Social Networks and Informal Insurance

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Abstract

VERY PRELIMINARY DRAFT.

This paper studies networks of informal insurance, and builds a model of risk-sharing which captures two basic characteristics. First, informal insurance is fundamentally *bilateral*, and rarely consists of an explicit arrangement across several people. Second, insurance is often based on *norms*. In the model studied here, only directly linked agents make transfers to each other, though they are aware of the (aggregate) transfers each makes to others. A *bilateral transfer arrangement* between two linked agents is viewed as a norm determining agent consumptions given their income realizations and the transfers made to or received from other agents. Based on these bilateral transfer arrangements between all pairs of agents, a *bilateral insurance scheme* is viewed as a fixed point of the resulting mapping.

With this setup as background, the paper then studies the stability of insurance networks, explicitly recognizing the possibility that the lack of commitment may destabilize insurance arrangements among the network. [These are the familiar self-enforcement constraints much studied in the literature, though not for networks.] We look at different punishment structures that determine the severance of links to a deviant. The weakest punishment scheme is one in which individuals break direct links only to agents who do not fulfill their obligations with respect to them. One can strengthen such schemes by asking that individuals who are connected directly to a deviant and are less than n links away from her victim, but not via the deviant, to also break links with the deviant, where n can be made progressively larger to capture wider information flows.

In such a framework, the density of links (as well as their specific placement in the network) has important consequences for network stability. In the equal sharing case, under strong punishment, we show that adding links always makes the network more stable, and that all decomposable networks have the same stability properties. However, when punishment is weaker, all fully decomposable networks continue to have the same stability properties, but adding new links can destabilize the network. Indeed, for high values of the discount factor, trees are the only stable network structures under weak punishment.

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1 Introduction

Informal risk-sharing arrangements exist in most developing countries – especially in rural areas where credit and insurance markets are scarce. In order to face income fluctuations due to a variety of exogenous factors, villagers enter into mutual insurance schemes, whose institutional details may vary from country to country. These institutions are often intertwined with the social networks of the community. It is now widely recognized that social networks (based on kin, gender or occupation) play a dominant role in the rural life of developing countries, and their neglect is a recurring criticism of development projects and policies.

The objective of this paper is to develop a model of risk-sharing network in order to explain the influence of social networks on informal insurance agreements and to derive endogenously the architecture of self-insurance networks. Most of the theoretical literature on mutual insurance schemes implicitly assumes either that institutions are based at the community level or among two individuals only. (Posner (1980), Kimball (1988), Coate and Ravallion (1993), Kocherlakota (1996), Kletzer and Wright (2000) and Ligon, Thomas and Worrall (2002)). Genicot and Ray (2003) go one step further, and suppose that informal insurance schemes may be formed by subgroups in the community. However, empirical evidence suggests that insurance schemes may in fact be designed at an even more disaggregated level, that social networks play an important role in the definition of mutual risk-sharing arrangements, and that agents with higher levels of social capital typically benefit from higher insurance possibilities. (See the studies by Fafchamps and Lund (2003) for the Philippines, de Weerdt (2000) and Dercon and de Weerdt (2000) for Tanzania and Murgai et al. (2002) for Pakistan).

Informal insurance arrangements typically suffer from the absence of enforcement by third parties. An individual agent cannot be forced to participate in the scheme and pay the transfers he is called to make. As a result, stable mutual insurance schemes must be *self-enforcing.* At no point must individuals called upon making a transfer have incentive to deviate and not make the transfer given that they will be punished by some sort of exclusion from the scheme in the future (and possibly other social exclusions). At the community level, the papers mentioned above suppose that a deviating agent is punished by being entirely barred from the scheme, and hence will have to bear all the fluctuations in income after a deviation. Genicot and Ray (2003) allow for group deviations, by which a subgroup of agents can still form a smaller mutual insurance scheme after the deviation, and find (rather unexpectedly) that this changes completely the picture, and that there is an upper bound on the size of the group which can form informal insurance arrangements. In the context of networks, the possibilities of punishment after a deviation are even more varied: an agent could only be punished by those agents to whom he has not transferred money, or by the entire community, or by any subset of agents in between.

In this paper, we study informal insurance networks, and builds a model of risk-sharing which captures two features. First, informal insurance in networks essentially results from a collection of *bilateral* arrangements rather than an explicit agreement across several people. In the model network studied here, only directly linked agents make transfers to each other, though they are aware of the (aggregate) transfers each makes to others. Linked agents have information only on each other's commitments, but not necessarily on the overall insurance scheme of the community. This assumption implies in particular that agents do not need to know the entire configuration of the network. Second, insurance is often based on internalized *norms* regarding mutual help. A *bilateral transfer arrangement* between two linked agents is viewed as a norm determining agent consumptions given their income realizations and the transfers made to or received from other agents. Based on these bilateral transfer arrangements between all pairs of agents, we define a *bilateral insurance scheme* as a fixed point of the resulting mapping. Examples of risk-sharing norms include *equal sharing*, in which each of a pair of linked agents receives the same consumption in all states, and *Nash bargaining*, in which consumptions are allocated to maximize the Nash product, given the outside options of each agent, which in turn is determined in a recursive way.

With this setup as background, the paper then studies the stability of insurance networks, explicitly recognizing the possibility that the lack of commitment may destabilize insurance arrangements among the network. Note that in the theoretical literature on norms within groups of agents or institutions, two different modelling approaches have been used. In the first one, as in this paper, institutions forms first and agents abide by the norms of that group, which may make that group unstable (Farrell and Scotchmer [1988], Hoff [1997]). Another possible approach would be to require the norm to preserve the stability of the network or group (such as in Dutta and Ray [1989]). However, evidence on informal insurance in traditional village communities (see Platteau [2004]) suggest that social norms are pervasive and relatively rigid so that traditional reciprocity networks can erode under the pressure of market integration and new opportunities.

We assess the stability of insurance networks under different possible punishment schemes that determine the severance of links to a deviant. The *weakest punishment* is one in which only individuals with respect to whom a deviant did not fulfill her obligations break links with her, and the best (from the deviant's viewpoint) stable subnetwork forms. One can strengthen such schemes by asking that individuals who are connected directly to a deviant and are n links away from her victim, but not via the deviant, to also break links with the deviant, where n can be made progressively larger to capture wider information flows. We call these punishment schemes *n-level gossips*. Finally, in the *strong punishment* case, a deviating agent believes that he will be excluded by the entire community, and receives after his deviation his autarchic allocation, bearing all income fluctuations alone.

In monotonic insurance schemes, only the size of the components he belongs to determines an individual's payoff. Looking at the sustainability of networks with monotonic insurance at high level of discount rate illustrates clearly the differences between the punishment schemes. For values of δ close to 1, all network are stable under strong punishment. In contrast, for weaker punishment schemes, the density of links (as well as their specific placement in the network) weakens punishments and has important consequences for network stability. In particular, for high values of the discount factor trees are the only stable networks.

For lower values of the discount rate, assessing the stability of mutual insurance schemes in the context of social networks is a difficult task. We do so assuming a specific risk-sharing norm: equal sharing – by which all agents divide equally income at every state. First, bilateral insurance schemes equalizing consumption across every component of the network always exist. Second, our analysis highlights two conflicting forces in the relation between the architecture of the network and the stability of insurance schemes. As transfers can only flow along the links in the network, in order to reduce agents' incentives to deviate from the insurance scheme, one ought to minimize the amount of transfers going through any particular agent. In particular, one could determine in any network the "bottleneck" agent as the agent who receives the highest amount of transfers in some state, and the enforcement constraint faced by this bottleneck agent defines the stability of the entire network. In the pessimistic beliefs case, this bottleneck effect is the only relevant feature of the network. We show that there is a class of "decomposable" trees (including stars and lines) for which the bottleneck effect is identical, and hence stability conditions are identical. Furthermore, the addition of new links can only relax the bottleneck effect, as new links can be used to reroute transfers at every state. Hence, adding links can only improve the stability of the network, and the complete network is stable for lower values of the discount factor than any other network. For weaker punishment schemes, a higher density in a network will have an ambiguous effect. On the one hand, it reduces the bottleneck effects, thereby helping stability, but it also reduces the potential punishment a deviant would suffer which hurts stability. Under weak punishment, we show that the stability conditions are identical for all fully decomposable trees (a class of trees including stars and lines).

2 Premises of the Model

We consider a community of n identical agents, indexed by $i \in N \equiv \{1, 2, ..., n\}$ At each period in time, agents receive a random income, $\widetilde{y_i}$ which can take on two values, $y_i = h$ with probability p and $y_i = l$ with probability $(1 - p)$ with $h > l \geq 0$. We let $\mathbf{y}$ denote the vector of income realizations for all the agents, and $p(\mathbf{y})$ the probability of income realization $\mathbf{y}$. All agents are endowed with a utility function u defined over consumption, which is smooth, strictly increasing and strictly concave, and have a common discount factor $\delta \in (0,1)$. We suppose that there is no credit market, and that consumption is perishable. Hence agents can only consume in period t their total income from that period.

Let g^N be the collection of all unordered pairs of agents in N. A social network in the community is a subset g of g^N, describing pairs of players who are directly linked to each other. We let $ij \in g$ denote the fact that community members i and j are linked. The network g^N is called the *complete network*, and the empty set the *empty network*. Players who are connected in the network have the ability to make transfers to each other. The presence of the network affects the players' ability to self-insure in two ways. First, we assume that transfers can only flow along the links of the network. Two players i and j who are not connected cannot negotiate any transfer. Second, we suppose that transfers are negotiated on a bilateral basis between linked players.

A *path* P between agents i and j in network g is a sequence of agents $i = i^0, i^1, ..., i^m = j$ such that $i^k i^{k+1} \in g$ for all $k = 0, .., m-1$. Two agents i and j are connected in graph g if there exists a path between them. The graph is *connected* if all players are connected. A component g' of a graph g is a maximally connected subgraph of g. The *size* of component g' is the number of agents in the component. The set of agents in component g' is denoted $N(g')$.

A *cycle* in network g is a sequence of $m \geq 3$ distinct agents such that $i = i^0, i^1, ..., i^m = i$ and $i^k i^{k+1} \in g$ for all $k = 0, .., m-1$. A graph is *acyclic* if it does not contain any cycle. An acyclic connected graph is called a *tree*. Clearly, there exists a unique path between two agents i and j in a tree. Furthermore, any tree of size m contains exactly $m-1$ links, and if g is a tree, then any subgraph $g \backslash ij$ is not connected.

Two special trees will play a prominent role in the analysis. A *star* is a graph such that there exists an agent i such that $ij \in g$ for all $j \in N(g)\backslash i$. Agent i is called the center of the star, and all other agents the periphery. A line is a tree such that there exists an ordering of the agents in $N(g), i^1, ..., i^m$ such that $i^k i^{k+1} \in g$ for all $k = 1, ..., m-1$.

3 Equilibrium Concept

3.1 Bilateral Transfers

As motivated and discussed in the Introduction, we focus on the theme that insurance arrangements within social networks are based on *bilateral* arrangements. We model the bilateral transfers between two agents i and j connected in the network in the following way. Let $N_i(g)$ and $N_j(g)$ denote the sets of direct neighbors of i and j in network g. Let x_{ik} denote a transfer to i from $k \in N_i(g)\backslash j$, with the convention that a positive transfer means a payment from k to i. Similarly, define x_{jk} for any $k \in N_j(g)\backslash i$. Let $\mathbf{y}$ be a realization of the income profile. Define a state $\theta = \{y_i, \{x_{ik}\}_{k \in N_i(g)\backslash j}, y_j, \{x_{jk}\}_{k \in N_j(g)\backslash i}\}$. Our objective is to determine, for any state θ, the bilateral transfer between players i and j, denoted $x_{ij}(\theta)$.

In order to determine this transfer scheme, we shall adopt different norms on the behavior of agents. The idea is that agents internalize norms of behavior practised in the community. Our objective is not to endogenize this norm, but rather to study different plausible norms, and in particular to contrast the results obtained under different assumptions. The stability in terms of incentive constraints of the network under a norm will determine the social networks that we expect to observe given the norms in place.

Two examples of such norms are:

Equal Sharing

Under equal sharing, the transfer between i and j is chosen to equalize consumption of the agents in all states. In other words, for all θ,

$$y_i + \sum_{k \in N_i(g) \setminus j} x_{ik} + x_{ij}(\theta) = y_j + \sum_{k \in N_j(g) \setminus i} x_{jk} - x_{ij}(\theta) \tag{1}$$

Nash Bargaining

Let $\underline{v_i}$ and $\underline{v_j}$ denote the disagreement points of agents i and j. These disagreement points can be defined in different ways, but are taken as exogenous here. Let $p(\theta)$ denote the probability distribution over states θ. In the Nash bargaining norm, transfers will be computed to maximize the Nash product of ex ante utilities. Let $\mathbf{x}_{ij}$ denote the vector of transfers for all θ,

$$\begin{aligned} \mathbf{x}_{ij} \quad = \quad & \arg\max \left(\sum_{\theta} p(\theta) u(y_i + \sum_{k \in N_i(g) \setminus j} x_{ik} + x_{ij}(\theta)) - \underline{v_i} \right) \\ & \left(\sum_{\theta} p(\theta) u(y_j + \sum_{k \in N_j(g) \setminus i} x_{jk} - x_{ij}(\theta)) - \underline{v_j} \right). \end{aligned} \tag{2}$$

It is important to note that the informational requirements to compute the transfers are minimal: a state θ only consists of the income realizations of the two agents, and the vector of transfers that they are supposed to make and receive to other agents. In order to compute transfers, agents do not need to know the entire vector of income realizations, nor the transfers to other agents in the network.

3.2 Multilateral Insurance Schemes

We now move from the definition of bilateral transfers to *multilateral* insurance schemes for all the agents. Instead of taking the vector of payments to other agents as exogenous, we know determine, for any vector of income realizations, $\mathbf{y} = (y_1, y_2, ..., y_n)$, the vector of transfers for all agents, $t_{ij}(\mathbf{y})$. An *insurance scheme* τ specifies a vector of transfers for each realization of incomes satisfying: (i) $t_{ij}(\mathbf{y}) = -t_{ji}(\mathbf{y})$, (i) there exists a state $\mathbf{y}$ such that $t_{ij}(\mathbf{y}) \neq 0$, and (iii) $t_{ij}(\mathbf{y}) = 0$ if $ij \notin g$. We let $v_i(\tau)$ denote the expected utility of agent i under the insurance scheme τ.

We now compute equilibrium insurance schemes, called *bilateral insurance schemes.* These transfers emerge as the *fixed point* of a simple mapping. Consider a pair of linked players, ij and define a vector of transfers $t_{ij}(\mathbf{y}))$ for all possible realizations of $\mathbf{y}$ as a function of the transfers for other linked pairs of players, $\mathbf{t}_{-ij}$. Formally, define

$$t_{ij}(\mathbf{y}, \mathbf{t}_{-ij}) = x_{ij}(\mathbf{y}, \{t_{ik}\}_{k \in N_i(g) \setminus j}, \{t_{jk}\}_{k \in N_j(g) \setminus i})$$

where $x_{ij}(\theta)$ is the bilateral transfer defined above.

Now consider a mapping $\Upsilon(\mathbf{t})$, associating to each vector of transfers for all linked pairs, the transfer $t_{ij}(\mathbf{y}, \mathbf{t}_{-ij})$:

$$\Upsilon(\mathbf{t}) = \times_{ij \in g} t_{ij}(\mathbf{y}, \mathbf{t}_{-ij})$$

Definition 1 *A bilateral insurance scheme for a network is a fixed point of the mapping* $\Upsilon_{\mathbf{y}}(\mathbf{t})$.

Finally, one property of some bilateral insurance scheme will prove useful. A bilateral insurance scheme is said to be *monotonic* if for any two connected networks g and g' such that $i \in g, g'$ and then $v_i(g) < (=, >) v_i(g') \Leftrightarrow |g| < (=, >) |g'|$.

4 Enforcement Constraints

The preceding definitions associate to each network g an insurance scheme for all the community members. These insurance schemes can only be enforced if every agent has an incentive to pay the transfer he is required to make. In a network setting, an agent could choose to renege on some – but not all – the transfers he is required to make. In words, he may choose to sever some links, and keep other links in the network. Furthermore, the consequences of a deviation cannot be ascertained unambiguously. If an agent chooses to sever some but not all links, what will the reaction of the community be? The community could choose to isolate the agent, or not. Our definitions of enforcement constraints will take into account these different possibilities.

In order to build our concept of stable networks, we use a recursive definition. For the empty graph, define

$$v_i^*(\emptyset) = pu(h) + (1-p)u(l)$$

to be the expected utility of an agent living in autarchy. Furthermore, the empty graph is clearly stable, as no transfer is required in the graph. For any other graph g, let $v_i^*(g)$ denote the expected utility of player i in graph g,

$$v_i^*(g) = \sum_{\mathbf{y}} p(\mathbf{y}) u(y_i + \sum_{ij \in g} t_{ij}(\mathbf{y})).$$

Next, consider a graph g and suppose that the set of stable subgraphs of g has been defined.

Consider a player i and any subset of players $S \subseteq N_i(g)$ to which i is linked in network g. For any realization $\mathbf{y}$, by abiding to the insurance scheme, agent i obtains an expected payoff (in present value terms) of

$$(1-\delta)u(y_i + \sum_{j \in N_i(g))} t_{ij}(\mathbf{y})) + \delta v_i^*(g)$$

We consider different levels of punishment which translate into different expected payoffs for an agent i who would renege on a subset $S \subset N_i$ of links. In the strong punishment case, a deviating agent is excluded from all insurance networks. In contrast, under weak punishment, a deviating agent is only excluded by the agents he deviated on and the best stable subnetwork will form. In the q-level gossip, a deviator is excluded by his victim and by all individuals who are connected directly to a deviant and are less than q links away from her victim, but not via the deviant. The idea being here that information regarding a deviant travels along the network itself.

Formally, let $\Gamma_i(g, S)$ denote the set of subgraphs of $g \backslash \{ij | j \in S\}$. Recursively, we can define the set of stable networks in $\Gamma_i(g, S)$, denoted $\Gamma_i^*(g, S)$ as follows.

Let i' s expected continuation utility from deviating on a subset $S \subset N_i$ of his neighbors be:

Strong Punishment

Let $v_i^p(g, S) = \max_{g' \in \Gamma_i^*(g, N_i)} v_i^*(g')$

Weak Punishment

Let $v_i^o(g, S) = \max_{g' \in \Gamma_i^*(g, S)} v_i^*(g')$

q-level Gossip

Let $v_i^q(g,S) = \max_{g' \in \Gamma_i^*(g,S_i^q)} v_i^*(g')$ where S_i^q is the set of all $k \in N_i$ such that there is a path $p_{jk} \equiv \left(j = j^0, j^1, ..., j^m = k\right)$ for some $j \in S$ of length $m < q$ such that $i \notin p_{jk}$.[1]

Hence, the payoff of reneging on a subset of links can be written, in per-period terms, as

$$(1-\delta)u(y_i + \sum_{j \in \{N_i(g) \backslash S\}} t_{ij}(\mathbf{y})) + \delta v_i^k(g,S))$$

for $k \in \{o, p\}$.

Definition 2 *A network g is stable if for every player i, every realization $\mathbf{y}$ and every $S \subseteq N_i(g)$,*

$$(1-\delta)u(y_i + \sum_{j \in \{N_i(g) \backslash S\}} t_{ij}(\mathbf{y})) + \delta v_i^k(g,S)) \leq (1-\delta)u(y_i + \sum_{j \in N_i(g))} t_{ij}(\mathbf{y})) + \delta v_i^*(g) \quad (3)$$

for $k = p$ (strong punishment) or $k = o$ (weak punishment) or $k = q$ (q-level gossip).

The first part of the definition indicates that, in a stable network, all links are used in the insurance scheme. This is justified by the fact that if links are costly to maintain, keeping unused links cannot be a rational decision. Furthermore, absent this requirement, any supergraph of a stable graph is stable, and the characterization of stable graphs becomes less crisp. The second requirement builds on a recursive definition of stability, and explicitly takes into account the fact that community members could choose to renege on any subset of links.

[1] Clearly, $S \subseteq S_i^q$.

5 Sustainability for High Discount Factors

Looking at the sustainability of networks with monotonic insurance at high level of discount rate illustrates very clearly the differences between the punishment schemes. For values of δ close to 1, all network are stable under strong punishment. In contrast, for weaker punishment schemes, the density of links weakens punishments and has important consequences for network stability. In particular, for high values of the discount factor trees are the only stable networks.

5.1 Weak Punishment

PROPOSITION **1** *For $\delta \sim 1$, an insurance network with a monotonic bilateral insurance scheme is sustainable under weak punishment iff it is a tree.*

Proof. To establish this result, the following preliminary lemma is useful.

LEMMA **1** *Let g be a stable network connecting k agents with a monotonic bilateral insurance scheme. Then any supergraph g' of g connecting k agents is unstable.*

Proof. Suppose by contradiction that g' is stable. Let ij be a link in $g' \backslash g$. By definition, there exists a realization of the state $\mathbf{y}$ for which there exists a nonzero transfer $t_{ij}(\mathbf{y})$. Suppose without loss of generality $t_{ij} < 0$. Consider a deviation by which player i refuses to make the transfer t_{ji} and let $t_i(\mathbf{y})$ denote the rest of the transfers he receives. Clearly, $g \in \Gamma(g, \{j\})$ and by assumption g is stable so $g \in \Gamma^*(g, \{j\})$. Now, given that g and g' both connect the same number k of agents, $v_i^*(g) = v_i^*(g')$. This establishes that,

$$(1-\delta)u_i(y_i + t_i(\mathbf{y}) + t_{ij}(\mathbf{y})) + \delta v_i^*(g') < (1-\delta)u_i(y_i + t_i(\mathbf{y})) + \delta v_i^*(g)$$

contradicting the fact that g' is stable. ■

The preceding Lemma shows that if a tree of size k is stable, no supergraph of this tree can be stable. The next lemma proves that any tree is stable for discount rates close to 1.

LEMMA **2** *For any tree with a monotonic bilateral insurance scheme, there exists $\underline{\delta}$ such that the tree is stable for $\delta \geq \underline{\delta}$*

Proof. Consider a tree g of size k. For g to be stable, it must be that for all players i, every realization $\mathbf{y}$ and every $S \subseteq N_i(g)$,

$$(1-\delta)\left(u(y_i + \sum_{j \in \{N_i(g) \setminus S\}} t_{ij}(\mathbf{y})) - u(y_i + \sum_{j \in N_i(g))} t_{ij}(\mathbf{y}))\right) \leq \delta\left(v_i^*(g) v_i^o(g,S))\right). \tag{4}$$

By definition, removing a link from a tree strictly increases the number of components. From this and from the monotonicity of the bilateral insurance scheme, it follows that $v_i^o(g,S) < v_i^*(g)$ for any non empty subset $S \subseteq N_i(g)$. Hence, when $\delta > 0$ the right-hand side is strictly positive. Since the left hand side in (4) tend to 0 when $\delta \to 1$, there exists $\underline{\delta}$ such that g is stable for $\delta \geq \underline{\delta}$. ■

From Lemma 1 and Lemma 2, we conclude that trees are sable and are the only stable networks for high values of the discount factor. ■

5.2 Gossips

We say that a graph g is *connected of degree* q if for all $ij \in g$ the graph formed by removing from g the links along all paths $i, i^1, ., i^k, .., i^{m-1}, j$ of size $m \leq q$ between i and j has strictly more components than g.

Note that all trees are connected of degree 1, and that is a graph os connected of degree q, it is connected of degree q' for all $q' \geq q$.

Proposition 2 *For $\delta \sim 1$, a network with monotonic bilateral insurance is sustainable under q-level gossip iff it is connected of degree q.*

Proof. [Sufficiency] Consider a network g connected of degree q. Under q-level gossip, if i deviates on any subset $S \subseteq N_i(g)$ all agents in S_i^q breaks their ties with i. Since g is connected of degree q, $g' = g \setminus S_i^q$ has more components than g and therefore, by monotonicity, $v_i^*(g) - v_i^k(g, S) > 0$. Since $(1 - \delta) \to 0$ as $\delta \to 1$, all incentive constraints (3) are satisfied for sufficiently high values of δ and g is stable.

[Necessity] Suppose that a network g of size n is stable but is not connected of degree q. Then there exist agents i and j with $ij \in g$ such that the graph $\widetilde{g}$ formed by removing from g the links along the paths of size q or less between i and j is still connected. Since g has a bilateral insurance scheme, there is a realization of $\mathbf{y}$ for which there is a nonzero transfer $t_{ij}(\mathbf{y})$. Without loss of generality, suppose that $t_{ij} < 0$.

Let S_i^q be the set of all $k \in N_i$ such that there is a path $p_{jk} \equiv \left(j = j^0, j^1, ..., j^m = k\right)$ for some $j \in S$ of length $m < q$ such that $i \notin p_{jk}$; and let $g'' = g \setminus S_i^q$. Since $\widetilde{g} \subseteq g''$, g'' is connected. To be sure, there exists a subnetwork $g' \subseteq g''$ of size n that is connected of degree q. Using the sufficiency part of the proof, we know that for high values of δ g' is stable, such that $g' \in \Gamma^*(g, \{j\})$. Moreover, $v_i^*(g') = v_i^*(g)$ by monotonicity.

To complete the proof of necessity, it suffices to consider i's incentive constraint (3) not to renege on this transfer t_{ji}:

$$(1-\delta)u_i(y_i + t_i(\mathbf{y})) + \delta v_i^*(g') \leq (1-\delta)u_i(y_i + t_i(\mathbf{y}) + t_{ij}(\mathbf{y})) + \delta v_i^*(g)$$

where $t_i(\mathbf{y})$ denote the sum of the net transfers received by i from all other agents than j. Clearly, this constraint is violated and this contradict the stability of g. ■

Clearly, a network of size n is connected of $n-1$. Hence, a simple consequence of Proposition 2 is that for ∞-level gossip, every network is sustainable.

5.3 Strong Punishment

Now, it is trivial to show that under strong punishment every network with monotonic bilateral insurance is sustainable.

Proposition **3** *For $\delta \sim 1$, all networks are sustainable under strong punishment.*

Proof. To be sure, the worst possible stable network for any player is the empty network. Take a network g with monotonic bilateral insurance. In the case of pessimistic beliefs, the continuation value $v_i^p(g,S) = v_i^*(\emptyset)$ independently of the graph g and of the subset S. It follows that, by monotonicity, $v_i^*(g) - v_i^p(g,S) > 0$. Since $(1-\delta) \to 0$ as $\delta \to 1$, all incentive constraints (3) are satisfied for sufficiently high values of δ and g is stable. ■

6 Threshold Discount Factors

To study the threshold discount factors at which some insurance network are stable, we need to precise the norm that we are studying. In the remaining of the paper we will study the stability of risk-sharing network that do follow an equal-sharing norm.

6.1 Equal Sharing Norms: Existence of a Bilateral Insurance Schemes.

If community members adopt the equal sharing norm, bilateral transfers are easily computed as:

$$x_{ij} = \frac{y_j - y_i + \sum_{k \in N_j(g) \setminus i} x_{jk} - \sum_{k \in N_i(g) \setminus j} x_{ik}}{2}.$$

This transfer scheme clearly exists and is ex post Pareto efficient. Furthermore, it results in an equal consumption for all agents belonging to the same component of the social network. It is easy to show that, for this particular definition of bilateral transfers, bilateral insurance schemes always exist.

PROPOSITION 4 *For the equal sharing norm, bilateral insurance schemes always exist.*

Proof. The proof is constructive. For a fixed network g we exhibit a vector of transfers resulting in equal consumption across all agents belonging to the same component in g. To this end, let g' be a component of g of size $|g'|$. Let $\mathbf{y}_{g'}$ denote the aggregate income in component g'. We construct a vector of bilateral transfers satisfying:

$$y_i + \sum_{k \in N_i(g) \setminus i} t_{ik} = \frac{\mathbf{y}_{g'}}{|g'|} \tag{5}$$

for all $i \in g'$ and all vectors of income realization $\mathbf{y}_{g'}$. For any vector of income realization, partition the set of nodes of g' into two subsets $\mathbf{B}$ and $\mathbf{G}$ corresponding to the players with bad and good shocks respectively. For any node i in $\mathbf{G}$ and j in $\mathbf{B}$, consider a path from i to j in g'. [The existence of this path is guaranteed because g' is connected.] For this path $P = i, i^1, ., i^k, .., i^{m-1}, j$ define transfers

$\tau_{i^k i^{k1}} = -(h-l)/|g'|$. Let

$$t_{ij} = \sum_{P|ij \in P} \tau_{ij}.$$

It is easy to check that this transfer scheme equalizes consumptions of all agents inside a component. ■

Finally, notice that the equal sharing insurance scheme is monotonic.

6.2 Stability with Strong Punishment

BOTTLENECK AGENTS. To be sure, the worst possible stable network for any player is the empty network. In the case of pessimistic beliefs, the continuation value $v_i^p(g,S) = v_i(\emptyset)$ is *independent* of the graph g, of the agent i, and of the subset S. Since furthermore consumption is equalized across all agents in a component, in any component g' of size m, and any state $\mathbf{y}$ with k good realizations,

$$y_i + t_{iN_i}(\mathbf{y}) = \frac{kh + (n-k)l}{m}$$

for all i in $g'$and $v_i^*(g)$ can easily be computed, by summing over all states. We thus conclude that, for all agents i, j belonging to the same component g' of g, $v_i^p(g,S) = v_j^p(g,S)$, $y_i + t_{iN_i}(\mathbf{y}) = y_j + t_{jN_j}(\mathbf{y})$ and $v_i^*(g) = v_j^*(g)$. Hence, for a given income realization, the only element differentiating incentives of different agents in a component is the maximal benefit an agent can obtain by reneging on his current transfers,

$$\max_{S \subseteq N_i} (y_i + t_{iS}(\mathbf{y}))$$

In order to check the stability of a graph, we thus need to check, component by component, and for every vector of income realization, the incentives of the agent for which $\max_{S \subseteq N_i}(y_i - t_{iS}(\mathbf{y}))$ is maximal. As this agent will always be an agent who receives transfers from many other agents, we call him the *bottleneck* of the transfer scheme.

In order to gain intuition about bottleneck agents, consider two different trees – a star and a line with four agents. Figure 1 illustrates for different state realizations (indexed by the number of good and high states) the bottleneck agents.

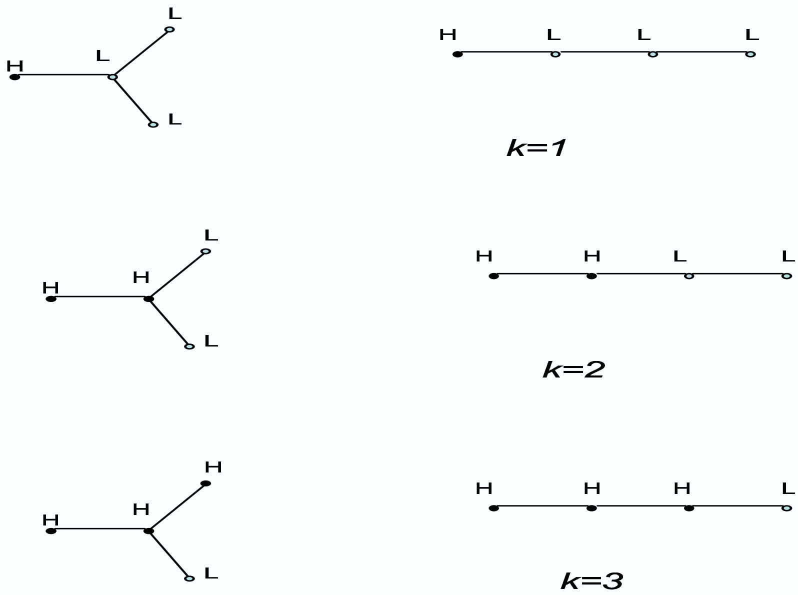

Figure 1: Bottleneck Agents.

In the star, not surprisingly, when the number of good states is greater than two, the bottleneck is the center of the star, which is called to make the maximal transfers. Similarly, in the line, the bottleneck agent is always one of the two central agents.

What is more surprising is that the constraints faced by the bottleneck agents is the same in the two graphs. For any k good realizations, the bottleneck agent receives transfer from $k-1$ agents both in the star and the line, and her maximal deviation payoff is given by

$$h + \frac{(k-1)(n-k)(h-l)}{n}$$

in both graphs.

The next result shows that this is not an accident: for a large class of trees, the maximal deviation payoffs will be equal so that stability conditions are identical.

Decomposable Trees. We define a *subnetwork* originating at i as a subgraph g' of a tree g satisfying the following conditions:

1. $i \in N(g')$
2. For any $j, k \neq i$. If $jk \in g$ and $j \in N(g')$ then $k \in N(g')$

A node i is said to be *critical of degree* k if there exists a subnetwork of size k originating at i.

A connected network g of size m is *decomposable* if for any $k = 1, ..., m-1$, there exists a critical node i of degree k.

Let $\delta(g)$ denote the minimal value of the discount factor for which graph g is stable.

Proposition **5** *For any two decomposable networks g and g', $\delta(g) = \delta(g')$.*

Proof. Consider a realization with k good shocks. Let i be a critical node of degree k. We claim that the tightest constraint is obtained for node i when all agents in the subnetwork of size k originating at i receive a high shock. In that case,

$$y_i + t_{iS} = h + \frac{(k-1)(n-k)(h-l)}{n}.$$

Clearly, $\frac{(k-1)(n-k)(h-l)}{n}$ is the highest possible transfer paid by all other players. The only other candidate would be a player with a low shock who receives exactly k transfers. But then

$$y_i + t_{iS} = l + \frac{k(n-k)(h-l)}{n} < h + \frac{(k-1)(n-k)(h-l)}{n}$$

for $k \geq 1$. Hence, for any realization with k good shocks, the tightest constraint will be the same in any decomposable network. To compute δ, one needs to compute the value of k which results in the tightest constraint, i.e. to solve the problem:

$$\max_k \ u(h + \frac{(k-1)(n-k)(h-l)}{n}) - u(\frac{kh + (n-k)l}{n}). \tag{6}$$

The solution k^* of this problem depends on the utility function. Once k^* is known, the threshold value $\delta(g)$ can easily be computed, and is denoted by δ^t. ■

PROPOSITION **6** *Suppose that g is a network which is not decomposable, then $\delta(g) \leq \delta^t$.*

Proof. Let k^* be the solution to the maximization problem (6). If the network g has a critical node of degree k then clearly $\delta(g) = \delta^t$. Otherwise, if g does not have a critical node of degree k, the highest transfer received by any agent in a realization with k^* good shocks must satisfy

$$y_i + t_{iS} < h + \frac{(k-1)(n-k)(h-l)}{n}$$

and $\delta(g) < \delta^t$. ■

PROPOSITION **7** *Consider two graphs g and g' with the same components and $g \subset g'$. Then $\delta(g) \geq \delta(g')$.*

Proof. Consider the binding constraint in graph g. Let $\mathbf{y}$ be the corresponding realization of the shock, and i the agent whose constraint is binding. Compute a transfer scheme for graph g' as follows. For any $\mathbf{y}' \neq \mathbf{y}$, $t(\mathbf{y}')$ is the same for the two graphs. For $\mathbf{y}$, one may be able to relax the transfer constraint, by using additional links. In other words, if new links permit a rerouting of the transfers to bypass i, i's constraint will necessarily be relaxed. ■

While very simple, Proposition 7 illustrates a very powerful fact. If agents hold pessimistic beliefs, the addition of new links will always result in more stable graphs, as new links relax the constraint of the bottleneck agent. As a consequence, for any connected graph g, $\delta(g) \geq \delta(g^c)$ where g^c is the complete graph. Examples can easily be provided to show that this inequality may be strict.

In a complete graph, no transfer is ever mediated. This implies that the binding constraint for an agent is whether to keep her income when it receives a high shock or not. In other words, the maximal deviation is given by

$$u(h) - u(\frac{kh + (n-k)l}{n})$$

This expression is clearly maximized when $k = 1$ so the constraint becomes

$$(1-\delta)u(h) + \delta v^*(1) \leq (1-\delta)u(\frac{h + (n-1)l}{n}) + \delta v^*(n)$$

where

$$\begin{aligned} v^*(1) &= pu(h) + (1-p)u(l) \\ v^*(n) &= \sum_{k=0}^{n} \begin{pmatrix} n \\ m \end{pmatrix} p^k (1-p)^{n-k} u(\frac{kh + (n-k)l}{n}). \end{aligned}$$

It is important to note that this remark does not show that the complete graph is the easiest graph to sustain. The inequality is only true among *connected* graphs and disconnected graphs, with smaller components, may be easier to sustain. In fact, there is no obvious reason why the enforcement constrained faced by community members should be monotonic in the number of agents in a component. The following example computes these enforcement constraints (and the resulting threshold value $\delta(g)$) for complete graphs as a function of the size of the graph. It appears that the threshold value is *non monotonic* in the size of the graph.

EXAMPLE **1** *Utility functions are given by* $U(c) = 2c^{1/2}$, $l = 0, h = 1$ *and the probabilities are given by* $p = 0.2$.

We compute

$$\delta = \frac{1-(1/n)^{1/2}}{1-(1/n)^{1/2} - 0.2 + \sum_{k=0}^{n} \binom{n}{k} 0.2^k 0.8^{n-k} (k/n)^{1/2}}$$

Figure 2 pictures δ *as a function of* n *for* $n \in \{1, ..., 100\}$.

DENSE NETWORKS. In this section we show that dense networks have no bottleneck agent and have the same stability condition as complete networks.

PROPOSITION **8** *For a graph* g *of size* n *and of density* $n-2$ *or more [all nodes have at least* $n-2$ *links], there is no bottleneck agent, and* $\delta(g) = \delta(g^N)$.

Proof. To establish this claim, we show that, although we are not in a complete graph, no transfer is ever mediated whenever there are more than one low and more than one high, and that in the latter cases the transit are not a problem.

Assume that we have a realization with k highs. Index all the highs from 1 to k and all the lows from 1 to $n-k$.

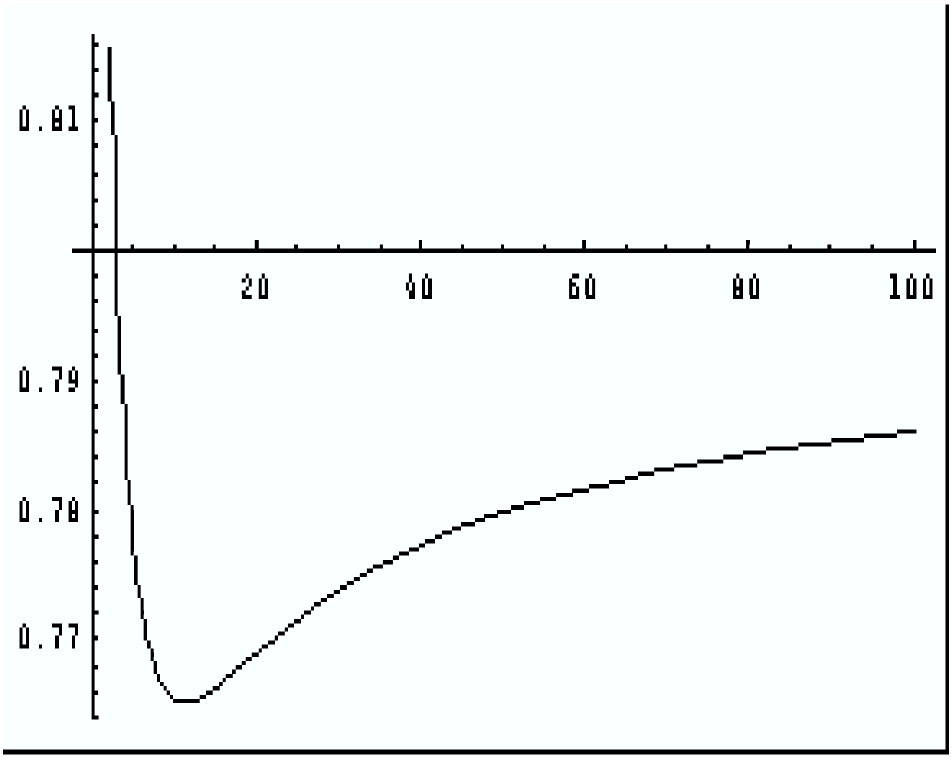

Figure 2: An Illustration of Example 1.

If every high are connected to all lows then no transfer is ever mediated and we are done. So clearly missing links between highs and lows are the worse. Note that in the complete graph $m = n - 1$. This means that in a graph of density $n - 2$ individuals are missing at most one link. To be sure, we want to consider the case in which as many possible missing links are between highs and lows.

First, assume that $1 < k < \frac{n}{2}$, that is there are less highs than lows. Let's distribute these links in a way that maximizes the number of highs and lows not linked with each other. Take the k highs and let them be each linked to the first k lows but one in the following way: the first low is not linked to the first high, the second low is not linked to the second high,... and so on until the k^th high not linked to the k^th low. Now, among the first k lows we have give them all their links, but we still have $n-2k$ lows and each of these must be linked to all the highs (since the highs are already missing one link each).

Now we want to show that all transfers can be distributed without transit. Let t_1 be the transfer that each high gives to the last $n-2k$ lows and let t_2 be the transfer that each h gives to all but one of the first k lows. We know that each low must receive $\frac{k}{n}(h-l)$ and that each high must give $\frac{k}{n}(h-l)$. With our transfers, the last $n-2k$ lows receive kt_1 such that $t_1 = \frac{1}{n}(h-l)$ and the first k lows receive $(k-1)t_2$ such that $t_2 = \frac{k}{(k-1)n}(h-l)$. Since each high gives $(k-1)t_2 + (n-2k)t_1 = \frac{k}{(n}(h-l) + \frac{n-2k}{n}(h-l) = \frac{n-k}{n}(h-l)$ we see that all transfers can be done without transit.

If $k = 1$ then one low could not be linked to any of the highs. However, we can show that the constraint for an agent with a low income receiving all possible transfers

$$(1-\delta)[u(l + \frac{n-1}{n}(h-l)) - u(\frac{h+(n-1)}{n})] \leq \delta[v^*(n) - v^*(1)]$$

is always less binding than the constraint for the one individual with a high income giving the full transfer

$$(1-\delta)[u(h) - u(\frac{h+(n-1)}{n})] \leq \delta[v^*(n) - v^*(1)] \tag{7}$$

since

$$u(h) - u(\frac{h+(n-1)l}{n}) \geq u(l + \frac{n-1}{n}(h-l)) - u(\frac{h+(n-1)}{n}).$$

Turn to the case where $n-1 > k \geq \frac{n}{2}$, that is there are at least as many highs than lows. Link all the $n-k$ lows to each but one of the first $n-k$ highs in the following way: for $i \in \{1, ...n-k\}$, the ith low is linked to all high with index in $\{1, 2..n-k\} \setminus i$. Now the $2k-n$ remaining highs have to be linked to all lows (since the lows already have one missing link).

We can show that all transfers can be distributed without transit. Let t_1 be the transfer that the first $n-k$ highs give the $n-k-1$ lows to whom they are linked and let t_2 be the transfer that the remaining $2k-n$ highs gives to all the lows. We know that each low must receive $\frac{k}{n}(h-l)$ and that each high must give $\frac{k}{n}(h-l)$. With our transfers, the first $n-k$ highs each give $(n-k-1)t_1$ such that $t_1 = \frac{n-k}{n(n-k-1)}(h-l)$ and the last $2k-n$ highs give $(n-k)t_2$ such that $t_2 = \frac{1}{n}(h-l)$. This implies that each low receives $(2k-n)t_2 + (n-k-1)t_1 = (2k-n)\frac{1}{n}(h-l) + \frac{n-k}{n}(h-l) = \frac{k}{n}(h-l)$. Therefore, all transfers can be done without transit.

Finally, we should look at the case were $k = n-1$, that is there is only one low. In the worse case, this low will be connected to all but one of the highs. Since the degree of this high is $n-2$, he must be connected to all other highs. This implies that the larger transfer that a high could receive is $\frac{1}{n(n-2)}(h-l)$ such that the constraint for a contact point is

$$u(h + \frac{1}{n(n-2)}(h-l)) - u(h - \frac{1}{n}(h-l)) \leq \frac{\delta}{1-\delta}[v^*(n) - v^*(1)].$$

Compare this with the incentive constraint (7) for a high when he is the only one to have received a good shock and we see that the latter constraint is stricter as

$$u(h) - u(\frac{h+(n-1)l}{n}) \geq u(h + \frac{1}{n(n-2)}(h-l)) - u(h - \frac{1}{n}(h-l)).$$

We conclude that networks of density $n-2$ or more have no bottleneck agent. Therefore, as for the complete graph, the incentive constraint (7) is the hardest to satisfied and $\delta(g) = \delta(g^N)$. ■

6.3 Stability with Optimistic Beliefs

When agents hold optimistic beliefs, the incentives to deviate are clearly larger than when they hold pessimistic beliefs, and insurance networks are harder to sustain. Our first result shows that again there is a family of networks (including stars and lines) for which the tightest incentive constraints are identical. As the argument relies on a recursive computation of stable networks, the property needed to show this equivalence is not decomposability but full decomposability – every subgraph of the graph must be decomposable.

PROPOSITION **9** *Consider two fully decomposable networks g and g' connecting the same number of players, then $\delta(g) = \delta(g')$.*

Proof. The proof is by induction on the size of the network, denoted n. If $n = 2$, the statement is clearly true, as there is only one graph connecting two players. Suppose that the statement is true for $m = 2, ..., n-1$. By definition, any subnetwork of a fully decomposable network is also fully decomposable. Hence, by the induction hypothesis, the threshold value of the discount factor only depends on the size of the network, and we let $\delta(m)$ denote the value of this discount factor. Furthermore, because the norm is an equal sharing norm, the continuation value of player i in a graph g only depends on the size of the graph, and hence we write $v_i^*(g) = v^*(m)$ where m is the size of graph g. Clearly, v^* is increasing in m. We also define $t(\delta, m) = \max\{t \leq m, \delta \geq \delta(t)\}$ the largest stable subgraph of size smaller or equal to m and $w(\delta, m) = v^*(t(\delta, m))$.

Now consider a fully decomposable network of size n. Pick any $k = 1, 2, ..., n-1$. Let δ be fixed. Suppose that a player plans to sever links to $n-k$ players. Then,

$$\max_{g' \in \Gamma^*(S)} v^*(g') = w(\delta, k).$$

Because the network is decomposable, the tightest constraint will be experienced by an agent who is critical of degree k, receives a high shock and keeps the transfer of $k-1$ other agents. Hence, the relevant constraint for stability is given by

$$(1-\delta)u(h + \frac{(k-1)(n-k)(h-l)}{n}) + \delta w(\delta, k) \leq (1-\delta)u(\frac{kh + (n-k)l}{n}) + \delta v^*(n).$$

This constraint is clearly the same for all decomposable networks showing that $\delta(g) = \delta(g')$ for any two fully decomposable networks. The exact value of δ is given by

$$\min\{\delta | \frac{\delta}{1-\delta}[v^*(n) - w(\delta, k)] \geq [u(h + \frac{(k-1)(n-k)(h-l)}{n}) - u(\frac{kh + (n-k)l}{n})] \forall k = 1, ...n-1\}$$

It is clear that δ exists, but the computation of the exact value of δ typically requires an algorithm. ■

Proposition 9 shows that Proposition 5 can be extended to the case of optimistic beliefs, and that the threshold value of the discount factor is identical for a large family of networks including stars and lines. As opposed to the case of pessimistic beliefs, the addition of new links typically does not result in a more stable insurance network. We saw that for high values of the discount factor trees are the only stable networks . In particular, the complete graph fails to be stable when agents hold optimistic beliefs, whereas it is the easiest graph to sustain when players hold pessimistic beliefs. For lower values of the discount factor, when trees cease to be stable, denser networks can reemerge as stable outcomes. This is illustrated in the following example with three agents.

Example **2** *The following parameters are set through the example: $l = 0$ and $h = 1$, and individuals are assumed to have utilities $U(c) = 2c^{1/2}$. In Figure 3, stable networks are computed for the entire range of discount factors δ for values of p equal to 0.2 in the upper part of the graph, then for $p = 0.5$ in the lower part.*

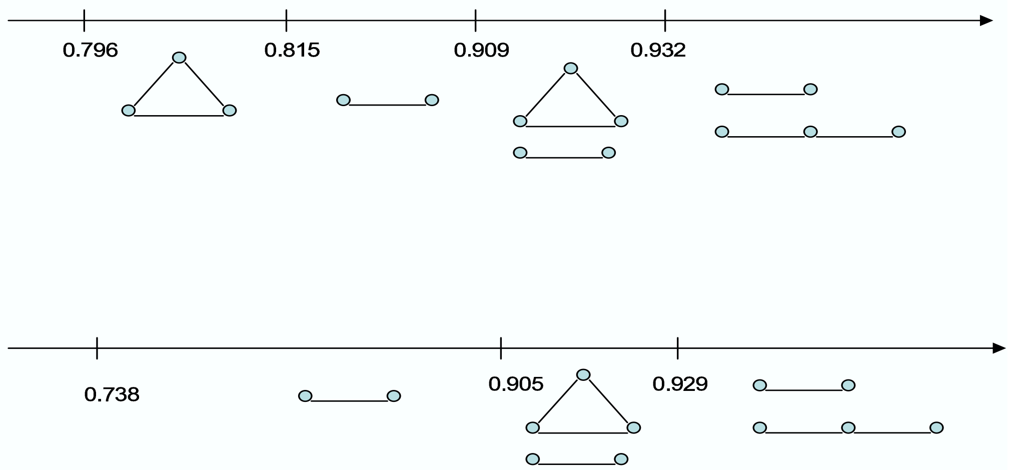

Figure 3: An Illustration of Example 2.

Figure 3 for instance shows that stability is a complex object to check for. When for $\delta = .8$ the complete graph is the only stable graph. Thereafter, for $\delta = .9$, a line of two is stable and the gain enjoyed by the former is large enough to render the complete graph unstable. Then, for values of δ of 0.91, the complete graph regains its stability. Yet, the fall is inevitable: for high values of δ only trees are stable, in *line with our previous results.*

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