Les Cahiers du CREF ISSN: 1707-410X

Risk Aversion and Full Rent Extraction in Mechanism Design

Moez Bennouri

CREF 06-07

February 2006

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Moez Bennouri

Professor
Finance Department

HEC Montréal, CREF and CIRANO

3000, chemin de la Côte-Sainte-Catherine
Montréal, Québec H3T 2A7

E-Mail : moez.bennouri@hec.ca

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Abstract

In this work, we analyze the problem of full rent extraction in mechanism design environments. We show that when agents' utility functions are concave in payments and allocations, the mechanism designer will always leave informational rents with agents. This result holds for almost all distributions of agents' private signals (even when signals are correlated). We show this result without having to tackle the complex issue of deriving the optimal mechanism when agents are risk averse and their private signals are correlated. This allows understanding the importance of several assumptions in the construction of fully extracting mechanisms in previous works.

JEL classification: C72, D44; D82

Keywords: Mechanism design; Surplus extraction; Risk aversion; Correlated information

Aversion au risque et extraction totale du surplus dans un design de mécanismes

Résumé

Dans ce travail, nous analysons le problème de l'extraction totale des rentes informationnelles des agents informés dans un problème général de design de mécanismes. Nous montrons que si les fonctions d'utilité des agents sont concaves dans les paiements et les allocations, le principal (designer) va toujours laisser des rentes informationnelles aux agents informés afin de les inciter à révéler leurs signaux privés. Ce résultat est valable pour presque toutes les distributions des signaux privés des agents. Nous montrons ce résultat sans devoir aborder la question complexe de dérivation du mécanisme optimal dans un environnement où les agents sont averses au risque et leurs signaux privés sont corrélés. Ce résultat permet de comprendre l'importance de certaines hypothèses concernant les fonctions d'utilités des agents pour la construction, dans des travaux précédents, de mécanismes permettant d'extraire tous les profits informationnels des informés.

Mots clés : Design de mécanisme, Extraction du surplus, Aversion au risque, Informations corrélées

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Acknowledgements: *I am greateful for their comments to participants at the meeting of the Société Canadienne des Sciences Économiques (Montréal, May 2003). I also gratefully acknowledge useful comments and suggestions from Narjess Boubakri, Dominique Demougin, Sonia Falconieri, Jacqueline Fleckinger, Patrick Gonzales, Angel Hernando, Jacques Robert and Jean-Charles Rochet. All errors are mine.*

1 Introduction

In a general mechanism design setting, where informed agents are risk averse and whose private signals are correlated, this paper analyzes the problem of extracting all information rents by the mechanism designer. The main result of this paper is that under some conditions on the utility functions and for almost all joint distributions of agents' signals, the full extraction result found in Crèmer and McLean (1985, 1998), McAfee and Reny (1992) and more recently Eső (2005) does no longer hold.

In their seminal works, Crèmer and McLean (1985, 1988) (CM hereafter) provide conditions on discrete joint distributions of agents' signals under which the seller would be able to extract all agents' informational rents. McAfee and Reny (1992) (MR herefater) generalize this result to a more general mechanism design set up and continuous distributions.[1] The linearity of bidders' utility functions in transfers is crucial for all these papers to construct the fully extracting mechanism.[2] Roughly speaking, the basic idea in both models is the following : they consider an incentive compatible mechanism to which they associate a set of functions representing the expected rents of each agent, given his private signal. Then, they fix a participation fee schedule for each player that depends only on his competitors' signals. The mechanism designer extracts all players' surplus if, for each of them, there exists a participation fee which is equal in expected value to his expected rents. The original mechanism and the associated participation fees (if they exist) represent the fully extracting mechanism.

The limit of these papers is that the participation fees used to construct the fully extracting mechanism may be so large that risk averse agents or those having limited liability would refuse to take part in the game. In an auction setting, Robert (1991) shows that under limited liability or risk aversion, the seller cannot extract all bidders' surplus even in some cases where the assumption of correlated signals, as suggested by CM, is

[1] See also McAfee, McMillan and Reny (1989) who design a Vickrey-like auction with common values where the auctioneer can get *almost* full extraction.

[2] It is important at this stage to precise that full extraction is defined in two different ways in CM and MR. CM define fully extracting mechanisms as those where the mechanism designer can obtain, in an incomplete information environment, the same expected revenues as in the *first-best*. However, MR define full extraction as the case where the mechanism designer does not leave surplus with the agents. When utility functions of bidders are linear in transfers, these definitions are equivalent.

satisfied.[3] Technically, Robert shows that the auctioneer's optimal expected revenues and the maximum total surplus are continuous functions in the set of possible information distributions (in the discrete case). However, the difference between these functions is strictly positive with independent distributions (*i.e.*, informed agents get positive rents when their signals are independently distributed). Thus, for almost independent distributions, *i.e.*, when we apply small perturbations to the initial independent distributions, the difference between these functions is strictly positive. This continuity result is due to the compactness of the set of feasible auctions in the case of limited liability and risk aversion.[4] Nevertheless, Robert (1991) only shows that correlation, as defined by CM, is not a sufficient condition for full extraction when agents are risk averse or have liquidity constraints. Based on this result one could question the existence of some level of correlation between signals that guarantee full extraction for some level of risk aversion or budget limit.

A partial response to this question is provided in Eső (2005). In a simple auction of an indivisible good, Eső (2005) shows that the seller can construct an auction allowing her to leave no surplus with the buyers when they are risk averse. This result holds when agents' signals are sufficiently correlated. Eső (2005)'s model has three main assumptions. First, there are only two bidders with binary distribution types (low and high). Second, buyers have CARA utility functions. This allows ruling out random transfers in the optimal auction. Third, and most importantly, with the indivisible good auction set up, buyers' expected utilities are linear in allocations (or probabilities of winning the auction).

The aim of this paper is to analyze whether the full extraction result of Eső (2005) holds if we relax these assumptions. We consider a general private values mechanism design set up which includes auctions of divisible goods, where agents are risk averse and their types are continuously distributed.

The fully extracting mechanisms constructed in CM, MR and Eső (2005) are based on the construction of perturbations over incentive compatible mechanisms so as to extract

[3] A similar result in a principal-agent framework was provided in Demougin and Garvie (1991).

[4] In the latter case, we should define a specific topology over the set of auctions so that the subset of feasible auctions is compact.

the agents' informational rents. These perturbations are risky since they depend, for each agent, on the announcements of his competitors. With risk aversion, agents need to be compensated for this risk. This reduces the likelihood of existence of fully extracting mechanisms. On the other hand, risk aversion of agents may help the mechanism designer to screen them and so decreases the cost of information gathering, and consequently increases the likelihood of existence of full extraction (see Maskin and Riley (1984)).

In a general set up, we show in this paper that the first impact dominates. More precisely, we demonstrate that for almost all distributions, the mechanism designer cannot design a mechanism that allows leaving no surplus with agents. Interestingly, this result is derived without having to tackle the complex issue of deriving the optimal mechanism when agents are risk averse and their private signals are correlated.

We argue in this paper that the existence of fully extracting mechanisms is sensitive to the leeway the mechanism designer has when she designs the optimal mechanism. With utility functions of agents that are linear in transfers, the mechanism designer has sufficient leeway to construct such mechanisms even with a minimum correlation between agents' private signals. When agents are budget constrained, the mechanism designer leeway is reduced and so she needs a larger level of correlation in order to construct the fully extracting mechanism. In Eső (2005)'s set up, even though agents are risk averse, because of the linearity of expected utilities in allocations, the mechanism designer has always some leeway. So provided there is enough correlation between agents' private signals, the mechanism designer would be able to leave agents with any surplus.[5] When this leeway disappears, as in the case of auctions of divisible goods to risk averse agents, the mechanism would not be able to extract agents' informational surplus for almost all distributions.

Besides the argument based on preferences used in this paper, different other arguments were advanced in the literature to show the non existence of fully extracting mechanisms. All these arguments question an implicit or an explicit assumption conside-

[5]Note that in the limited liability and auction of divisible good for risk averse buyers there is less leeway for the seller because in both cases perturbations are constrained. In the first case, payments are limited because of budget constraints. In the latter case, perturbations on allocations are constrained because of the feasibility constraints.

red in CM and MR. Laffont and Martimort (2000) argue that the full extraction result ignores the existence of collusion among players which will prevent the seller from proposing the fully extracting mechanism. They show that whatever the positive amount of correlation, the optimal *collusion-proof* mechanism calls for distortions away from the *first-best* efficiency obtained without collusion. Neeman (2004) argues that full extraction hinges on the implicit assumption that the agents' beliefs uniquely determine their preferences. He proves that if this assumption is relaxed, it is impossible for the seller to extract informational rents. Similarily, Parreiras (2005) shows that the auctionner may fail to extract all the surplus when she cannot recognize how noisy is the signal of an agent. Finally, considering a competitive economy in which sellers offer alternative direct mechanisms to buyers who have correlated information about their valuations, Peters (2001) demonstrates that the sellers cannot afford to propose fully extracting mechanisms.

This paper is organized as follows. In Section 2, the model and some notations are presented. Full extraction is then defined and characterized in Section 3. In section 4, the main result of the paper is derived. Finally, Section 5 concludes. All proofs about deterministic mechanisms are in Appendix A. In Appendix B, we define stochastic mechanisms and show that fully extracting mechanisms in our set up must be deterministic.

2 The Model and notations

We consider a standard auction design set up.[6] A risk neutral seller whose objective is to maximize her expected profit by selling one unit of a perfectly divisible good. There are n bidders indexed by $i \in N = \{1, 2, ..., n\}$. Each bidder receives a private signal about the final value of the good. It is commonly known that the final value belongs to $[0, 1]$, and each private signal to $\Omega_i = [a_i, b_i] \subset [0, 1]$. We assume that the seller's uncertainty about bidders' signals or types can be described by a probability distribution over $\Omega = \prod \Omega_i$. Let F be the c.d.f of the random vector $\widetilde{t} = (\widetilde{t_1}, \widetilde{t_2}, ..., \widetilde{t_n})$ of bidders' private informations and, f the density function.

[6]In this work, we will use the auction environment terminology. The model may however be applied to several environments in which an uninformed principal seeks to design an optimal mechanism that incites informed agents to truthfully reveal their private information.

Bidder i's subjective density function over $\prod_{j\neq i}\Omega_j = \Omega_{-i}$, conditional on having type t_i, are derived using the Bayes rule. Then, $f_i(t_{-i}|t_i) = f(t_i, t_{-i})/f_i(t_i)$ where $t_{-i} = (t_1, t_2, ..., t_{i-1}, t_{i+1}, ..., t_n)$ and $f_i(t_i) = \int_{\Omega_{-i}} f(t_i, t_{-i})dt_{-i}$. His utility function is $u^i(-x, q, t_i)$ where t_i is his type, x is what he pays for a share q of the good. We assume that for all $i \in N$, u^i satisfies the following assumptions :

A.1 u^i is twice continuously differentiable.

A.2 $u^i(0, 0, t_i) = 0$, for all $t_i \in \Omega$.

A.3 $u^i_1 > 0$; $u^i_2 > 0$ and $u^i_3 > 0$.

A.4 $u^i_{11} < 0$, $u^i_{22} < 0$ and $u^i_{33} < 0$.

Subscripts denote the arguments with respect to which the partial derivative is taken. Assumption A.2 is just a convenient normalization of preferences. Assumptions A.3 and A.4 stipluate that each bidder's utility function is concave in payments, in allocated shares and in types. Clearly, linear utility functions do not satisfy A.4. Moreover, environments where utility functions are linear in allocations (as in auctions of indivisible goods) are excluded.

We consider in this work the Bayesian-Nash equilibrium concept. By the revelation principle, we can restrict attention to direct truthful mechanisms. Hence, the auction unfolds as follows : the seller asks each player to reveal his type. If the vector $t = (t_1, ..., t_n)$ $\in \Omega$ is announced, then for each i, the seller allocates to buyer i a share $q^i(t)$ of the good and asks him to make payment $x^i(t)$. The optimal mechanism designed by the seller is the set of functions[7] $\{x^i(\cdot),\ q^i(\cdot)\}_{i\in N} \in [C_p(\Omega)]^{2n}$ that satisfy the usual constraints : feasibility, individual rationality and incentive compatibility constraints.

1. *Feasibility Constraints* (FC) : These constraints insure that allocations are non negative (*i.e.*, $q^i(t) \geq 0$ for all $i \in N$ and all $t \in \Omega$) and the sum of all allocations is lower than the whole quantity (i.e., $\sum_{i=1}^{n} q^i(t) \leqslant 1$ for all $t \in \Omega$). Denote by M the set of mechanisms satisfying the feasibility constraints :

$$M = \{X = \{x^i(\cdot), q^i(\cdot)\}_{i\in N} \in [C_p(\Omega)]^{2n} |\ \text{(FC) are satisfied}\}.$$

[7] $C_p(\Omega)$ is the set of piecewise C^1 functions over Ω_j for all $j \in N$.

2. *Individual Rationality Constraints* (IRC) : Each bidder can choose to drop out from the auction if his expected utility, conditional on his private information, is lower than his reservation utility. We normalize this level of utility to zero.

$$E_i[u^i(-x^i(t), q^i(t), t_i)|t_i] = \int_{\Omega_{-i}} u^i(-x^i(t), q^i(t), t_i)\ f_i(t_{-i}|t_i)\ dt_{-i}\ \geq 0 \qquad (1)$$

for all $i \in N$ and all $t_i \in \Omega_i$. Let $I'(f)$ denote the set of feasible mechanisms satisfying the (IRC) for all i and all t_i; so, $I'(f) = \{X \in M \mid \text{(IRC) are satisfied }\}$.

3. *Incentive Compatibility Constraints* (ICC) : for each buyer i, when he observes the signal t_i, his expected utility if he announces $t'_i \neq t_i$ is always lower than his expected utility if he truthfully reveals his type t_i. Formally, this means

$$U_i(X^i, t'_i, t_i) \leq U_i(X^i, t_i, t_i) \qquad (2)$$

where

$$\begin{aligned} U_i(X^i, t'_i, t_i) &= E_i[u^i(-x^i(t'_i, t_{-i}), q^i(t'_i, t_{-i}), t_i)|t_i] \\ &= \textstyle\int_{\Omega_{-i}} u^i(-x^i(t'_i, t_{-i}), q^i(t'_i, t_{-i}), t_i)\ f_i(t_{-i}|t_i)\ dt_{-i} \end{aligned}$$

for all $i \in N$ and all t_i and $t'_i \in \Omega_i$; with $X^i = \{x^i(\cdot), q^i(\cdot)\}_{i\in N} \in [C_p(\Omega)]^2$. Let denote by $I(f) = \{X \in M \mid \text{(IRC) and (ICC) are satisfied}\}$.

Note that $I(f)$ and $I'(f)$ depend on the information structure represented by the density function f. The optimal auction depends on the characteristics of this function f which belongs to the set :

$$\Phi = \{f : \Omega \rightarrow R | \int_{\Omega} f(t)\ dt\ = 1 \text{ with } f(t) > 0 \text{ almost everywhere over } \Omega\}.$$

The set of distributions where $f_i(t) = 0$ for some i and some t is excluded. This case includes the set of distributions where signals are perfectly correlated (*i.e.*, the case where observing the signal t_i is sufficient to deduce the value of t_{-i}). In fact, when signals are perfectly correlated, say when $f_i(t^*_{-i}|t^*_i) = 1$ for some $(t^*_i, t^*_{-i}) \in \Omega$, one can easily show that the seller can extract all the agent i's profits regardless of their preferences.

Then, for some $f \in \Phi$, the auctioneer's problem is to select an optimal auction $X = \{x^i(\cdot), q^i(\cdot)\}_{i\in N} \in M$ which maximizes his expected utility. The seller's program is :

$$\max_{X\in I(f)} W(X, f)$$

where $W(X,f)$ is the seller's expected revenues if he chooses the mechanism[8] X, i.e.,

$$W(X,f) = \int_{\Omega} (\sum_{i=1}^{n} x^i(t)) f(t) dt. \quad (3)$$

Let also consider the set of *ex-ante* optimal mechanisms under complete information for the seller, *i.e.,* when the seller observes traders' private information but each agent cannot observe his competitors' types. The optimal mechanism is selected from $I'(f)$ since in this case the incentive compatibility constraints are not needed. The seller's program in this case is

$$\max_{X \in I'(f)} W(X,f).$$

Given a prior distribution function f, the maximum expected revenue the seller can attain in the asymmetric information and the complete information for the seller environments are $V(f)$ and $V'(f)$ respectively. More formally, let $V(f) \equiv sup\{W(X,f)|X \in I(f)\}$ and $V'(f) \equiv sup\{W(X,f)|X \in I'(f)\}$. Furthermore, define the following sets :

$$\Re'(f) = \{X \in I'(f)|W(X,f) = V'(f)\}$$

and

$$\Re(f) = \{X \in I(f)|W(X,f) = V(f)\}.$$

$\Re'(f)$ and $\Re(f)$ are sets of mechanisms which are optimal under respectively complete information for the seller and asymmetric information.

To complete notations, we consider the full information environment. For a vector of types t, we denote by $V_F(t)$ the maximum revenue the seller can attain in the full information environment. Let I_F denote the set of feasible mechanisms under full information, i.e., $I_F = \{\{x^i(t), q^i(t)\}_{i \in N} \in M | u^i(-x^i(t), q^i(t), t_i) \geq 0 \text{ for each } t\}$. Then define Σ as the set of optimal mechanisms in the full information environment, so, $\Sigma = \{\{x^i(\cdot), q^i(\cdot)\}_{i \in N} \in M | V_F(t) = \max_{X \in I_F} \sum_i x^i(t) \text{ for each } t\}$.

Before turning to the definition of full rent extraction, note that all definitions are based on deterministic mechanisms. We will show in the following section that with risk

[8]Notice that in this work, to simplify computations and to make exposition clearer, we do not consider production costs for the seller.

averse bidders, the fully extracting mechanism cannot be stochastic. So, without loss of generality, we focus on deterministic mechanisms.

3 Definition and characteristics of full extraction

The construction of fully extracting auctions in MR (1992) and Eső (2005) is based on the use of an incentive compatible mechanism X. Full extraction will be ensured by creating perturbations on X without altering the incentive compatibility constraints. This could be done by restricting attention to perturbations (on payments or allocations) that depend, for each bidder, on his competitors' announcements. Perturbations on payments may be seen as participation fees that should be chosen so that all potential rents for bidders participating in the auction X are extracted. Technically, participation fees are selected so that, for each bidder and each type, the individual rationality constraints (IRC) are bind. Showing the existence of fully extracting auctions is consequently equivalent to show the existence of these fees. We define full extraction by following this reasoning.

Definition 1 *A fully extracting mechanism under asymmetric information is defined as a mechanism X such that $X \in \Re(f)$ and the (IRC) for this mechanism are bind for all i and all $t_i \in \Omega_i$.*

Thus, we define fully extracting mechanims as optimal mechanisms where the seller leaves no surplus with the buyers.[9] Note that this definition of full extraction is different from the one used in CM. CM consider that the seller can extract all players' rents under asymmetric information if and only if she can attain the weighted revenues under full information. Using our notations, full extraction is possible if $V(f) = \int_\Omega V_F(t) f(t) dt$. In other words, for CM, full extraction is possible when the seller obtains the same expected revenues as in the first-best while we consider that full extraction is equivalent to the case where the seller does leave no surplus with the buyers (by making the (IRC)s bind). These definitions are equivalent when the bidders' utility functions are linear in transfers. However, with risk aversion, it matters which definition we consider. For CM's definition,

[9] Our definition focuses on the case of full rent extraction. In a continuous distributions set up, McAfee and Reny (1992) explore the case of $\varepsilon - full$ extraction, i.e., the case where the seller can extract all but ε of buyers' informational rents, for an arbitrarily small ε.

since the fully extracting mechanism involves risky transfers for which risk averse buyers need compensation, it is clear that full extraction is not possible.

Proposition 1 *A fully extracting mechanism under asymmetric information exists if and only if* $V'(f) = V(f)$.

In Proposition 1 we show that despite the fact that full extraction is defined is terms of leaving no surplus with bidders, it can be related to the expected revenues of the seller. Indeed, full extraction is possible if and only if the seller can achieve under asymmetric information the same expected revenues as when there is complete information for the seller.

Proposition 2 *For some distribution* $f \in \Phi$, *a fully extracting mechanism exists if and only if* $\Re'(f)$ *and* $I(f)$ *share a common element. More formally,* $V(f) = V'(f)$ *if and only if* $\Re'(f) \cap I(f) \neq \emptyset$.

Proposition 2 states that the fully extracting mechanism should be optimal under complete information for the seller and satisfy the incentive compatibility constraints for each agent. Henceforth, the analysis of the problem of full rent extraction can be undertaken in two steps. First, we characterize the set of optimal mechanisms under complete information for the seller. Second, we verify whether this set contains at least one mechanism satisfying the incentive compatibility constraints for each agent. If such mechanism exists, we conclude that full extraction is possible, otherwise the seller cannot design a mechanism that extracts all the agents' expected rents.

Before studying the existence of fully extracting mechanisms, we show that if such mechanisms exist, they should be deterministic. Stochastic mechanisms involve risky outcomes for buyers, so they need to be compensated for such risks by leaving them some rents.

Lemma 1 *All mechanisms belonging to* $\Re'(f)$ *are deterministic.*

From Proposition 2, we have that fully extracting mechanisms should allow the seller to attain the expected revenues when she is informed about bidders' signals. However, Lemma 1 states that such level of expected revenues cannot be achieved with a stochastic mechanism. Therefore we can, without loss of generality, focus on deterministic mechanisms. This result is an artifact of the concavity of bidders' utility functions, and consequently the convexity of the (IRC)s. Note finally that Lemma 1 does not imply that the optimal mechanism with asymmetric information (if full extraction is not possible) cannot be stochastic. This is true only if we consider additional assumptions about bidders' utility functions in order to guanantee the convexity of the (ICC)s (see Maskin and Riley (1984)).

4 Full extraction with risk averse buyers

4.1 Characterization of $\Re'(f)$

In this subsection we characterize the set $\Re'(f)$. As mentioned above, the components of this set may be seen as mechanisms maximizing the seller's "weighted" revenues when he observes players' types whereas each agent cannot observe his competitors' types.

Proposition 3 *If $X = \{(x^i(\cdot), q^i(\cdot)\}_{i \in N} \in \Re'(f)$, then : $X \in M$ and*

$$U_i(X^i, t_i, t_i) = 0, \forall i \in N and\ \forall t_i \in \Omega_i \quad (4)$$

$$\sum_{i=1}^{n} q^i(t) = 1, \forall t \in \Omega \quad (5)$$

$$u_1^i(-x^i(t), q^i(t), t_i) = u_1^i(-x^i(t_i, t'_{-i}), q^i(t_i; t'_{-i}), t_i), \forall t_i \quad and \quad \forall t_{-i} \neq t'_{-i} \quad (6)$$

and

$$\frac{u_2^i(-x^i(t), q^i(t), t_i)}{u_1^i(-x^i(t), q^i(t), t_i)} \leq \delta_0(t), \forall i,\ \forall t_i\ and\ \forall t \in \Omega. \quad (7)$$

and, the equality holds for equation (7) when $q^i(t) > 0$. $\delta_0(t)$ is the Lagrangian multiplier associated with the constraint $\sum_{i=1}^{n} q^i(t) \leq 1$.

Proposition 3 states that necessary conditions for a mechanism X to maximize the seller's expected revenues when agents are expected to truthfully announce their types are : (i) participation constraints bind for each buyer (Equation (4)) ; (ii) the seller allocates the total quantity of the good (Equation (5)) ; (iii) the marginal utility for each type does not depend on the others' types (Equation (6)) ; (iv) for each vector of types, all bidders with strict positive allocation (*i.e.*, when $q^i(t) > 0$) have the same value of the marginal rate of substitution between allocations and payments (Equation (7)) ; and (v) X is in the set of feasible mechanisms. Intuitively, to maximize his revenues, the seller should maximize buyers' payments. The optimal mechanism should be such that participation constraints bind otherwise he can switch to another auction yielding higher expected proceeds, and the quantity should be wholly sold otherwise he can require higher payments and thus increase his expected payoffs.

4.2 The impossibility of full extraction

Under risk neutrality, fully extracting mechanism is a perturbed version of the optimal mechanism under full information. The perturbations are introduced to extract the bidders' private information. Even with risk aversion, Eső (2005) shows that these perturbations may exist. However the auction of an indivisible good used in Eső (2005) exhibits a linearity of the utility function with respect to allocations. In the following, we argue that the existence of such perturbations is due to the linearity of the utility function and that with risk averse bidders, the introduction of such perturbations on the optimal mechanism under full information will decrease the expected profits for the seller.

From the definition of Σ as the set of optimal mechanisms under full information, we can easily check that the elements of Σ satisfy the following equations for all t and all i :

$$\begin{cases} u^i(-x^i(t), q^i(t), t_i) = 0 \\ q^i(t) \geq 0 \\ \sum\limits_{i=1}^{n} q^i(t) = 1 \\ [u_2^i(-x^i(t), q^i(t), t_i) / u_1^i(-x^i(t), q^i(t), t_i)] < \delta(t); \text{ the equality holds when } q^i(t) > 0. \end{cases}$$

For each t, $\delta(t)$ is the Lagrangian multiplier related to the constraint $\sum q^i(t) \leq 1$; $\delta(t)$ is different from zero for all $t \in \Omega$.

Given this characterization of Σ, we prove in a first step the following proposition :

Proposition 4 *For all information structure $f \in \Phi$, when bidders' utility functions satisfy conditions A-1—A-4, then $\Re'(f) = \Sigma$.*

In Proposition 4, we demonstrate that for almost all possible priors about the distributions of types and when buyers' utility functions satisfy conditions A.1 - A.4, the set of optimal mechanisms under complete information for the seller coincides with the set of optimal mechanisms under full information. Thus, if we take an optimal mechanism under full information, we cannot enhance the level of expected revenues under complete information for the seller by applying some perturbations over the first mechanism. Notice that Proposition 4 shows also that $\Re'(f)$ does not depend on f, so the distribution of types affects the optimal mechanism only through the incentive compatibility constraints.

This also means that small perturbations of the optimal full information mechanism does not lead to the same expected revenue as the case of complete information for the seller. Henceforth, we cannot introduce the perturbations used in CM and MR to construct the fully extracting mechanism.

This result is only due to buyers' risk aversion. Indeed, take a mechanisms belonging to Σ and consider the relation between $x^i(t)$ and $q^i(t)$ (see Figure 1) for some i. This relation is derived by applying the implicit function theorem to the equation[10] : $u^i(-x^i(t), q^i(t), t_i) = 0$.

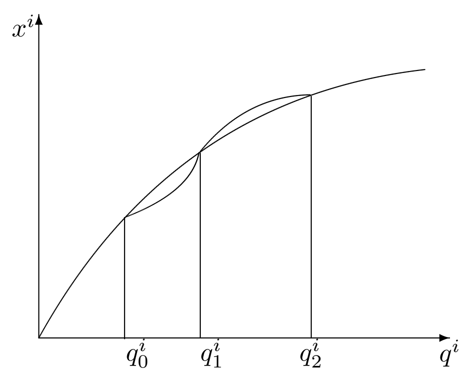

Figure 1 : The set of functions where $u^i(-x^i(t), q^i(t), t_i) = 0$

[10]Let consider that $x^i = \varphi(q^i)$ for some i and some t_i. Then, from the implicit function theorem we can easily prove that $\frac{\partial \varphi}{\partial q^i} > 0$ and $\frac{\partial^2 \varphi}{\partial (q^i)^2} \leq 0$ so that φ is concave in q^i.

After applying the perturbations, we need to have mechanisms satisfying :

$$\int_{\Omega_{-i}} u^i(-x^i(t), q^i(t), t_i) f_i(t_{-i}|t_i) dt_{-i} = 0;$$

which is the participation constraint when the seller observes the buyers' types.

As represented in Figure 1, increase the buyers' utility for $q^i \in [q_0^i, q_1^i]$ and decrease it for $q^i \in [q_1^i, q_2^i]$ by, respectively, decreasing and increasing his payments. We are interested only in mechanisms satisfying equations (6) and (7). By the concavity of u^i, when we apply these perturbations, the seller's expected revenues are lower than the expected revenues he would get under full information.

On the contrary, with risk neutrality, the linearity of buyers' utility functions in payments allows the achievement of the same level of revenues if we undertake some perturbations over the optimal mechanism under full information. The intuition is that the seller can impose participation fees having zero expected value for all types and, by linearity, the difference between the seller's expected revenues in the full information and the case when he observes the buyers' types is also equal to zero. In an auction of an indivisible good setting, the linearity of the utility functions in allocations allow also to construct such perturbations. Because of this linearity, Proposition 4 does not hold and we have only that $\Sigma \subsetneq \Re'(f)$.

So far we showed that every $X = \{x^i, q^i\}_{i\in N} \in \Re'(f)$ belongs also to Σ. From Proposition 2, a mechanism X fully extracts buyer's surplus if and only if X belongs to $\Re'(f) \cap I(f)$. This means that the fully extracting mechanism must satisfy the following system of equations :

$$(S) \begin{cases} u^i(-x^i(t), q^i(t), t_i) = 0, \forall i,\ \forall t \in \Omega \\ u_1^i(-x^i(t), q^i(t), t_i) = u_1^i(-x^i(t_i, t'_{-i}), q^i(t_i; t'_{-i}), t_i), \forall t_i \text{ and } \forall t_{-i} \neq t'_{-i} \\ \sum_{i=1}^{n} q^i(t) = 1, \forall t \in \Omega \\ \frac{u_2^i(-x^i(t), q^i(t), t_i)}{u_1^i(-x^i(t), q^i(t), t_i)} \leq \delta(t), \forall i \text{ and } \forall t. \text{ The equality holds when } q^i(t) > 0 \\ U_i(X^i, t'_i, t_i) \leq 0 \text{ for all } t_i \text{ and } t'_i \in \Omega_i \text{ and for all } i \in N, \end{cases}$$

where the last equality is derived from (2) and (4). $\delta(t)$ is the Lagrangian multiplier associated to the constraint $\sum_{i=1}^{n} q^i(t) \leq 1$. By Assumption A.3, u^i is increasing in its third argument, then for all i we have :

$$0 = u^i(-x^i(t_i', t_{-i}), q^i(t_i', t_{-i}), t_i') < u^i(-x^i(t_i', t_{-i}), q^i(t_i', t_{-i}), t_i), \ \forall t_{-i} \text{ and } \forall t_i > t_i'.$$

Since the left hand side of this inequality is equal to zero, we have for all $t_i > t_i'$:

$$u^i(-x^i(t_i', t_{-i}), q^i(t_i', t_{-i}), t_i) > 0;$$

taking expectations over Ω_{-i} gives : $U_i(X^i, t_i', t_i) > 0$ for all $t_i > t_i'$ which violates the incentive compatibility constraints. Thus, for all distributions f in Φ, the system (S) has no solution and the intersection between $\Re'(f)$ and $I(f)$, when buyers are risk averse, is always empty. We sum up this result in the following proposition.

Proposition 5 *When buyers' utility functions satisfy conditions A.1 - A.4, and for all prior distribution function f in Φ, the seller cannot extract all buyers' informational rents.*

In order to prove Proposition 5, we just need to show that all the *first-best* mechanisms do not satisfy the incentive compatibility constraints. Indeed, each bidder is better off by announcing the lowest type in his range (*i.e.,* each bidder will announce a_i). This result is true for all distribution function f in the set Φ.

Therefore, for all distribution functions, the seller cannot extract the risk averse agents' expected profits. This result is due to the fact that the perturbations the seller would apply to the optimal mechanism under full information cannot enhance his expected profits. Indeed, these perturbations will increase the riskiness of the game. So, risk averse agents will have positive expected profits representing the risk premia they get as a compensation for their participation in the game.

5 Conclusion

In this work, we analyze the problem of full extraction in a general mechanism design framework. We show that for almost all distribution, the mechanism designer should always leave some rents to risk averse informed agents in order to incite them to truthfully reveal their private information. This result is directly related to the concavity of agents' utility functions.

The result of this paper argues that, besides the requirement of some level of correlation, full extraction is possible only when the agents' utility functions are linear in payments and/or allocations. Linearity and correlation are indeed necessary conditions to the existence of perturbations (on payments and/or allocations) that allow leaving agents with no surplus. In the cases of CM and MR, where agents' utility functions are linear in payments, perturbations on payments (participation fees) are not constrained and so a minimum level of correlation is sufficient to construct fully extracting mechanisms. However, when perturbations are constrained because of agents' limited liability (as in Robert (1991)), a higher level of correlation should be needed in order to guarantee the feasibility of perturbations. Equivalently, when perturbations ought to be applied to allocations, in which case perturbations are constrained (because of feasibility constraints), a higher level of correlation should be required to construct fully extracting mechanisms (as in Eső (2005)).

The main result of this paper is derived by showing the non existence of perturbations (on both payments and allocations) applied to some incentive compatible mechanism that allow to leave agents with no surplus. Let $X = \{x^i(\cdot), q^i(\cdot)\}_{i \in N} \in I(f)$ be such mechanism and let $\epsilon^i(t)$ and $\eta^i(t)$ be such perturbations (respectively for payments and allocations) for each buyer i and each $t \in \Omega$. Mathematically, we claim in this paper the non existence of functions $\epsilon^i(\cdot)$ and $\eta^i(\cdot)$ belonging to $C_p(\Omega)$ such that : $\int_{\Omega_{-i}} u^i(-x^i(t) + \epsilon^i(t), q^i(t) + \eta^i(t), t_i)\ f_i(t_{-i}|t_i)\ dt_{-i} = 0$ for all $i \in N$, all $t_i \in \Omega_i$ and all $X = \{x^i(\cdot), q^i(\cdot)\}_{i \in N} \in I(f)$.[11] An important feature of the methodology followed in this paper is that the result is derived without having to tackle the complex problem of deriving the optimal mechanism under risk aversion and correlation.

Finally, the fact that buyers' signals are private values and hence utility functions do not depend directly on their competitors' signals is an important assumption in this model. Whether this result may be generalized to common values mechanisms is an interesting question that should be addressed in future research.

[11] We should also precise that $q^i(t) + \eta^i(t)$ should satisfy the feasibility constraints.

Appendix A

Proof of Proposition 1

"⇒" We proceed by contradiction. Assume that $V'(f) > V(f)$ and consider some $X = \{x^i, q^i\}_{i \in N} \in \Re(f)$. By construction X belongs also to $I'(f)$. We can construct $X_1 = \{x_1^i, q_1^i\}_{i \in N}$ such that :

$$\begin{array}{rl} q_1^i(t) = & q^i(t) \text{ for all } t \text{ and all } i \\ x_1^i(t) = & x^i(t) + \varepsilon^i(t) \text{ for all } t \text{ and all } i \end{array}$$

where $\varepsilon^i(\cdot)$ for all i could be sufficiently small so that $0 \leq \int_\Omega \sum_i \varepsilon^i(t) f(t) dt < V'(f) - V(f)$ and $X_1 \in I'(f)$. Therefore, we have $V'(f) > W(X_1, f) \geq V(f)$. By Assumption A.1, we can also choose the perturbations $\varepsilon^i(\cdot)$ such that for some j, some t^0 and some neighborhood of t^0, $\mathcal{N}(t^0)$, we have $\varepsilon^j(t) > 0$, for all $t \in \mathcal{N}(t^0)$. Now consider the mechanism $X_2 = \{x_2^i, q_2^i\}_{i \in N}$ such that

$$\begin{array}{rl} q_2^i(t) = & q^i(t) \text{ for all } t \text{ and all } i \\ x_2^j(t) = & x^j(t) \text{ for all } t \text{ and all } j \neq i \\ x_2^i(t_i, t_{-i}) = & x^i(t_i, t_{-i}) \text{ for all } t_{-i} \in \Omega_{-i} \text{ and all } t_i \neq t_i^0 \\ x_2^i(t_i^0, t_{-i}) = & x^i(t_i^0, t_{-i}) \text{ for all } t_{-i} | (t_i^0, t_{-i}) \notin \mathcal{N}(t^0) \\ x_2^i(t_i^0, t_{-i}) = & x^i(t_i^0, t_{-i}) + \varepsilon^i(t) \text{ for all } t_{-i} | (t_i^0, t_{-i}) \in \mathcal{N}(t^0). \end{array}$$

Therfore, we have $U^i(X^i, t_i^0, t_i^0) > U^i(X_2^i, t_i^0, t_i^0) \geq 0$. Which shows that the (IRC) are not bind for all $X \in \Re(f)$.

"⇐" We also proceed by contradiction. Assume that for all $X \in \Re(f)$, the (IRC) are not bind. So for some X, there exists j and t_j such that $U^j(X^j, t^j, t^j) > 0$. Consider now the mechanism $X_1 = \{x_1^i, q_1^i\}_{i \in N}$ such that :

$$\begin{array}{rl} q_1^i(t) = & q^i(t) \text{ for all } t \text{ and all } i \\ x_1^i(t) = & x^i(t) \text{ for all } t \text{ and all } i \neq j \\ x_1^j(t) = & x^j(t) + \varepsilon^j(t) \text{ for all } t. \end{array}$$

So the mechainsm X_1 is just a perturbation of X at the payments of bidder j. We can choose $\varepsilon^j(\cdot)$ so that $U^j(X^j, t^j, t^j) > U^j(X_1^j, t^j, t^j) > 0$. By Assumption A.3, we have $V(f) < W(X_1, f)$. Furthermore, $X_1 \in I'(f)$ by construction, so $W(X_1, f) < V'(f)$. Thus, $V(f) < V'(f)$. ■

Proof of Proposition 2 :

"$\Rightarrow$" We proceed by contradiction. Assume that $\Re'(f) \cap I(f) = \emptyset$, then for each $X \in \Re'(f)$ and for each $Y \in I(f)$ we have $W(X,f) > W(Y,f)$. Since $\Re(f) \subset I(f)$, then for all $Y \in \Re(f)$ and all $X \in \Re'(f)$ we have $W(X,f) > W(Y,f)$. Consequently, $V'(f) > V(f)$.

"$\Leftarrow$" Consider $X^\circ \in \Re'(f) \cap I(f)$, then $W(X^\circ, f) = V'(f)$ since $X^\circ \in \Re'(f)$; so,

$$W(X^\circ, f) \geq W(Y,f), \text{ for all } Y \in I'(f);$$

however, $I(f) \subset I'(f)$, so,

$$W(X^\circ, f) \geq W(Y,f), \text{ for all } Y \in I(f).$$

Since $X^\circ \in I(f)$, this proves that $W(X^\circ, f) = \max\limits_{X \in I(f)} W(X,f) = V(f)$. ■

Proof of Proposition 3 :

Proof of equations (4) and (5) :

For equations (4) and (5) we proceed by contradiction. Consider some $X \in \Re'(f)$ such that either $U_j(X^j, t_j^0, t_j^0) > 0$ for some $j \in N$ and $t_j^0 \in [a_j, b_j]$ or $\sum\limits_{i=1}^{n} q^i(t^0) < 1$ for some $t^0 \in \Omega$.

(i) *First case :* Assume that $U_j(X^j, t_j^0, t_j^0) = \varepsilon > 0$ for some $j \in N$ and $t_j^0 \in \Omega_j$. Since $X \in \Re'(f)$, then

$$X \in Arg \max_{Y \in I'(f)} W(Y,f).$$

By continuity of X and U_j, there exists some $0 < \varepsilon^0 < \varepsilon$ and some $\alpha^0 > 0$ such that : $\forall t_j \in]t_j^0 - \alpha^0; t_j^0 + \alpha^0[$ we have, $U_j(X^j, t_j, t_j) > \varepsilon^0$. Now consider $X_* \in M$ such that :

$$X_*(t_j, t_{-j}) = \begin{cases} \{x_*^j; x^{-j}; q^j; q^{-j}\} \text{ if } t_j \in]t_j^0 - \alpha^0; t_j^0 + \alpha^0[\\ \\ X(t_j, t_{-j}) \qquad\qquad\qquad\qquad \text{otherwise}; \end{cases}$$

where x_*^j is defined as follows : consider $\bar{x}^j(t_j, t_{-j}) > x^j(t_j, t_{-j})$ for all $t_j \in]t_j^0 - \alpha^0; t_j^0 + \alpha^0[$ and $t_{-j} \in \Omega_{-j}$. Since u^j is increasing in $(-x)$, we have $u^j(-\bar{x}^j(t), q^j(t), t_j) < u^j(-x^j(t), q^j(t), t_j)$, for all $t_j \in]t_j^0 - \alpha^0; t_j^0 + \alpha^0[$ and $t_{-j} \in \Omega_{-j}$. If we denote $\bar{X}^j = (\bar{x}^j, q^j)$, taking expectation conditional on t_j gives :

$$U_j(\bar{X}^j, t_j, t_j) < U_j(X^j, t_j, t_j), \forall t_j \in]t_j^0 - \alpha^0; t_j^0 + \alpha^0[.$$

Then, choose some $\bar{X}^j$ such that

$$0 \le U_j(\bar{X}^j, t_j, t_j) < \varepsilon' < U_j(X^j, t_j, t_j), \forall t_j \in]t_j^0 - \alpha^0; t_j^0 + \alpha^0[;$$

finally, consider x_*^j given by

$$x_*^j(t_j, t_{-j}) = \begin{cases} \bar{x}^j(t_j, t_{-j}) \text{ if } t_j \in]t^\circ{}_j - \alpha^\circ; t^\circ{}_j + \alpha^\circ[\\ x^j(t) \qquad\qquad\qquad \text{otherwise.} \end{cases}$$

Note that we can choose $\bar{x}^j$ to be piecewise smooth over Ω.

So, if we consider $X_* = (X_*^j, X^{-j})$, we have $W(X_*, f) > W(X, f)$ from the definition of x_*^i. This contradicts the fact that $X \in \Re'(f)$ and proves that if $X \in \Re'(f)$, then equation (4) is satisfied.

(ii) *Second case :* Assume that $\sum q^i(t^0) < 1$ for some $t^0 \in \Omega$. Since q^i is piecewise smooth for all $i \in N$, there exists an $\varepsilon > 0$ such that for all[12] $t \in B^n(t^0, \varepsilon) \subset \Omega$, we have $\sum_{i=1}^{n} q^i(t) < 1$.

consider a function $\eta \in [C_p(\Omega)]^n$ such that : $0 < \eta(t) < 1 - \sum_{i=1}^{n} q^i(t)$ for all $t \in B^n(t^0, \varepsilon)$ and $\eta(t)$ is equal to zero elsewhere. For some $i \in N$, consider the allocation function $\bar{q}^i(t) = q^i(t) + \eta(t)$.

Since u^i is increasing with respect to its second argument, then

$$u^i(-x^i(t), \bar{q}^i(t), t_i) > u^i(-x^i(t), q^i(t), t_i) \text{ for all } t \in B^n(t^0, \varepsilon);$$

denote $\bar{X}^i = (x^i, \bar{q}^i)$. Then, if we take conditional expectation and consider, from the first case, that $U_i(X^i, t_i^0, t_i^0) = 0$, we get : $U_i(\bar{X}^i, t_i^0, t_i^0) > 0$.

Since u^i is increasing with respect to its first argument, we can find $\bar{x}^i \in C_p(\Omega)$ such that : $\bar{x}^i(t_i^0, t_{-i}) > x^i(t_i^0, t_{-i})$ for each t_{-i} such that $(t_i^0, t_{-i}) \in B^n(t^0, \varepsilon)$, and $U_i(X_*^i, t_i^0, t_i^0) = 0$ where $X_*^i = (\bar{x}^i, \bar{q}^i)$. Thus, if we denote $X_* = (X_*^i, X^{-i})$, we have : $W(X_*, f) > W(X, f)$, which contradicts the fact that $X \in \Re'(f)$.

Proof of equations (6) and (7) :

Let us now prove that for all $X \in \Re'(f)$, equations (6) and (7) should be satisfied. The generalized Lagrangian function for the optimization problem is

$$L(X, \lambda, \delta, \theta) = \int_\Omega H(X, \lambda, \delta, \theta) f(t) dt;$$

[12] $B^n(t^\circ, \varepsilon)$ is the ball with radius ε, centered in t° ; we take a sufficiently small ε so that $B^n(t^\circ, \varepsilon)$ is included in Ω.

where :

$$H(X,\lambda,\delta,\theta)=\sum_{i=1}^{n}H^i(X^i(t),\lambda^i(t_i),\delta(t),\theta^i(t))$$

and,

$$H^i(X^i(t),\lambda^i(t_i),\delta(t),\theta^i(t))=x^i(t)+\lambda^i(t_i)u^i(-x^i(t),q^i(t),t_i)+\delta(t)(1-\sum_{i=1}^{n}q^i(t))+\theta^i(t)q^i(t);$$

with $\lambda^i(t_i)$, $\theta^i(t)$ and $\delta(t)$ are multipliers or conjugate variables.

If $X^0\in\Re'(f)$, then there exists a set of functions $(\lambda_0^1,..,\lambda_0^n,\theta_0^1(t),..,\theta_0^n(t),\delta_0)$ such that : $L(X^0,\lambda_0,\delta_0,\theta_0)\geq L(X,\lambda_0,\delta_0,\theta_0)$ for all $X\in I'(f)$. Applying the Euler equation[13] gives :

$$\frac{\partial H^i(X^0,\lambda_0,\delta_0,\theta_0)}{\partial x^i}=0,\ \text{for all } i\in N$$

and

$$\frac{\partial H^i(X^0,\lambda_0,\delta_0,\theta_0)}{\partial q^i}=0,\ \text{for all } i\in N.$$

After computations, this gives :

$$1-\lambda_0^i(t_i)u_1^i(-x_0^i(t),q_0^i(t),t_i)=0 \text{ for all } i\in N \text{ and all } t\in\Omega$$

and,

$$\lambda_0^i(t_i)u_2^i(-x_0^i(t),q_0^i(t),t_i)-\delta_0(t)+\theta_0^i(t)=0, \text{for all } i\in N \text{ and all } t\in\Omega.$$

Therefore, we get :

(i) $u_1^i(-x_0^i(t),q_0^i(t),t_i)=\dfrac{1}{\lambda_0^i(t_i)}$, which proves that $u_1^i(-x_0^i(t),q_0^i(t),t_i)$ does not depend on t_{-i} as stated in equation (6) ; and,

(ii) $[u_2^i(-x^i(t),q^i(t),t_i)/u_1^i(-x^i(t),q^i(t),t_i)]\leq\delta_0(t)$ for all $i\in N$. When $q^i(t)>0$, the equality holds since $\theta_0^i(t)=0$. This proves equation (7). ■

Proof of Proposition 4

The proof of this proposition is divided in two steps. First, we prove in Lemma 2 that $V'(f)=\int_\Omega V_F(t)f(t)dt$. Then, in Lemma 3, we demonstrate that for all $X=\{\{x^i,q^i\}_{i\in N};i\in N\}\in\Re'(f)\backslash\Sigma$, we have $W(X,f)>\int_\Omega V_F(t)f(t)dt$.

[13]Notice that this equation is applied in the case of multidimensional problem since mechanisms are defined over $\Omega\subset I\!R^n$. See Gelfand and Fomin (1969) for more details about the conditions of applying Euler equations in the multidimensional case.

Lemma 2 *The seller's expected revenues from the optimal mechanism under complete information for the seller are equal to his weighted revenues under full information, i.e.,* $V'(f) = \int_\Omega V_F(t)f(t)dt$.

Proof. Consider an $X \in \Re'(f)$; from the joint concavity of $u^i(\cdot)$ in payments and allocations, we have for each i and t_i :

$$\begin{array}{c} 0 = \int_{\Omega_{-i}} u^i(-x^i(t), q^i(t), t_i)\ f_i(t_{-i}|t_i)\ dt_{-i} < \\ u^i(-\int_{\Omega_{-i}} x^i(t) f_i(t_{-i}|t_i) dt_{-i}, \int_{\Omega_{-i}} q^i(t) f_i(t_{-i}|t_i) dt_{-i}, t_i). \end{array} \tag{8}$$

The mechanism $X_0 = \{x_0^i, q_0^i\} = \{\int_{\Omega_{-i}} x^i(t) f_i(t_{-i}|t_i) dt_{-i}, \int_{\Omega_{-i}} q^i(t) f_i(t_{-i}|t_i) dt_{-i}\}$, belongs to I_F and it yields the same expected revenues as the optimal mechanism X. This proves that $V'(f) \le \int_\Omega V_F(t)f(t)dt$. However, we know that $I_F \subset I'(f)$, so that $V'(f) \ge \int_\Omega V_F(t)f(t)dt$. This proves the equality between $V'(f)$ and $\int_\Omega V_F(t)f(t)dt$. ■

Lemma 3 *For all* $X = \{(x^i, q^i); i \in N\} \in \Re'(f) \backslash \Sigma$, *we have* $W(X, f) > \int_\Omega V_F(t)f(t)dt$.

Proof. Consider an $X = \{\{x^i, q^i\}_{i \in N}; i \in N\} \in \Re'(f) \backslash \Sigma$ and $X_0 = \{(x_0^i, q_0^i); i \in N\} \in \Sigma$. The mechanism X may be seen as a perturbation of X_0 by defining $\varepsilon^i(t)$ and $\eta^i(t) \in C_p(\Omega)$ such that :

$$x^i(t) = x_0^i(t) + \varepsilon^i(t),$$

and

$$q^i(t) = q_0^i + \eta^i(t), \text{for all } i \text{ and } t.$$

By Assumption A.4, we have for all $t \in \Omega$ and all i :

$$\begin{aligned} u^i(-x_0^i(t), q_0^i(t), t_i) \quad &< \quad u^i(-x_0^i(t) - \varepsilon^i(t), q^i(t) + \eta^i(t), t_i) + \varepsilon^i(t) u_1^i(-x^i, q^i(t), t_i) \\ &\quad -\eta^i(t) u_2^i(-x^i, q^i(t), t_i), \end{aligned} \tag{9}$$

Substitution in Equation (9) of the properties of X as proposed in Proposition 3, gives

$$0 < u^i(-x^i(t), q^i(t), t_i) + [\varepsilon^i(t) - \eta^i(t)\delta(t)]\xi^i(t_i); \tag{10}$$

we can easily derive this condition when $q^i(t) > 0$. On the other hand, if $q^i(t) = 0$ we use the definitions of X and X_0 to conclude that $\eta^i(t) \le 0$. Then, we have from Proposition 3 that

$$-\eta^i(t) u_2^i(-x^i, 0, t_i) < -\eta^i(t)\delta(t) u_1^i(-x^i, 0, t_i).$$

Substitution in Equation (9) yields Equation (10) for $q^i(t) = 0$.

Notice that $\xi^i(t_i)$ and $\delta(t)$ are the Lagrangian multipliers associated respectively to the participation constraints and the constraint $\sum_{i=1}^{n} q^i(t) \leq 1$.

Then, multiplying both sides by $f_i(t_{-i}|t_i)$, integrating over Ω_{-i} and using the fact that the participation constraints are bind gives :

$$\int_{\Omega_{-i}} [\varepsilon^i(t) - \eta^i(t)\delta(t)] f_i(t_{-i}|t_i) dt_{-i} > 0. \tag{11}$$

After taking expectation over Ω_i and summing over i, we get :

$$\int_{\Omega} \sum \varepsilon^i(t) f(t) dt - \int_{\Omega} (\sum \eta^i(t)) \delta(t) f(t) dt > 0;$$

Now, since $\sum \eta^i(t) = 0$ for all t ; this yields :

$$\int_{\Omega} \left[\sum \varepsilon^i(t)\right] f(t) dt = \int_{\Omega} \left[\sum (x^i(t) - x_0^i(t))\right] f(t) dt > 0;$$

which shows that $W(X, f) > \int_{\Omega_{-i}} V_F(t) f(t) dt$. ■

In Lemma 2, we showed that $V'(f) = \int_{\Omega} V_F(t) f(t) dt$. This means that $\Sigma \subset \Re'(f)$. In order to prove the result of Proposition 4, we should demonstrate that $\Re'(f) \subset \Sigma$. In Lemma 3, we proved that for all $X \in \Re'(f) \backslash \Sigma$, the seller's optimal revenue is greater than the optimal expected revenue for the seller under full information. This contradicts the result of Lemma 2. Therefore, $\Re'(f) \backslash \Sigma = \emptyset$ and consequently $\Re'(f) = \Sigma$. This ends the proof of Proposition 4. ■

Appendix B

We consider $\widetilde{X}$ a stochastic mechanism. This mechanism could be defined as a joint distribution over the set of all feasible mechanisms. So we can identify $\widetilde{X}$ by defining a couple of distributions (φ, χ) over the set M. Let decompose M by defining M_x and M_q such that $M = M_x \times M_q$ and $M_x = \{x^i(\cdot) \in C_p(\Omega)$ for all $i \in N\} = [C_p(\Omega)]^n$and $M_q = \{q^i(\cdot)$ for all $i \in N|$ (FC) are satisfied$\} \subset [C_p(\Omega)]^n$. So $\varphi = (\varphi_1, \varphi_2, .., \varphi_n)$ is a probability measure defined over $[C_p(\Omega)]^n$ and $\chi = (\chi_1, \chi_2, .., \chi_n)$ is a probability measure defined over M_q. Furthermore, for the distribution φ define the expectation operator $E^{\varphi}_{M_x}$ such that

$$E^{\varphi}_{M_x}(U_i(X^i, t'_i, t_i)) = \int_{C_p(\Omega)} U_i(X^i, t'_i, t_i) d\varphi_i(x^i).$$

The expectation operator $E^{\chi}_{M_q}$ is defined differently :

$$E^{\chi}_{M_q}(U_i(X^i, t'_i, t_i)) = \int_{M_x} U_i(X^i, t'_i, t_i) d\chi_i(q).$$

Note that there is a slight difference between definitions of expectation operators because of the dependence between allocation functions caused by the constraint on the sum of allocations for all t. So, $E^{\chi}_{M_q}$ is defined as the an integration over the whole set M_q because this latter cannot be defined as a product of independant spaces, as was the case for M_x.

Then, given these notations, the *ex ante* (IRC) and (ICC) would be defined as follows :

1. *Individual Rationality Constraints* (IRC) :

$$E^{\varphi}_{M_x}(E^{\chi}_{M_q} U_i(X^i, t_i, t_i)) \geq 0 \tag{12}$$

   for all $i \in N$ and all $t_i \in \Omega_i$.

2. *Incentive Compatibility Constraints* (ICC) :

$$E^{\varphi}_{M_x}(E^{\chi}_{M_q} U_i(X^i, t'_i, t_i)) \leq E^{\varphi}_{M_x}(E^{\chi}_{M_q} U_i(X^i, t_i, t_i)) \tag{13}$$

   for all $i \in N$ and all t_i and $t'_i \in \Omega_i$.

Now that the notion of stochastic mechanisms is well defined, we prove Lemma 1.

Proof of Lemma 1

From Proposition 2, we have that fully extracting mechanisms should allow the seller to attain the expected revenues when she is informed about bidders' signals. So a fully extracting mechanism should also be optimal in the case of complete information for the seller. Let consider a stochastic mechanism $\widetilde{X} = (\varphi, \chi)$ that is optimal in such environment. Now define $\overline{X} = \{\overline{x}_i(\cdot), \overline{q}_i(\cdot)\}_{i \in N}$ as follows :

$$\begin{aligned} \overline{x}^i(t) &= \int_{C_p(\Omega)} x^i(t) d\varphi_i(x^i) \text{ for all } i \text{ and all } t \in \Omega \\ \overline{q}^i(t) &= \int_{M_q} q^i(t) d\chi(q^i) \text{ for all } i \text{ and all } t \in \Omega. \end{aligned}$$

We can easily verify that $\overline{X} \in M$ and that $\widetilde{X}$ and $\overline{X}$ generate the same expected revenue to the seller. Furtherome, by definition $\widetilde{X}$ satisfies the incentive compatible constraints :

$$E^{\varphi}_{M_x}(E^{\chi}_{M_q} U_i(X^i, t_i, t_i)) \geq 0$$

and by Assumption A.4 we have

$$0 \leq E^{\varphi}_{M_x}(E^{\chi}_{M_q} U_i(\widetilde{X^i}, t_i, t_i)) < U_i(\overline{X}^i, t_i, t_i) \text{ for all } i \text{ and all } t_i \in \Omega_i,$$

which shows that $\overline{X} \in I'(f)$. So, we can create a small perturbation on $\overline{X}$ so as to remain in $I'(f)$ and enhance the generated expected revenue to the seller. This contradicts the fact that a stochastic mechanism may be optimal when the seller observes the agents' signals. This ends the proof of Lemma 1. ∎

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