Backstop Technology Adoption

Matti Liski and Pauli Murto*

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Abstract

To analyze how markets solve the problem of reducing dependence on volatile factors such as oil, we model the competitive equilibrium adoption of factor-free, backstop technologies. A general property of efficient backstop technology adoption is the expansion of the technology portfolio – new technology entry rate initially exceeds old technology exit rate. From this follow the unique characteristics of the backstop technology transition such as the prolonged coexistence of old and new technologies, and increase in output price volatility despite the lower use of the volatile factor in production. The properties of the transition require no market failures and they depend on (i) the option to remain idle rather than exit, (ii) heterogeneity in factor supply, and (iii) factor market volatility.

JEL Classification: D1; D9; O30; Q40.

Keywords: technology adoption; factor markets; uncertainty; irreversible investment; energy

*<liski@hkkk.fi> Helsinki School of Economics, Department of Economics, Arkadiankatu 7, P.O. Box 1210, 00101 Helsinki, Finland, www.hkkk.fi/~liski. <murto@hkkk.fi> Visiting MIT, Dept. of Economics, 50 Memorial Drive, Cambridge, MA 02142, USA
"The concept that is relevant to this problem is the backstop technology, a set of processes that (1) is capable of meeting the demand requirements and (2) has a virtually infinite resource base".

(William D. Nordhaus, 1973, pp. 547-548)

1 Introduction

More than 30 years have passed since the first oil price shock but the dependency on oil is still at the forefront of public concern. It is perhaps no longer the finiteness of long-term factor supply but the risk of economic disruption due to volatility of prices that is concerning. Will the alternative technologies – backstop technologies – that reduce the dependence on the volatile fossil fuel markets ever enter the market in large scale? To analyze such questions, we develop a competitive equilibrium model where factor-free technologies are adopted through irreversible investments under factor market uncertainty. A general feature of efficient backstop technology adoption is the expansion of the technology portfolio – new technology entry rate exceeds old technology exit rate. In this sense it is socially efficient to build the backstop capacity to coexist with the factor demand infrastructure.

From this follow the unique characteristics of the backstop technology transition such as the prolonged coexistence of old and new technologies, and increase in the industry output price volatility despite the lower use of the factor in production.

William Nordhaus (1973) introduced the concept of backstop technology and analyzed the timing of entry of such technologies in markets for factors that are finite in supply, a feature of most energy commodities. Following his reasoning, it is usual to think that the backstop technology entry depends on the overall factor supply that is exhausted before it is profitable.

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1 Average year-to-year fluctuation of oil price was within 1% of the price level during the years 1949-1970, whereas this number jumbed to 30% from 1970 to date (Smith, 2002).
for the new technology to enter. While scarcity rents may ultimately become important, it
seems far less obvious today than in the 1970s that scarcity rents alone could be important
for technology choices.\textsuperscript{2} In contrast, most economists agree that factor markets, oil market
in particular, are characterized by supply-side shocks. Yet, factor price volatility has no role
in the existing elaborations of the backstop technology adoption. This seems potentially
a serious omission since the volatility clearly affects the profitability of production using
the volatile factor while backstop technologies are, by definition, free from factor market
volatility. This asymmetry with which uncertainty enters together with the fact that the
decisions to reduce the dependence on the factor market are irreversible suggest that the
transition to backstop technologies may not be well understood without the factor market
uncertainty.

While the energy sector is our prime motivation, we make general conclusions for the
factor-market induced technology adoption under the following preconditions. First, factor
demand infrastructure is long-lived and costly to maintain. When factor market conditions
turn unfavorable, utilization of the technology can be adjusted or technology units can exit
irreversibly. We thus consider situations where idleness, while costly, is an alternative to exit,
which seems a particularly relevant case in the energy sector. Second, there is heterogeneity
among factor supply sources, implying an upward sloping supply curve and ensuring that
those who reduce the usage of the factor-dependent technology, all else equal, relax the
factor market conditions for those who still use the technology. Third, the factor market
is subject to supply-side shocks. In the oil market such shocks are related to wars and
political instability, uncertain reservoir levels, accidents in refineries, sporadic success in
market power, hurricanes hitting oil fields, etc. These may occur around a deterministic
trend reflecting the presence of scarcity rents if the overall factor supply is finite.\textsuperscript{3} Finally,

\textsuperscript{2}See Krautkramer (1999) for a survey of the empirical success of the Hotelling model.
\textsuperscript{3}However, we do not explicitly model the factor as an exhaustible resource.
there is an alternative technology which can serve the same output market without using the volatile factor. We thus consider relatively mature technologies that can be irreversibly adopted by incurring a costly up-front investment. In the energy sector such technologies are nuclear, solar, wind, geothermal, and biomass, proving a backstop for fossil fuels.\footnote{In general, adoption of energy saving technologies shows up in cross-section data across countries: energy use or investments in capital goods with different energy intensities are responsive to permanent differences in energy prices (Berndt and Wood, 1975; Atkenson and Kehoe, 1999).}

Under these circumstances, we show that as factor market conditions get worse and new technology starts to enter, the entry rate must exceed the old technology exit rate, implying that the aggregate portfolio of technology units is expanded. Due to the uncertainty regarding the future profitability of production forms, the factor-free units are built to co-exist together with the factor demand infrastructure to the extent that the overall capacity overshoots the level that would prevail under one-to-one replacement of technology units. This prolonged coexistence can, in expectations, be of temporary or permanent nature.

As the adoption progresses, aggregate output becomes less and less factor intensive – the market share for the new technology increases – and yet the factor-induced output price volatility increases during the transition. Factor market shocks are transmitted to the output market to a greater extent, the larger is the fleet of remaining but idle old technology units that can respond to factor market conditions. It is a general property of backstop adoption paths that large scale idleness precedes the final decline of the old technology, leading to the necessary existence of this volatile capacity.\footnote{Macroeconomists find it puzzling that the oil prices have an aggregate effect despite the low cost share of oil in GDP (See, e.g., Barsky and Kilian (2004) and Hamilton (2005)). One potential explanation is that factor price changes are propagated through movements in other factor prices they induced. We do not consider macroeconomic effects but do intend the price volatility result to be suggestive of a propagation channel.}

The main difference between our work and the earlier literature on technology adoption
is that we do not consider one-to-one replacement of technologies by assumption\(^6\). Instead, we model the equilibrium exit and entry of technologies, implying that the aggregate availability of technology units can change along the equilibrium path. However, we obtain the one-to-one replacement paths in equilibrium, for example, if volatility is sufficiently small or if the old technology does not have the option to remain idle but can only exit. Also, most of the adoption literature considers adoption in environments where strategic issues and externalities are important whereas we consider a competitive equilibrium without distortions; our backstop adoption paths maximize the social surplus. Atkeson and Kehoe (1999) argue that the long-run adoption of factor-saving technologies can be understood using an approach where the technology portfolio embodied in capital goods is responsive to differences in factor prices. This "putty-clay" approach to adoption delivers inelastic short-run factor demands and allows the long-run demand to be more responsive to permanent changes in factor prices and thereby it allows better calibration to data, as opposed to the putty-putty model by Pindyck and Rotemberg (1983). Our model produces very similar pattern: in the early phase of the technology transition the factor demand is relatively insensitive to changes in factor prices, but when the new technology increases its market share, the factor demand becomes more responsive.

There is a large but somewhat dated literature on backstop technologies (for example, Nordhaus 1973, Heal 1976, Dasgupta and Heal 1974). Without exceptions known to us, this research casts the adoption problem in an exhaustible-resource framework without uncertainty. The models from the 70s typically feature a switch to the backstop as soon as the resource is physically or economically depleted. While such models are helpful in gauging the limits to resource prices using the backstop cost data (see Nordhaus), the predictions for the backstop technology entry are not entirely plausible if one accepts the characteristics of factor markets and energy demand infrastructure that we have outlined above. A more

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\(^6\)See Reinganum (1989) and Hoppe (2002) for surveys of the technology adoption literature.
realistic backstop technology entry is obtained in Charkravorty et al. (1997) where the demand for exhaustible factors is heterogenous and backstop technologies such as solar energy have a declining trend in adoption costs. We provide an alternative approach to gradual backstop technology transition where factor market price trends and volatility are distinct determinants of the expected long-run market shares for the technologies.

The paper is organized as follows. In Section 2, we introduce the agents, technologies, markets, and define the equilibrium. We also state the main Theorem of existence which is proved in Appendix. Section 3 then progresses as a sequence of propositions characterizing the equilibrium. We explain how volatility determines the nature of the transition (Section 3.1), characterize the output price risk (Section 3.2.), and discuss the determinants of the long-run market shares for technologies (Section 3.3.). In Section 4, we conclude by discussing the robustness of the qualitative features, and the lessons for energy policies.

2 Model

2.1 Production technologies

Consider two technologies, the old and new, for producing the same homogenous output. The old technology is a fixed proportions technology using one unit of a factor (say, oil) for one unit of output. The old technology is embodied in old capacity units that constitute the demand infrastructure for the factor. The demand infrastructure is given by history and it can respond to output and factor market conditions by adjusting utilization and scrapping capacity units.

The new technology is embodied in backstop capacity units that do not use the factor – one installed backstop unit produces one unit of output for free but the installation of such a unit requires a costly up-front investment.
Consider now firms behind these capacities. There is a continuum of infinitesimal firms, and each active firm has one unit of capital of either type. If we let $k^f_t$ and $k^b_t$ denote the respective total factor-dependent and backstop capacities at time $t$, then $k^f_t$ and $k^b_t$ denote also the numbers of firms at $t$. Each factor-dependent firm that is still in the industry at some given $t$ must choose one of the following options: produce, remain idle, or exit. To make the choice between idleness and exit interesting, we assume that staying in the industry implies an unavoidable cost per period. Let $c > 0$ denote this fixed flow cost. A producing unit in period $t$ thus incurs cost $c + p^f_t$, where $p^f_t$ is the factor price. An idle unit pays just $c$. An exiting unit pays a one-time cost $I_f > 0$ and, of course, avoids any future costs. Note that, in equilibrium, firms (discrete) choices between production and idleness determine the overall utilization of the old capacity. Let $q^f_t$ denote the total output from the factor dependent capacity. Then, $q^f_t$ is also the number of producing firms which satisfies $q^f_t = k^f$ if all remaining firms produce, and $0 \leq q^f_t < k^f$ if utilization is adjusted.

Consider then the build-up of the backstop capacity. We assume infinitely many potential entrants which can adopt the backstop technology by paying the up-front investment cost $I_b > 0$. Once installed, a backstop unit produces output without using the factor or other variable inputs. Without losing generality, we normalize the unavoidable cost flow from running a backstop plant to zero.\textsuperscript{7} While in principle idleness is an option for also these units, the plant utilization will not be an issue in equilibrium. Thus, if we let $q^b_t$ denote the total output from backstop capacity units in period $t$, we can also write $q^b_t = k^b_t$.

All agents are risk neutral, have infinite time horizons, and discount with rate $r$ (time is continuous). The following restriction holds throughout the paper:

$$I_f < \frac{c}{r} < I_f + I_b.$$ 

The first inequality implies that exit saves on unavoidable costs for an old capacity unit.

\textsuperscript{7}Because the entry is irreversible, the firm’s post entry behavior does not depend on such cost.
The second inequality implies that replacing an old unit by a new unit is costly. Without
the former restriction, old plants would never exit. Without the latter, the factor-dependent
capacity would be scrapped and new capacity built immediately.

2.2 Output and factor markets

Ignore the firms entry and exit decisions for a while and suppose that capacities \((k_f^t, k_b^t) =
(k_f^t, k_b^t)\) are fixed over time. Consider then the output market. Denote the total output
from both capacities by \(q_t = q_f^t + q_b^t\). In period \(t\), output price \(P_t\) that clears the market is
given by inverse demand \(P_t = D(q_t)\) that is continuously differentiable, and decreasing in
\(q_t\). Since the supply is always at least \(q_t = k_b^t\), it is natural to assume that the market can
absorb this quantity, \(P_t = D(k_b^t) > 0\) (this will, of course, always hold in equilibrium where
\(k_b^t\) is determined by investments). Then, the utilization of the old capacity depends on the
residual demand \(P_t = D(q_f^t + k_b^t)\) where we assume that the full utilization is an option,
\(P_t = D(k_f^t + k_b^t) > 0\) (this will also hold in equilibrium). To see how the utilization of plants
is determined, consider next the factor market.

We assume the following inverse supply curve for the factor:

\[
p_f^t = x_t + C(q_f^t) \geq 0
\]

where intercept \(x_t \geq 0\) is a stochastic variable capturing the factor market volatility,\(^8\) and
\(C(q_f^t)\) is continuous and strictly increasing in \(q_f^t\). Variable \(x_t\) follows Geometric Brownian

\(^8\)We seek to model supply-side shocks such as those caused by hurricanes hitting oil fields, wars and
political instability, fires in refineries, uncertain reservoir levels, sporadic successes in exploiting market
power, technological improvements, etc. We assume that \(x_t\) is expected to follow a positive trend because of
future scarcity rents, although we do not explicitly model the factor as an exhaustible resource. The solution
of the model does not require the positive trend but some qualitative results depend on the assumption.
These will be discussed below.
Motion with drift $\alpha \geq 0$ and standard deviation $\sigma$,

$$dx_t = \alpha x_t dt + \sigma x_t dz_t.$$  \hfill (1)

We will use notation \{\(x_t\)\} to denote the stochastic process defined by (1), while \(x_t\) refers to the value of this process at time \(t\). 9 Keeping the capacities still fixed at \((k^f, k^b)\), the equilibrium output process depends on \(x_t\) because factor-dependent capacity units must reduce utilization for sufficiently high realizations for \(x_t\). To describe the price process, we define the following critical values for \(x\):

$$\underline{x}(k^f, k^b) = D(k^f + k^b) - C(k^f),$$

$$\overline{x}(k^f, k^b) = D(k^b).$$

We will say that the factor market conditions are favorable to the old capacity if \(x < \underline{x}(k^f, k^b)\), because then the overall capacity constraint is binding, driving a wedge between the equilibrium output and factor prices, \(P_t > p_t^f\). See also Fig. 1. We will say that the factor market conditions are unfavorable if \(x \geq \underline{x}(k^f, k^b)\), because then a fraction of capacity \(k^f\) must remain idle and \(P_t = p_t^f\), implying no flow surplus covering the unavoidable cost \(c\).

The equilibrium output by factor-dependent producers is implicitly defined by the condition

$$D(q^f + k^b) = x_t + C(q^f).$$

Let \(q^f(k^b, x_t)\) denote this quantity. If \(x_t > \overline{x}(k^f, k^b)\), all capacity units \(k^f\) must remain idle, that is, \(q^f(k^b, x_t) = 0\). The output price process, for given \((k^f, k^b)\), is thus

$$P_t = P(x_t; k^f, k^b) = \begin{cases} 
D(k^f + k^b), & \text{when } x_t < \underline{x}(k^f, k^b) \\
D(q^f(k^b, x_t) + k^b), & \text{when } \underline{x}(k^f, k^b) \leq x_t \leq \overline{x}(k^f, k^b) \\
D(k^b), & \text{when } x_t > \overline{x}(k^f, k^b). 
\end{cases} \hfill (2)$$

9Formally, \(\{x_t\}\) is a sequence of random variables indexed by \(t > 0\) defined on a complete probability space \((\Omega, F, P)\). We denote by \(\{F_t\}\) the filtration generated by \(\{x_t\}\), i.e. \(F_t\) contains the information generated by \(\{x_t\}\) on the interval \([0, t]\).
Since an active backstop capacity has no production costs, the cash-inflow of such a unit is simply equal to the output price (2). Note that this captures the idea that the new technology’s payoff is uncertain because of the factor market condition determining the competitiveness of the old technology. We can also see at this point that when \( k_f \) capacity goes to zero so does the output price uncertainty.

\[
\pi_f(x_t; k_f, k_b) = \begin{cases} 
  x(k_f, k_b) - x_t - c, & \text{when } x_t < x(k_f, k_b) \\
  -c, & \text{when } x_t \geq x(k_f, k_b)
\end{cases}
\]  

(3)

Let us now pull together the basic assumptions as follows.

**ASSUMPTIONS**: We consider a competitive industry where the following hold:

1. All agents are risk neutral and discount with rate \( r > 0 \).

2. There is continuum of factor-dependent firms, each choosing one of the following options per period: produce a unit of output, remain idle, or exit. Production cost is the factor price, \( p_t \geq 0 \). Staying in the industry costs \( c > 0 \) per period for both producing and idle firms, and irreversible exiting costs \( I_f > 0 \).

3. There is a continuum of potential entrants to the industry. Entry is irreversible and costs \( I_b > 0 \). Each entrant produces a unit of output for free.

4. Exit saves on unavoidable costs but replacing technologies is costly:

\[
I_f < \frac{c}{r} < I_f + I_b.
\]

5. Inverse demand for output, \( D(q) \), is continuously differentiable and decreasing in \( q \).
6. Inverse supply for the factor is \( x + C(q^f) \), where \( x \) follows Geometric Brownian Motion with a positive drift and \( C(q^f) \) is continuous and strictly increasing in \( q^f \).

### 2.3 Equilibrium capacity paths

Let us now allow the capacities \( k^f_t \) and \( k^b_t \) change over time as new plants are built and old ones are scrapped. The information on which the firms base their behavior at period \( t \) consists of the historical development of \( x_t, k^f_t, \) and \( k^b_t \) up to time \( t \). This means that the resulting capacity paths are stochastic processes \( \{k^f_t\} \) and \( \{k^b_t\} \) such that their values at time \( t \) depend on the history of \( \{x_t\} \) up to that moment\(^{10} \). Since factor dependent capacity (backstop capacity) can only be decreased (increased), we must impose a restriction on the set of admissible capacity paths according to which \( \{k^f_t\} \) (\( \{k^b_t\} \)) must be non-increasing (non-decreasing).

Even with this restriction the capacity levels at time \( t \) could in principle depend in complicated ways on the entire history of \( \{x_t\} \) up to \( t \). However, what matters to the firms’ behavior is the current value of this process, not its entire history. The higher the value of \( x_t \), it becomes not only more attractive to exit or remain idle but also invest in backstop technology because entrants face less competition. In any sensible description of the firms’ behavior, it will always be the case that the capacities only change when \( x_t \) reaches new historical maximum values, and thereby, the capacities at time \( t \) will depend on the history of \( x_t \) only through the historical maximum value, which we denote by

\[
\hat{x}_t \equiv \max_{\tau \leq t} \{x_\tau\}. 
\]

In this paper, we only need to consider capacity paths that describe the evolution of the capacities as functions of \( \hat{x}_t \). Using boldface notation to denote capacities as such functions, we define admissible capacity paths as follows:

\(^{10}\)That is, they are stochastic processes adapted to the filtration \( \{F_t\} \).
Definition 1 An admissible capacity path is a pair $k = (k^f, k^b)$ consisting of two mappings: a non-increasing, right-continuous function $k^f : \mathbb{R}^+ \to \mathbb{R}^+$ and a non-decreasing, right-continuous function $k^b : \mathbb{R}^+ \to \mathbb{R}^+$. These mappings define capacity processes $\{k^f_t\} \equiv \{k^f(\hat{x}_t)\}$ and $\{k^b_t\} \equiv \{k^b(\hat{x}_t)\}$ where $k^f(\hat{x}_t)$ gives the level of factor dependent capacity and $k^b(\hat{x}_t)$ gives the level of backstop capacity at time $t$ as functions of the historical maximum for $x_t$.

[Fig. 2]

In fig. 2, we depict a sample path for the factor market condition and an admissible capacity pair. Let us now consider individual firms’ optimal investment and scrapping decisions. Consider first a firm, which owns a unit of factor dependent capacity. Assume that this firm anticipates correctly the capacity path $k = (k^f, k^b)$ induced by the behavior of all other firms, and chooses the optimal time to scrap its own capacity unit at cost $I_f$. The value of this firm at $t$ is a function of the current value $x_t$ and the historical maximum value $\hat{x}_t$:

$$V_f(x_t, \hat{x}_t; k) = \sup_{\tau^* \geq t} E \left[ \int_t^{\tau^*} \pi_f(x_\tau; k^f(\hat{x}_\tau), k^b(\hat{x}_\tau)) e^{-r(\tau-t)} d\tau - I_f e^{-r\tau^*} \right],$$

where $\tau^*$ is an optimally chosen scrapping time$^{11}$. Note that all active units are alike and therefore solve the same exit problem, but as will be formalized shortly, in equilibrium there is rationing of exit such that the firms staying and leaving make the same ex-ante profit.$^{12}$

$^{11}$ $\tau^*$ is a stopping time adapted to the filtration $\{F_t\}$.

$^{12}$ Without affecting the equilibrium we can also assume that factor-dependent firms are heterogeneous and produce the factor "in house" rather than buy it from the market. Then, heterogeneity is equivalent to assuming an upward sloping supply curve for the factor. In this interpretation, $x_t$ is a productivity shock common to all firms. Also, we could let $x_t$ affect firms asymmetrically by introducing it multiplicatively into the model. This would not affect the main Theorem of the paper.
On the other hand, the owner of backstop capacity has no decisions to make, and hence the value of an infinitesimal unit of such capacity is given by:

$$V_b(x_t, \hat{x}_t; k) = E \int_t^{\infty} P(x_\tau; k^f(\hat{x}_\tau), k^b(\hat{x}_\tau)) e^{-r(\tau-t)} d\tau.$$ (5)

One unit of the backstop technology can be adopted by paying cost $I_b > 0$. All potential entrants to the backstop sector are effectively holding an option to install one unit, so they solve the following stopping problem:

$$F(x_t, \hat{x}_t; k) = \sup_{\tau^* \geq t} \left[ E \int_{\tau^*}^{\infty} P(x_\tau; k^f(\hat{x}_\tau), k^b(\hat{x}_\tau)) e^{-r(\tau-t)} d\tau - I_b e^{-r\tau^*} \right],$$ (6)

where $F(\cdot)$ is the value of the option to enter. Again, all the potential entrants are alike and solve the same entry problem, but in equilibrium with unrestricted entry there is rationing that makes each entrant indifferent between entering and staying out. Of course, this means that $F(\cdot) = 0$ in equilibrium.

Let us now define formally the competitive equilibrium as a rational expectations Nash equilibrium in entry and exit strategies such that, given the entry and exit points of all firms, no firm can find any strictly more profitable entry and exit points (including the possibility of not entering or exiting at all). More formally, we want to find capacity path $k$ such that when firms take it as given, entering firms are indifferent between investing and remaining inactive, and exiting firms are indifferent between staying and leaving.

**Definition 2** An admissible capacity path $k = (k^f, k^b)$ is an equilibrium, if for all $t$, $x_t$, and $\hat{x}_t$:

$$V_b(x_t, \hat{x}_t; k) - I_b \leq 0 \quad (= 0 \text{ if } x_t = \hat{x}_t \text{ and } k^b(\cdot) \text{ increases at } x_t) \text{ and}$$

$$V_f(x_t, \hat{x}_t; k) \geq -I_f \quad (= -I_f \text{ if } x_t = \hat{x}_t \text{ and } k^f(\cdot) \text{ decreases at } x_t).$$ (7) (8)

For intuition, we will now discuss equilibrium conditions for entering and exiting firms. Consider first entry and suppose that current period $t$ is an entry point, i.e., factor market
condition hits a new record, $x_t = \hat{x}_t$, and thus $V_b(\hat{x}_t, \hat{x}_t; k) - I_b = 0$ by (7). Because entrants must be indifferent between entering now or at the next expected entry time, we have

$$E\{I_b(1 - e^{-r(\tau^{**} - \tau^*)})\} = E \int_{\tau=\tau^*}^{\tau^{**}} P_\tau e^{-r(\tau - \tau^*)} d\tau. \quad (9)$$

where $P_\tau = P(x_\tau; k^f(\hat{x}_t), k^b(\hat{x}_t))$ and $\tau^*, \tau^{**}$ are two consecutive entry points. Note that capacities are constants between the two time points since $k$ changes only when $x_t$ reaches a new record value, which occurs at the next expected entry time. The LHS is what, in expectations, the firm could save in costs by postponing entry to the next point at which factor market conditions favor entry. Because this reasoning must hold between any two consecutive entry points, we can write the indifference condition (9) as follows

$$I_b = E \int_{\tau^*}^{\infty} P_\tau e^{-r(\tau - \tau^*)} d\tau. \quad (10)$$

There will be no exceptions to this rule: the discounted value of the equilibrium price process will be equal to the entry cost at any equilibrium entry point.

Consider now exit and suppose that current period $t$ is an exit point, i.e., factor market condition hits a new record, $x_t = \hat{x}_t$, and thus $V_f(\hat{x}_t, \hat{x}_t; k) = -I_f$ by (8). Following the logic from above, an exiting firm must be indifferent between two consecutive exit points:

$$E\{(\frac{c_r}{r} - I_f)(1 - e^{-r(\tau^{**} - \tau^*)})\} = E \int_{\tau=\tau^*}^{\tau^{**}} \{P_\tau - p_\tau\} e^{-r(\tau - \tau^*)} d\tau, \quad (11)$$

where $P_\tau = P(x_\tau; k^f(\hat{x}_t), k^b(\hat{x}_t))$ and $p_\tau^f = x_\tau + C(q_f^f(k^b(\hat{x}_t), x_\tau))$. Again, $k$ is fixed between two consecutive entry and exit points. The LHS is the expected cost from delaying exit optimally, recall that $\frac{c_r}{r} - I_f > 0$, and the RHS is the expected surplus from this delay (see Fig. 1 to see why the flow payoff takes this form). Note that the reason to stay in the industry is that in expectations the total capacity constraint will bind before the next entry point, implying rents for the old capacity units under favorable factor market conditions, i.e., when $x_\tau < \underline{x}(k^f(\hat{x}_t), k^b(\hat{x}_t))$ and thus $P_\tau - p_\tau > 0$. This same reasoning holds for any two consecutive equilibrium exit points: the cost from staying rather than exiting at an
equilibrium exit point equals the expected present value of rents from being able to produce in under favorable factor market conditions.

While the indifference conditions for marginal entering and exiting firms are intuitive, they are not yet helpful in characterizing the technology transition, i.e., the entire capacity path $k$. The key to the characterization is the observation that a marginal firm which understands the stochastic process $\{x_t\}$ but disregards the other firms’ entry and exit decisions will choose the same entry or exit time as a firm that optimizes against the equilibrium capacity path $k$. For example, an exiting firm that thinks the current capacities $(k^f(\hat{x}_t), k^b(\hat{x}_t)) = (k^f, k^b)$ remain unchanged in the future solves the exit time from

$$V_f^m(x_t; k) = \sup_{\tau^* \geq t} E \left[ \int_t^{\tau^*} \pi_f(x_{\tau'}, k^f, k^b) e^{-r(\tau-t)} d\tau - I_f e^{-r\tau^*} \right]$$

and finds the same exit time as the sophisticated firm that solves (4) with the understanding of the aggregate capacity development. This myopia result is due to Leahy (1993).\(^{13}\) It can be used to transform each firm’s problem into a simple Markov decision problem determining entry and exit thresholds in terms of $\hat{x}$, for any given pair $(k^f, k^b)$. Alternatively, we can take any $\hat{x} \in \mathbb{R}^+$ as given and look for capacity pairs $(k^f, k^b)$ that make $\hat{x}$ the investment threshold for myopic firms. This way we can map from all conceivable myopic thresholds $\hat{x} \in \mathbb{R}^+$ to an equilibrium path $k = k^*$. In Appendix, we show that the equilibrium is unique and that it can indeed be computed solving the myopic problems.

**Theorem 1** The model has a unique equilibrium $k = (k^f, k^b)$ with the following properties:

- $k^f$ is everywhere continuous, strictly decreasing on some interval $(a_f, b) \subset \mathbb{R}^+$, and constant on $\mathbb{R}^+ \setminus (a_f, b)$.

- $k^b$ is everywhere continuous, strictly increasing on some interval $(a_b, b) \subset \mathbb{R}^+$, and constant on $\mathbb{R}^+ \setminus (a_b, b)$.

\(^{13}\)Although our extension of the result to a two-dimensional model is non-trivial.
Proof. See Appendix.

Before turning to characterization, let us note two basic implications of the Theorem. The exit of the old technology may start before or after the entry of the new one (i.e., \( a_f \neq a_b \)), but both transitions end at the same factor market condition, \( \hat{x} = b \). Theorem also implies that as long as the transition is going on for both technologies, there is both exit and entry every time \( \hat{x} \) reaches a new record value.

3 Characterization

3.1 Volatility and the transition

In this section, we describe the trajectory that the equilibrium capacity pair \( k(\hat{x}) = (k^f(\hat{x}), k^b(\hat{x})) \) must follow when the underlying factor market condition \( \hat{x} \) runs from zero to infinity. We do this for all degrees of uncertainty \( \sigma \), starting with the deterministic case \( \sigma = 0 \). However, before progressing we want to limit attention to technology transitions that are relevant for technology adoption. For example, we are not particularly interested in situations where there is so much initial backstop capacity that the transition is merely about the exit. We are also not interested in transitions that more or less jump to the long-run equilibrium (small initial \( k^f \)). Interesting transitions are such that there is both entry and exit as \( \hat{x} \) reaches new values. Along such a path it makes sense to talk about technology replacement.

Definition 3 Adoption path is an equilibrium path \( k(\hat{x}) \) with both entry and exit for all \( \hat{x} \in (0, b) \subset \mathbb{R}^+ \).

Recall from Theorem 1 that \( b \) is the factor market condition at which the transition is over for both technologies. The definition of the adoption path confines attention to an equilibrium path \( k(\hat{x}) \) with the property that \( a_f = a_b = 0 \) in Theorem 1, i.e., the transition in both technologies should start already at \( \hat{x} \) arbitrarily close to zero. The adoption path is a
generic equilibrium path in the sense that any given equilibrium starting from arbitrary initial conditions \((x_0, k_f^0, k_b^0)\) will ultimately coincide with the adoption path for all \(\hat{x} \in (a, b) \subset \mathbb{R}^+\) where \(a = \max\{a_f, a_b\}\). That is, as soon as the transition has started for both technologies, the equilibrium coincides from that point on with the adoption path. By confining attention to the path along which there is entry and exit for all \(\hat{x} \in (0, b)\), we can find the equilibrium path that is not constrained by the starting point and this way characterize all cases at once. Note that \(a_f = a_b = 0\) requires that the old technology units cannot meet the demand alone at \(\hat{x} = 0\) so that some entry must take place already at very favorable factor market conditions. For ease of exposition, let us assume that demand is large enough for the adoption path, as defined above, to exist.\(^{14}\)

The nature of the adoption path will depend critically on whether old technology units exit without adjusting technology utilization. For if there is no utilization adjustment between two consecutive exit points, the output market is isolated from the factor market volatility, implying that the new technology entrants face no uncertainty. Recall that \(z(k^f(\hat{x}), k^b(\hat{x})) = D(k^f(\hat{x}) + k^b(\hat{x})) - C(k^f(\hat{x}))\) is the critical value determining whether the capacity utilization is adjusted at given \(\hat{x}\).

**Remark 1** *Along the adoption path, exit from activity (idleness) implies*

\[z(k^f(\hat{x}), k^b(\hat{x})) - \hat{x} \geq 0 (\leq 0).\]

Whether or not all firms exit before factor the market condition reaches the critical value \(z(k^f(\hat{x}), k^b(\hat{x}))\) depends on factor market volatility, \(\sigma\). Let us now consider the deterministic benchmark case.

\(^{14}\)The assumption is not needed for Theorem 1. This assumption is without loss of generality because it can be relaxed by adding more initial backstop capital \(k_b^0\), leading to lower residual demand to start with. Therefore, if the assumption on demand is not satisfied, the effect on the equilibrium is the same as that from a large \(k_b^0\). Any equilibrium that is constrained by excessively large \(k_f^0\) or \(k_b^0\) will ultimately follow the path \(k\) identified by the adoption path.
Proposition 1 Assume $\sigma = 0$. Then, all factor-dependent units exit from activity. Along the adoption path, each entrant replaces one factor-dependent unit.

Proof. For $\sigma = 0$, we can write the exit condition as

$$c - r I_f = D(k_f^f(\hat{x}) + k^b(\hat{x})) - C(k_f^f(\hat{x})) - \hat{x}$$

$$= x(k_f^f(\hat{x}), k^b(\hat{x})) - \hat{x} > 0,$$

meaning that if there is exit, the old capacity units exit from activity. We can write the entry condition (10) as

$$r I_b = D(k_f^f(\hat{x}) + k^b(\hat{x})),$$

implying that if there is entry, the total capacity must be a constant between two entry points; the entry-exit ratio is one.

Let us now consider uncertainty and the nature of the transition. The degree of volatility determines whether firms exit from activity to the very end of the transition, which in turn determines whether entrants face uncertainty or not. For this reason, we consider now the stopping problem for the last exiting firm and whether this firm exits from activity or idleness. The value of the old technology unit satisfies

$$\frac{1}{2} \sigma^2 x^2 V''_f + \alpha x V'_f - r V_f + \pi_f = 0,$$

where arguments are omitted and primes denote derivatives with respect to $x$. Note that because we are considering this problem for the last exiting unit, there are no subsequent exit or entry decisions\(^{15}\) and, therefore, the myopic and sophisticated approaches to the firm’s problem are trivially equivalent. Also, this infinitesimal firm is the last one on the factor demand curve, implying that the factor price is just the intercept, $p_f = x + C(0) = x$. Let $k_\infty$ denote the long-run capacity when only backstop units produce, i.e., $k_\infty$ is given by

\(^{15}\)Recall that exit and entry stop at the same factor market condition (Theorem 1).
$D(k_\infty) = rI_b$. Noting that for the last firm $x(0,k_\infty) = \pi(0,k_\infty) = D(k_\infty)$ and solving the firm’s stopping problem under the assumption that the firm exits from activity,\(^{16}\) we find the following exit threshold:

$$b(\sigma) = \frac{\beta_1(\sigma)}{\beta_1(\sigma) - 1} (r - \alpha)(I_b + I_f - \frac{c}{r})$$

where $\beta_1(\sigma) > 1$ and is given by

$$\beta_{1,2}(\sigma) = \frac{1}{2} - \frac{\alpha}{\sigma^2} \pm \sqrt{\left[ \frac{\alpha}{\sigma^2} - \frac{1}{2} \right]^2 + \frac{2r}{\sigma^2}}.$$ (12)

Note that as $\sigma \to 0$, the expression for $b(\sigma)$ approaches the exit condition in Proposition 1. Since all firms exit from activity under certainty and $b(\sigma)$ is continuously increasing in $\sigma$, it is clear that $x(0,k_\infty) - b(\sigma) > 0$ holds also for small $\sigma$. Let $\sigma = \sigma^* > 0$ denote the unique solution to

$$x(0,k_\infty) - b(\sigma) = 0.$$ 

**Proposition 2** For any given $\sigma \leq \sigma^*$, Proposition 1 holds.

For small volatility, the equilibrium transition is thus qualitatively equivalent to that in the deterministic case. Because the last old technology firm exits before factor market conditions turn unfavorable, all factor-dependent firms must exit from activity. By this, the old capacity does not adjust utilization between consecutive entry and exit points, meaning that the output market is isolated from the factor market volatility. With $P_t$ fixed at $rI_b = D(k_\infty)$, the factor-dependent technology is scrapped such that expected rents from producing under binding total capacity are equal to the expected costs of remaining active at each equilibrium exit point.

\(^{16}\)The solution procedure is standard if the firm exits from activity. If the firm exits from idless, the value matching and smooth pasting conditions change and there are also boundary conditions for values of $x$ at which the firm switches from production to idleness. But the procedure is still standard and not reported here.
For larger factor market volatility, $\sigma > \sigma^*$, the last exiting firm cannot exit from activity. We now solve the same stopping problem as above but this time under the assumption that the last firm exits from idleness. Without reporting the routine details we note that the exit threshold for the last factor-dependent unit is

$$b(\sigma) = \left[ \frac{(r - \alpha)(r I_f)^{\beta_2(\sigma) - 1}(c - r I_f)}{r \beta_1(\sigma) - \alpha} \right]^{\frac{1}{\beta_2(\sigma)}}$$

for $\sigma > \sigma^*$.

It is straightforward to verify that $b(\sigma)$ increases in $\sigma$ and that the above two expressions for $b(\sigma)$ coincide when $\sigma = \sigma^*$.

Before the exit, the last factor-dependent firm remaining in the industry faces no competition in case the factor market turns favorable. Therefore, for the last firm $\bar{x}(0, k_\infty) = \bar{x}(0, k_\infty)$, i.e., there is no phase where this firm would produce jointly with other firms such that the flow surplus is zero. Before exiting, the last firm is thus either idle, $x > \bar{x}(0, k_\infty)$, or active and running a flow surplus, $x < \bar{x}(0, k_\infty)$. Consider then the current $\hat{x}$ slightly below $b(\sigma)$. Now, there is still a group of old-technology firms remaining in the industry such that the wedge

$$\bar{x}(k_f^f(\hat{x}), k_b^b(\hat{x})) - \bar{x}(k_f^f(\hat{x}), k_b^b(\hat{x})) = C(k_f^f(\hat{x})) > 0$$

exists. This means that before exiting the old-technology units compete away any flow surplus from production if factor market conditions improve to reach the range

$$\bar{x}(k_f^f(\hat{x}), k_b^b(\hat{x})) < x \leq \bar{x}(k_f^f(\hat{x}), k_b^b(\hat{x})),$$

or they run a surplus at full capacity if $x < \bar{x}(k_f^f(\hat{x}), k_b^b(\hat{x}))$. However, when they exit, they exit from an environment where all capacity is idle, because

$$b(\sigma) > \hat{x} > \bar{x}(k_f^f(\hat{x}), k_b^b(\hat{x})).$$

We call the phase when the entry-exit point satisfies $\hat{x} > \bar{x}(k_f^f(\hat{x}), k_b^b(\hat{x}))$ the idle capacity phase.
From the above follows that when volatility is large enough, $\sigma > \sigma^*$, the entry-exit points along the adoption path must pass through the idle capacity phase. However, keeping still $\sigma > \sigma^*$, the first entry-exit points occur when all capacity active, since (13) holds at $\hat{x} = 0$ and entry and exit starts at arbitrarily small $\hat{x}$ along the adoption path. Thus, the entry-exit points along the adoption path must also pass through an active capacity phase.

Finally we argue that the adoption path entry-exit points must pass through the volatile capacity phase, $\underline{\pi}(k^f(\hat{x}), k^b(\hat{x})) < \hat{x} \leq \overline{\pi}(k^f(\hat{x}), k^b(\hat{x}))$. In particular, $\hat{x}$ enters and leaves this phase only once.

**Lemma 1** Assume $\sigma > \sigma^*$. Then, the adoption path defines unique thresholds $X$ and $\overline{X}$ in $(0, b) \subset \mathbb{R}^+$ such that

$$\hat{x} = \underline{\pi}(k^f(\hat{x}), k^b(\hat{x})) \text{ for } \hat{x} = X$$

$$\hat{x} = \overline{\pi}(k^f(\hat{x}), k^b(\hat{x})) \text{ for } \hat{x} = \overline{X} > X.$$

**Proof.** See Appendix □

The result requires a proof because Theorem 1 does not pin down how $\underline{\pi}(k^f(\hat{x}), k^b(\hat{x}))$ develops in equilibrium. In Appendix we use the exiting firm’s indifference condition (11) to show that the equilibrium $\underline{\pi}(k^f(\hat{x}), k^b(\hat{x}))$ must increase in $\hat{x}$ and also the gap $\hat{x} - \underline{\pi}(k^f(\hat{x}), k^b(\hat{x}))$ must increase in $\hat{x}$ so that the exit point $\hat{x}$ is increasingly "further away" from the factor market condition at which the remaining capacity starts to bind.

**Proposition 3** Assume $\sigma > \sigma^*$. Then, the adoption path entry-exit points pass through three phases:

1. **active capacity phase**, $0 < \hat{x} \leq X$;

2. **volatile capacity phase**, $X < \hat{x} \leq \overline{X}$;

3. **idle capacity phase**, $\overline{X} < \hat{x}$.
To grasp the precise picture, see Fig. 3, which is the same Fig. 2 but now we assume that the admissible capacity paths are the equilibrium paths. In the active capacity phase, the old technology is fully used at the entry-exit points. Utilization is depicted by the shaded area under path $k^b(\hat{x})$. In the volatile capacity phase, utilization is always less than 100 per cent at the entry-exit points. Note that full utilization can be reached at other than entry-exit points. Finally, in the idle capacity phase, utilization drops to zero at the entry-exit points. In this sample path, utilization never becomes positive once the equilibrium enters the last phase, although in expectations positive utilization is an option.

**Proposition 4** Assume $\sigma > \sigma^*$. Then, the adoption path exhibits capacity overshooting:

1. $k^f(\hat{x}) + k^b(\hat{x}) = k_\infty$ is a constant in active capacity phase;

2. $k^f(\hat{x}) + k^b(\hat{x})$ increases and stays above $k_\infty$ in volatile capacity phase;

3. $k^f(\hat{x}) + k^b(\hat{x})$ declines back to $k_\infty$ at the end of the idle capacity phase.

**Proof.** See Lemma 2 in Appendix. ■

Proposition 4 means that the total capacity peaks somewhere along the way, but it does not specify whether this happens in the volatile or idle capacity phase. With linear demand and supply we are able to show that the capacity always peaks in the idle capacity phase, but we can not rule out the possibility of capacity peaking in the volatile capacity phase with some highly non-linear demand and supply curves (although we do not think this would be a typical case). See Fig. 4, which is the same as Fig. 3 but now the total capacity included. The
aggregate capacity overshooting implies that the technology portfolio is expanded during the transition. Intuitively, when there is sufficient uncertainty regarding the future profitability of the production forms, it makes sense to invest in the alternative technology without one-to-one scrapping of the old technology. The result can be better understood by considering what destroys it.

**Factor market volatility.** One-to-one replacement is obtained with low factor market volatility, \( \sigma \leq \sigma^* \), as already explained. It is thus the sufficient uncertainty that makes the old technology units to choose idleness rather than exit in expectations of more favorable factor market conditions. This contributes to the adoption overshooting.

**Option to remain idle.** If idleness is ruled out by assumption, leaving exit as the only response option to worsening factor market conditions, then replacement is again one-to-one. Clearly, if the old technology units cannot adjust utilization, the output price cannot not change between consecutive entry points and, by the equilibrium entry condition, price must be a constant along adoption paths. Hence, the entry-exit ratio is one and there is no adoption overshooting.

**Heterogeneity.** Heterogeneity in the factor supply is necessary for adoption overshooting. To see this, suppose temporarily that the cost of supplying a marginal unit of the factor is zero, \( C(\cdot) = 0 \), so that the inverse factor supply curve is horizontal, \( p^f = x \). Consider then the first replacement of an old technology unit by a new one. The cost of this irreversible replacement is \( I_f + I_b - \frac{c}{r} > 0 \), and the expected benefit is \( \frac{\hat{x}}{r-\alpha} \) where \( \hat{x} \) is the factor market condition at which the replacement is chosen to occur. The solution to this standard stopping problem satisfies

\[
\hat{x} = \frac{\beta_1}{\beta_1 - 1}(r - \alpha)(I_f + I_b - \frac{c}{r})
\]  

(14)
where $\beta_1$ is given by (12).\footnote{Note that this is the same threshold as for the last exiting firm when $\sigma \leq \sigma^*$. This is because the equilibrium factor supply curve is horizontal for the last exiting firm.} However, since the factor market price is just $p^f = \hat{x}$, it is independent of exit, meaning that all remaining old capacity producers can be profitably and instantaneously replaced as well. Just before the replacement these units produced $\hat{x} = D(q^f)$, satisfying (14), so the optimal amount of new capacity is given by

$$k^b = D^{-1}(\frac{\beta_1}{\beta_1 - 1}(r - \alpha)(I_f + I_b - \frac{c}{r})).$$

After this large scale one-to-one replacement of technologies, the overall capacity declines for all $\hat{x} > D(k^b)$. Therefore, heterogeneity in factor supply is necessary for the adoption overshooting.

We now turn to elaborate additional implications of the capacity overshooting.

### 3.2 Output price volatility

In this section we describe how the factor market volatility is transmitted into the output price along the adoption path that exhibits capacity overshooting. In particular, we want to demonstrate that it is a feature of the equilibrium transition that the factor price volatility translates into a larger output price volatility as the equilibrium progresses through the volatile capacity phase although production becomes less intensive in the factor.

We are first interested in describing the set of output prices that are achievable for each $\hat{x}$. Let $\underline{P}(\hat{x})$ and $\bar{P}(\hat{x})$ denote the maximum and minimum output prices that can be observed at current capacities, i.e., without strictly exceeding the current factor market record $\hat{x}$. Clearly,

$$\underline{P}(\hat{x}) = D(k^f(\hat{x}) + k^b(\hat{x})), $$

$$\bar{P}(\hat{x}) = D(q^f(k^b(\hat{x}), \hat{x}) + k^b(\hat{x})).$$
where \( q^f = 0 \) if \( \hat{x} > \mathbf{f}(\hat{x}) \). That is, the lowest price is achieved when the current capacity is in full use, and the highest when the factor market condition is so bad that the technology utilization could not be adjusted further without triggering exit. In equilibrium, any price from the set \([\mathcal{P}(\hat{x}), \mathcal{F}(\hat{x})]\) can be reached depending on the realization \( x \leq \hat{x} \).

To describe how \([\mathcal{P}(\hat{x}), \mathcal{F}(\hat{x})]\) develops as \( \hat{x} \) runs through the phases identified in Proposition 3, we must impose more structure on the model. Rather than imposing tedious curvature restrictions on demand and supply relations, we choose to assume linearity:

\[
D(q) = A - Bq \quad (15)
\]
\[
C(q^f) = x + Cq^f \quad (16)
\]

where \( A, B, C \) are strictly positive constants.

**Proposition 5** Assume \( \sigma > \sigma^* \) and \( D(q) \) and \( C(q^f) \) satisfying (15)-(16). Then,

1. \( \mathcal{P}(\hat{x}) = \mathcal{P}(\hat{x}) = rI_b \) in the active capacity phase;
2. \( \mathcal{P}(\hat{x}) \) increases and \( \mathcal{P}(\hat{x}) \) decreases throughout the volatile capacity phase;
3. \( \mathcal{P}(\hat{x}) \) decreases throughout the idle capacity phase, and \( \mathcal{P}(\hat{x}) \) increases in the end of the idle capacity phase and \( \mathcal{P}(b) = \mathcal{P}(b) = rI_b \).

**Proof.** The price set is depicted in Fig. 5. We prove the result by studying the entrants’ indifference condition (10) along the adoption path. Let \( \tau^* \) and \( \tau^{**} > \tau^* \) denote two equilibrium entry points in the volatile capacity phase, \((X, X)\). By the indifference condition, the expected present-value revenues between consecutive entry points must be the same at the two entry points. Therefore, we can write

\[
E_{t=\tau^*} \int_{\tau^*}^{\tau^{**}} P(x_\tau; k^f(x_\tau^*), k^b(x_\tau^*)) e^{-r(\tau-\tau^*)} d\tau
\]
\[
= E_{t=\tau^{**}} \int_{\tau^{**}}^{\tau^{***}} P(x_\tau; k^f(x_\tau^{**}), k^b(x_\tau^{**})) e^{-r(\tau-\tau^{**})} d\tau
\]
\[
= E_{t=\tau^{**}} \int_{\tau^{**}}^{\tau^{***}} P(\frac{1}{\gamma} y_\tau; k^f(x_\tau^{**}), k^b(x_\tau^{**})) e^{-r(\tau-\tau^{**})} d\tau
\]
where $\tau^*$ and $\tau^{**} > \tau^*$ are two equilibrium entry points, $\tau^{*'}$ and $\tau^{**'}$ are the corresponding next entry points. The first and second lines follow from the entrants’ indifference condition, and the third is simply the second rewritten after defining $y_\tau \equiv \gamma x_\tau$, where $\gamma \equiv \frac{x_\tau}{x_{\tau^{*'}}} < 1$. We make use of the fact that a variable following a Geometric Brownian Motion can be scaled without changing the process. That is, when $x \sim GBM(\alpha, \sigma)$, also $y \equiv \gamma x \sim GBM(\alpha, \sigma)$. The idea is to normalize process at the entry point $\tau^{*'}$ such that the starting value of the process is the same as at $\tau^*$, that is $y_{\tau^{*'}} = x_{\tau^*}$. In this way we are replicating time $\tau^*$ entry problem at time $\tau^{*'}$ but with two changes in the price function: 1) the argument has been scaled by term $\frac{1}{\gamma}$, and 2), the capacities have changed by time $\tau^{*'}$. Now we can consider how the equilibrium price function must change to retain equality between (17) and (19). By the linearity, the price function in the volatile capacity phase is

$$P(x_\tau; k^f(\hat{x}), k^b(\hat{x})) = \begin{cases} 
A - B(k^f(\hat{x}) + k^b(\hat{x})), & \text{when } x_\tau < y(x^f(\hat{x}), x^b(\hat{x})) \\
Q(k^b(\hat{x})) + Rx_\tau, & \text{when } x(x^f(\hat{x}), x^b(\hat{x})) \leq x_\tau \leq \hat{x}
\end{cases}$$

where

$$Q(k^b(\hat{x})) = \frac{C(A - Bk^b(\hat{x}))}{B + C}$$

$$R = \frac{B}{B + C}.$$ 

Now consider the lines (17) and (19) above. By the fact that $k^f(\hat{x}) + k^b(\hat{x})$ increases in the volatile capacity phase (Proposition 4), we must have $P(x_{\tau^{*'}}) < P(x_{\tau^*})$, which is equivalent to $P(\frac{1}{\gamma} \cdot y_{\tau^{*'}}) < P(x_{\tau^*})$. To retain the equality of (17) and (19) it must then be that

$$P(\frac{1}{\gamma} \cdot y_{\tau^{*'}}) > P(x_{\tau^*})$$

otherwise the scaled price function associated with $\tau^{*'}$ would be strictly worse than the original for the excursion of (This argument rests on the linearity of the price function, and the fact that scaling the argument by the term $\frac{1}{\gamma}$ increases its slope). We have now shown that $P(x_{\tau^{*'}}) = P(\frac{1}{\gamma} \cdot y_{\tau^{*'}}) > P(x_{\tau^*})$ for arbitrary entrypoints $x_{\tau^{*'}} > x_{\tau^*}$, meaning that $P(\hat{x})$
must be increasing for all $\hat{x} \in (\underline{X}, \overline{X}]$. The remaining cases follow trivially from Proposition 4.

The set of volatile prices thus expands during the volatile capacity phase. We will next demonstrate that also the price volatility increases: for any given $\hat{x} \in (\underline{X}, \overline{X}]$ and price $P$ from the set $(\underline{P}(\hat{x}), \overline{P}(\hat{x}))$, a change in $x < \hat{x}$ translates into a greater change in price $P$ as the equilibrium progresses through the volatile capacity phase. Using Ito’s Lemma, we can write the price process under the linear structure as follows:

$$dP = \begin{cases} 0 & \text{for } P < P(\hat{x}) \\ (P - Q(k^b(\hat{x}))(\alpha dt + \sigma dz) & \text{for } P(\hat{x}) < P < \overline{P}(\hat{x}) \end{cases}$$

Now note that, given $x < \hat{x}$, $Q(\cdot)$ is a constant, and the volatility of $P$ is

$$\frac{P - Q(k^b(\hat{x}))}{P} \sigma \text{ for } \underline{P}(\hat{x}) < P < \overline{P}(\hat{x}).$$

Now, when a new entry point is reached, $Q(\cdot)$ drops to a lower level and therefore the expression for volatility, which holds till the next entry point, is larger. We can thus conclude that the output prices become more volatile during the volatile capacity although production becomes less intensive in the factor.

Consider now Fig. 6 which is the same as Fig. 4 but now the output price is included. During the active capacity phase, the output market is isolated from the factor market volatility. This follows because the old technology is fully used and therefore absorbing the volatility. When the old capacity turns volatile, we see a considerable transmission of uncertainty to the output sector. Finally, during the idle capacity phase the volatility gradually levels off.

\textit{Figs. 5 – 6}
3.3 Probability of the transition

How likely is it that the technology transition is completed in the sense the backstop technology takes over the market and eliminates the dependence on the volatile factor entirely in the long run? To get an idea of this, consider the probability of reaching the factor market condition $b$ at which the process is completed within $T$ periods. Let $\Phi$ denote the cumulative distribution function for the standard normal distribution. Then, starting from $x_0 < b$, the probability of $\hat{x} \geq b$ at $T$ is

$$\text{Prop}(\hat{x}_T \geq b) = \Phi\left(\frac{\ln(x_0/b) + (\alpha - \sigma^2/2)T}{\sigma\sqrt{T}}\right) + \left(\frac{b}{x_0}\right)^{2\alpha/\sigma^2} - 1 \Phi\left(\frac{\ln(x_0/b) - (\alpha - \sigma^2/2)T}{\sigma\sqrt{T}}\right).$$

**Proposition 6** The backstop technology takes over the market with probability one if $\alpha \geq \sigma^2/2$ and with probability $\left(\frac{b}{x_0}\right)^{2\alpha/\sigma^2} - 1$ if $0 < \alpha < \sigma^2/2$, where $x_0 < b$.

**Proof.** As $T \to \infty$,

$$\text{Prop}(\hat{x}_T \geq b) \to 1, \text{ if } \alpha \geq \sigma^2/2$$

$$\text{Prop}(\hat{x}_T \geq b) \to \left(\frac{b}{x_0}\right)^{2\alpha/\sigma^2} - 1, \text{ if } 0 < \alpha < \sigma^2/2.$$ 

While the result follows directly from the properties of the stochastic process $\{x_t\}$, it gives some idea of the distinct roles of scarcity rents and volatility in the technology transition. Although we do not explicitly link the overall availability of the factor and the factor price trend $\alpha$, the above result suggests that the trend must be sufficiently large relative to the factor price volatility for the backstop to fully take over the market. In other words, volatility satisfying $\sigma^2/2 > \alpha$ tends to protect the old technology in the sense that, in expectations, the old technology has a market share.
4 Concluding Remarks

We considered socially efficient adoption of technologies that reduce dependence on volatile factors of production. Three assumptions are essential for the nature of the technology transition. First, factor-dependent technology units have the option to remain idle rather than exit when factor markets develop unfavorably. Thus, the aggregate technology utilization can be adjusted. Second, the factor supply sources are heterogenous so that the supply curve is upward sloping. Third, factor market is subject to sufficient supply-side uncertainty.

Under these circumstances, we found that the technology adoption process had three qualitatively distinct phases, depending on the historical performance of the factor market. In the active capacity phase, the history of the factor market is still relatively favorable to the factor-using production but continually worsening. Then, the adoption process has the traditional form where the existing technology units are replaced one-to-one by the new units.

In the volatile capacity phase, the factor use becomes so expensive that the old capacity cannot be fully utilized. However, because the factor use may still become cheaper in the near future, it is a profitable option to leave some capacity idle rather than scrap the old units. A general property of this phase is that the new technology units are built to coexist with the old ones so that the total availability of production units increases. It is important to emphasize that it is socially efficient to expand the portfolio of production forms in this way since both scrapping and adoption are irreversible decisions which are made under uncertainty about the future profitability of both production forms. As a result of this capacity expansion, the factor market uncertainty is increasingly transmitted to the output market.

If the volatile capacity phase is about the old technology’s fight against its decline, the final phase, the idle capacity phase, is about the decline. The old technology exit rate exceeds
new technology entry rate, and the output market volatility gradually diminishes. Yet, the factor-dependent technology may have a positive long-run market share because, depending on the factor price trend and volatility, there may be a persistent possibility of improving factor market conditions.

At the theoretical level, there are some obvious sources of criticism. For tractability, we could not allow expansion of the factor-dependent capacity. We do not believe that this restriction is central to the results. This holds in particular if the factor price process has a trend large enough to imply no long-run market share for the old technology. Moreover, the explicit inclusion of the option to remain idle serves as a partial substitute for the option to expand: under improving factor market development new production capacity comes from the idle reserve before any new investment should take place. Another shortcoming is the fact that the factor price trend is exogenous. Ideally, the trend should reflect the Hotelling-type rents due to the finiteness of the overall factor supply. Making this link explicit would allow addressing the roles of scarcity rents and volatility in the backstop technology adoption in detail.

The most recent revival of interests in reducing dependence on some key factors such as energy commodities is due to various externalities caused by the use of these factors. Reducing dependence on oil may contribute to road safety through the reduced size of the vehicles. In general, fossil fuels cause local and global pollution problems. We deliberately excluded any externalities from the analysis to provide insights regarding the determinants of the prolonged transition to the factor-free environment in a well-functioning market economy. However, these insights remain intact under an alternative interpretation of the model that incorporates the externality pricing. Without affecting the equilibrium we can think that factor users face a horizontal supply curve, \( p^f = x \), but are heterogenous in their efficiency of using the factor. The efficiency in factor use may relate to emission rates and thereby to externality payments, making firms to exit the industry in the order given by their emission.
rates.

This alternative interpretation of the model can provide important policy implications. Penalizing the use of factors causing pollution or other externalities may not cause a decline of the factor demand infrastructure but only its utilization decline. If externalities are correctly priced, the persistence of the polluting technology together with the new clean technology is socially optimal for the reasons that we have underscored in this paper.

References


APPENDIX: Theorem 1.

**Proof.** We will proceed as follows. We first show that the family of solutions to the stopping problems of myopic agents that disregard the future capacity adjustments define a unique admissible capacity path with the properties defined in Theorem 1. We then show that this capacity path satisfies the conditions of Definition 2, which establishes the existence of equilibrium. The uniqueness follows from the uniqueness of the myopically optimal capacity path and necessity of the myopic optimality along the equilibrium path.

We start by investigating the optimal behavior of myopic agents that solve the following stopping problems with fixed \((k^f, k^b)\):

\[
\sup_{\tau^* \geq t} E \left[ \int_t^{\tau^*} \pi_f(x_{\tau}; k^f, k^b)e^{-r(\tau-t)}d\tau - I_f e^{-r\tau^*} \right],
\]

\[
\sup_{\tau^* \geq t} E \left[ \int_{\tau^*}^{\infty} P(x_{\tau}; k^f, k^b)e^{-r(\tau-t)}d\tau - I_b e^{-r\tau^*} \right].
\]

These are standard problems, and it can be shown that the optimal solution to both problems is a threshold level such that it is optimal to stop whenever the current \(x\) is at or above the threshold. Denote by \(x_{k^f, k^b}^m\) and \(x_{k^f, k^b}^b\) the optimal thresholds for problems (20) and (21), respectively. Note that a stopping threshold may get value 0, in which case it is always optimal to stop, or it may get value \(\infty\) in which case it is never optimal to stop.

The first thing to note is that the indifference conditions (9) and (11) must hold at \(x_{k^f, k^b}^m\) and \(x_{k^f, k^b}^b\). This is because at the optimal stopping threshold a myopic firm must be indifferent between stopping now or at the next moment when an infinitesimally higher threshold level is reached from below. Conversely, because \(\pi_f(x_{\tau}; k^f, k^b)\) and \(P(x_{\tau}; k^f, k^b)\) are monotonic functions in \(x\), the indifference conditions (9) and (11) imply the optimality of stopping, and hence (9) and (11) are essentially equivalent with the optimality of the problems (20) and (21), respectively. This is the key to understanding the
myopia result explained in Leahy (1993), see also Baldursson and Karatzas (1997) for a more
general and rigorous analysis. Because (9) and (11) must hold when capacitites adjust in
equilibrium, by tracking in \((x, k^f, k^b)\) space all points where a myopic agent finds it optimal
to stop we necessarily also track all points where capacities adjust in equilibrium.

Let us work in the \((k^f, k^b)\) plane and consider the optimal behavior of myopic agents.
Define two curves. First, denote by \(k^\infty\) the set of points on the 45-degree line along which it
holds that \(k^f + k^b = k^\infty\). Below this curve the total capacity is so low that there is immediate
investment in \(k^b\) at all values of \(x\), that is, \(x^m_\beta(k^f, k^b) = 0\). Second, denote by \(\tilde{k}\) the set
of points for which it holds that a myopic owner of a unit of \(k_f\) is just indifferent between
scraping the unit and continuing at the most favorable factor market condition, that is
when \(x_t = 0\). This curve is defined by the condition \(c + C(k^f) - D(k^f + k^b) - r I_f = 0\). To
the right of this curve the total capacity is so high that there is immediate scrapping of \(k_f\)
at all values of \(x\), that is, \(x^m_f(k^f, k^b) = 0\). The curves \(k^\infty\) and \(\tilde{k}\) are depicted in Figure 7.

For the time being, assume that demand is so high that the following holds:

\[ C(k^b_\infty) > r (I_b + I_f) - c. \]  (22)

This condition implies that \(k^\infty\) and \(\tilde{k}\) intersect for some \(k^b > 0\). Define point \(\overline{k}^f\) as the
solution to \(C(\overline{k}^f) = r (I_b + I_f) - c\). Denote by \(\overline{k}_b = k^b_\infty - \overline{k}^f\). Then \(\overline{k} \equiv (\overline{k}_f, \overline{k}_b)\) denotes the
intersection of \(k^\infty\) and \(\tilde{k}\).

Let us first assume that initial capacities are \(k_0 = \overline{k}\) and consider the capacity path that
is implied by a mass of agents behaving according to the myopic behavior. Let us denote by
\(k^m_m(\hat{x}_t; \overline{k}) = (k^f^m(\hat{x}_t; \overline{k}), k^b^m(\hat{x}_t; \overline{k}))\) the resulting capacity path as a function of \(\hat{x}_t\).

Since \(\overline{k}\) is at the intersection of the borders of areas, where myopic firms adjust \(k_f\) and
\(k_b\) at \(x_t = 0\), it must be that as soon as \(\hat{x}_t\) rises even slightly above 0, \(k_b\) increases and \(k_f\)
decreases, and hence \(k_t\) must move towards up-left. Note that the curve \(k^\infty\) represents a
discontinuous threshold for myopic \(k_b\) investors such that whenever \(k_t\) is below that curve, a
myopic firm is strictly willing to invest at any \( x_t \), but when \( k_t \) is above this curve, \( x_t \) must rise above \( x(k^f, k^b) \) in order to induce a myopic agent to invest. When \( k_t \) lies exactly along \( k_\infty \), a myopic \( k_b \) investor is indifferent between investing and waiting as long as \( 0 \leq x_t \leq x(k^f, k^b) \). This means that as long as \( x_t^m(k^f, k^b) < x(k^f, k^b) \) for the current \( (k^f, k^b) \in k_\infty \), every new record setting of \( x_t \) moves \( k_t \) along the curve \( k_\infty \) to up-left. If \( x_t^m(k^f, k^b) < x(k^f, k^b) \) for all \( (k^f, k^b) \in k_\infty \), then \( k^t \) eventually moves to \( (0, k_b^\infty) \) all the way along \( k_\infty \). If, on the other hand, there is some \( (k^f, k^b) \in k_\infty \) such that \( x_t^m(k^f, k^b) = x(k^f, k^b) \), then \( x^t \) reaches such a high level that myopic investors are willing to move \( k^t \) above the curve \( k_\infty \). In that case, \( k^t \) makes an excursion along the intersection of the surfaces \( x_t^m(k^f, k^b) \) and \( x_b^m(k^f, k^b) \) between the curves \( k_\infty \) and \( \tilde{k} \), and finally ends up at \( (0, k_b^\infty) \), as illustrated in Figure 2. To confirm this, we should show that the intersection of \( x_t^m(k^f, k^b) \) and \( x_b^m(k^f, k^b) \) is a continuous curve along which \( k^f \) decreases and \( k^b \) increases continuously as \( \tilde{x}_t \) is increased. It follows from the properties of problems (20) and (21) that between \( k_\infty \) and \( \tilde{k} \) we must have:

\[
\frac{\partial x_b^m(k^f, k^b)}{\partial k^b} > \frac{\partial x_t^m(k^f, k^b)}{\partial k^f} > 0, \\
\frac{\partial x_b^m(k^f, k^b)}{\partial k^f} < \frac{\partial x_t^m(k^f, k^b)}{\partial k^b} < 0.
\]

The desired properties of the intersection of \( x_t^m(k^f, k^b) \) and \( x_b^m(k^f, k^b) \) follow from these relationships and the fact that both \( x_t^m(k^f, k^b) \) and \( x_b^m(k^f, k^b) \) are continuous surfaces between \( k_\infty \) and \( \tilde{k} \). Note that the intersection of those surfaces ends at the point \( (0, k_b^\infty) \), which is reached as soon as \( x^t \) hits the point \( b = x_t^m(0, k_b^\infty) \) for the first time. To see that \( k^t \) must indeed move along this intersection curve, assume that for some reason \( k^t \) departs slightly below it. Then, we know that \( x_b^m(k^f, k^b) < x_t^m(k^f, k^b) \), which means that there will be investments in \( k^b \) without any scrapping of \( k^f \), until the equality of \( x_t^m(k^f, k^b) \) and \( x_b^m(k^f, k^b) \) is restored. A similar reasoning holds if \( k^t \) departs above the intersection curve.

We have now seen that along \( k_m(\tilde{x}_t; k) \), \( k_f \) increases and \( k_b \) decreases at each moment when \( x \) hits a new historic maximum, until \( x^t \) hits a point \( b = x_t^m(0, k_b^\infty) \), at which point
the transition is over. This means that $k_m (\hat{x}_t; \overline{k})$ is an admissible capacity path such that $k_m^f$ is everywhere continuous, strictly decreasing on some interval $(0, b) \subset \mathbb{R}^+$, and constant on $[b, \infty)$ and $k_m^b$ is everywhere continuous, strictly increasing on some interval $(0, b) \subset \mathbb{R}^+$, and constant on $[b, \infty)$.

Let us now establish that $k_m (\hat{x}_t; \overline{k})$ is an equilibrium. Denote by $\Upsilon$ the set of all stopping times at which $x_t$ hits a new record value within interval $(0, b)$. Along $k_m (\hat{x}_t; \overline{k})$, $k_f$ decreases at every $\tau \in \Upsilon$, and hence, the indifference condition (11) holds between any consecutive time points $\tau^*$ and $\tau^{**}$ in $\Upsilon$. Starting from some $\tau^* \in \Upsilon$ and summing (11) over all consecutive time points at which $x_t$ hits new record values until any other $\tau^{***} \in \Upsilon$, we find that

$$E \left[ \int_{\tau^*}^{\tau^{**}} \pi_f (x_\tau; \k_m^f (\hat{x}_i; \overline{k}), \k_m^b (\hat{x}_i; \overline{k})) e^{-r(\tau - t)} d\tau - I_f e^{-r\tau^{**}} \right] = 0,$$

which means that the value of a unit of $k_f$ is $-I_f$ if its owner scraps the unit at any myopically optimal scrapping time. Could the owner increase the value of the unit by choosing a different scrapping time? Assume that the owner scraps at some $\tau'' \notin \Upsilon$. Then the value of the unit can be written as:

$$E \left[ \int_{\tau^*}^{\tau''} \pi_f (x_\tau; \k_m^f (\hat{x}_i; \overline{k}), \k_m^b (\hat{x}_i; \overline{k})) e^{-r(\tau - t)} d\tau - I_f e^{-r\tau^*} \right] + E \left[ \int_{\tau^*}^{\tau''} \pi_f (x_\tau; \k_m^f (\hat{x}_i; \overline{k}), \k_m^b (\hat{x}_i; \overline{k})) e^{-r(\tau - t)} d\tau - I_f e^{-r\tau^*} \right],$$

where $\tau' = \sup \{ \tau \in \Upsilon \mid \tau < \tau'' \}$. Since $\tau' \in \Upsilon$, the first term in (23) gets the value 0, and since $\tau'' \notin \Upsilon$, the second term must be less than 0. Hence, the value of the unit must be $-I_f$ when the owner scraps at any $\tau \in \Upsilon$, and less than $-I_f$ otherwise. This means that $V_f (\hat{x}_t, \hat{x}_t; k_m) = -I_f$, where the value is evaluated at some $\tau \in \Upsilon$ at which $k_m^f$ decreases, and obviously $V_f (x_t, \hat{x}_t; k_m) < -I_f$ whenever $x_t$ is below the historic maximum, that is, whenever $k_m^f$ stays constant. This means that $k_m^f$ satisfies the equilibrium condition given in Definition 2.

Let us check the same for the backstop technology. Along $k_m (\hat{x}_i; \overline{k})$, $k_b$ increases every
time \( x_t \) hits a new record within \((0, b)\), and hence, the indifference condition (9) holds between any consecutive time points \( \tau^* \) and \( \tau^{**} \) at which \( x_t \) hits a new record. Starting from some \( \tau^* \in \Upsilon \) and summing (9) over all consecutive time points at which \( x_t \) hits new record values until \( \tau^b \), we find that

\[
E \left[ \int_{\tau^*}^{\tau^b} \left( P(x_t; k^f_m(\hat{x}_t; \bar{k}), k^b_m(\hat{x}_t; \bar{k})) e^{-r(\tau-t)} - rI_b \right) d\tau \right] = 0.
\]

Since along \( k_m(\hat{x}_t; \bar{k}) \), price will be constant at \( rI_b \) for all \( t > \tau^b \), we may extend the integral to infinity to get:

\[
E \left[ \int_{\tau^*}^{\infty} \left( P(x_t; k^f_m(\hat{x}_t; \bar{k}), k^b_m(\hat{x}_t; \bar{k})) e^{-r(\tau-t)} - rI_b \right) d\tau \right] = 0,
\]

which may as well be written as:

\[
E \left[ \int_{\tau^*}^{\infty} P(x_t; k^f_m(\hat{x}_t; \bar{k}), k^b_m(\hat{x}_t; \bar{k})) e^{-r(\tau-t)} d\tau \right] = I_b,
\]

which means that the value of a unit of \( k_b \) is \( V_b(\hat{x}_t, \hat{x}_t; k_m) = I_b \) if it is built at any \( \tau \in \Upsilon \). Obviously, the value must be \( V_b(x_t, \hat{x}_t; k_m) < I_b \) if such a unit is built at any moment where \( x_t \) is below its historic maximum, that is, whenever \( k^b_m \) stays constant. This means that also \( k^b_m \) satisfies the equilibrium condition given in Definition 2. We have hence shown that \( k_m(\hat{x}_t; \bar{k}) \) is an equilibrium path starting from \( \bar{k} \). Further, there can not be other equilibria starting from \( \bar{k} \), because there are no other paths satisfying myopic optimality, which is a necessary condition for equilibrium.

Theorem 1 is now proved for a specific initial condition \( k_0 = \bar{k} \) and assuming that (22) holds. Relaxing these is easy. Start by taking any initial condition, but still assuming (22). As before, define \( k_m(\hat{x}_t; k_0) \) as the capacity path induced by myopic behavior starting form some \( k_0 \). Consider various possibilities. First, if \( k_0 \) is below \( k_\infty \), then there is an immediate investment in \( k^b \), which moves \( k^t \) directly to \( k_\infty \). From there on, \( k^t \) moves upwards along the surface \( x^m_f(k^f, k^b) \), until it hits \( k_m(\hat{x}_t; \bar{k}) \) from which on it stays on that path. Second,
if \( k_0 \) is to the right of \( \tilde{k} \), then there is immediate scrapping of \( k^f \), which moves \( k^t \) directly to \( \tilde{k} \). From there on, \( k^t \) moves left along the surface \( x_b^m (k^f, k^b) \), until it hits \( k_m (\tilde{x}_t; \tilde{k}) \) from which on it stays on that path (except if \( k_0^b > k_0^\infty \), in which case \( k^t \) only moves horizontally to the left, until the point \((0, k_0^b)\) is reached). If \( k_0 \) is below \( k_\infty \) and to the right of \( \tilde{k} \), then there is immediate simultaneous investment and scrapping, which moves \( k^t \) directly to point \( \tilde{k} \), and thereafter \( k \) stays on path \( k_m (\tilde{x}_t; \tilde{k}) \). Note that \( k_m (\tilde{x}_t; \tilde{k}) \) is a path that essentially describes the capacity adjustment process, because starting from any initial value the adjustment path reaches \( k_m (\tilde{x}_t; \tilde{k}) \) at some point, and coincides with it thereafter (except when \( k_0^b > k_0^\infty \)). Proving that \( k_m (\tilde{x}_t; k_0) \) is an equilibrium is in all these cases similar to proving that for \( k_0 = \bar{k} \). It is also clear that equilibrium is unique in all these cases. Finally, it is straightforward to see that the properties given in Theorem 1 hold for \( k_m (\tilde{x}_t; k_0) \) with all possible \( k_0 \).

Finally, let us relax assumption (22). Note that a model with any given demand function is equivalent to another model with an expanded demand, but initial backstop capacity expanded so that the residual demand with this increased initial \( k_b \) is equal to the full demand of the original model (this transformation is equivalent to shifting the horizontal axis in Figure 2). Hence, any model where (22) does not hold is equivalent to a demand-expanded model such that (22) holds, but with a higher initial \( k_b \). Since the Theorem 1 holds for any initial capacity for the demand-expanded model, it must also hold for the original model where (22) does not hold. ■
Figure 1
Figure 2
Figure 4
Figure 5
Figure 7