Technological Paradigms and the Measurement of Innovation*

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Résumé / Abstract

Cet article propose une définition formelle de la notion de paradigme technologique. Cette définition s'avère compatible avec la proposition fondamentale de l'approche Kuhnienne du développement scientifique, à l'effet que le progrès à long terme des connaissances ne survient que grâce à de brusques changements de paradigme. La présente définition permet aussi de clarifier plusieurs notions, comme celle d'innovation tirée par la demande ou bien poussée par l'offre, d'innovation incrémentale ou bien radicale, ou encore de générations de produits.

This paper proposes a formal definition of the notion of technological paradigm. This definition is consistent with the fundamental proposition of Kuhnian philosophy of science, that progress only happens through successive and abrupt shifts of paradigm. It also helps clarifying a number of other notions, such as demand-pulled vs. supply-driven innovation, incremental vs. radical innovation, and present vs. next-generation product.

Mots Clés : Théorie des treillis, connections de Galois, treillis de concepts, modularité

Keywords: Lattice theory, Galois connections, concept lattices, modularity

JEL: O31, C60

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I wish to thank David Encaoua and Pierre Mohnen for helpful comments on an earlier draft.

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> "Effective research scarcely begins before a scientific community thinks it has acquired firm answers to questions like the following: What are the fundamental entities of which the universe is composed? (...) What questions may legitimately be asked about such entities and what techniques employed in seeking solutions?"
>
> [Thomas S. Kuhn, *The Structure of Scientific Revolutions*, 1962.]

1. Introduction

The economic literature on innovation has developed considerably over the past decade, following significant advances in the understanding of imperfect competition and economic growth. With a few notable exceptions, research has centered - and still does - on the role of market-based incentives. This brought forth a consistent and thorough perspective on the type of economic landscape most likely to favor innovation, one valuable outcome now being a set of concrete propositions regarding, for example, intellectual property and competition policy.

Incentives, however, constitute only a necessary condition for innovation. They provide little insights on the content and process of innovative activity, or on the existence of persistent differences in the volume, scope and quality of innovation across countries or even firms. As already stressed, for instance, by Rosenberg (1976),[1]

> The ultimate incentives are economic in nature; but economic incentives to reduce cost always exist in business operations and precisely because *such incentives are so diffuse and general*, they do not pertain much in terms of the *particular* sequence and timing of innovative activity. [p. 110; emphasis added]

Some economists interested in technological forecasting, R&D policy and the management of innovation have therefore turned to a complementary view of innovation. Their approach often draws on Thomas Kuhn's seminal thesis that scientific progress is mainly driven by *paradigms*, i.e. social constructs made of consensual expectations, conventions, rules and heuristics which characterize professional practice. Technological paradigms are indeed seen to be a key ingredient for the study of technological trajectories (see Dosi (1988) and the references therein). They may also account for success or failure in product development within business firms (see Tabrizi and Walleigh (1997)). Despite these findings and its plausible relevance to research and policy making, however, the notion of technological paradigm remains an elusive one and still lies at the fringe of mainstream economics. The objective of this paper is to improve on the current situation by proposing a first formal definition of technological paradigm.

Our development borrows extensively from *formal concept analysis*, a recently created mathematical field at the crossroads of lattice theory, computer science and data analysis (Ganter and Wille (1999), Wille (2000)). Straightforward interpretation of certain objects of this field fits the intuitive meaning of technological paradigm; it also yields rigorous definitions of commonly used terms such as

[1] To be sure, a similar view is implicit in the more recent work of Aghion and Tirole (1994). They motivate their incomplete-contract approach to the study of the allocation of intellectual property rights by the very fact that the exact nature of innovations cannot be contracted upon ex ante.

incremental and radical innovation, or demand-pulled and supply-pushed innovation. Our definition of technological paradigm allows us to demonstrate Kuhn's celebrated statement that the pursuit of innovation requires occasional shifts of paradigm. It furthermore opens a promising avenue to the construction of operational metrics of innovativeness.

The following section introduces the basics of formal concept analysis and presents the proposed definition of technological paradigm. Section 3 contains the formal statement that technological progress is driven by the "destruction" of old paradigms and their replacement by new ones; the underlying argument makes use of a powerful fixed point theorem from lattice theory which has to our knowledge never been applied in economics. Section 4 deals with the identification of new generations of products. It is seen that a function which identifies and ranks successive generations of products can be constructed from paradigms; furthermore, this function is submodular in the various possible paradigms. Section 5 contains concluding remarks.

2. Technological paradigm - a definition

2.1 Partially-Ordered Sets and Lattices[2]

A partially-ordered set, or *poset*, is a set X with a binary relation $\leq$ which satisfies the following properties: for any three elements x, y, z $\in$ X,

(i) $x \leq x$,

(ii) $x \leq y$ and $y \leq x$ imply $x = y$,

(iii) $x \leq y$ and $y \leq z$ imply $x \leq z$.

Properties (i), (ii), and (iii) are respectively referred to as *reflexivity*, *antisymmetry*, and *transitivity*. The relation $\leq$ is called a *partial order* on the set X. The notation $x < y$ indicates that $x \leq y$ but not $y \leq x$. A subset of X where for any two of its elements x and y, $x \leq y$ or $y \leq x$, is called a totally ordered set or a *chain*.

A set of subsets with the inclusion relation $\subseteq$ is an example of a poset. A representation of posets that can sometimes be useful is the *Hasse diagram* (see Davey and Priestley (1990)). It associates with each member of a poset X a point in the plane, and $x \leq y$ corresponds to an ascending line segment from x to y. The Hasse diagram of the poset X = {x, y, z} with $x \leq y$, for instance, is drawn as Figure 1.

Let X be a poset and Y be a subset of X. An element x of X such that $x \leq y$ ($y \leq x$) for all y $\in$ Y is a *lower (upper) bound* for Y. If x also belongs to Y, then it is the *least (greatest)* element of Y. If x $\in$ Y is such that $y \leq x$ for y $\in$ Y implies that x = y, then x is a *minimal (maximal)* element of Y. Least (greatest) elements are necessarily unique; this is not so for minimal (maximal) elements.

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[2] Unless it is said explicitly, this subsection follows Topkis's (1998; ch. 2) presentation of the main notions of lattice theory.

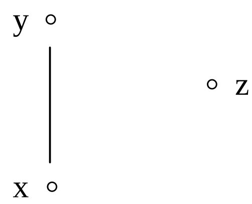

Figure 1. - Hasse diagram of $X = \{x,y,z\}$ with $x \leq y$.

If the set of lower (upper) bounds of Y has a greatest (least) element, this element is called the *greatest lower bound (least upper bound)* of Y, or the *infimum (supremum)*. The infimum (supremum) of any pair $\{x,y\}$, if it exists, is denoted as $x \wedge y$ ($x \vee y$) and referred to as the *meet (join)* of x and y. Similarly, the infimum (supremum) of a subset Y, if it exists, is denoted $\wedge Y$ ($\vee Y$).

A poset L where $x \wedge y$ and $x \vee y$ always exist is called a *lattice*. If in addition $x \leq y$, $z \wedge x = z \wedge y$ and $z \vee x = z \vee y$ always imply $x = y$, then L is *modular*. If $\wedge S$ and $\vee S$ exist for every $S \subseteq L$, then L is a *complete* lattice. Clearly, a complete lattice must have a bottom and a top element, which are respectively noted $\perp$ and $\top$ (To see this, notice that $\emptyset \subseteq L$ and $\perp = \vee\emptyset$, $\top = \wedge\emptyset$.). A poset X that has a bottom element $\perp$ and where $\vee C$ exists for every nonempty chain $C \subseteq X$ is called a *complete partially ordered set, or a CPO*.

Let us now briefly consider functions over posets and lattices. A mapping $\psi:X \rightarrow Y$ from a poset X to a poset Y is *increasing (decreasing)* if $x \leq y$ in X entails $\psi(x) \leq \psi(y)$ ($\psi(y) \leq \psi(x)$) in Y. Two posets X and Y are *isomorphic* when there exists a one-to-one mapping $\psi:X \rightarrow Y$ such that $x \leq y$ in X if and only if $\psi(x) \leq \psi(y)$ in Y for all $x, y \in X$. Finally, a numerical function $f:L \rightarrow \mathbb{R}$ over a lattice L is supermodular (submodular) if $f(x) + f(y) \leq (\geq) f(x \wedge y) + f(x \vee y)$ for all x, y in L.

A *fixed point* of a mapping $\psi:X \rightarrow X$ is an element x of X such that $\psi(x) = x$. The following theorem, which concludes this subsection, asserts the existence of fixed points under relatively mild assumptions. A proof can be found in Davey and Priestly (1990).

THEOREM 1: *Let the set* X *be a CPO. If the mapping* $\psi:X \rightarrow X$ *is such that* $x \leq \psi(x)$ *for all* x *in* X*, then* $\psi(\cdot)$ *has a fixed point.*

2.2 Concept Lattices[3]

From now on we shall focus on a peculiar type of lattices that is currently finding a lot of applications in computer science and data analysis. The study of concept lattices is now regarded as a new branch of applied lattice theory. The theory of concept lattices developed as an alternative to

[3] This subsection borrows without restraints from Ganter and Wille (1999).

treelike structures was increasingly needed, in response to growing demands to classify data whilst keeping as much information as possible. The appeal of concept lattices partly lies in their fulfilment of the general precept that there are two ways to characterize a given notion: one consists in directly stating some key property or feature, the other is to illustrate it with a proper list of examples.

Formally, let us first define a *context* as a triple K = (Z,Y;R), where Z is interpreted as a set of *objects*, Y as a set of *attributes*, and R is a relation between Z and Y such that oRa means that object o possesses attribute a. Subset of Z and Y are respectively denoted by the capital letters O and A. Now, a *formal concept* in the context K is now a pair (I,J) such that:

(i) $I = \{o \in Z \mid oRa \text{ for all } a \in J\}$,

(ii) $J = \{a \in Y \mid oRa \text{ for all } o \in I\}$.

Take the set $\mathbb{C}(K)$ of all concepts in the context K. Two concepts c = (I,J) and c′ = (I′,J′) are such that $c \leq c'$ if and only if $I \subseteq I'$ (in which case it can easily be checked that $J' \subseteq J$). The set $\mathbb{C}(K)$ ordered in this fashion is called a *concept lattice*.

Concept lattices can be plotted as Hasse diagrams, as in Figure 2 (from Mephu Nguifo (1993)). Alternatively, let the associated context coincide with the one depicted in table 1, then the boxed area, which constitutes a "maximal rectangle" of the table, corresponds to the concept (1234,abcd). There are several algorithms for retrieving, browsing through, or drawing concept lattices; a presentation of these is beyond the scope of this paper (but see Ganter and Wille (1999; chapters 2,4,5) and Guénoche (1990)). Moreover, the "Fundamental Theorem of Concept lattices" (see Ganter and Wille (1999; chapter 1) - a precise statement of which would require additional definitions and bring us too far away from our present topic - gives necessary and sufficient conditions for a complete lattice to be isomorphic to, hence transformable into, a concept lattice.

	Y	a	b	c	d	e	f	g	h
Z									
1		×	×	×	×	×	×	×	
2		×	×	×	×	×	×		×
3		×	×	×	×	×		×	×
4		×	×	×	×		×		
5		×	×		×	×		×	
6		×	×	×		×			×
7		×		×			×		

Table 1. – Table representation of a context.

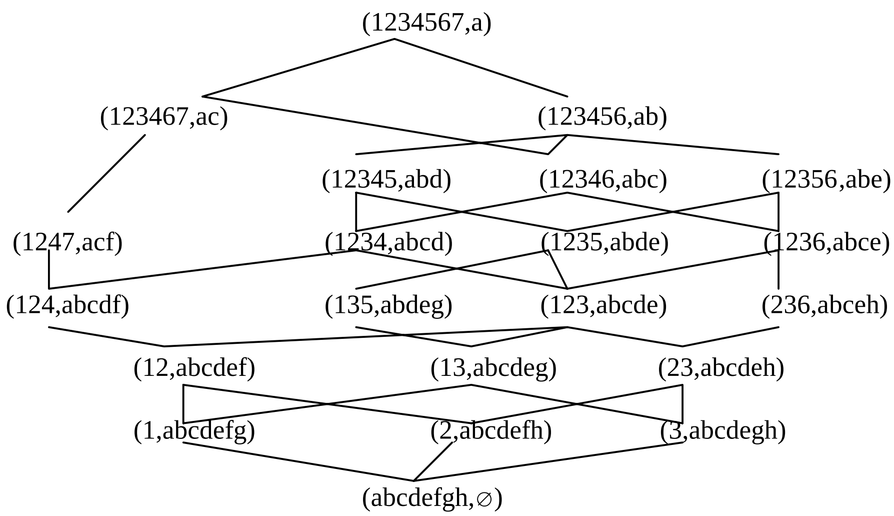

Figure 2. - Hasse diagram for the concept lattice of Table 1, with objects $Z = \{1,2,3,4,5,6,7\}$ and attributes $Y = \{a,b,c,d,e,f,g,h\}$

Consider finally two transformations $\alpha:Z\to Y$, $\beta:Y\to Z$ satisfying the following properties:

(i) $O_1 \subseteq O_2$ entails that $\alpha(O_2) \subseteq \alpha(O_1)$,

(ii) $A_1 \subseteq A_2$ entails that $\beta(A_2) \subseteq \beta(A_1)$,

(iii) $O \subseteq \beta\circ\alpha(O)$ and $A \subseteq \alpha\circ\beta(A)$.

Such a pair is a well-known and useful mathematical object that is called a *Galois connection* (see Ore (1944), Everett (1944), Guenoche (1990)). It is easy to get a relation between objects and attributes using a Galois connection (and vice-versa): simply define

$$R_{(\alpha,\beta)} = \{(o,a) \in Z\times Y \mid o = \beta(a)\} = \{(o,a) \in Z\times Y \mid a = \alpha(o)\} \quad .$$

Hence, one might as well write a context as the triplet $K = [Z,Y;(\alpha,\beta)]$ where $\alpha(\cdot)$ and $\beta(\cdot)$ are Galois connections.

2.3 Transformations

Let us now return to the main topic of this paper. In the present framework, one can think intuitively of innovation as happening in two major ways. Innovation may first consist in finding out new attributes for a given object; it may also be the discovery that another object carries some desired attributes. Whatever the type of innovation that is being considered, however, it usually happens along some inference rule or pattern. This rule corresponds to a *paradigm*, which we now formally define as follows.

DEFINITION: *A paradigm is a context* K = [X,V;(α,β)], *where* X *contains some specific objects,* V *is a set of valuable attributes those objects may have, and* (α,β) *is a Galois connection on* X *and* V.

To fix ideas, it may help to be given an example of a concrete context. A good illustration would be the rapidly expanding area of nanoscale technologies. An object here would be, for instance, a quantum wave switch or a nanocircuit (for more examples of this sort, see Meyer (2000)). Valuable attributes or properties of these objects are information transmission (quantified) and information storage (also quantified) respectively. And the pair (α,β) captures the past and present research seeking relationships between nanoscale objects and nanoscale attributes. The mapping $\alpha:X\rightarrow V$, which corresponds to existing and new applications of nanoscacle objects, should in fact be seen as a formal representation of *supply-driven* innovation. Similarly, the mapping $\beta:Y\rightarrow Z$, which takes desirable attributes on to existing or newly discovered objects that deliver them, might be seen as representing *demand-pulled* innovation.

The pair (α,β) in the above definition captures the intuitive notion of *incremental innovation*. Its counterpart – *radical innovation* – would rather enlarge the set of objects, the set of attributes, and/or the set of relationships between objects and attributes. To formalize this, let $\mathfrak{C}$ be the set of all contexts with number of objects and attributes lower than some (large) integer N.[4] We define a partial order on $\mathfrak{C}$ by saying that two paradigms K = [X,V;(α,β)] and K′ = [X′,V′;(α′,β′)) are such that $K \leq K'$whenever

(i) $X \subseteq X'$ and $V \subseteq V'$,

(ii) for all $O \subseteq X$, $\alpha(O) \subseteq \alpha'(O)$,

(iii) for all $A \subseteq V$, $\beta(A) \subseteq \beta'(A)$.

The set $\mathfrak{C}$ with this partial order is again a complete lattice. *Radical innovation* can now be identified with a mapping $\Psi:\mathfrak{C}\rightarrow\mathfrak{C}$ such that, for all K in $\mathfrak{C}$, $\Psi(K) \geq K$.

3. The necessity scientific revolutions

In the present framwork, successive waves of innovation can be captured by repeatedly applying the mappings defined in the preceding section. It is intuitive that such a repetition might produce decreasing returns. Kuhn's thesis goes one step further in asserting that scientific progress would actually stall unless society periodically wipes out some existing knowledge and switches to a

[4] Computing the number of contexts with n objects and m attributes is a nontrivial task. The following table, however, can be found in Ganter and Wille (1999; p. 59).

n \ m	**1**	**2**	**3**	**4**	**5**
1	2	3	4	5	6
2	3	7	13	22	34
3	4	13	36	87	190
4	5	22	87	317	1053

radically new paradigm. This statement can be formally made as follows.

THEOREM 2: (i) *The composite mappings* $\alpha\circ\beta$ *and* $\beta\circ\alpha$ *are such that* $\alpha\circ\beta\circ\alpha\circ\beta = \alpha\circ\beta$ *and* $\beta\circ\alpha\circ\beta\circ\alpha = \beta\circ\alpha$.. (ii) *The sequence* $\Psi(K)$, $\Psi\circ\Psi(K)$, ... , $\Psi\circ...\circ\Psi(K)$ *converges to a fixed point of the mapping* $\Psi(\cdot)$.

The second part of this theorem is a straightforward consequence of theorem 1. The first part is a well-known property of Galois connections (see Everett (1944)) and can be easily checked. The compositions $\alpha\circ\beta$ and $\beta\circ\alpha$ actually bear the evocative name of *closures*, which suggests that by applying them successively one indeed "closes up" on the underlying relation R and the corresponding lattice of concepts. The upshot of this theorem is that the long-run pursuit of technological innovation entails the episodic deletion of some knowledge.[5]

4. Innovation metrics

One way to assess the extent of innovation is to measure the "distance" between existing and new products. A crude measure of such a distance might be the notion of product generation: intuitively, an innovation is more significant if it yields higher-generation products. This notion turns out to be formalizable in the present framework.

Consider two Galois connections (α_1,β_1) and (α_2,β_2) on the sets of objects and attributes X and V. Let us write $(\alpha_1,\beta_1) \leq (\alpha_2,\beta_2)$ if the concept lattice generated by (α_1,β_1) is a sublattice of the concept lattice generated by (α_2,β_2). If X and V are finite sets, the set of Galois connections on Z and Y endowed with this partial order forms a complete lattice, noted G. One can now get the following result establishing the existence of a (submodular) ranking function on the pairs (α,β). This function acts as a generation label. The result itself is a straightforward consequence of theorem 42 in Ganter and Wille (1999, p. 226).

THEOREM 3: If the lattice G of Galois connections is modular, then there exists a ranking function r assigning a natural number to each member of G such that $r(\alpha_1,\beta_1) \leq r(\alpha_2,\beta_2)$ whenever $(\alpha_1,\beta_1) \leq (\alpha_2,\beta_2)$. Moreover, for all (α_1,β_1) and (α_2,β_2), we have that

$$r(\alpha_1,\beta_1) + r(\alpha_2,\beta_2) \geq r((\alpha_1,\beta_1)\wedge(\alpha_2,\beta_2)) + r((\alpha_1,\beta_1)\vee(\alpha_2,\beta_2)) \ .$$

5 It would not stretch the meaning of this proposition too far if we said that it reinforces the commonly held belief that continuous economic growth results from a process of *creative destruction*.

5. Conclusion

This paper introduced a formal definition of the notion of technological paradigm, based on lattice theory and formal concept analysis. The definition is theoretically appealing, for it captures some fundamental ideas of the philosophy of science such as Thomas Kuhn's thesis that scientific progress is paradigm-dependent. It also help clarifying several key notions such as demand-pulled and supply-driven innovation, and incremental and radical innovation. It might also yield practical methods for the measurement of innovative activity, a first one being a rigorous method for identifying successive generations of products.

At this point the paper presents a rather high ratio of definitions over results. Its contribution might be to have brought up and offered some grasp at an important yet elusive notion, thereby opening up a challenging research agenda.

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