Credit Barrier Models

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Summary

I discuss credit barrier models which can be fitted to both the credit transition matrix and credit spread curves under the real-world and risk-neutral measures, respectively. The underlying stochastic process is in a newly discovered class which is solvable in analytically closed form and is characterized by state dependent volatility and jumps and can accomodate stochastic volatility. This class extends at once most such models in the literature. I discuss the estimation procedure of the real-world measure based on the credit transition matrix and the calibration procedure for the risk neutral measure based on spread curve data. I present new results on implied credit migration rates and estimate liquidity convenience yields for forward yield spreads.

Contents

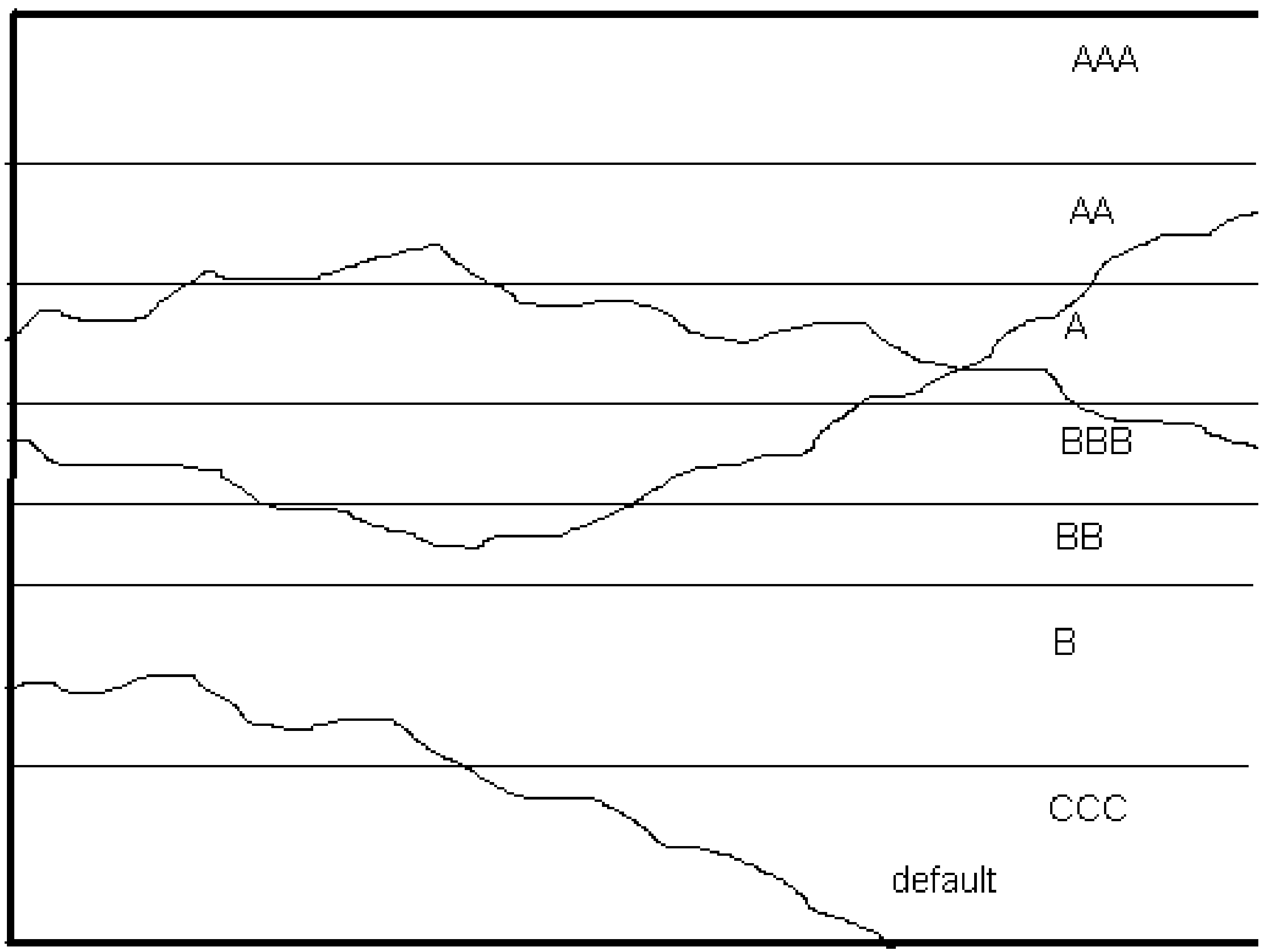
References

I will cover the content of a paper distributed at the conference and mention results from several other papers posted on my web page:

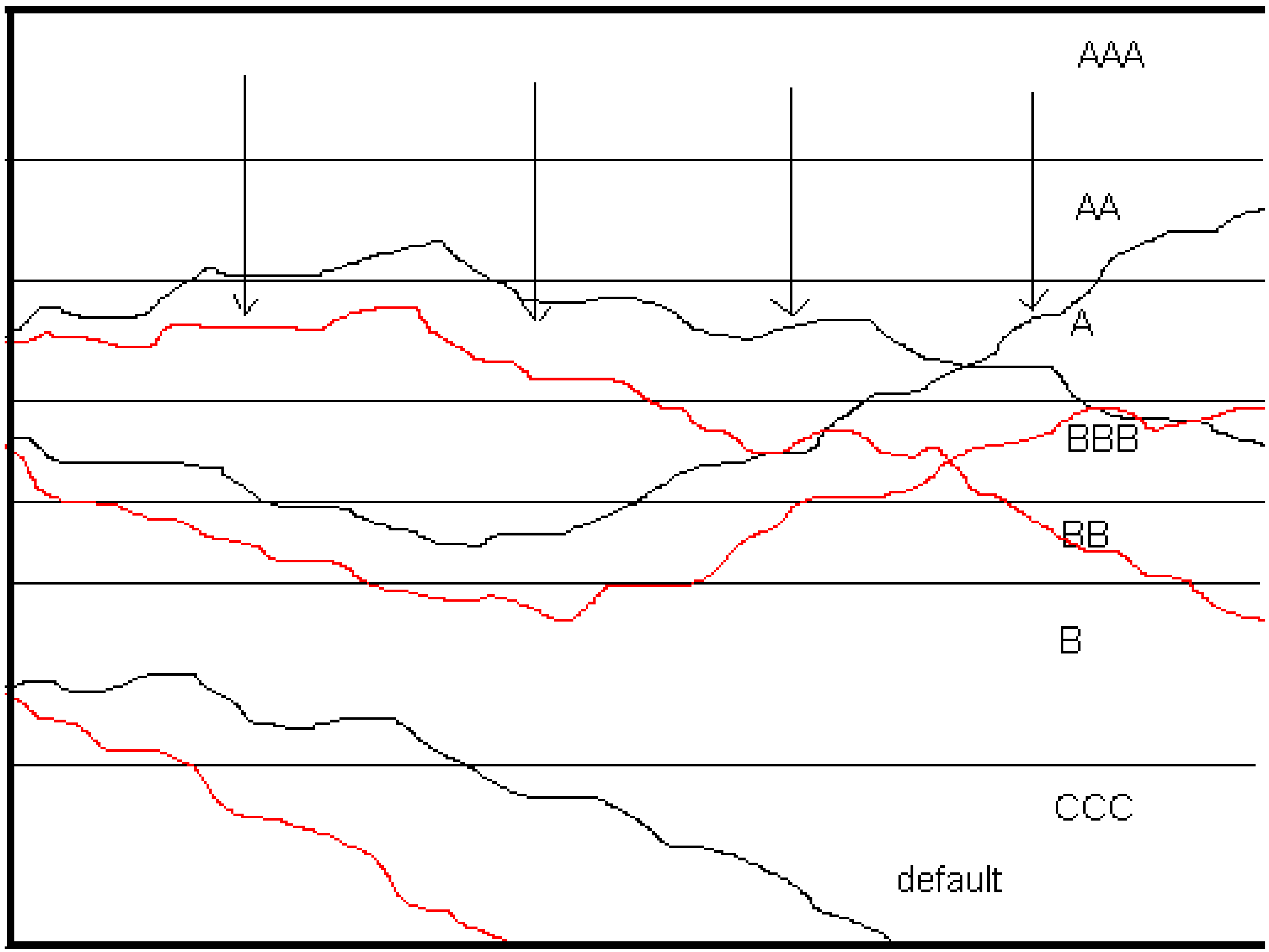
www.ma.ic.ac.uk/∼calban

My main co-author on the topic of Credit Barrier models is Oliver X. Chen. Other co-authors on related work are Peter Carr, Giuseppe Campolieti, Alexey Kuznetsov, Stephan Lawi, Alex Lipton.

The Big Picture: Real-world scenarios in our Credit Barrier Model



The Big Picture: Risk-neutral scenarios in the equivalent pricing measure



Dual real-world/risk neutral estimation

Our credit barrier models are not estimated directly on bond data (e.g. the Lehman database). Instead, they are estimated based on aggregate data, namely:

- One year credit migration probabilities

- One, three and five year historical default frequencies

- Aggregate spread curves for all rating categories (with taxation adjustment)

Disentangling liquidity spreads

We observe that real world data can be reobtained with great accuracy, while pricing data can be reobtained only on average across ratings.

The price mismatches across ratings are then interpreted as a measure of the *liquidity spreads*.

This framework can be regarded as a tool for relative value analysis which reveals a rich and instructive structure for liquidity spread across obligors and maturities.

PART I: Four classes of credit risk models

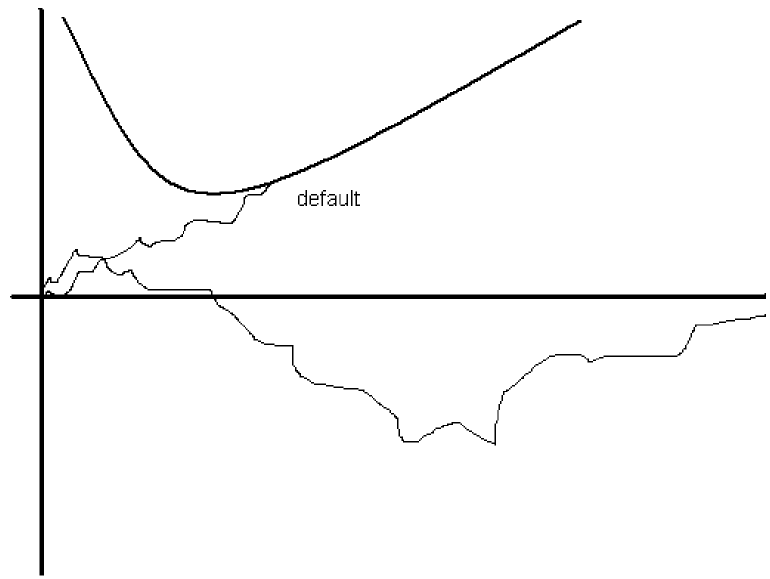
(A) **Structural models** have an explanatory variable that undergoes a diffusion process and default events occur whenever a barrier is crossed

References: Merton, Black-Cox, Brennan-Schwarz, Kim-Ramaswamy-Sundaresa, Titman-Totous, Shimko, Teima, Van Deventer-Geske, Huang-Huang.

Good for the fundamental economic description especially under the real-world measure and to establish connections with equity prices (see Dan Galai's talk). Correlations can be accounted for in the KMV-CrediMetrics scheme by using equity as a proxy for asset values.

Absolute *liquidity effects are estimated starting from first principle (as opposed to our credit barrier models where we aim at estimating* **relative** *liquidity spreads by comparing across different asset classes such as bonds of different ratings. .*

(B) **Single obligor rating credit barrier models** were introduced by Lipton et al. (1998), Hull-White (2002). They are variations on structural models estimated under the pricing measure.



Easy to calibrate with respect to the risk-neutral measure for an individual credit spread curve. No real-world connection and no consistency between different credit ratings. Correlations can be modeled using equity proxies and provide an alternative to copula models.

The case for state dependent local volatility

As outlined by several authors (including Martijn Cremers at this conference), lower quality ratings appear to be more volatile than higher quality ratings. This represents a limitation for single obligor credit barrier models and structural models whereby the spread process is only adjusted to the initial condition. As time evolves and the credit quality changes, the spread process can potentially step out of calibration, revealing initial condition biases.

(C) In **reduced form models**, credit events occur unexpectedly according to a Poisson process.

References: Duffie-Singleton, Schoenbucher

Easy to calibrate to spread data under the risk-neutral measure, difficult to estimate under the real world measure.

Correlations can be modeled with copulas but are (i) hard to put in relation with equity correlations and (ii) hard to estimate in a consistent way among large collections of reference names.

(D) **Rating based models** propose to risk-neutralize the credit migration matrix.

References: Jarrow-Lando-Turnbull, Das-Tufano, Kijima-Koboribayas

Risk neutralization schemes for the transition probabilities $p_{i,j}$ are obtained by multiplying real world probabilities by factors $\pi_{i,j}$ while keeping positivity and probability conservation:

$$q_{i,j} = \pi_{i,j}(t) p_{i,j} \quad 1 \leq i, j \leq K. \tag{1}$$

Under all proposed schemes, the factors $\pi_{i,j}$ are chosen empirically to match spread data, they attribute higher probabilities to default events but do not differentiate between up-moves and down-moves.

Our model yields a distinctly different structure for implied migration rates (where upgrades are systematically less likely under the pricing measure, downgrades are more probable and inter-temporal consistency is achieved by considering a continuous time model).

Why using a continuous rating variable

To extrapolate multi-year data to construct a continuous time model we choose to use a *continuous rating variable* (as also discussed by Gorieraux) as opposed to a discrete variable as discussed by (Christensen and Lando).

The advantage of using a continuous time model is that it gives a more natural framework for capturing correlations across obligors by regression analysis. Our analysis in this respect is based on a mapping between credit quality and equity also used to build a model for convertible bonds (coming up).

Lattice approximation to the Jacobi model

In our first papers on this topic we considered strictly continuous models. In the more recent approaches (aimed at addressing structures with optionality such as convertible bonds), we consider a lattice discretization which can be arbitrarily fine. Our model can be regarded as a

(i) Bochner subordinated

(ii) measure changed

(iii) coordinate transformed

(iv) discretized

version of the Jacobi process referred to also by Gorieraux. Our lattice approximations are also solvable in analytically closed form and are based on the Hahn process (more below).

Subordination: jumps versus stochastic volatility

The credit quality process shows leptokurtotic features because of a combination of two effects:

- **Stochastic volatility** triggered by the correlation to the economic cycle on a time scale

- **Jumps** linked to sudden changes in liquidity levels (as in Newman's talk) or to news events such as sovereign defaults that trigger collective flight to quality because of portfolio insurance schemes.

Both stochastic volatility and jumps can be modeled by means of a form of stochastic subordination (i.e. a stochastic time change). We choose to limit ourselves to Bochner subordinators (i.e. consider only jumps) for purely technical reasons and as a way of achieving dimensional reduction (bypassing the need of keeping track of an instanteneous volatility parameter).

The role of measure changes and coordinate transformations

We use a combination of measure changes an coordinate transformations in such a way to construct models which can be solved in analytically closed form. We find that analytical solvability is essential as it enables us to carry out high precision statistical estimations effectively. (We would not be able to reobtain our results by using more generic numerical methods.)

The methods we use are a variation of those in articles I co-authored with Campolieti, Carr and Lipton (2001-02) which led to what we called *hyergeometric Brownian motions*.

Background on integrable models

Among the martingale diffusions of equation:

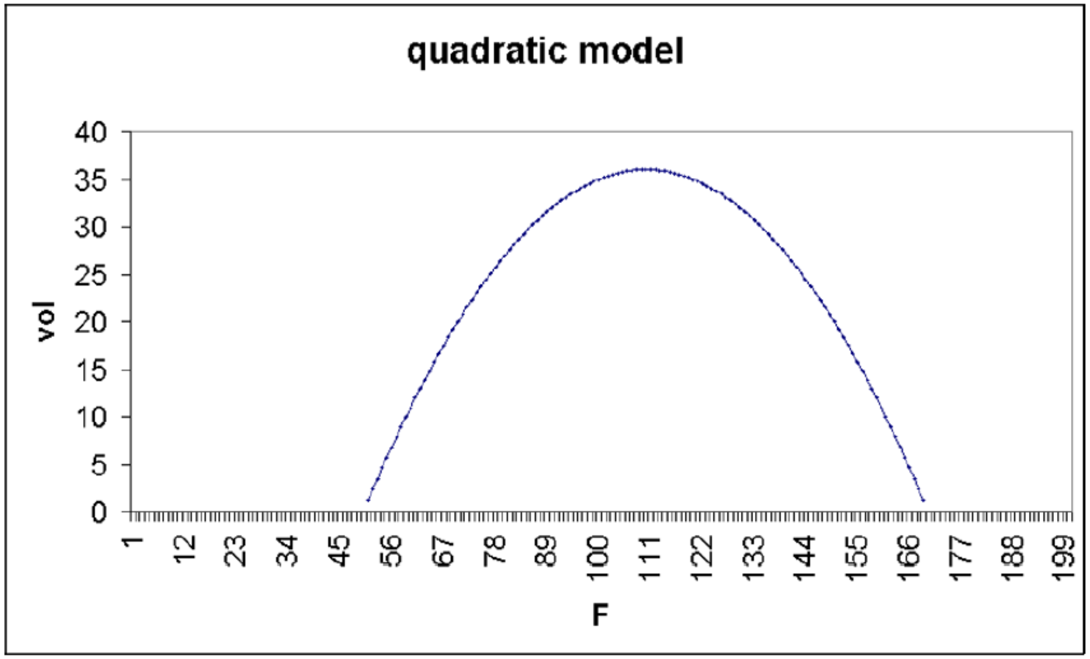
$$dF = \sigma(F)dW. \tag{2}$$

well known integrable families are given by the quadratic and CEV models

$$\sigma(F) = \sigma_0 + \sigma_1 F + \sigma_2 F^2, \qquad \sigma(F) = \frac{\sigma_0}{\theta}(F - \bar{F})^{1+\theta} \tag{3}$$

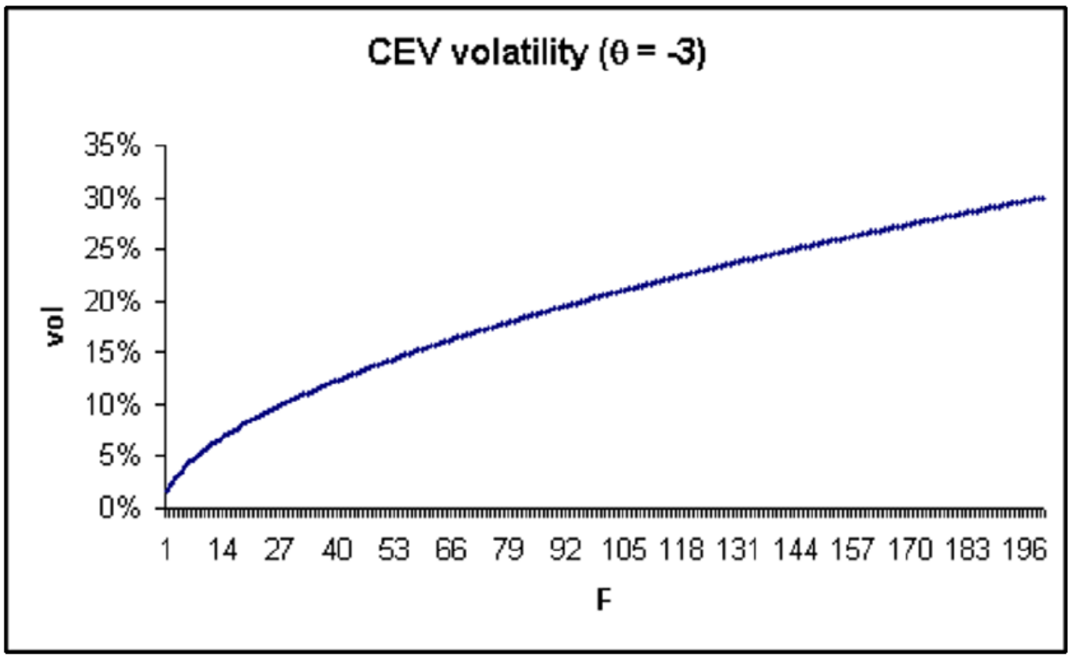
The quadratic volatility model

Quadratic volatility models with $\sigma(F) = \sigma_0 + \sigma_1 F + \sigma_2 F^2$ are often used to model interest rate derivatives. Similarly to the Black-Scholes model, quadratic volatility models reduce to ordinary Brownian motion by means of a coordinate transformation.



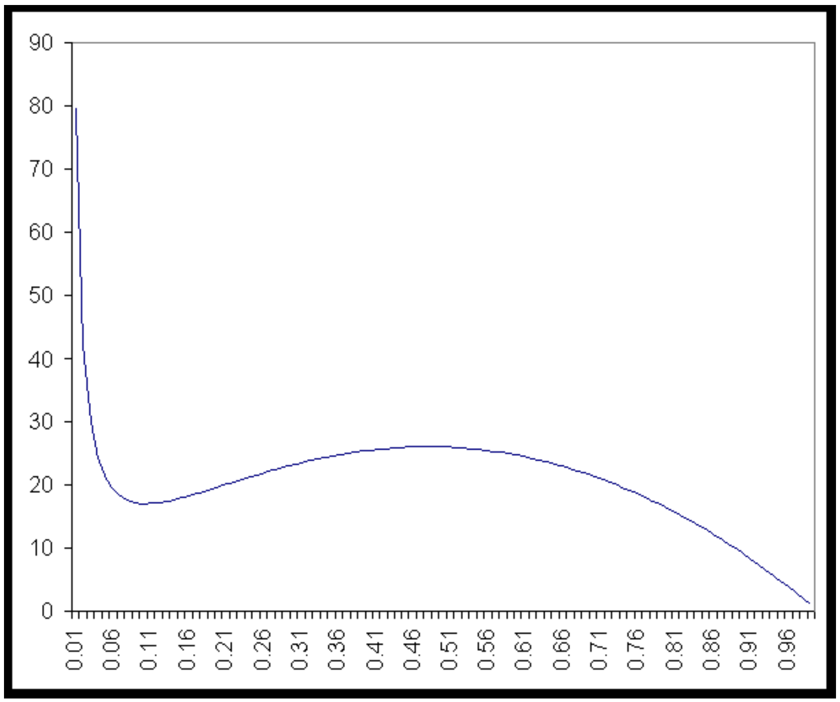
Models with constant elasticity of variance

CEV models have volatility function $\sigma(F) = \frac{\sigma_0}{\theta}(F - \bar{F})^{1+\theta}$. Cox and Ross (1978) show they can be solved by reduction to the Feller (CIR) process.



Hypergeometric Brownian motions

Hypergeometric Broanian motions are a new integrable family with 7 independent parameters:



C.Albanese, G. Campolieti, P.Carr, A. Lipton **Risk Magazine**, *December Issue, 2001*.

Further extensions

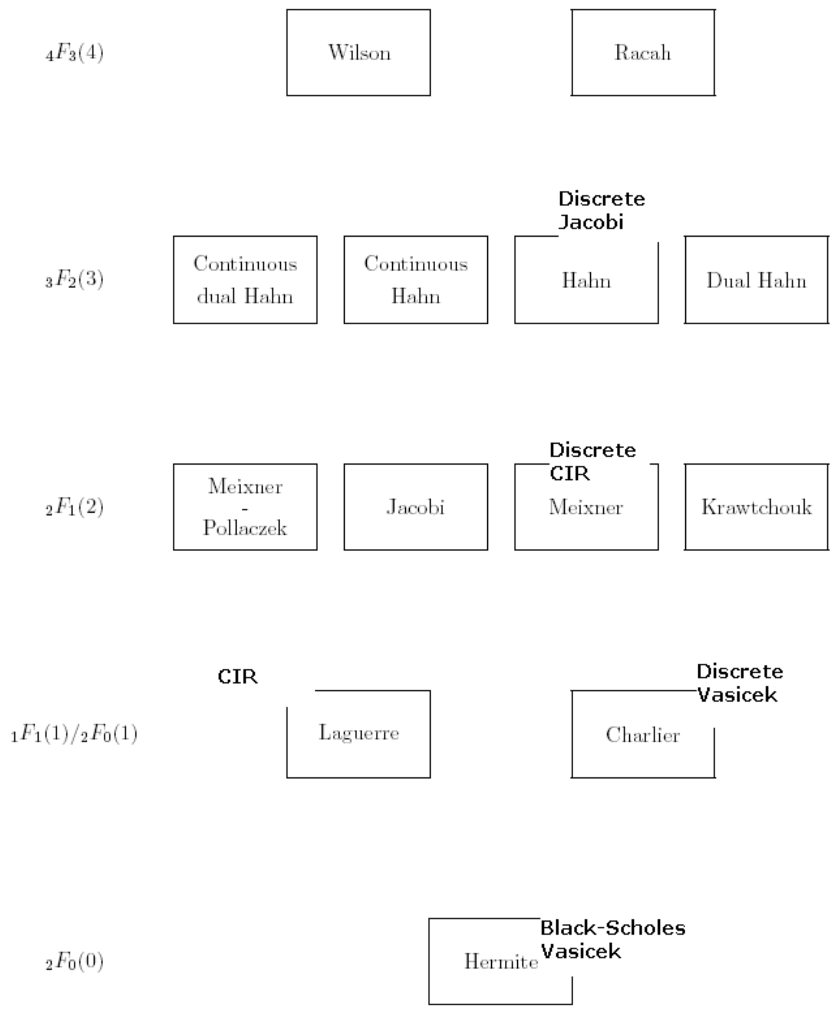
Hypergeometric Brownian motions are integrated by reduction to the CIR process

$$dx = (a - bx)dt + \sigma\sqrt{x}dW. \tag{4}$$

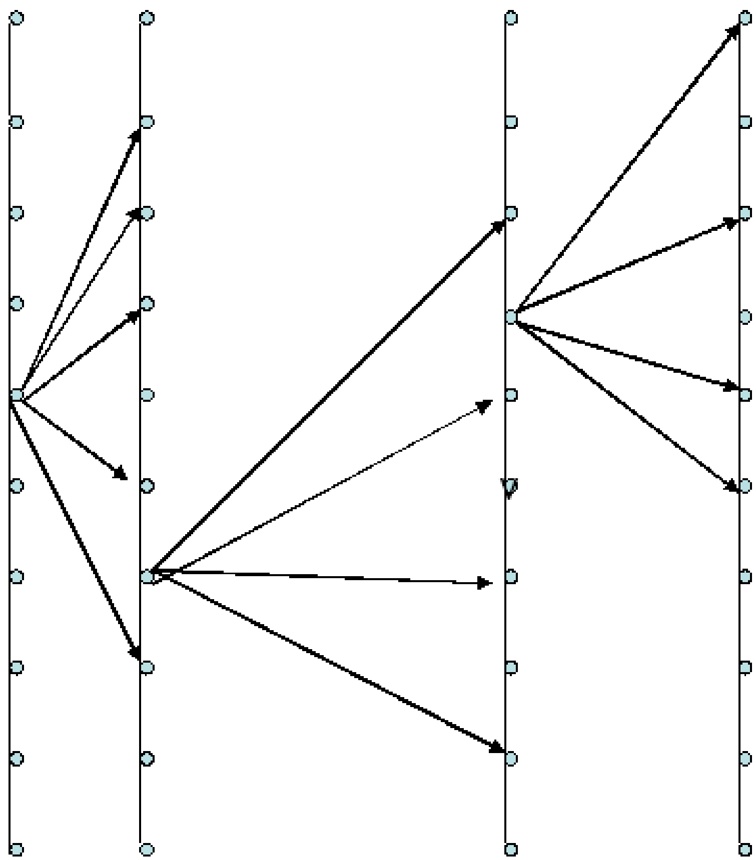
Extensions can be achieved by replacing this with more general integrable processes such as for instance the Jacobi process of equation

$$dx = (a - bx)dt + \sigma\sqrt{x(1 - x)}dW. \tag{5}$$

Model Building with the Askey-Wilson Tree

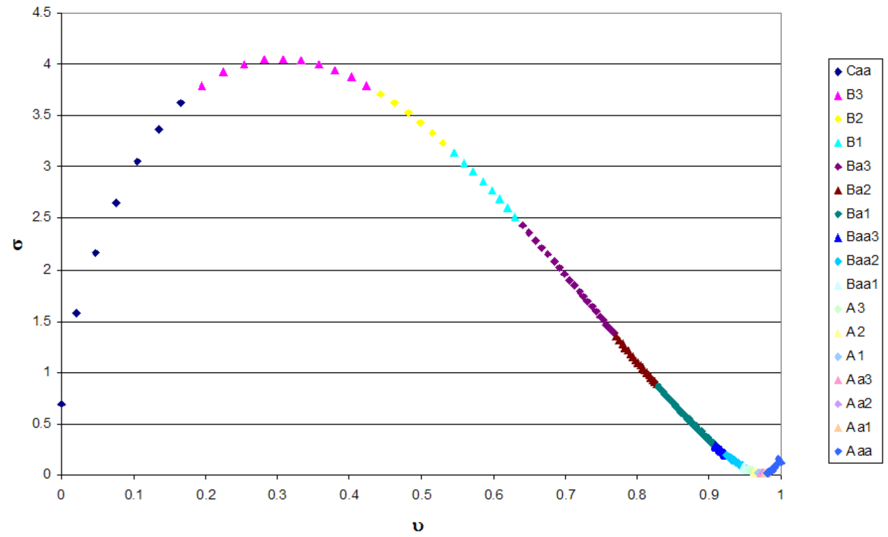


Lattice Structures for the Discretized Models



PART II: Estimation of the Real-World Measure

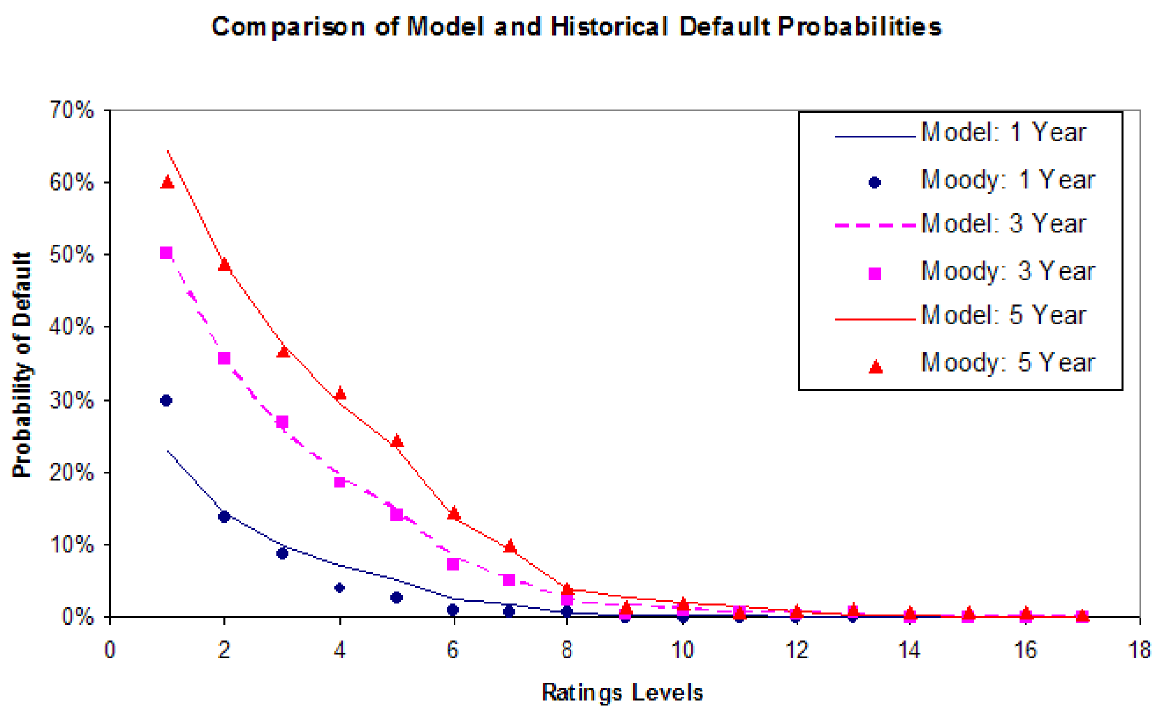
Local volatilities



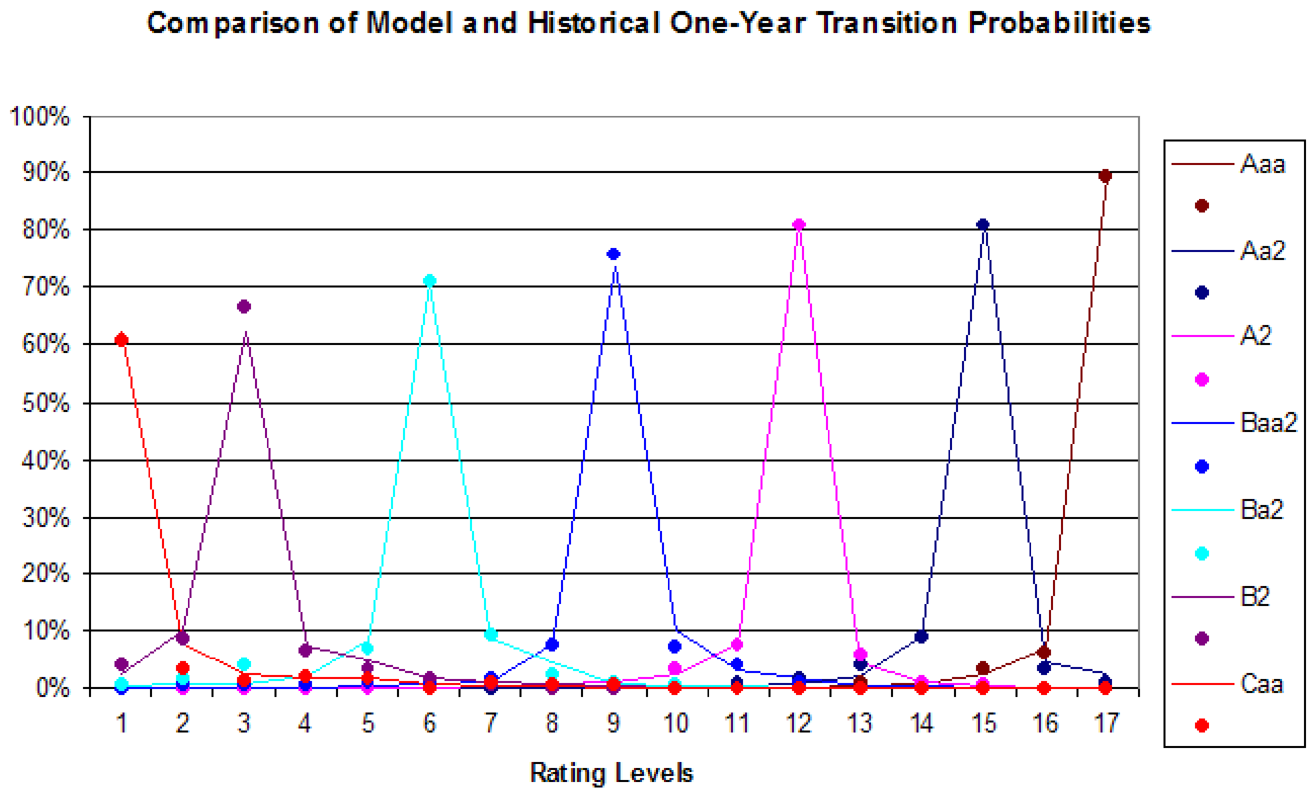
This estimated shape shows greater volatility for lower quality ratings.

Matching Moody's default probabilities.

Here, $1->Aaa$, $2->Aa1$, ..., $17->Caa$. Also notice that we *estimate* our continuous time model by forcing agreement with Moody's default frequencies across time horizons (as opposed to what done by Lando et al.)



A comparison with Moody's conditional transition probabilities for credit migrations for 1 year horizons



PART III: Risk-Neutral Calibration

Disentangling liquidity from credit spreads is perhaps the single biggest modelling challenge.

Liquidity effects can be absorbed only in part in the risk neutral measure and justified in terms of market price of credit risk; however, this approach won't capture the term structure and credit quality structure of liquidity effects. We considered the following three modelling alternatives:

- Absorb liquidity in implied recovery rates (good to start with but yields inconsistent prices for out-of-the-money default swaps);

- Model liquidity by means of forward liquidity yields to be added to risk-neutral prices (gives a consistent albeit empirical picture, is a tool for relative value analysis).

Forward liquidity yields

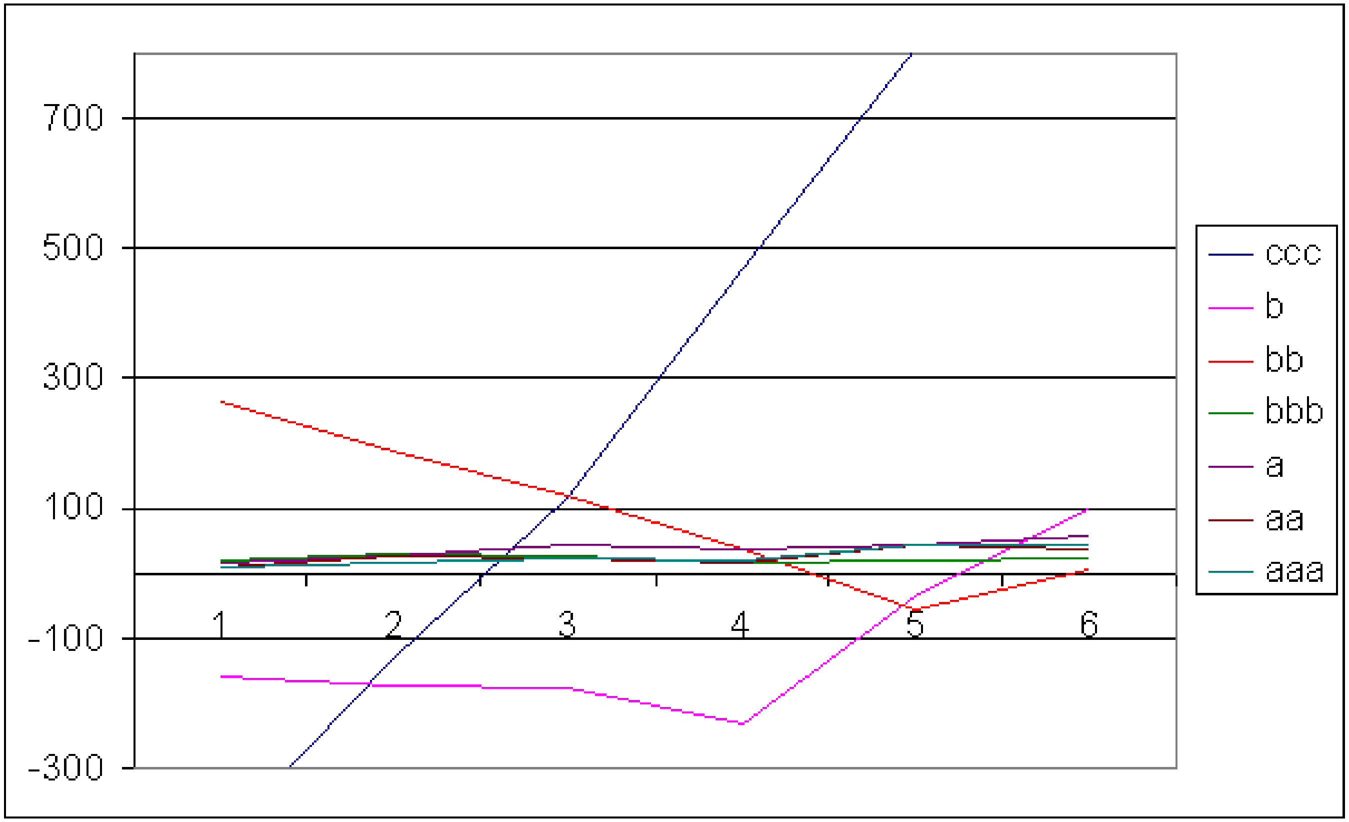
The forward liquidity yield $l(T, T+\tau, i)$ for the period $[T, T+\tau]$ and the average spread curve of rating i, computed with simple compounding over the period τ, is defined so that

$$f^{\mathsf{mkt}}(T, T+\tau, i) = l(T, T+\tau, i)) + f^{\mathsf{mdl}}(T, T+\tau, i) \qquad (6)$$

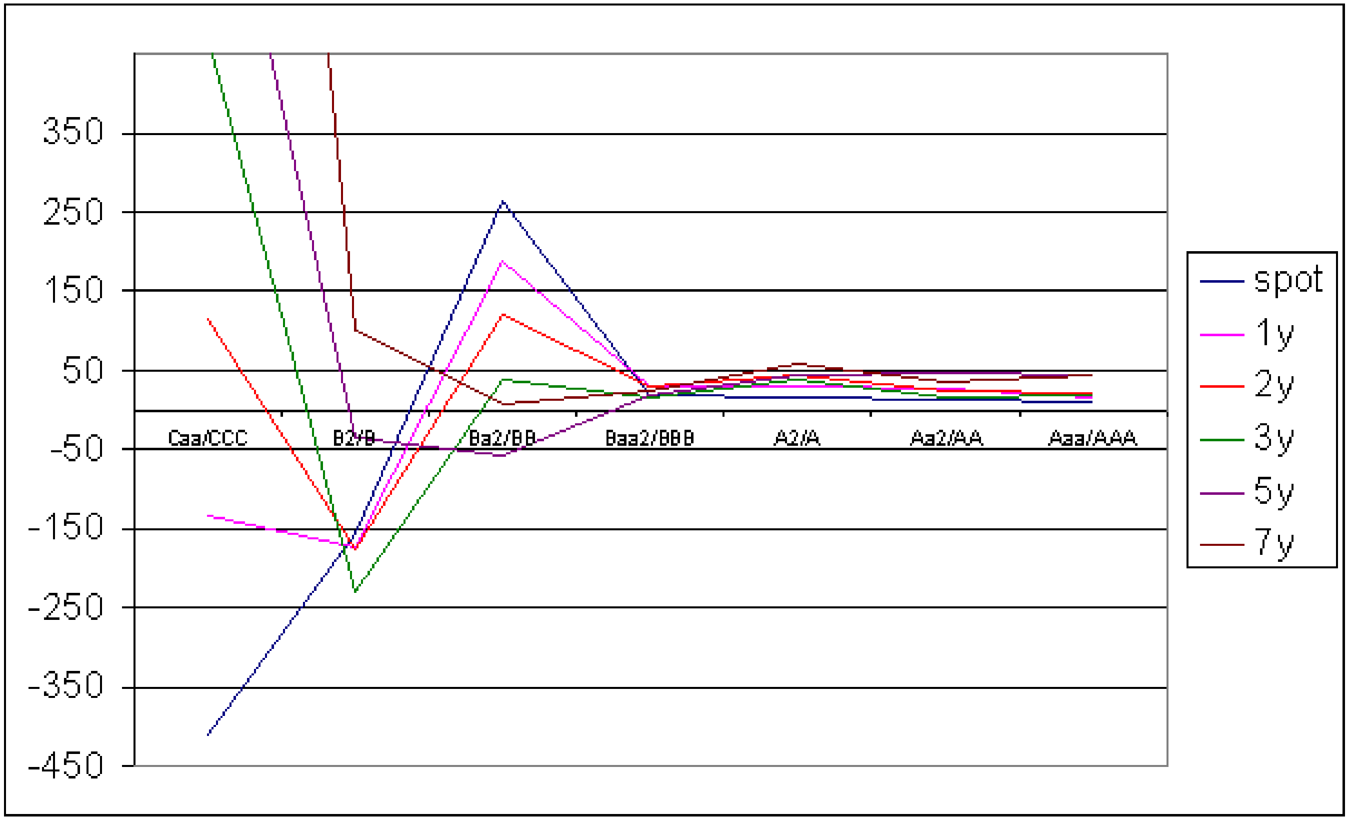
where $f^{\mathsf{mkt}}(T, T+\tau, i)$ and $f^{\mathsf{mdl}}(T, T+\tau, i)$ are the model and market forward rates, respectively.

The estimation criterion and term structure of liquidity forward yields

Objective: forward liquidity yields for investment grade bonds are strictly negative and the sum of their squares is minimum:

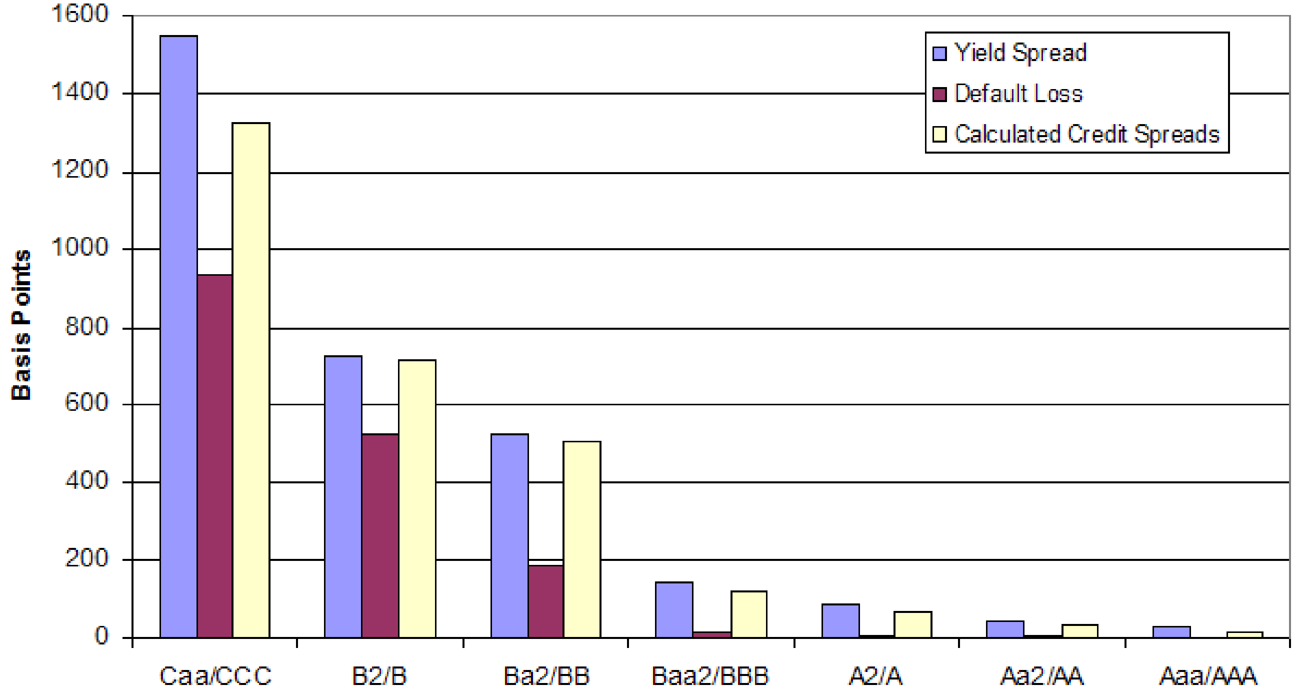


Forward liquidity yields by credit quality

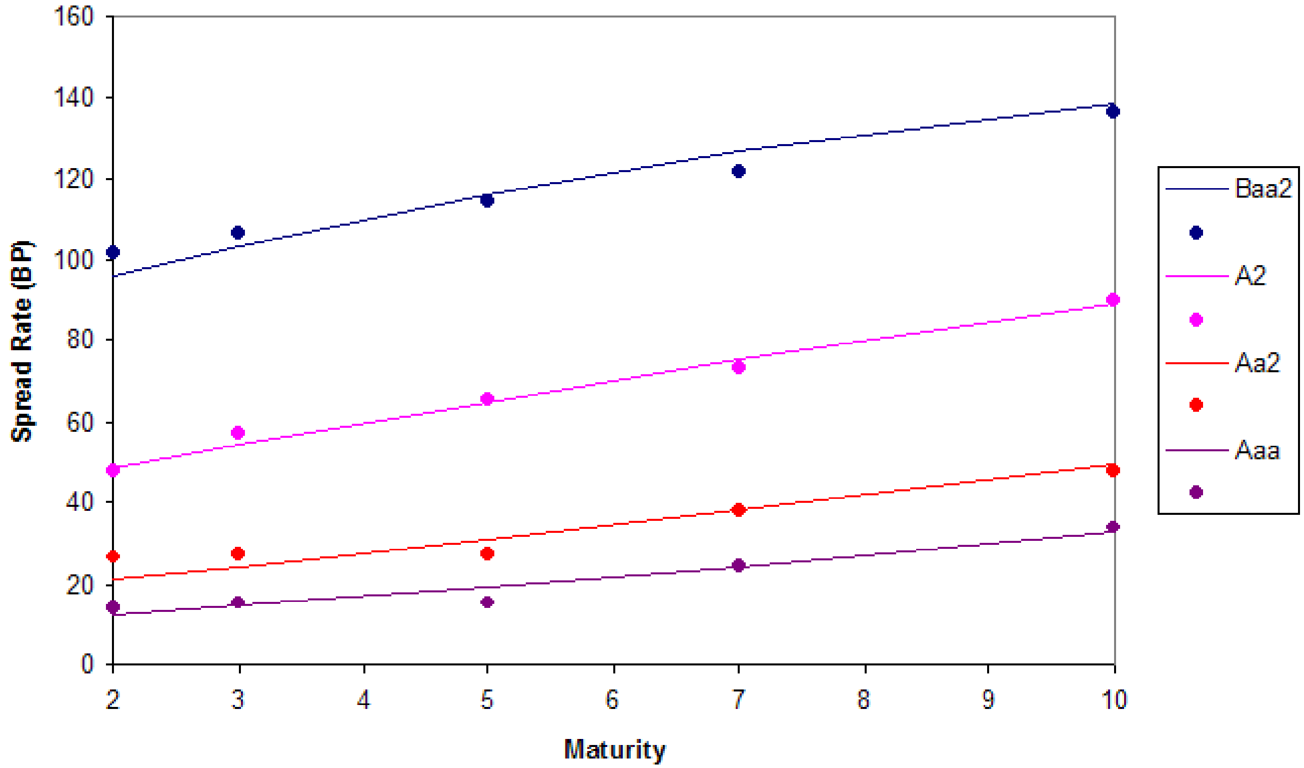


Liquidity and Credit spreads versus Real-world probabilities (5-year horizon)

An intuitive way of graphing liquidity versus credit spreads is the following picture picture for default loss probabilities within 5 year time horizons:



Example: Fitting Recent Market Spread Curves (absorbing the liquidity spread into a constant implied recovery rate)



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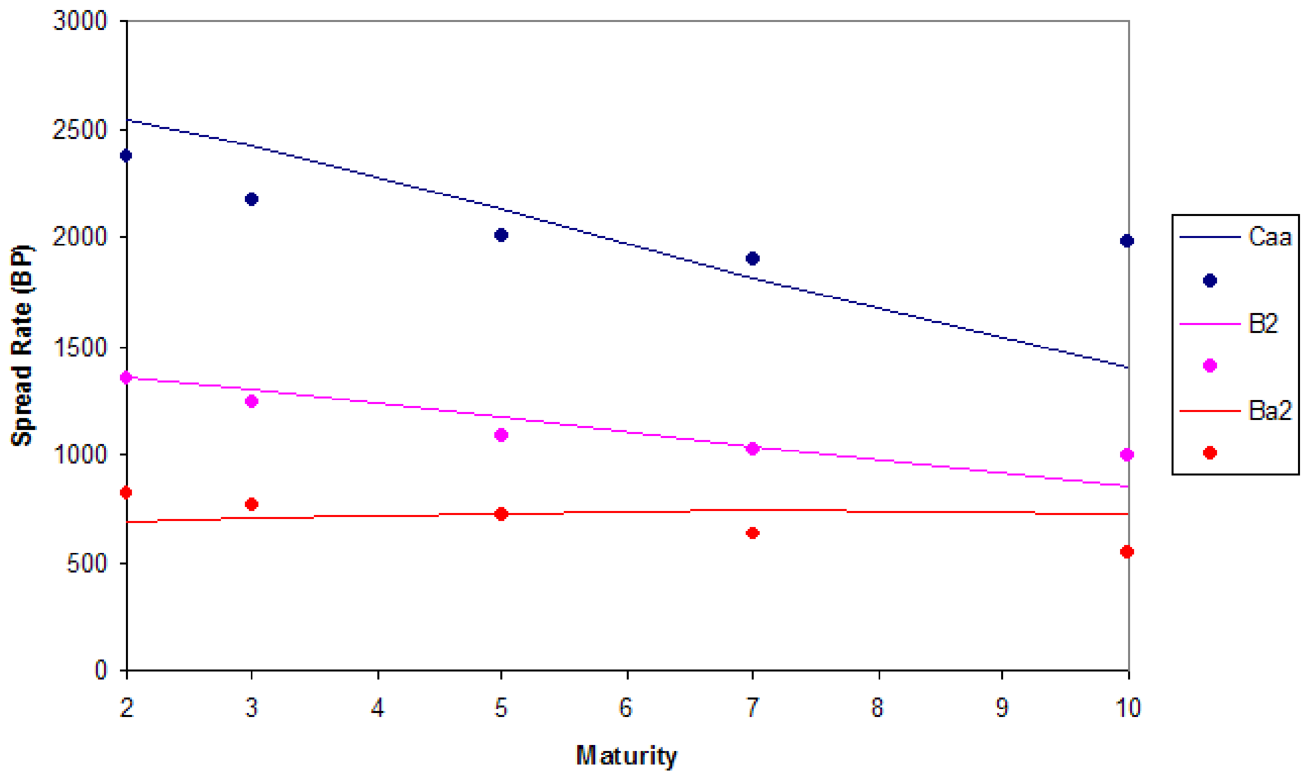
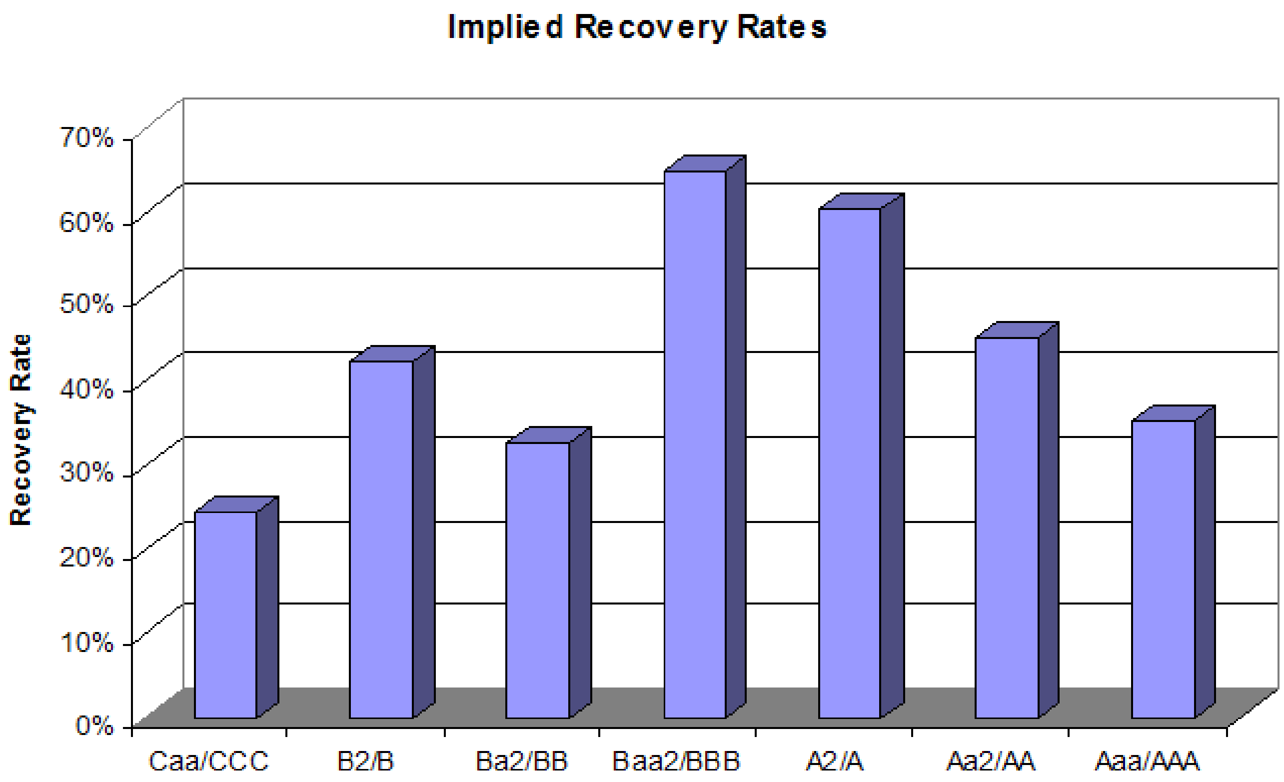
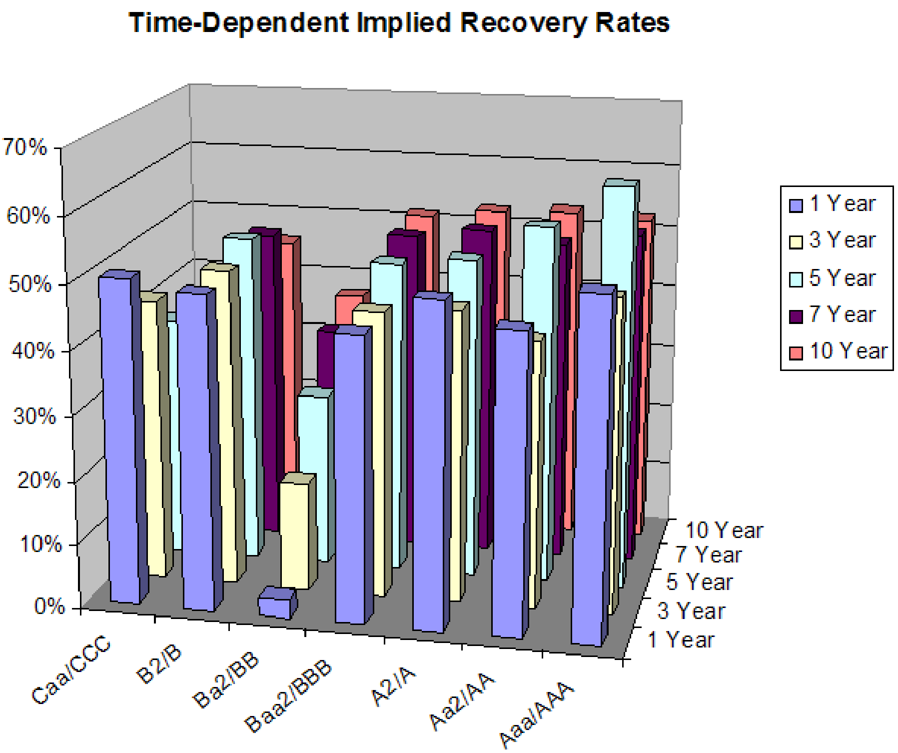


Example: Implied Recovery Rates (assumed constant and including liquidity effects)

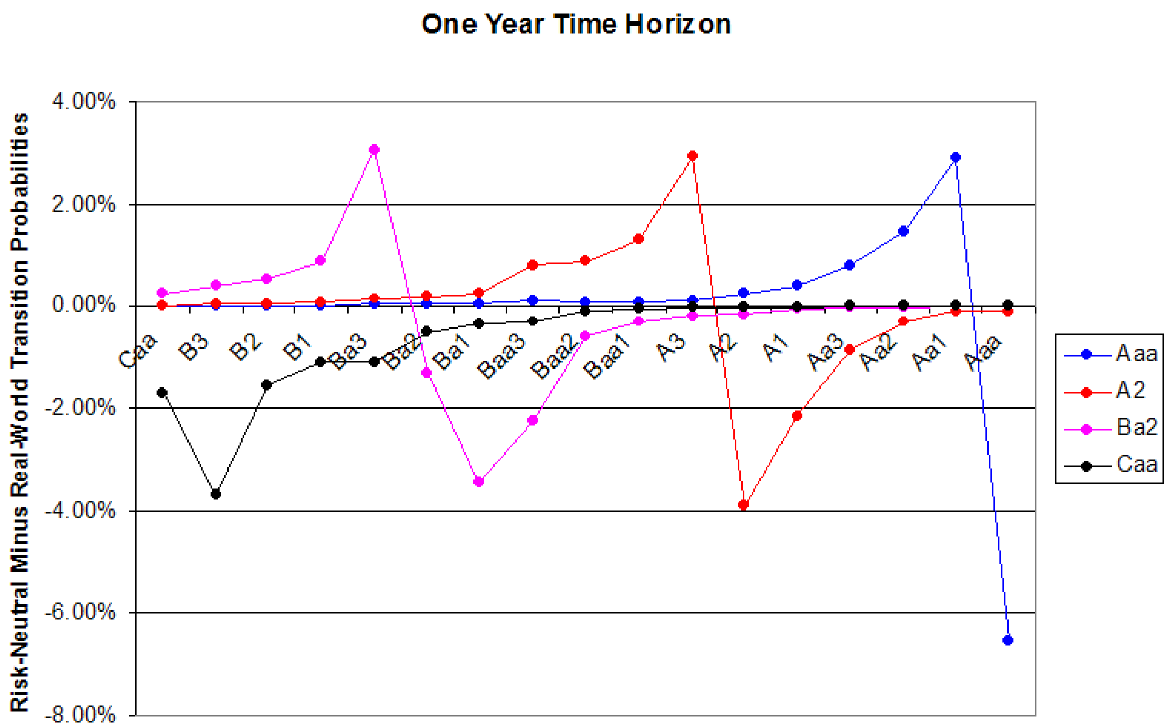


Example: Term Structure of Implied Recovery Rates (to achieve perfect fit)

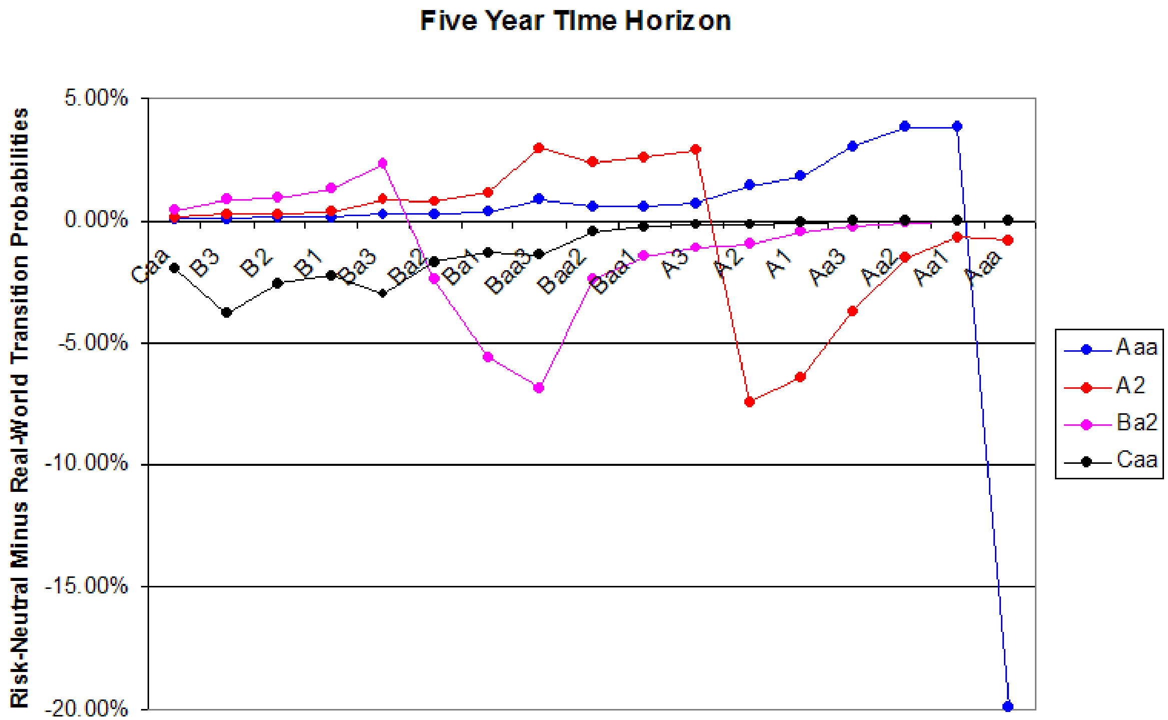


The spread between implied and historical recovery rate term structures can be interpreted as a direct measurement of liquidity premia.

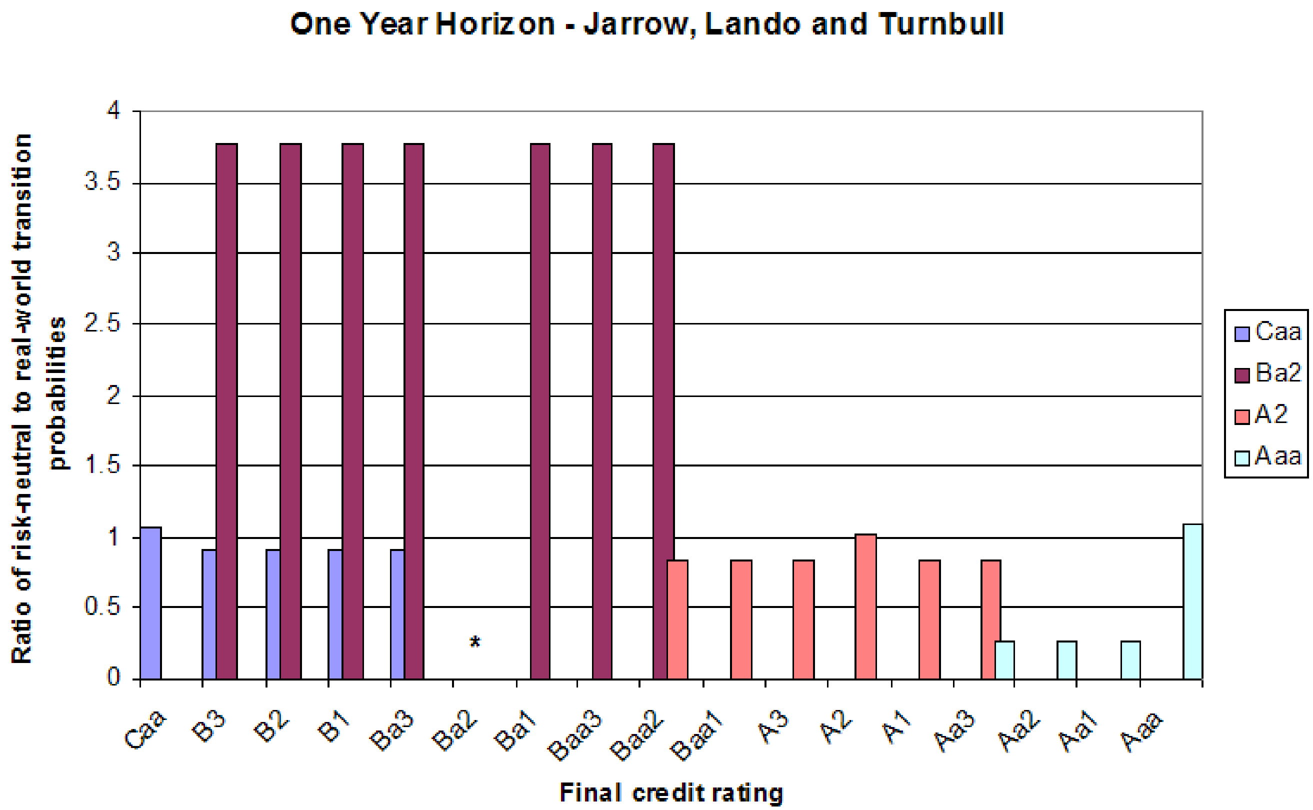
PART IV: Implied Credit Migration Rates (One year risk-adjustments

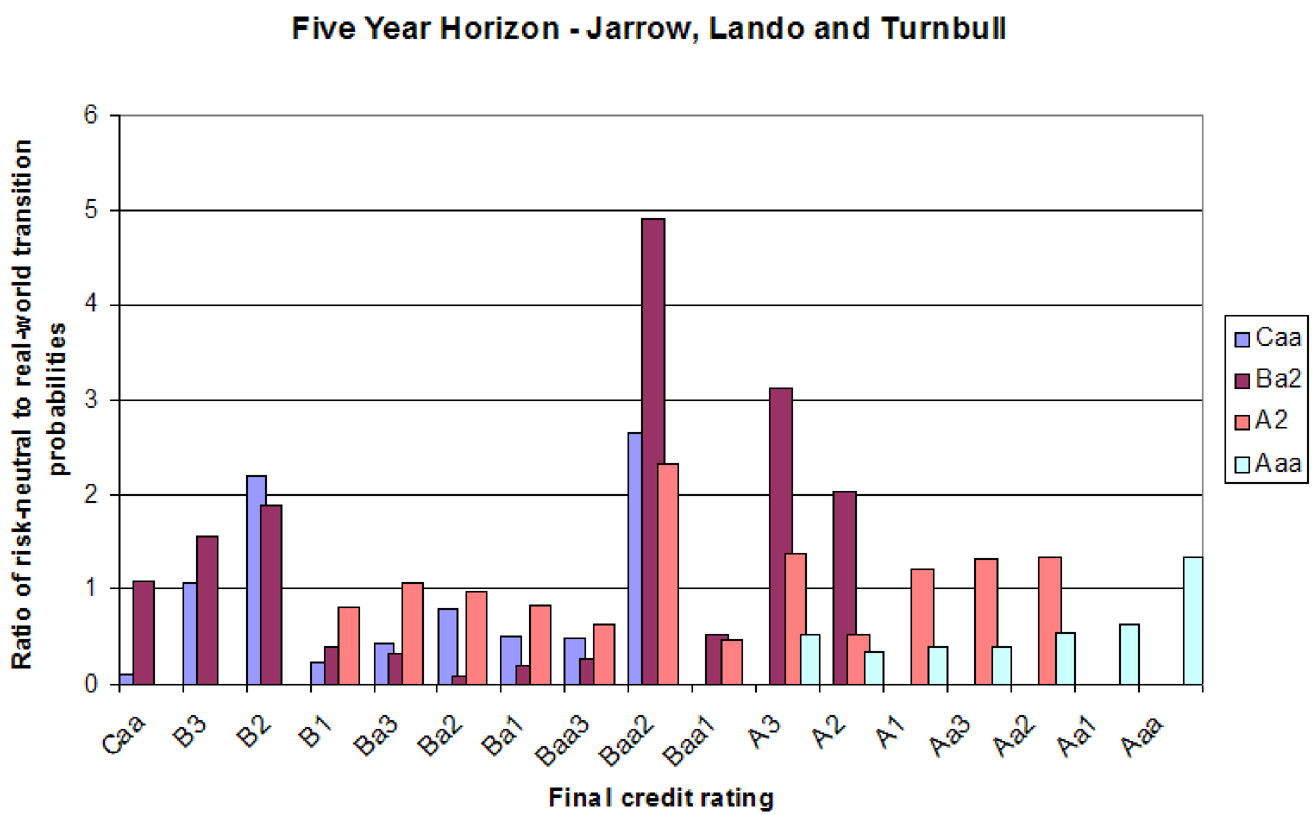


Implied Credit Migration Rates (Five year risk-adjustments)



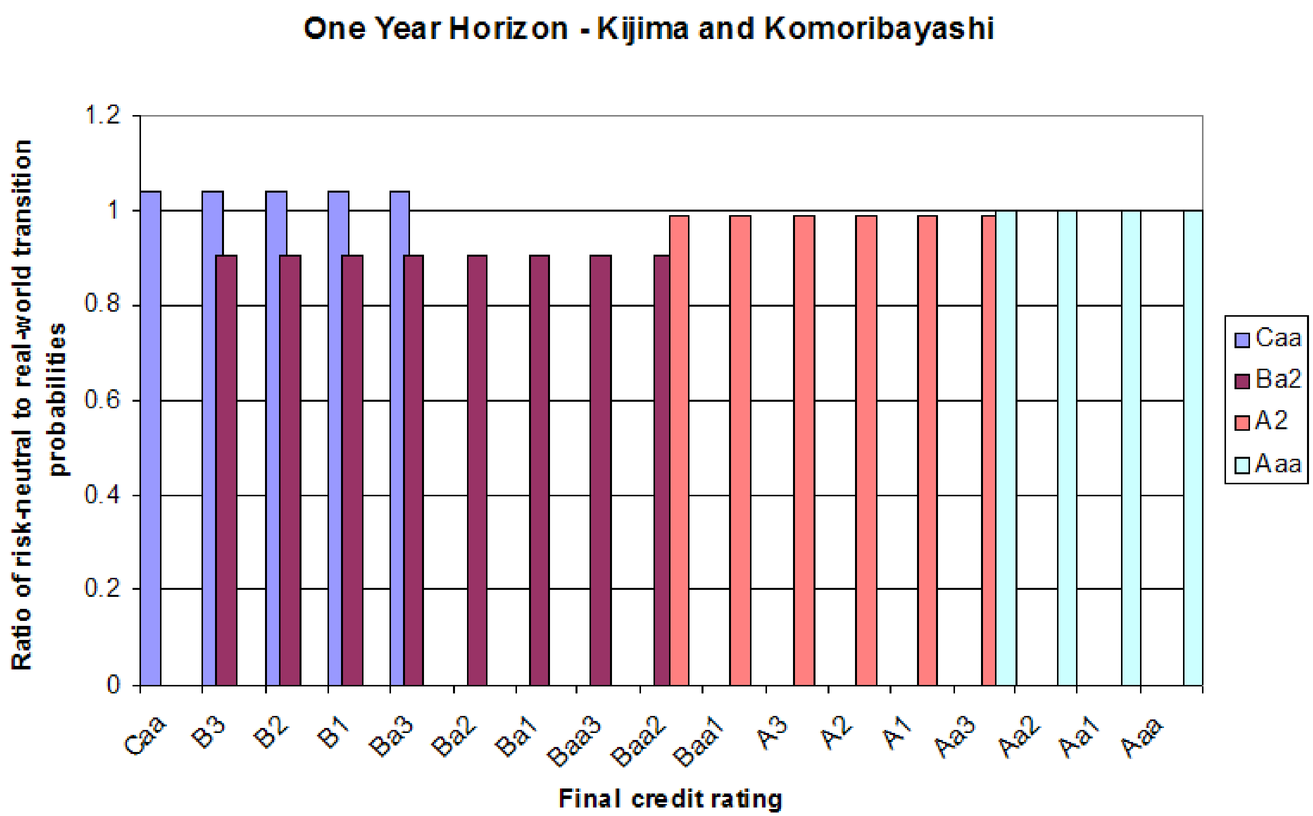
Risk-neutral migration rates according to Jarrow-Lando-Turnbull

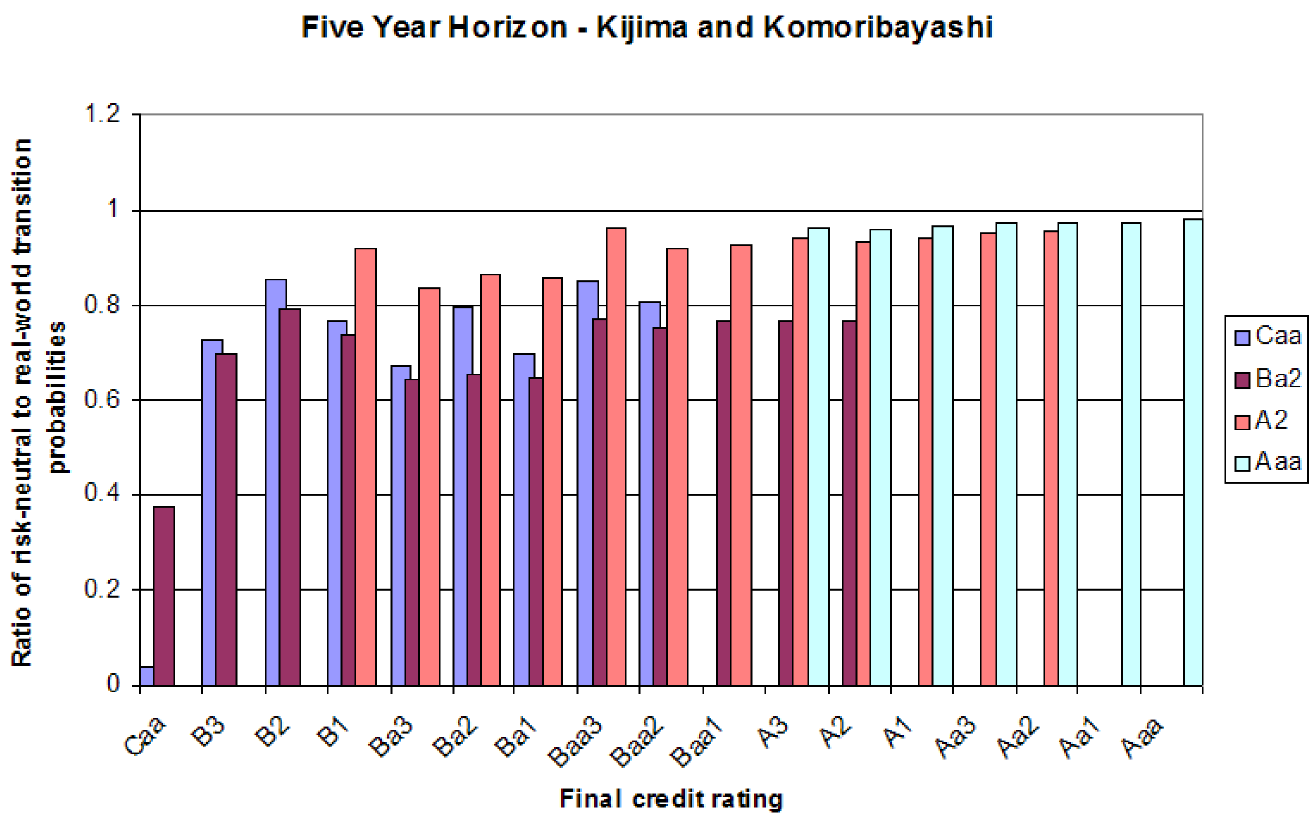




Five Year Horizon - Jarrow, Lando and Turnbull
Ratio of risk-neutral to real-world transition probabilities
0
1
2
3
4
5
6
Caa
B3
B2
B1
Ba3
Ba2
Ba1
Baa3
Baa2
Baa1
A3
A2
A1
Aa3
Aa2
Aa1
Aaa
Final credit rating
Caa
Ba2
A2
Aaa

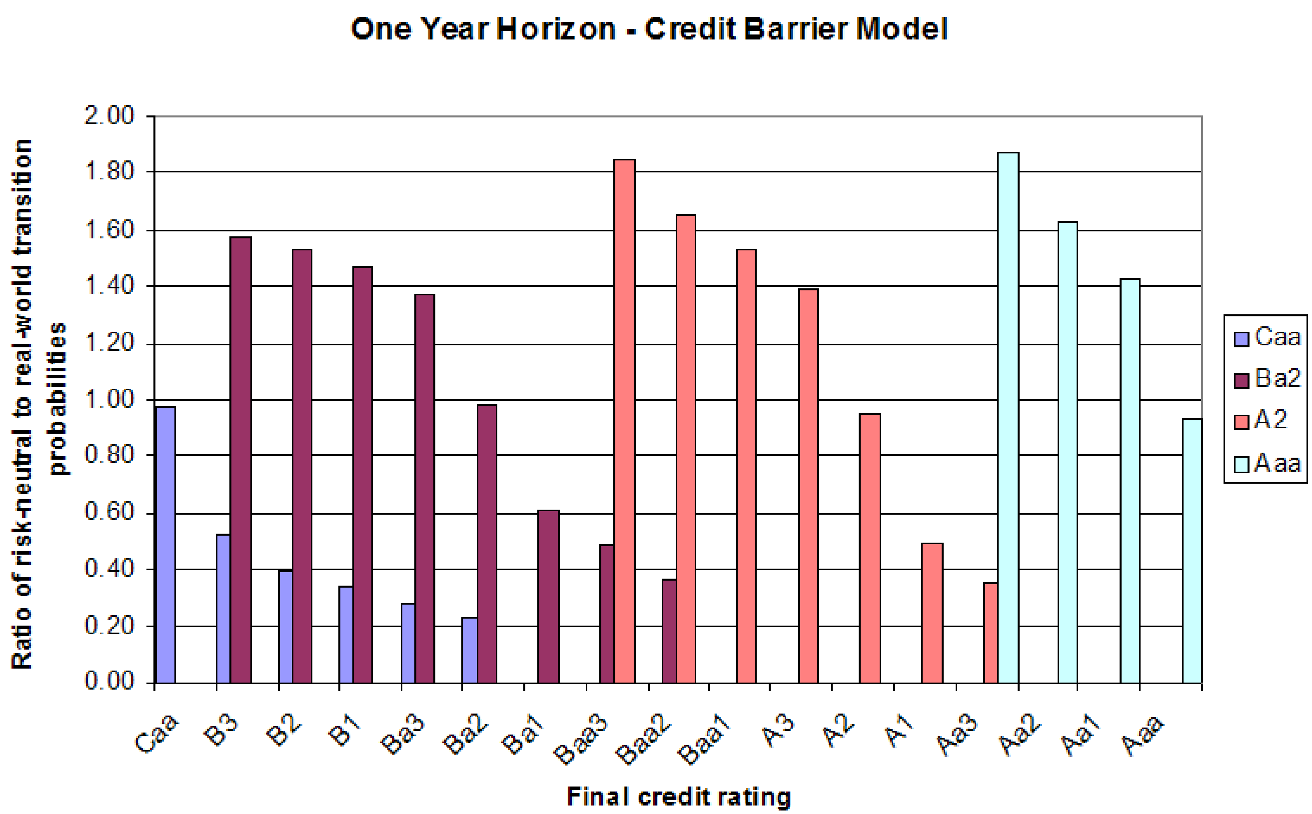
Risk-neutral migration rates according to Kijima-Koboribayashi.

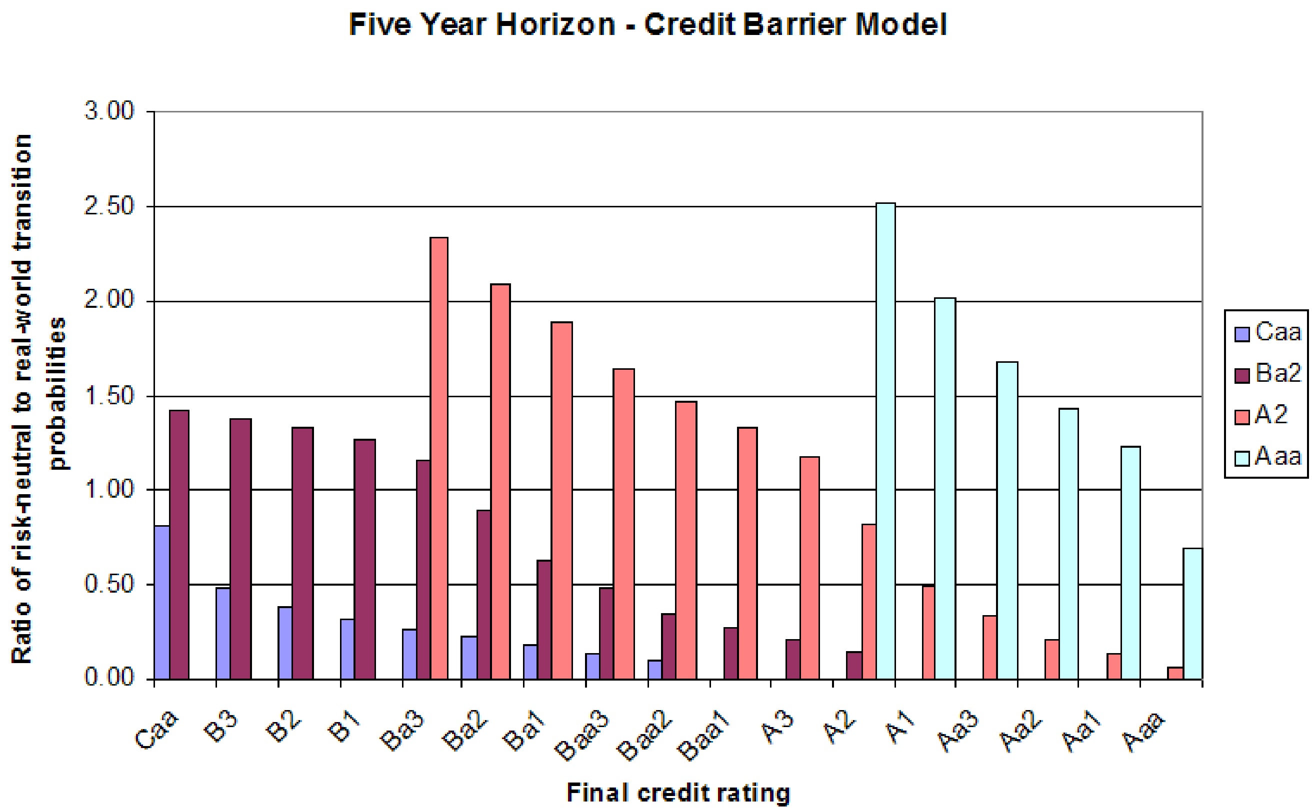




Five Year Horizon - Kijima and Komoribayashi
Ratio of risk-neutral to real-world transition probabilities
Final credit rating
1.2
1
0.8
0.6
0.4
0.2
0
Caa
B3
B2
B1
Ba3
Ba2
Ba1
Baa3
Baa2
Baa1
A3
A2
A1
Aa3
Aa2
Aa1
Aaa
Caa
Ba2
A2
Aaa

Risk-neutral migration rates according to us




Five Year Horizon - Credit Barrier Model
Ratio of risk-neutral to real-world transition probabilities
3.00
2.50
2.00
1.50
1.00
0.50
0.00
Caa
B3
B2
B1
Ba3
Ba2
Ba1
Baa3
Baa2
Baa1
A3
A2
A1
Aa3
Aa2
Aa1
Aaa
Final credit rating
Caa
Ba2
A2
Aaa

What's next

Establishing a **mapping to equity** is for us the next challenge in order of importance. This problem is addressed in a fortcoming paper with Oliver X. Chen which (i) introduces a one-factor model for convertible bonds (addressing some of the inconsistencies in the Black-Derman model and achieving consistent calibration across equity options and credit spreads) and (ii) allows for a 2-factor extension whereby credit liquidity can be hedged by means of the TRAXX.

Next, we plan to carry out a correlation analysis, related to the one presented by Prof. Goriereaux but making use of the equity mapping from our convertible bond analysis.

Conclusion

We introduce an integrable model for a continuous time process that reproduces with high accuracy historical credit transition matrices.

The model has a pure jump stochastic process, state dependent volatility, an absorbing boundary corresponding to default and a reflecting boundary corresponding to the highest ratings.

Thanks to integrability, Monte-Carlo scenario generation and to the calculation of first passage times are numerically efficient.

The model allows one to calibrate spread curves accounting for tax and liquidity effects, generate processes for spread curves under both the risk neutral and the real world measure and estimate the matrix of credit transition probabilities under the risk neutral measure.