Information Acquisition in a Limit Order Market

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- How valuable is information in a financial market?
- Are markets informationally efficient?
  and are they allocationally efficient?
- Specific context: Limit Order Market
  - How traders benefit from private information determines if they acquire it
  - How investors trade on private information affects informationally efficiency
- We numerical determine:
  - How is information incorporated into market outcomes?
  - What effect does information acquisition have on equilibrium outcomes?
  - Which investors acquire costly information? and how does it affect their trading strategies?
  - Do these markets work?

- Two ways to think about the gains from trade:
  1. Agents Trade for Risk Sharing Reasons.
     - This leads to the Hirshleifer effect: prices that reveal too much can destroy risk-sharing gains.
     - Information is not socially useful.
     - Grossman-Stiglitz (1980) paradox: if prices are fully revealing, no one will acquire costly information.
  2. Gains to trade are fixed
     - Akerlof: adverse selection deters trade

Limit Order Markets

- Limit Order Markets: Continuous double auction in which agents can either post prices or accept posted priced.
- Two important reasons to study them:
  1. They are becoming the dominant market form
     - Some exchanges are exclusively LO markets, e.g., Paris, Toronto, Sydney.
     - Most exchanges e.g. NYSE, Nasdaq have incorporated the LO book in to their design.
  2. Theoretically:
     - Blume and Easley showed that generically there is no game that has the Rational Expectations equilibrium as an outcome.
     - Perry and Reny showed a double auction with discrete prices that converges to the Rational Expectations outcome.

What is information?

- Different types are valuable:
  - Information about the book (trading opportunities)
  - Publically available information — earnings announcements and news.
  - Private information (about idiosyncratic shocks) — illegal to trade on $\Longrightarrow$ appears in other prices.
- Information substitutes, and seen by a lot of traders

Key Results

- Adverse selection matters for individuals but not for the market as a whole
  - Trade is not deterred by adverse selection.
- The limit order book is remarkably successful at consummating trade: is achieves 92% of a theoretical frictionless benchmark.
- For a given acquisition cost, there are multiple equilibria, in which different sets of investors are informed.
  - Uninformed agents change strategies based on how many agents are informed.
  - Equilibria can be Pareto-ranked.
- Prisoner's dilemma: there exists an equilibrium such that:
  - it is a best response for all agents to acquire information.
  - all agents are worse off than if no agent had acquired information.

Literature Review

- GE rational expectations models:
  - Grossman (1976)
  - Grossman and Stiglitz (1980)
  - Admati and Pfleiderer (1987)
  - Ausubel (1990)
- Noisy results:
  - Admati (1985) demonstrates that intuition obtained from models with single risky asset may not go through with multiple risky assets.
  - Barlevy and Veronesi (2000) show that CARA-normal phenomena may be reversed in more general models.
- Microstructure models with strategic traders:
  - Kyle (1985)
  - Taub, Bernhardt, and Seiler (2004)
  - Holden and Subrahmanyam (1992)
  - Back, Cao and Willard (2000)
  - Mendelson and Tunca (2004).
- Market maker sets prices equal to expected value of asset, conditional on all public information.
- Dynamic microstructure models.
  - Foucault, Kadan and Kandel (2003), Rosu (2004), Goettler, Parlour and Rajan (2004)

- Glosten (1994)

Framework

1. Numerically solve for Equilibrium of the Trading sub-game
2. Using these payoffs compute equilibria in the Information acquisition game.

Structure

- Market for single financial asset.
- Time is continuous, starting at time 0.
- Poisson arrival process for new traders. Arrive at rate λ.
- Traders can post orders at discrete prices $p^0, p^1, \cdots$.
  - Evenly-spaced price grid.
  - Defined tick size.
- At each t, each price has a backlog of outstanding orders, ℓ_t^i.
  - Collection of orders defines the limit order book $L_t = \{\ell_t^0, \ell_t^1, \cdots\}$.
- Asset has a common value v_t (present value of future cash flows).
  - Changes over time. Innovations process has Poisson distribution, mean μ.
  - At each innovation, probability $\frac{1}{2}$ of v increasing/decreasing by 1 tick.

- "Information" here refers to knowledge of v_t.
  - E.g. earnings reports, option prices.
  - Potentially public information, but cost to acquire it (pay a news service, opportunity cost of time).
- Each trader has a type $\theta = \{\rho, \beta\}$.
  - $\rho =$ continuous discount rate.
  - β = private value to trade: reflects liquidity motive, hedging needs.
- Before entering market, trader is allowed to purchase information about v at a cost c.
  - Paying this cost is akin to purchasing a subscription service.
  - Informed traders observe current v_t on each entry to market
  - Uninformed traders observe v with a lag Δ_t.

Trading Sub-game

- Fix information acquisition choices.
- On arrival at the market, a trader can submit one of :
  1. a market order (executes against a previously posted limit order)
  2. a limit order (order to buy or sell at a particular price p^j)
  3. no order.
- In cases 2 and 3, trader stochastically re-enters the market.
  - Re-entry time drawn from Poisson process.
  - may execute before he gets a chance to re-enter.
  - on re-entry, can change his order, or leave it in place.
- Each trader allowed to trade one share of the asset
  - Buy or sell is endogenous.

- Suppose a trader j first enters at time τ, and buys a share at price p^i at time t.
- Payoff: $= e^{-\rho(t-\tau)} \, (\beta + v_t - p^i)$.
- Traders are risk neutral; maximize expected payoff.
  - Since they can re-enter, solve a dynamic program to decide optimal order.
- Consider an informed trader who re-enters the market at time t.
  - State space $= (s, a)$.
  - $s =$ market state $= (L_t, v_t)$.
  - $a = (p, q, x) =$ status of previous order.

- Consider a trader who enters at time $t = 34.2$, with $v_t = 37$, and L_t given by:

Price	Depth	
39	-2	
38	-2	$\leftarrow$ *Ask*
37	3	$\leftarrow$ *Bid*
36	4	

- Suppose this is a first entry (so no previous order).
- Suppose he places a buy order at 37; this is 4^{th} in the buy queue at this price.
  - Queue served on basis of price-priority, with first-come, first-served at each price.

- At some random time, he re-enters the market (say this is time $t = 45.1$).
- Before he re-enters, the following events may happen:

  1. His time priority in the queue at $p = 37$ improves.

  2. He executes at $p = 37$—another trader submits a market sell that executes against his order.

  3. His overall price priority decreases (e.g., the Bid moves to 38).

  4. v changes, to (say) 33.

- If he executes: earns his payoff and leaves the market.
  - Execution time is random.
  - Depends on types of new traders + strategies.
- Otherwise, he re-enters the market, examines status of his old order, and either leaves it in place, or cancels it and submits a new order.
- A strategy for a trader is a mapping from type and state to action.

- Let $\pi(s, \tilde{a}, w, \sigma)$ denote the expected payoff to an order due to execution prior to re-entry at some time w.

- Then, the Bellman equation for his dynamic programming problem is

$$\begin{aligned} J(s, \sigma) \; = \; & \max_{\tilde{a} \in A(s)} \int_0^\infty \; \Big\{ \pi(s, \tilde{a}, w, \sigma) \; + \\ & e^{-\rho w} \int_{s' \in S_\theta} J(s', \sigma) \; h(s' \mid s, \tilde{a}, w, \sigma) ds' \Big\} \, dG(w). \end{aligned} \qquad (1)$$

Existence

- Trading game: restrict action space to be k ticks on either side of belief about v (in practice, no orders are submitted further away).
- Finite action space, countable state space (changes in state space happen after discrete intervals).

- Existence of Markov-perfect equilibrium follows from standard results (e.g. Reider, 1979).

- Information acquisition game: finite action space, existence is immediate.

Solving for equilibrium

- Numerical solution for trading game, along the lines of Pakes and McGuire (*Econometrica* 2001).

- Directly solve for value of each state.

- Usual guess-and-update procedure.

- Two main features:
  1. asynchronous updating: update value for each state in single simulation.
     - Start with some initial beliefs for each state, pick a random state.
     - Traders enter, takes optimal action given beliefs.
     - Each time payoff is realized, update belief for the corresponding state.
  2. Determine values only on recurrent class of states.

- While solving, allow for small probability of trembles to update utilities of actions not taken.
  - Take tremble probability to zero as we converge to solution.
- Only numeric uniqueness.
- Usual convergence tests:

  1. Den Haan & Marcet (1994): χ^2 test on $($ believed value $-$ actual value $)^2$.

  2. Pakes and McGuire (2001):

     (a) Mean absolute error < 0.01

     (b) Correlation between beliefs and actuals > 0.99.

Simulation Results

- Rate of new trader arrival $= 1$.
- Re-entry interval $= 4$, on average
- On average, v changes every 8 units of time.
- $\rho = 0.05$.
- β distribution:

β	-8	-4	-0.1	0.1	4	8
Probability	0.15	0.2	0.15	0.15	0.2	0.15

- Consider four possible information regimes:

  1. No agents informed about current v (i.e., all observe v with 16-period lag).
  2. Agents with $|\beta| = 0.1$ informed about current v
  3. Agents with $|\beta| \in \{0.1, 4\}$ informed about current v
  4. All agents informed about current v.

- In each regime, determine payoff to agents who deviate in information acquisition, and play optimally thereafter.

Information Structure			Value of $\|\beta\|$		
			0.1	4	8
None Informed	Equilibrium	Mean	0.403	3.515	7.333
		Std. Dev	0.968	1.499	1.607
	Deviation	Mean	*1.178*	*3.652*	*7.353*
		Std. Dev	0.772	1.342	1.558
	Value of Information	Mean	0.776	0.138	0.020
Speculators Informed	Equilibrium	Mean	*0.628*	3.413	7.204
		Std. Dev	0.662	1.040	1.137
	Deviation	Mean	0.413	*3.499*	*7.228*
		Std. Dev	0.779	0.955	1.072
	Value of Information	Mean	0.215	0.086	0.024
$\beta \in \{.1, 4\}$ Informed	Equilibrium	Mean	*0.495*	*3.508*	7.234
		Std. Dev	0.542	0.792	0.965
	Deviation	Mean	0.287	3.414	*7.279*
		Std. Dev	0.587	0.877	0.864
	Value of Information	Mean	0.207	0.094	0.044
All Informed	Equilibrium	Mean	*0.447*	*3.510*	*7.311*
		Std. Dev	0.469	0.766	0.855
	Deviation	Mean	0.244	3.430	7.251
		Std. Dev	0.459	0.804	0.895
	Value of Information	Mean	0.203	0.080	0.060

Notes
(i) Reported means and standard deviations are averages and sample standard deviations from market simulations over 100 million arrivals (new and returning traders).
(ii) Reported numbers exclude agents who trembled to suboptimal actions.
(iii) Standard errors on means are less than .0005 for equilibrium strategies and less than .0020 for deviator strategies (for which only 2% of traders deviate).
(iv) Payoffs in *italics* indicate informed agents.

Table 1: **Payoffs (in ticks) in different information structures**

Endogenous Information Acquisition

- Speculators have the greatest incentive to acquire information.

Observation 1 *If any agent acquires information, agents with* $|\beta| = 0.1$ *acquire information.*

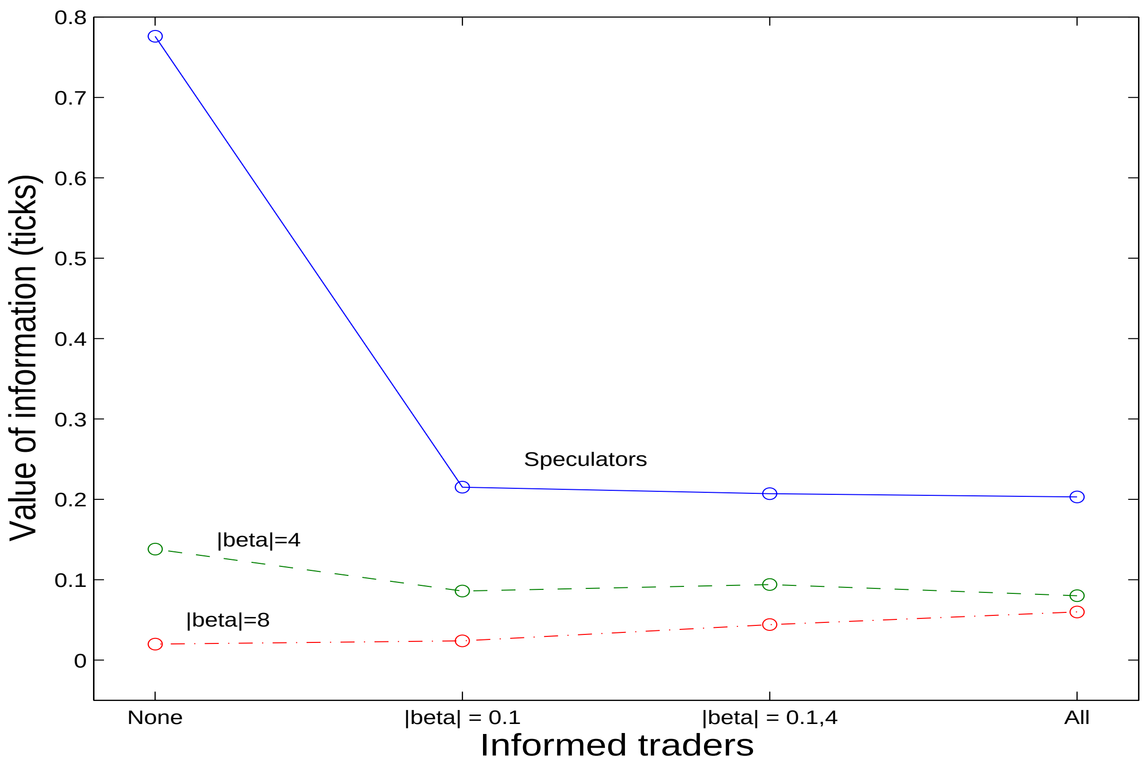

Figure 1: **Value of information**

Observation 2 *The following are equilibria in the information acquisition game:*

$$c \in \begin{cases} [0, 0.060) & \text{all agents acquire information} \\ (0.044, 0.094) & |\beta| \in \{0.1, 4\} \ \text{ acquire information} \\ (0.086, 0.215) & |\beta| = 0.1 \ \text{ acquire information} \\ (0.776, \infty) & \text{no agent acquires information} \end{cases}$$

- Multiple equilibria in information acquisition game.

Observation 3
(a) For $c \in (0.044, 0.060)$ *there are two equilibria:*
(i) $|\beta| \in \{0.1, 4\}$ *acquire information.*
(ii) All agents acquire information.
(b) For $c \in (0.086, 0.094)$*, there are two equilibria:*
(i) Only speculators acquire information.
(ii) $|\beta| \in \{0.1, 4\}$ *acquire information.*

- Prisoner's dilemma in information acquisition.

  Observation 4 *For $c \in (0.044, 0.060)$, the equilibrium in which all agents are informed reflects a prisoner's dilemma—each type of agent would prefer to be in the regime in which no agents are informed.*

- Best response for some agents to acquire information.
- Leads to all agents being worse off as a result.
- Similar to Hirshleifer (1971): more information can make all agents worse off.
- Different from Hirshleifer: potential gains to trade are constant across information acquisition regimes.
- Uninformed agents worse off due to adverse selection.

- Informed agents do benefit from this, in terms of gross payoff.
- Worse off after accounting for information acquisition cost.

Informed Agents	Value of $\|\beta\|$ 0.1	4	8
$\|\beta\| = 0.1$	17.22	2.19	0.62
Deviators	25.73	2.46	0.58
$\|\beta\| \in \{0.1, 4\}$	19.41	2.13	0.56
All	19.64	2.09	0.59

Table 2: **Average time to execution**

- Speculators skew terms of trade in their favor when informed.

Informed Agents	$\beta = 0.1$ Buy Orders	$\beta = 0.1$ Sell Orders	$\beta = 4$ Buy Orders	$\beta = 8$ Buy Orders
Speculators:				
Equilibrium	−1.21	1.30	0.21	0.57
Deviators	−1.30	1.18	0.00	0.62
$\lvert\beta\rvert \in \{0.1, 4\}$	−1.17	1.19	0.11	0.57
All agents	−1.14	1.07	0.10	0.48

Table 3: **Average of (price minus common value) for executed orders, in ticks**

Allocative Efficiency

- Optimal incentive compatible mechanism is an open question.

- Consider two benchmarks for planner:

1. Frictionless benchmark W_f.
   - Suppose all agents in market at the same time.
   - Consummate all trades at price $= v$.
   - Agents with $\beta < 0$ are sellers, $\beta > 0$ are buyers.
   - Clearly IC.
   - Ignores all frictions.

2. Naïve IC mechanism (W_ℓ)
   - LIFO rule for trading.
   - All trades occur at price $= v$.
   - Respects discounting, trader arrival sequence.
   - Also IC.

- In terms of gross surplus, market does better than naïve mechanism

	Frictionless: W_f	LIFO: W_ℓ	All Informed	$\|\beta\| = \{0.1, 4\}$ Informed	$\|\beta\| = 0.1$ Informed	None Informed
Gross surplus	4.030	3.482	3.734	3.724	3.718	3.730
Net surplus	4.030	3.482	$3.734{-}c$	$3.724{-}0.7c$	$3.718{-}0.3c$	3.730

Table 4: **Welfare gain per trader, and benchmarks**

Informational Efficiency

	All Informed	$\lvert\beta\rvert \in \{4, 0.1\}$ Informed	Speculators Informed	None Informed
Standard deviation of $(p - v)$	0.80	0.91	1.11	1.61
Mean absolute error in uninformed agents' belief about v	0.31	0.39	0.49	1.05

Table 5: **Volatility in prices and errors in beliefs**

Independent Variable	Correlation with dependent variable	Regression 1	Regression 2
Constant	0	-0.00 (0.61)	0.00 (1.87)
Lagged common value, $v_{t-\Delta_t}$	0.39	0.06 (114.86)	0.08 (140.47)
Signed order, b_t	0.01	-0.06 (80.49)	
Last transaction price, $\hat{p}_t$	0.68	0.22 (231.73)	0.21 (202.57)
Bid price, B_t	0.72	0.25 (204.40)	0.22 (166.73)
Ask price, A_t	0.72	0.25 (203.65)	0.21 (164.78)
Bid depth	0.00	0.13 (120.86)	
Ask depth	-0.00	-0.13 (120.29)	
Total buy depth	0.01	-0.04 (71.38)	
Total sell depth	-0.00	0.04 (72.12)	
No. of observations		146,069	146,069
R^2		0.81	0.74

Note: t-statistics in parentheses

Table 6: **Regression of change in belief about v, when only speculators are informed**

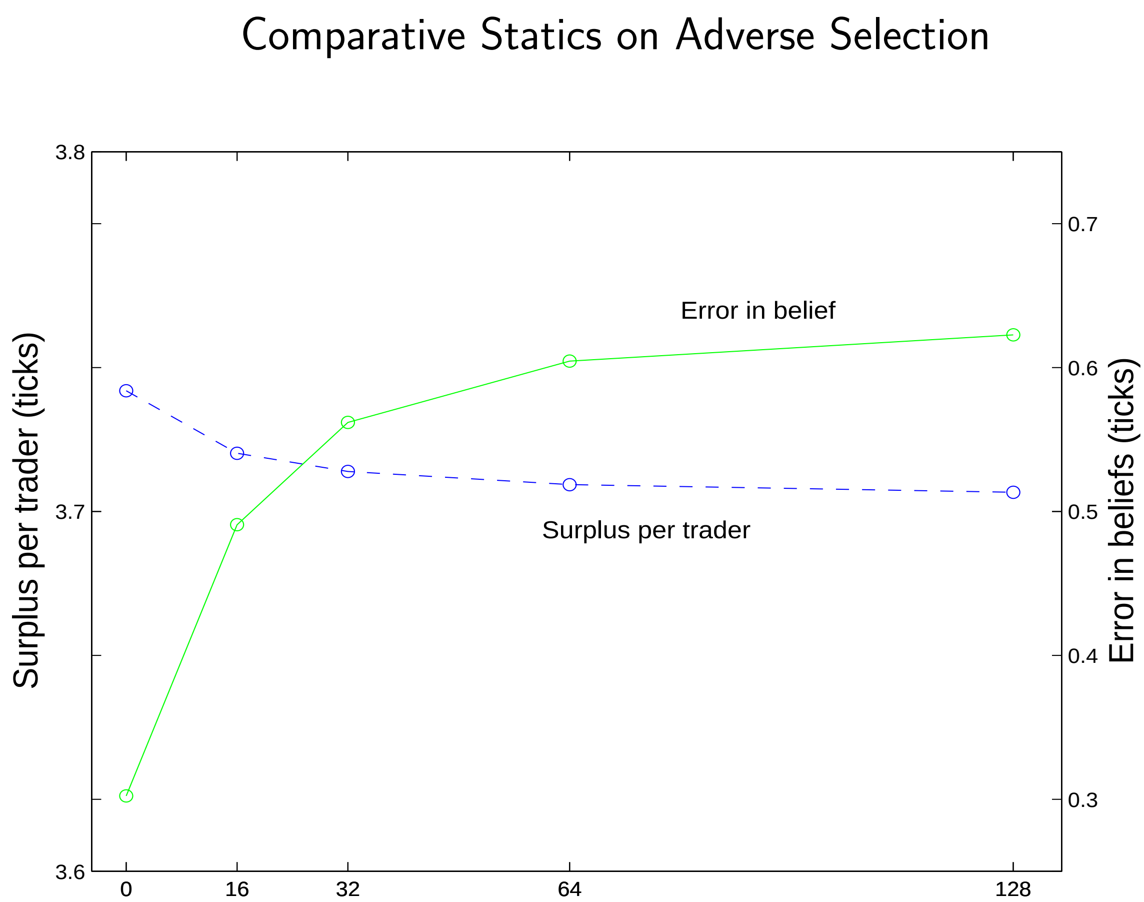

Figure 2: **Aggregate surplus and mean absolute error in belief, as Δ_t changes**

Conclusion

- Speculators most likely to be informed.
- Information does find its way into market observables.

- Prisoner's dilemma: information may lead to all agents being worse off.

- Adverse selection important at individual level, not at aggregate level.

- The transparent limit order book is an effective mechanism for consummating trade.