A Spatiotemporal Model of Shifting Cultivation and Forest Cover Dynamics

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Abstract

Shifting cultivation is the primary means of livelihood for subsistence farmers throughout the humid forests of the tropics. They rely on the forest landscape as a source of fertile land to sustain their livelihood. Sustainable use of the resource base requires long periods of fallow and the ability to move the zone of active cultivation from one location to another over time. At the individual patch or field level, shifting cultivation is essentially a resource extraction problem somewhat akin to a pulse fishery – intensive use of the stock of soil fertility for a short period followed by a long idle period to allow regeneration of the stock. This paper describes a spatiotemporal model of resource extraction adapted to the use of forest resources by shifting cultivators. In contrast to other models of spatial resource exploitation, decision criteria depend on a nonseparable agricultural household model extended to accommodate both the temporal and spatial dimensions. The paper focuses on the theoretical issues related to modelling shifting cultivation. It concludes with a brief discussion of the development and implementation of a simulation model based on the theoretical approach described herein.

Key words: spatial resource modelling, spatial dynamics, spatiotemporal modelling, nonseparable agricultural household model, shifting cultivation, subsistence agriculture, bioeconomic model, sustainable resource use

1 Introduction

Shifting cultivation is one of many phenomena that involve the interaction of humans with their natural environment for which the spatial and temporal dimensions are essential to understanding the system. Many resource management issues involve both spatial and temporal dimensions. Fishermen expend varying levels of effort to search over spatially heterogeneous patches of ocean for their catch, itself a population that varies over space and time [1]. Forest resources are found over vast areas, some of which are difficult (and therefore costly) to access. The timing of cutting and the methods used to do so have a significant impact on costs of extraction, long-term forest regeneration as well as the population dynamics of associated plant and animal species. In the Serengeti, wildlife migrates over vast distances to follow seasonal variations in rainfall and pasture. Pastoralists engage in similar movements over space and time. Even the classic von Thünen model [2] of rural-urban land use and land values is based on the fact that land use varies over space and time.

Until recently, the tools of resource economists have been primarily focused on just one dimension of resource allocation problems: on exploring optimal allocation or use of resources over time. While biologists are making progress at modelling the spatial dimension explicitly, economists have not, generally, addressed the spatial dimension [1]. In fact, the treatment of space in economic analysis has been largely superficial and, with few exceptions, spatial dynamics have been ignored by economists [3] until recently. However, ignoring the spatial dimension in resource management problems that are inherently spatial in nature can lead to incorrect conclusions about the viability of alternative management strategies [4]. On the other hand, when the spatial dimension is taken into consideration, it becomes clear that land is not a costless factor of production even when it is in abundant supply [5]. In contrast to the often cited observation that population growth results in intensification of agricultural land use [6], the intensification of production through the shortening of fallow cycles depends not only on the limits of cultivable land, but also on the degree of social cohesion in the village [5]. The implication is that where there is a strong sense of community cohesiveness (or other reasons that limit establishment of new villages) agricultural production can intensify (as measured by shorter fallow periods) even when there are abundant forest resources not too far away. In other words, the spatial dimension, social norms and individual preferences have implications for land use intensification.

In shifting cultivation, the unit of analysis is a small household that relies almost entirely on its own labour resources to exploit the natural resources at its disposal and earn its livelihood. Modeling of decision-making by agricultural households in the developing world often relies on the use of separable household models [7]. However, in the context of subsistence agriculture, where some markets may be either incomplete, fragmented or missing, household consumption decisions are not often separable from agricultural production decisions. Non-separable household models [8] have been used to better understand the dynamics of household decision-making in this context [9-13] and shed light on the observation that subsistence households frequently appear not to respond to incentives as a profit-maximizing producer might. While a short-fallow system may appear to be optimal (cost minimizing) for the production of subsistence household needs [14] under certain restrictive assumptions, Holden [12, 13], using a nonseparable agricultural household model, demonstrates that there are sound economic reasons for households to continue to employ traditional shifting cultivation systems of production. Ignoring the impact of high trading costs on farm-gate production incentives can lead to incorrect conclusions about the rationality of smallholders' crop choices [15] as well as their willingness to respond to market signals [8]. In fact, diversification and self-sufficiency (seemingly "inefficient" food-dominated cropping patterns in comparison to market-oriented cash crop production) are optimal responses to high trading costs [15, 16] and therefore perfectly rational livelihood strategies in this context. The assumption of nonseparability is essential in this setting since models that rely on inappropriate simplifying assumptions, and thereby fail to account for important aspects of the decision-making context, can lead to incorrect conclusions [17-19].

However, few have explored the spatial dimension of shifting cultivation beyond accounting for distance from markets [15, 16] or the location of the agricultural frontier [17-19]. Shifting cultivation is a spatiotemporal phenomenon which differs significantly from other forms of subsistence agriculture in that it requires the selection and clearing of new patches of land on an annual or semi-annual basis. It uses forest resources to provide the means for human sustenance, both from the temporary use of patches of forest for agricultural production and from the harvest of non-wood forest products from that same mosaic of forest and agricultural land. In its "purest" form, the area of active cultivation literally shifts across the landscape. In other circumstances, where households or extended families retain rights of access and use to land over a period of years, the pattern is different. In either case, however, the system consists of temporary forest clearing for one to three years cultivation followed by extended periods of forested fallow. The result is a unique and dynamic mosaic of forest, fallow and cultivated land at the forest margin. Assessment of the long-term sustainability of farming systems and the ecological integrity of the forest landscape requires an understanding of the dynamics of this mosaic of land use.

This paper presents a model that facilitates this sort of analysis by extending the essential elements of a nonseparable household model to incorporate a dynamic model of spatial resource exploitation. In so doing, it addresses the limitations of current household modelling techniques as well as those of bio-economic models of resource use. The modelling technique outlined has broader application to modelling spatio-temporal phenomena in general.

2 The model

The model described here addresses several gaps in the literature. To date, decision-making behaviour has been limited to maximization of the net returns to resource exploitation while accounting for the variation of cost over space and time. To be suitable in the context of shifting cultivation, and other resource management problems that require nonseparable modelling of household production and consumption decisions, a more refined set of decision-making criteria are required. By considering insights from the household modelling literature in the context of a dynamic model of resource use, this is possible.

2.1 A spatial model of forest resource use

This model builds on the work of Sanchirico and Wilen [1], who described a model of renewable resource exploitation that incorporates both intertemporal dynamics and spatial movement, and adapts it to the context of shifting cultivation. They structure their spatial resource model as follows:

$$\dot{x}_i = f_i(x_i)x_i + d_{ii}x_i + \sum_{\substack{j=1 \\ i \neq j}}^{n} d_{ij}x_j \text{ , for } i = 1,\ldots, n$$

where

$\dot{x}_i$ is $\frac{\partial x_i}{\partial t}$, the instantaneous rate of change in biomass in patch *i,*

x_i is the biomass in patch *i* in time *t*,

$f_i(x_i)$ is the per unit growth rate in patch *i,*

d_{ii} is the rate of emigration from patch *i* and

d_{ij} is the rate of dispersal between patches *i* and *j*.

In the context of shifting cultivation, the principal natural resource on which agricultural productivity depends is the stock of biomass, soil fertility or nutrients, *N*, available at time *t*. Since soil nutrients do not disperse, one can eliminate the second and third elements ($d_{ii}x_i$ and $d_{ij}x_j$) of the above general model.[1] The model of nutrient accumulation over time for a particular patch of forest or fallow, *s*, is therefore:

$$\dot{N}_{st} = g(N_{st})$$

where

[1] This does not mean that location is irrelevant. Even though nutrient accumulation from forest regrowth is not effected by a dispersal mechanism, location is important for other reasons – for example, seed dispersal of forest species important as non-wood forest products and for wildlife populations. A more fully developed model of the forest resource can build on this basis and incorporate the dynamics of other plant and animal populations that are important for forest health and for the livelihoods of subsistence farmers.

$g(N_{st})$ is the nutrient growth function $f_i(x_i)x_i$ in Sanchirico and Wilen's model and

$g_N > 0, g_{NN} < 0, g \leq \overline{g}$.[2]

In addition to the dynamics of forest regeneration, it is essential to account for use or extraction of nutrients by agricultural production. When the patch is in fallow, forest-fallow or forest, there is growth of the biomass or nutrient stock (the function *g*(.) above).[3] Conversely, during clearing and subsequent cultivation, the nutrient stock ceases to accumulate and depletion occurs. There are two major sources of nutrient loss or use. Burning, *b*(*N*), the principal means of land preparation, amounts to a significant loss of biomass and the nutrients it contains, although it also makes a considerable amount available as ash for more or less immediate uptake by crops. Subsequent cultivation, *c*(*N*) also depletes the remaining nutrient stock, the rate of decrease depending upon the stock available for use.[4] If we assume that forest regrowth does not occur at the same time as cultivation (extraction), then for patch *s* at time *t* we have:

$$\dot{N}_{st} = g(N_{st}) - b(N_{st}) - c(N_{st})$$

where

$b(N_{st})$ is nutrient loss due to clearing and burning where $b_N > 0, b_{NN} < 0, b < \overline{b}$

$c(N_{st})$ is nutrient use during cultivation where $c_N > 0, c_{NN} < 0, c < \overline{c}$

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[2] With the exception of *i, j, s* and *t*, subscripts reflect partial derivatives. The upper bar (i.e. $\overline{g}$) represents an upper bound while a lower bar represents a lower bound.

[3] I use the terms fallow, forest-fallow and forest to represent a continuum of ages of regrowth after agricultural use with fallow being the most recently abandoned and forest having been in fallow for the longest.

[4] This assumes that there is no net fixation of atmospheric elements (e.g., nitrogen). One might argue that, while this may be true for cassava, it does not apply to peanuts (or groundnuts), which fix nitrogen. However, given that the principle field type in shifting cultivation in the Congo Basin of Central Africa, for which this model was conceptualized, is a mixed cropping system that includes cassava, plantain and maize intercropped with peanuts, it is probably safe to say that there is no net fixation of atmospheric elements during the cultivation phase.

The result, over time, gives the typical pattern of soil fertility illustrated in Figure 1. This is essentially similar to the Faustmann [20] optimal rotation model found in the resource economics literature [21] and used by Angelsen in modified form [18]. However, in the case of subsistence agricultural production, the objective is to choose the length of fallow that maximizes utility derived from consumption and leisure for a particular patch of forest or fallow land (see Appendix 1 for the derivation of the optimal fallow length). In order to ensure a steady supply of food to meet subsistence and cash income requirements several patches at various stages of cultivation and regeneration (fallow) are required.

It is most straightforward to implement this in discrete time,[5] where the model of nutrient accumulation and extraction over time for a particular patch of forest, s, is:

$$\Delta N_{st} = (1-u_{st})g(N_{st}) - u_{st}\left[(1-u_{st-1})b(N_{st}) + c(N_{st})\right]$$

where

$$u_{st} \in \{0,1\} \text{ such that } u_{st} = \begin{cases} 0 & \mathit{If}\ s = \quad \mathit{forest} / \mathit{fallow} \\ 1 & \mathit{If}\ s = \quad \mathit{cultivated} \end{cases}$$

$g(N_{st})$, $b(N_{st})$ and $c(N_{st})$ are as defined above.

When u_{st} = 0 and u_{st-1} = 0 (i.e., when the patch is in fallow), only the forest growth function, $g(N_{st})$, is operable. When u_{st} = 1 and u_{st-1} = 0 (i.e., the plot was in forest in the previous period, but is now cleared for cultivation), the nutrient use/loss functions for burning and cultivation, $b(N_{st})$ and $c(N_{st})$, are operable. Finally, when both u_{st} = 1 and u_{st-1} = 1 (i.e., the plot was cultivated in the previous period as well as the current period), only the nutrient use function for cultivation, $c(N_{st})$, is operable.

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[5] The model is described in discrete time given that subsistence-farming households make major land use decisions on a crop-seasonal basis for the most part.

2.2 Household resource use in a spatial setting

I will assume that a particular household, *i*, in a community has a stock of land, *S*, at its disposition:

where $S = \{s\}_{i=1}^{S}$

That stock of land is made up of a finite, but potentially variable, number of patches or plots of land, *s*, that are in various stages of development along a continuum from mature forest, to forest-fallow, young fallow and active agricultural fields. The household decision-maker's task is then to allocate available labour resources for agricultural work to their most efficient use across the available patches of land while ensuring that subsistence needs are met and the long-term maintenance of the labour stock of the household.

As previously mentioned, the principal emphasis of subsistence agricultural households is on food production for home consumption plus an additional small marketable surplus[6] for sale to provide cash for other household needs. For this reason, the model of resource exploitation developed here focuses on the mixed food crop field, which, in the Congo Basin of Cameroon at least, is the primary source of food for the subsistence household.[7] The objective of the household is to maximize the discounted utility, *U*, derived from food surplus, q_t^f, (for consumption and sale over and above a

[6] Research by the International Institute of Tropical Agriculture has shown that the majority of households in the area for which this model is conceptualized are net sellers of subsistence food crops [22].
[7] Subsequent work will develop the livelihood choice aspect of household resource use that logically precedes the decision about where to cultivate subsistence food crops. However, given that there are different criteria for each of the principal field types, it is not unreasonable to examine the most common one in isolation from the others. Nearly everyone has at least one mixed food crop field. This is not the case for the other field types.

subsistence constraint) and leisure, l_t, (actually non-labour activities; i.e., activities not associated with agricultural work)[8] in each period t:

$$\underset{\{q_t^f, l_t\}\forall t}{Max} \sum_{t=0}^{\infty} \rho^t U\left(q_t^f, l_t\right)$$

where

$\rho = \dfrac{1}{1+\delta}$ is the discount factor and δ is the discount rate.

Since labour requirements for clearing and burning of fallow, forest fallow and forest generally increase with the age of the fallow or forest stand (whether measured in terms of age, biomass or nutrient stock) it is possible to model labour requirements for clearing as a function of N. However, the rate of increase is not constant, but assumed to decline as the age of the fallow or forest stand increases. Therefore, we have:

$L_{st}^b = e\left(N_{st}\right)$, where $e_N^b > 0, e_{NN}^b < 0, e < \overline{e}$

On the other hand, labour requirements for cultivation (weeding in particular) vary inversely with the age (or amount of biomass or nutrients) of the fallow or forest stand that was cleared for cultivation. One of the major limitations to long-term cultivation of forest plots in addition to declining fertility is the cost of weed control. Fallow periods are as much to control weeds as to restore fertility. After a short fallow, there is still a large stock of weed seeds lying dormant that quickly germinate when the land is cleared again. However, with a longer fallow period, many of the annual weeds will have died out. When such a forest-fallow plot is cleared, the labour cost of weed control will be significantly reduced. Since higher soil nutrient status and fallow age are directly related, we can combine the two and say that labour for weeding and the nutrient

[8] More generally, this can represent all goods and services (including leisure) that take labour time as the sole, monotone input.

status of the fallow are inversely related. Therefore, labour requirements for weeding are effectively a function of nutrient status, N:[9]

$$L^c_{st} = w(N_{st}), \text{ where } w^c_N < 0, w^c_{NN} > 0, w > \underline{w}$$

Labour requirements for travel to and from a particular patch of land are also an important factor in labour use. The total labour required for fieldwork is adjusted by a travel time factor based on the distance, d, from the village to the particular patch as well as the proximity to other fields, f, or patches being actively cultivated. The travel time adjustment factor is:

$$k(d_s, f_s)$$

where $k \in [1, \bar{k}), k_d > 0, k_f > 0$ and $\bar{k}$ is the maximum feasible time to walk to do a day's work.

Substituting the labour requirements into the labour constraint:

$$\sum_{s=1}^{n} k_s(d_s, f_s) u_{st}\left[(1 - u_{st-1})L^b_{st} + L^c_{st}\right] + l_t \le L^T_t$$

gives

$$\sum_{s=1}^{n} k_s(d_s, f_s) u_{st}\left[(1 - u_{st-1})e(N_{st}) + w(N_{st})\right] + l_t \le L^T_t$$

which can be solved for l and substituted into the utility function.

The evolution of the stock of available labour depends on three factors. The rate of natural change of the household labour force, γ, captures the impact of births, deaths and aging on the household and is essentially the net change in working-age population

[9] They also vary with the length of the period of cultivation of a patch of land. In other words, if the same crop were grown again in the second year of cultivation there would be more weeds to remove and greater effort required to do so. However, in the mixed food cropping systems associated with shifting cultivation in the Congo Basin the length of the period is not normally a choice variable.

on a year-to-year basis. The general state of health and well-being of the household members influences the amount of time available for work and leisure, η, and means that fewer labour resources are available as health or general well-being decline. Increases in household size, m, arise through marriage or in-migration.[10] I assume γ, η and m to be exogenous to the model itself.[11] The labour stock law of motion is:

$$\Delta L_t^T = L_{t+1}^T - L_t^T = \gamma L_t^T - \eta L_t^T + m_t \text{, where } \gamma \in (-1,1) \text{ and } \eta \in (0,1]$$

Food production (yield) on a particular patch of land depends on the level of nutrients in the soil at the time of cultivation. While it is also influenced by the amount of labour input, for the purposes of decision-making, it is assumed that households have an intuitive knowledge[12] of the optimal labour allocation associated with different levels of soil fertility as represented by fallow or forest types and ages.[13] As a result, the patch-level production function is:

$$y_{st} = f(N_{st}) \text{, where } y_N > 0, y_{NN} < 0$$

Therefore, the food surplus over-and-above the subsistence constraint is:

$$q_t^f = \sum_{s=1}^{S} u_{st} f(N_{st}) - Q_t^f \geq 0 \text{, where } Q_t^f \text{ is the subsistence requirement.}^{14}$$

[10] Negative m, therefore, implies net out-migration or net movement away from the household through marriage.
[11] For the moment, these are exogenously determined. A potential extension of the model is to specify one or more of these parameters as endogenous. Consumption will no doubt have an impact on labour effort, as well as the net rate of population increase. Similarly, one could specify marriage as a function of the socio-economic (and more specifically, demographic) profile of the household.
[12] To put it another way, households use heuristics in their decision-making to approximate solving complex problems [23].
[13] In other words, I assume that there is effectively a set of fixed coefficient (i.e., Leontief) production technologies. For a given set of patch characteristics (approximated by fallow age or fertility), there is a particular amount of labour required and specific yield or output produced.
[14] The subsistence requirement assumes that household demand for food is perfectly inelastic.

2.3 The household's problem and the first order necessary conditions

The complete nonseparable model for a subsistence household is therefore:

$$\underset{\{u_{st}\}\forall s\in S,t}{Max}\sum_{t=0}^{\infty}\rho^t U\left(\sum_{s=1}^{S}u_{st}f\left(N_{st},e(N_{st}),w(N_{st})\right)-Q_t^f,L_t^T-\sum_{s=1}^{S}k_s(d_s,f_s)u_{st}\left[(1-u_{st-1})e(N_{st})+w(N_{st})\right]\right)$$

subject to:

$$\Delta N_{st}=(1-u_{st})g(N_{st})-u_{st}\left[(1-u_{st-1})b(N_{st})+c(N_{st})\right]$$

$$\Delta L_t^T=\gamma L_t^T-\eta L_t^T+m_t$$

$$N_{so}>0,\forall s$$
$$L_0^T>0$$

To summarize, the subsistence household maximizes the sum of the discounted utility derived from food consumption and leisure subject to law of motion constraints on patch-specific nutrient stocks and on household labour supply when deciding the location of food crop production among the patches of land available to the household (i.e. the choice set *S*).

The current value Hamiltonian for the problem is therefore:

$$H_t=U_t\left(\begin{array}{l}\sum_{s=1}^{S}u_{st}f\left(N_{st},e(N_{st}),w(N_{st})\right)-Q_t^f,\\ L_t^T-\sum_{s=1}^{S}k_s(d_s,f_s)u_{st}\left[(1-u_{st-1})e(N_{st})+w(N_{st})\right]\end{array}\right)$$
$$+\rho\sum_s\lambda_{st+1}\left[(1-u_{st})g(N_{st})-u_{st}\left[(1-u_{st-1})b(N_{st})+c(N_{st})\right]\right]$$
$$+\rho\varphi_{t+1}\left[\gamma L_t^T-\eta L_t^T+m_t\right]$$

where the co-state multipliers $\rho\lambda_{st+1}$ and $\rho\varphi_{t+1}$ are, respectively, the present value (in terms of discounted utility) of an additional unit of the soil resource and of the labour resource in the next period.

Taking the FOCs and rearranging gives:

– the maximal condition:

$$U_q(\cdot)f(\cdot)-U_l(\cdot)k_s(\cdot)[(1-u_{st-1})e(\cdot)+w(\cdot)]-\rho\lambda_{st+1}[g(\cdot)+(1-u_{st-1})b(\cdot)+c(\cdot)]=0\,,\ \forall s\in S,\forall t$$

– the co-state equations for soil nutrients:

$$\begin{aligned}&\rho\lambda_{st+1}\left[1+(1-u_{st})g_{N_{st}}(\cdot)-u_{st}\left((1-u_{st-1})b_{N_{st}}(\cdot)+c_{N_{st}}(\cdot)\right)\right]-\lambda_{st}\\&+U_q(\cdot)u_{st}\left[f_{N_{st}}(\cdot)+f_e(\cdot)e_{N_{st}}(\cdot)+f_w(\cdot)w_{N_{st}}(\cdot)\right]\\&-U_l(\cdot)k(d_s,f_s)u_{st}\left[(1-u_{st-1})e_{N_{st}}(\cdot)+w_{N_{st}}(\cdot)\right]=0\end{aligned}\quad,\ \forall s\in S,\forall t$$

– the co-state equation for labour:

$$\rho\varphi_{t+1}[1+(\gamma-\eta)]-\varphi_t+U_l(\cdot)=0\,,\ \forall t$$

– the nutrient stock laws of motion:

$$N_{st+1}-N_{st}=(1-u_{st})g(N_{st})-u_{st}[(1-u_{st-1})b(N_{st})+c(N_{st})]\,,\ \forall s\in S,\forall t$$

– the labour stock law of motion:

$$L_{t+1}^T-L_t^T=(\gamma-\eta)L_t^T+m_t\,,\ \forall t$$

The maximal condition states that the marginal utility of output over and above the subsistence requirement (the first term) net of the marginal disutility of work (the second term) must equal the discounted foregone utility from not having the particular patch of land available to cultivate later. This must be true for all S patches in equilibrium.

Usually, the first-order conditions are simplified and rewritten in the following form to facilitate interpretation [21]. For the maximal condition:

$$\frac{\partial H(\cdot)}{\partial Y_t}=\rho\lambda_{t+1}$$

the above simplifies to:

$$\frac{U_q(\cdot)f(\cdot)-U_l(\cdot)k_s(\cdot)[(1-u_{st-1})e(\cdot)+w(\cdot)]}{[g(\cdot)+(1-u_{st-1})b(\cdot)+c(\cdot)]}=\rho\lambda_{st+1},\ \forall s\in S,\forall t$$

The left hand side of this equation is the marginal net utility of an additional unit of the soil resource and must equal the right hand side, the opportunity cost equal to the discounted value of having an additional unit of the soil resource available for cultivation in the next period, for the solution to be optimal.

For the co-state equations:

$$\lambda_t=\frac{\partial H(\cdot)}{\partial X_t}+\rho\lambda_{t+1}[1+F'(\cdot)]$$

the above simplify to:

$$\begin{aligned}\lambda_{st}=&U_q(\cdot)u_{st}\left[f_{N_{st}}(\cdot)+f_e(\cdot)e_{N_{st}}(\cdot)+f_w(\cdot)w_{N_{st}}(\cdot)\right]\\&-U_l(\cdot)k(d_s,f_s)u_{st}\left[(1-u_{st-1})e_{N_{st}}(\cdot)+w_{N_{st}}(\cdot)\right]+\\&\rho\lambda_{st+1}\left[1+(1-u_{st})g_{N_{st}}(\cdot)-u_{st}\left((1-u_{st-1})b_{N_{st}}(\cdot)+c_{N_{st}}(\cdot)\right)\right]\end{aligned},\ \forall s\in S,\forall t$$

$$\varphi_t=U_l(\cdot)+\rho\varphi_{t+1}[1+(\gamma-\eta)],\ \forall t$$

The left hand side the first equation is the utility of an additional unit of the soil resource in the current period. At the optimal solution, this is equal to the marginal net utility for the soil resource in the same period plus the marginal utility that would be derived from the unused soil resource in the next period. Similarly, the left hand side of the second equation is the utility of an additional unit of the labour resource in the current period. At the optimal solution, this is equal to the marginal net utility of an additional unit of the labour resource in the same period plus the marginal utility that the additional unit of the labour resource would bring in the next period.

For the laws of motion:

$$X_{t+1}=X_t+F(X_t)-Y_t$$

the above take the form:

$$N_{st+1} = N_{st} + (1 - u_{st})g(N_{st}) - u_{st}[(1 - u_{st-1})b(N_{st}) + c(N_{st})],\ \forall s \in S, \forall t$$

$$L_{t+1}^{T} = L_{t}^{T} + (\gamma - \eta)L_{t}^{T} + m_{t},\ \forall t$$

Typically, one derives the steady state by evaluating the first-order necessary conditions when the control and state variables and the multipliers λ and φ are unchanging. In this problem, one would normally do this by eliminating the time subscripts from *u, N,* λ and φ. This is possible for the co-state equation and law of motion involving labour:

$$\rho\varphi((\gamma - \eta) - \delta) = -U_{l}(\cdot)$$

$$(\gamma - \eta)L^{T} = -m$$

However, due to the nature of shifting cultivation, which is more like a "pulse" fishery[15] or the periodic clear cutting of a stand of forest, eliminating the time subscripts from the maximal condition, the co-state equation for *N* and the nutrient stock law of motion is not advisable. The very nature of resource extraction in this context means that there is not a steady-state level of harvest on an individual patch of land, although there should be a long-term average level of the nutrient stock (see Figure 1).[16]

While it is conceivable that one could solve for the dynamic equilibrium, it is not practicable due to problems with tractability. Deriving the first order conditions required an implicit assumption that the indicator variable u_{st} is continuous, in essence indicating the proportion of a particular patch of land that would be cultivated in a particular year. This would work well should a patch be defined to be a reasonably large plot of land, at a

[15] A pulse fishery is one where the stock is harvested very intensively (at an unsustainable level) for a short period of time and then left to recover on its own.
[16] This merits further study – exploring the nature of a dynamic equilibrium or steady state over all patches taken together for each household.

scale where one would not clear the entire area in any one year. However, to facilitate explicit modelling of the spatial dynamics, it is important to choose a relatively small patch size. By doing so, any one patch would either be cultivated or not in any one year, with the farmer choosing the most preferable patches to cultivate in their entirety and leaving the rest for future years. The decision-maker, therefore, chooses based on that which is most preferable according his or her own criteria.

Rearranging the maximal condition, I have the basis for this comparison:

$$\frac{U_q(\cdot)f(\cdot)-U_l(\cdot)k_s(\cdot)[(1-u_{st-1})e(\cdot)+w(\cdot)]}{[g(\cdot)+(1-u_{st-1})b(\cdot)+c(\cdot)]}=\rho\lambda_{st+1}$$

For any pair of patches, if

$$\frac{U_q(\cdot)f(\cdot)-U_l(\cdot)k_s(\cdot)[(1-u_{st-1})e(\cdot)+w(\cdot)]}{[g(\cdot)+(1-u_{st-1})b(\cdot)+c(\cdot)]}\geq\frac{U_q(\cdot)f(\cdot)-U_l(\cdot)k_{-s}(\cdot)[(1-u_{-st-1})e(\cdot)+w(\cdot)]}{[g(\cdot)+(1-u_{-st-1})b(\cdot)+c(\cdot)]}$$

where *s* and -*s* refer to any two plots and where –*s* denotes all plots that are $\in S$, but $\neq s$, then the patch with the greatest marginal net benefit when put into use should be chosen over all of the others in any particular year. Clearly, the relative attractiveness of any one patch will depend on a number of factors:

- the relative value attached to food in comparison to leisure time because of differences in labour requirements by patch;
- declining productivity of a patch over time as compared to the labour saved by not moving to a new patch and clearing it for cultivation;
- different rates of nutrient accumulation for patches of different fallow age.

The attractiveness or suitability of any particular patch of land will depend on the characteristics of the patch itself (relative to other patches available to the household) and personal preferences with respect to the patch-specific characteristics. If I hypothesize that each household chooses to cultivate the patches *s* that give the greatest marginal net benefit or utility, I can state the reduced form of the model for household *i* as:

$$u_{st}^{*} = f\left(\boldsymbol{Z}_{st}, \boldsymbol{X}_{it} \mid \boldsymbol{W}_{it}\right)$$

where

u_{st}^{*} is a measure of the overall suitability of patch *s* in time *t*

$\boldsymbol{Z}_{st}$ is a vector of plot specific-factors or characteristics

$\boldsymbol{X}_{it}$ is a vector of household-specific factors

$\boldsymbol{W}_{it}$ is a vector of exogenous socioeconomic variables specific to the economy, the village or the household.[17]

The vector $\boldsymbol{Z}_{st}$ includes factors such as the age of the fallow at clearing, fertility status, *N*, length of time under cultivation, proximity to the village, *d*, proximity to other currently cultivated fields, *f*. It also includes variables such as the proximity to neighbours' fields, presence of indicator species for fertility, etc. The proximity to neighbours' fields serves as a proxy for the risk of encroachment by neighbours and the potential for reducing the size of the choice set in future periods. The others provide additional information about the fertility status of a patch of land. The vector $\boldsymbol{X}_{it}$ includes the stock of household labour, *L*, available at time *t*, in or out-migration of labour resources, *m*, the size of the household's land holdings, *S*, availability of additional forested land within the clan or village and other household-specific demographic or socioeconomic variables. Finally, the vector $\boldsymbol{W}_{it}$ includes the subsistence food requirement, *Q*, per person, the rate of natural change of the household labour force, γ, the impact of the general state of health and well-being of the household members on the

[17] Note that a boldfaced letter refers to a vector (***X*** or ***r***) while the regular letter (*X* or *r*) refers to an individual element of the vector. Unless otherwise noted, a vector is assumed to be a column vector.

amount of time available for work and leisure, η, the period for which a household retains land use rights to fallow patches $\left(s \in \{S\} \ni u_{st} = 0\right)$ of land within their stock of land, S, the set of characteristics associated with the location of the village in terms of market access, costs of transport to market and the relative prices of market goods.

Changes in factors ($m>0, \gamma$) which directly or indirectly increase the stock of labour, L, will have an effect on both the total number of patches that can be cultivated in any year and indirectly on the utility derived from cultivation. This occurs either through increased leisure, as there is more time left over after doing the same amount of work, or through a larger food surplus, q, above subsistence needs, Q, as food production increases without sacrificing leisure. The reverse will be true for factors ($m<0, \eta$) that directly or indirectly decrease the stock of labour, L. Migration, m, is a function of price ratios for agricultural products and prevailing urban wage rates and employment opportunities as well as movements related to marriage of members of the household. Each of these influence livelihood choice and have a direct impact on the amount of labour devoted to subsistence agriculture.

The location of a patch of land relative to the village, d, and relative to other patches under cultivation, f, has a direct impact on the perceived suitability of the patch for cultivation. Other things being equal, a patch that is further from the village is less attractive due to the increased cost of travel to and from home each day. Similarly, patches located in close proximity to others that are currently being cultivated result in reduced travel costs since it is possible to carry out various tasks with less time spent on going from one patch to another. This is particularly true for patches in which, for example, the harvest of previously planted cassava and plantain takes place gradually as household needs dictate. Fields located closer to these are more attractive since they do not require a special trip to harvest them when they mature.

The size of the discrete choice set, *S*, among which a household can choose patches for cultivation at any one time, will also influence the returns to cultivation. With a larger choice set, for example, a household can leave land in forest-fallow longer and thereby benefit from improved fertility (higher yields) and reduced weeding labour requirements.[18] The dynamics of the choice set will depend on particular customs related to land tenure and effective use of the land by the household. Depending on these customs, some patches may be lost to the household if they are not cultivated often enough while, on the other hand, the choice set could expand if there is additional forest belonging to the clan or village that the household could clear and claim.

Technological changes may affect the relative costs of labour devoted to land clearing, $b(N)$, as compared to the labour cost of cultivation (especially weeding), $c(N)$, and may therefore change the suitability of different patches of forest or fallow land. Since costs of land clearing increase with N or biomass and therefore with the age of the fallow, anything that reduces the cost of clearing older biomass relative to younger fallows (e.g., a chain saw) will make older fallows relatively more attractive since labour costs associated with cultivation, and weeding in particular, decline as N increases. The reverse will apply for technological changes that reduce the labour cost of weeding younger fallows relative to the cost of older ones since $c(N)$ is decreasing in N or biomass and therefore fallow age. Similarly, the way in which household decision-makers assess the relative importance of time spent in either land clearing or cultivation will also change the relative suitability of different patches.

[18] Utility is nondecreasing in the size of the choice set.

2.4 Decision criteria and model implementation

In this section, I lay out a method by which one can implement this model using survey-based data. Ultimately, what matters for the pattern of land use over space and time is the relative attractiveness or utility of individual patches of land from the point of view of decision-makers. It is therefore important to understand the decision criteria used by households when deciding where to cultivate each year among the available forest or fallow lands. For each cropping season, farmers make a series of decisions with respect to where they will cultivate, how much land to clear, and the particular mixture of crops they will plant. The actual decisions that they make depend upon criteria that include not only the physical characteristics of the natural resources at their disposal (forest, forest-fallow and fallow land patches) but on personal preferences and household-specific demographic characteristics. These decisions result in the particular mosaic of land use observed today and determine how it evolves over time.

One approach to estimating the above reduced-form of the model is to take a revealed-preferences approach and determine the likelihood of clearing a patch in relation to the attributes of households and of specific land use choices. Another approach is to use stated preferences.[19] Since it is possible to ask decision-makers directly about the importance they place on different attributes and generally much cheaper and easier to survey households rather than every plot in a vast landscape, in the empirical

[19] It is most common to use revealed rather than stated preferences in economic analysis since stated preferences may not correspond to actual behaviour. It is possible to compare the preferred age of fallow as stated by decision-makers with the age actually chosen and determine whether or not this problem exits. There are indications that in some circumstances households actually clear younger fallows than they consider optimal [24]. However, this may not necessarily indicate a disconnect between revealed and stated preferences – but rather that there are other factors that weigh heavily in the actual choice of a patch of land for cultivation and which outweigh the stated age preference. By incorporating these factors in the decision model, it is therefore possible to explain any divergence that occurs.

implementation I use survey-based rather than regression-based weights on the vector of patch characteristics.[20]

Each patch of land has observable characteristics, the vector $\boldsymbol{Z}_{st}$, which can be objectively measured and compared to other patches. These observable characteristics include the age of the forest or fallow regrowth, the fertility of the soil as measured by the presence of indicator species, the travel time or distance to the village, the proximity to other cultivated fields, etc. By converting each characteristic to a ratio on a scale from zero to one, the different measures are without specific units.

At the same time, each household attaches differing levels of importance to these characteristics, represented by the vector $\boldsymbol{r}_{it}$, according to household-specific characteristics $\boldsymbol{X}_{it}$. The vector $\boldsymbol{r}_{it}$ is a time-varying preference parameter vector that is a function of household-specific characteristics.[21] As a result, stated preferences vary over time according to changes in household-specific characteristics. In other words, the overall suitability of each patch available to a household is:

$$u^*_{st} = \hat{\boldsymbol{r}}'_{it}\boldsymbol{Z}_{st}\text{, where } \hat{\boldsymbol{r}}_{it} = f\left(\boldsymbol{X}_{it}\right)$$

The elements of $\hat{\boldsymbol{r}}_{it}$ can be estimated individually using a series of ordered logit models relating the importance of a particular patch characteristic r_{it} in the vector $\boldsymbol{r}_{it}$ to the household-specific $\boldsymbol{X}_{it}$'s. The value for u^*_{st} is then determined based on $\hat{\boldsymbol{r}}_{it}$ and the plot-specific characteristics $\boldsymbol{Z}_{st}$ at the beginning of period t. A household will choose to cultivate those patches with the highest scores according to what is feasible as determined by the labour resources available, L. Figure 2 illustrates the sequence diagrammatically.

[20] Although I recognize this practice also introduces the possibility of interviewer bias.
[21] By time-varying preferences, I mean that the estimated preferences vary over time as household characteristics change (especially those related to age and the agricultural labour force).

A household survey conducted in three communities in southern Cameroon during the spring of 2003 [25] quantified individuals' assessments of the relative importance of the various factors influencing their decisions (the vector $\boldsymbol{r}_{it}$). It considered decisions about field location and size as well as decision-makers' assessments of the suitability for cultivation of different aged patches of fallow and forest for the main agricultural field types, including the mixed food crop field cultivated for subsistence food crop production. The factors evaluated in terms of their importance in decision-making included not only those normally considered to be of economic importance, i.e., fallow age (and, therefore, soil fertility) and distance from the field to the village. It also included the importance of non-traditional factors such as the proximity of the patch to the area currently being cultivated, the presence of an invasive weed species, the strategic choice of location in order to protect long-term land use rights to one's land holdings and whether or not the patch could be cleared and still leave adequate time for drying down in advance of burning. Most of the responses to each part were solicited using a modified form of an indigenous board game [26] common in the area.[22] The households interviewed were a subset of those who participated in a resource management survey conducted previously by the International Institute of Tropical Agriculture [22]. For this reason, I was able to link these responses to the household-level socio-economic and demographic data in the resource management survey and estimate the relationship between $\boldsymbol{r}_{it}$ and $\boldsymbol{X}_{it}$.

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[22] In the traditional board game, players move seeds among a matrix of pockets carved in a board which usually consists of two rows of six to eight pockets each. In the method used in this research, one of the decision criteria was assigned to each pocket and farmers were asked to put from one to ten seeds in each pocket according to the importance they attached to that criteria when deciding where to locate an agricultural field.

Having estimated the stated preferences as a function of household-specific characteristics, it is now possible to determine the relative suitability for subsistence food crop production of the different patches of forest that are available to a household. That is, to derive u^*_{st} as a function of the estimated preference parameter vector $\hat{\boldsymbol{r}}_{it}$ and plot-specific characteristics $\boldsymbol{Z}_{st}$ for a specific set of household characteristics $\boldsymbol{X}_{it}$. Ranking of the relative suitability of patches of forest and fallow land is household-specific and will differ across households and within households over time. It is therefore possible to characterize different household types within each of the communities and model their land use choices over space and time.

3 Discussion and conclusion

This paper has discussed the essential issues related to the modelling of forest clearing and cultivation decisions in space and time. By incorporating the spatial dimension into dynamic resource use decisions it is possible to model their landscape-level impacts over time. In addition to this, I have discussed the essential elements for a model of human decision-making where consumption and production decisions are non-separable, an essential feature for dynamic models in the context of subsistence agricultural production. Simplifying assumptions which neglect these considerations for the sake of analytical tractability unfortunately risk failing to capture elements that are essential to understanding human behaviour when resource use decisions are made over space and time by subsistence farming households, as at the forest margins in central Africa, for example.

This paper has developed an analytical model of subsistence agricultural production that is both spatially explicit and dynamic. While it has not been possible to solve it analytically due to issues of tractability, it has been helpful in demonstrating the essential elements of decision-making and the key factors at the household level that might influence the spatial location of agricultural production. Based on these insights, it has been possible to develop an estimable reduced form model. In essence, it uses stated preferences for different characteristics of the resource (forest and fallow land for subsistence food production) and links them to household-specific characteristics, some of which themselves vary over time. In so doing, I develop a time-varying preference parameter vector that, when multiplied by a vector of plot-specific characteristics, assigns a suitability score to each of the plots of land available to household. The household then chooses to cultivate those plots that are most suitable in any one year within the constraints of labour availability and subsistence food needs.

This decision-making model has been incorporated into a simulation model of subsistence agricultural production [25]. Given the significant heterogeneity of household characteristics, the model simulates several households simultaneously. By proceeding in this fashion, it is possible to simulate the differential impact of changes in exogenous parameters on household decisions (and hence the pattern of land use) as well as how patterns change over time as household-specific characteristics change. Further development will enable modelling either individual households or a collection of representative households at the village scale in order to effectively to describe the spatial mosaic of forest resource use that evolves over time. Ultimately, this work should serve to develop an effective tool for analysis of the sustainability of subsistence agricultural practices at the household level, in terms of livelihood, as well as at the landscape-scale, in terms of land cover change and biodiversity conservation.

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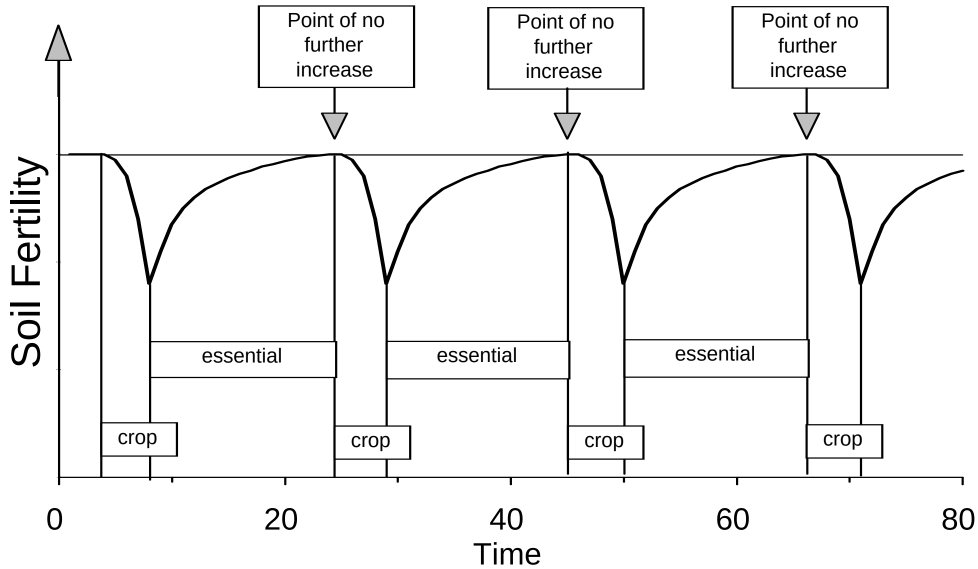

Figure 1: Nutrient dynamics with alternate cycles of cultivation and forest regrowth [after 27].

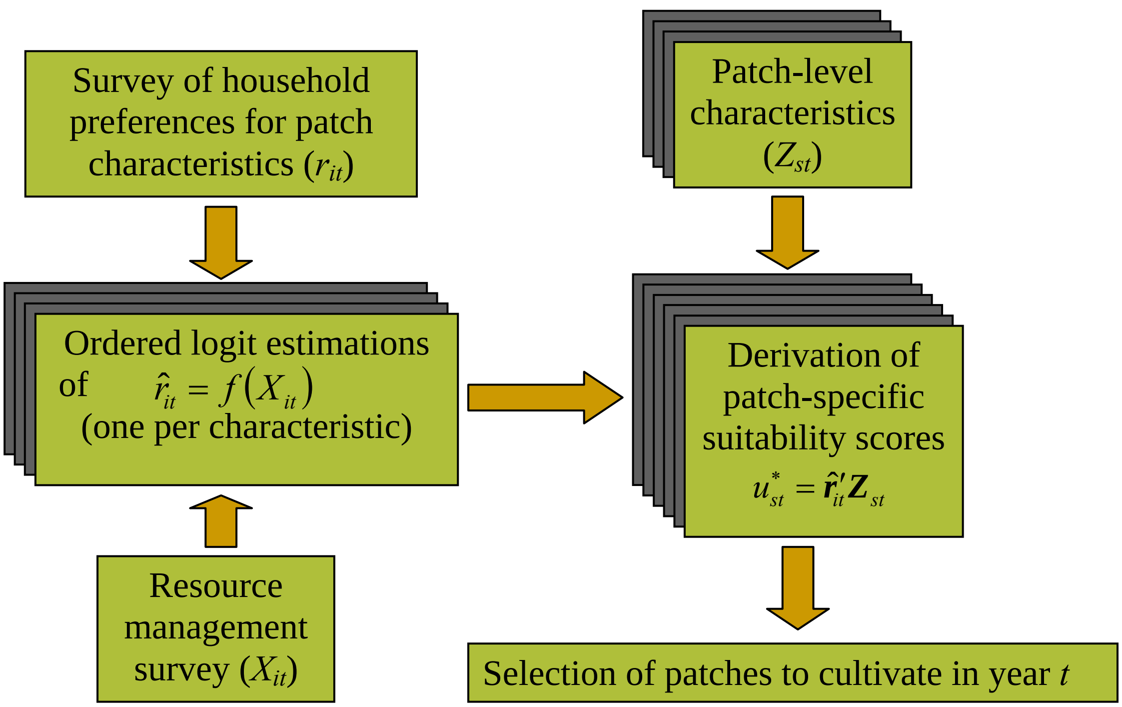

Figure 2: Schematic diagram of model estimation and implementation.

Appendix 1 – Optimal rotation period for a pure subsistence household

Perhaps the simplest approach is to take the purely dynamic problem faced by the subsistence household. In this case, the problem becomes one of determining the optimal length of fallow, *T*, for a subsistence household so as to maximize the discounted utility, *U*, derived from food surplus, q^s, (for consumption and sale over and above a subsistence constraint, Q^f), and leisure, *l*, when the plot is cleared and cultivated.[23] The optimal rotation problem, for one rotation, is:

$$\underset{\{T\}}{Max}\, U\left(q^s, l\right) e^{-\delta t}$$

where δ is the discount factor.

If we consider a stock of land that has recently been abandoned to fallow after a period of cultivation, the stock of nutrients accumulates as a function of the time *t* in fallow, asymptotically approaching an upper limit:

$$N(t) \text{ where } \frac{\partial N}{\partial t} > 0, \frac{\partial^2 N}{\partial t^2} < 0, N \leq \overline{N}$$ [24]

The patch generates a benefit in the form of food production, *y*, following clearing, which is assumed a function of labour input, *L*, and the stock of available nutrients, *N*. However, as the labour required for clearing *e(N(t))* and cultivation *w(N(t))* is itself a function of the nutrient status (in particular biomass, which increases with the duration of the fallow, and of weeds, which decrease over the fallow duration) food production is:

[23] This is essentially a modified version of the Faustmann rotation [20], as commonly referred to in the resource economics literature [21, pages 63-64].

[24] The upper bar (i.e., $\overline{N}$) represents an upper bound, while a lower bar (i.e., $\underline{w}$) represents a lower bound.

$y = f[N(t), L(N(t))]$

where

$L(N(t)) = e(N(t)) + w(N(t))$

$e(N(t))$ is labour for clearing and $e_N > 0, e_{NN} < 0, e < \bar{e}$[25]

$w(N(t))$ is labour for cultivation and $w_N < 0, w_{NN} > 0, w > \underline{w}$

The total labour required for fieldwork is adjusted by a travel time factor based on the distance, *d*, from the village to the particular patch as well as the proximity to other fields, *f*, or patches being actively cultivated:

$k(d_s, f_s)$, where $k \in [1, \bar{k}), k_d > 0, k_f > 0$ and $\bar{k}$ is the maximum feasible time to walk to do a day's work.

Since the amount of labour required to cultivate a patch cleared at instant *t* is essentially fixed based on the amount of *N* at clearing, we can abbreviate *y* to:

$y = f[N(t)]$

and *L* to:

$L(t) = e(t) + w(t)$

The optimal rotation problem, for one rotation, is therefore:

$$\underset{\{T\}}{Max}\, U(q^s, l)e^{-\delta t}$$

subject to the subsistence and labour requirements:

$$q^s = f[N(t), e(N(t)), w(N(t))] - Q^f$$
$$l = L^T - k(d, f)[e(N(t)) + w(N(t))]$$

This simplifies to:

[25] Subscripts reflect partial derivatives.

$$\underset{\{T\}}{Max}\, U\left(f\left[N(t), e(N(t)), w(N(t))\right] - Q^f, L^T - k(d,f)\left[e(N(t)) + w(N(t))\right]\right)e^{-\delta t}$$ and then

to:

$$\underset{\{T\}}{Max}\, U\left(f\left[N(t)\right] - Q^f, L^T - k(d,f)\left[e(t) + w(t)\right]\right)e^{-\delta t}$$

For an infinite number of rotations, the problem becomes:

$$\begin{aligned}\underset{\{T\}}{Max}\, U_{\infty}(.) &= U(.)e^{-\delta T} + U(.)e^{-\delta 2T} + \ldots \\ &= U(.)e^{-\delta T}\frac{1}{1-e^{-\delta T}}\end{aligned}$$

The FOC for this problem is:

$$\frac{\partial U_{\infty}(.)}{\partial T} = 0 \Rightarrow U_T(.) = \frac{\delta U(.)}{1-e^{-\delta T}}$$

and thus:

$$\left(1-e^{-\delta T}\right)U_T(.) - \delta U(.) = 0$$

Substituting in for *U* gives:

$$\begin{aligned}&\left(1-e^{-\delta T}\right)\left[U_q f_N N_t(.) - k(d,f)\left[U_l e_t(.) + U_l w_t(.)\right]\right] \\ &-\delta U\left[f(N(t)) - Q^f, L^T - k(d,f)\left[e(t) + w(t)\right]\right] = 0\end{aligned}$$

Defining the above as

$$g\left(T^*(\alpha), \alpha\right) \equiv 0$$

with α any parameter in the model, by the implicit function theorem, I have:

$$g_T \frac{\partial T^*}{\partial \alpha} + g_{\alpha} \equiv 0 \Rightarrow \frac{\partial T^*}{\partial \alpha} = \frac{-g_{\alpha}}{g_T}$$

Using the implicit function theorem it is possible to examine the relationship of distance, *d*, and labour availability, L^T, to the optimal length of fallow, *T*. In other words:

for $\alpha = d$, $\frac{\partial T^*}{\partial d} = \frac{-g_d}{g_T}$

for $\alpha = L^T$, $\frac{\partial T^*}{\partial L^T} = \frac{-g_{L^T}}{g_T}$

The required derivatives are:

$$g_T = \left(1 - e^{-\delta T}\right)\left[U_{TT}(.) - \delta U_T(.)\right]$$

$$g_T = \left(1 - e^{-\delta T}\right)\begin{pmatrix} U_{qq}\left[f_N N_t\right]^2 + U_q f_{NN}\left[N_t\right]^2 + U_q f_N N_{tt} \\ -k(d,f)\left[U_{ll}\left[e_t\right]^2 + U_l e_{tt} + U_{ll}\left[w_t\right]^2 + U_l w_{tt}\right] \\ -\delta\left[U_q f_N N_t - k(d,f)\left[U_l e_t + U_l w_t\right]\right] \end{pmatrix}$$

which can be rearranged as:

$$g_T = \left(1 - e^{-\delta T}\right)\begin{pmatrix} U_{qq}\left[f_N N_t\right]^2 + U_q f_{NN}\left[N_t\right]^2 + U_q f_N N_{tt} - \delta U_q f_N N_t \\ -k(d,f)\left[U_{ll}(e_t)^2 + U_l e_{tt} + U_{ll}(w_t)^2 + U_l w_{tt} - \delta\left(U_l e_t + U_l w_t\right)\right] \end{pmatrix}$$

For $\alpha = d$:

$$g_d = -\left(1 - e^{-\delta T}\right)\left[k_d(d,f)\left[U_l e_t + U_l w_t\right]\right] + \delta U_l(.)\left[k_d(d,f)\left[e(t) + w(t)\right]\right]$$

For $\alpha = L^T$:

$$g_{L_T} = -\delta U_l(.) < 0$$

Without some very specific assumptions regarding the relative marginal costs of labour for clearing, *e*, and for weeding, *w*, as well as an indication of the relative contributions to utility of the food surplus, q^f, and leisure, *l*, the sign on g_T is indeterminate. The same situation applies to the sign on g_d. Even though the sign on g_{L_T} is negative, the sign of the derivative of T with respect to L^T remains indeterminate due to the denominator, g_T. A similar conclusion applies to the proximity to other fields, *f*.

As can be seen, even with a very simplified nonseparable model, which only accounts for the time dimension, the analytics do not give clear results.[26] This is not only because both food consumption and leisure appear in the objective function. The problem would also exist in a separable model that only looked at net returns to food production, for example, because of the trade-off between labour requirements for clearing and for weeding. As fallow or forest age increases, there are increasing requirements for labour for clearing, but declining requirements for labour for weeding. It is certainly possible to derive an estimable reduced form. However, it would not address the problem of the spatial dimension and, due to the long periods of fallow required, one household would have several patches of forest for which it would simultaneously need to determine the optimal fallow length since the same labour and subsistence constraint would apply to them all.

[26] Except for the inclusion of the impact of distance on travel time and labour availability, we have not yet made the model spatially explicit.