Stock Market Performance and the Term Structure of Credit Spreads*

Andriy Demchuk† and Rajna Gibson‡

First draft: April 2003
Current draft: April 2004

Abstract

We build a structural two-factor model of default where the stock market index is one of the stochastic factors. We allow the firm to adjust its leverage ratio in response to changes in the business climate, for which the past performance of the stock market index acts as a proxy. We assume that the firm's log-leverage ratio follows a mean-reverting process and that the past performance of the stock index negatively affects the firm's target leverage ratio. Our model shows that the past performance of the stock index returns and the correlation between the firm's assets and index returns have a significant impact on credit spreads. Hence, our model can explain why credit spreads may be different within the same credit-rating groups and why spreads are lower during economic expansions and higher during recessions. We also show that our model may explain actual yield spreads better than other well known structural credit risk models.

Key words: credit risk, capital structure, stock market performance

*We thank Pierre Collin-Dufresne, René Stulz, Ernst-Ludwig von Thadden, the participants of FAME and NCCR workshops, session participants of the European Investment Review Meeting, the participants of the 7th Conference of the Swiss Society for Financial Market Reseach and of the 2nd International Conference on Credit Risk for valuable comments. This research has been carried out within the project on "Conceptual Issues in Financial Risk Management" of the National Centre of Competence in Research "Financial Valuation and Risk Management" (NCCR FINRISK). NCCR FINRISK is a research program supported by the Swiss National Science Foundation.

†Swiss Banking Institute, Plattenstrasse 14, 8032 Zürich, Switzerland. E-mail: demchuka@isb.unizh.ch.

‡Swiss Banking Institute, University of Zürich, Plattenstrasse 14, 8032 Zürich, Switzerland. E-mail: rgibson@isb.unizh.ch.

Theoretical models of default can be classified as either structural or reduced-form ones. In structural models (see, for instance, Merton (1974), Longstaff and Schwartz (1995), Leland (1994), Leland and Toft (1996), Collin-Dufresne and Goldstein (2001)), default occurs when the value of a latent variable, usually the firm value, hits the default boundary. In the above mentioned structural models the default boundary is set exogenously. For example, in the Merton model, the default boundary is equal to the promised principal payment at maturity, and default occurs if the value of the firm at maturity is below the principal bond value. In Leland (1994) and Leland and Toft (1996), the firm's default boundary is endogenously defined. Given the fact that the firm value is modelled as a continuous diffusion process, default is a fully predictable event in structural models. To the contrary, reduced form models (see, for instance, Jarrow, Lando, and Turnbull (1997), Duffie and Singleton (1999)) abstract from the firm value process. In this class of models default is treated as an unpredictable event and is driven by a jump process. Reduced form models are primarily designed to fit the observed credit spreads, while structural models rather look at the fundamentals of default.

Structural models have been extended to allow for a second state variable. In most cases, this second state variable is the short-term interest rate (see for instance Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001)). However, the empirical evidence on these two factor bond pricing models remains rather mixed (see Huang and Huang (2002)).

Our study is inspired by the empirical evidence that corporate bond prices and credit spreads significantly depend on the overall business climate. For example, Chen (1991), Fama and French (1989), Friedman and Kuttner (1992) and Guha and Hiris (2002) find that credit spreads behave counter-cyclically i.e. credit spreads tend to increase during recessions and narrow during expansions. Collin-Dufresne, Goldstein, and Martin (2001) include the S&P500 return to proxy for the overall state of the economy and find a highly statistically significant and negative relationship between credit spreads' changes and the index returns. Landschoot (2003) studies the determinants of the Euro term structure of credit spreads and also finds that DJ Euro Stoxx returns significantly and negatively affect credit spreads.

Therefore, we propose a two factor structural model of default which incorporates both firm's specific risk and systematic, e.g. overall business climate, risk. In our model, the first factor is the stochastic value of the firm's assets and the second factor is the value of the stock market index. Our model aims to explain how the performance of the stock market index (which is a proxy for the business climate) and the correlation between the firm's assets and index returns directly[1] affect corporate bond prices and resulting credit spreads.

In the paper, we allow the firm to adjust its capital structure, and hence its leverage ratio, dynamically. We model the dynamics of the firm's log-leverage ratio as a mean-reverting process, where the target (or long-term) leverage ratio is assumed to be negatively affected by the stock market performance. Even though our modeling is similar to the one in Collin-Dufresne and Goldstein (2001), our assumption about the mean-reverting log-leverage ratio is based on market timing and trade-off theories of corporate debt-equity issuance policies, whereas Collin-Dufresne and Goldstein derive the log-leverage dynamics endogenously by assuming that default threshold follows a mean-reverting process. Thus, our model is consistent with the facts that firms have target leverage ratios and time their equity and debt issues according to the business climate. In the following section, we provide theoretical and empirical evidence which supports such a modeling.

Our model shows that credit spreads tend to widen when the performance of the stock market index is weak, and vice-versa. This agrees with the empirical evidence that credit spreads tend to be lower during economic expansions and higher during recessions. Also, we show that credit spreads are higher when the correlation between the firm's assets and index returns is higher. This may explain why bonds with similar credit ratings but in different industries can have significantly different credit spreads. Finally, when comparing the performance of our model to the one of Merton (1974), Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001) models, we find that our model is generally superior especially when we look at credit spreads of medium- and long-term investment grade bonds.

The rest of the paper is organized as follows. In section I, we make a brief review of theoretical and empirical studies on firms' capital structure choices which support our modeling of the dynamics of the leverage ratio. In section II, we present the model and its assumptions. We describe the bond pricing methodology is section III. In section IV, we discuss the numerical results of our study and conduct a sensitivity analysis. We also compare the credit spreads generated by our model with the average market yield spreads and with the spreads resulting from Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001) and Merton (1974) models. Section V concludes the paper.

[1]There is also an indirect effect: from the CAPM, it follows that firms' asset values depend on their covariance with the stock market returns.

I. Capital structure choices: theory and empirical evidence

Our theoretical model of corporate default is based on the assumption that firms continuously adjust their capital structure in response to changes in the business climate and also to changes in their assets' value. Namely, we assume that the firm's log-leverage ratio follows a mean-reverting process where the long-term mean leverage is negatively related to the performance of the stock market index (see equation (6) below). In fact, the latter assumption rests on two hypotheses. The first one is that firms tend to decrease their leverage (for example, by issuing new equity) if their current leverage ratio is above the target value, and to increase their leverage (for example, by issuing new debt) if their current leverage ratio is below that target. This implies mean-reversion in the dynamics of leverage ratios. The second hypothesis about a negative relationship between firms' target leverage ratios and the stock index performance implies that firms adjust their target leverage ratios over time: they set their target leverage ratios at lower levels during periods of economic expansions and raise them during recessions. In order to see whether our assumption is consistent with the capital structure theory and empirical facts, we now look at the existing theoretical and empirical literature on firms' capital structure choices.

The first hypothesis is in line with the so-called trade-off theory of the capital structure. This theory is based on the view that firms trade-off tax benefits of debt financing and costs of financial distress. Such a trade-off leads to an "optimal" capital structure choice. A number of empirical studies have shown that firms tend to make financing choices based on target leverage ratios (see, for example, Hovakimian, Opler, and Titman (2001) and Marsh (1982)).

The second hypothesis is in line with timing models of corporate capital structure. Those models are based on the evidence that firms tend to time their issuance of corporate securities. For example, Moore (1980), Marsh (1982) and Taggart (1977) document that firms prefer to issue equity when the value of equity is relatively high (which are typically periods of economic expansion), and to issue debt when interest rates are relatively low (which are periods of economic contraction). Similarly, Choe, Masulis, and Nanda (1993) show that firms tend to issue more equity than debt in expansionary periods of the business cycle. They explain these financing patterns by changes in adverse selection costs. That is, during periods of economic upturns, i.e. when the relative value of the firm's equity is typically high, the negative price reaction associated with an equity offering announcement is smaller. Other studies on timing of securities issuances focus on firm-specific variables to explain variations in firms' leverage ratios[2].

In addition to theoretical and empirical studies on the capital structure, we would also like to cite the study by Bancel and Mittoo (2002) who surveyed managers of firms in sixteen European countries to examine the link between theory and practice of capital structure choices. The results of this survey (see Table I) are also in favor of our modeling of the dynamics of the leverage ratio. Namely, nearly 60% of the respondents admitted that they consider a target debt ratio when issuing equity, and that they prefer to issue equity when the stock price has recently risen. Nearly 45% of surveyed managers said that they would rather issue debt when interest rates are low or when their equity is undervalued. Finally, nearly two-thirds of surveyed CFO in the U.S. admitted to pursue a market timing strategy (Graham and Harvey (2001)).

[Insert Table I]

II. The Model

In this section we present our modeling assumptions and develop a continuous-time valuation framework for risky debt. This framework is then used in the next section to price risky corporate bonds.

We build a structural two-factor model of default where the value of the firm's assets and the stock market index are the underlying stochastic factors. In the model, we assume that the firm continuously adjusts its capital structure, i.e. issues either equity or debt, in response to the observed past returns of the stock market index and to changes in its leverage ratio. Our modeling approach is similar to the one of Collin-Dufresne and Goldstein (2001), which in turn extended and corrected the paper by Longstaff and Schwartz (1995). In the above two studies, a stochastic risk-free interest rate, along with the stochastic firm's value, determine the pricing of risky bonds and corresponding credit (yield) spreads. In our model, the risk-free rate is assumed to be constant. Instead, we introduce a stochastic stock

[2] Baker and Wurgler (2002) study the effect of equity market timing on capital structure. Their findings indicate that following periods of high equity valuations, when measured by market-to-book ratio, the firm's leverage decreases, and vice-versa. Hovakimian, Opler, and Titman (2001) found that the probability of issuing equity vis-a-vis issuing debt increases with the firm's market-to-book ratio. This is consistent with the hypothesis that high-growth firms have low leverage ratios and low-growth firms have high leverage ratios (Stulz (1990)).

market index with the goal to study the impact of the business climate (as measured by the index performance) and the correlation between the index and firm's assets returns on corporate bond prices. Our main assumptions are discussed below.

Assumption 1: *The risk-free rate r is constant for all maturities.*

The above assumption about the constant risk-free interest rate is made solely for the purpose of keeping the model tractable. It is possible, however, to make it stochastic, but since the impact of stochastic interest rates on risky bond prices is well documented[3], we prefer not to do so.

Assumption 2: *The uncertainty in the model is characterized by the probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the state space, $\mathcal{F}$ is filtration, and $\mathcal{P}$ is the probability measure.*

Assumption 3: *The total value of the firm's assets V follows the diffusion process:*

$$\frac{dV_t}{V_t} = (\mu_V - \delta_V)dt + \sigma dW_t \tag{1}$$

where μ_V is the instantaneous expected return and δ_V *is the rate of all possible firm's cash outflows per unit of time, σ is the volatility of assets' returns, all μ_V, δ_V and σ are assumed to be positive constants; dW_t is the increment of a standard Wiener process under the true probability measure $\mathcal{P}$.*

Assumption 4: *There is a stock market index, and its value I evolves as:*

$$\frac{dI_t}{I_t} = (\mu_I - \delta_I)dt + \gamma dZ_t$$

where μ_I is the expected drift, δ_I is the dividend yield, *γ is the volatility of the index returns, μ_I, δ_I and γ are all positive constants, dZ_t is the increment of a standard Wiener process under $\mathcal{P}$ such that $dW_t dZ_t = \rho dt$.*

One can think about the stock market index as a broadly diversified stock index (like the S&P 500) or a sector (industry) specific index, depending

[3]See, for instance, Collin-Dufresne and Goldstein (2001) or Longstaff and Schwartz (1995).

upon whether the firm adjusts its leverage based on market or industry specific economic conditions.

Since the main goal of our paper is to derive risky bond prices and the corresponding credit spreads, we work with the risk-neutral probability measure instead of the historical one. Hence, we assume that:

ASSUMPTION 5: *The financial market is frictionless and complete. Therefore, there exists a unique martingale probability measure Q equivalent to $\mathcal{P}$.*

Under the risk-neutral probability measure, the dynamics of the firm's value and the stock market index become:

$$\begin{aligned} \frac{dV_t}{V_t} &= (r - \delta_V)dt + \sigma dW_t^Q \\ \frac{dI_t}{I_t} &= (r - \delta_I)dt + \gamma dZ_t^Q \end{aligned}$$

where $W_t^Q = W_t + \Lambda t$ and $Z_t^Q = Z_t + \Lambda t$ and Λ is the market price of risk[4].

As in Collin-Dufresne and Goldstein (2001), it is convenient to define the log-firm value and log-index value variables: $y_t = \log V_t$ and $x_t = \log I_t$. Applying Ito's lemma, we obtain

$$dy_t = (r - \delta_V - \frac{1}{2}\sigma^2)dt + \sigma dW_t^Q \tag{2}$$

$$dx_t = (r - \delta_I - \frac{1}{2}\gamma^2)dt + \gamma dZ_t^Q \tag{3}$$

As already mentioned, the performance of the stock market index is assumed to affect the firm's leverage ratio in our model. Therefore, we introduce the variable ψ_t which refers to the *"recent"* performance of the stock market index:

$$\psi_t = \log I_t - \log \bar{I}_t = x_t - \bar{x}_t$$

where $\bar{x}_t = \log \bar{I}_t$ is the logarithm of the *"historical average"* value of the index. In our model, the variable ψ_t aims to reflect the business climate:

[4] Λ is equal to: $\Lambda \equiv \frac{\mu_V - r}{\sigma} = \frac{\mu_I - r}{\gamma}$.

High values of ψ (i.e. strong performance of the stock index) would imply economic improvements, and low values of ψ (i.e. weak index performance) would imply economic slowdowns. We will show below that the initial value of the stock index performance ψ_0 has an important impact on credit spreads.

The *"historical average"* value of the stock index is defined as follows:

$$\bar{I}_t = \exp\left(\theta \int_{-\infty}^{t} e^{-\theta(t-s)} x_s ds\right) \tag{4}$$

The above specification of the "average" historical index value implies that the latter is a geometric average of the past index values[5], where the parameter θ represents the weight with which the average is calculated: the higher the value of θ, the more weight is put on the most recent history of the index[6]. Also, notice that the setting of the lower integration limit in (4) to minus infinity grants us the Markovian property of the "recent" index performance variable ψ_t [7].

Using (4), we derive the dynamics of the market index performance:

$$d\psi_t = dx_t - \theta\psi_t dt$$

[5]Formula (4) is a continuous-time analog to the definition of the geometric average:

$$\bar{I}_0 = I_{-n}^{\alpha_n} \cdot I_{-n+1}^{\alpha_{-n+1}} \cdot ... \cdot I_0^{\alpha_0}$$

where $\sum \alpha_i = 1$.

[6]To explain the role of θ, assume that for a specific firm the relevant time horizon for the computation of the index performance is equal to one year. Then, by requiring from the last one year index values to enter the historical average (4) with the cumulative weight of, let's say, 0.99, we obtain:

$$\theta \int_{t-1}^{t} e^{-\theta(t-s)} ds = 1 - e^{-\theta} = 0.99$$

Solving the above equation, we obtain that the value of parameter θ should be equal to 4.6.

[7]It can be easily shown that if in (4) one takes a finite lower boundary, let say $t - \bar{t}$, instead of $-\infty$ (with the appropriate adjustment of the weights), then at each point of time t the innovations in ψ_t will depend on the value of the index not only at time t, but also on its value at time $t - \bar{t}$. In that case ψ_t is non-Markovian and the further analysis becomes much more complex. In order for formula (4) to make sense, we have to define the "distant" history of the stock index. For example, we do not know the value of S&P 500 at time $t = -\infty$, or 500 years ago. Without loss of generality, we assume that at times prior to the appearance of the first data point of the index value, all the index values are equal to the initial index value. However, the "distant" index history can be irrelevant if one properly chooses the value of parameter θ. For example, if $\theta = 4.6$ (i.e. the performance of the index is calculated during the last one year), then the "distant" historical index values enter the average with a cumulative weight of 0.01 (see the previous footnote).

If we substitute in the above equation expression (3) for dx_t, we obtain:

$$d\psi_t = \theta \left(\frac{r - \delta_i - \frac{1}{2}\gamma^2}{\theta} - \psi_t \right) dt + \gamma dZ_t^Q, \qquad \psi_0 = \log \frac{I_o}{\bar{I}_0} \tag{5}$$

The above equation implies that the performance of the stock market index follows a mean-reverting process. Also, we can see that the longer the time horizon during which the index performance is calculated, that is the smaller θ, the higher is the long-run mean of the index performance and the lower is the speed of adjustment of the above mean-reverting process.

In our further analysis, the log-firm value $y_t = \log V_t$ and the stock market index performance $\psi_t = \log(I_t/\bar{I}_t)$ affect the assumed dynamics of the firm's leverage ratio and the probability of default. Thus, we first describe the leverage ratio process and provide our motivation for it, and then we present our modeling of the default event.

ASSUMPTION 6: *The firm's log-leverage ratio $l_t = \log(L_t) = \log(D_t/V_t)$, where L_t is the firm's leverage ratio and D_t is the book value of the firm's outstanding debt, evolves as*

$$dl_t = \lambda[\bar{l}^Q - \phi\psi_t - l_t]dt - \sigma dW_t^Q \tag{6}$$

In other words, we assume that the firm's leverage ratio follows a mean-reverting process, where the target, or long-run mean, leverage ratio is time-dependent and is equal to $\bar{l}^Q - \phi\psi_t$ under the risk-neutral probability measure. Such a formulation corresponds to the so-called trade-off theory of firms' capital structure, which is built on the concept of target capital structure that balances costs and benefits of debt and equity[8]. We further assume that there is a negative relationship between the stock market performance ψ_t and changes in the leverage ratio, i.e. $\phi > 0$ [9], which implies that the long-run mean leverage ratio declines with the stock market performance.

Thus, our modeling of the dynamics of the log-leverage ratio (6) is in line with the so-called timing theory of capital structure and is in line with studies of debt vs. equity choice that have found that firms prefer to issue equity rather than debt, i.e. the leverage ratio tends to decline, after periods of economic expansion (when ψ_t is high)[10], and vice versa.

[8]See, for example, Hovakimian, Opler, and Titman (2001) or Marsh (1982). Also, see our discussion in section I.

[9]For example, Fama and French (2002) find a negative relationship between target market and book leverage and the firm's asset returns, and this is in line with our assumption given a positive correlation between assets' and index returns.

[10]See, for instance, Choe, Masulis, and Nanda (1993), Marsh (1982), Taggart (1977).

Therefore, the drift term in equation (6) incorporates a hybrid hypothesis that firms have target leverage ratios but also time their equity and debt issuance.

In our risk neutral specification of the dynamics of the log-leverage (6), $\bar{l}^Q$ stands for the long-run mean log-leverage ratio when the firm is not concerned about timing of equity and debt issuances, i.e. $\phi = 0$. The risk-neutral value $\bar{l}^Q$ and the true value $\bar{l}$ of the long-run mean log-leverage ratio are related as follows:

$$\bar{l}^Q = \bar{l} + \frac{\sigma}{\lambda}\Lambda \tag{7}$$

where Λ is the market price of risk, σ is the assets' volatility and λ is the speed of adjustment of the process in (6). We will use this relationship in our numerical analysis to get an estimate of $\bar{l}^Q$ from the historical (or true) values of the firms' target leverage ratios.

The diffusion term in (6) is derived from the definition of the leverage ratio and is based on the natural assumption that the firm's book value of debt evolves deterministically[11]. To conclude the discussion of Assumption 6, we would like to point out that Collin-Dufresne and Goldstein (2001), having a stochastic short-term risk-free interest rate as a second factor, derive similar dynamics of the log-leverage ratio endogenously. Their derivation is based on the assumed negative relationship between the firm's default threshold and the level of interest rates, where the default threshold is implicitly associated with the firm's book value of debt. Therefore, it follows from their model that changes in the firm's leverage are due to changes in the firm' assets value and in the firm's debt issuance policy. In contrast, our model explains changes in the firm's leverage ratio by the firm's equity and debt issuance policies.

The existing structural models define the event of default in two distinct ways. For example, in Merton (1974), the firm defaults if, at the time of maturity of the bond, the value of the firm's assets is below the face value of the bond. In Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001), default occurs when the firm's assets decline to an

[11] The market value of the risky debt may evolve stochastically, but the book value of debt, as follows from its definition, evolves deterministically in our framework. Assuming that

$$d \log D_t = g(t, y_t, \psi_t)dt,$$

and then using the definition of the log-leverage, i.e. $l_t = \log(D_t/V_t) = \log D_t - y_t$, we obtain that the diffusion term of dl_t is equal to $-\sigma dW_t^Q$.

exogenously specified boundary (threshold). On the other hand, Leland (1994) and Leland and Toft (1996) model a firm which defaults strategically, and, by maximizing the firm's equity, they derive the default boundary endogenously.

In the paper, we assume a very general firm's debt structure consisting of different bonds with different maturities. The total time-t book value of the firm's outstanding debt is denoted by D_t and we specify the default event as follows:

Assumption 7: *The firm defaults when the value of its assets V hits an endogenously specified boundary (threshold) for the first time. This threshold is assumed to be equal to the firm's book value of total debt D. Equivalently, default occurs when the firm's log-leverage ratio l_t becomes equal to zero for the first time. The corresponding stopping time is defined as:*

$$\tilde{\tau} = \inf\{t > 0, V_t = D_t \Leftrightarrow l_t = \log \frac{D_t}{V_t} = 0\}$$

The firm continues to service its debt as long as the value of its assets V_t is greater than the book value of its debt D_t. If V hits D, the firm is assumed either to be unable to meet some of its debt obligations or violates minimum net worth or working-capital requirements. Then, the firm is assumed to default on all its debt obligations because of, for instance, cross-default provisions. Although we assume that the default boundary is equal to the book value of debt, we could extend our analysis to allow the default threshold to be, for example, a linear function of D [12]. However, since it is the leverage ratio, rather than the actual value of the default boundary, that plays a major role in our analysis, allowing a linear specification of the default boundary will not bring additional insights into the valuation of the risky bond.

To price a risky bond, we have to specify its payoff in case of default. Namely, we must specify how much a bondholder recovers when the firm is in distress. In order to keep our model simple, we assume that in the case of default a bondholder recovers a constant fraction of the face value of the

[12] For example, in the KMV model, the default boundary is equal to the sum of the face value of the short-term debt and half of the face value of the long-term debt. In Huang and Huang (2002), the default boundary is equal to 60% of the book value of a firm's debt. However, they report that their estimated credit spreads do not change significantly when they change the default boundary to 100% of the book value of debt (table 12 in Huang and Huang (2002)).

risky bond[13], and we set this fraction exogenously[14].

Assumption 8: *Following Longstaff and Schwartz (1995), we assume that in the case of default each bondholder receives* $(1 - w)$ *units of the identical risk-free bond (i.e. with the same face value and maturity) in exchange for the defaulted risky bond at the time of default.*

The above assumption implies that at the time of maturity of a risky bond, an investor will recover only a fraction $(1 - w)$ of the face value of his risky bond portfolio, where the factor w represents a loss-given-default fraction of the face value of the defaulted security. Note, that companies normally have several categories of bonds outstanding. Therefore, bonds which belong to the same category are expected to have the same recovery rate in the case of reorganization. Thus, only a few values of w are necessary in valuing a firm's debt. Empirical studies on recovery rates report that they can vary significantly. For example, Altman, Brady, Resti, and Sironi (2002) find that recovery rates of defaulted bonds vary between 25% and 65% in their sample, which implies that the loss rate at default is between 35% and 75%[15].

III. Pricing corporate bonds

Based on Assumption 8 and given that the default-free interest rate r is constant, we use risk-neutral valuation to price a zero-coupon bond which promises to pay one dollar at time T:

$$B(0,T) = e^{-rT} E_0^Q [\mathbf{1}_{\tilde{\tau}>T} + (1 - w)\mathbf{1}_{\tilde{\tau}\leq T}] \tag{8}$$

[13] One could also assume a more complex structure of the recovery rate. For example, the recovery rate can depend on the bond's seniority, which would imply that the strict absolute priority rule (APR) holds. However, empirical evidence shows that APR is frequently violated. For example, Franks and Torous (1989) and Weiss (1990) find that APR is violated in about 75% of bankruptcies they study.

[14] Several studies model the recovery rate as the outcome of the bargaining between different corporate claimholders. See, for example, Anderson and Sundaresan (1996).

[15] We would like to note that, following Merton (1974), we could extend our model to treat the recovery value as a constant fraction of the value of the firm's assets at the time of default. Also, one can make the recovery rate time-dependent to reflect bonds' seniority levels or to reflect the recovery rate's dependence with respect to the business cycle variable (see Altman, Brady, Resti, and Sironi (2002)).

where $(1-w)$ is a constant recovery rate in case of default. The above pricing formula can be rewritten in the following form

$$B(0,T) = e^{-rT}(1 - wQ(l_0,T)) \tag{9}$$

where $Q(l_0,T)$ is the risk-neutral probability that default occurs before time T, where the event of default is defined by Assumption 7. In order to define the probability of default, we have to estimate the hitting-time density function of the process governing the dynamics of the log-leverage ratio l_t. From (6) it follows that $l_t = l_t(\psi_t, W_t)$ is a Markov process. Unfortunately, there is no known closed form solution for the density of the hitting time for such a process. Therefore, we derive the risk-neutral probability of default $Q(l_0,T)$ numerically by using the technique proposed by Collin-Dufresne and Goldstein (2001)[16].

Proposition 1 *Given the initial value of the firm's log-leverage l_0^Q and the initial distance between the current and the average index value ψ_0, the risk-neutral probability that default occurs before maturity of the bond T is equal to*

$$Q(\psi_0, l_0^Q, T) = \sum_{i=1}^{N_\psi}\sum_{j=1}^{N_T} q(\psi_i, t_j)$$

where

$$q(\psi_i, t_1) = \Delta\psi G(\psi_i, t_1) \qquad i = 1, 2, ..., N_\psi$$

$$q(\psi_i, t_j) = \Delta\psi \left[G(\psi_i, t_j) - \sum_{v=1}^{j-1}\sum_{u=1}^{N_\psi} q(\psi_u, t_v) g(\psi_i, t_j|\psi_u, t_v)\right]$$

$$i = 1, ..., N_\psi \text{ and } j = 2, ..., N_T$$

$$G(\psi, t) \equiv \pi(\psi_t|\psi_0) N\left(\frac{\mu(\psi_t, t|l_0^Q, \psi_0)}{\Sigma(\psi_t, t|l_0^Q, \psi_0)}\right)$$

$$g(\psi_t, t|\psi_s, s) \equiv \pi(\psi_t, t|\psi_s, s) N\left(\frac{\mu(\psi_t, t|l_s = 0, \psi_s, s)}{\Sigma(\psi_t, t|l_s = 0, \psi_s, s)}\right) \qquad \forall t > s$$

[16] See their proposition 2.

where $\pi(\psi_t, t|\psi_s, s)$ *is the transition density of the process* ψ_t*,* μ *and* Σ *are the conditional mean and the variance of the log-leverage,respectively;* $\Delta\psi$ *and* Δt *are discretization intervals.*

Proof. *See Appendix A.* ■

Based on the theoretical bond prices, defined by equation (9), we can compute the term structure of credit spreads (S_T) using the following expression:

$$S_T = -\frac{\log B(0,T)}{T} - r \tag{10}$$

In the following section we present numerically simulated credit spreads and analyze their sensitivities to a set of parameters. We also compare the credit spreads generated by our model to actual credit spreads and to those derived from other well known structural corporate debt pricing models.

IV. Numerical results

In this section, we analyze the performance of our model numerically. For that purpose, we compute credit spreads for representative firms from different investment-grade groups (firms are grouped using Moodys' credit ratings Aaa, Aa, A, Baa and Ba) and compare them with observed market yield spreads. Next, we study the sensitivity of the credit spreads with respect to the stock market index volatility and the correlation between the assets' and index returns. We finally compare term structures of credit spreads generated by our model to those derived from Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001) and Merton (1974) models.

We begin with the description of the model's base-case parameter values.

A. The base-case: parameter value choices

Below, we present the base-case parameters of our model and discuss each parameter's choice individually.

a) Parameters governing the evolution of the assets' value and the stock market index dynamics (see Assumptions 1 and 2)

- the value of assets' volatility σ is set to 0.30. This value is taken exogenously, however, it is close to the implied volatility values obtained by Huang and Huang (2002) (HH). They derived values for the assets' volatility endogenously by calibrating Longstaff and Schwartz (1995) (LS) and Collin-Dufresne and Goldstein (2001) (CDG) models.

- S&P 500 index is taken as a proxy for the market index. Using monthly time-series data for S&P 500, which we obtain from Datastream, we estimate that the annualized standard deviation of the index returns during 1999-2002 was close to 20%. Thus, we set $\gamma = 0.20$;

- the risk-free rate r is assumed to be 3%, and this figure is close to the average of Treasury notes yields during the period 1999-2002. Both the assets' payout ratio δ_V and the index dividend yield δ_I are assumed to be equal to 1.0%;

- in our analysis we do not need the values of expected assets' and index returns μ_V and μ_I. Instead, we need an estimate of the market unitary price of risk Λ, which is equal to the ratio of the market risk premium to the market returns volatility. In our base-case, we set Λ equal to 0.2. This value is within the range of estimated Sharpe ratios in Lustig (2002).

- the correlation coefficient ρ is assumed to be the same for each rating group, and we set it to $\rho = 0.5$. This value is derived from the relationship

$$\beta = \frac{\sigma}{\gamma}\rho \Longrightarrow \rho = \beta\frac{\gamma}{\sigma}$$

where the average of the rating-groups' betas is from Barnhill and Maxwell (2002): β =0.8.

b) Parameters governing the dynamics of the log-leverage ratio (6)

- the base-case value of parameter λ we set to 0.05[17]. This means that the speed of adjustment of the log-leverage ratio to its long-term mean value equals to 0.05. However, we will study the sensitivity of credit spreads with respect to changes in the speed of adjustment λ;

- we set the value of parameter ϕ to 10. This means that the instantaneous sensitivity of leverage growth rate to the market return equals to $\phi\lambda = 0.5$.

- values of historical (true) target leverage ratios $\bar{L}$ are taken from Standard and Poor's (2002) and presented in Table II. The risk-neutral log-leverage ratios are then defined by using formula (7).

[Insert Table II]

c) The other parameters affecting risky bonds' prices

[17] This value is close to the estimate of Fama and French (2002) ($\lambda \approx 0.1$).

- recovery rate $(1-w)$ is taken from HH and is equal to 0.51. This value also agrees with observed recovery rates[18];

- the initial value of the index performance ψ_0 [19] is assumed to be 0.2. However, we will study the impact of the initial index performance on credit spreads;

- the initial true leverage ratio L_0 is assumed to equal to 80% of the target leverage ratio, i.e. $L_0 = 0.8\bar{L}$. Below, we will show that changes in the initial leverage ratio have a significant impact on credit spreads;

- the value of parameter θ, which models the length of a time period during which the index performance is calculated is equal to 2. This value implies that the performance of the stock market index is computed over the previous 2.5 years.

B. Credit spreads under the base case parameter values

In Table III, we present credit spreads implied by our model under the base-case parameter values. We can see that our model yields very low credit spreads for Aaa-grade bonds, for short maturity Aa-grade bonds and for all one-year maturity bonds substantially. We would like to mention that this under-estimation is common to all structural models of default which model the dynamics of the assets' value as a continuous diffusion process. The continuity of the assets' value dynamics yields negligible default probabilities for short-maturity bonds and for highly-graded bonds of all maturities. For example, in our case the long-term leverage ratio of Aaa-rated firms equals to 0.13, and those firms, according to our modeling of the leverage dynamics, balance their leverage ratios around this value. Therefore, given the continuity of their assets' value, the probability of default (i.e. the probability that the leverage ratio approaches unity) for Aaa firms is relatively low, and hence credit spreads are low. However, this effect can be corrected if, for instance, one introduces jumps in the assets' value process (see Zhou (2001)). Also, a significant part of actual yield spreads can be explained by liquidity risk, whereas our model aims to study the contribution of default risk to yield spreads.

Table III shows that our model yields increasing term-structures of credit spreads for Aaa, Aa, A and Ba bonds. For Baa bonds we obtain a humped term-structure.

[Insert Table III]

[18]For example, see Altman, Brady, Resti, and Sironi (2002) who find that recovery rates of defaulted bonds vary between 25% and 65%.

[19]See equation (5).

C. Sensitivity analysis

C.1. The impact of the firm's initial leverage (L_0) on credit spreads.

The firm's initial leverage ratio (L_0) is one of the factors that plays a crucial role in the bond pricing, and hence in the determination of initial credit spreads. This factor is a kind of *"distance to default"* measure. Intuitively, the higher is the initial leverage, the smaller is the "distance to default", and thus, the higher is the probability that the firm will be in distress during a given time horizon. This in turn induces higher credit spreads. Indeed, credit spreads of lower-grade bonds, which are characterized by high leverage ratios, are much higher than those of high-grade bonds, whose leverage ratios are low. Our model fully confirms this intuition. In Table IV we compare credit spreads when the initial leverage ratio is set to 80% (the base-case), 100% and 120% of the target leverage ratio. We see that this increase in the initial leverage induces credit spreads to become substantially higher. Also, we see that short-maturity bonds are much more sensitive to changes in the initial leverage ratio than long-maturity bonds. Namely, a given increase in the initial leverage ratio induces credit spreads to at least double for bonds with a maturity of up to two years. This difference in sensitivities of short and long maturity bonds to the initial leverage is due to the mean reversion of the leverage ratio and due to the independence of the long-run mean leverage from its initial level (see equation (6)).

[Insert Table IV]

C.2. The impact of the stock index performance (ψ_0) on credit spreads.

In Table V, we report resulting credit spreads under different values of the performance of the stock market index ψ_0. We can see that credit spreads tend to decrease with ψ_0. That means that credit spreads are lower when the "recent" performance of the stock index is high, and they are higher when the index performance is low. This agrees with the empirical evidence that yield spreads tend to be lower during economic expansions and higher during recessions.

[Insert Table V]

C.3. The impact of the speed of adjustment of the log-leverage process λ on credit spreads

In Table VI, we report credit spreads for different values of the parameter λ, which is a speed of adjustment of the mean-reverting process (6) governing the dynamics of the log-leverage ratio. It turns out that changes in the speed of adjustment may have a two-sided effect on credit spreads. On the one side, when the speed of adjustment is low, one would expect the firm's leverage ratio to exhibit higher variation around the target leverage ratio than when the speed is high. This implies that slower mean reversion leads to higher volatility of the leverage ratio, and hence to higher credit spreads. On the other hand, when the initial leverage ratio is below the target one, lower speed of adjustment of the leverage ratio implies that the latter will converge to its higher target level slower, and hence credit spreads should decrease. We can see that these two effects indeed take place when the initial leverage ratio equals to 80% of the target level (panel A). For most bonds credit spreads tend to widen when the speed of adjustment decreases from 0.15 to 0.015, and that means that the first effect dominates the second one. Only for seven- and ten-year Ba bonds does the second effect dominate the first one. However, when the initial leverage ratio is set equal to the target level (panel B), then only the first effect is present, and hence credit spreads widen when the speed of adjustment decreases.

[Insert Table VI]

C.4. The impact of the stock market index volatility γ on credit spreads.

The stock market index volatility has a weak effect on credit spreads. Table VII shows the change in credit spreads when the stock market index volatility changes from 0.20 (base case) to 0.10 and 0.30. We can observe that in all cases, credit spreads slightly increase with the volatility of the stock index. For example, when the index volatility increases from 10% to 30%, two-year Ba bonds exhibit the highest increase in credit spreads (40 basis points). This week effect results from our modeling assumptions. Namely, it is the volatility of the stock index performance (i.e. volatility of ψ_t) rather than the volatility of the index which matters for bond pricing. By examining the volatility of the index performance (see formula (A2) in Appendix A), we can see that a factor $\frac{(1-e^{-2\theta\tau})}{2\theta}(< 1)$ is multiplying the stock index volatility, which means that $Var[\psi_t]$ can be much smaller than $Var[I_t] = \gamma$. This is why the effect of γ on credit spreads is weak.

[Insert Table VII]

C.5. The impact of the correlation ρ on credit spreads.

It turns out that the correlation ρ between the assets' returns and the index returns has a much stronger impact on credit spreads than the stock index volatility. Table VIII shows that credit spreads widen when the correlation increases, and that this effect is particularly strong for short maturity bonds. In order to explain the positive relationship between credit spreads and the correlation, it is useful to prove the following proposition.

Proposition 2 *The variance of the log-leverage ratio, $Var_0[l_t]$, increases with the correlation between the asset and the stock index returns if $0 < \lambda < \theta$ and the parameters ϕ, σ and γ are positive.*

Proof. *See Appendix B.* ■

As one can see, our base-case parameters satisfy the conditions of the above proposition, and therefore, the correlation between the assets' and index returns positively contributes to the volatility of the log-leverage ratio. Note that the dependence of credit spreads on the correlation between the firm assets's and the index returns can explain why bonds with similar credit ratings but in different industries or sectors have different credit spreads.

[Insert Table VIII]

D. The comparison with other structural models

In Table IX, we compare the explanatory power of our model with the one of Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001) and Merton (1974) structural models. To make this comparison, we run our model under the same parameter values (which are common to all models) as Huang and Huang (2002). Namely, we take the risk-free rate $r = 8\%$, the assets' payout ratio δ_V and the index dividend yield δ_I are both equal to 6%, the recovery rate at default is 51.31%, and the long-term mean leverage ratio is 38%. For each credit rating group, the initial leverage ratio is equal to 80% of the corresponding target leverage ratio[20], where the latter corresponds to the base case leverage value in Huang and Huang (2002). For each credit-rating group, we take the assets' volatility value as the average of implied volatilities from (HH) (for LS (with stochastic interest rate) and CDG models). The default boundary equals 60% of the firms' face value of total debt. All the models are compared to the same average market yield spreads, which are also taken from (HH). The values of those parameters which are not common to all four models correspond to our base case choice. We compare credit spreads generated by the four models relative to the same average market yield spreads, which are also taken from Huang and Huang (2002).

[Insert Table IX]

We can see that under the same sub-set of parameters, our model explains actual credit spreads better than the LS, CDG and Merton models for all investment-grade bonds with all maturities (except four-year Aa bonds). Therefore, this comparison suggests that the stock market dynamics matter more for corporate bond pricing than the dynamics of interest rates.

V. Concluding remarks

We build a structural two-factor model of default where the stock market index is one of the stochastic factors. In our model, the performance of the stock market index serves as a proxy for the business climate. We allow the firm to adjust its capital structure, and hence its leverage ratio, in response to changes in the firm value and to changes in the business climate. Our modeling of the dynamics of the firm's leverage ratio is based on the theoretical and empirical findings regarding the firm's capital structure choices. In particular, it captures the fact that firms have mean-reverting leverage ratios and tend to issue equity rather than debt during economic expansions.

We find that the correlation between the firm asset and index returns and the performance of the stock market index have a strong impact on

[20] Notice that setting the initial leverage ratio to 80%, we put ourselves in a disadvantageous position (comparing with setting the former to 100%), because credit spreads increase with the initial leverage (see Table IV).

credit spreads. Therefore, our model has the potential to explain why credit spreads are different within the same credit-rating groups and why spreads are lower during economic expansions and higher during recessions. We also find a weak impact of the volatility of the stock market index returns on credit spreads.

We compare the explanatory power of our model with the one of Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001) and Merton (1974) models. This comparison shows that under the base-case parameters, our model explains actual credit spreads better than those three models for all investment-grade bonds irrespective of their maturities (except four-year Aa bonds). Therefore, this comparison suggests that the stock market dynamics matter more for corporate bond pricing than the dynamics of interest rates. Further empirical studies should be required in order to test the validity of this conjecture.

One of the limitations of our model is that it relies on the estimation of a large set of parameters. Namely, in order to apply our model, one must estimate the parameters governing the firm value and the stock index dynamics, the correlation coefficient between asset and index returns and all the parameters entering the log-leverage dynamics (6). Hence, this study emphasizes the need for more studies on the empirical properties of firms' leverage ratios dynamics. Another limitation of this study is that it relies on a constant recovery ratio, whereas empirical studies suggest that it is time-varying and that it depends on the business climate (see, for example, Keenan (1999)). It is however possible to extend the results along these lines by allowing the recovery rate to depend on the business cycle. This extension is left for future research.

Appendix A. Proof of Proposition 1

See the proof of proposition 2 in Collin-Dufresne and Goldstein (2001). Below we present the expressions for first two moments of l_t and ψ_t.

Calculation of the first two moments of l_t and ψ_t

Solving equations (6) and (5) we obtain:

$$l_t = e^{-\lambda(t-s)}\left[l_s + \lambda\int_s^t e^{\lambda(u-s)}\bar{l}(\psi_u)du - \sigma\int_s^t e^{\lambda(u-s)}dW_u^Q\right] \qquad s < t$$

$$\psi_t = e^{-\theta(t-s)}\left[\psi_s + (r-\delta_i-\frac{1}{2}\gamma^2)\int_s^t e^{\theta(u-s)}du + \gamma\int_s^t e^{\theta(u-s)}dZ_u^Q\right] \quad s < t$$

Then:

$$E_s[l_t] = e^{-\lambda(t-s)}\left[l_s - (1-e^{\lambda(t-s)})\left(\bar{l}^Q - \phi\frac{r-\delta_i-0.5\gamma^2}{\theta}\right) + \right.$$
$$\left. + \frac{\lambda\phi}{\lambda-\theta}\left(\frac{r-\delta_i-0.5\gamma^2}{\theta} - \psi_s\right)\left(e^{(\lambda-\theta)(t-s)}-1\right)\right];$$

$$E_s[\psi_t] = e^{-\theta(t-s)}\left[\psi_s + \frac{(r-\delta_i-\frac{1}{2}\gamma^2)}{\theta}\left(e^{\theta(t-s)}-1\right)\right];$$

$$Var_s[l_t] = \left[\frac{K\sigma\rho}{\lambda} - \frac{K^2+\sigma^2}{2\lambda}\right]e^{-2\lambda(t-s)} + 2\left[\frac{K^2-K\sigma\rho}{\lambda+\theta}\right]e^{-(\lambda+\theta)(t-s)} - \frac{K^2}{2\theta}e^{-2\theta(t-s)} +$$
$$+K^2\left(\frac{1}{2\theta} - \frac{2}{\lambda+\theta} + \frac{1}{2\lambda}\right) + \frac{\sigma^2}{2\lambda} + 2K\sigma\rho\left(\frac{1}{\lambda+\theta} - \frac{1}{2\lambda}\right); \qquad (A1)$$

$$Var_s[\psi_t] = \frac{\gamma^2}{2\theta}\left(1-e^{-2\theta(t-s)}\right); \qquad (A2)$$

$$Cov_s[l_t, \psi_t] = \gamma\left[\frac{K}{2\theta}(e^{-2\theta(t-s)}-1) + \frac{K-\sigma\rho}{\lambda+\theta}\left(1-e^{-(\lambda+\theta)(t-s)}\right)\right];$$

where $K = \frac{\phi\gamma\lambda}{\lambda-\theta}$.

Appendix B. Proof of Proposition 2

For notational simplicity, we study here the unconditional variance (i.e.$s = 0$). For the conditional variance all the derivations can be easily performed in the same way. Collecting terms which stand for the correlation coefficient ρ in the equation (A1), we obtain

$$Var_0[l_t] = \{...\} + K\sigma \left[\frac{1}{\lambda}(e^{-2\lambda t} - 1) + \frac{2}{\lambda+\theta}(1 - e^{-(\lambda+\theta)t}) \right] \rho$$

If $\phi > 0$, $\gamma > 0$, $\sigma > 0$ and $0 < \lambda < \theta$, then $K = \frac{\phi\gamma\lambda}{\lambda-\theta} < 0$, and it remains to show that the expression in brackets on the right hand side of the above expression is negative. Consider the following function

$$f(\theta, \lambda) = \frac{1}{\lambda}(e^{-2\lambda t} - 1) + \frac{2}{\lambda+\theta}(1 - e^{-(\lambda+\theta)t})$$

For any fixed λ, the above function decreases with θ. Indeed,

$$\frac{df(\theta,\lambda)}{d\theta} = -\frac{1}{(\lambda+\theta)^2} \left[1 - \frac{1 + (\lambda+\theta)t}{e^{(\lambda+\theta)t}} \right] < 0$$

because of well known result that

$$e^x > 1 + x \quad \forall x > 0$$

We just have shown that function f is strictly decreasing in θ for each fixed λ. Also, for every λ,

$$f(\theta = \lambda, \lambda) \equiv 0.$$

Given the above results, we conclude that

$$f(\theta > \lambda, \lambda) < 0$$

and by this we complete the proof.

References

Altman, Edward I., Brooks Brady, Andrea Resti, and Andrea Sironi, 2002, The link between default and recovery rates: implications for credit risk models and procyclicality, Working paper, New York University.

Anderson, Ronald W., and Suresh Sundaresan, 1996, Design and valuation of debt contracts, *Review of Financial Studies* 9, 37–68.

Baker, Malcolm, and Jeffrey Wurgler, 2002, Market timing and capital structure, *Journal of Finance* 57, 1–32.

Bancel, Franck, and Usha R. Mittoo, 2002, The determinants of capital structure choice: a survey of european firms, Working paper, AFA 2003 Washington, DC Meetings; EFMA 2002 London Meetings.

Barnhill, Theodore M., and William F. Maxwell, 2002, Modeling correlated market and credit risk in fixed income portfolios, *Journal of Banking and Finance* 26, 347–374.

Chen, Naifu, 1991, Financial investment opportunities and the macroeconomy, *Journal of Finance* 46, 529–554.

Choe, Hyuk, Ronald W. Masulis, and Vikram Nanda, 1993, Common stock offerings across the business cycle, *Journal of Empirical Finance* 1, 3–31.

Collin-Dufresne, Pierre, and Robert S. Goldstein, 2001, Do credit spreads reflect stationary leverage ratios?, *Journal of Finance* 56, 1929–1957.

———, and Spencer J. Martin, 2001, The determinants of credit spread changes, *Journal of Finance* 56, 2177–2208.

Duffie, Darrell, and Kenneth J. Singleton, 1999, Modeling term structures of defaultable bonds, *Review of Financial Studies* 12, 687–720.

Fama, Eugene F., and Kenneth R. French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics* 25, 23–49.

———, 2002, Testing tradeoff and pecking order predictions about dividends and debt, *Review of Financial Studies* 15, 1–33.

Franks, Julian R., and Walter N. Torous, 1989, An empirical investigation of u.s. firms in reorganization, *Journal of Finance* 44, 747–769.

Friedman, Benjamin M., and Kenneth N. Kuttner, 1992, Money, income, prices and interest rates, *American Economic Review* 82, 472–492.

Graham, John R., and Campbell R. Harvey, 2001, The theory and practice of corporate finance: evidence from the field, *Journal of Financial Economics* 60, 187–243.

Guha, Debashis, and Lorene Hiris, 2002, The aggregate credit spread and the business cycle, *International Review of Financial Analysis* 11, 219–227.

Hovakimian, Armen, Tim Opler, and Sheridan Titman, 2001, The debt-equity choice, *Journal of Financial and Quantitative Analysis* 36, 1–24.

Huang, Jingzhi, and Ming Huang, 2002, How much of the corporate-treasury yield spreads is due to credit risk, Working paper, New York University.

Jarrow, Robert A., David Lando, and Stuart M. Turnbull, 1997, A markov model for the term structure of credit risk spreads, *Review of Financial Studies* 10, 481–523.

Keenan, Sean C., 1999, Historical default rates of corporate bond issuers, 1920-1998, Moody's Global Credit Research.

Landschoot, Astrid Van, 2003, The determinants of the euro term structure of credit spreads, Working paper, Ghent University.

Leland, Hayne E., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.

———, and Klaus Toft, 1996, Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads, *Journal of Finance* 51, 987–1019.

Longstaff, Francis A., and Eduardo Schwartz, 1995, A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance* 50, 789–821.

Lustig, Hanno, 2002, Market price of aggregate risk and the wealth distribution, Working paper, University of Chicago.

Marsh, Paul, 1982, The choice between equity and debt: an empirical study, *Journal of Finance* 37, 121–144.

Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.

Moore, Geoffrey H., 1980, Business cycles, inflation and forecasting, National Bureau of Economic Research Studies in Business Cycles, 24, Ballinger Publishing Company.

Standard, and Poor's, 2002, Corporate ratings criteria.

Stulz, Ren M., 1990, Managerial discretion and optimal financing policies, *Journal of Financial Economics* 26, 3–28.

Taggart, Robert A., 1977, A model of corporate financing decisions, *Journal of Finance* 32, 1467–1484.

Weiss, Lawrence A., 1990, Bankruptcy resolution: Direct costs and violation of priority claims, *Journal of Financial Economics* 27, 285–314.

Zhou, Chunsheng, 2001, The term structure of credit spreads with jump risk, *Journal of Banking and Finance* 25, 2015–2040.

Table I.
The determinants of securities' issuance policies

The results of a survey of European firms on the determinants of capital structure choice: the three most frequent answers regarding equity and debt issuances. Source: Bancel and Mittoo (2002), tables IV and VI).

		Important or very important (%)
Most important factors influencing ***common stock*** *issuance*		
a)	Earning per share	66.04
b)	Maintaining a target dent-to-equity ratio	59.26
c)	If the stock price has resently risen, the price at which we can issue is "high"	59.26
Most important factors influencing ***debt*** *issuance*		
a)	With the use of debt we try to minimize the weighted average cost of capital	69.77
b)	We issue debt when interest rates are low	44.83
c)	We issue debt when our equity is undervalued by the market	43.68

Table II.
Target leverage ratios.

The base-case values for the target leverage ratios. The data are from Standard and Poor's (2002).

	Aaa	Aa	A	Ba	B
Target Leverage ratio	0.133	0.282	0.339	0.425	0.572

Table III.
Credit spreads under the base-case parameter values

Credit spreads (in basis points) resulting from our model under the base-case parameters choice (see subsection A of section IV). For each credit rating group, the initial leverage ratio L_0 is equal to 0.8 times the corresponding target leverage ratio (the target leverage ratios are defined in Table II).

	Aaa	Aa	A	Baa	Ba
Time to maturity					
T=1	0.0	0.0	0.1	2.0	50.8
T=2	0.0	2.2	9.5	44.2	230.0
T=4	0.6	33.8	71.4	159.9	385.2
T=7	7.8	89.4	141.7	233.1	401.8
T=8	11.5	102.7	155.1	242.1	393.2
T=10	19.6	122.1	172.2	249.2	371.3

Table IV.
The impact of the initial leverage ratio on credit spreads

Credit spreads (in basis points) resulting from our model when the initial leverage ratio equals to 80% of the target leverage ratio (base-case), 100% and 120% of the target leverage ratio. The other parameters are base-case ones (see subsection A of section IV). The target leverage ratios are defined in Table II.

	Aaa	Aa	A	Baa	Ba
Initial leverage					
			Time to maturity: T=1		
80% of target	0.0	0.0	0.1	2.0	50.8
100% of target	0.0	0.1	1.7	22.3	308.8
120% of target	0.0	1.5	12.9	115.1	1016.5
			Time to maturity: T=2		
80% of target	0.0	2.2	9.5	44.2	230.0
100% of target	0.0	10.9	38.4	143.9	576.2
120% of target	0.1	34.4	104.4	330.4	1090.9
			Time to maturity: T=4		
80% of target	0.6	33.8	71.4	159.9	385.2
100% of target	1.9	72.5	141.3	287.7	619.1
120% of target	4.5	127.6	233.4	442.3	874.1
			Time to maturity: T=7		
80% of target	7.8	89.4	141.7	233.1	401.8
100% of target	14.4	136.7	207.9	325.6	527.4
120% of target	23.0	188.8	277.9	418.7	645.7
			Time to maturity: T=8		
80% of target	11.5	102.7	155.1	242.1	393.2
100% of target	19.6	148.4	216.3	323.6	497.8
120% of target	29.5	196.8	278.8	403.2	593.7
			Time to maturity: T=10		
80% of target	19.6	122.1	172.2	249.2	371.3
100% of target	29.6	163.1	223.7	312.9	445.8
120% of target	40.9	204.0	273.5	372.3	511.5

Table V.
The impact of the stock index performance (ψ_0) on credit spreads

Credit spreads (in basis points) resulting from our model when the performance of the stock market index equals to 50%, 20% (base-case) and -50%. The other parameters are base-case ones (see subsection A of section IV).

	Aaa	Aa	A	Baa	Ba
			Time to maturity: T=1		
Psi (0) = 0.5	0.0	0.0	0.0	0.9	28.1
Psi (0) = 0.2 (base)	0.0	0.0	0.1	2.0	50.8
Psi (0) = -0.5	0.0	0.1	0.7	10.7	173.5
			Time to maturity: T=2		
Psi (0) = 0.5	0.0	1.3	5.7	28.6	163.8
Psi (0) = 0.2 (base)	0.0	2.2	9.5	44.2	230.0
Psi (0) = -0.5	0.0	7.9	28.7	111.8	467.5
			Time to maturity: T=4		
Psi (0) = 0.5	0.4	25.5	55.5	128.5	322.9
Psi (0) = 0.2 (base)	0.6	33.8	71.4	159.9	385.2
Psi (0) = -0.5	1.6	62.7	124.1	256.9	562.0
			Time to maturity: T=7		
Psi (0) = 0.5	6.2	76.6	123.3	206.4	363.6
Psi (0) = 0.2 (base)	7.8	89.4	141.7	233.1	401.8
Psi (0) = -0.5	12.8	126.1	193.3	305.4	499.4
			Time to maturity: T=8		
Psi (0) = 0.5	9.5	89.9	137.5	217.8	360.7
Psi (0) = 0.2 (base)	11.5	102.7	155.1	242.1	393.2
Psi (0) = -0.5	17.7	138.4	203.0	306.0	474.8
			Time to maturity: T=10		
Psi (0) = 0.5	16.9	110.1	156.8	229.5	347.3
Psi (0) = 0.2 (base)	19.6	122.1	172.2	249.2	371.3
Psi (0) = -0.5	27.4	154.3	212.8	299.5	429.8

Table VI.
The impact of the speed of adjustment of the log-leverage process (λ) on credit spreads

Credit spreads (in basis points) resulting from our model when the speed of adjustment of the log-leverage process (6) equals to: $\lambda = 0.15$, $\lambda = 0.05$ (base-case) and $\lambda = 0.015$. The other parameters are base-case ones (see subsection A of section IV).

Panel A: Initial leverage ratio is 80% of the target leverage ratio

	Aaa	Aa	A	Baa	Ba
			Time to maturity: T=1		
Lam=0.15	0.0	0.0	0.1	2.2	45.3
Lam=0.05 (base)	0.0	0.0	0.1	2.0	50.8
Lam=0.015	0.0	0.0	0.1	2.1	55.6
			Time to maturity: T=4		
Lam=0.15	0.6	33.2	70.7	158.8	382.3
Lam=0.05 (base)	0.6	33.8	71.4	159.9	385.2
Lam=0.015	0.8	36.7	76.2	167.4	396.5
			Time to maturity: T=7		
Lam=0.15	4.5	78.0	131.7	229.4	413.1
Lam=0.05 (base)	7.8	89.4	141.7	233.1	401.8
Lam=0.015	10.8	99.7	152.5	242.2	405.0
			Time to maturity: T=10		
Lam=0.15	9.4	103.3	158.6	247.9	390.6
Lam=0.05 (base)	19.6	122.1	172.2	249.2	371.3
Lam=0.015	28.5	136.1	184.1	255.9	368.9

Panel B: Initial leverage ratio is equal to the target leverage ratio

			Time to maturity: T=1		
Lam=0.15	0.0	0.2	1.5	17.3	223.2
Lam=0.05 (base)	0.0	0.2	1.7	22.3	308.8
Lam=0.015	0.0	0.2	2.0	26.0	358.1
			Time to maturity: T=4		
Lam=0.15	1.3	58.3	117.2	246.9	550.2
Lam=0.05 (base)	1.9	72.5	141.3	287.7	619.1
Lam=0.015	2.6	84.5	160.1	316.5	660.6
			Time to maturity: T=7		
Lam=0.15	6.8	104.9	172.2	290.1	500.4
Lam=0.05 (base)	14.4	136.7	207.9	325.6	527.4
Lam=0.015	21.4	159.6	233.0	350.4	547.0
			Time to maturity: T=10		
Lam=0.15	12.3	125.2	188.8	288.2	439.6
Lam=0.05 (base)	29.6	163.1	223.7	312.9	445.8
Lam=0.015	45.2	188.3	246.9	330.8	454.3

Table VII.
The impact of the stock index volatility (γ) on credit spreads.

Credit spreads (in basis points) resulting from our model when the stock index volatility (γ) equals to: $\gamma = 0.30$, $\gamma = 0.20$ (base-case) and $\gamma = 0.10$. The other parameters are base-case ones (see subsection A of section IV).

	Aaa	Aa	A	Baa	Ba
			Time to maturity: T=1		
Gama=0.30	0.0	0.0	0.2	2.7	58.6
Gama=0.20 (base)	0.0	0.0	0.1	2.0	50.8
Gama=0.10	0.0	0.0	0.1	1.5	43.6
			Time to maturity: T=2		
Gama=0.30	0.0	3.2	12.4	52.6	250.0
Gama=0.20 (base)	0.0	2.2	9.5	44.2	230.0
Gama=0.10	0.0	1.5	7.2	36.7	210.2
			Time to maturity: T=4		
Gama=0.30	1.0	40.8	82.1	174.8	401.0
Gama=0.20 (base)	0.6	33.8	71.4	159.9	385.2
Gama=0.10	0.4	27.7	61.6	145.4	368.9
			Time to maturity: T=7		
Gama=0.30	10.3	99.4	153.0	244.4	409.3
Gama=0.20 (base)	7.8	89.4	141.7	233.1	401.8
Gama=0.10	5.8	80.0	130.8	221.9	393.9
			Time to maturity: T=8		
Gama=0.30	14.7	112.5	165.6	251.9	399.1
Gama=0.20 (base)	11.5	102.7	155.1	242.1	393.2
Gama=0.10	8.9	93.3	144.8	232.2	387.1
			Time to maturity: T=10		
Gama=0.30	23.9	131.1	181.1	256.6	374.8
Gama=0.20 (base)	19.6	122.1	172.2	249.2	371.3
Gama=0.10	16.0	113.4	163.5	241.8	367.6

Table VIII.
The impact of the correlation between assets' and index returns (ρ) on credit spreads.

Credit spreads (in basis points) resulting from our model when the correlation between the firm's assets and stock index returns (ρ) equals to: $\rho = 0.9$, $\rho = 0.5$ (base-case) and $\rho = -0.5$. The other parameters are base-case ones (see subsection A of section IV).

	Aaa	Aa	A	Baa	Ba
			Time to maturity: T=1		
Corr =0.9	0.0	0.0	0.2	3.0	62.4
Corr =0.5 (base)	0.0	0.0	0.1	2.0	50.8
Corr = -0.5	0.0	0.0	0.0	0.6	26.5
			Time to maturity: T=2		
Corr =0.9	0.0	3.5	13.3	55.4	257.6
Corr =0.5 (base)	0.0	2.2	9.5	44.2	230.0
Corr = -0.5	0.0	0.5	3.0	20.4	157.1
			Time to maturity: T=4		
Corr =0.9	1.1	42.4	84.7	178.7	406.5
Corr =0.5 (base)	0.6	33.8	71.4	159.9	385.2
Corr = -0.5	0.1	15.0	39.1	108.3	321.3
			Time to maturity: T=7		
Corr =0.9	10.7	101.4	155.5	247.2	412.1
Corr =0.5 (base)	7.8	89.4	141.7	233.1	401.8
Corr = -0.5	2.4	57.2	102.5	190.7	370.0
			Time to maturity: T=8		
Corr =0.9	15.3	114.5	167.9	254.4	401.4
Corr =0.5 (base)	11.5	102.7	155.1	242.1	393.2
Corr = -0.5	4.1	69.9	117.7	204.6	368.4
			Time to maturity: T=10		
Corr =0.9	24.6	132.9	183.0	258.5	376.3
Corr =0.5 (base)	19.6	122.1	172.2	249.2	371.3
Corr = -0.5	8.6	90.8	139.8	220.6	356.3

Table IX.
The comparison of the performance of structural models

We compare how much (in %) of the actual credit spreads can be explained by our model, Longstaff and Schwartz (1995) (LS), Collin-Dufresne and Goldstein (2001) (CDG) and Merton (1974) models. The performance of LS and CDG models is taken from Huang and Huang (2002) (HH). The parameters in our model correspond to those in (HH): risk-free rate $r = 8\%$, payout ratios $\delta_V = \delta_I = 6\%$, the recovery rate at default is 51.31%, and the long-term mean leverage ratio is 38%. For each credit rating group, the initial leverage ratio is equal to 80% of the corresponding target leverage ratio, where the latter corresponds to the base case leverage value in (HH). For each credit-rating group, we take the assets' volatility value as the average of implied volatilities from (HH)(for LS (with stochastic interest rate) and CDG models). The default boundary equals 60% of the firms' face value of total debt. All the models are compared to the same average market yield spreads, which are from (HH). All the other parameters are base-case ones (see subsection A of section IV).

	Aaa	Aa	A	Baa	Ba
			Time to maturity: T=4		
Average market yield spread	55	65	96	158	320
			% of spreads due to default		
Our model	1.1%	4.1%	12.6%	36.5%	84.5%
LS model	1.5%	7.0%	7.8%	16.1%	46.6%
CDG model	0.1%	9.7%	10.3%	19.7%	52.5%
Merton model	0.2%	1.1%	5.0%	21.0%	65.5%
			Time to maturity: T=10		
Average market yield spread	63	91	123	194	320
			% of spreads due to default		
Our model	40.7%	44.1%	53.9%	65.4%	84.9%
LS model	9.6%	9.4%	11.8%	19.9%	48.1%
CDG model	18.2%	16.4%	18.3%	26.9%	57.1%
Merton model	11.9%	16.6%	26.3%	41.6%	73.6%