growth follows from (28), in the Cobb-Douglas case, as

$$\gamma^* = \psi \left( \frac{g^* - \psi g^*}{1 + 2\phi} \right) - (\rho + \delta). \quad (31)$$

The first equality here shows the growth rate as the marginal product of infrastructure capital less the standard $\rho + \delta$. Compared to the first-best growth rate, which satisfies $\gamma^{fb} = \psi \left( g^{fb} \right) - (\rho + \delta)$, it is immediate that

$$\gamma^* < \gamma^{fb}.$$
since $g^* > g^{fb}$, and $\psi$ is concave. The planner always chooses a growth rate that is less than the first-best solution. In this Cobb-Douglas case where, the diversion coefficient cannot be manipulated through $G$ (that is, $\phi_g \equiv 0$), the difference in growth rates is seen to be attributable entirely to the fact that the second-best capital ratio is higher than first-best efficient.

The result that imperfect property rights result in a less than first-best growth rate may seem banal. It is not, for two reasons. First, the mechanism emphasized here works through the diversionary externality, rather than through the more familiar investment externality. While the diversionary externality is widely recognized in the literature (see for example Hall and Jones (1999)) it is less widely analyzed (see for example Barro (1990), who looks at property rights in a model of government expenditure and taxation, but considers only the investment externality). And second, we are not describing a private economy with unameliorated property rights problems; what is shown here is that a planner who is explicitly optimizing utility in the presence of imperfect property rights will choose a well-defined, but less than first-best, growth rate. An economic advisor who knew the fundamentals of such an economy but ignored the property rights issue would recommend growth at the rate $\gamma^{fb}$; this would be excessive in the presence of imperfect property rights.

The solution value for consumption at any time $t$ can be written as

$$c = y - I^k - I^x - I^f - I^G = k\psi - (1 + g + 2\phi)(k + \delta k).$$

(32)

Using the solution growth rate from (31), the transversality conditions and the initial conditions $x(0) = z(0) = k(0) = k_o$ and $G(0) = gk_o$, the value of initial consumption is derived as

$$c^*_o = k_o(1 + g^* + 2\phi)\rho.$$  (33)

The corresponding first-best value is

$$c^{fb}_o = k_o(1 + g^{fb})\rho.$$  (34)

It is immediate that

$$c^*_o > c^{fb}_o,$$

the level of consumption in the second-best is higher than in the first-best. This reflects both the direct impact of the diversionary externality, $2\phi$, together with the induced effect of a relatively higher level of government infrastructure, $g^* > g^{fb}$. As with the growth rate comparison, it is not surprising that imperfect property rights are associated with high current consumption. In
the model current consumption comes from income to which property claims have already been established, whereas investment will produce output that will be contested in the future. This imparts a “consume it or lose it” bias in favor of current consumption. What is surprising is that second-best optimality calls for current consumption to be higher than it would be in the first best case. Along a constant, balanced growth path the aggregate present value of utilities can be written simply as

\[ W = \gamma + \rho \log(c_o). \] (35)

A given second-best value \( W^* \) can be generated by an infinite set of growth rate and consumption pairs, \((\gamma, c_o)\), including in principle the set for which both the second-best growth rate and consumption level are below the respective first-best values. However, what is in fact chosen is a pair such that growth is lower and consumption higher in the second best than in the first best.

In fact the second best growth rate is in essence the growth rate that maximizes welfare, taking into account the diversion externality. From \((32)\) write initial consumption as

\[ c_o(\gamma) = k_o(\psi(g^*) - (1 + g^* + 2\phi)(\gamma + \delta)) \] (36)

where \( \gamma \) is treated for the moment as a parameter\(^{12}\). If \( \gamma \) could be varied parametrically then initial consumption would decrease as \( \gamma \) increased. This trade-off between initial consumption and growth occurs both because of infrastructure capital, and because of the diversion externality. Then we can ask how utility would vary with a parametric change in the growth rate, taking into account both the direct positive effect of growth on welfare, and the indirect negative effect of growth on welfare through its negative impact on initial consumption. Substitute the expression for \( c_o(\gamma) \) into \((35)\) and differentiate to obtain

\[ \frac{\partial W(\gamma)}{\partial \gamma} = 1 - \frac{k_o}{c_o} (1 + g^* + 2\phi) = \frac{k_o}{c_o} [\psi(g^*) - (1 + g^* + 2\phi)(\gamma + \delta + \rho)]. \] (37)

Now evaluate this derivative at the second best. Substituting the expression for \( g^* \) from \((31)\) into the bracketed RHS term here, and using condition \((30)\) that defines \( g^* \), allows this RHS term to be simplified to zero, so that \( \partial W(\gamma^*)/\partial \gamma = 0 \). In other words, we can think of the growth rate at the second-best optimum as having been adjusted in such a way as to maximize welfare, taking account of both direct and indirect effects. In addition, it is evident that a zero value of the bracketed term in \((37)\) immediately implies that the expression for \( c_o(\gamma) \) in \((36)\) reduces to that in

\(^{12}\)When this expression is evaluated at \( \gamma^* \) the result is \( c_o^* \) which is independent of \( \gamma \). However this independence is a feature of the second best solution, and is not true for an arbitrary value of \( \gamma \). See below.
which is independent of the growth rate. That the growth rate maximizes welfare and that the initial consumption is independent of growth are equivalent statements.

These features of the second-best stand in sharp contrast to what occurs in a private, no-intervention equilibrium. González (2004) shows in that case that (1) equilibrium involves the dependence of initial consumption on growth; and (2) growth may be above or below the level that optimizes welfare when taking the trade-off between consumption and growth into account. Comparative statics of the model with respect to an exogenous productivity increase, or a change in the state of law parameter, $\pi$, may have a perverse impact on welfare. If the growth rate is already above the welfare-maximizing level then these apparent improvements in the parameters, which both raise the equilibrium growth rate, actually make society worse off by lowering welfare.

These perverse comparative statics are eliminated from the second-best solution analyzed here because the planner always adjusts the optimal allocation to ensure that the growth rate is welfare maximizing, given the underlying parameters.

In summary, in the second-best solution constrained by the fact of diversionary investment by the agents the growth rate is chosen such that welfare is maximized, given both the direct and indirect impact of growth on welfare. In consequence, the growth rate is lower and the consumption level is higher than in the first-best solution. Imperfectness of property rights results in a situation where relatively high current consumption and low long-run growth are welfare-optimizing. Low current savings and long-run poverty appear as challenges for the economic advisor to remedy; in fact they may be second-best optimal outcomes so long as the condition of imperfect property persists. This model therefore shows, in a clear and consistent fashion, one interpretation of Bates’ fundamental ideas that prosperity brings conflict and that poverty is the price of peace.

3.3 CES extension

The solution of the primal problem above was carried out in general terms, but the subsequent discussion has been carried out in the context of a Cobb-Douglas technology. We have proceeded this way for the sake of informality and because the solutions are very simple. Before looking at the implementability of this solution through a linear income tax scheme, we state a proposition that extends the above results to the case of CES technology. This establishes the robustness of these second-best comparisons.
Write a CES production function as

\[ F(k, G) = \left( (1 - \alpha)k^{-\varsigma} + \alpha G^{-\varsigma} \right)^{-\frac{1}{\varsigma}}. \]

This technology is characterized by two parameters \((\varsigma, \alpha)\) drawn from the set \(C = [-1, \infty) \times [0, 1]\).

For any CES the solution values for \(g\) can be written as \(g^*(\varsigma, \alpha)\) and \(g^{fb}(\varsigma, \alpha)\).

**Proposition 1.** A solution to the primal problem described by (18) must satisfy

a. the second-best ratio of infrastructure to private capital is strictly larger than the first best:

\[ g^*(\varsigma, \alpha) > g^{fb}(\varsigma, \alpha); \]

b. the second-best consumption level is strictly larger than the first best:

\[ c_o^*(\varsigma, \alpha) > c^{fb}_o(\varsigma, \alpha); \]

c. the second-best growth rate is strictly less than the first best:

\[ \gamma^*(\varsigma, \alpha) < \gamma^{fb}(\varsigma, \alpha); \]

for all CES technologies, with the exception of \((\varsigma, \alpha) = (-1, 1/2)\).

**Proof:** Given (a), (b) follows by comparison of (33) and (34). Given (a) and (b), (c) follows because, by definition, \(W^{fb} = \gamma^{fb} + \rho \log(c^{fb}_o) > W^* = \gamma^* + \rho \log(c^*_o);\) and \(c^*_o > c^{fb}_o\). To prove (a) note that the solutions \(g(\varsigma, \alpha)\) are continuous functions. In the Cobb-Douglas case we have shown by computation that \(g^*(0, \alpha) > g^{fb}(0, \alpha).\) If \(g^*(\varsigma, \alpha) \leq g^{fb}(\varsigma, \alpha),\) by continuity there must exist a technology \((\varsigma_o, \alpha_o)\) such that first and second best \(g\)'s are equal: \(g^*(\varsigma_o, \alpha_o) = g^{fb}(\varsigma_o, \alpha_o) \equiv g_o.\)

First best \(g\) equates the LHS of (30) to 1 so \(g_o = (\alpha_o/(1 - \alpha_o))^{1/1+\varsigma_o}.\) If \(g_o\) is also the second-best solution it must, therefore, equate the RHS of (30) to 1. Simplifying, RHS equals 1 if and only if \((1 + \varsigma_o)(1 + g_o) = 0,\) which is true if and only if \((\varsigma_o, \alpha_o) = (-1, 1/2).\)

### 3.4 Implementation through taxes

The second-best solutions described in the previous section can be implemented through a linear income tax system. In a decentralized economy with an income-tax of the form \(T(t, y_i(t)) = \ell(t) + \tau(t)y_i(t)\) and a government infrastructure plan \(G^*(t)\) growing at rate \(\gamma^*,\) the solution to the individual agent’s optimization problem, in a symmetric equilibrium, can be shown from (5)-(9) to involve a growth rate given by

\[ \gamma = (1 - \tau)\frac{\partial y_i}{\partial k_i} - (\rho + \delta) = (1 - \tau)p(\pi)(\psi(g^*) - g^*\psi_g(g^*)) - (\rho + \delta). \]
The second best growth rate from (31) is given by\footnote{This is the Cobb-Douglas case. More generally $\gamma^* = (\psi(g^*) - g^*\psi'(g^*)/(1 + 2\phi(g^*) - g^*\phi'(g^*)) - \rho - \delta$. Accounting for the additional term $-2g^*\phi'(g^*)$ in the denominator, which is non-zero in the CES case, would add nothing to the discussion.}

$$\gamma^* = \frac{\psi(g^*) - g^*\psi'(g^*)}{1 + 2\phi(g^*)} - (\rho + \delta).$$

It is immediate that the second-best solution can be implemented in the decentralized economy if the marginal tax rate $\tau$ is set such that

$$1 - \tau = \frac{1}{p(\pi)(1 + 2\phi(g^*))}. \quad (38)$$

This time-independent marginal tax rate induces the agent to value capital at the margin at its appropriate second-best level. This tax takes into account both externalities: $p(\pi)$ reflects the impact of the investment externality on the agent’s decentralized investment decision, while the $2\phi$ term reflects the impact of the diversion externality.

The tax system also needs to achieve budget balance. The income tax collects instantaneous revenue $\tau k^*(t)\psi(g^*)$. This, combined with a lump-sum tax $\ell^*(t)$, must finance government expenditure

$$I_G(t) = \frac{\partial G}{\partial t} + \delta G(t) = G(t)(\gamma^* + \delta) = g^*k^*(t)\left(\psi(g^*) - \rho\right)$$

in each period. The lump-sum tax component therefore solves

$$\ell^*(t) = k^*(t)\left(g^*\left(\psi(g^*) - \rho\right) - \tau\psi(g^*)\right).$$

With perfect property rights $\gamma^* \rightarrow \gamma^{fb}$, $g^* \rightarrow g^{fb}$ and these taxes reduce to

$$\tau^{fb} = 0 \quad \text{and} \quad \ell^{fb}(t) = g^{fb}k^{fb}(t)\left(\psi(g^{fb}) - \rho\right) > 0.$$  

There are no externalities, and so no income tax is needed as a corrective. The lump-sum tax is required only to finance government infrastructure investment and so is positive.

Returning to the second-best solution the central question is whether the marginal tax rate that supports this solution is positive or negative. If negative it indicates that income is being subsidized in the second-best, which raises investment and growth above what they would have been in a no-tax economy. We expect a subsidy aspect to $\tau$ in response to the investment externality, ceteris paribus. On the other hand, an income subsidy that raises growth also raises the levels of diversionary investment by $2\phi$. On this account, ceteris paribus, welfare would be increased by a
tax on income rather than by a subsidy. The instrument \( \tau \) thus has to deal with both externalities, and will be chosen to trade off the two conflicting impacts on welfare.

From (38) it is clear that

\[
\tau \geq 0 \iff p(\pi)(1 + 2\phi) \geq 1. \tag{39}
\]

The less important is the investment externality (high \( p(\pi) \)), and the more important is the diversion externality (high \( \phi \)) the more likely it is that the marginal tax rate \( \tau \) will be positive, a growth reducing tax. Higher \( p(\pi) \) means a relatively unimportant investment externality, ceteris paribus. A subsidy on output, to ameliorate low growth due to this externality, would bring too much additional diversionary investment, due to a high diversion coefficient \( \phi \), to be worth it. In this case the trade-off between controlling the impact of the diversionary externality and controlling the impact of the investment externality falls in favor of the former which calls for a tax.

Conversely, the more important is the investment externality (low \( p(\pi) \)), and the less important is the diversion externality (low \( \phi \)) the more likely it is that \( \tau \) will be negative, a growth inducing subsidy. Lower \( p(\pi) \) means a relatively important investment externality, ceteris paribus. A subsidy on output, to ameliorate low growth due to this externality, would not bring sufficient additional diversionary investment to outweigh its value, due to low \( \phi \).

We have seen that the second-best planner chooses a welfare-maximizing level of growth that is strictly less than the first-best level. The fact that this second-best level of growth might be implemented by a growth-reducing tax implies that in the no-tax economy growth is inefficiently high from the second-best viewpoint (though still less than the first-best level), and so requires to be reduced. When an income subsidy is required to implement the second best growth rate it implies that the no-tax growth rate is sub-optimally low relative to the second best, and the subsidy is to raise growth to this level.

The subsidy case, or more generally a pro-growth case, is the conventional prescription, and is supported by models that focus on the investment externality to the neglect of the diversionary one, and which see growth and prosperity as outcomes that will directly supplant diversion. The growth-reducing tax case is less conventional, and is the one that we wish to highlight here. It stems from the fact that in this model where property rights are endogenous, and must be purchased at a cost, the resulting diversion externality inevitably means that growth and prosperity directly induce the waste of diversionary investment. If growth would bring too much diversion then welfare may be increased by reducing growth rather than by increasing it. This is a second-
best consequence, however unpleasant it may seem, of the fact of incomplete property rights and it formalizes what Bates describes as the key problem of development, that of achieving growth without inducing diversion and conflict.

The interpretation of the sign of the marginal tax rate from (39) has been presented in terms of a trade-off of the two externalities. Because the diversion coefficient depends on \( m, p(\pi) \) and \( \alpha \), the discussion has given little indication of the sets of parameter values for which a second-best income tax rather than subsidy is called for. In particular, the property rights parameter \( p(\pi) \) has a dual role: it measures the investment externality directly, but is also a component of the diversion coefficient. An increase in it reduces the investment externality, favoring a higher tax rate, but it simultaneously reduces the diversion externality, favoring a lower tax rate. To illustrate the dependence of the sign of the marginal tax rate on the fundamental parameters \( m, p(\pi) \) and \( \alpha \) simplify (39) to show that

\[
14 \tau \geq 0 \iff p(\pi)m \geq \frac{1 - \alpha}{2}.
\]

Thus whether income taxation to decrease growth or income subsidy to increase growth is required in the second-best as compared to the no-tax economy depends in a simple way on the key parameters.

The parameter \( m \) measures the effectiveness of conflict. The larger is the value of \( m \) the greater the loss due to diversionary investment, and the more likely it is that a growth reducing tax will be required in the second-best. The better is state of the law – higher \( \pi \) – and equivalently, the more secure are property rights – higher \( p(\pi) \) – the smaller are the losses from both investment and diversion externalities. Nonetheless, the formula indicates that, on balance, a higher value of \( p(\pi) \) increases the likelihood that a tax on income is the optimal second-best policy. Finally, an increase in \( \alpha \), the weight of infrastructure capital in production, increases the diversion coefficient. The higher this value is the more likely the marginal tax rate is to be positive, to counter diversionary investment by reducing growth.

The possibilities are best illustrated by a figure. The value of \( p(\pi) \) lies between 0.5 and 1, of \( \alpha \) between 0 and 1, and of \( m \) between 0 and 1. The contours of \( p(\pi) \) and \( m \) that give a zero marginal tax rate are shown in figure 1 for three values of \( \alpha \). All \((p(\pi),m)\) values north-west of an \( \alpha \)-contour indicate that a growth-reducing positive tax rate is called for, given \( \alpha \). It is clear that there is nothing perverse or marginal about the possibility of this outcome given the allowed values

\[14 \text{ In the CES case this is } pm \geq 1/2\theta \text{ where } \theta \equiv \psi/(\psi - g\psi_g) - g\left( \partial (\psi/(\psi - g\psi_g))/\partial g \right) \text{.} \]
for the three parameters. For each value of $\alpha$ there is a large enough value of the effectiveness of conflict $\hat{m}(\alpha)$ to ensure that a growth reducing tax is optimal for all values of the property rights parameter $p(\pi)$. By the same token, if conflict effectiveness is low enough, $\check{m}(\alpha)$ at given $\alpha$, then a growth-enhancing subsidy may be appropriate for all values of $p(\pi)$. Finally, for intermediate values of conflict effectiveness, $\bar{m}$, subsidies are appropriate at lower values of effective property rights while taxes are appropriate at higher values. In this way, an exogenous improvement in the state of the law parameter $\pi$ might induce a move from a subsidy regime to a taxation regime. In the boundary case where $p(\pi)m = (1-\alpha)/2$ then $\tau = 0$ and $f^*(t)$ finances infrastructure investment.

Finally, note that the sign of the marginal tax rate is independent of the level of infrastructure capital; this indicates that it is chosen to ameliorate externalities only, not to finance infrastructure. Likewise, whether there is a tax or subsidy in place has no direct effect on the level of consumption, which is set at $k_o(1 + g^* + 2\phi)\rho$. This is a corollary of the point made in the previous section, that second-best growth is designed to maximize welfare, taking into account both its direct effect and its indirect effect through consumption. Initial consumption in this case is independent of the growth rate, and is therefore independent of the tax or subsidy rate on which the growth
rate depends. We can therefore think of the income tax rate as being set to achieve the level of growth that maximizes welfare, or equivalently as set to make initial consumption independent of growth.\footnote{Re-solve the level of consumption in the second-best solution using the decentralized growth rate \(\gamma(\tau) = (1 - \tau)p(\pi)(\psi - g\psi_g) - (\rho + \delta)\). This gives \(c_o(\tau) = k_o(\psi - (1 + g + 2\phi)(\gamma(\tau) + \delta)) = c_o^* + k_o[\psi - (1 + g + 2\phi)(1 - \tau)p(\pi)(\psi - g\psi_g)]\). Evaluate \(\frac{\partial W}{\partial \tau} = \left(\frac{\partial \gamma}{\partial \tau} + \frac{1}{c_o(\tau)} \frac{\partial c_o}{\partial \gamma}\right) \frac{\partial \gamma}{\partial \tau} = \frac{\rho k_o}{c_o^*} [\psi - (1 + g + 2\phi)(1 - \tau)p(\pi)(\psi - g\psi_g)] \frac{\partial \gamma}{\partial \tau}\). It is easy to check that, at \(\tau = \tau^*\) and \(g = g^*\), the RHS term in brackets in these two equations is zero, showing that growth maximizes welfare, and consumption is independent of growth.}

4 Adding an investment subsidy

The main result above, that there is an optimal ceiling on growth which lies below the first-best, depends on the fact that the tax structures to which the government has access were restricted by assumption. In particular while the investment externality was addressed by the income tax, no effective means of altering the diversionary coefficient was available.\footnote{For a CES production function this coefficient could be manipulated through the value of the capital ratio \((\phi_g \neq 0)\) but not in a way that would lead to the first-best outcome.} The first-best allocation can be trivially implemented if an unrestricted system of costless and well-functioning taxes is in place. Even if the diversion of resources cannot be targeted directly, the effects of a tax on appropriative activities can be replicated if all non-appropriative activities can be taxed or subsidized.

In particular, a subsidy to private productive investment will increase the private marginal product of productive relative to diversionary capital. Productive investment is encouraged relative to diversionary investment, ceteris paribus, effectively reducing the value of the diversion coefficient. Thus the diversion externality can be manipulated directly by an investment subsidy. The income tax system must also adjust. An investment subsidy that is high enough to reduce the diversion coefficient towards zero, as required by the first-best, will result in too high a level of growth and output, unless a marginal income tax at a sufficiently high rate is imposed to counteract it. The lump-sum tax must then change to allow for budget balance.

To see the effects of this redefine the tax system to include an investment subsidy at a time-independent rate \(\kappa\) on private productive investment. This is an arbitrary subsidy specification that may not be locally optimal, but we use it because it allows achievement of the first-best.
Budget balance for the government requires
\[ \int_0^1 \left( \tau y_j(t) + \kappa \ell_j(t) + \ell(t) \right) \, dj = f^G(t) \]
at each time \( t \).

Inserting the subsidy into the agents' equilibrium allows (10) above to be rewritten as
\[ \frac{1}{1 + \kappa} \frac{\partial y_i}{\partial k_i} = \frac{\partial y_i}{\partial x_i} = \frac{\partial y_i}{\partial z_i} = \frac{1}{1 - \tau} \left( \frac{\dot{c}_i}{c_i} + \rho + \delta \right). \]  \hspace{1cm} (40)
and (14) as
\[ p(\pi) = \frac{\pi}{\pi + 1} \quad \text{and} \quad z = x = \Phi k \quad \text{for} \quad \Phi \equiv (1 + \kappa)\phi. \]

The primal problem, now including the investment subsidy \( \kappa \) as a parameter, can be solved explicitly in the Cobb-Douglas case:
\[ g^*(\kappa) = g^{fb}(1 + 2\Phi(\kappa)) \]  \hspace{1cm} (41)
\[ \gamma^*(\kappa) = \psi_g(g^*(\kappa)) - (\rho + \delta) \]  \hspace{1cm} (42)
\[ c^*_0(\kappa) = k_o\rho \left( 1 + g^*(\kappa) + 2\Phi(\kappa) \right) \]  \hspace{1cm} (43)
\[ W(\kappa) = \gamma^*(\kappa) + \rho \log (c^*_0(\kappa)) \]  \hspace{1cm} (44)

It is immediate that, as \( \kappa \) approaches -1, \( \Phi \) approaches zero; then \( (g^*, \gamma^*, c^*_0) \) approaches \( (g^{fb}, \gamma^{fb}, c^{fb}_0) \) and so \( W(\kappa) \) approaches \( W^{fb} \). The first-best can be approximated arbitrarily closely as the subsidy rate on productive investment approaches 100%. The marginal return to diversion is infinity at zero diversion in our model and so total subsidy is required to eliminate all diversion. This full subsidy of private investment is not a palatable solution. It becomes even less so when the implied income taxes for a decentralized solution are considered. To equate the private return and the social return on productive investment the marginal income tax rate must satisfy
\[ \frac{1 - \tau}{1 + \kappa} p(\pi) = 1 \]  \hspace{1cm} (45)
requiring the marginal income tax rate to approach 100%. Under a 100% investment subsidy growth would be too high; a 100% income tax is needed to suppress this excessive growth. Finally, in the presence of a 100% income tax, first-best consumption will be fully subsidized by the lump-sum amount \( \ell(t) = k(t)\rho(1 + g^{fb}) \).

This taxation solution to the imperfect property rights problem is presented here for completeness, since it is not one that can be recommended seriously as a policy alternative. As noted
above, with a benevolent government and misbehaving agents the externalities can be dealt with by crowding out the agents’ ability to invest in diversion. This underlies the fact that the second-best infrastructure to private capital ratio, $g^*$, is higher than in the first-best. This taxation solution leaves the state of property rights unchanged at $p(\pi)$, but avoids the externalities by controlling agents’ decisions to the exclusion of diversion investments. However when all output, consumption and productive investment are cycled through the government budget the result is a Soviet-like authoritarian system. As a practical matter it is likely to be even less capable of achieving the first-best than the actual Soviet system was. It requires a well-functioning tax system whose absence is conspicuous precisely in societies that suffer property insecurity and conflict over economic distribution. In addition, it requires an ideal government that is benevolent, provides productive services efficiently and can commit to future government spending and tax policies. In particular, counting on benevolence in a government that controls the quantities so completely would be naïve.

In fact the model presented here is of relevance to conceptualizing the manner in which collapsing authoritarian regimes may give rise to large-scale diversionary activity in their successor states. Yugoslavia, the Soviet Union, Afghanistan and Iraq are instances of this phenomenon. The problems associated with flawed property rights, weakness in the rule of law, emergent mafia behavior, disintegration into smaller political units etc. in transition economies of the former Soviet Union has received considerable attention in the literature (Johnson et al. (1998), Roland (2000), Roland and Verdier (2003) etc.). These problems have not yet been worked out to a satisfactory extent. In terms of the present model, social order can be maintained in an authoritarian structure through extensive, exclusionary government decision-making. Despite the fact of order, the underlying state of the law parameter $\pi$ can remain completely undeveloped because it plays no role in the maintenance of order. In (45) for example, the magnitude of $p(\pi)$ is irrelevant; the first-best is achieved, not by providing the legal and political institutions that leverage the agents’ own defensive investments (i.e. pushing $p(\pi)$ to 1, as would be achieved by a ‘nation-building’ program), but by implicitly controlling quantities to the point where diversionary investment is crowded down to zero. However, once the authoritarian structure collapses self-enforcement of property rights at the low value of $\pi$ results in an immediate, dramatic readjustment of individual investments away from productive capital towards diversionary capital. The habit of order in the previous regime has no persistence in a decentralized successor regime in the face of low $\pi$. Just as the rapidity of the collapse of political order in the Soviet Union surprised most observers, so
also the depth of disruption in the property system and the widespread lawlessness that followed
decentralization of the old regime were a surprise to many. The distinction between order through
central control, and the failure of order under decentralized decision-making when legal support
for individual property rights protection is absent, is useful in understanding this emergence of
chaos.

Despite the conclusion drawn above that achieving a first-best allocation through the fiscal
system is impractical, nonetheless the exercise demonstrates that there may be important welfare
benefits associated with income taxes and investment subsidies in a world with imperfect prop-
erty rights. Many societies have poorly developed income and investment taxes, collecting most
revenue from sales taxes and excise taxes. Arguably, this is also a reflection of poorly developed
institutions. In this regard, our analysis of income taxation and government spending indicates
the potential benefits associated with responsible fiscal policy. Income taxes help mitigate the ad-
verse effect of excessive growth when property rights are inadequate. Investment subsidies are
in effect a differential tax against appropriative activities, which helps discourage the diversion of
resources.

5 Further remarks and conclusions

This paper has emphasized the key insight that derives from our model of growth with imperfect
property rights: second-best optimality places a ceiling on growth. This observation suggests
considerable caution about the nature of policy interventions that are appropriate to economies
with imperfect property rights systems.

Weak institutions make growth beyond the second-best level undesirable and policy needs to
account for this. Relative to the first-best, government tax and infrastructure policy will always in-
volve an inefficiently high ratio of infrastructure to private productive capital, and an externality-
ameliorating marginal income tax rate that can be positive or negative. The second-best trades off
low growth for high consumption and this balance should not be disturbed unless institutions are
improved. Institutional reform enables growth to take place, but more importantly it is necessary
in the first place to make further growth desirable, by increasing the second-best growth rate.

The fact that an income tax in particular can support the second-best indicates that growth
in the no-intervention equilibrium would be too high in that case. Policy intervention in the
no-intervention status quo must seek to reduce growth to increase welfare. This case is not ex-
ceptional in the model, where we have seen it to depend on perfectly plausible specifications of the parameters \( \pi, m \) and \( \alpha \). It is also not exceptional in the real world where, as Bates (2001, p. 47) reports, in the face a system of private and violent provision of security “people may seek to increase their welfare by choosing to live in poverty ... egalitarianism becomes a strategy in which people forgo consumption for the sake of peaceful relations with neighbors. To forestall predation, they may simply choose to live without goods worth stealing. In such a setting, poverty becomes the price of peace.”

That policies that reduce growth may be welfare preferable is a surprising result, especially in view of the current state of the literature that analyzes the impact of institutional imperfections on growth. Typically in this literature imperfections give rise to constraints that impose a ceiling on growth. The policy problem then is how to relax these constraints so that further growth can take place.

That growth may be not be welfare improving in general is not controversial. Many analyses of the problem of development begin precisely with the observation that growth is not Pareto-improving because it has the potential to create both winners and losers. The idea of gain and loss here may be quite broad, including narrow economic profitability, and broader social and political concerns, as when economic growth might threaten a group’s political power. Whether growth actually occurs will then depend on the relative organizational abilities and political strengths of the winners and losers. Political transactions costs are the central theme. (See e.g. Olson (1982), Robinson (1998), Dixit (2003), Acemoglu and Robinson (2000, 2002), Mokyr (1990), Shleifer and Treisman (2002).) What these models suggest overall is that growth is lower than it ought to be in welfare terms, and that if growth could be engineered despite the vested interests this would be desirable. Institutional reform is aimed at removing barriers to growth.

Our paper, whose theme of imperfect property rights falls within the general transactions cost paradigm, is complementary to the literature, but differs in specifying the source of the problem and in pointing to remedies. We assume symmetry of the agents precisely to abstract from distributional considerations that are not directly the focus of agents’ decision-making. Asymmetry of the benefits and costs of growth are not an issue; what is at issue is that, in the absence of the abstraction of ‘perfect’ property rights, the benefits of growth are contestable, and the agents’ ability to control resources is endogenous to the investments they make in securing their own and others’ property. Symmetry means that either all benefit or all lose equally from growth. What is surprising is that all can lose equally, because of the externalities they impose through the self-provision
of property rights. Institutional reform, here reflected in improvements in the state-of-law parameter, is intended not to remove barriers to achievement of desirable growth, but rather to make additional growth desirable by raising the second-best growth rate.

A related observation in this context is that inequality is often seen a source of divisiveness and conflict in society, and that insofar as growth brings about inequality and conflict it may be undesirable. The development policy of institutions such as the World Bank is more nuanced now than in previous decades on the question of how aggressively growth ought to be pursued, and the syllogism of growth-inequality-conflict appears to be a component of this revision. No doubt inequality is an important ingredient in provoking conflict but in our model it is potential inequality rather than actual inequality that is the key factor. In equilibrium in this model all agents are symmetric with respect to outcomes, so there is ex post equality. What drives the model however is the fact that the shares of output that an agent can achieve, $p^{ij}$ and $1 - p^{ij}$, are manipulable ex ante, and each agent invests in diversion precisely because the non-cooperative expectation is that the agent will become better off at the expense of another. It is the expectation of inequality, made possible by the imperfection and endogeneity of property rights, that results in diversion being a consequence of growth. In a non-symmetric equilibrium, of course, ex post inequality would exist and intuition suggests it may well be associated with a greater degree of conflict. However, the analytical point is that while a program of egalitarian income redistribution might ameliorate the aggregate of diversionary investment it would not erase the link between growth and conflict analyzed here. Imperfect property rights rather than inequality is the fundamental problem.

Another literature that considers failures in development looks at the possibility of multiple equilibria (e.g. Murphy et al. (1993), Hoff and Stiglitz (2001), Roland and Verdier (2003)). There are several possible mechanisms underlying the strategic complementarities that drive these models, but the common feature is that the same fundamentals can support good equilibria, which exhibit high growth and low conflict, as well as bad equilibria, which exhibit low growth and high conflict. In this context, the policy focus is on how the implicit coordination failure can be solved to reach the good equilibrium. Again the situation is one where growth is constrained below its best attainable level, so that an increased growth rate is the policy desideratum. Moreover, an increase in growth will be accompanied by a reduction in conflict in moving from bad to good equilibrium. In viewing institutional problems as being inimical to economic development, the present paper is closely related in spirit to this literature. Furthermore, our results should not be interpreted as implying that coordination problems are unimportant. However, our conclusions
are quite distinct from the existing literature, even though there is nothing unusual in our modeling of property rights. The equilibrium we analyze is unique. For any value of the property rights parameter $\pi$, an increase in growth brings along an increase in diversionary expenditure, and precisely for this reason the second-best solution may call for a decrease in growth from the no-intervention status quo. Given the property rights parameter there is an interior solution for the second-best growth rate and policy that attempts to push it or maintain it above that level will reduce welfare.

The results on the use of fiscal policy in a growth context extend those of Barro (1990) and Barro and Sala-I-Martin (1992). Barro's seminal paper considered the possibility of endogenous growth in a context where private investment and government infrastructure expenditure are complements, and looked at the tax requirements this would call for. Barro and Sala-I-Martin extend this to look at the tax regimes, typically income tax versus lump sum tax, that would be appropriate in the case of various externality and public goods aspects of the infrastructure expenditure. We allow both an income tax and a lump-sum tax so that issues of dealing with the imperfect property rights situation do not become entangled with the question of how infrastructure investment is financed. And we assume non-rivalry of the infrastructure capital to abstract from these issues already dealt with in Barro and Sala-I-Martin (1992).

We have emphasized throughout the paper properties of the equilibrium that occurs when the state of the law parameter $\pi$ is fixed. We have looked at fiscal policies, which are implementable, as a gauge of how the policy maker should react to growth in the presence of imperfect property rights. What is clearly missing from this framework is an analysis of how the state of the law parameter might be endogenized. This can be done by allowing $\pi$ to depend on the level of government expenditure or investment in a legal etc. system. This extension is beyond the scope of the current paper but is an area of current research.

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