Default Risk in Corporate Yield Spreads

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HEC MONTRÉAL

Outline of the presentation

- Motivation
- Overview of the approach
- The discrete-time model
- Estimation of zero-coupon yield curves
  - Assumptions; Default probabilities; Results;
- The continuous-time model
  - Assumptions; Estimation of intensities; Results;
- Extensions

Motivation

- **Corporate yield spreads composition:**
  - Default
  - Liquidity
  - Taxes

- **Current literature :**
  - Default represents approximately 25% of the spreads !
  - Taxes also represent a substantial portion !

Motivation (continued)

- **This study :**
  - Empirical study
  - Re examination of the estimation of the proportion attributable to default risk
  - Alternative measures of default probabilities

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Overview of the approach

Treasury and corporate zero-coupon yield curves

Exogenous estimates of default probabilities or intensities

Recovery rates

↓

Model

↓

Proportions of corporate rate spreads attributable to default

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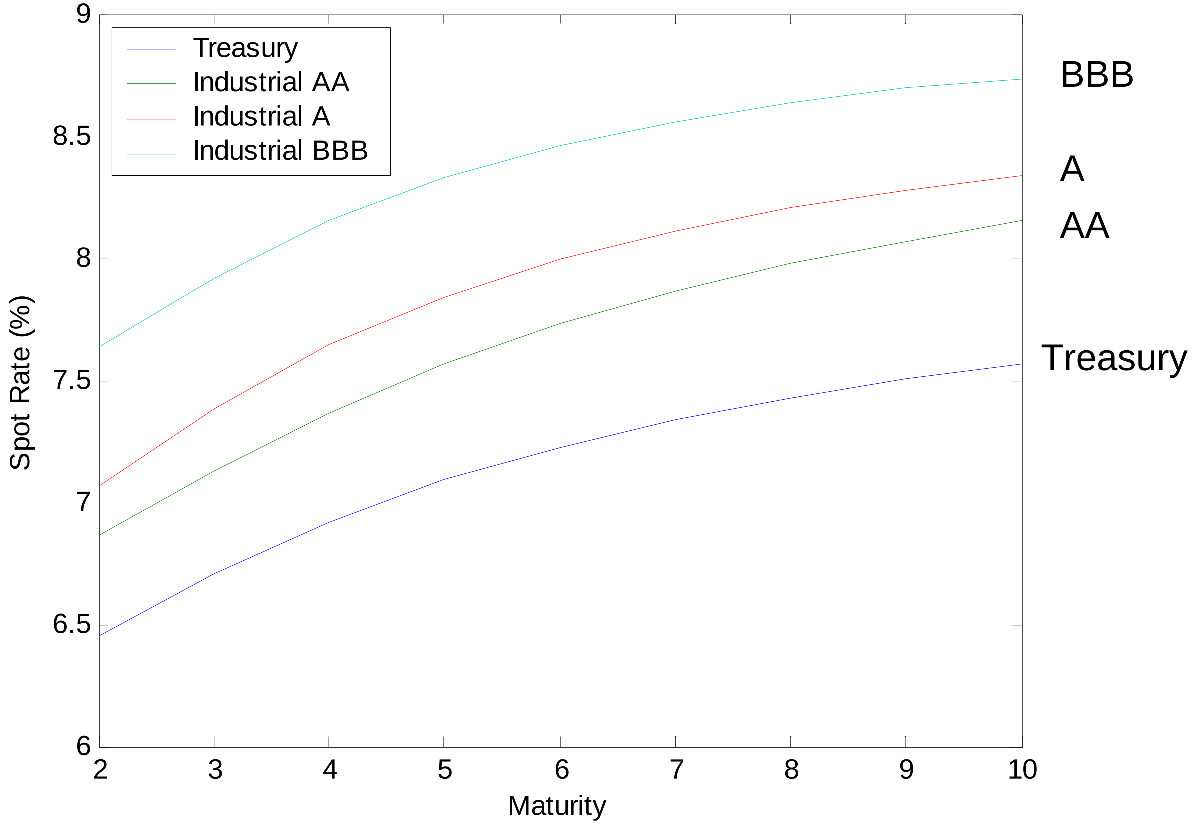
Yield curves estimation

- **Data :**
  - Lehman Brothers Fixed Income data base
  - Monthly price quotes
  - Categories: AA, A, BBB
  - Removal of all bonds with matrix price and options
  - Industrial

- **Estimation :**
  - Nelson and Siegel (1987) approach

Yield curve estimation (continued)

- **Results (averages for 1987-1996)**



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Discrete-time model

- **Assumptions:**
  - Existence of a risk-neutral measure
  - Independence of risk free rate and default time
  - Recovery of a fraction of the market value
  - Constant probabilities
  - Risk neutrality

Discrete-time model (continued)

$$\tilde{P}_T = P_T \times p_T$$

$\tilde{P}_T$: price of a corporate zero-coupon

P_T : price of a risk-free zero-coupon bond

$$p_T = \rho \sum_{u=1}^{s} p_{s-u} q_u + \left(1 - \sum_{u=1}^{s} q_u\right)$$

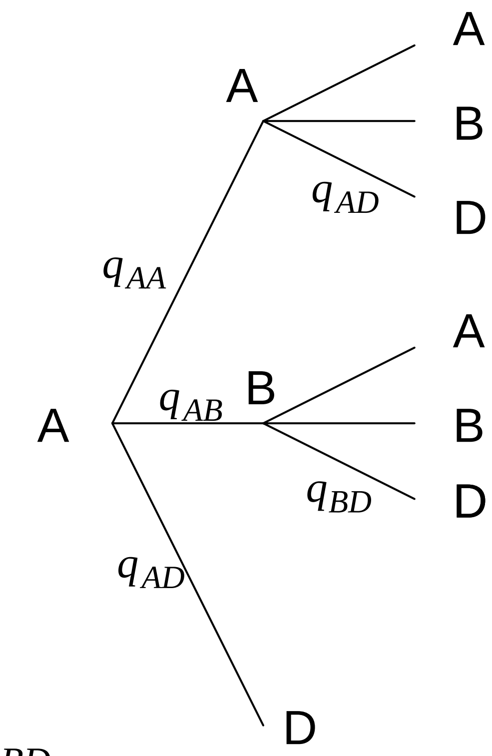
ρ : recovery rate

q_u : probability of default in u periods from now

Discrete-time model (continued)

- **Default probabilities estimation:** **A simplified example**

	A	B	D
A	q_{AA}	q_{AB}	q_{AD}
B	q_{BA}	q_{BB}	q_{BD}
D	-	-	1.0



default in one year: $q_1 = q_{AD}$

default in two years: $q_2 = q_{AA} \times q_{AD} + q_{AB} \times q_{BD}$

Discrete-time model (continued)

- **Moody's transition matrix (one year horizon)**

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.9189	0.0738	0.0071	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
AA	0.0113	0.9126	0.0709	0.0030	0.0020	**0.0000**	**0.0000**	**0.0000**
A	0.0010	0.0256	0.9118	0.0532	0.0061	0.0020	**0.0000**	**0.0000**
BBB	**0.0000**	0.0020	0.0536	0.8793	0.0546	0.0082	0.0010	0.0010
BB	**0.0000**	0.0010	0.0042	0.0499	0.8512	0.0733	0.0042	0.0159
B	**0.0000**	0.0010	0.0010	0.0054	0.0597	0.8219	0.0217	0.0890
CCC	**0.0000**	0.0043	0.0043	0.0087	0.0251	0.0589	0.6779	0.2205
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

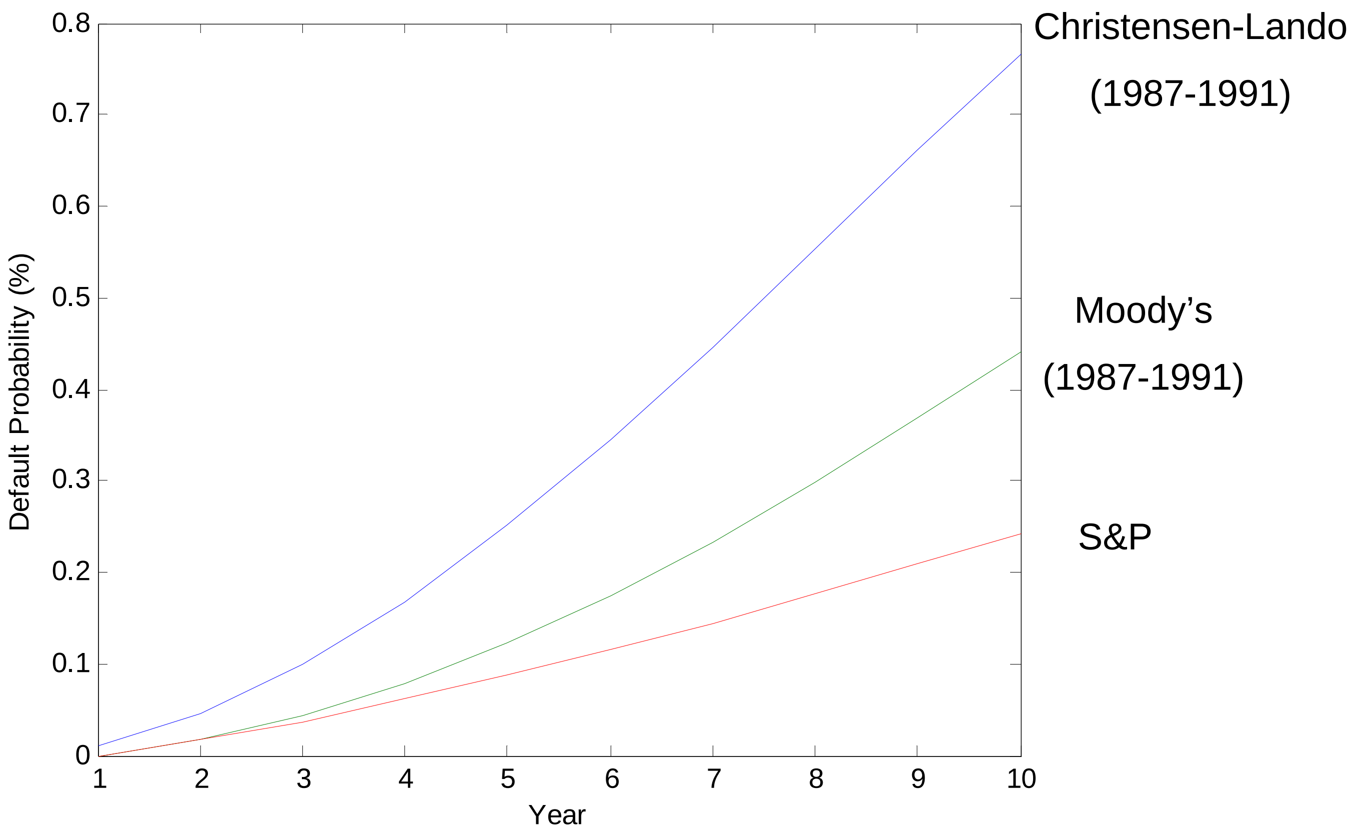
Discrete-time model (continued)

- **Christensen and Lando (2002) (one year horizon)**

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.9308	0.0604	0.0050	**0.0033**	**0.0002**	**0.00009**	**.000005**	**.000005**
AA	0.0159	0.8320	0.1386	0.0090	0.0024	**0.0016**	**0.00007**	**0.0001**
A	0.0015	0.0196	0.8778	0.0797	0.0151	0.0053	**0.0002**	**0.0004**
BBB	**0.0011**	0.0086	0.0591	0.8223	0.0810	0.0234	0.0017	0.0025
BB	**0.0001**	0.0029	0.0116	0.1173	0.7110	0.1221	0.0084	0.0263
B	**0.00001**	0.0002	0.0045	0.0223	0.0617	0.7479	0.0604	0.1027
CCC	**0.00005**	0.0005	0.0187	0.0504	0.0190	0.0323	0.2669	0.6117
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

Discrete-time model (continued)

- **Term structure of default probabilities (AA)**



Discrete-time model (continued)

- **Recovery rate assumption**

  – From Altman and Kishore (1998)

  – AA: 59.59
  – A: 60.63
  – BBB: 49.42

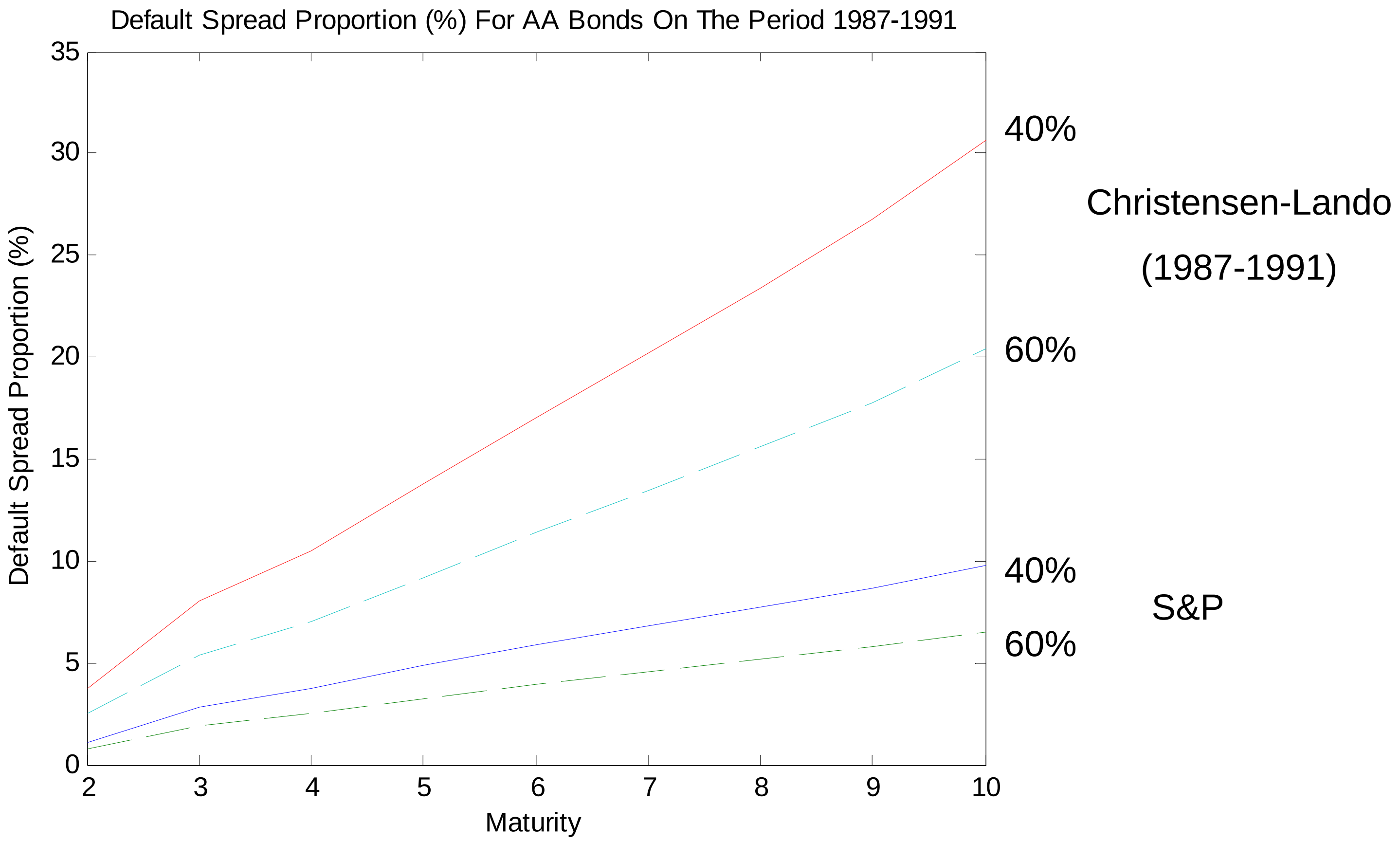
Discrete-time model (continued)

- Results (proportion of default for a maturity of 10 years)

Period / Matrix	SPEG	MDEG	MDD	MDCL
AA				
1987-1996	8.41	4.29	4.56	--
1987-1991	6.61	3.37	10.81	20.39
1992-1996	10.22	5.21	0.31	--
A				
1987-1996	17.69	11.11	9.87	--
1987-1991	15.06	9.46	22.79	36.47
1992-1996	20.31	12.75	1.15	--
BBB				
1987-1996	36.07	35.15	31.61	--
1987-1991	32.18	31.36	64.01	78.97
1992-1996	39.96	38.94	7.35	--

Discrete-time model (continued)

- Sensitivity to recovery rate assumption



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Continuous-time model

- **Assumptions:**
  - Existence of a risk-neutral measure
  - Independence of risk free rate and default time
  - Recovery of a fraction of the market value
  - Deterministic time varying intensity process for the defaut time
  - Risk neutrality

Continuous-time model (continued)

- Special case of Duffie-Singleton :

$$\widetilde{P}(0,T) = P(0,T) \times \exp\left[-(1-\rho)\int_t^T \lambda_s ds\right]$$

- Additional assumption for tractability:
  - Default-time driven by a time-homogeneous Markov process with a diagonable generator matrix
  - Intensity under this additional assumption:

$$\lambda_t = \frac{\sum_{k=1}^m a_k d_k \exp(d_k t)}{1 - \sum_{k=1}^m a_k \exp(d_k t)}$$

Continuous-time model (continued)

- **Estimates of the generator :**
  - Existence of a generator for transition matrix from Moody's or S&P is not garanteed
  - Israel et al (2001) :
    - Verification of the existence of a generator (Israel et al (2001))
    - Recovery of a generator producing matrices close to the original matrices (Israel et al (2001))

Continuous-time model (continued)

- **Results**
  - Similar to the discrete-time model results i.e.

    Sensitivity to the transition matrix used

    Sensitivity to recovery rate assumption

    Sensitivity to the time period examined

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Extensions

- **Inference**

- **Relaxing the risk-neutrality assumption**
  - premium for default
  - premium for recovery

- **Liquidity**