Liquidation Triggers and the Valuation of Equity and Debt

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Abstract

Net-worth covenants provide the firm's bondholders with the right to force reorganization or liquidation if the value of the firm falls below a certain threshold. In the event of default, however, many bankruptcy codes stipulate an *automatic stay* of assets that prevent bondholders from triggering liquidation. To consider this impact on the pricing of corporate securities we develop a model where liquidation is driven by a state variable that accumulates with time and severity of distress. In addition, current distress periods may have greater weight than old ones. The liquidation trigger can be adjustable for different bankruptcy codes and jurisdictions.

Net-worth covenants provide the firm's bondholders with the right to force reorganization or liquidation if the value of the firm falls below a certain threshold. However, the creditors' ability to extract value at the onset of financial distress depends heavily on the nature of the bankruptcy code and the jurisdiction under which the borrower operates.

Bankruptcy procedures, as noted by White (1996), LaPorta et al. (1998) and Franks and Sussman (2002), diverge often across countries due to political and social considerations. For example, the state of affairs in the UK is in stark contrast to the US. The differences may originate in the fundamentally different approaches to bankruptcy procedures in the two countries: strict enforcement versus judicial discretion. US law imposes mandatory procedures to prevent liquidation, even at the expense of some violations of the lenders' contractual rights. Once an insolvent company files for reorganization under Chapter 11, an "automatic stay" prevents debtholders from seizing assets until a reorganization plan is adopted, or till the supervising court converts the bankruptcy proceedings to a Chapter 7 liquidation of the firm's assets.[1] Empirical studies have found that the average time period between the indication of financial distress and its resolution ranges between two to three years for firms that renegotiate their claim under Chapter 11 of the U.S. Bankruptcy Code.[2] In contrast, UK insolvency law is not codified. It is simply a procedure of enforcing debt contracts. The approach is characterized by the strict enforcement of creditors' contractual rights, including the liquidation rights of secured creditors.[3]

The nature of the different bankruptcy procedures has important ex-ante consequences, since participants in the market agree to finance the company on terms that reflect the possible ex-post outcomes, which may include reorganization or liquidation of a firm's assets. This paper presents a general and adjustable structural model for pricing corporate securities under a wide array of bankruptcy codes and law enforcement. Each legal and economic environment can be efficiently translated into an appropriate bankruptcy procedure.

Previous pricing models have restricted the relationship between default events and liquidation procedure to one specific regime or to a limited set of scenarios. Our proposed pricing model is more general since it encompasses these models as particular cases.

The classic structural approach pioneered by Black and Scholes (1973) and Merton (1974), assumes that default can only occur in the event that the total value of the firm at maturity is less than the contractual payment due on the debt. To cope with the possibility of early default prior to bond maturity, Black-Cox (1976, hereafter BC) developed a "first passage" model, which assumes that the time of default is the first instance that the market value of the assets of the issuer has fallen below a specified distress threshold that precipitates immediate liquidation of the firm's assets. This distress threshold marks the trigger for the liquidation of firm's assets in subsequent structural models as well [see Brennan and Schwartz (1978), Longstaff and Schwartz (1995), Leland (1994), Ericsson and Reneby (1998) and others].[4]

In practice the onset of financial distress does not necessarily lead to immediate change of control or liquidation of the firm's assets by its debtholders since several bankruptcy laws favor firm continuation and creditors cannot immediately liquidate the firm. [5] As a result, asset pricing models of the firm's financial claims based on "first passage", as introduced first by BC, may be invalid, since reaching the threshold does not automatically trigger either liquidation or reorganization of the firm's assets.

To address the discrepancy between default event and the liquidation of the firm's assets recent works on capital structure and securities valuation suggest that liquidation occurs only if the value of the firm's assets has reached the distress threshold and remains below this threshold for a prolonged period of time. Fan and Sundaresan (2000) suggest that when the firm is in default, borrowers stop making the contractual coupon and start servicing the debt strategically until the firm's asset value returns to a level above the distress threshold.

In this spirit, a valuation model developed by François and Morellec (2002, hereafter FM), assumes that the firm issues perpetual debt with contractual coupon payments, and liquidation occurs when the value of its assets dips below the distress threshold and remains below that level for an interval exceeding a pre-determined 'grace' period. If the value of the firm's assets rebounds and rises above the distress threshold before the pre-determined grace period has elapsed, the procedure is discontinued and the invisible "distress clock" is reset to zero. According to this approach, while debt is strategically serviced automatically after the value of the firm crosses the distress threshold, liquidation is declared only after the predetermined grace period has elapsed. As opposed to previous pricing models of the firm capital structure, FM's model can adjust itself to a variety of bankruptcy procedures by varying the pre-determined grace period

Moraux (2002) points out that according to FM, each time the firm value falls below the threshold level an additional grace period is granted without reference to previous instances of insolvency and thus FM implicitly assume that the bankruptcy procedure is automatically stopped at each time that the value of the firm's assets has rebounded above the threshold level. Asset value could theoretically remain below the threshold level for the majority of the duration of debt without the firm being liquidated. To overcome this disadvantage, Moraux (2002) assumes that the firm issues zero-coupon debt, and proposes that liquidation is triggered when the *total* time that the firm's asset value spends under the distress threshold ("excursion time") exceeds a pre-determined grace period. Consequently, the previous "distress clock" is not reset to zero when the value of the firm's assets rebounds above the threshold. In this manner, the liquidation decision becomes highly path-dependent, since it accumulates the entire history of a firm's financial distress. However, by simply accumulating excursion time this model gives a company's history of financial distress an equal weight in triggering liquidation. This description of the bankruptcy process implicitly assumes that the procedure is never stopped even if the value of the firm's assets has rebound above the distress threshold for a prolonged consecutive period.

Both models (Moraux and FM) do not differentiate between cases in which the firm value is below but close to the distress threshold and cases where the firm value falls far below the threshold level. The severity of the distress event has no influence on the decision to liquidate a firm.

We present a model in which liquidation is executed when a state variable, the *cumulative distress time*, exceeds a pre-determined grace period. This state variable accumulates the distress periods, which are defined as any period that the value of the firm's assets has spent under the distress threshold. The influence of each distress event on the liquidation state variable, and thus on the liquidation decision, may depend on the severity of the distress period, on its length, consecutiveness and the distance of the distress event from current time.

By applying this process, we can increase the weight of recent and/or severe distress events over old and/or mild distress events on the decision to liquidate the firm's assets. Our model is more flexible than previous models since we assume that the impact of each distress event on the liquidation trigger is a function of the degree of insolvency and the distance from current time and thus we can adjust the modeling of the bankruptcy procedures to the multiplicity of legal regimes and contractual agreements. At one extreme, we can exclude or significantly reduce the impact of distress episodes in the distant past on the liquidation trigger. At the other extreme, we can weigh each distress observation equally. Our model, therefore, is a general model incorporating both FM and Moraux (2002).

With our approach, we can directly value different types of corporate securities and analyze complex capital structure scenarios for various bankruptcy procedures. We provide numerical examples to investigate how the length of the grace period, the liquidation decays factors, the distress severity, the leverage ratios, and the firm's asset volatility affect both asset prices and credit spreads.

The model presented in this paper assumes a simple capital structure with one type of zero-coupon debt. However, it can be extended to a case in which the firm has issued both senior and junior debt, convertible bonds or warrants.

The remainder of the paper is organized as follows: Section (I) specifies the assumptions, and describes the liquidation procedure according to the model. Section (II) derives the valuation of equity and debt. Section (III) is devoted to explaining bankruptcy procedures of previous pricing models for corporate securities and to highlighting the advantage of our model over these models. In section (IV), we numerically analyze the main implications of the asset pricing models. We present Conclusions in section (V).

I. Pricing Model for corporate securities with Adjustable distress Memory

In this section we construct a general pricing model with adjustable distress memory to estimate the value of various corporate securities under a wide array of bankruptcy codes. According to our model, liquidation is triggered when the weighted cumulative time that firm value has spent under the distress threshold exceeds a fixed exogenous amount of time. Using our model, it is possible to increase the impact of late and/or severe distress events over old and/or mild distress events on the decision to liquidate. We rely on standard structural approach assumptions: assets are continuously traded in an arbitrage-free and complete market with riskless borrowing or lending at a constant rate r. The instantaneous standard deviation of the rate of return of the firm, σ, is constant; the value of the firm's assets, V_t, is independent of the capital structure of the firm, and is well described under the risk neutral measure Q, by the following stochastic differential equation:

$$dV_t = (r - \delta)V_t dt + \sigma V_t dW_t \tag{1}$$

where W is a standard Brownian motion and δ is the firm's payout ratio.

We suppose that the firm has only equity and a single bond issue with a promised final payment of P and maturity T outstanding. The firm goes bankrupt in one of two ways: either the value of the firm's assets falls below a time dependent threshold level, denoted by K_t, at any time prior to debt maturity, or the value of the assets is less than some constant F at debt maturity.[6]

According to the BC model, the default event allows the creditor to force immediate liquidation through the bond's safety covenant. In our model, as in FM and Moraux (2002) models, distress and liquidation might diverge. We assume that liquidation is declared when the liquidation state variable (i.e., the "weighted cumulative distress time") exceeds a pre-determined grace period, denoted by d. In order to determine the liquidation state variable we define the time dependent threshold level K_t according to BC:

$$K_t = \omega F e^{-r(T-t)} \quad \text{where}: 0 \le \omega \le 1 \tag{2}$$

In this exponential form the reorganization/liquidation value specified in the safety covenants is a fraction of the promised final payment. Under the BC assumption of immediate liquidation of the firm's assets at the threshold level, as ω goes to one the debt becomes riskless. Let us define the following random variable:

$$g_t^K = \sup\left\{ s \le t \middle| V_s = K_s \right\} \tag{3}$$

when g_t^K is the last time before t that the firm value crossed the then-prevailing threshold K_s. The state variable for the liquidation trigger is I_t^K, which is defined at date t as:

$$I_t^K = \int_0^{g_t^K} e^{-\beta(t-s)} f(V_s)ds + \int_{g_t^K}^{t} e^{-\gamma(t-s)} f(V_s)ds$$

(4)

where β is the decay factor for previous distress periods and γ is the decay factor of the last distress period. As β and γ increase, the impact of past distress events become decreasingly meaningful to the decision to liquidate the firm. For example, when $\beta = 3$ the effect of distress event that has occurred a year ago on the liquidation state variable is negligible (equal to 0.05). However, when $\beta = 0.5$ this effect is more than twelve times larger. These parameters enable us to adjust the survey period and the length of the bankruptcy procedures to the multiplicity of legal regimes and contractual agreements.

The function $f(V_t)$ defines the impact of the severity of the distress event on the liquidation state variable. We model $f(V_t)$ as follows:

$$f(V_t) = \begin{cases} e^{\alpha\left[\left(\frac{K_t - V_t}{K_t}\right) - \lambda\right]} & V_t \le K_t \\ 0 & V_t > K_t \end{cases}$$

(5)

where α determines the slope of the function and the parameter λ determines the point of intersection with the value of one. To make certain that the liquidation state variable would increase with the severity of the distress event we set $\alpha \ge 0$, and to ensure that the function would intersect the value of one we set $0 \le \lambda \le 1$.

Accordingly, the decision to liquidate a firm's assets does not depend solely on the duration of the distress events or its continuity, as described in Moraux (2002) and FM (2002) respectively, but also on the distance of past distress events from the present and on the severity of distress, i.e., the degree to which firm value falls below the threshold. Liquidation occurs the first time that the cumulative distress time extends beyond d. The liquidation time is denoted by θ^K, and it is defined mathematically by:

$$\theta^K = \inf\left\{t > 0 \middle| I_t^K \geq d, V_t \leq K_t\right\} \tag{6}$$

In the particular case where α=0, the severity of the distress event has no impact on the liquidation decision and the liquidation state variable can be calculated by the expression:

$$I_t^K = \int_0^{g_t^K} e^{-\beta(t-s)} 1_{\{V_s \leq K_s\}} ds + \int_{g_t^K}^{t} e^{-\gamma(t-s)} 1_{\{V_s \leq K_s\}} ds \tag{7}$$

where $1_{\{V_S \leq K_B\}}$ is the characteristic function that receives the value of one if firm value is below the distress threshold level, and zero otherwise.

In our setup, shareholders have a residual claim on the cash flows generated by the firm's assets unless the cumulative distress time has reached the pre-determined grace period, *d*. The bondholders receive the debt's face value at maturity if liquidation has not occurred before. In the event of liquidation the debtholders would receive the remaining assets of the firm. The following examples are of special interest since they pertain to previous contributions of the literature. In all of these examples α=0.

Example 1. *When $\beta \to +\infty$ and $\gamma = 0$, the liquidation procedure occurs at the first point in time when the firm value process has spent consecutively more than the pre-specified grace period below the threshold K_t. Thus, when $\beta \to +\infty$ and $\gamma = 0$, we get the François and Morellec (2002) bankruptcy procedure.*

In this example the liquidation state variable is accumulated only during the current distress period, where past distress periods do not influence the liquidation state variable. At the one extreme, when $d = 0$, the FM model is similar, as a special case, to the standard modeling of default and liquidation [see Leland (1994)]. At the other extreme, when $d > (T - t)$, i.e. the grace period is longer than the maturity of debt, default never leads to liquidation before debt maturity and the FM (2002) model takes on, as a special case, the standard model for default and reorganization [see Anderson and Sundaresan (1996) or Fan and Sundaresan (2000)].

Example 2. *When $\beta = 0$ and $\gamma = 0$, liquidation occurs the first time the firm value spends a total time greater than the pre specified grace period below K_t. Thus, when $\beta = 0$ and $\gamma = 0$, we have the Moraux (2002) bankruptcy procedure.*

Under this parameterization, each distress period is weighted equally and each period has the same influence on I_t^K. At the one extreme, when $d = 0$, no extra survival time below the distress threshold is allowed, default leads to immediate liquidation of the firm's assets and we get, as a special case, the BC liquidation model. At the other extreme, when $d \geq (T-t)$, liquidation can occur only at debt maturity, and the model collapses to the basic structural approach introduced by Merton (1974).

II. The Valuation of Defaultable Bonds

In this section we evaluate the various corporate securities by considering the simple case of a firm with market value of assets V_t, which is financed by equity, S_t, and one debt obligation, maturing at time T, with par value P, and market value B_t. The bond contract gives debtholders, under a *protective covenant*, the right to force liquidation at any time $t \in [0, \mathrm{T}]$, if asset value equals or is lower than an exogenous threshold level K_t. However, the debtholders succeed to force liquidation only when the liquidation trigger, I_t^K, exceeds the predetermined grace period, d. At liquidation, debtholders would receive V_{θ^K} at time θ^K; and equityholders would receive nothing. At debt maturity, T, assuming no early liquidation has been declared, equityholders would receive the maximum between zero and the difference between the firm's assets value, V_T, and the promised face value, P. Using the indicator function $1_{\{\theta^K > T\}}$ the equityholders payoff is given by the following function:

$$\mathrm{S}(V_T, T, I_T^K, T) = (V_T - P)^+ \, 1_{\{\theta^K > T\}} = \begin{cases} V_T - P & \text{if } V_T > P \text{ and } \theta^K > T \\ 0 & \text{otherwise} \end{cases}$$

(

8

)

The value of the equityholders claim at any time prior to debt maturity $t \in [0,T]$, provided that default has not occurred by time t, is expressed by:

$$S(V_t, t, I_t^K, T) = e^{-r(T-t)} E_t^Q[(V_T - P)^+ 1_{\{\theta^K > T\}}]$$

(9)

where $E_t^Q[\cdot]$ denotes the conditional expectation under a risk neutral measure Q, given the available information at time t.

The value of the zero-coupon bond is decomposed in two sources of value: first, its value at maturity, assuming the firm is not prematurely liquidated, and second, its value if the firm is liquidated before debt maturity, T, since the pre-determined grace period d was exceeded by the cumulative distress period. As noted by BC, although those two components are mutually exclusive, they are both possible outcomes. Accordingly each contributes to the present value of both equity and debt.[7] The price of a zero coupon bond, B, with maturity $T > t$, is given by the expected discounted cash flows under the risk neutral probability measure:

$$B(V_t, t, I_t^K, T) = \mathrm{E}_t^Q[\min(V_T, P) e^{-r(T-t)} 1_{\{\theta^K > T\}}] + \mathrm{E}_t^Q[V_{\theta^K} e^{-r(\theta^K - t)} 1_{\{\theta^K \le T\}}] \quad (10)$$

Roughly speaking, the payoff at time $\theta_d \wedge T$ is given according to the no liquidation scenarios (the left expression at the right hand side of the equation), in which debtholders receive the minimum between the value of the firm's assets and the par value of debt, or alternatively, should early liquidation take place (the right expression at the right hand side of the equation), debtholders receive the value of the firm's assets at that time.

The next step in evaluating the firm's capital structure is to calculate the zero-coupon yield spread. Practitioners typically quote corporate bond prices in terms of the spread of their yield-to-maturity over the riskless interest rate. The firm credit spread at time t, denoted by sp_t is calculated as:

$$sp_t = \frac{\ln\left(\frac{B_t}{P}\right)}{-(T-t)} - r \qquad (11)$$

Given the above assumptions, we can derive the governing partial differential equations and boundary conditions that should be solved to value the firm's stocks and bonds as a function of the three state variables $V, t,$ and I.

The relevant form of the valuation equation for the stock, S, will be:

$$\frac{\partial S}{\partial t} + \frac{\sigma^2 V^2}{2}\frac{\partial^2 S}{\partial V^2} + (r-\delta)V\frac{\partial S}{\partial V} - rS + \frac{\partial S}{\partial I} = 0 \qquad (12)$$

The boundary conditions are as follows:

$$S(V_T, T, I_T^K, T) = \max(V_T - P, 0) \qquad \textit{for} \;\; 0 \le I_T^K < d \tag{13}$$

$$S(V_t, t, d, T) = 0$$

(14)

The relevant form of the valuation equation for the bond, B, will be:

$$\frac{\partial B}{\partial t} + \frac{\sigma^2 V^2}{2}\frac{\partial^2 B}{\partial V^2} + (r - \delta)V\frac{\partial B}{\partial V} - rB + \frac{\partial B}{\partial I} = 0$$

(15)

The boundary conditions are as follows:

$$B(V_T, T, I_T^K, T) = \min(V_T, P) \qquad \textit{for}: \;\; 0 \le I_T^K < d$$

(16)

$$B(V_t, t, d, T) = V_t$$

(17)

Bergman (1985) has developed a general procedure for pricing path-dependent contingent claims and applied the procedure to the case of the averaging claims. A new term that is proportional to the rate of change of the average is introduced in the Black-Scholes equation. Haber, Schönbucher and Wilmott (1999) have used this extension for the pricing of Parisian options, where a new state variable I_t^K gives rise to a modified form of the Black-Scholes equation. In a standard Parisian option, the knocked-out feature is activated if the value of the firm's assets has spent a pre specified consecutive time below K_t, and thus the clock variable $I_t^{K_B}$ is reset to zero once the asset value rises above K_t. However, in a Parasian contract the knocked-out feature is activated only if the cumulative time spent below K_t exceeds some prescribed value. The type of option that we value in our model can be considered as a hybrid of the Parasian contract and the Parisian options.

III. A comparison with Past Liquidation Models Based on the Excursion Time

In this section we describe the two existing pricing models for corporate securities that are based on excursion time as developed by FM and Moraux (2002). By using two numerical examples we illustrate the anomalous behavior that may stem from each of them, and demonstrate how, through determination of the parameters, our general model can prevent such anomalies.

In both examples, we consider a leveraged firm that issues only one stock and one zero-coupon bond maturing in 10 years. The debtholders are protected by a safety covenant that allows them to force liquidation when the value of the firm's assets is less than the distress threshold K_t. For each of the models, the distress period before liquidation lasts at most one year, so $d = 1$. However, the state variable that triggers liquidation, I_t^K, is treated differently in each model. According, to Moraux's (2002) *cumulative excursion method,* liquidation occurs when the value of the firm's assets accumulates more than one year under the threshold level, even with interruptions, and thus in our setting equation (7) is parameterized as follow: $\beta = \gamma = 0$. According to FM's *consecutive excursion method*, liquidation occurs when the value of the firm's exceeds uninterruptedly a consecutive one-year period under the distress threshold, and thus: $\gamma = 0$ and $\beta \to \infty$. To illustrate our *adjustable excursion method* we have chosen a third set of parameters: $\gamma = 0,$ and $\beta = 0.5$. This parameterization constitutes a particular case of our method.

In the first example, depicted in Figure 1 and 2, the value of the firm's assets between the middle of the second year and the beginning of the seventh year has accumulated three and half years under the distress threshold. Figure 2 shows the value of the state variable that triggers liquidation according to each method. After two years and nine months, the cumulative distress time is greater than a year and liquidation is triggered according to the *cumulative excursion method.* Our *adjustable excursion method* reduces the impact of previous distress periods and liquidation is postponed by seven months, since the excursion periods are not consecutive. In the *consecutive excursion method* the value of the firm's assets value falls below the threshold level during eleven non-consecutive periods, which means that default occurs no less than eleven times, however, liquidation is avoided since none of these comprise a consecutive twelve-month period and the bankruptcy procedure is stopped at each time that the firm's assets value rebounds above the threshold. The safety covenant is not respected despite the fact that the firm had been in dire financial straits for a prolonged period of time.

In the second example, as described in Figures 3 and 4, the value of the firm's assets crosses the distress threshold at the end of the third year for the first time and stays there for a consecutive time of ten months until firm value rebounds above the threshold. Liquidation is not triggered under any of the three models since the liquidation state variable I_t^K is less than one (10/12). In the middle of the ninth year the value of the firm once again falls below the threshold level and stays there two consecutive months. According to the *consecutive excursion method,* the distress clock is reset and liquidation procedures are not initiated after two months. We receive similar results for the chosen parameterization of the *adjusted excursion method,* since the liquidation state variable has fallen to the value of 0.05 from 0.83 given the fact that firm value remained above the threshold for more than five years. In contrast, according to the *cumulative excursion method,* liquidation is warranted. The distress clock is not reset or "moved back" and liquidation is declared immediately after two months. The state variable that triggers default has not forgotten or reduced the impact of the distress period that occurred in the distant past. Since the sensitivity of the liquidation state variable in this framework exaggerates distress experienced in the previous episodes, one can conclude that the consecutive excursion model may have too strong a memory.

IV. Numerical Implementation and Sensitivity Analysis

We now turn to the implementation of the model for calculating bond prices, equity prices and the credit spread of a levered firm. We describe the procedure for model implementation, then, comparative statics are presented.

A. Numerical implementation

A first step toward numerical solution is discretizing the partial differential equations for V, and I . Since in most cases, an analytical solution is not available, we need to employ a numerical solution. We follow the Monte-Carlo simulation approach since it is easy to implement and applicable for a wide range of problems presented in this paper. We briefly describe the method, which is discussed extensively in Boyle (1977), Broadie and Glasserman (1996) and Boyle et al. (1997).

Our procedure is as follows:

A. We generate independent replications of the firm's asset value, V, for n short intervals of length Δt and approximate equation (1) as:

$$V_{t+\Delta t}^{(i)} = V_t \exp\left[\left(r - \delta - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}Z^{(i)}\right] \tag{18}$$

for $i = 1,...,n$, where $Z^{(i)}$ is a standard normal random variable.

B. At each time step, we calculate the value of the discrete threshold level as: $K_i^D = Pe^{-r(T-i\Delta t)}$. To replace the discrete monitored threshold level with a continuous threshold, we use the Broadie, Glasserman and Kou (1995) approximation:

$$K_i^C = K_i^D e^{\rho\sigma\sqrt{\Delta t}} \tag{19}$$

where $\rho = -\zeta\left(\frac{1}{2}\right)\Big/\sqrt{2\pi} \approx 0.5826$ and ζ is the Riemann Zeta function.

C. At each time step along the price path, i, we calculate the cumulative distress time (the liquidation state variable) $I_t^{V_B}$ by approximating equation (3) and (4), and then check if the cumulative distress time (the liquidation state variable) exceeds the pre- determined grace period, d .

D. The firm value is divided among the various claimants according to equations (9) and (10), based on the earlier of the two events T (bond maturity) and θ^K (end of the grace period). Each claim payoff is discounted at the risk free rate.

E. Repeat steps (A)-(C) to collect samples of the bond in a risk- neutral world.

F. Calculate the mean of the samples to estimate the value of the various claims.

B. Application and Analysis

We consider some examples and perform a sensitivity analysis of the bond price, the equity price and the credit spread with respect to a number of parameters. In order to emphasize the impact of our method on the value of the various corporate claims we also compare our results to existing structural methods for modeling credit spreads.

As the base case, we assume a firm with capital structure comprised of one stock and one zero-coupon bond with $P = 109.926$ and $T = 5$. The firm value comes to 100 and, as a result the quasi leverage ratio, which is defined as $LR = Pe^{-rT} / V_t$, is equal to $LR = 0.9$. The risk free interest rate is $r = 4\%$, the pre-determined grace period is $d = 0.25$, the firm asset volatility is 30%, and no payout is expected $(\delta = 0)$. The parameter α is set to zero, and as a result, the distress severity has no influence on the liquidation state variable (no matter which value λ receives). The parameter γ equals zero as well, which means that each observation on the last distress period has an equal impact on the decision to liquidate the firm's assets. Bondholders hold a contract which enables them to take the firm over at the time the value of the firm's assets is smaller than the discounted face value of debt, and as a result, the distress threshold parallels the secured discounted balance and equals:

$K_t = Pe^{-r(T-s)}$. To isolate the impact of the deviations from the provisions of the bondholder's contract on claim value, we assume an absence of costs pertaining to liquidation and bankruptcy.

We now analyze the determinants of the level of credit spreads and corporate securities values. Table 1 lists the numerical estimates of corporate securities within various structural frameworks of default and liquidation. The credit spread according to the Merton model comes to 5.1%. This high spread stems from the model's underlying assumption that neither liquidation nor default can occur before the contractual maturity of debt, and thus, in instances of financial distress, debtholders cannot extract value from the firm prior to maturity. At the other extreme, BC assume that the firm's assets are immediately liquidated upon hitting the distress threshold. If this threshold is equal to the secured discounted debt balance, there is no effective credit risk and the credit spread is equal to zero. Figure 7 and table 1 show that as the decay factor of previous distress events increases, the value of debt decreases. At the extreme, as in FM, when $\beta \to \infty$, the distress clock is reset whenever the firm's asset value crosses the distress threshold. When the grace period is shorter, and is equal to one month, the gap between the credit-spread according to the *cumulative excursion method* $(\beta = 0, \gamma = 0)$ and the *consecutive excursion method* $(\beta \to \infty, \gamma = 0)$ is relatively small and equal to 46 basis points. However, when the grace period increases to three months, and the violation of the terms of the safety covenant are more severe, the gap becomes larger, reaching 70 basis points. In this case, modeling the true nature of the bankruptcy procedure becomes of the essence; intermediate values of β may capture the true nature of bankruptcy procedure more appropriately. When the grace period rises to one year, the gap between the two extreme cases declines to 67 basis points, since the probability of early liquidation by any method is extremely decreased.

Figure 5 and table 2 show that credit spreads increase with asset volatility. As the volatility increases (potentially hurting bondholders), having a liquidation procedures with long memory, i.e., better bondholder protection (small β) is of the essence: when β is equal to zero an increase of 10% in asset's volatility cause to increase of 77 basis points, while a similar increase of the volatility when $\beta \to \infty$ cause to increase of 106 basis points.

Figure 6 and table 2 provide estimated credit spreads for a combination of financial quasi-leverage ratios (LR) and decay factors for previous distress periods (β). As β increases, the gap between the credit-spreads of the two leveraged firms increases. When β=0, the credit spread of a firm with quasi leverage ratio of 90% is equal to 1.84% and the credit spread of a firm with quasi leverage ratio of 95% is equal to 1.97%. However, when $\beta \to \infty$, the credit spreads amount to 2.54% and 3.76% respectively. The gap between the credit spreads of the two leveraged firms according to the Merton model comes to 2.87%. This important outcome emphasizes the fact that as the liquidation state variable is less sensitive to the impact of past distress periods, an increase in financial leverage has a larger impact on debtholders protection.

To highlight the effect of the severity of the distress event (i.e., the degree of insolvency) on the liquidation state variable we present in Figure 8 the value of the function $f(t)$, which appeared in equations (4) and (5), for different levels of the parameters α and λ. If λ is set to 0.4 the distress event has an effect which is equal to one when the value of the firm's assets is lower than the distress threshold by 40%. However, when this parameter is equal to 0.2, the same effect is achieved when the value of the firm's assets is lower than the threshold level by only 20%, and thus creditor rights are more respected under this regime. Table 3 emphasizes the influence of these parameters on the firm's liabilities. When the parameter λ is set to 0.4 and α increases from 0.5 to 1.5, we observe an increase in the credit spread from 239 and 257 basis points to 282 and 298 basis points respectively for β values of 1.5 and 3. The parameter λ has a similar effect on the credit spread: when the parameter α is set to 1.5, and λ increases from 0.2 to 0.4, we observe an increase in the credit spread from 244 and 260 basis points to 282 and 298 basis points respectively for β values of 1.5 and 3.

V. Conclusion

We present a simple and general structural model for the valuation of corporate securities where the bondholders' right to force immediate reorganization or liquidation of a distressed firm may consume time. To evaluate the impact on the firm's capital structure, we develop a general and adjustable pricing model for corporate securities driven by a liquidation state variable. Unlike other models, our state variable, which may trigger liquidation of the firm's assets, accommodates a greater number of scenarios and yield a more accurate assessment of financial distress. The liquidation trigger accumulates over distress time, but is also dependent on the degree to which the threshold is violated. In addition, recent distress episodes can carry a higher influence than older episodes.

We show that by applying the appropriate liquidation parameters, our model converges to François and Morellec (2002) bankruptcy model, in which liquidation is triggered if the value of the firm's assets has exceeded a consecutive excursion time. Moreover, our general model also accommodates Moraux's (2002) bankruptcy model, which assumes that liquidation occurs if total excursion time exceeds a pre-determined grace period. While these two models may describe the bankruptcy procedure accurately for a specific set of legal regime and low enforcement, our model, as illustrated in this paper, covers a wider array of legal precepts and contractual arrangements. All of the liquidation models presented above, may be viewed as a middle ground approach between the Merton's framework, where liquidation occurs only upon debt maturity, and the Black-Cox model, where reorganization of the firm's assets is invoked when a minimum threshold is violated during the lifetime of the debt.

We illustrate the applicability of our model for the valuation of firms with simple capital structures and we present both comparative statics and sensitivity analysis of the various corporate claims for different legal regimes and corporate capital structures.

A natural direction for future research is to apply the model to environments characterized by empirically supported dynamics of risk- free short rates and observed credit spreads. Additional features such as interim payments, taxes, liquidation costs, debt subordination and alternative bond indentures can be incorporated as well. Although not a trivial task, exploring these directions may be rewarding in providing new guidance for risk measurement and pricing, as well as for supporting empirical findings and observed behavior patterns in the fixed income and equity markets.

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Table 1

Corporate credit spread and the value of the firm's capital structure

This table presents the corporate credit spread and the value of the firm's capital structure for various grace periods and past period decay factors. Parameters for the base case are the risk free interest rate $r = 4\%$, the volatility of the firm's assets volatility $\sigma = 30\%$, $P = 109.926$ and $T = 5$. The firm asset value equals 100, and as a result, the leverage ratio, which is defined as $LR = Fe^{-rT} / V_t$, equals $LR = 0.9$. The pre- determined grace period: $d = 0.25$, no payout is delivered $(\delta = 0)$. The liquidation model parameters α and γ are set at zero.

Scenario	*β*	*Equity value*	*Debt Value*	*Credit spread*
Base case	β=0	17.93	82.07	1.84%
	β=1.5	19.31	80.69	2.19%
	β=3	20.03	79.97	2.36%
	β→∞	20.74	79.26	2.54%
d=0.083 (One month)	β=0	14.46	85.54	1.02%
	β=1.5	15.01	84.99	1.14%
	β=3	15.27	84.73	1.21%
	β→∞	16.41	83.59	1.48%
d=1 (One year)	β=0	24.17	75.83	3.43%
	β=1.5	26.53	73.47	4.06%
	β=3	26.66	73.34	4.09%
	β→∞	26.67	73.33	4.10%
d=T (Merton 1974)		30.25	69.75	5.10%
d=0 (BC 1976)		10.00	90.0	0.0%

Table 2

Credit spread and the value of the firm's capital structure

This table presents the corporate credit spread and the value of the firm's capital structure for various past period decay factors, asset volatilities and leverage ratios. All other parameter values are the same as in Table 1.

Scenario	*β*	*Equity value*	*Debt Value*	*Credit spread*
Base case	β=0	17.93	82.07	1.84%
	β=1.5	19.31	80.69	2.19%
	β=3	20.03	79.97	2.36%
	β→∞	20.74	79.26	2.54%
d=T (Merton 1974)		30.25	69.75	5.10%
d=0 (BC 1976)		10.0	90.0	0.0%
σ=40%	β=0	21.02	78.98	2.61%
	β=1.5	22.90	77.10	3.09%
	β=3	23.89	76.11	3.35%
	β→∞	24.82	75.18	3.60%
d=T (Merton 1974)		36.21	63.78	7.97%
LR=0.95 (*P=116.0 3*)	β=0	13.93	86.07	1.97%
	β=1.5	15.46	84.54	2.33%
	β=3	16.27	83.73	2.53%
	β→∞	21.28	78.72	3.76%
d=T (Merton 1974)		28.18	71.82	5.60%
d=0 (BC 1976)		5.00	95.00	0.00%

Table 3

Corporate credit spread and the value of the firm's capital structure

This table presents the corporate credit spread and the value of the firm's capital structure for various α and λ, parameters that determine the impact of the severity of the distress event on the liquidation trigger. All other parameter values are the same as in Table 1.

Scenario		*β*	*Equity value*	*Debt Value*	*Credit spread*
λ=0.2	α=0.5	β=0	18.42	81.58	1.90%
		β=1.5	19.85	80.15	2.26%
		β=3	20.56	79.44	2.44%
		β→∞	21.12	78.88	2.58%
	α=1.5	β=0	18.96	81.04	2.04%
		β=1.5	20.56	79.44	2.44%
		β=3	21.20	78.80	2.60%
		β→∞	21.60	78.40	2.71%
λ=0.4	α=0.5	β=0	18.79	81.21	2.00%
		β=1.5	20.35	79.65	2.39%
		β=3	21.08	78.92	2.57%
		β→∞	21.58	78.42	2.70%
	α=1.5	β=0	20.18	79.82	2.34%
		β=1.5	22.06	77.94	2.82%
		β=3	22.69	77.31	2.98%

		β→∞	22.86	77.14	3.03%

Figure 1

Example 1: Simulation of the firm's asset value and the distress threshold.

In Figure 1 we simulate one path of the distress threshold and firm value over a ten- year period, as discussed in example 1 in section 4. The distress threshold is worth $K_t = Pe^{-rt}$, where $r = 0.04$ and $P = 100$.

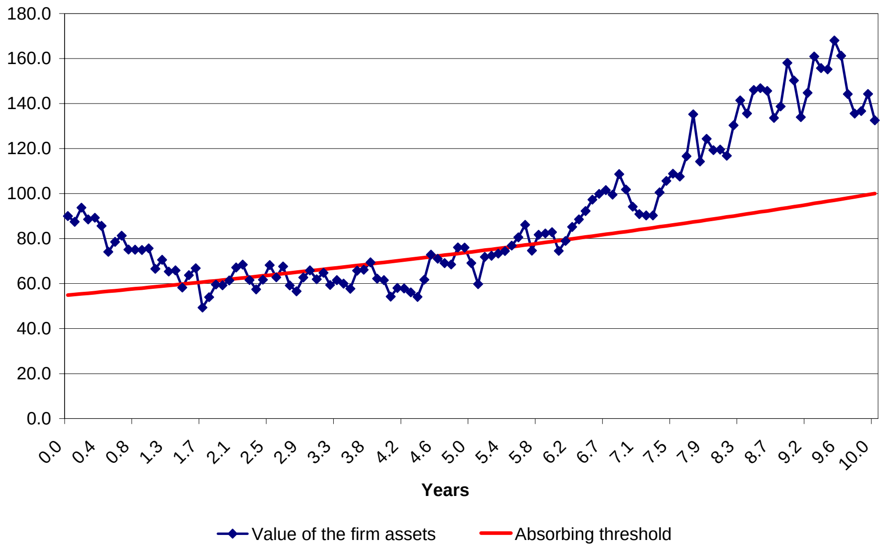


Figure 2

Example 1: Simulation of excursion time according to alternative trigger models.

In Figure 2 the weighted excursion period is calculated for the value of the firm's path presented in Figure 1. The grace period is set at $d = 1$. Liquidation is not triggered according to the FM model, while according to Moraux (2002) liquidation is triggered after two years and nine months. With $\beta = 0.5$ and $\gamma = 0$ the liquidation is triggered after three years and four months.

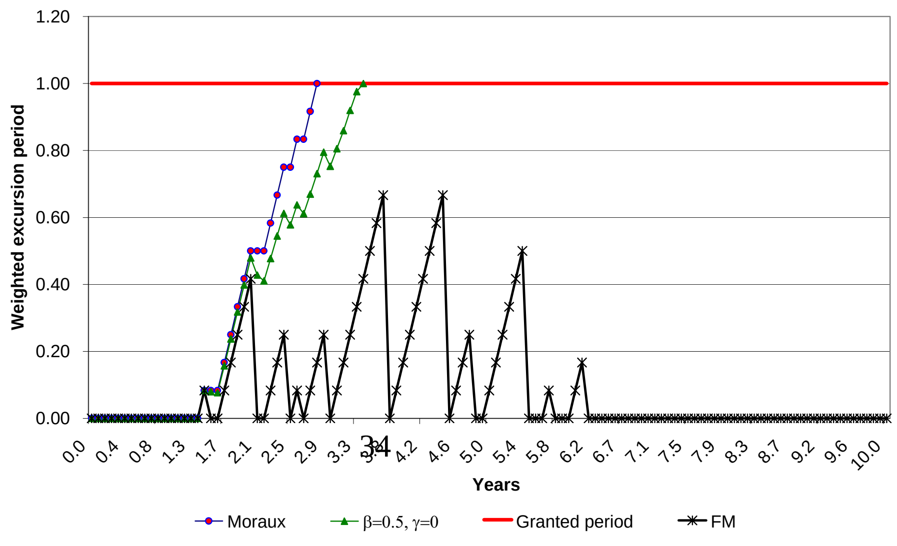


Figure 3

Example 2: Simulation of firm value and the distress threshold.

In Figure 3 we simulate one path of the distress threshold and the value of the firm's asset over a ten-year period, as discussed in Example 2 in Section 4. The distress threshold is $K_t = Pe^{-rt}$, where $r = 0.04$ and $P = 100$.

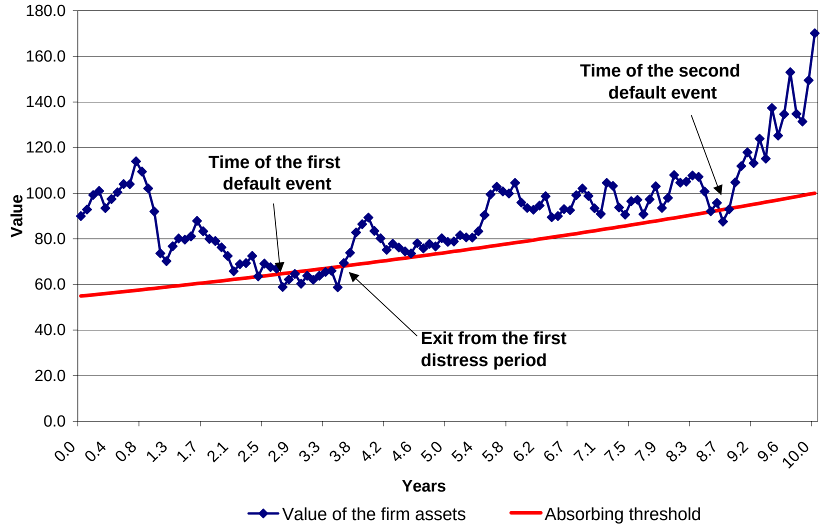


Figure 4

Example 2: Simulation of excursion time according to alternative trigger models.

In Figure 4, the weighted excursion period is calculated for the firm's path value presented in Figure 3. The grace period is set at $d = 1$. Liquidation is not triggered according to the FM model, while according to Moraux (2002) liquidation is triggered eight years and eleven months. With $\beta = 0.5$ and $\gamma = 0$ liquidation is not triggered too.

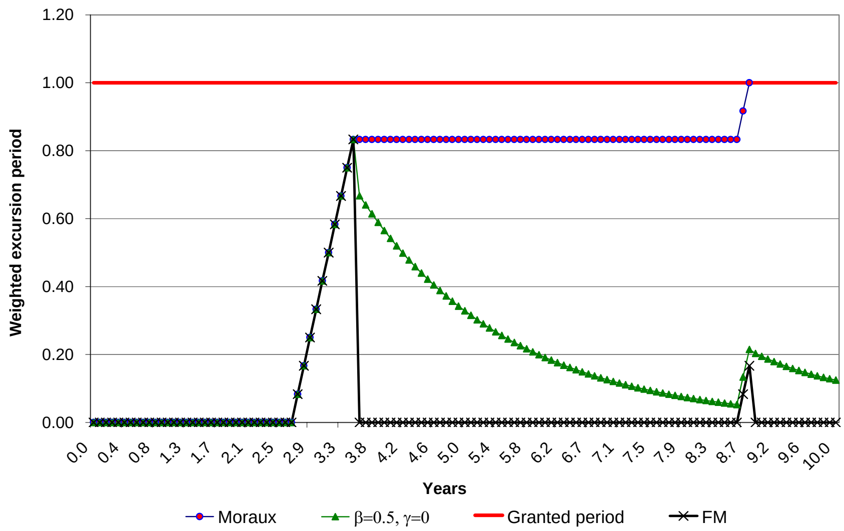


Figure 5

Corporate credit spread as a function of past distress decay factor (β) and asset volatility (σ).

Parameters: See table 1.

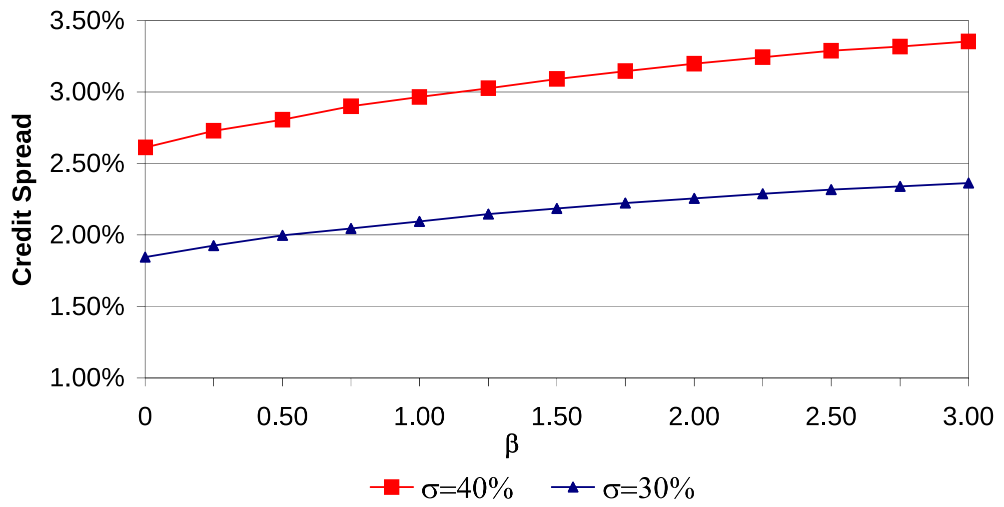


Figure 6

Corporate credit spread as a function of past distress decay factor (β) and leverage ratio (LR).

Parameters: See table 1.

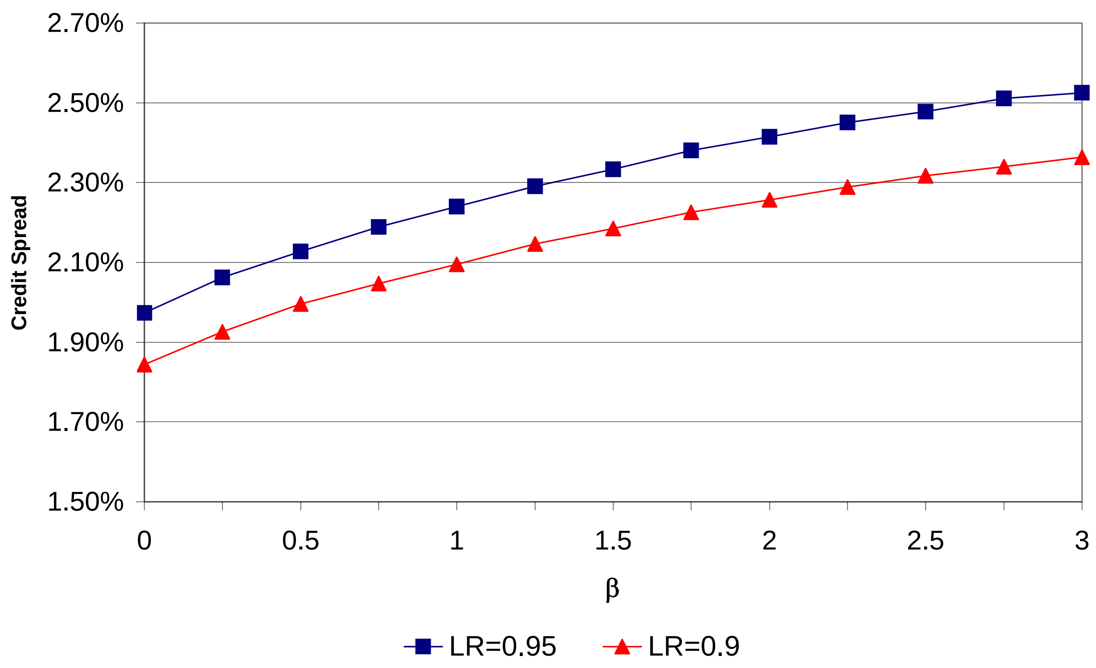


Figure 7

Corporate credit spread as a function of past distress decay factor (β) and grace period (d).

Parameters: See table 1.

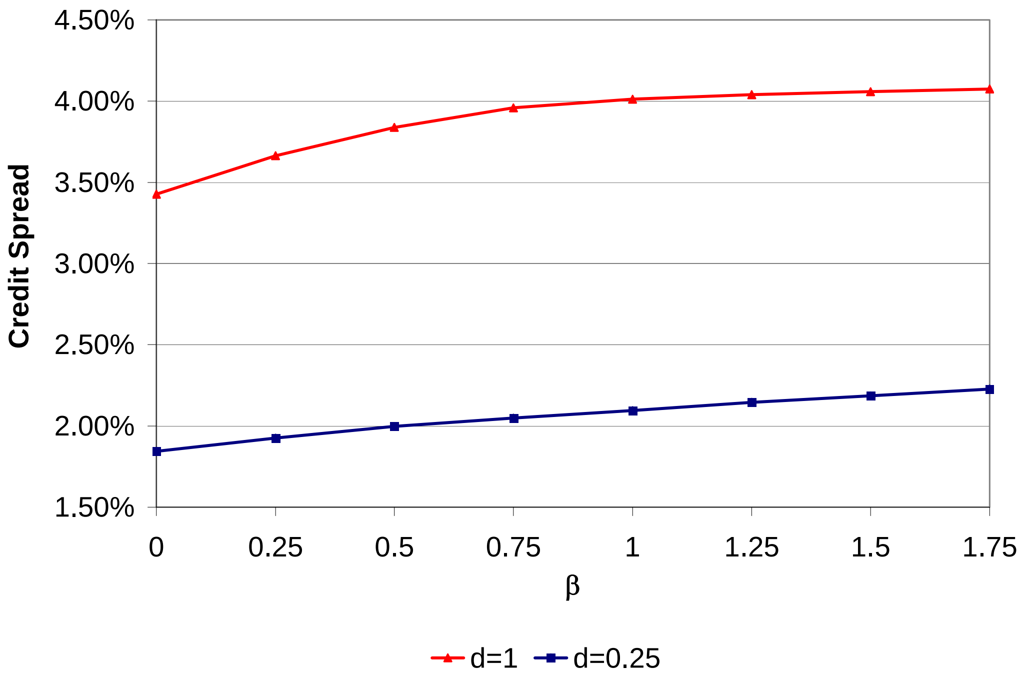
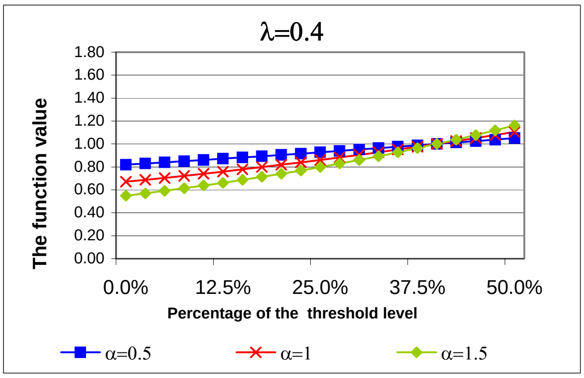
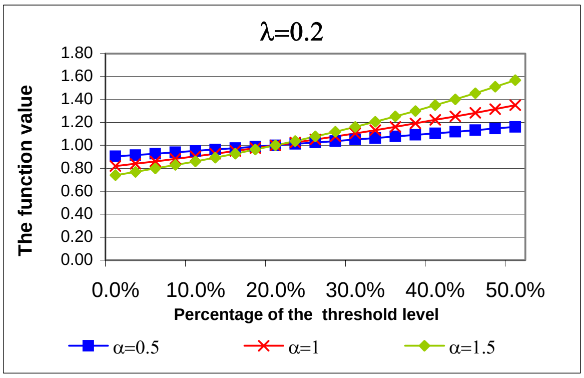


Figure 8

The value of the function *f*(t) against the percentage difference between the value of the firm's assets and the threshold level for different level of α and λ





Endnotes

[1] See Bebchuk and Chang (1992) and Bebchuk (1998, 2002).

[2] See Frank and Torous (1989), Betker (1995), Gilson (1997) and Helwege (1999). According to Weiss (1990) and Gilson, John and Lang (1990), only 5% of the firms were liquidated under Chapter 7 after filing for Chapter 11 protection, while according to Morse and Show (1988) 15% - 25% of these firms are liquidated.

[3] Franks and Sussman (2002), notice that English bankruptcy procedure is simply a practice of enforcing commercial contracts rather than a specialized bankruptcy law since the English courts are obliged to enforce the debt contract strictly.

[4] There are several ways to determine and justify a distress threshold. A legal interpretation is based on the practice in many countries to determine the financial distress threshold in corporate law. The minimum value of the firm's assets signaling legal financial distress is usually related to the total nominal value of outstanding debt. An economic approach views the distress threshold as the level of asset value necessary for the firm to retain sufficient credibility to continue operations. A contractual interpretation for the existence of the distress threshold is based on positive net worth covenants that enable debtholders to force reorganization or liquidation in the event the value of the firm falls below a pre-determined threshold.

[5] In our model, liquidation might occur in or outside bankruptcy proceedings. We refer to liquidation and reorganization interchangeably.

[6] Usually this parameter is set to equal the principal P of debt as in Merton (1974) and in subsequent models. However, if liquidation costs are incurred at maturity, as in Anderson and Sundaresan (1996), this may not accurately reflect the value of debt.

[7] Black and Cox (1976) decompose firm value into two additional components: the upper boundary of the security value if the firm is reorganized and the value of the payouts it will potentially receive.