The Impact of Simple Fiscal Rules in Growth Models with Public Goods and Congestion

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Abstract

In this paper we examine the implication of a simple class of fiscal rules for long-run economic growth and welfare. In particular, the golden rule of public finance (GRPF) that we examine is motivated by institutional arrangements in countries such as Germany and the UK. We find that rules which seek to limit government borrowing to productive investment spending have a clear justification in terms of growth and welfare, when government provided goods are otherwise excessively provided. Even in the case where it is private consumption that is excessive, the GRPF is likely to be good from a growth perspective, but the welfare effects are more ambiguous.

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1. Introduction

More and more countries are adopting fiscal rules. They may become an important feature of the macroeconomic landscape in the same way as central bank independence has emerged as a dominant institutional arrangement for monetary policy across an increasing number of countries. Some argue that fiscal rules are a complement to monetary rules–both ultimately aimed at price stability. However, fiscal rules may also have longer term, growth, implications. For example, recently the Prime Ministers of Italy, Spain, Poland, The Netherlands, Portugal and Estonia wrote to Bertie Ahern Irish holder of the rotating EU Presidency. In that letter[1] the premiers argued that "[T]he Stability and Growth pact is an essential element of economic governance...and *a necessary condition to sustained economic growth*" (emphasis added)[2]. And in fact the endogenous growth literature has indicated that there are indeed long-run growth implications of fiscal policies. But what are the growth and welfare implications of such rules, and what are the underlying distortions that they seek to address? In this paper we take a preliminary look at these issues.

1.1. Related Literature

We construct a series of model economies in which the decentralised equilibria reflect a number of externalities. We then identify the circumstances in which what we call the Golden Rule of Public Finance (motivated by actual 'rules' adopted in several countries, notably Germany and the UK), can improve on the decentralised outcome.

These externalities reflect situations where private and government consumption are, in turn, excessive relative to the social optimum, and where the marginal product of capital is low, while nevertheless reflecting a situation of *underinvestment* in private capital. For simplicity we build these distortions directly into our baseline model via our assumptions on preferences and production technology. We view these distortions as proxies for richer political economy-type features. For

[1] This correspondence was reported in the Financial Times, Tuesday February 17 2004 (Page 8).

[2] The letter was widely viewed as a rebuke to France and Germany who were breaching at the time the 3% EU deficit guidelines. These two countries persuaded the EU to suspend the sanctions against excessive deficit countries in November 2003, to the dismay of many of the other EU countries. The European Central Bank subsequently also made clear its dismay at the sidelining of the Pact.

instance, as Drazen (2000, page 380) notes, "... there is no presumption about whether decision making by majority voting leads to a level of public good provision either systematically above or below the [social] optimum level."[3] In practice, the arguments are made both ways. For instance, some have argued that market economies have a built in dynamic that constrains (some) public good provision to fall increasingly behind private consumption to the detriment of economic welfare; this is in part what lies behind Galbraith's quip concerning 'private opulence and public squalor'.[4] On the other hand, some have argued that the public sector in market economies, under the influence of pressure group activity, has a tendency to grow too big (James Buchanan, 1972[5]). Ultimately, governments may be tempted to supply goods that the market could otherwise supply, or government activity may result in the crowding out of productive private investment. More formally, there has been much empirical and theoretical work aimed at understanding the size and scope of government, see, for instance, Alesina and Perotti (1995), and Drazen (2000).

1.1.1. The Golden Rule of Public Finance

In this paper we focus on a simple class of fiscal rules among which is the Golden Rule of Public Finance, where government borrowing is constrained for investment purposes only. In other words, the government cannot borrow to boost its nondurable consumption. Ultimately, fiscal rules of the sort analysed here have two effects. First, they may constrain the overall size of the public sector as measured by the sum of government spending. In our set up the government supplies

[3] See also the discussion in Atkinson and Stiglitz (1980).

[4] See, for example, J. K. Galbraith, The Affluent Society (1987) or Economics and the Public Purpose (1974) for an articulation of this type of concern. That said, to our knowledge Galbraith has never recommended fiscal rules of the sort analysed in this paper.

[5] Consider the following remarks by Buchanan: '... If political reality is recognized at all (and it seldom is by academic scribblers), surely it suggests the strong bias of fiscal policy toward the creation of budget deficits rather than budget surpluses. Governments, that is to say, politicians, faced with any sort of responsive citizenry or electorate, are surely cognizant of two powerful and ever-present forces. Constant pressure is exerted upon them to reduce (not to increase) the level of taxes, and, at the same time, to expand (not to reduce) both the range and the extent of the various public services. ...

Both of the dominant pressure groups, the tax reducers and the expenditure expanders, direct their fire at the politicians, who must, other things equal, respond (otherwise, they will not remain politicians for long). These pressures assume especial importance in an economy where tax rates are already prohibitively high in the view of many people, and where ever-expanding public spending programmes have been firmly "built in" to the structure of expectations. ...'

a non-durable, non-rival and non-excludable consumption good, and a rival but non-excludable investment good. Second, by changing the social cost of a unit of public investment the GRPF, ceteris paribus, reduces the amount of the investment good necessarily below the optimal level. However, the implications for growth and welfare of our class of fiscal rules are not clear cut. For example, if government spending on the non-durable, non-rival and non-excludable consumption good is sufficiently high, the imposition of the GRPF may result in a sufficiently low tax rates such as to compensate for the lower level of the non-excludable investment good (an input in the production technology). However, the lower level of the government investment good will also have a direct effect on growth via the interest rate. Tracing out how these complicated general equilibrium effects work themselves out in this and the other cases we analyse is a main contribution of this paper.

On the issue of fiscal rules, Greiner and Semmler (2000) is an important paper, which investigates long-run growth performances under alternative budgetary regimes (in particular, the golden rule of public finance), in an endogenous growth model with public capital and public debt. They show that the growth effects of an increase in public investment depend on the exact budgetary regime the government operates within. In particular, they demonstrate that less strict budgetary regimes do not necessarily imply higher rates of long-run growth. These authors do not, however, analyse the welfare implications of such budgetary rules. Ghosh and Mourmouras (2004), who extend the Greiner and Semmler framework to include welfare analysis, have studied precisely this aspect. They demonstrate analytically that welfare-maximising fiscal rules differ, depending on whether or not government borrowing is earmarked to finance only productive public spending, and this is in line with the Greiner and Semmler result of growth effects depending on which particular regime is in place.

However, these papers set to one side two important issues. First, how do these simple rules affect growth and welfare when congestion effects are present? Second, in the face of what distortions does a fiscal rule (like the GRPF) make sense? Turning to the first question, we first need to establish why it is important to study congestion-type effects in the first place, and what the prominent effects of fiscal policies in such settings are. As argued by Barro and Sala-i-Martin (1995), virtually all public services – including perhaps national defence – are characterised by some degree of congestion. The classic article by Barro and Sala-i-Martin (1992) demonstrates that income taxation operates as a user fee for rival but non-excludable public goods and prevents the growth rate from being too high, something that lump-sum taxation cannot achieve. Turnovsky (1997) captures congestion effects in a model with public capital, and characterises the transitional dynamics under alternative fiscal policies. He also derives a time-varying income tax that could enable the decentralised economy to replicate both the short- and long-run behaviour achievable under a social planner. Fisher and Turnovsky (1998) show how the effect of government investment on private capital formation involves a trade-off between the degree of substitution between private and public capital in production and the degree of congestion. Here, neither lump-sum nor distortionary tax financing of public investment is optimal.

As is clear from above, an important aspect of endogenous growth theory over the years has been the study of how fiscal variables, both on the expenditure and revenue sides, affect the long-run growth rate of an economy. On the expenditure side, Barro (1990) shows how the presence of productive public services, as an input in the production function, can affect steady state growth. This seminal paper by Barro (1990) considers the flow of public services rather than the stock of public capital. Futagami, Morita and Shibata (1993) consider the latter, and demonstrate the existence of a unique steady growth equilibrium with private and public capital. They also analyse the transitional dynamics of their model. Departing from the balanced budget set-up of the two papers cited above, Bruce and Turnovsky (1999), considering an array of fiscal instruments, identify the conditions under which a tax cut (by itself, or with accompanying expenditure cuts) can improve the long-run government budget balance. They do not, however, focus on the impact of aggregative fiscal rules, despite the growing prominence of such rules in policy debates. This is what we aim to do in this paper, within a Brice and Turnovsky (1999)-type framework.

1.2. Outline of Paper

In the next section we set out a baseline model with conventional assumptions as regards the preferences of agents, and with congestion effects in the production technology, as in Barro and Sala-i-Martin (1992). We then examine the equilibrium paths of the model's key variables and calculate the present value of utility under a number of scenarios. First, we analyse the decentralised equilibrium when the purpose of government borrowing is unconstrained. Second, we examine what happens when the fiscal authorities are constrained to borrow only for productive purposes (that is to boost the level of the non-excludable investment good). Finally, we compare these outcomes to the social optimum.

In section 3 we analyse the situation when the decentralised equilibrium is characterised by excessive private consumption. Again we compare this outcome to that when the government follows a fiscal rule, and under the social optimum. In section 4 we examine an alternative scenario when the decentralised equilibrium is characterised by excessive government supply of the non-durable, non-rival and non-excludable consumption good. In section 5 we look at the case where the decentralised equilibrium features underinvestment. (in the private stock of capital) as compared with the social optimum, and the baseline case of congested production technology. Section 6 sums up our key results and concludes.

2. Basic Model with Congested Technology

We start off with a conventional preference and production set up in order to establish some baseline results. There are a large number of individuals in this economy, say n. The size of the population, for simplicity, is fixed for all time at this number. The present value of utility, V, and the flow utility, U, for the i^{th} individual are given by,

$$V = \int_0^\infty U(c, G_c)e^{-\rho t}dt; \quad \rho > 0;$$

$$U = \frac{(c^{1-\eta}G_c^\eta)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, 0 < \eta < 1. \tag{2.1}$$

Here c denotes per capita consumption, G_c denotes consumption of the government supplied public good, and ρ is the rate of time preference. Generally we adopt the convention that $X = nx$, where lower case letters denote per capita values while upper case denote economy-wide aggregates. The budget constraint for this individual is

$$\dot{k} + \dot{b} = (1-\tau)(y + rb) - c. \tag{2.2}$$

A dot above a variable denotes a time derivative. k denotes the capital stock, b denotes government bonds, τ is the tax rate, y is output, and r is the interest rate. The production technology is described by

$$y = Ak(G_I/K)^\beta, \quad \beta > 0. \tag{2.3}$$

The production function captures the sense in which there is congestion in public services, as in Barro and Sala-i-Martin (1992). G_I is aggregate government expenditure on the investment goods. Hence, since it is G_I/K that appears in the production function, the public investment good is rival but not excludable; we cannot preserve a portion of G_I for our own use (hence non-excludable), and the higher our capital stock is relative to another producer, the smaller are the facilities available for production to that producer (hence rival). As a consequence of this, we may think of G_I as both the per capita level of government expenditure on the investment good, and the aggregate level of this expenditure. We may also consider G_c, which is non-rival *and* non-excludable, to be a per capita and aggregate quantity.

2.1. The Decentralised Outcome

Throughout the paper we shall assume that initial debt is positive, $nb(0) > 0$. We assume that the usual transversality conditions with respect to bonds and capital hold. The optimality conditions of the representative household maximising (2.1) subject to (2.2) and (2.3) include:

$$(1-\eta)c^{(1-\eta)(1-\sigma)-1}G_c^{\eta(1-\sigma)}e^{-\rho t} = \lambda; \tag{2.4}$$

$$\dot{\lambda} = -\lambda(1-\tau)\frac{\partial y}{\partial k}; \tag{2.5}$$

$$\dot{\lambda} = -\lambda(1-\tau)r. \tag{2.6}$$

In an appendix we set out the optimisation problem in detail and indicate why λ is related to both the dynamic behaviour of bonds and capital. The decentralised interest rate is related to the evolution of the costate variable as follows

$$r = \frac{\partial y}{\partial k} = -\frac{\dot{\lambda}}{\lambda(1-\tau)}, \tag{2.7}$$

where we have that,

$$\frac{\partial y}{\partial k} = A(G_I/K)^{\beta}. \tag{2.8}$$

In a symmetric equilibrium we then have that

$$r = An^{-\beta}k^{-\beta}G_I^{\beta}. \tag{2.9}$$

Balanced growth is derived by taking logs of equation (2.4) and differentiating with respect to time:

$$\frac{\dot{\lambda}}{\lambda} = [(1-\eta)(1-\sigma)-1]\frac{\dot{c}}{c} + \eta(1-\sigma)\frac{\dot{G}_c}{G_c} - \rho. \tag{2.10}$$

Along the balanced growth path $(\dot{c}/c) = (\dot{G}_c/G_c)$. Hence, from (2.5) and (2.6) we may write

$$\begin{aligned} \phi &= \frac{(1-\tau)r - \rho}{\sigma}; \\ &= \frac{(1-\tau)An^{-\beta}k^{-\beta}G_I^{\beta} - \rho}{\sigma}, \end{aligned} \tag{2.11}$$

where $\phi \equiv \dot{c}/c = \dot{G}_c/G_c$. To make progress on this expression, we shall need to solve for the equilibrium behaviour of G_I. We do this under various assumptions about the constraints that impinge on the government's behaviour. The first regime we analyse is one where the government chooses the fiscal variables to maximise the utility of a representative agent, respecting the agent's and its own flow budget constraints. We label this the DGBC regime (standing for the dynamic government budget constraint). The second regime we analyse is one where the Golden Rule of Public Finance (GRPF) is in place. This rule in effect places an additional restriction on the government such as to constrain the level of G_c at some arbitrary level. Finally we compare the outcome under these decentralised equilibria with the outcome under the social optimum and enquire whether the GRPF is welfare enhancing or not.

2.1.1. The Benevolent Government's Problem under the DGBC

We note that the flow budget constraint, at the economy-wide level, is:

$$\dot{B} = rB + G_c + G_I - \tau(Y + rB). \tag{2.12}$$

This expression, in per capita terms, enters the maximisation of the benevolent government. The Hamiltonian may then be written as

$$\begin{aligned} H = (1-\sigma)^{-1}\left(c^{1-\eta}G_c^{\eta}\right)^{1-\sigma} e^{-\rho t} + \lambda\left[(1-\tau)(y+rb) - c\right] \\ + \mu\left[rb + G_I + G_c - \tau(y+rb)\right], \end{aligned}$$

where the two side constraints may be combined to show that when aggregated up, $Y = \dot{K} + G_c + G_I + C$, as we require of any fiscal plan. As we show below at an optimum $\lambda = -\mu$, so that the government's plan does indeed respect the economy-wide budget constraint. The optimality conditions are

$$\eta c^{(1-\eta)(1-\sigma)} G_c^{\eta(1-\sigma)-1} e^{-\rho t} = -\mu; \tag{2.13}$$

$$\mu + \lambda(1-\tau)\frac{\partial y}{\partial G_I} = \mu\tau\frac{\partial y}{\partial G_I}; \tag{2.14}$$

$$-\lambda(y + rb) = \mu(y + rb). \tag{2.15}$$

Since $\lambda = -\mu$ implies $(\partial y/\partial G_I) = 1$,we find that

$$G_I = \beta Y; \tag{2.16}$$

$$G_c = \frac{\eta}{1-\eta}C. \tag{2.17}$$

(2.13) can be manipulated as we did above using (2.4) to yield an expression for the balanced growth path of the economy. Since this remains a decentralised equilibrium we continue to find:

$$\phi = \frac{(1-\tau)Ak^{-\beta}\beta^{\beta}y^{\beta} - \rho}{\sigma}, \tag{2.18}$$

where we have used (2.16) to substitute out for G_I.

2.1.2. The Benevolent Government's Problem under the GRPF

Under what we here call the 'golden rule of public finance' (GRPF) we constrain the government such that it can only borrow for productive spending purposes, i.e., to boost the supply of G_I. The quantity $G_c + rB$ must then be met out of period taxation. If we assume that $G_c + rB$ does not exhaust all the period tax take then it follows that

$$G_I = (1-\theta)\tau(Y + rB) + \dot{B}, \tag{2.19}$$

where $0 < \theta < 1, \dot{B} \geq 0$. Our formulation of the GRPF follows Ghosh and Mourmouras (2004), which in turn follows the formulation in Greiner and Semmler (2000). It is motivated directly by the institutional arrangements in Germany and the UK. The Hamiltonian for this problem is

$$H = (1-\sigma)^{-1}\left(c^{1-\eta}G_c^{\eta}\right)^{1-\sigma}e^{-\rho t} + \lambda\left[(1-\tau)(y+rb)-c\right] + \nu\left[G_I - (1-\theta)\tau(y+rb)\right].$$

We note that we may not now regard G_c as a choice variable. The equilibrium value of bond holdings (as captured by the equilibrium value of the costate variable), and the equilibrium value of r (as determined by the production technology), combine with θ (which is parametrically given) and means that G_c is effectively determined by residual. In this case, we find that $\lambda = -\nu(1-\theta)$. It follows then ,

$$G_I = \beta(1-\theta)Y, \tag{2.20}$$

and

$$G_c = \theta\tau(Y + rB) - rB. \tag{2.21}$$

Along the balanced growth path, as before $(\dot{c}/c) = (\dot{G}_c/G_c)$, we have that

$$\phi = \frac{(1-\tau)(1-\theta)^{\beta}Ak^{-\beta}\beta^{\beta}y^{\beta} - \rho}{\sigma}. \tag{2.22}$$

The key distortion associated with the GRPF is that the social cost of a unit of public investment is higher under the GRPF as opposed to the DGBC (compare (2.20) with (2.16)). Under the GRPF a higher θ is associated with a higher marginal social cost as it implies higher (non-productive) current spending. When $\theta = 0$, the social cost is the lowest $(= 1)$ because all spending by the government is for productive purposes. Next, comparing the GRPF (with $\theta = 0$, where all spending is for productive purposes) with the balanced budget case of Barro (1990) with only productive spending, the social cost is 1 for both, as should intuitively be the case. In terms of social cost, the 'problem' with the GRPF is that (unless $\theta = 0$), the government is earmarking some expenditure for non-productive purposes at the outset, whereas under the DGBC, it is not. We note, however, that this does not mean that G_c cannot be higher under the DGBC, ex post. This will be apparent in the simulations that we report below. This also does not imply that the optimal value of θ is zero, since G_c enters utility directly and so the marginal utility of G_c rises as G_c falls. It is also intuitively clear that $\theta = 1$, is also not optimal as that implies a zero level of output. Consequently, optimal θ lies in the open unit interval.

2.2. The Social Optimum

We now compare the above outturns with the result of a social planning optimum. The Hamiltonian for this problem is

$$H = (1-\sigma)^{-1} \left(C^{1-\eta} G_c^{\eta}\right)^{1-\sigma} e^{-\rho t} + \lambda_p \left[Y - C - G_c - G_I\right]. \tag{2.23}$$

The optimality conditions include:

$$(1-\eta) C^{(1-\eta)(1-\sigma)-1} G_c^{\eta(1-\sigma)} e^{-\rho t} = \lambda_p; \tag{2.24}$$

$$\eta C^{(1-\eta)(1-\sigma)} G_c^{\eta(1-\sigma)-1} e^{-\rho t} = \lambda_p. \tag{2.25}$$

The preceding equations imply that

$$G_c = \frac{\eta}{1-\eta} C. \tag{2.26}$$

$$\lambda_p \frac{\partial Y}{\partial G_I} - \lambda_p = 0; \tag{2.27}$$

Combining (2.27) with aggregate production function ($Y = AK^{1-\beta} G_I^{\beta}$) yields

$$G_I = \beta Y. \tag{2.28}$$

$$\dot{\lambda}_p = -\lambda_p \frac{\partial Y}{\partial K}; \tag{2.29}$$

The interest rate is given by

$$r = (1-\beta) A \beta^{\beta} y^{\beta} k^{-\beta}.$$

Balanced growth is derived from (2.24) or (2.25) and using our expression for the interest rate and optimal G_I we find that

$$\phi_{SO} = \frac{(1-\beta) A k^{-\beta} \beta^{\beta} y^{\beta} - \rho}{\sigma}. \tag{2.30}$$

We compare the balanced growth rate under the social optimum (ϕ_{SO}) with the out turn under the DGBC (ϕ_{DGBC}) and the GRPF (ϕ_{GRPF}):

$$\phi_{DGBC} = \frac{(1-\tau) A k^{-\beta} \beta^{\beta} y^{\beta} - \rho}{\sigma}; \tag{2.31}$$

$$\phi_{GRPF} = \frac{(1-\tau')Ak^{-\beta}\beta^{\beta}(1-\theta)^{\beta}y'^{\beta} - \rho}{\sigma}. \tag{2.32}$$

We find that growth under the social planner is higher than the other regimes so long as the following conditions obtain. If the capital share is larger than the tax distortion

$$(1-\beta) > (1-\tau), \tag{2.33}$$

then we find that $\phi_{SO} > \phi_{DGBC}$. Furthermore, if it is also the case that

$$(1-\theta) > \left(\frac{1-\tau}{1-\tau'}\right)^{\frac{1}{\beta}} \frac{y}{y'}, \tag{2.34}$$

then we find that

$$\phi_{SO} > \phi_{GRPF} > \phi_{DGBC}. \tag{2.35}$$

This ordering of the growth rates is plausible but by no means inevitable. If the debt level under the DGBC is substantially higher than under the GRPF, then taxes will need to be somewhat higher to repay the debt plus interest payments on the outstanding stock of debt. Moreover, the GRPF, insofar as it constrains public consumption, ought to reduce taxes which are the only source of financing such spending. Besides, the fact that the public investment to output ratio is smaller under the GRPF should also have an affect in lowering taxes. Indeed, numerical simulations of the model suggest that the ordering in (2.35) is the likely ordering, as we shall see below.

2.3. Simulations and discussion of results

Table 1 displays the results of some simulations of the above model economies (the appendix explains how we constructed these numerical solutions). As compared with the case of the DGBC and the Social Optimum, the level of productive investment is somewhat lower under the GRPF, which implies for a given capital stock a somewhat lower level of period output. This lower level of G_I is consistent with a higher level of private consumption, ceteris paribus, but can imply a sharp contraction in G_C, as in Table 1, suggesting offsetting implications for utility. The smaller size of the public sector under the GRPF may imply lower period taxation, and higher equilibrium growth. But a lower level of G_I implies a lower real interest rate. Table 1 displays, for a particular parameterization, how these various factors play out.

Note that even though the level of output is the same under the social optimum (SO) and the DGBC (factor inputs are the same – capital is inherited from the last period while the optimal level of G_I is the same in both set-ups), the rate of interest differs. Under the DGBC, people do not take into account the fact that their individual production decisions create congestion effects, which explains the higher real interest rate, while under the social planner, internalisation of the congestion effect causes a lower interest rate, but a higher long-run growth rate. (Under the DGBC, distortionary taxation scales down the growth effect considerably, as the numbers demonstrate.) Private consumption is consequently lower under the SO, but still the present discounted value of utility is higher under the planner because of the higher growth rate.

Table 1. Congested Production Only

	DGBC	GRPF	Social Optimum
Output	2.873	2.354	2.873
Private Consumption	1.172	1.237	1.022
Government Consumption	0.293	0.042	0.256
Government Investment	0.718	0.324	0.718
Real Interest Rate	0.144	0.118	0.108
Tax Rate	0.380	0.192	
Growth Rate	0.035	0.038	0.044
Value Function	-20.659	-27.603	-20.215

Parameter values/initializations: $A = 0.33, \beta = 0.25, \sigma = 2, \eta = 0.2, \rho = 0.02, \theta = 0.45, K(0) = 20$ and $B(0) = 1.5$.

In this set-up, the GRPF can actually deliver a higher growth rate (which is closer to the SO) than under the DGBC, so long as the ratio of current spending to taxes (θ) is not too high. Given our other parameters/initial values, even a value of θ close to 0.60 can result in a growth rate that is higher than under the DGBC. This is because of the much lower government consumption[6] and tax rate implied by the GRPF. Clearly, the value of θ has a bearing on the present discounted value (PDV) of utility through its impact on C and G_C. A higher value of θ will tend to increase utility directly by raising, ceteris paribus, G_C (although it also crowds out private consumption). A higher value of θ also reduces output below what it would otherwise have been, by reducing G_I, and it also acts directly to lower the growth rate, by lowering the rate of interest. However, since $G_c = \theta\tau(Y + rB) - rB$, this implies a somewhat lower level of taxes, and growth ends up higher under the

[6]Note that the link between C and G_C as given by eq. (2.17) no longer exists, and G_C in general rises with θ.

GRPF than under the DGBC.

While the numbers presented in the table show that welfare is higher under the DGBC than under the GRPF, it is theoretically possible to have the opposite for certain values of θ. One thing that emerges quite clearly from the numbers for the model with congestion in production is that under the GRPF, for a large range of plausible values for θ, the real interest rate and growth rate are closer to the SO than under the DGBC.

3. A Simple Model of Excessive Private Consumption

As we indicated in the introduction, the public finance literature suggests majority voting can lead to a level of public good provision either systematically above or below the social optimum level. In practice this means the sum of marginal utilities will differ from that implied under the 'Samuelson Rule'. In our set up we model this as a deviation in the marginal rate of substitution from what it would be under the social optimum. For a given level of (G_c/c), we have that

$$MRS_c < MRS_{SO},$$

where MRS_c denotes the marginal rate of substitution in the excess private consumption case, and MRS_{SO} denotes the marginal rate of substitution in the social optimum. We therefore replace (2.1) with

$$U = \frac{\left(c^{1-\eta+\gamma} g(G_c/C) G_c^{\eta-\gamma}\right)^{1-\sigma}}{1-\sigma},$$

where

$$g(G_c/C) \equiv (G_c/C)^{\gamma}.$$

It follows then that

$$\begin{aligned} MRS_{SO} &= -\left(\frac{1-\eta}{\eta}\frac{G_c}{c}\right); \\ MRS_c &= -\left(\frac{1-\eta+\gamma}{\eta-\gamma}\frac{G_c}{c}\right). \end{aligned}$$

With these preliminaries in place, we now investigate the impact of the GRPF in a situation where underlying preferences result in excessive private consumption. In practice, to do this we replace (2.1) with

$$U = \frac{\left(c^{1-\eta+\gamma} (G_c/C)^{\gamma} G_c^{\eta-\gamma}\right)^{1-\sigma}}{1-\sigma}. \tag{3.1}$$

This utility function will generate a decentralised equilibrium, under benevolent government, where *private* consumption is excessive.[7] In such an environment we might not expect to see beneficial results from fiscal rules where the ultimate aim is to constrain the government in some way. One may think this since it will be private behaviour, not the government's behaviour, that is different from the outcome under the social optimum. However, that intuition may not go through. We are in a second-best world and as the previous simulation results demonstrated the GRPF may act to reduce private consumption.

3.1. The Decentralised Outcome

The representative agent's optimality conditions in this case include

$$(1-\eta+\gamma)c^{-\sigma-(1-\sigma)(\eta-\gamma)}(G_c/C)^{\eta(1-\sigma)}G_c^{(1-\sigma)(\eta-\gamma)}e^{-\rho t}=\lambda; \tag{3.2}$$

$$\lambda(1-\tau)\frac{\partial y}{\partial k}=-\dot{\lambda}; \tag{3.3}$$

$$\lambda(1-\tau)r=-\dot{\lambda}. \tag{3.4}$$

Once again, these last two equations can be combined to yield an equation for the interest rate that we may use when we calculate the balanced growth path from the optimality condition for consumption,

$$r=-\frac{1}{1-\tau}\frac{\dot{\lambda}}{\lambda}. \tag{3.5}$$

An expression for balanced growth is found using (3.2) and the interest rate. Since $y=Ak(G_I/K)^{\beta}$, we have that $r=An^{-\beta}k^{-\beta}G_I^{\beta}$, so

$$\phi=\frac{(1-\tau)An^{-\beta}k^{-\beta}G_I^{\beta}-\rho}{\sigma}. \tag{3.6}$$

[7] The following quote from Galbraith is a somewhat colourful exposition of these types of concerns. "The family which takes its mauve and cerise, air-conditioned, power-steered and power-braked automobile out for a tour passes through cities that are badly paved, made hideous by litter, blighted buildings, billboards and posts for wires that should long since have been put underground. They pass on into a countryside that has been rendered largely invisible by commercial art...They picnic on exquisitely packaged food from a portable icebox by a polluted stream and go on to spend the night at a park which is a menace to public health and morals. Just before dozing off on air mattress, beneath a nylon tent, amid the stench of decaying refuse, they may reflect on the curious unevenness of their blessings." (The Affluent Society, page 192).

Once we find the optimal G_I we can clarify the implications of this expression further.

3.1.1. The Benevolent Government's problem under the DGBC

In this case we envisage a benevolent government choosing government expenditure and taxation such as to maximise the representative agent's utility, as we did above. In this case the we find that, at an optimum, we have:

$$(\eta-\gamma)c^{(1-\sigma)(1-\eta+\gamma)}(G_c/C)^{\eta(1-\sigma)}G_c^{(\eta-\gamma)(1-\sigma)-1}e^{-\rho t}+\mu=0; \tag{3.7}$$

$$\lambda(1-\tau)\frac{\partial y}{\partial G_I}+\mu-\mu\tau\frac{\partial y}{\partial G_I}=0; \tag{3.8}$$

$$-\lambda(y+rb)-\mu(y+rb)=0. \tag{3.9}$$

These are combined to yield, at the aggregate level

$$G_I=\beta Y; \tag{3.10}$$

$$G_c=\frac{\eta-\gamma}{1-\eta+\gamma}. \tag{3.11}$$

Equation (3.11) reveals a key result: private consumption is excessive when compared to (2.17), and as compared with the social optimum for this economy. The balanced growth rate of the economy can be found in the usual way:

$$\phi=\frac{(1-\tau)Ak^{-\beta}\beta^{\beta}y^{\beta}-\rho}{\sigma}. \tag{3.12}$$

3.1.2. The Benevolent Government's Problem under the GRPF

In this section we employ the GRPF, as in section 2, to analyse the implications for growth and welfare. As before θ pins down, in effect, the feasible level of G_c, which is now no longer a choice variable for the government. We find that

$$-\nu(1-\theta)\tau\frac{\partial y}{\partial G_I}+\nu+\lambda(1-\tau)\frac{\partial y}{\partial G_I}=0; \tag{3.13}$$

$$-\nu(1-\theta)(y+rb)-\lambda(y+rb)=0. \tag{3.14}$$

Since $\lambda = -\nu(1-\theta)$, we have that

$$G_I = \beta(1-\theta)Y', \tag{3.15}$$

and

$$G_c = \theta\tau(Y' + rB) - rB. \tag{3.16}$$

Balanced growth is different to the decentralised solution under the DGBC since

$$r = A(1-\theta)^\beta \beta^\beta k^{-\beta} y'^\beta. \tag{3.17}$$

It follows then that,

$$\phi = \frac{(1-\tau')A(1-\theta)^\beta \beta^\beta k^{-\beta} y'^\beta - \rho}{\sigma}. \tag{3.18}$$

3.2. The Social Optimum

The results under the social optimum are as before. The optimality conditions include:

$$G_I = \beta Y, \tag{3.19}$$

and

$$G_c = \frac{\eta}{1-\eta} C. \tag{3.20}$$

Since the social planner internalises all externalities in this economy, the production technology is described by $Y = AK^{1-\beta}G_I^\beta$. The interest rate is then given by

$$r = (1-\beta)AG_I^\beta n^{-\beta} k^{-\beta}. \tag{3.21}$$

Balanced growth is derived in the usual way. We compare the the balanced growth rates under the social optimum, the GRPF and the DGBC, as we did above.

$$\phi_{SO} = \frac{(1-\beta)Ak^{-\beta}\beta^\beta y^\beta - \rho}{\sigma}; \tag{3.22}$$

$$\phi_{GRPF} = \frac{(1-\tau')(1-\theta)^\beta Ak^{-\beta}\beta^\beta y'^\beta - \rho}{\sigma}; \tag{3.23}$$

$$\phi_{DGBC} = \frac{(1-\tau)Ak^{-\beta}\beta^\beta y^\beta - \rho}{\sigma}. \tag{3.24}$$

The same basic considerations seem to apply here, as before in section 2. The important, and perhaps surprising, thing to note here is that although the GRPF appears to be addressing the 'wrong' problem (it is constraining G_c when it is c that is excessive), it nevertheless has the ability to deliver a higher growth rate, as before, because it makes possible a lower level of distortionary taxes. However, as before, this higher growth rate need not be informative as to the welfare rankings of these fiscal regimes, as the simulation results in the next section make clear.

3.3. Simulations and discussion of results

Comparing the numbers in Table 2 with those in Table 1 for the case where the DGBC is in place, it is clear that private consumption is higher and this is quite intuitive, as in this case agents do not take into account the fact that if they increase their individual consumption (c) out of the public good (G_C), then this increases the overall (C/G_C) ratio for the economy. So C is higher and G_C lower than the benchmark case of Table 1. The nature of the congestion on the production side remains the same as in Table 1, which gives rise to identical real rates of interest in the two cases. Given this (together with identical Y and G_I and lower G_C), the tax rate has to be lower in Table 2, which implies that the growth rate is higher. The values with the GRPF in place will, of course, remain unchanged from Table 1, but it is clear from the numbers that if there is congestion in consumption as well as in production, then this gives rise to values of C, G_C, τ and ϕ that are closer to the GRPF numbers than where there is congestion in production alone. The PDV of utility could go either way. Where the DGBC is in place, C is higher and G_C lower in Table 2 than in Table 1. This means that the values of U and ϕ are closer to those attained under the GRPF[8].

Table 2. Congested Production and Excess Private Consumption ($\gamma = 0.1$)

[8] The possibility of the two fiscal regimes delivering very similar outcomes is clearly enhanced when we have congestion in both production and consumption.

	DGBC	GRPF
Output	2.873	2.354
Private Consumption	1.252	1.237
Government Consumption	0.139	0.042
Government Investment	0.718	0.324
Real Interest Rate	0.144	0.118
Tax Rate	0.329	0.192
Growth Rate	0.038	0.038
Value Function	-21.299	-27.603

See Table 1 for additional parameter settings

4. A Simple Model of Excessive Public Consumption

The utility function of the previous section was intended to capture what many might argue is a risk in modern economies; private agents not internalising all the implications of their consumption plans. One important upshot of this was that, relative to the social optimum, we encountered excessive private consumption.

However, a natural question to ask is how the economy behaves in the presence of the GRPF when the decentralised equilibrium is characterised by excessive public consumption, i.e., excessive G_c. In other words, here the GRPF not only may allow higher growth via lower distortionary taxation, but may also address directly an externality. In this section we work a utility function which implies the decentralised equilibrium is characterised by excessive G_c, for which it is the case that

$$MRS_{G_c} = -\left(\frac{1-\eta-\gamma}{\eta+\gamma}\frac{G_c}{c}\right) > MRS_{SO}.$$

Figure 8.1 displays the differing $MRS's$ in the present case, the case of excessive private consumption and under the social optimum. The utility function is written as

$$g(G_c/C) \equiv (G_c/C)^{-\gamma}$$

$$\begin{aligned} U &= \frac{(c^{1-\eta-\gamma}g(G_c/C)G_c^{\eta+\gamma})^{1-\sigma}}{1-\sigma}; \qquad (4.1) \\ g(G_c/C) &\equiv (G_c/C)^{-\gamma}. \end{aligned}$$

We assume that $1-\eta-\gamma>0$. This utility function basically implies that private agents underestimate the marginal utility of private consumption relative to that of the public good–the congestion effect goes in the opposite direction to the previous section.

The decentralised equilibrium is characterised by the following pair of relations:

$$G_c = \frac{\eta+\gamma}{1-\eta-\gamma}C; \tag{4.2}$$

$$G_I = \beta Y. \tag{4.3}$$

As suggested, the implication is that, relative to the social optimum, there is an excess of government consumption.

Under the GRPF the following pair of equilibrium relations obtain:

$$G_I = \beta(1-\theta)Y; \tag{4.4}$$

$$G_c = \theta\tau(Y+rB)-rB. \tag{4.5}$$

4.1. Simulations and discussion of results

Comparing Table 3 with Table 1, it is clear that as regards the DGBC case, excessive unproductive government spending shows up in higher G_C, lower C, higher τ, and lower ϕ in Table 3. Theoretically, the value function could go either way. In this case, the GRPF makes a bigger difference to the outcome in terms of reducing government consumption and the tax rate, as is expected of a restrictive fiscal rule in the presence of excessive public consumption. So the numbers clearly indicate that if a higher growth rate is the objective, then there is a strong case in favour of the GRPF when there is excessive G_C.

Table 3a. Congested Production and Excessive $\mathbf{G}_c$ ($\gamma = 0.1$)

	DGBC	**GRPF**
Output	2.873	2.354
Private Consumption	1.083	1.237
Government Consumption	0.464	0.042
Government Investment	0.718	0.324
Real Interest Rate	0.144	0.118
Tax Rate	0.438	0.192
Growth Rate	0.030	0.038
Value Function	-21.711	-27.603

See Table 1 for additional parameter settings

Table 3b. Congested Production and Excessive $\mathbf{G}_c$ ($\gamma = 0.6$)

	DGBC	**GRPF**
Output	2.873	2.354
Private Consumption	0.431	1.237
Government Consumption	1.723	0.042
Government Investment	0.718	0.324
Real Interest Rate	0.144	0.118
Tax Rate	0.860	0.192
Growth Rate	0.000	0.038
Value Function	-87.787	-27.603

See Table 1 for additional parameter settings

The larger is the value taken by γ, the stronger is the case for the GRPF. As regards the impact on welfare, the effects of higher ϕ and C attainable under the GRPF are balanced by the lower G_C, as such public services add to utility. But even here, a sufficiently large value of γ can generate quite small utility-values in the DGBC case, and this can be bettered under the GRPF regime. This is clear from Table 3b, where $\gamma = 0.6$ results in a utility value that is much lower than is achievable under the DGBC.

5. A Simple Model on Insufficient Private Investment

We considered – till section 3 – various cases where the benevolent government was choosing fiscal policy to maximise the welfare of the representative agent, and was in many cases not able to internalise the externalities, where there was congestion in production and some distortion to consumption, or both. Section 4 considered the case where the government was spending excessively on public consumption compared to what it would have spent, had the social planner undertaken the same activity. This provides a rationale for having the GRPF in the first place, as one of the key elements of the GRPF is to restrict the source of funding of government consumption to taxes alone. The analysis of what happens under the GRPF is carried out through simulations, as under the GRPF – with G_C not being a choice variable – we do not have the proportional relationship between G_C and C, that we have under the DGBC (and the social planner's outcome).

In this section, we deal with the other issue in the context of the DGBC: under what circumstances is there insufficient private investment under the benevolent government (when compared to the planning outcome), and can the GRPF bring public investment more in line with what the social planner would achieve? This situation is reflected in Figure 8.2 where the 'distorted' marginal rate of technical substitution is lower than in the baseline case of the congested production function. The marginal rate of technical substitution would lie in between these two cases, at the point of intersection. (IS THERE SOME FURTHER JUSTIFICATION THAT WE MAY OFFER HERE, AKIN TO THE VOTING STORY WE ALLUDED TO ABOVE?) First, we formulate a certain specification of production function with congestion which gives rise to the interest rate (i.e., marginal productivity of capital) being lower under the DGBC than under the planner. When we involve the GRPF, the first order condition for optimal public services (i.e., link between G_I and Y) remains the same as before, and the issue has to be sorted through simulations.

First, let us formulate a production function with congestion:

$$y = Ak^{1-\beta-\zeta}(G_I/K)^{-\zeta}G_I^{\zeta+\beta}, \quad 1-\beta-\zeta > 0. \tag{5.1}$$

This implies that the interest rate is given by:

$$r = (1-\beta-\zeta)Ak^{-\beta}G_I^{\beta}. \tag{5.2}$$

Clearly, since the social planner internalises the externalities, his production function would look like:

$$y = AK^{1-\beta}G_I^{\beta}. \tag{5.3}$$

The interest rate in this case is given by:

$$r = (1-\beta)AK^{-\beta}G_I^{\beta}. \tag{5.4}$$

Comparing (5.4) with (5.2), it is clear that the benevolent government, operating under the DGBC, achieves a lower marginal return to capital compared to the social planner. Note that in both cases, the first order condition for public services is given by $\partial Y/\partial G_I = 1$, but the interest rate differs because the production functions are different. Note that in Model 1, the individual production functiondiffers from the aggregate, and yet we have $(G_I/y)^*$ being the same in the DGBC regime as in the SO. However, here with similar production function characteristics (as regards disortions), we now have $(G_I/y)^*_{DGBC} > (G_I/y)^*_{SO}$.

Using $\partial Y/\partial G_I = 1$ in both cases gives us expressions for r that are as follows:

$$r = (1-\beta-\zeta)Ak^{-\beta}[(\beta+\zeta)y]^{\beta}, \tag{5.2'}$$

under the DGBC, and

$$r = (1-\beta)Ak^{-\beta}(\beta Y)^{\beta}, \tag{5.4'}$$

under the social planner.

Now, when we introduce the GRPF, the (congested) production function remains as in (5.1), but the government budget constraint must take into account the fact that borrowing (if any) is for public investment purposes only. Because of this, the optimality condition for public services becomes $\partial Y/\partial G_I = 1/(1-\theta)$. Consequently, the expression for the interest rate is:

$$r = (1-\beta-\zeta)Ak^{-\beta}[(\beta+\zeta)(1-\theta)y']^{\beta}. \tag{5.5}$$

Comparing (5.5') with (5.2'), it appears that r is lower in (5.5) than (5.2'). But this is not necessarily true, despite the fact that it is lower under the GRPF than under the DGBC (and under the social planner). The reason for this is that output is endogenous, and $y\prime$ could be higher or lower than y in equilibrium.

5.1. Simulations and discussion of results

First, comparing the DGBC with the SO, it is clear that the growth rate and the real interest rate are both lower under the DGBC, which is quite intuitive, given the 'underinvestment'. Mainly as a result of the considerably lower growth rate, the PDV of welfare is lower under the DGBC (although C and G_C are higher). Comparing the values for the different variables under the DGBC with those under congested production only, our benchmark case (Table 1), we find that ϕ is much further away from the SO in Table 4 than Table 1. Also, while r is higher than the SO value in Table 1, here it is lower. This too is intuitive, given that here there is underinvestment, while in the previous case there was too much private investment as agents were ignoring the congestion effects. As expected, the 'underinvestment' case generates higher private consumption than the 'overinvestment' case.

Table 4. Insufficient Private Investment

	DGBC	GRPF	Social Optimum
Output	3.495	2.864	2.873
Private Consumption	1.360	1.452	1.022
Government Consumption	0.340	0.446	0.256
Government Investment	1.573	0.709	0.718
Real Interest Rate	0.096	0.079	0.108
Tax Rate	0.561	0.420	
Growth Rate	0.011	0.013	0.044
Value Function	-31.183	-26.566	-20.215

$\zeta = 0.2$,See Table 1 for additional parameter settings

Comparing now the outcomes under the DGBC and the GRPF in Table 4, we find that in this case the GRPF seems to deliver a significantly better outcome, both in terms of growth and welfare. For a plausible value of θ equal to 0.30, the tax rate is lower, the growth rate is higher, consumption is higher and welfare higher than under the DGBC. The lower real interest rate (and consequently, lower interest payments on debt) under the GRPF enables the tax rate to be lower, and this leads to both a higher growth rate and higher current consumption. It is interesting to see that in the underinvestment case, the GRPF (for a wide range of values of θ) gives rise to values of the growth rate and welfare that are closer to the social optimum than is possible under the DGBC.

6. Conclusions

In this paper we attempted to analyse the effects of two different fiscal regimes on key macroeconomic variables within a framework of congestion in production and/or consumption services, and to relate these to the social optimum. First, we introduced congestion in production only, and then we added in distortions (relative to the social optimum) in consumption. Comparing outcomes under the DGBC it is clear that private consumption is higher and government consumption lower when there is congestion in production and distortions to consumption, than when there is congestion in production alone (the benchmark case). The GRPF regime can make a significant difference to the growth rate in the benchmark case, though the DGBC, by 'trading' more private for public consumption comes closer to the GRPF in terms of the effects on key macroeconomic variables. We then introduced the excessive government consumption case, and found that a restrictive fiscal regime like the GRPF can bring about a significant difference to the growth rate. Despite public consumption being in the utility function, the GRPF regime can, through a rise in private consumption, a fall in the tax rate and rise in the growth rate, bring about higher welfare in certain cases. Finally, the underinvestment case shows that the GRPF can perhaps make the biggest difference in terms of its effects on growth and welfare, and the values of the key variables are more likely to be closer to the social optimum than when the unrestricted DGBC is in place.

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7. Appendix: Optimisation Problem in Detail

In the text we noted that the same costate variable applied to the dynamic behaviour of bonds and capital. This appendix sets out the class of problems in which we are interested and clarifies this point regarding the costate variable. First, we analyse a slightly simpler example of the class of problems we are interested in, and then we turn to a specific example that appears in the text.

Consider the following maximisation problem:

$$\text{Max} \int_0^\infty u(c_t) e^{-\rho t} dt \tag{7.1}$$

subject to

$$f(k_t) = c_t + i_t \ \forall t, \tag{7.2}$$

where $i_t \equiv dk_t/dt$, k_0 is given, and $\lim_{T\to\infty} e^{-\rho t} \lambda_T k_T = 0$. The Lagrangian for this problem can be written as

$$J(t) = \int_0^\infty \left[u(c_t) + \lambda_t \left(f(k_t) - c_t - dk_t/dt \right) \right] e^{-\rho t} dt. \tag{7.3}$$

We optimise with respect to $c(t)$ and $k(t)$. Clearly, the complication is the term in dk/dt. We can simplify it, however. Take the term,

$$\int_0^\infty \lambda_t (dk_t/dt) e^{-\rho t} dt. \tag{7.4}$$

It follows that

$$\int_0^\infty \lambda_t (dk_t/dt) e^{-\rho t} dt = e^{-\rho t} \lambda_t \int_0^\infty i_t dt - \int_0^\infty k_t \left[\frac{d\lambda_t}{dt} - \rho \lambda_t \right] e^{-\rho t} dt. \tag{7.5}$$

Note that,

$$\int_0^T i_t dt = k_t|_0^T = k_T - k_0.$$

So, the first term on the right hand side of (7.5) may be written as

$$\lim_{T\to\infty} e^{-\rho T}\lambda_T k_T - e^{-\rho\cdot 0}\lambda_0 k_0.$$

The transversality condition 'knocks out' the first part of that expression, while k_0 is given by initial conditions. So (7.5) becomes

$$\int_0^\infty \lambda_t (dk_t/dt)e^{-\rho t}dt = -\lambda_0 k_0 - \int_0^\infty k_t \left[\frac{d\lambda_t}{dt} - \rho\lambda_t\right] e^{-\rho t}dt. \tag{7.6}$$

The Lagrangian then becomes,

$$J(t) = \int_0^\infty \left[u(c_t) + \lambda_t \left(f(k_t) - c_t\right) + k_t \left[\frac{d\lambda_t}{dt} - \rho\lambda_t\right]\right] e^{-\rho t}dt + \lambda_0 k_0. \tag{7.7}$$

Differentiating this expression with respect to c_t and k_t is more straightforward.

Now we turn to an version of the problem we encounter in the body of the paper:

$$J(t) = \int_0^\infty \left[u(c_t) + \lambda_t \left((1-\tau)(y+rb) - c - \dot{k} - \dot{b}\right)\right] e^{-\rho t}dt, \tag{7.8}$$

where we encounter two state variables. As before, the terms of 'annoyance' are

$$\int_0^\infty \lambda_t (dk_t/dt)e^{-\rho t}dt;$$

$$\int_0^\infty \lambda_t (db_t/dt)e^{-\rho t}dt.$$

However, following the above line of argument (7.8) may be written as

$$J(t) = \int_0^\infty \left[u(c_t) + \lambda_t \left((1-\tau)(y+rb) - c_t\right) + k_t \left[\frac{d\lambda_t}{dt} - \rho\lambda_t\right] + b_t \left[\frac{d\lambda_t}{dt} - \rho\lambda_t\right]\right] e^{-\rho t}dt$$
$$+ \lambda_0 k_0 + \lambda_0 b_0.$$

Hence, the same costate applies to both our state variables.

8. Numerical Solution of the Model

The models that we analyse in this paper have a simple structure (that is almost recursive) that we can exploit in our numerical calculations. First we note that from period zero onwards, the economy always lies on the balanced growth path. Consequently, once we have found the growth rate for the economy, and given $k(0)$ and $b(0)$, we can easily solve for the path of all of the prices and quantities in our model, reducing the model ultimately to two equations in two unknowns.

We consider the case of the decentralised equilibrium under the dynamic government budget constraint. The other cases that we analyse in the paper result in more or less straightforward changes to this example.

Given the capital stock, $k(0)$, and the fact that $G(t)_I = \beta y(t)$, we may calculate $y(t)$ and $r(t)$ using the production technology. We then find it useful to define

$$\phi' = \frac{A(t)k(t)^{-\beta}\beta^{\beta}y(t)^{\beta} - \rho}{\sigma}$$

such that

$$\phi = [(1-\tau(t))\phi' - (\rho/\sigma)\tau(t)].$$

Along the balanced growth path it follows that for a variable $X(t)$,

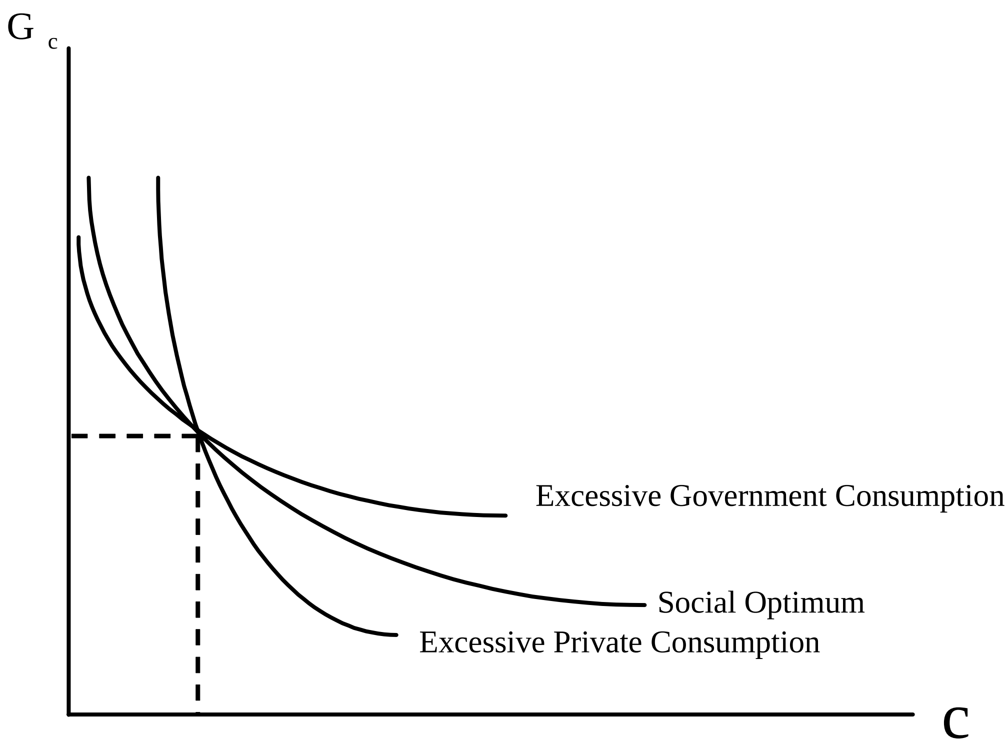
$$\dot{X}(t)/X(t) = [(1-\tau(t))\phi' - (\rho/\sigma)\tau(t)].$$

Hence, we may write the agent's budget constraint and the government's budget constraint as follows:

$$[(1-\tau(t))\phi' - (\rho/\sigma)\tau(t)](k(t)+b(t)) = (1-\tau(t))[y(t)+r(t)b(t)] - c(t), \quad (8.1)$$

$$[(1-\tau(t))\phi' - (\rho/\sigma)\tau(t)]b(t) = r(t)b(t) + (\eta/1+\eta)c(t) + \beta y(t) + \tau(t)[y(t)+r(t)b(t)], \quad (8.2)$$

where we have used $G(t)_I = \beta y(t)$ and $G(t)_c = (\eta/1+\eta)c(t)$. (8.1) and (8.2) comprise two (nonlinear) equations in two unknowns, $c(t)$ and $\tau(t)$ and are numerically easily solved.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Excessive Government Consumption
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Figure 8.1:

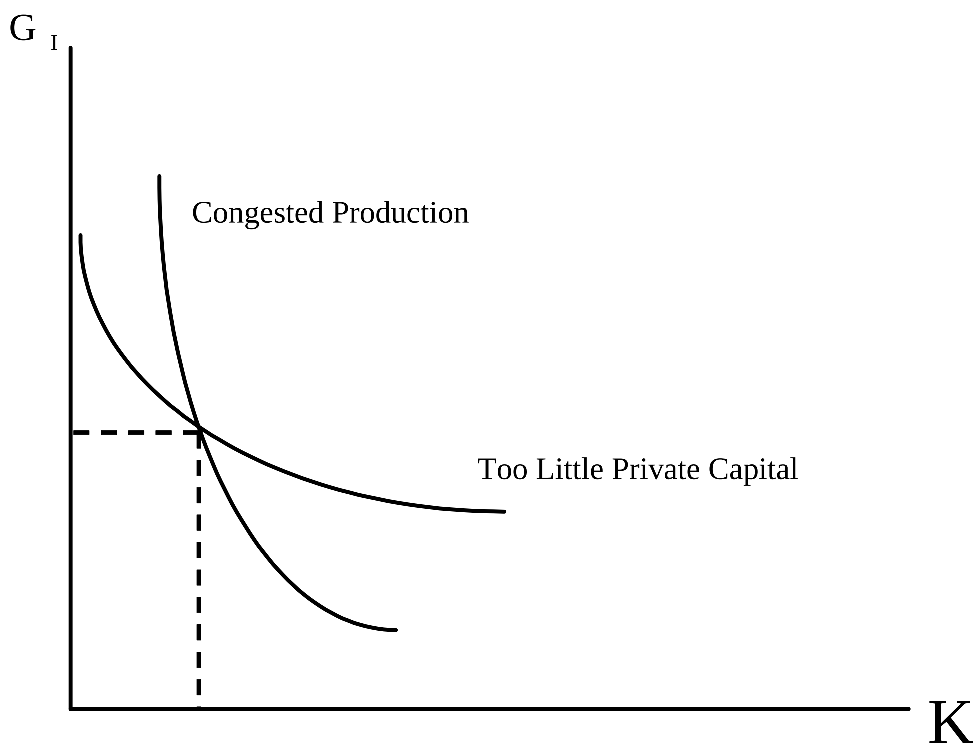

Figure 8.2: