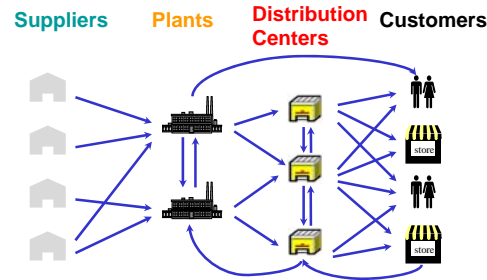


# Location Problems in Supply Chain Management

Stefan Nickel

Institute for Operations Research  
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## Location Problems in SCM

### Contents

- A Supply Chain Design Problem with facility location, production and transportation activities
- The real problem: Multi-Period Relocation Problems
- Strategic Supply Chain Model from Practice
- Heuristic Solution Procedures
- (Applications and Software)

## Location Problems in SCM

### Contents

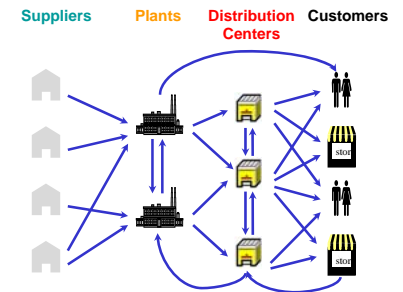
A Supply Chain Design Problem with facility location, production and transportation activities

- Strategic planning
- MIP formulation
- Numerical results

## Location Problems in SCM

### Strategic Planning of Supply Chains

- **procurement:**  
set of suppliers to select & amount of raw materials to acquire?
- **production:**  
which products to manufacture, where and in which amount?
- **facility location:**  
number, location, capacity of **new service facilities** to open?  
Number, location of **existing service facilities** to keep?
- **transportation:**  
which transportation channels to use to ship products between facilities?
- **customer allocation:**  
which customers to serve by which facility?



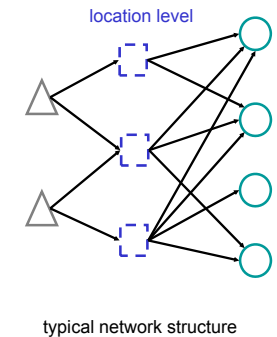
## Characteristics of Strategic Planning

- Highly aggregated data
- High uncertainty in data or even no data
- Decisions involve the upper management
- Specialized and well trained users
- Running time of algorithms is non-critical
- No classical fixed costs
- Operational decisions (like „make or buy“) should not be modeled

## Facility Location Models in SCM

### Typical features found in the literature

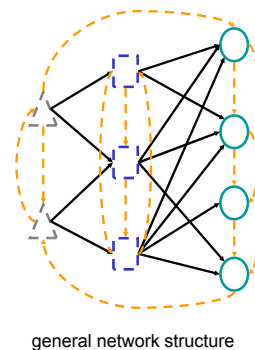
- Specific network: categorization of facilities into levels, usually maximum 3 levels
- product flow from one level to the next (e.g. plants → DCs → customers)
- strategic decisions focus on facility location and allocation
- facility location restricted to one or two levels (e.g. locate new DCs; locate new plants and DCs)
- demand occurs in the lowest level of the network



## Facility Location Models in SCM

### Practical needs

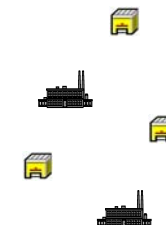
- general network: no echelon structure required, no restriction on the type of facilities
- product flow allowed between any type of facility and in any direction (e.g. Inter-depot transportation, recycling)
- additional strategic planning possible, e.g. production, procurement
- no restrictions on the type of facilities to open/close
- demand can occur in any type of facility



## Supply Chain Networks

### Practical needs

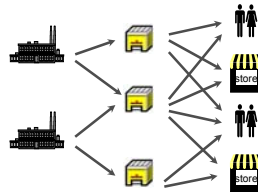
- Different types of facilities



## Supply Chain Networks

### Practical needs

- Different types of facilities
- Multiple facility layers



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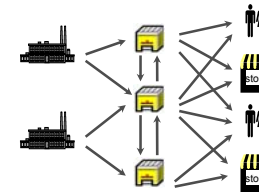
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## Supply Chain Networks

### Practical needs

- Different types of facilities
- Multiple facility layers
- Flow between facilities in the same layer



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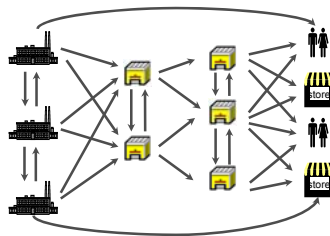
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## Supply Chain Networks

### Practical needs

- Different types of facilities
- Multiple facility layers
- Flow between facilities in the same layer
- Direct shipments



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## Location Problems in SCM

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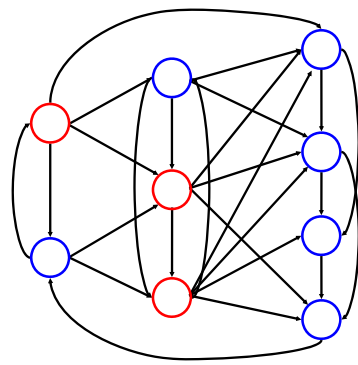
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## A General Supply Chain Design Model



○ non-selectable facility

○ selectable facility

decision: operate a **selectable facility** (y/n)?

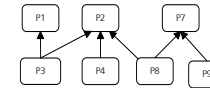
| selectable facilities | fixed costs |         |
|-----------------------|-------------|---------|
|                       | closing     | opening |
| existing              | X           | -       |
| potential             | -           | X       |

## A General Supply Chain Design Model

### Activities

■ **procurement**

■ **production**



← **modelling production processes**

■ **transportation**

■ **facility operation**

■ **capacity expansion**

■ **unsatisfied demand**

### Decision Variables (product $p$ , facility $i$ )

$b_{i,p}$  ≡ amount of  $p$  bought by facility  $i$

$h_{i,p}$  ≡ amount of  $p$  produced by facility  $i$

$v_{i,j,p}$  ≡ amount of  $p$  shipped from facility  $i$  to facility  $j$

$\delta_i \equiv \begin{cases} 1 & \text{if selectable facility } i \text{ operates} \\ 0 & \text{otherwise} \end{cases}$

$x_r, y_r$  ≡ amount of additional production /handling resource  $r$  used

$z_{i,p}$  ≡ amount of  $p$  not delivered to facility  $i$

## Capacity Modelling

resource consumption in

- production, e.g. equipment, number of working shifts ( $\mu_{i,r,p}$ )
- handling of goods, e.g. pallet loader for incoming/outgoing goods in a facility ( $\bar{\lambda}_{i,r,p}, \tilde{\lambda}_{i,r,p}$ )

resource – facility combination

- one – to – many → same resource used in several facilities
- one – to – one → same resource used by all products in a facility
- many – to – one → different resources used by several products in a facility

resources have limited availability

resource extension, e.g. through overtime work

→ additional costs charged

## A Mixed Integer Program

### Objective function

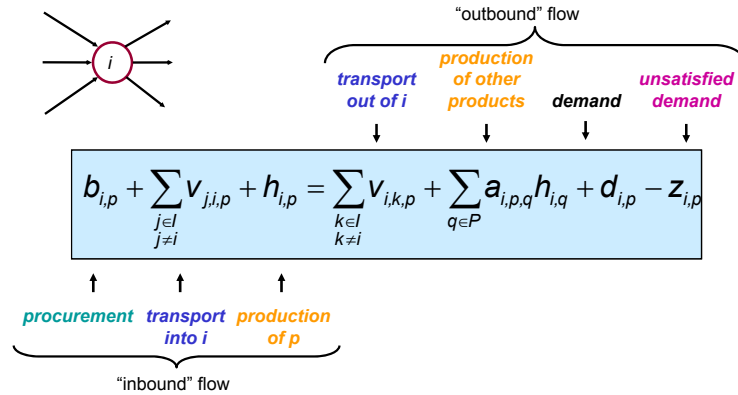
$$\begin{aligned}
 \min \quad & \sum_{i \in I} \sum_{p \in P} BC_{i,p} b_{i,p} + \sum_{i \in I} \sum_{p \in P} HC_{i,p} h_{i,p} \\
 & + \sum_{i \in I} \sum_{j \in I, j \neq i} \sum_{p \in P} TC_{i,j,p} v_{i,j,p} \\
 & + \sum_{r \in R^p} EPC_r x_r + \sum_{r \in R^h} EHC_r y_r \\
 & + \sum_{i \in I} \sum_{p \in P} PDC_{i,p} z_{i,p} \\
 & + \sum_{i \in S} OC_i \delta_i + \sum_{i \in S} CC_i (1 - \delta_i)
 \end{aligned}$$

procurement costs (BC)      production costs (HC)  
 transportation costs (TC)  
 capacity expansion costs (EPC, EHC)  
 penalty demand costs (PDC)  
 opening costs new facilities (OC)      closing costs existing facilities (CC)

## A Mixed Integer Program

### Constraints I

- general flow conservation for every product  $p$ , in every facility  $i$



## A Mixed Integer Program

### Constraints II

- resource consumption and capacity expansion

$$\begin{aligned}
 &\sum_{i \in I} \sum_{p \in P} \mu_{i,r,p} h_{i,p} \leq PK_r + x_r, & r \in R^p \\
 &0 \leq x_r \leq \overline{PK}_r, & r \in R^p \\
 &\sum_{i \in I} \sum_{j \in I, j \neq i} \sum_{p \in P} (\bar{\lambda}_{i,r,p} + \bar{\lambda}_{j,r,p}) v_{i,j,p} + \sum_{i \in I} \sum_{p \in P} \bar{\lambda}_{i,r,p} b_{i,p} \leq HK_r + y_r, & r \in R^h \\
 &0 \leq y_r \leq \overline{HK}_r, & r \in R^h
 \end{aligned}$$

production resources → available capacity

handling resources →

## A Mixed Integer Program

### Constraints III

- activities in **selectable facilities**

$$\begin{aligned}
 b_{i,p} &\leq M_{i,p} \delta_i, & i \in S, p \in P \\
 h_{i,p} &\leq M_{i,p} \delta_i, & i \in S, p \in P \\
 v_{i,j,p} &\leq M_{i,j,p} \delta_i, & i \in S, j \in I (j \neq i), p \in P \\
 v_{i,j,p} &\leq M_{i,j,p} \delta_j, & i \in I \setminus S, j \in S, p \in P
 \end{aligned}$$

procurement  
production  
transportation **out of**  
a selectable facility  $i$   
transportation **into** a  
selectable facility  $j$

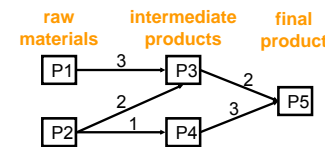
( $M$ : upper bound depending on material requirements & resource availability)

- integrality and non-negativity conditions

$$\begin{aligned}
 b_{i,p} &\geq 0, h_{i,p} \geq 0, 0 \leq z_{i,p} \leq d_{i,p} & i \in I, p \in P \\
 v_{i,j,p} &\geq 0 & i, j \in I, (i \neq j), p \in P \\
 \delta_i &\in \{0, 1\} & i \in S
 \end{aligned}$$

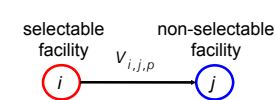
## Choosing suitable upper bounds $M$

### Taking material requirements into account



$MD_p \equiv \text{max. amount of product } p \text{ required in the network}$

### Taking resource availability in each **selectable facility** into account



$$v_{i,j,p} \leq M_{i,j,p} \delta_i$$

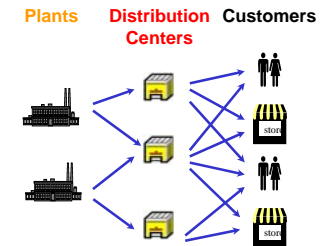
$$M_{i,j,p} = \min \left\{ MD_p, \min_r \frac{\text{capacity } r \text{ in } i}{\text{consumption } r}, \min_r \frac{\text{capacity } r \text{ in } j}{\text{consumption } r} \right\}$$

## Contents

A Supply Chain Design Problem with facility location, production and transportation activities

- Strategic planning
- MIP formulation
- Numerical results

- 72 test problems randomly generated
- structure of distribution network
  - 50 – 200 customers
  - 5, 10, 15 product types
  - one-level: 30 selectable DCs (10 existing + 20 potential)
  - two-level: 5 plants, 30 selectable DCs (10 existing + 20 potential)
  - dense transportation network (70-80%)
  - all costs randomly generated (production, transportation, capacity expansion) (uniform dist.)
  - incapacitated & capacitated instances



# Numerical Results

| problem type |         | #var    | #const  | root LP gap(%) | CPU (sec.) | root feas. sol gap(%) | CPU (sec.) | #B&B nodes | total CPU (sec.) |
|--------------|---------|---------|---------|----------------|------------|-----------------------|------------|------------|------------------|
| uncap.       | min.    | 2439.0  | 2542.0  | 0.0            | 0.0        | 0.0                   | 0.0        | 0.0        | 2.0              |
|              | average | 14088.3 | 14528.8 | 4.2            | 14.7       | 5.3                   | 49.2       | 30.7       | 139.6            |
|              | max.    | 29345.0 | 30414.0 | 16.4           | 15.0       | 16.0                  | 573.2      | 68.0       | 149.2            |
| large cap.   | min.    | 2598.0  | 2578.0  | 0.0            | 0.0        | 0.0                   | 0.0        | 0.0        | 4.5              |
|              | average | 13268.3 | 13430.4 | 8.4            | 54.4       | 2.0                   | 279.8      | 207.2      | 3818.0           |
|              | max.    | 30909.0 | 31470.0 | 20.6           | 264.5      | 12.3                  | 1546.3     | 1141.0     | 34030.0          |
| medium cap.  | min.    | 2598.0  | 2579.0  | 0.0            | 0.9        | 0.0                   | 4.7        | 0.0        | 7.4              |
|              | average | 13268.4 | 13430.7 | 12.6           | 40.1       | 3.7                   | 226.0      | 1784.8     | 8187.2           |
|              | max.    | 30909.0 | 31471.0 | 44.2           | 196.5      | 31.6                  | 1276.6     | 18578.0    | 59796.3          |

Problems modeled with ILOG Concert (C++ interface) and solved optimally with CPLEX® 7.5 in a Pentium III, 850 MHz, 1 GB RAM

# Location Problems in SCM

## Contents

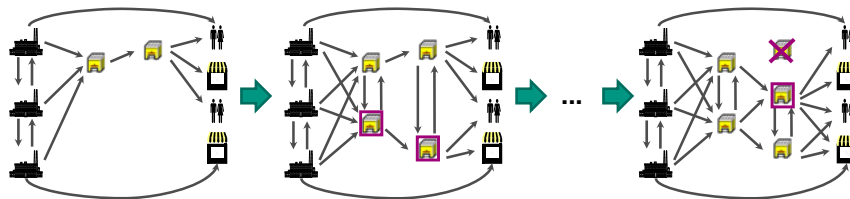
- A Supply Chain Design Problem with facility location, production and transportation activities
- The real problem: Multi-Period Relocation Problems
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- Heuristic Solution Procedures
- (Applications and Software)

## Supply Chain Networks

### Practical needs

- Different types of facilities
- Multiple facility layers
- Flow between facilities in the same layer
- Direct shipments
- Network redesign  
→ Multi-period process

Flexibility!



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## Dynamic relocation of facilities

### Motivation

Melo, Nickel,  
Saldanha da Gama,  
2006, COR

#### Facility location

- Long-term project
- Robust facilities → cope with current and future system conditions
- Large investment capital → financial strain on the company
- Time-consuming activities → new infrastructure, equipment supply, employee training, ...
- Gradual setup of new facilities and phase-out of existing facilities
- Coordination of all operational aspects → supply chain activities cannot be disrupted
- Practical situation: relocation of production facilities, new target markets

➔ gradual location / relocation of facilities through capacity shifts

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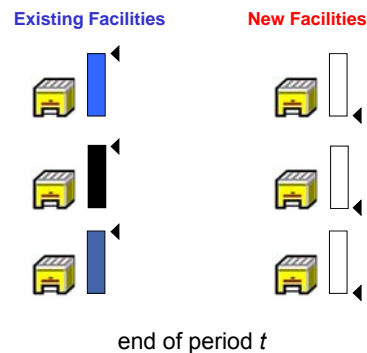
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## Model features

### Facility relocation through capacity shifts

(Gradual) capacity transfers from existing to new facilities during the planning horizon



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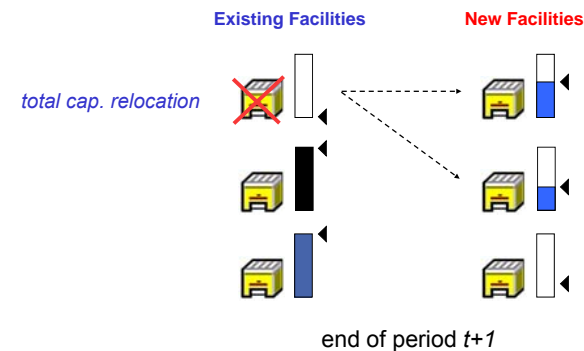
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## Model features

(Gradual) capacity transfers from existing to new facilities during the planning horizon



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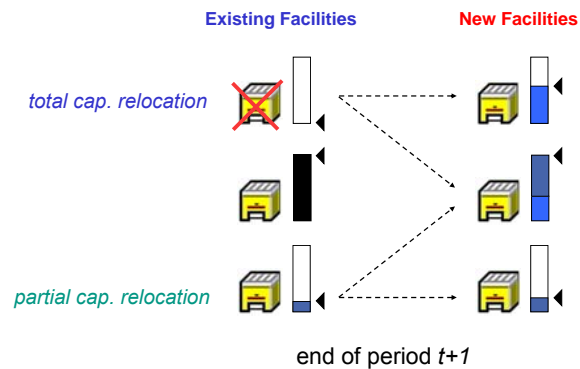
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## Model features

(Gradual) capacity transfers from existing to new facilities during the planning horizon



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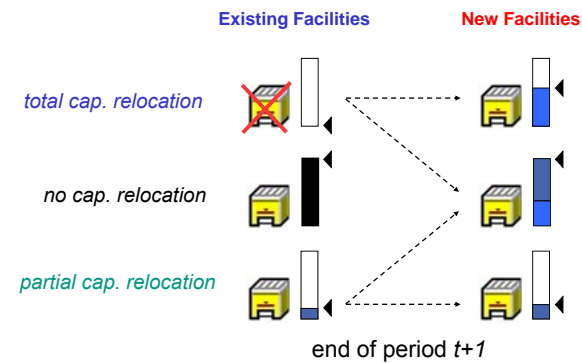
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## Model features

(Gradual) capacity transfers from existing to new facilities during the planning horizon



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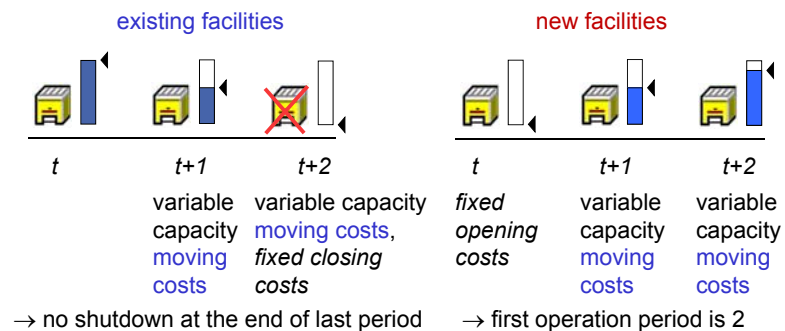
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## Model features

### Financial budget

Availability of investment capital in each period for opening new facilities, moving capacity and closing existing facilities



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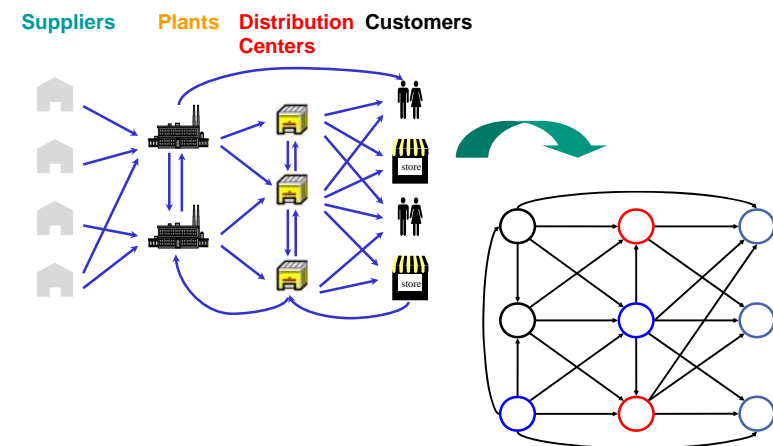
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## Model features

### General Supply Chain Structure



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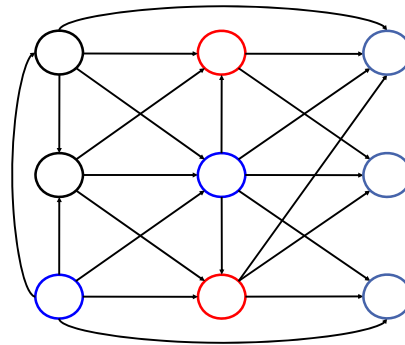


## Model features

not restricted to an echelon structure

shipping lanes in any direction, e.g. inter-facility transportation, reverse logistics

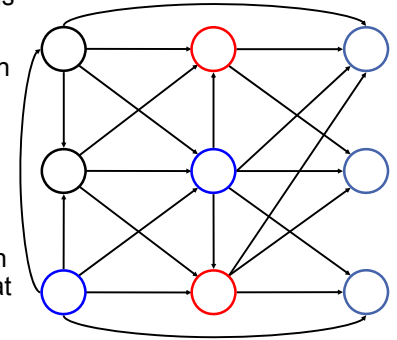
-  fixed facility, e.g. plant
-  fixed facility, e.g. customer
-  existing facility
-  new facility
-  capacity transfers
-  commodity flow



## Model features

### Additional Aspects

- Available budget per period:  
use non-invested capital in previous periods subject to interest rate
- External supply of products through procurement / production (supply nodes)
- Facilities with limited capacity
- Min. throughput at facilities
- Inventory opportunities for goods in each period (availability of stocks at the beginning of the planning horizon)



## Dynamic relocation of facilities

### Decisions

- **Supply**  
Amount of product  $p$  purchased by facility  $i$  in period  $t$  ?
- **Transportation**  
Amount of product  $p$  shipped from facility  $i$  to facility  $j$  in period  $t$  ?
- **Inventory**  
Amount of product  $p$  held in stock in facility  $i$  at the end of period  $t$  ?
- **Facility operation**  
Should facility  $i$  be operated in period  $t$  ?
- **Relocation**  
Amount of capacity to shift from the existing facility  $i$  to a new facility  $j$  at the beginning of period  $t$  ?
- **Capital investment**  
Capital available at the end of period  $t$  ?

## Dynamic relocation of facilities

### Model

- **Objective function**  
Minimize total costs
  - external supply, transportation, inventory holding costs
  - fixed facility operating costs
- **Constraints**
  - general conservation of flow per facility, product, period
  - feasible capacity transfers from existing to new facilities
  - capacity limits per facility, period
  - min. throughput per facility, period
  - open / close a facility at most once during planning horizon
  - budget limitations for investments on opening & closing facilities and capacity shifts per period
- Large MIP model
- NP-hard: reduction to dynamic model of Van Roy & Erlenkotter (MS, 1982)

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The **facilities modeled** in the supply chain do not necessarily have to be part of the **own organization**. E.g. external supplier.

### Notation

- $L$  Set of all **facilities**
  - $S$  Set of all **selectable** facilities
  - $S^o$  Set of all **selectable** facilities, which could be **opened**
  - $S^c$  Set of all **selectable** facilities, which could be **closed**
- $\Rightarrow S = S^o \cup S^c$ .

$\mathcal{P} = \{1, \dots, P\}$  Set of all products

$\mathcal{T} = \{1, \dots, T\}$  Set of all time periods

### Location decisions

#### Open

$$\ell \in S^o, t \in \mathcal{T} \quad y_\ell^t := \begin{cases} 1 & \text{if facility } \ell \text{ is opened} \\ & \text{at the beginning of period } t \\ 0 & \text{otherwise} \end{cases}$$

#### Close

$$\ell \in S^c, t \in \mathcal{T} \setminus \{T\} \quad y_\ell^t := \begin{cases} 1 & \text{if facility } \ell \text{ is closed} \\ & \text{at the end of period } t \\ 0 & \text{otherwise} \end{cases}$$

$$\ell \in S^c \quad y_\ell^T := \begin{cases} 1 & \text{if facility } \ell \text{ is not closed} \\ 0 & \text{otherwise} \end{cases}$$

$OC_\ell^t$  **Opening Costs** for the facility  $\ell \in S^o$  at the beginning of period  $t$  and for its **operation** for the rest of the planning period

$CC_\ell^t$  **Costs** for **closure** of the facility  $\ell \in S^c$  at the end of period  $t$  and their **operation** until then.

### Demand

- Facilities can have **demand** for products.
- **Forecast** future demands.
- If the forecasts are not exact enough, then consider the problem **several times** for **different scenarios** of **demand trends**
  - pessimistic (worst-case)
  - normal (average-case)
  - optimistic
  - ...
- Notation

$D_{\ell,p}^t$  **Demand** in quantity units for product  $p$  at facility  $\ell$  in period  $t$ .

## Strategic Supply Chain Model from Practice

### Satisfaction of customer demands

It may be that the **demand** can/shall **not** be (completely) **satisfied**.

Example:

- **Costs** for the **satisfaction** of demand is too high (compared to profit)
- Supply within the given **service time** is not possible, or just with very high efforts
- **Capacities** are not sufficient

Unsatisfied demand incurs **penalty costs**.

However, they are **difficult to quantify**.

**Possibility:** lost profits, service level which has to be satisfied

$z_{\ell,p}^t$  Number of quantity units of **demand** at facility  $\ell$  for product  $p$  in period  $t$ , which were **not delivered**.

$PDC_{\ell,p}^t$  **Penalty costs** per quantity unit of product  $p$ , which were **not delivered** to facility  $\ell$  in period  $t$  to **satisfy the demand**.

## Strategic Supply Chain Model from Practice

### Procurement

Facilities can **buy** products from „external“, i.e. from **external suppliers**.

Example:

- **raw material** or **semi-finished goods**, which **cannot be produced at their own facilities**
- **products**, that are **cheaper to buy** than to produce them (Out-Sourcing)

Notation

$b_{\ell,p}^t$  Amount of product  $p$ , which is **procured** at facility  $\ell$  in period  $t$ .

$BC_{\ell,p}^t$  **Costs** for **procurement** of one unit of product  $p$  at facility  $\ell$  in period  $t$ .

## Strategic Supply Chain Model from Practice

### Production

Manufacturing of **finished goods** from different **inputs**.

Example:

- „classical“ **manufacturing** of finished goods in factories from raw material and intermediate goods
- **packaging** or **picking** products in distribution centers. E.g. drill machine from factory A with boring head from factory B packed together.

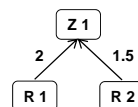
Manufacturing processes are specified by **bills of materials**.

Example:

Intermediate product **Z1** is manufactured by raw material **R1** and **R2**.

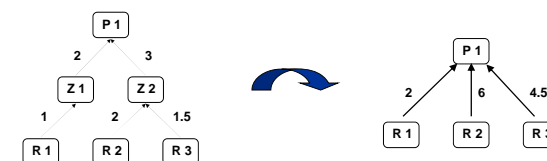
The **numbers** on the **arcs** indicate the related material consumption factors.

The production of one unit of **Z1** needs **2** and **1.5** units of raw materials **Z1** and **Z2**, respectively



## Strategic Supply Chain Model from Practice

Simplify multi-stage lists of material to single-stage ones.



Notation

$a_{\ell,p,q}$  Number of units of product  $q$ , needed to **manufacture** one unit of product  $p$  in facility  $\ell$ .

$h_{\ell,p}^t$  Amount of product  $p$ , **produced** in facility  $\ell$  in period  $t$ .

$HC_{\ell,p}^t$  **Costs** for manufacturing one unit of product  $p$  in facility  $\ell$  in period  $t$ . Includes costs for material, machine utilization, ....

### Storage

Products (raw material, intermediate products, finished goods) can be **stored** in facilities from **one period to the next**.

#### Notation

$inv_{\ell,p}^t$  Amount of product  $p$  stored at facility  $\ell$  in period  $t$ .

$IC_{\ell,p}^t$  Costs for storing one unit of product  $p$  at facility  $\ell$  in period  $t$ .  
Include costs for inventory, stock ground, ...

### Distribution

Transportation links **between all facilities** possible.

#### Notation

$x_{\ell,\ell',p}^t$  Amount of product  $p$  transported from facility  $\ell$  to  $\ell'$  in period  $t$ .

$TC_{\ell,\ell',p}^t$  Transportation costs for one unit of product  $p$  from facility  $\ell$  to  $\ell'$  in period  $t$ .

- The **transportation costs** depend on the **distance**, but also on the **product** and the **means of transportation**.
- Include often **costs** for **goods issue** (e.g. order picking, shipment) at the starting facility and for **incoming goods** (warehousing) at the destination facility.
- Sometimes **costs** for **storage** (within a period) at the starting location, too.

### Capacities

Displayed via **resources**.

Example:

- machine, stockyard
- storage, order picking system
- staff, shift

### Resources characterized by

- **Base capacity** (e.g. production capacity of a machine, maximal throughput of the picking system per period).
- **Consumption factor** states for each product the consumption of resources in resource units per quantity unit of a product.
- **Expansible capacity** of the resource (e.g. overtime, leasable storage or production capacity).
- **Penalty costs** per unit, that extend (overload) the base capacity.

### Relations between facilities and resources

- **one – to – many**
  - The same resource can be used on several facilities.
  - Example: executive producer, which is responsible for several production lines
- **one – to – one**
  - The same resource is used by all products of one facility.
  - Example: flexible configurable machine
- **many – to – one**
  - Several resources attached in the same facility.
  - Example: facilities correspond to production lines and resources to executive producers

### Consider resources for

- **production** and
- **incoming goods** and **goods issue** (handling)

## Notation

- $R^p$  Set of production resources
- $R^h$  Set of handling resources
- $\mu_{\ell,r,p}$  Consumption factor of production resource  $r \in R^p$  per unit of product  $p$  at facility  $\ell$ .
- $\lambda_{\ell,r,p}^i$  Consumption factor of production resource  $r \in R^h$  per unit of product  $p$  at goods receipt respectively issue at facility  $\ell$ .
- $\lambda_{\ell,r,p}^o$
- $v_r^t$  Number of units, the resource  $r \in R^p \cup R^h$  has been extended in period  $t$ .
- $RK_r^t$  Base capacity of resource  $r \in R^p \cup R^h$  in period  $t$ .
- $ERK_r^t$  Maximally allowed extension of the capacity of the resource  $r \in R^p \cup R^h$  in period  $t$ .
- $RC_r^t$  Penalty costs per extended resource unit of resource  $r \in R^p \cup R^h$  in period  $t$ .

## Mixed integer linear program

### Objective function

$$\begin{aligned} \min \quad & \sum_{t \in T} \sum_{\ell \in L} \sum_{p \in P} (BC_{\ell,p}^t b_{\ell,p}^t + HC_{\ell,p}^t h_{\ell,p}^t) + && \text{procurement and production} \\ & \sum_{t \in T} \sum_{\ell, \ell' \in L} \sum_{p \in P} TC_{\ell,\ell',p}^t x_{\ell,\ell',p}^t + && \text{distribution} \\ & \sum_{t \in T} \sum_{r \in R^h \cup R^p} RC_r^t v_r^t + && \text{resource extension} \\ & \sum_{t \in T} \sum_{\ell \in L} \sum_{p \in P} PDC_{\ell,p}^t z_{\ell,p}^t + && \text{unsatisfied demand} \\ & \sum_{t \in T} \sum_{\ell \in L} \sum_{p \in P} IC_{\ell,p}^t inv_{\ell,p}^t + && \text{inventory costs} \\ & \sum_{t \in T} \sum_{\ell \in S^p} OC_{\ell}^t y_{\ell}^t + \sum_{t \in T \setminus \{T\}} \sum_{\ell \in S^c} CC_{\ell}^t y_{\ell}^t && \text{location decisions} \end{aligned}$$

## Constraints

### Flow conservation

$$\begin{aligned} & \text{procurement} \quad \downarrow \quad \text{incoming transports} \quad \downarrow \quad \text{production} \quad \downarrow \quad \text{inventory last period} \quad \downarrow \quad \text{incoming goods} \\ & b_{\ell,p}^t + \sum_{\ell' \in L, \ell' \neq \ell} x_{\ell',\ell,p}^t + h_{\ell,p}^t + inv_{\ell,p}^{t-1} = \\ & \sum_{\ell' \in L, \ell' \neq \ell} x_{\ell,\ell',p}^t + \sum_{q \in P} a_{\ell,p,q} h_{\ell,q}^t + inv_{\ell,p}^t + D_{\ell,p}^t - z_{\ell,p}^t \quad \forall \ell, p, t \\ & \quad \quad \quad \uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow \\ & \quad \quad \quad \text{outgoing transports} \quad \text{consumption to production} \quad \text{inventory this period} \quad \text{demand} \quad \text{unsatisfied demand} \end{aligned}$$

## Resources

### Production

$$\sum_{t \in T} \sum_{\ell \in L} \sum_{p \in P} \mu_{\ell,r,p} h_{\ell,p}^t \leq RK_r^t + v_r^t \quad \forall r \in R^p$$

### Handling

$$\sum_{t \in T} \sum_{p \in P} \left( \sum_{\ell, \ell' \in L, \ell \neq \ell'} (\lambda_{\ell',r,p}^i + \lambda_{\ell,r,p}^o) x_{\ell,\ell',p}^t + \sum_{\ell \in L} \lambda_{\ell,r,p}^i b_{\ell,p}^t \right) \leq RK_r^t + v_r^t \quad \forall r \in R^h$$

↑ Incoming goods and goods issue, respectively, by transports
 ↑ goods issue by procurement

### Feasibility of extension

$$0 \leq v_r^t \leq ERK_r^t \quad \forall r \in R^p \cup R^h$$

## Location decisions

Selectable facilities can be opened and closed, respectively, only once

$$\sum_{t \in T} y_{\ell}^t \leq 1 \quad \forall \ell \in S^o \quad \text{and} \quad \sum_{t \in T} y_{\ell}^t = 1 \quad \forall \ell \in S^c$$

Define

$$T_{\ell}^t = \begin{cases} \{1, \dots, t\} & \text{if } \ell \in S^o \\ \{t, \dots, T\} & \text{if } \ell \in S^c \end{cases}$$

## Activities at selectable facilities

### Procurement

$$b_{\ell,p}^t \leq M \sum_{\tau \in T_{\ell}^t} y_{\ell}^{\tau} \quad \forall \ell \in S, t \in T, p \in \mathcal{P}$$

## Production

$$h_{\ell,p}^t \leq M \sum_{\tau \in T_{\ell}^t} y_{\ell}^{\tau} \quad \forall \ell \in S, t \in T, p \in \mathcal{P}$$

## Inventory

$$inv_{\ell,p}^t \leq M \sum_{\tau \in T_{\ell}^t} y_{\ell}^{\tau} \quad \forall \ell \in S, t \in T, p \in \mathcal{P}$$

## Outgoing distribution

$$x_{\ell,\ell',p}^t \leq M \sum_{\tau \in T_{\ell}^t} y_{\ell}^{\tau} \quad \forall \ell \in S, \ell' \in L, t \in T, p \in \mathcal{P}$$

## Incoming distribution

$$x_{\ell',\ell,p}^t \leq M \sum_{\tau \in T_{\ell}^t} y_{\ell}^{\tau} \quad \forall \ell \in S, \ell' \in L \setminus \{S\}, t \in T, p \in \mathcal{P}$$

## Integer and non-negativity constraints

$$inv_{\ell,p}^0 = 0 \quad \forall \ell \in L, p \in \mathcal{P}$$

$$b_{\ell,p}^t, h_{\ell,p}^t, inv_{\ell,p}^t \geq 0 \quad \forall \ell \in L, t \in T, p \in \mathcal{P}$$

$$0 \leq z_{\ell,p}^t \leq D_{\ell}^t \quad \forall \ell \neq \ell' \in L, t \in T, p \in \mathcal{P}$$

$$x_{\ell,\ell',p}^t \geq 0 \quad \forall \ell \neq \ell' \in L, t \in T, p \in \mathcal{P}$$

$$y_{\ell}^t \in \{0, 1\} \quad \forall \ell \in L, t \in T$$

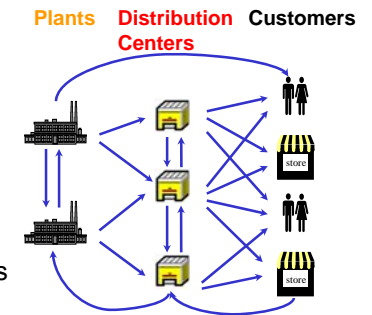
## Example

- 10 periods
- 10 products
- 5 plants
- 50 customers
- existing facilities: 10 DCs
- new facilities: 20 potential sites for DCs

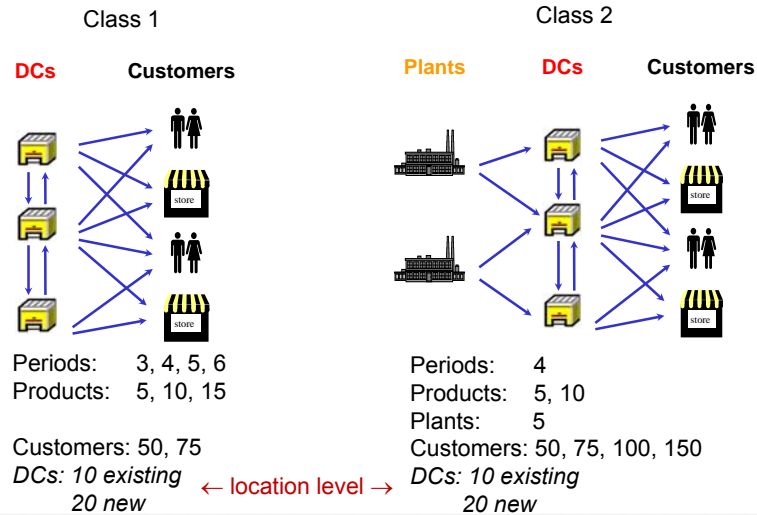
→ 10 270 constraints

→ 732 810 non-negative variables (dense network)

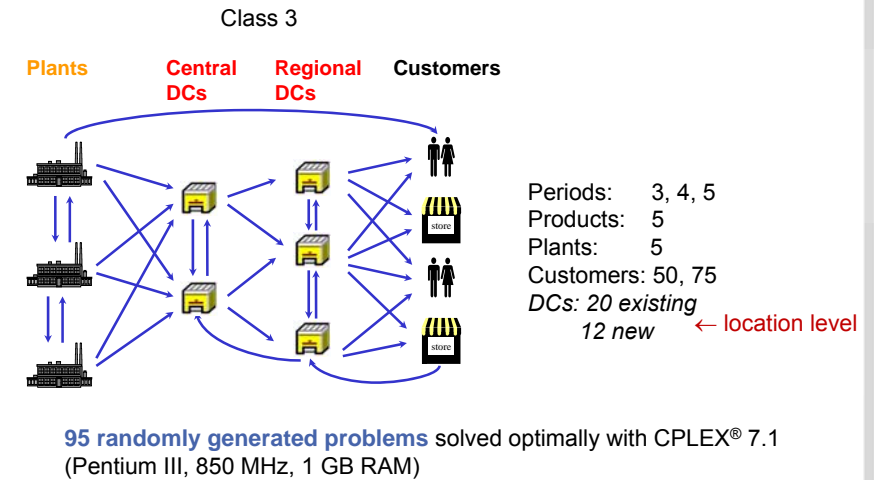
→ 270 binary variables status of facilities



## Numerical tests

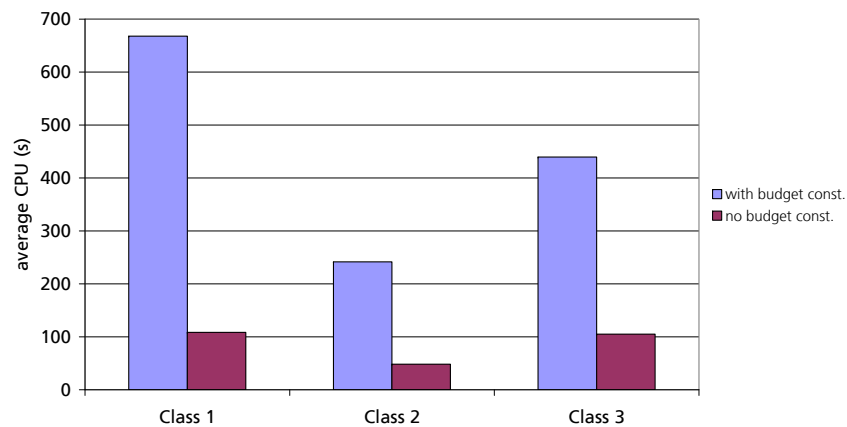


## Numerical tests



## Numerical tests

### Impact of budget constraints



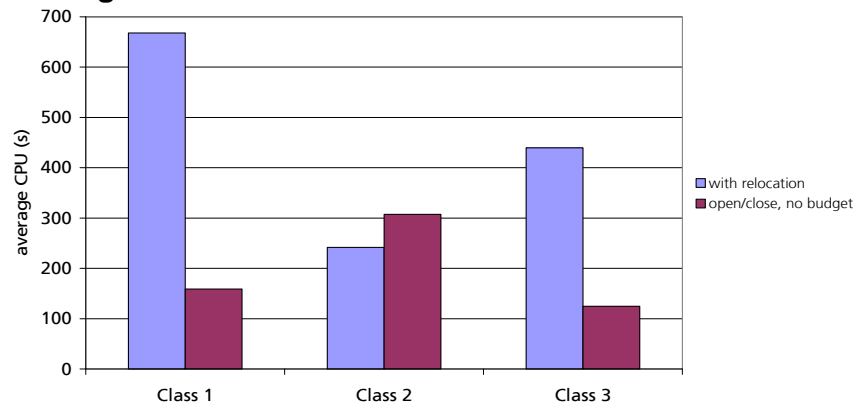
## Numerical tests

### Facility relocation vs “pure” open / close facilities



## Numerical tests

### Facility relocation vs “pure” open / close facilities & no budget constraints



## Location Problems in SCM




### Contents

- A Supply Chain Design Problem with facility location, production and transportation activities
- The real problem: Multi-Period Relocation Problems
- Strategic Supply Chain Model from Practice
- Heuristic Solution Procedures
- (Applications and Software)

## Problem characteristics

### Two types of decision variables:

- Non-negative continuous
  - capacity transfers, transportation, inventory, investments
- Binary → Status change of a facility in a period

|                   |   | 1  | 2  | 3  | 4  | 5  | periods |
|-------------------|---|----|----|----|----|----|---------|
| existing facility |  | ✓0 | ✓0 | ✓1 | ✗0 | ✗0 |         |
| new facilities    |  | ✗0 | ✗0 | ✗0 | ✗0 | ✗0 |         |
|                   |  | ✗0 | ✗0 | ✓1 | ✓0 | ✓0 |         |

0 no status change  
1 status change

✓ open  
✗ closed

for given status matrix the resulting problem is a LP !

## Heuristics Approach

### LP-based heuristic

#### Phase 1: (Fast) Construction

- All status variables are fixed to 0 or 1

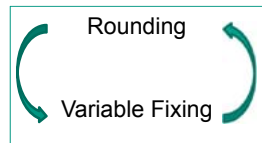
#### Phase 2: Repair / Improvement

- Some fixing decisions made in phase 1 are changed



## Heuristics Approach

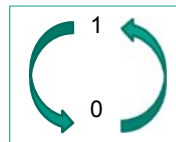
### Phase 1: Construction



- Feasible solution with good quality → STOPP
- Feasible solution with bad quality
- Infeasible solution

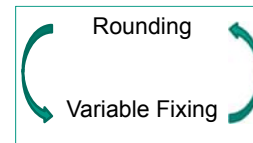


### Phase 2: Repair / Improvement



## Heuristics Approach

### Phase 1: Construction



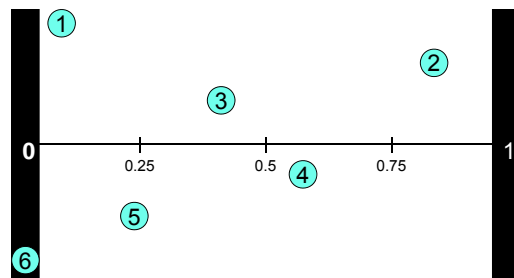
#### 4 Strategies for variable fixing:

- Fixing variables to 0
  - S1 Use lower threshold
  - S2 Select an existing facility
- Fixing variables to 1
  - S3 Use upper threshold
  - S4 Select a potential facility

## Heuristics approach

### Phase 1: Construction

- Start by solving LP-relaxation considering all facility status variables in  $[0, 1]$

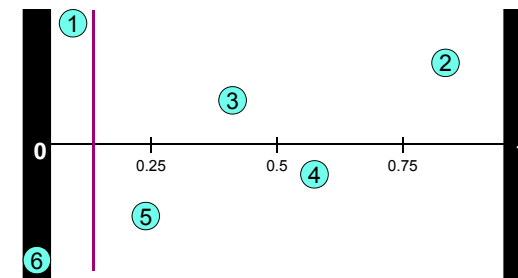


● = (facility, period)

## Heuristics approach

### Phase 1: Construction

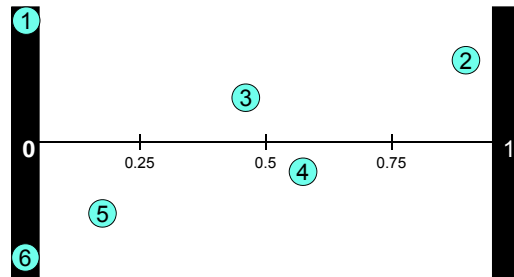
- Variable fixing strategy 1



## Heuristics approach

### Phase 1: Construction

- Solve LP considering all non-fixed facility status variables in  $[0,1]$



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05/13/2010

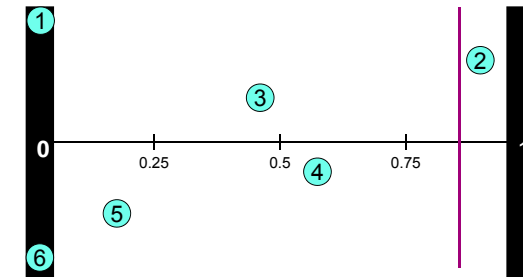
Professor Dr. Stefan Nickel - Location Problems in Supply Chain Management

Institute for Operations Research  
Discrete Optimization and Logistics

## Heuristics approach

### Phase 1: Construction

- Variable fixing strategy 3



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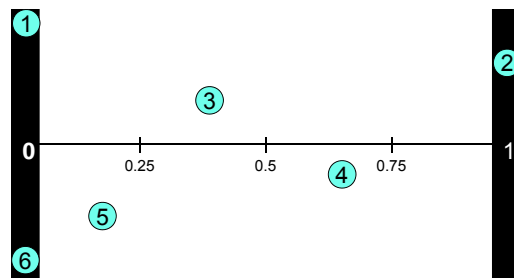
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## Heuristics approach

### Phase 1: Construction

- Solve LP considering all non-fixed facility status variables in  $[0,1]$



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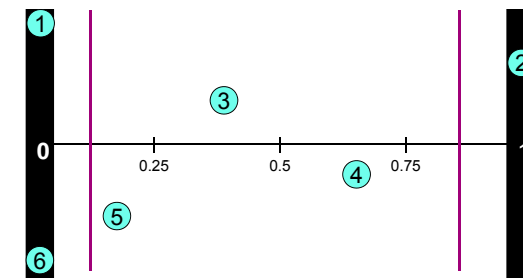
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## Heuristics approach

### Phase 1: Construction

- Variable fixing strategy 2



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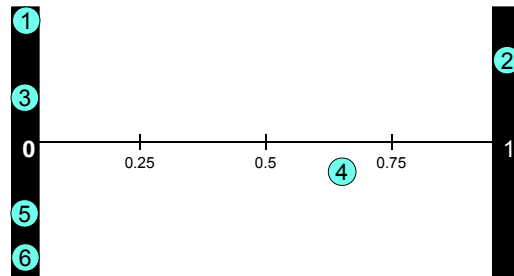
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## Heuristics approach

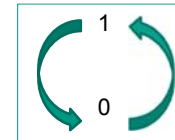
### Phase 1: Construction

- Variable fixing strategy 4



## Heuristics approach

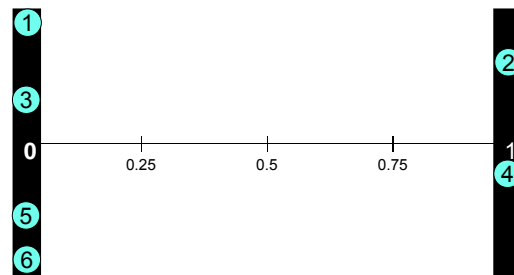
### Phase 2: Repair / Improvement



- One or more variables are randomly selected to change from 1 to 0
- One or more variables are randomly selected to change from 0 to 1

## Heuristics approach

### Phase 2: Repair / Improvement



## Heuristic approach

### Tuning the heuristic:

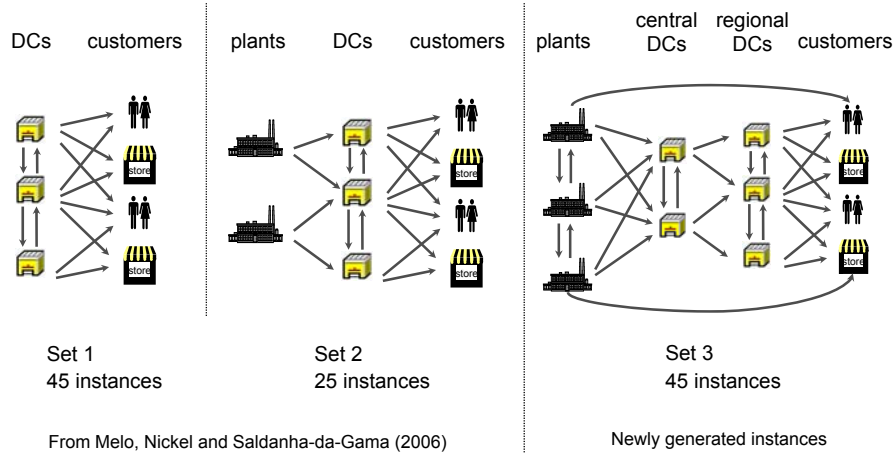
- Phase 1:
  - How many variables should be fixed to 0 before a variable is fixed to 1?
  - Values for the upper & lower thresholds: should they change during the procedure?
- Phase 2:
  - How to choose variables for repair / improvement?
- Phases 1 & 2:
  - ...

Many degrees of freedom!



A set of preliminary computational experiments were performed

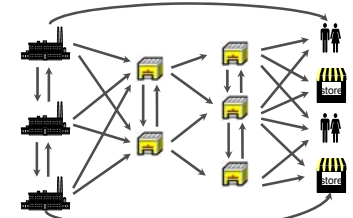
## Computational experience



## Computational Experience

115 randomly generated instances:

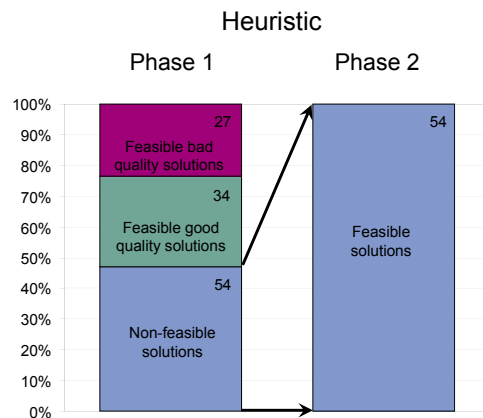
- Periods: 3, 4, 5, 6, 8
- Products: 5, 10, 15, 20, 50
- Plants: 1, 5
- Customers: 50, 75, 100, 150, 200



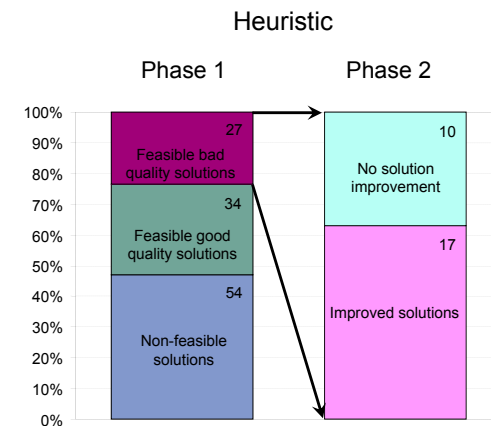
|            |              | Existing | New       |
|------------|--------------|----------|-----------|
| Relocation | Central DCs  | 4, 8, 10 | 8, 12, 20 |
| Levels     | Regional DCs | 10, 20   | 20, 30    |

- Continuous variables: 29000 – 370000
- Binary variables: 90 – 560
- Constraints: 1700 – 20000

## Computational experience

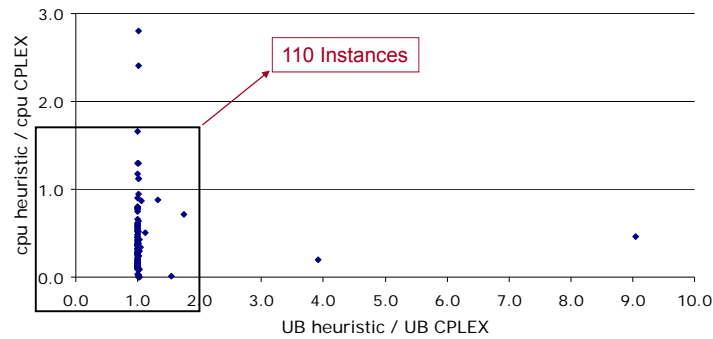


## Computational experience



## Computational experience

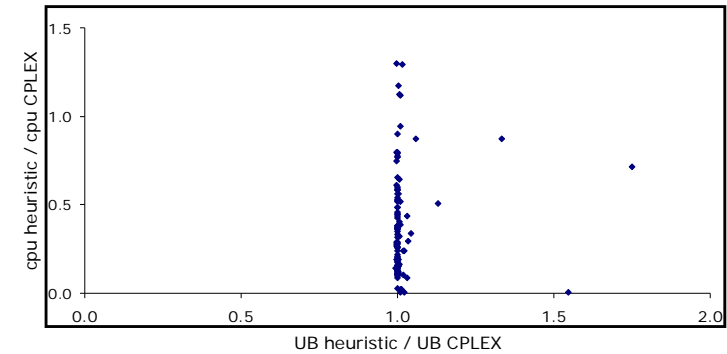
### Execution time and solution quality



CPLEX used as a heuristic  
(1% gap or time limit 5h)

## Computational experience

### Execution time and solution quality



Heuristic: - solution quality as good as CPLEX  
- much less cpu time than CPLEX

## Further Model Enhancements

- Include uncertainties – Stochastic Programming
- Include Nonlinearities
- Location-Routing (from a location perspective)

## Solution Approach for a special Multiperiod Case

### Lagrangian Relaxation

#### Assumptions

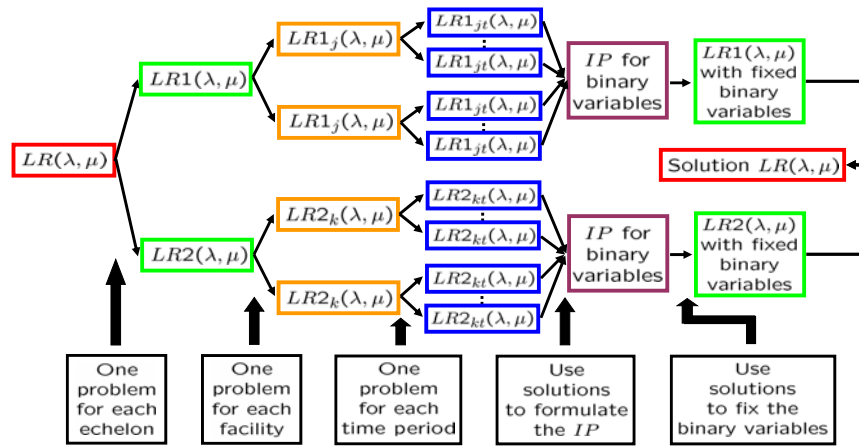
- Two-echelon structure
- inventory is allowed in the warehouses
- location decisions are final:  
if a facility is opened/closed once it can never be closed/reopened again
- Constraints on the minimum number of open facilities at the beginning and the end of the planning horizon

- Relax the demand satisfaction

and

- the flow conservation between the planning periods.

## Lagrangian Relaxation



Reference: Hinojosa, Kalcsics, Nickel, Puerto, Velten, 2008, COR

## Solution Approach for a special Multiperiod Case

### Problem Types

|    | Customers       | Commodities     | Warehouses         |                    |                  | Plants             |                    |                  |
|----|-----------------|-----------------|--------------------|--------------------|------------------|--------------------|--------------------|------------------|
|    | $ \mathcal{C} $ | $ \mathcal{P} $ | $ \mathcal{CW}_o $ | $ \mathcal{CW}_c $ | $ \mathcal{CW} $ | $ \mathcal{LP}_o $ | $ \mathcal{LP}_c $ | $ \mathcal{LP} $ |
| P1 | 50              | 2               | 14                 | 6                  | 20               | 14                 | 6                  | 20               |
| P2 | 75              | 2               | 25                 | 15                 | 40               | 25                 | 15                 | 40               |
| P3 | 100             | 10              | 10                 | 3                  | 13               | 7                  | 2                  | 9                |
| P4 | 125             | 12              | 12                 | 4                  | 16               | 8                  | 2                  | 10               |

| Value | Description   |
|-------|---|
| H-Gap | Average percentage gap between the objective value of the heuristic solution and the lower bound resulting from the <i>Subgradient Optimization</i> |
| E-Gap | Average percentage gap between the objective values of the heuristic and the optimal solution found by <i>CPLEX</i>                                 |
| CPU-E | Average CPU time needed to find the optimal solution  |
| CPU-H | Average CPU time needed to find the heuristic solution  |
| N     | Average number of iterations performed to find the heuristic solution   |

## Solution Approach for a special Multiperiod Case

### Summary of Computational Results

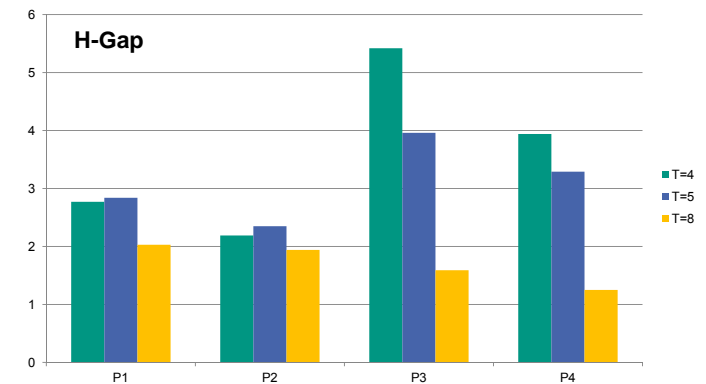
■ (Average over 5 Instances / Problem Type and Time Periods)

|         |       | P1    | P2    | P3     | P4     |
|---------|-------|-------|-------|--------|--------|
| $T = 4$ | H-Gap | 2,77  | 2,19  | 5,42   | 3,94   |
|         | E-Gap | 1,09  | 1,07* | 0,12   | 0,099  |
|         | CPU-E | 1523  | 7200* | 2331   | 3369   |
|         | CPU-H | 48,72 | 587,1 | 161,2  | 926,2  |
|         | N     | 271   | 736   | 44     | 427    |
| $T = 5$ | H-Gap | 2,84  | 2,35  | 3,96   | 3,29   |
|         | E-Gap | 1,19* | 1,19* | 0,18*  | 0,075* |
|         | CPU-E | 5095* | 7200* | 3577*  | 5033*  |
|         | CPU-H | 68,88 | 1048  | 307    | 1035   |
|         | N     | 316   | 657   | 44     | 52     |
| $T = 8$ | H-Gap | 2,03  | 1,94  | 1,59   | 1,25   |
|         | E-Gap | 0,53* | 0,61* | 0,098* | 0,02*  |
|         | CPU-E | 6148* | 7200* | 3620*  | 6768*  |
|         | CPU-H | 236   | 1871  | 1053   | 3938   |
|         | N     | 293   | 441   | 44     | 52     |

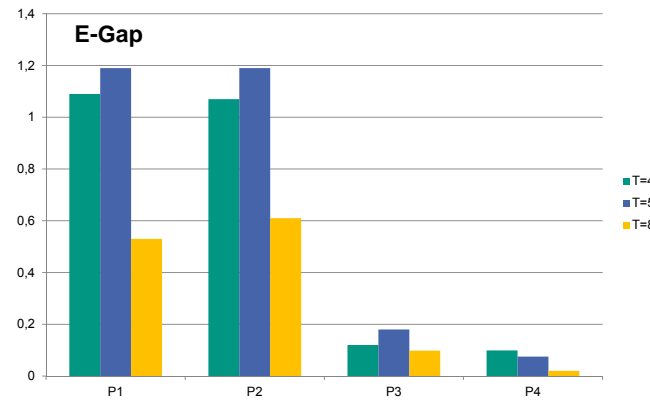
\* at least one example could not be solved to within the time limit optimality

## Solution Approach for a special Multiperiod Case

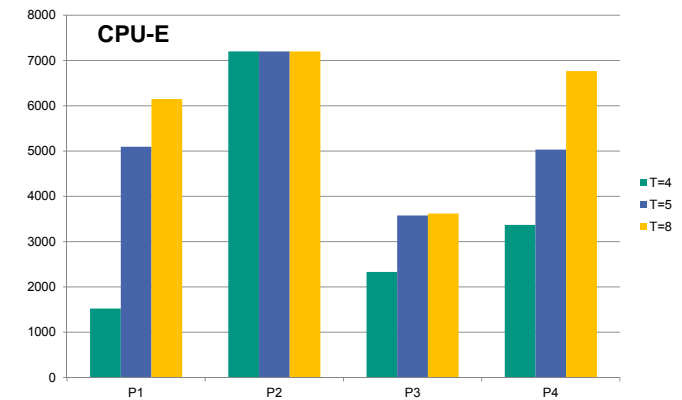
### Summary of Computational Results



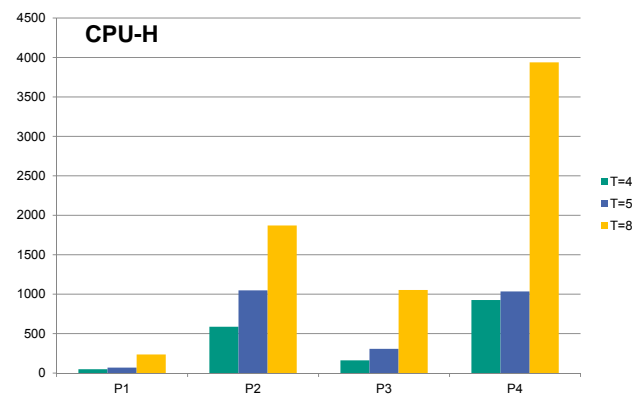
## Summary of Computational Results



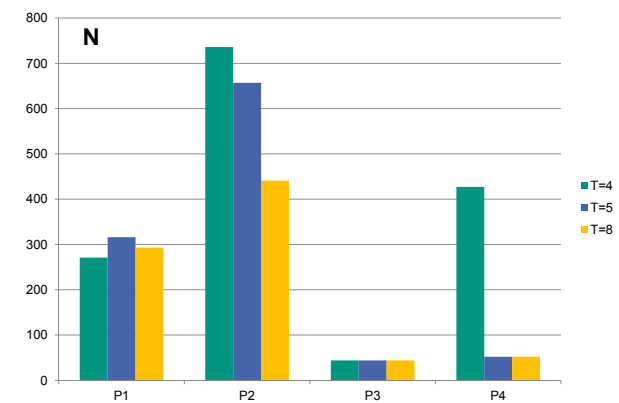
## Summary of Computational Results



## Summary of Computational Results



## Summary of Computational Results



## Location Problems in SCM

### Contents

- A Supply Chain Design Problem with facility location, production and transportation activities
- The real problem: Dynamic Relocation Problems
- Strategic Supply Chain Model from Practice
- Heuristic Solution Procedures
- Applications and Software

## Take Over Scenario in the Steel Industry

Nickel, Velten, Weimerskirch,  
2005 and 2007

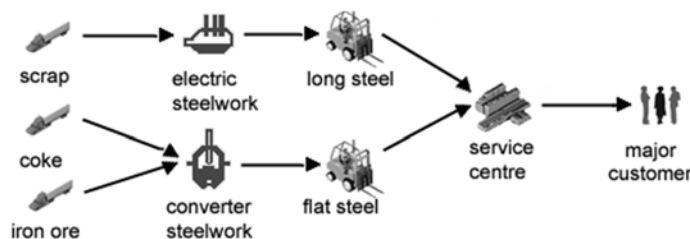


### Problem Description

- Steel company is a merger of 3 huge steel companies
- Multiple synergy effects are expected
- World leader in steel production p.a. but less than 5 % market share → no monopoly
- Entering new markets is an ultimate strategic objective
- Possibility to take over another steel company
- Where and when should capacity be shifted? Which facilities should be opened and closed, respectively?
- How should the capacity be shifted from existing facilities to new ones without interrupting the supply chain activities or without breaking the budget and capacity restrictions?
- Is it profitable to take over the foreign company in order to use its additional capacities and to include its supply chain?

## Take Over Scenario in the Steel Industry

### Supply Chain



- Steel company SC: 63 operating facilities
- Potential new facilities: 13 locations
- Take over candidate SC: 13 operating facilities

## Analytical Approach for WEEE Network Design

Nickel, Velten, Verter,  
Weimerskirch, 2008



### Problem Description

- WEEE Directive (Waste Electrical and Electronic Equipment) is setting collection, recycling and recovery targets for all types of electrical goods
- Existing infrastructure incorporating collection points, inspection and recycling facilities needs to be redesigned
- The responsibility of the WEEE is imposed on the producers
- The municipalities are responsible for the drop off centers
- Do we need more or less, bigger or smaller inspection centers?
- Is it economically reasonable to operate a demanufacturing facility?
- Where and when should capacity be shifted?
- Which facilities should be opened and closed, respectively?

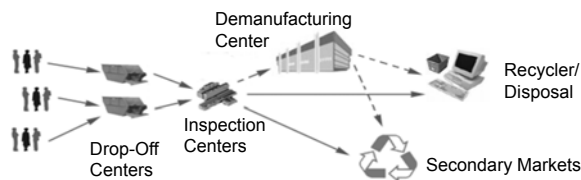


## Analytical Approach for WEEE Network Design

**Min.** objective function  
(disposal / treatment + collecting + inventory + operating costs)

→ Constraints categories:

- |                                 |  |
|---------------------------------|--|
| a) flow conservation            | e) econ. reasonable minimum throughput |
| b) quotas for secondary markets | f) facility location                   |
| c) capacity relocation          | g) budget restrictions                 |
| d) capacity restriction         | h) non-negativity and integrality      |



## Location Planning for Petrol Stations

### Consideration of Geo-Data

Horstmann, Nickel, van Bonn,  
Ziegler 2007

#### Questions

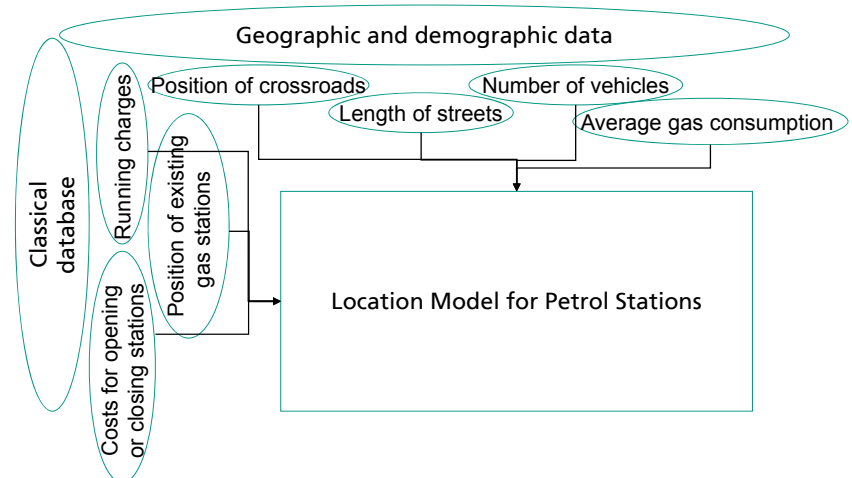
- Which data may support location planning and how can we integrate these data in existing location models?
- Which geographic and demographic data are usually available?
- Where can we get these data from?
- How can we govern the huge amount of geographic and demographic data?
- For how far into the past do we need the data and how far into the future is our location planning reliable?

## Location Planning for Petrol Stations

### Advantages

- Some geographic and demographic data are widely available, all over the world.
- These data provide a wider data base, if the classical database is unsatisfyingly.
- They allow better forecasts of the expected trends, since they were elevated and saved for years.
- The additional information may help the user to come to more reliable and safer decisions.

## Location Planning for Petrol Stations

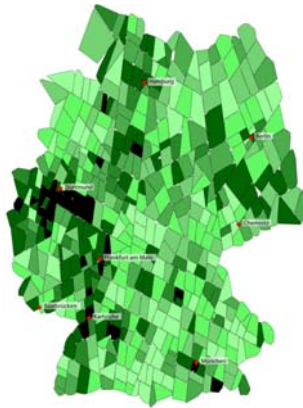


## Case Study - Gas Stations

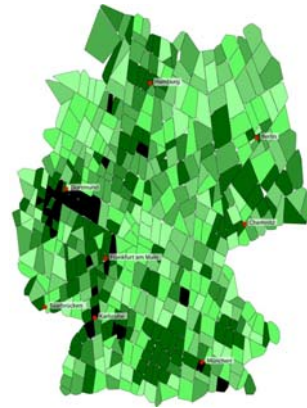
Number of Gas Stations

|         |
|---------|
| 0       |
| 1       |
| 2       |
| 3       |
| 4       |
| 5 - 9   |
| 10 - 19 |

Before Optimization



After Optimization



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