TRANSPORTATION@MIT

Transportation Service Network Design

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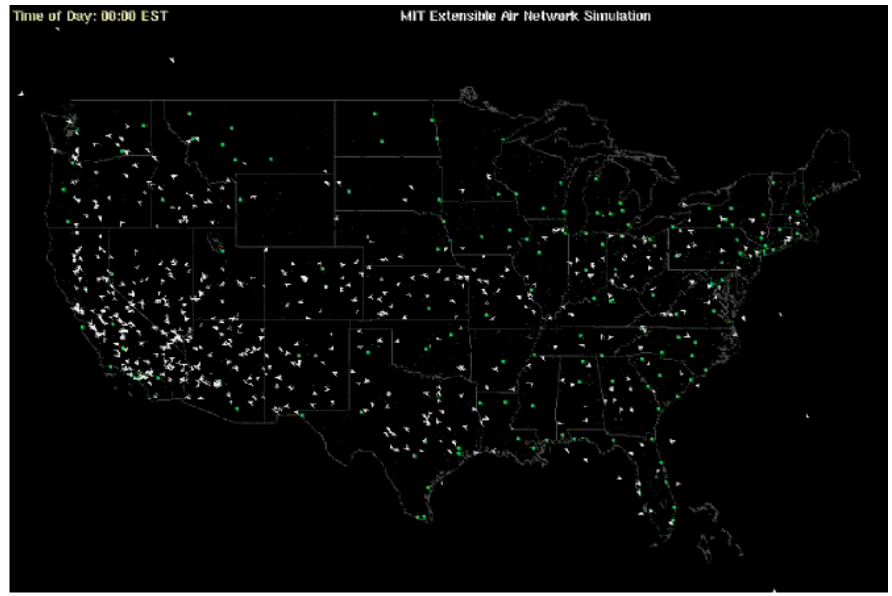
Montreal Spring School on Supply Chain and Transportation Network Design
May 12-14, 2010

Outline

- Transportation Service Network Design
  - Airline scheduling examples
  - Interplay between service network design and schedule reliability
- Impact of service network design and scheduling on aviation system performance
  - Cost of congestion
- Service network design and scheduling responses to reduce congestion and delays
  - The role of schedule slack
  - The role of demand management and competition



Airline Schedule Design

- Schedule design addresses the questions of:
  - Where to fly?
  - How frequently to fly?
  - When to fly?
  - How much capacity to provide on each flight leg?
- The output is a set of scheduled flight legs, with assigned capacity, that forms the input to all subsequent planning operations
  - Operating and profitability consequences

Optimization of Airline Service Network Design

- Optimization approaches to schedule design face numerous challenges
  - The ‘tractability’ issue
    - Need to determine where to assign ‘lumpy’ capacity and how to flow individual demands
    - Very large-scale nature of the problems
  - The ‘reliability’ or ‘robustness’ issue
    - Schedules are never executed as planned, and the cost of recovering from unplanned disturbances is huge
  - The competition issue
    - Optimization models ignoring competitive factors result in service network designs that might overestimate revenue capture, and hence, profitability of the service network

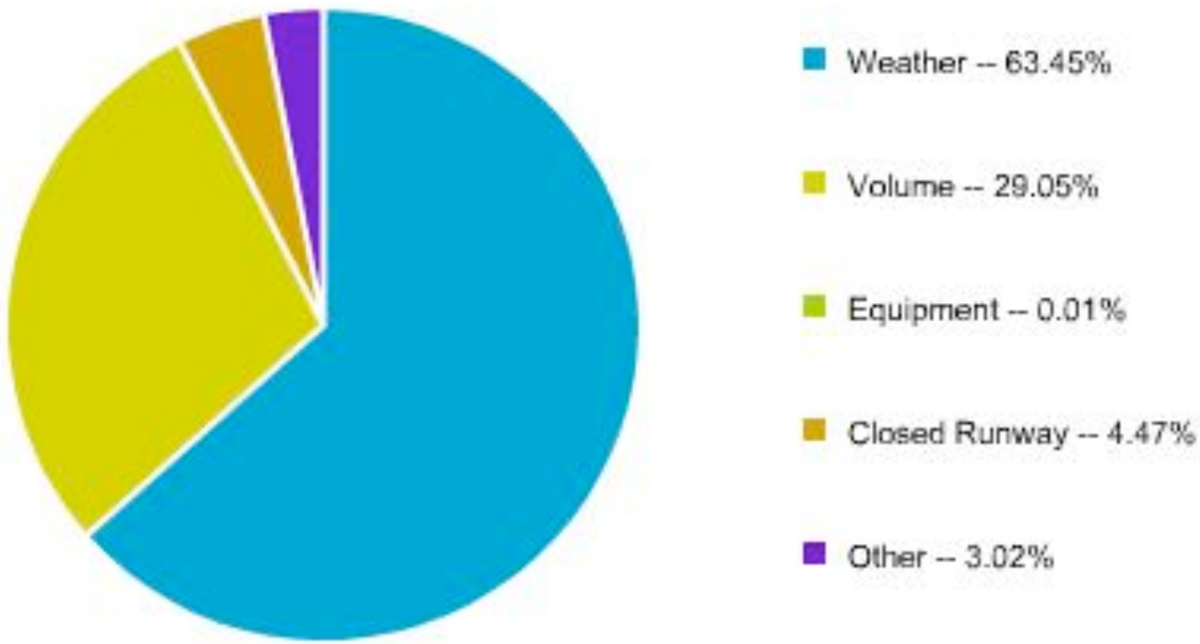
Service Network Design and Schedule Reliability: The Fundamental Issue

- Demand for aviation system capacity exceeds amount available
  - At airports in US
    - During operations, the number of operations (flight departures and arrivals) can exceed the capacity at airports
    - Airport capacity is stochastic
      - Bad weather results in reduced airport capacities, allowing fewer departures and arrivals per unit time, to ensure safe operations
    - In US, number of **scheduled** operations at most airports is NOT limited by airport capacity

- Result of demand-capacity imbalance in aviation system
  - During operations, schedule delays imposed to ensure number of operations does not exceed airport capacity during operations
  - Delays propagate to downstream flights
  - Flights are canceled
  - Passengers misconnect and must be re-accommodated on later flights with available seats- resulting in long passenger delays

Airline Delays

- Nearly 50% of the delays were due to National Aviation System (NAS)
- Majority of National Aviation System (NAS) delays attributed to scheduling more than the realized capacity
  - 90% + of NAS delays



Causes of National Aviation System Delays

[[1]Source: Air Transport Association, 2008; [2]Source: U.S. Airline Passenger Trip Delay Report, 2008; [3]Source: Bureau of Transportation Statistics, 2009]

Cost to Airlines

Cost to airlines of flight delays ($ Billions):

	Delay Against Schedule	Buffer	Total
7 major airlines	3.3	2.6	5.9
Industry wide	4.6	3.7	8.3

Passenger Trip Delays (2007)

- 7.45 million flights
- 487.2 million passengers
- 4437 direct routes between 267 airports.
- Average number of flights between O/D pairs in 2007 was 4.57
- Total Passenger Trip Delay - 28,539 years
- Average Passenger Trip Delay - 31 min/pax
  - Average Passenger Trip Delay for Delayed or Disrupted Passengers

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  - Interplay between service network design and schedule reliability
- Impact of service network design and scheduling on aviation system performance
  - Cost of congestion
- Service network design and scheduling responses to mitigate delays
  - The role of schedule slack
  - The role of demand management and competition

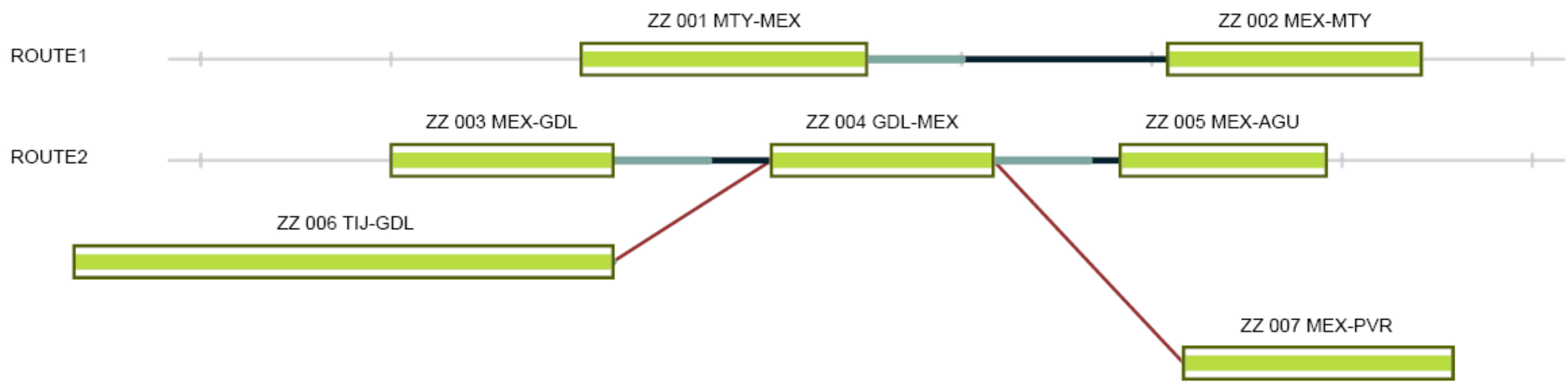
Service Network Design and Schedule Reliability: Possible Solutions

- The Airline:
  - Build ‘robust’ or ‘reliable’ flight networks that can absorb delays
  - *The role of schedule slack*
- The Aviation Authority (FAA in US)
  - Limit number of scheduled operations at airports (and in airspace)
  - *The role of competition*

Slack in Airline Schedules

- Slack is additional time beyond the minimum requirement
  - Aircraft connection, passenger connection, and flight block time
- Slack is desirable in robust schedules
  - Absorb delays in the airline network
  - Reduce the likelihood of downstream propagation of disruptions
  - Obviates need to recover, or provides recovery options
- Slack can be very costly
  - Decreases the utilization of resources in airline operations
- We seek to re-allocate the existing slack such that ..
  - The resulting distribution of slack is more effective in minimizing delays and disruptions

Schedule Aircraft and Passenger Slack



Aircraft Routing/ Scheduling Approach

Jan-Feb 2008 Data

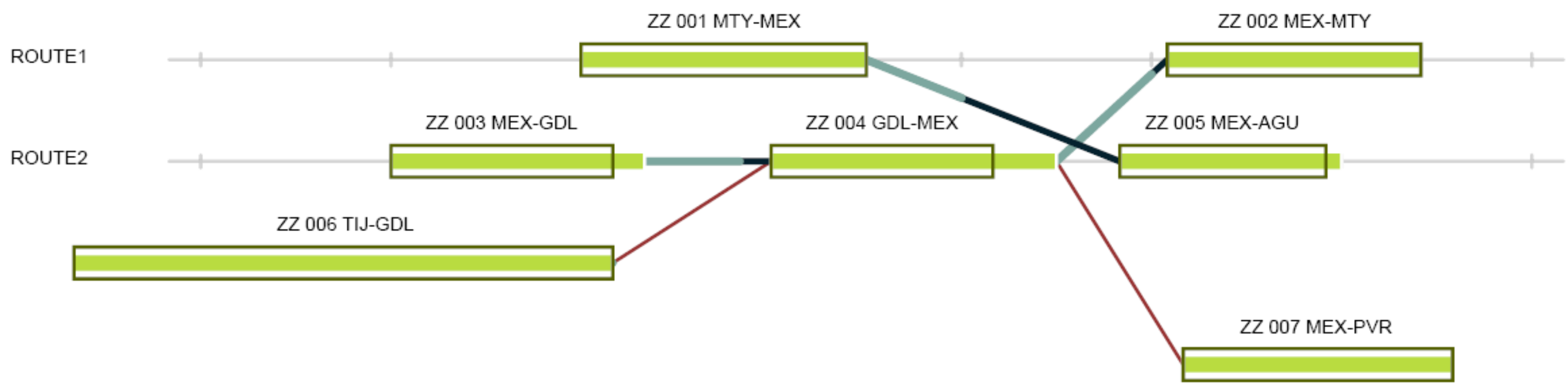
↓

Aircraft routes: slack allocation models

- Each day of operation in Jan-Feb represents one instance of delay scenario ω (|Ω|=60)
- Assume each delay scenario is equally likely
- Solve 3 different slack allocation models over each day of operation

Robust Aircraft Re-routing (AR)

- Flight schedule and fleet assignments are fixed
- Aircraft assignment of each flight can be changed
  - Affect aircraft connection slack



AR : Optimization Model

- We focus on designing aircraft routes (daily)
  - Assuming that the maintenance feasibility is preserved
- **Goal:** Minimize expected total propagated delay
  - Equivalent to minimizing total delay
- **Constraints:**
  - Every flight is assigned to exactly one aircraft
  - Only use available aircraft
- Use a string-based formulation
  - Each flight string represents a set of flight legs that are operated by a single aircraft on a given day of operation

AR : Optimization Model

$$\text{Minimize} \quad \mathbb{E}\left[\sum_{s \in S}\left(x_s \times \sum_{(i,j) \in S} pd_{ij}^s\right)\right]$$

$$\text{subject to} \quad \sum_{s \in S} a_{is} x_s = 1 \qquad \forall i \in F$$

$$\sum_{s \in S_{m^+}} x_s = N_{m^+} \qquad \forall m^+ \in M^+$$

$$x_s \in \{0, 1\} \qquad \forall s \in S$$

AR : Optimization Model

Rewrite the objective function:

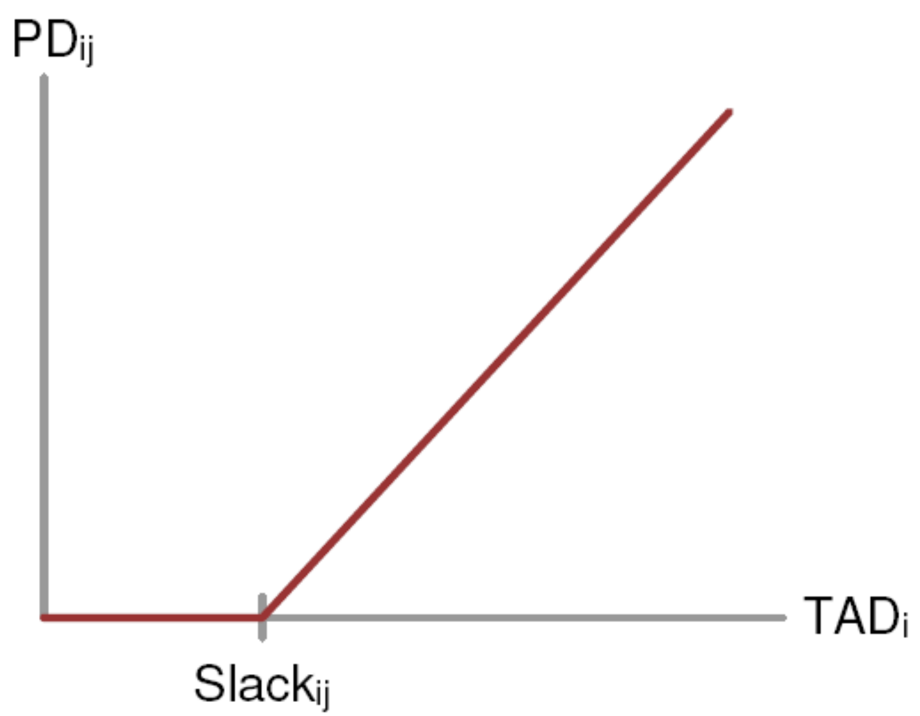
$$\mathbb{E}\left[\sum_{s \in S}\left(x_s \times \sum_{(i,j)\in S} pd_{ij}^s\right)\right] = \sum_{s\in S}\left(x_s \times \underbrace{\mathbb{E}\left[\sum_{(i,j)\in S} pd_{ij}^s\right]}_{}\right)$$

$$= \sum_{\omega \in \Omega} p_\omega \sum_{(i,j)\in S} pd_{ij}^s(\omega)$$

Can be computed offline!

AR : Alternative Objective Function

- From $PD_{ij} = \max(TAD_i - Slack_{ij}, 0)$

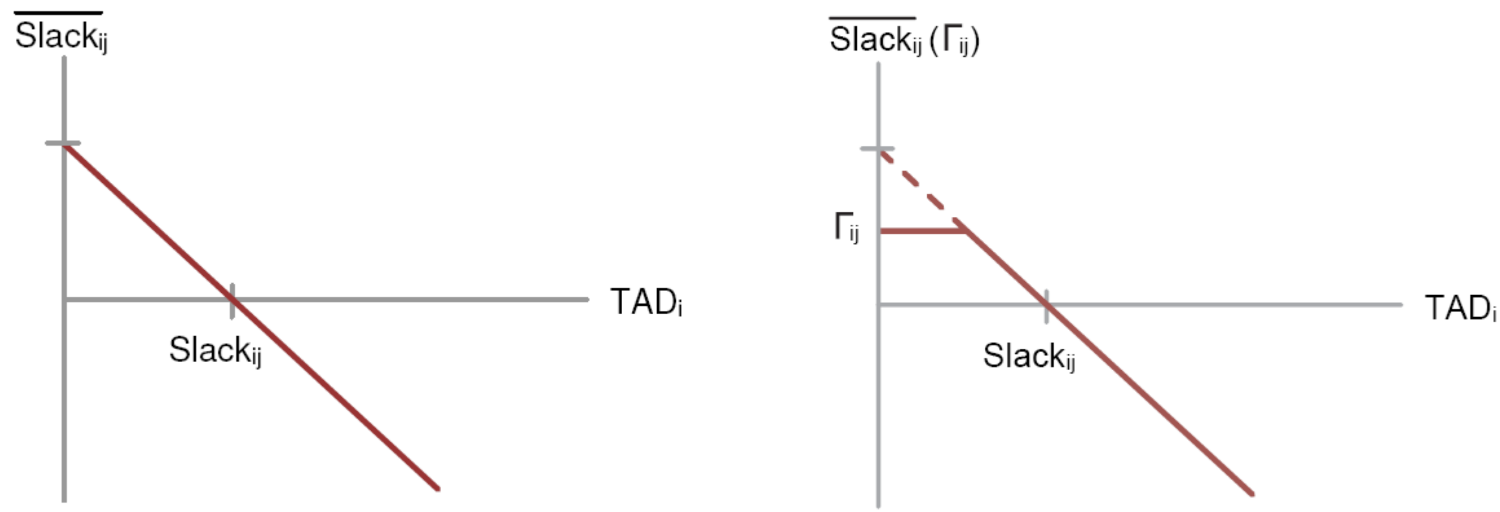


AR : Alternative Objective Function

- Maximizing the total **effective slack**

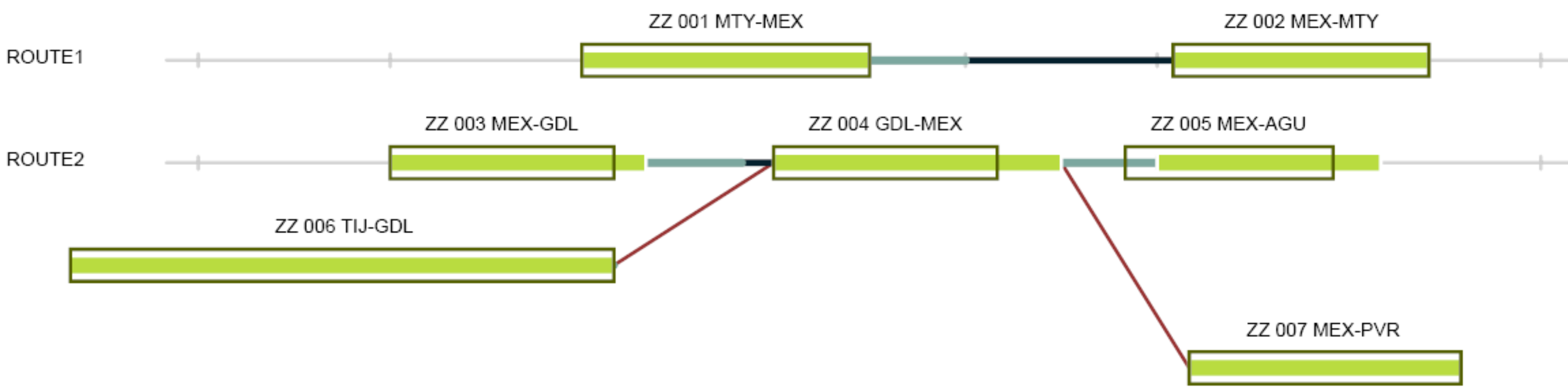
$$\overline{Slack}_{ij} = Slack_{ij} - TAD_i$$

$$\overline{Slack}_{ij}(\Gamma_{ij}) = \min(Slack_{ij} - TAD_i, \Gamma_{ij})$$



Robust Flight Schedule Re-timing (FR)

- Aircraft routing is fixed
- Departure times are allowed to shifted earlier or later, but block times are fixed
  - Also affect passenger connection slack



FR : Optimization Model

$$\text{Minimize} \quad \sum_{(i,j)\in A} \mathbb{E}\left[pd_{ij}\right] = \sum_{(i,j)\in A} \left(\sum_{\omega\in\Omega} p_\omega pd_{ij}^\omega \right)$$

$$aSlack'_{ij} = aSlack_{ij} - x_i + x_j \qquad \forall (i,j) \in A$$

$$aSlack'_{ij} \geq 0 \qquad \forall (i,j) \in A$$

Aircraft Connection Slack

$$pSlack'_{ij} = pSlack_{ij} - x_i + x_j \qquad \forall (i,j) \in P$$

$$pSlack'_{ij} \geq 0 \qquad \forall (i,j) \in P$$

Passenger Connection Slack

$$pd_{ij}^\omega \geq tad_i^\omega - aSlack'_{ij} \qquad \forall (i,j) \in A, \forall \omega \in \Omega$$

$$pd_{ij}^\omega \geq 0 \qquad \forall (i,j) \in A, \forall \omega \in \Omega$$

Propagated Delay

$$tad_i^\omega \geq IAD_i^\omega \qquad \forall i \in F_0, \forall \omega \in \Omega$$

$$tad_j^\omega \geq pd_{ij}^\omega + IAD_j^\omega \qquad \forall (i,j) \in A, \forall \omega \in \Omega$$

$$tad_i^\omega \geq 0 \qquad \forall i \in F, \forall \omega \in \Omega$$

Total Arrival Delay

$$l_i \leq x_i \leq u_i \qquad \forall i \in F$$

Time Window Size

$$x_i \in \mathbb{Z}^n \qquad \forall i \in F$$

FR : Alternative Objective Functions

- Maximizing the total expected effective **aircraft** connection slack:

$$\text{Maximize} \quad \sum_{(i,j)\in A}\left(\sum_{\omega\in\Omega} p_\omega \overline{aSlack}_{ij}^{\omega}\right)$$

$$\overline{aSlack}_{ij}^{\omega} \leq aSlack'_{ij} - tad_i^{\omega} \qquad \forall (i,j)\in A$$

$$\overline{aSlack}_{ij}^{\omega} \leq \Gamma_{ij} \qquad \forall (i,j)\in A$$

- Maximizing the total expected effective **passenger** connection slack:

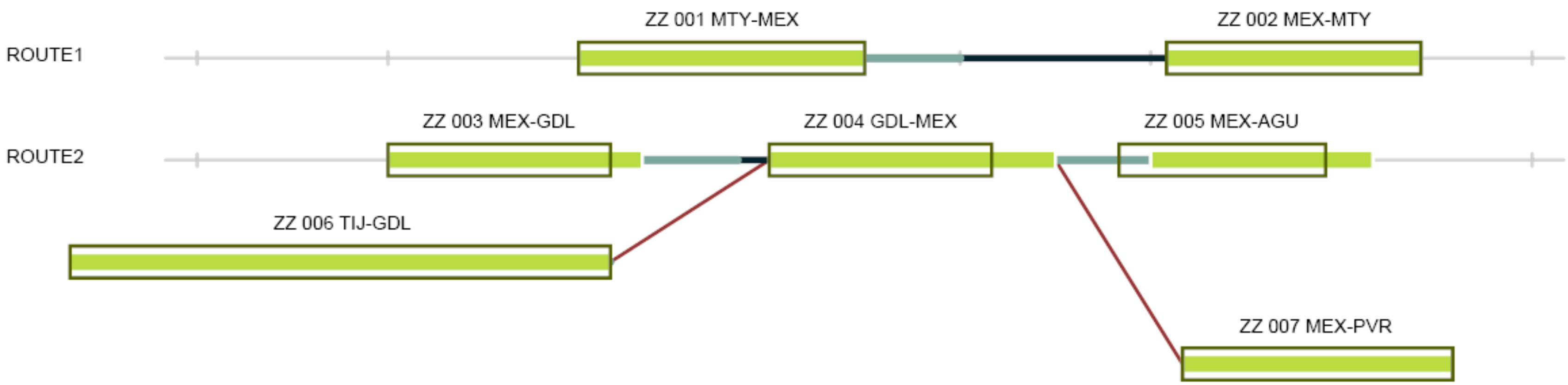
$$\text{Maximize} \quad \sum_{(i,j)\in P}\left(\sum_{\omega\in\Omega} p_\omega \overline{pSlack}_{ij}^{\omega}\right)$$

$$\overline{pSlack}_{ij}^{\omega} \leq pSlack'_{ij} - tad_i^{\omega} \qquad \forall (i,j)\in P$$

$$\overline{pSlack}_{ij}^{\omega} \leq \Gamma_{ij} \qquad \forall (i,j)\in P$$

Robust Block Time Adjustment (BA)

- Aircraft routing is fixed
- Departure and arrival times are allowed to change independently
  - Also affect block time slack



BA : Optimization Model

$$\text{Minimize} \qquad \sum_{i \in F} \mathbb{E}\left[tad_i\right] = \sum_{i \in F} \left(\sum_{\omega \in \Omega} p_\omega tad_i^\omega \right)$$

$$\left.\begin{aligned} aSlack'_{ij} &= aSlack_{ij} - y_i + x_j && \forall (i,j) \in A \\ aSlack'_{ij} &\geq 0 && \forall (i,j) \in A \end{aligned}\right\} \text{Aircraft Connection Slack}$$

$$\left.\begin{aligned} pSlack'_{ij} &= pSlack_{ij} - y_i + x_j && \forall (i,j) \in P \\ pSlack'_{ij} &\geq 0 && \forall (i,j) \in P \end{aligned}\right\} \text{Passenger Connection Slack}$$

$$\left.\begin{aligned} pd_{ij}^\omega &\geq tad_i^\omega - aSlack'_{ij} && \forall (i,j) \in A, \forall \omega \in \Omega \\ pd_{ij}^\omega &\geq 0 && \forall (i,j) \in A, \forall \omega \in \Omega \end{aligned}\right\} \text{Propagated Delay}$$

$$\left.\begin{aligned} tad_i^\omega &\geq IAD_i^\omega + x_i - y_i && \forall i \in F_0, \forall \omega \in \Omega \\ tad_j^\omega &\geq pd_{ij}^\omega \boxed{+ IAD_j^\omega + x_j - y_j} && \forall (i,j) \in A, \forall \omega \in \Omega \\ tad_i^\omega &\geq 0 && \forall i \in F, \forall \omega \in \Omega \end{aligned}\right\} \text{Total Arrival Delay}$$

$$\left.\begin{aligned} l_i \leq y_i - x_i &\leq u_i && \forall i \in F \\ l_{x_i} \leq x_i &\leq u_{x_i} && \forall i \in F \\ l_{y_i} \leq y_i &\leq u_{y_i} && \forall i \in F \end{aligned}\right\} \text{Allowable Changes}$$

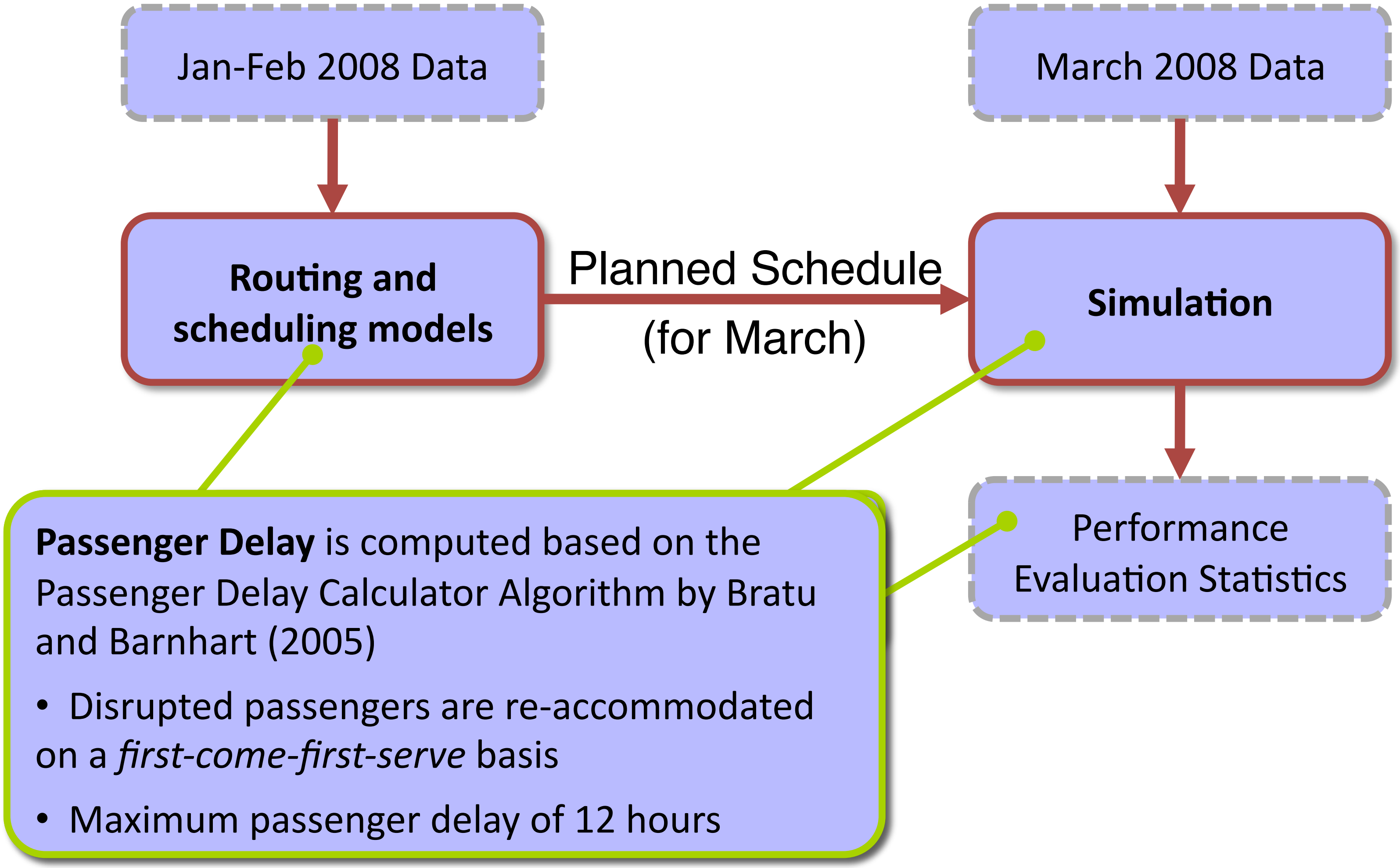
$$x_i, y_i \in \mathbb{Z}^n \qquad \forall i \in F$$

BA : Optimization Model

The polyhedron formed by the constraints in the BA formulation is ***integral****, given that all data and parameters in those constraints are integral.*

- **Proof:** use Ghouila-Houri's characterization to show that the coefficient matrix is totally unimodular
- Consequently, we can relax the integrality constraint and solve the problem as an LP

Data and Evaluation Process



AR : Results

Average performance evaluation statistics over 25 days (March 1-25, 2008) for the AR models

	Original	AR_minPD		AR_maxEffACSlack15	
Flight Delay Statistics					
Total Propagated Delay (mins)	1009.60	818.60	-18.92%	781.40	-22.60%
% of Flights with PD > 0	17.74%	14.86%		14.41%	
Total Arrival Delay (mins)	3141.16	2965.56	-5.59%	2929.16	-6.75%
15-min On-Time Performance	76.53%	77.82%		78.02%	
60-min On-Time Performance	96.89%	97.26%		97.36%	
Passenger Delay Statistics					
Total Pax Delay (mins)	260565	250325	-3.93%	246903	-5.24%
Total Disrupted Pax (pax)	47.56	45.16	-5.05%	44.72	-5.97%

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FR : Results

Average performance evaluation statistics over 25 days (March 1-25, 2008) for the FR models

	Original	FR_minPD		FR_maxEffACSlack15		FR_maxEffPaxSlack15	
Flight Delay Statistics							
Total Propagated Delay (mins)	1009.60	756.24	-25.10%	741.48	-26.56%	1076.40	+6.62%
% of Flights with PD > 0	17.74%	11.97%		11.34%		20.12%	
Total Arrival Delay (mins)	3141.16	2967.40	-5.53%	2952.52	-6.01%	3219.36	+2.49%
15-min On-Time Performance	76.53%	78.26%		78.48%		76.08%	
60-min On-Time Performance	96.89%	97.01%		97.04%		96.86%	
Passenger Delay Statistics							
Total Pax Delay (mins)	260565	256540	-1.54%	256211	-1.67%	260854	+0.11%
Total Disrupted Pax (pax)	47.56	64.52	+35.66%	67.16	+41.21%	36.80	-22.62%

Assume :

1) Each flight is allowed to move at most 15 minutes earlier or later
2) The first and last flights of each string are not allowed to move earlier and later, respectively

FR : Results

Average performance evaluation statistics over 25 days (March 1-25, 2008) for the FR models

	Original	FR_minPD		FR_maxEffACSlack15		FR_maxEffPaxSlack15	
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BA : Results

Average performance evaluation statistics over 25 days (March 1-25, 2008) for the BA models

	Original	BA_minTAD	
Schedule Statistics			
Total A/C Connection Slack (mins)	6676.76	4122.60	-38.25%
Average Block Time Change (mins)		10.55	
Flight Delay Statistics			
Total Propagated Delay (mins)	1009.60	827.76	-18.01%
% of Flights with PD > 0	17.74%	14.18%	
Total Arrival Delay (mins)	3141.16	1873.48	-40.36%
15-min On-Time Performance	76.53%	87.49%	
60-min On-Time Performance	96.89%	97.69%	
Passenger Delay Statistics			
Total Pax Delay (mins)	260565	178004	-31.69%
Total Disrupted Pax (pax)	47.56	62.12	+30.61%

1) Each flight is allowed to move at most 15 minutes earlier or later, and the maximum total change in block time is 15 minutes
2) The first and last flights of each string are not allowed to move earlier and later, respectively

Cost to Airlines

Cost to airlines of flight delays ($ Billions):

	Delay Against Schedule	Buffer	Total
7 major airlines	3.3	2.6	5.9
Industry-wide (US)	4.6	3.7	8.3

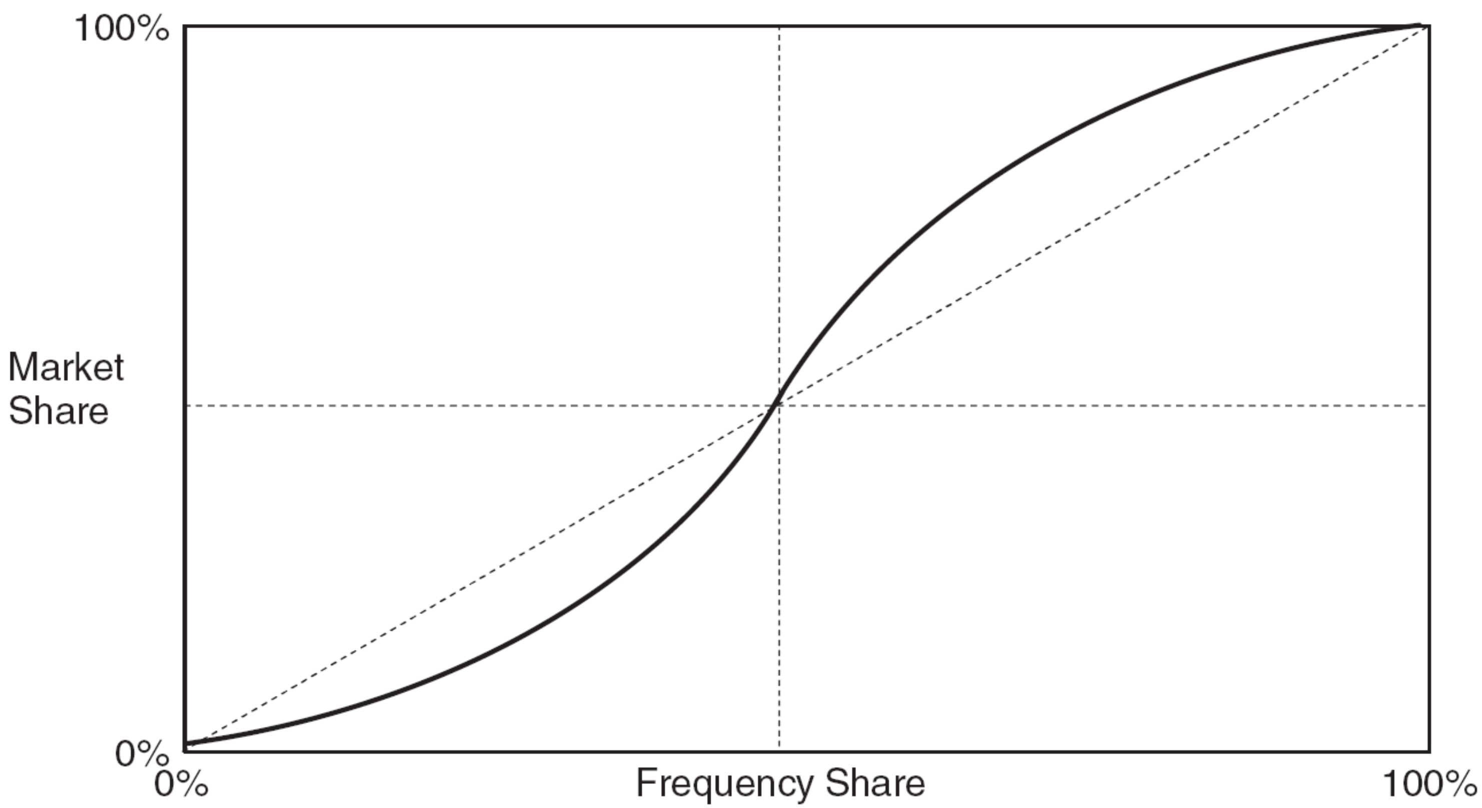
Service Network Design and Schedule Reliability: Possible Solutions

- The Airline:
  - Build 'robust' or 'reliable' flight networks that can absorb delays
  - *The role of schedule slack*

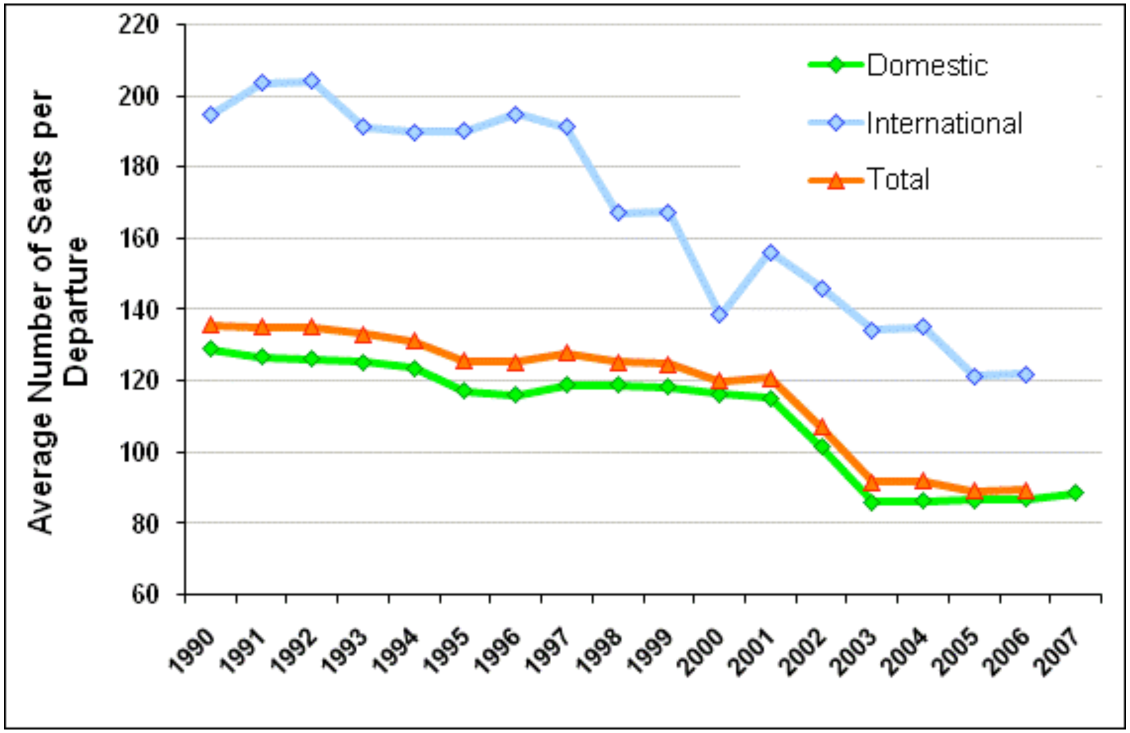
- The Aviation Authority (FAA in US)
  - Limit number of scheduled operations at airports (and in airspace)
  - *The role of competition*

Frequency Competition

- S-curve relationship between market share and frequency share
- Higher frequency shares associated with disproportionately higher market shares
  - And higher delays… but reductions in frequency and hence delays, result in loss of competitive position and profits



Airline Scheduling under Competition



[Source: Bonnefoy and Hansman, 2008]

LGA-BOS: 40 direct flights/day

Carrier	Flight No.	Dep. Time	Arr. Time
DL	1906	6:00	7:00
US	2114	6:00	7:00
DL	1908	6:30	7:34
MQ	4803	7:00	8:15
US	2116	7:00	8:12
DL	1910	7:30	8:37
US	2118	8:00	9:12
MQ	4802	8:20	9:30
DL	1912	8:30	9:40
US	2120	9:00	10:16
DL	1914	9:30	10:46
US	2122	10:00	11:15
DL	1916	10:30	11:47
MQ	4805	10:50	12:05
US	2124	11:00	12:15
DL	1918	11:30	12:46
US	2126	12:00	13:10
DL	1920	12:30	13:39
US	2128	13:00	14:11
DL	1922	13:30	14:39

Landing Slot Reduction Schemes

- Number slots available at an airport reduced by *x*%
  - Total number of allocated slots might equal IFR (bad weather) capacity at the airport
- Mechanisms for allocating the capacity to airlines
  1) Proportionate slot reduction
     - Number of slots at an airport allocated to each airline is *(100-x)*% of the number of flights the airline currently has scheduled at that airport
  2) Reward based slot reduction
     - Slot reduction for each carrier proportional to inverse of passengers/slot
     - Idea is to reward those who are using their slots efficiently

Our Goal

- To understand airline scheduling decisions under competition and to assess the impact of demand-management based congestion mitigation strategies on various concerned stakeholders

Objectives

1) To assess the *maximum* possible impact of demand management strategies on congestion and delays
2) To model airline competition using game theory and to provide theoretical justification of how competition aggravates the congestion problem
3) To investigate empirically the suitability of the Nash equilibrium solution concept for describing airline decisions
4) To investigate airline response to slot allocation strategies under competition
5) To quantify the benefits and costs of slot allocation strategies to various different stakeholders and overall social welfare

To assess the *maximum* possible impact of demand management strategies on congestion and delays

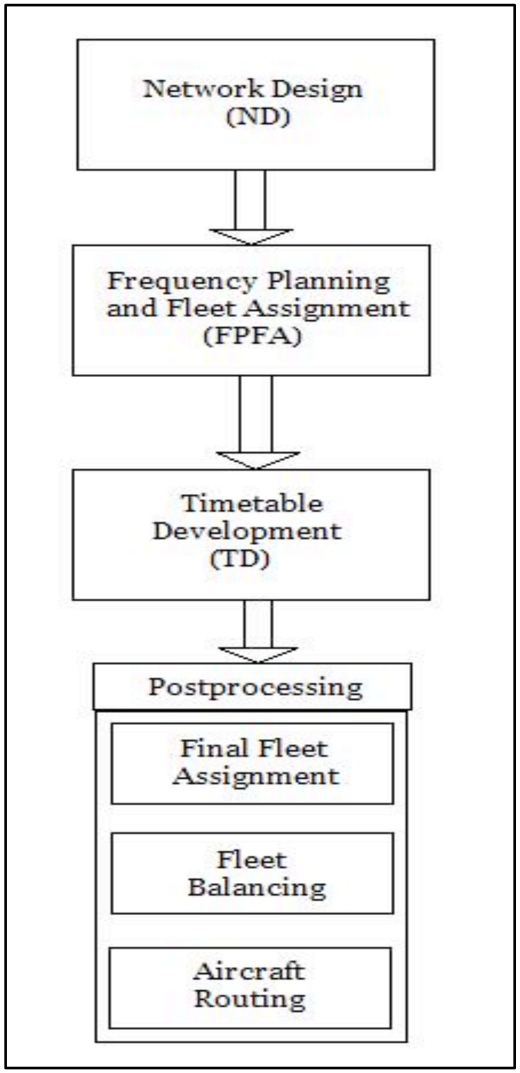
OBJECTIVE 1

Computation of a Lower Bound on Airport Congestion

- For the entire US aviation network
  - Design a schedule to minimize airport congestion
  - Assume a single monopolistic airline
  - Carry as many passengers as being carried currently for each market for each time of the day
  - Provide a daily frequency equal to the effective maximum daily frequency provided currently in that market

- Problem solved in 3 stages
  1. Network Design (ND): number of hubs, candidates for non-stop service and allowable airports where passengers can connect
  2. Frequency Planning and Fleet Assignment (FPFA)
  3. Timetable Development (TD)

  - FA is readjusted in the post processing
  - Decisions at a stage may be modified based on feedback from later stages



Integer Programming Formulation for Timetable Development

$$\text{Minimize } r_{max}$$

Minimize maximum utilization ratio

Subject to:

$$\sum_{i \in I(l)} y_i = D_l, \forall\, l \in L(m), m \in M$$

Demand constraint

$$x_f * C_f \geq \sum_{i \in I} \delta_f^i * y_i, \forall\, f \in F$$

Seat capacity constraint

$$\sum_{f \in F(s)} x_f \geq f_s, \forall\, s \in S$$

Minimum frequency constraint

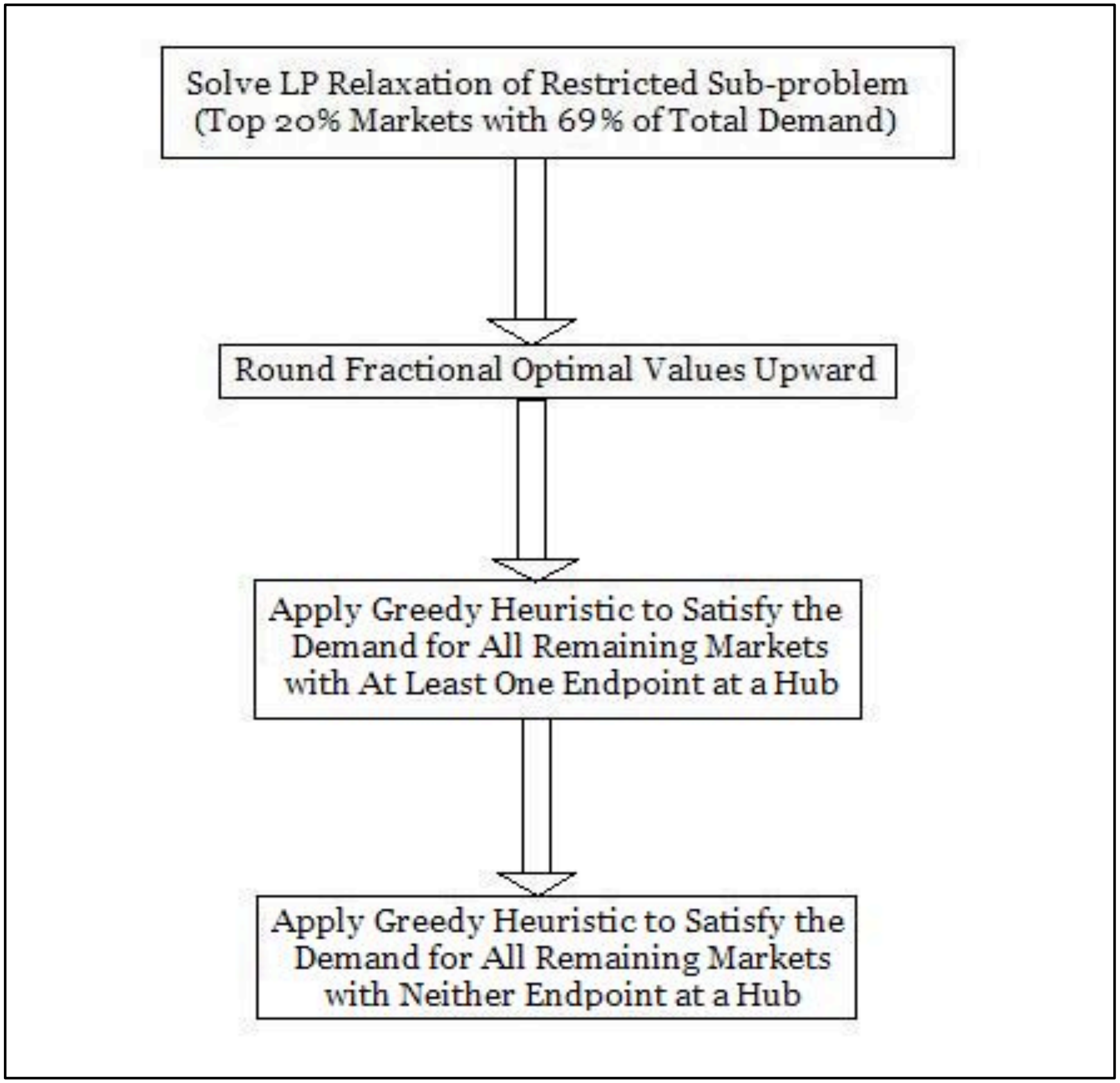
$$\sum_{f \in F} \gamma_f^t * x_f \leq r_{max} * HC_t\ \forall\, t \in T(a), a \in A$$

Utilization ratio cannot exceed maximum utilization

$$x_f \in \mathbb{Z}^+, \forall\, f \in F$$

$$y_i \in \mathbb{Z}^+, \forall\, i \in I$$

TD Solution Methodology



Results:
Cumulative Distribution of Utilization Ratios

Utilization Ratio	Actual Network	Optimized Network
>150%	1	0
> 140%	3	0
> 130%	8	0
> 120%	19	0
> 110%	28	0
> 100%	55	0
> 90%	76	35
> 80%	133	84
> 70%	196	153
> 60%	275	212
> 50%	350	309

- No more than 92% of bad-weather capacity (IFR) is required
- Substantial reduction in airport congestion can be achieved with existing capacity

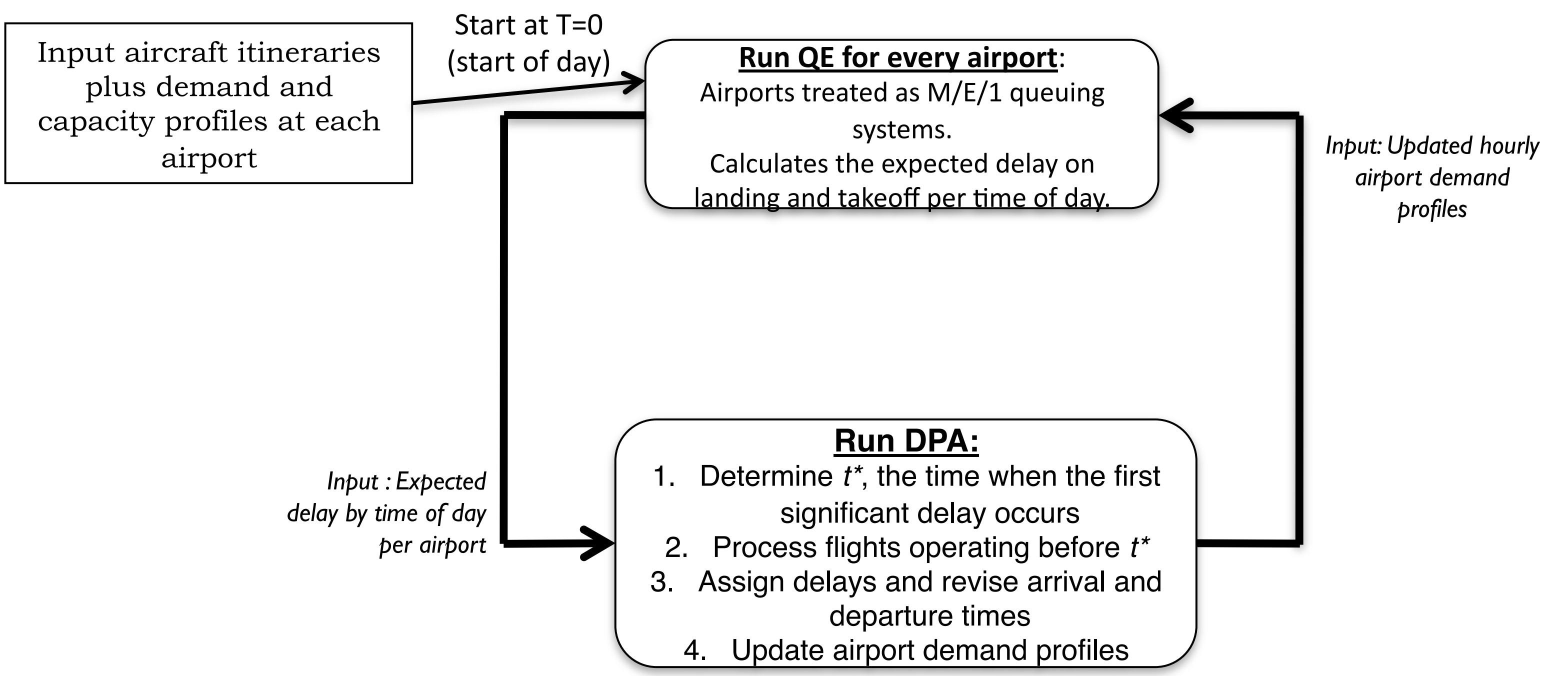
Delay Calculation

Goal: Given airline networks, calculate expected aircraft delays, capturing effects of delay propagation

- Calculate delays for current airline networks and 'single airline network' using actual airport capacity values
  - Airport capacity values available for an entire year
    - Assign each day to one of 5 equal sized buckets
    - 5 realistic scenarios: one corresponding to the median of each bucket

Delay Impact of Airport Congestion: Quantifying the Propagation of Delays in a Network of Airports

- Odoni and Pyrgiotis (2009): Network model to estimate:
  - delays to every flight at individual airports,
  - how these delays propagate through the network of airports, and
  - the effect of real-time mitigation actions, e.g., ground delay programs, airline schedule recovery intervention



Total Delay Comparison

Scenario	Existing Network Delay (aircraft-min)	Single Airline Network Delay (aircraft-min)	Reduction
Very Good	7495.05	3552.97	52.60%
Good	14682.30	4090.06	72.14%
Normal	27998.76	5940.40	78.78%
Bad	35081.44	6289.88	82.07%
Very Bad	64026.52	7421.76	88.41%
Average	29856.82	5459.02	81.72%

On the order of an 80% reduction in delays could have been achieved, had there been no competition

To model airline competition using game theory and to investigate the suitability of the Nash equilibrium solution concept for describing airline decisions

OBJECTIVES 2 AND 3

Optimization Under Slot Constraints

$$Maximize: \sum_{s \in S} (P_{a,s} * Q_{a,s} - C_{a,s} * f_{a,s})$$

Maximize total profit = fare revenue – operating cost

Subject to:

$$Q_{a,s} \leq \frac{f_{a,s}{}^{\propto}}{\sum_{a' \in A} f_{a',s}{}^{\propto}} * M_s \; \forall s \in S$$

S-curve relationship between market share and frequency share

$$Q_{a,s} \leq Seats_{a,s} * f_{a,s} \; \forall s \in S$$

Seating capacity constraint

$$\sum_{s \in S} f_{a,s} \leq MAX\_SLOTS_a$$

Maximum number of available slots

$$\sum_{s \in S} f_{a,s} \geq MIN\_SLOTS_a$$

Minimum number of slots that must be utilized (Use-It-Or-Lose-It)

$$f_{a,s} \in \mathbb{Z}^+ \; \forall s \in S$$

Multi-Agent Model

- A system of profit maximizing agents
- Optimal frequency decision (f_{as}) for an airline *a* on segment *s* depends on actions by other airlines (f_{-as}) and is constrained by number of available slots at airport *a*
- Nash Equilibrium:

  A frequency profile *f* is a Nash Equilibrium if for every airline *a*,

  f_a is the best response to f_{-a}

- Solution Methodology: "Myopic Best Response"
  - While there exists a carrier whose current decision is not optimal in relation to others' decisions, re-optimize for that carrier

Solution Algorithm

- **Myopic Best Response** algorithm for equilibrium computation
- Successive optimizations using **Dynamic Programming**
  - Slot restrictions are the only coupling constraints across segments
  - Objective function is additive across segments
- No. of stages = No. of segments
- No. of states per stage = Maximum no. of slots

$$\text{Profit}(s, n) = \text{Segment s profit due to exactly n flights per day}$$

$$R(0,0) = 0, \quad R(0,n) = -\infty \; for \; n \geq 1$$

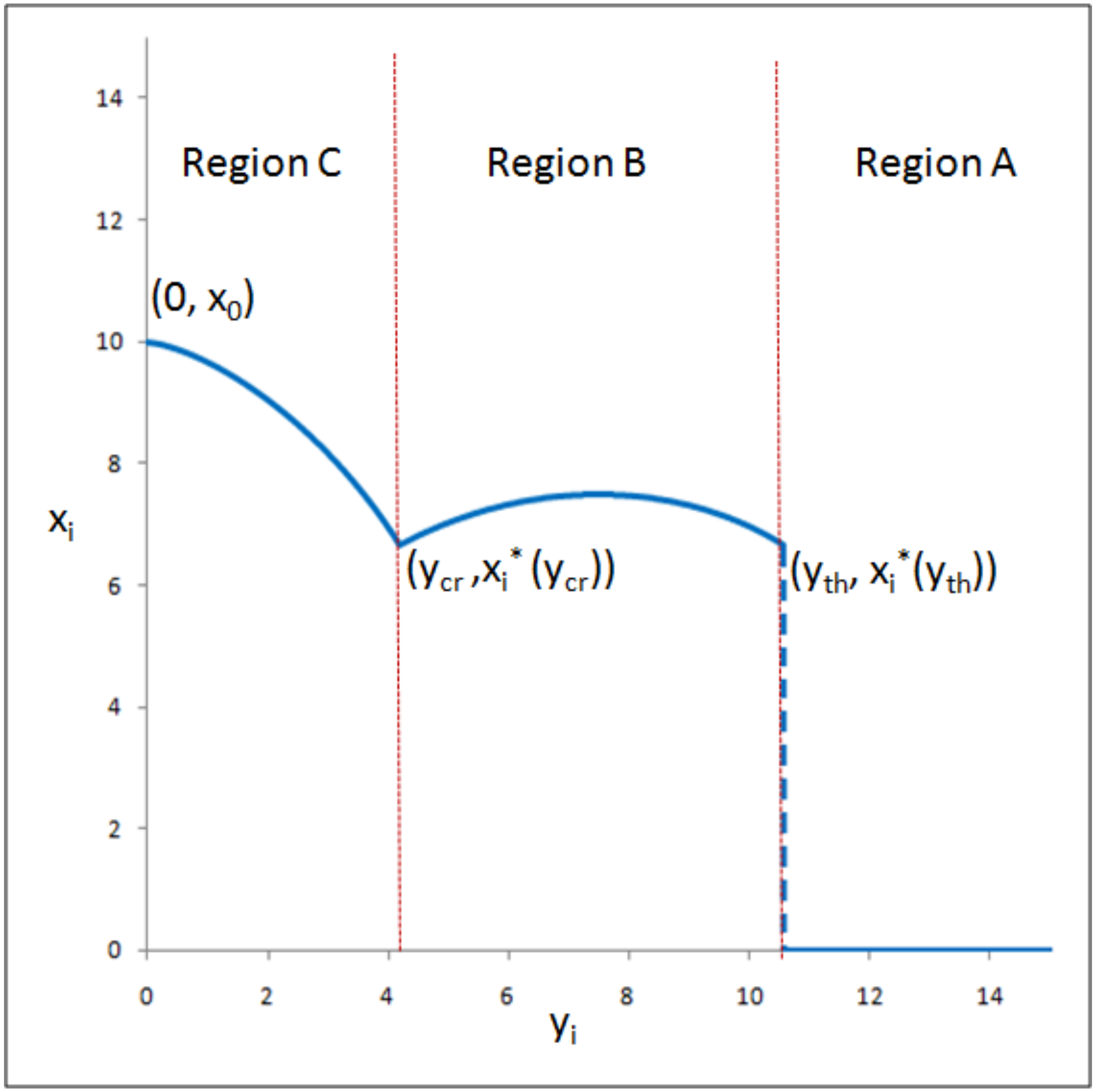
$$R(s,n) = \max_{0 \leq n' \leq n} \left(R(s-1, n') + Profit(s, n-n') \right)$$

$$Optimal\ total\ profit = \max_{MIN\_SLOTS_a \leq n \leq MAX\_SLOTS_a} R(|S|, n)$$

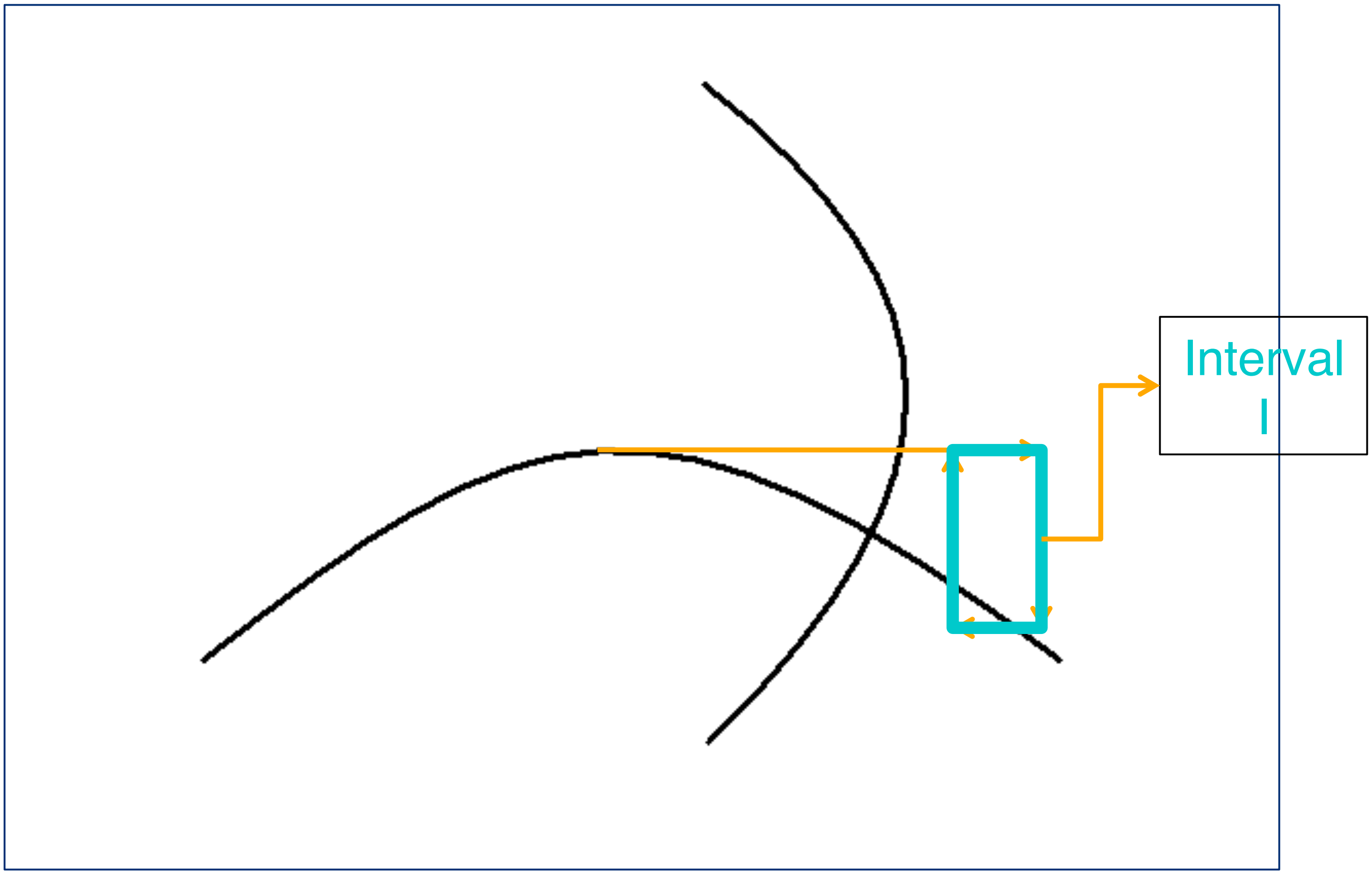
Model Limitations

- Leg-based demand assumption
  - Ignores passengers connections
- Constant average fares assumption
  - Fare assumed constant for each carrier in each market
  - But in reality there is differential pricing and revenue management
  - Pricing used as a competitive tool and interacts with frequency competition
- Aircraft availability and rotation constraints ignored
- Assumption of constant aircraft sizes for each carrier on each leg
  - This assumption is partially relaxed later

A Typical Best Response Function



Central Idea of the Convergence Proof



Convergence Proof

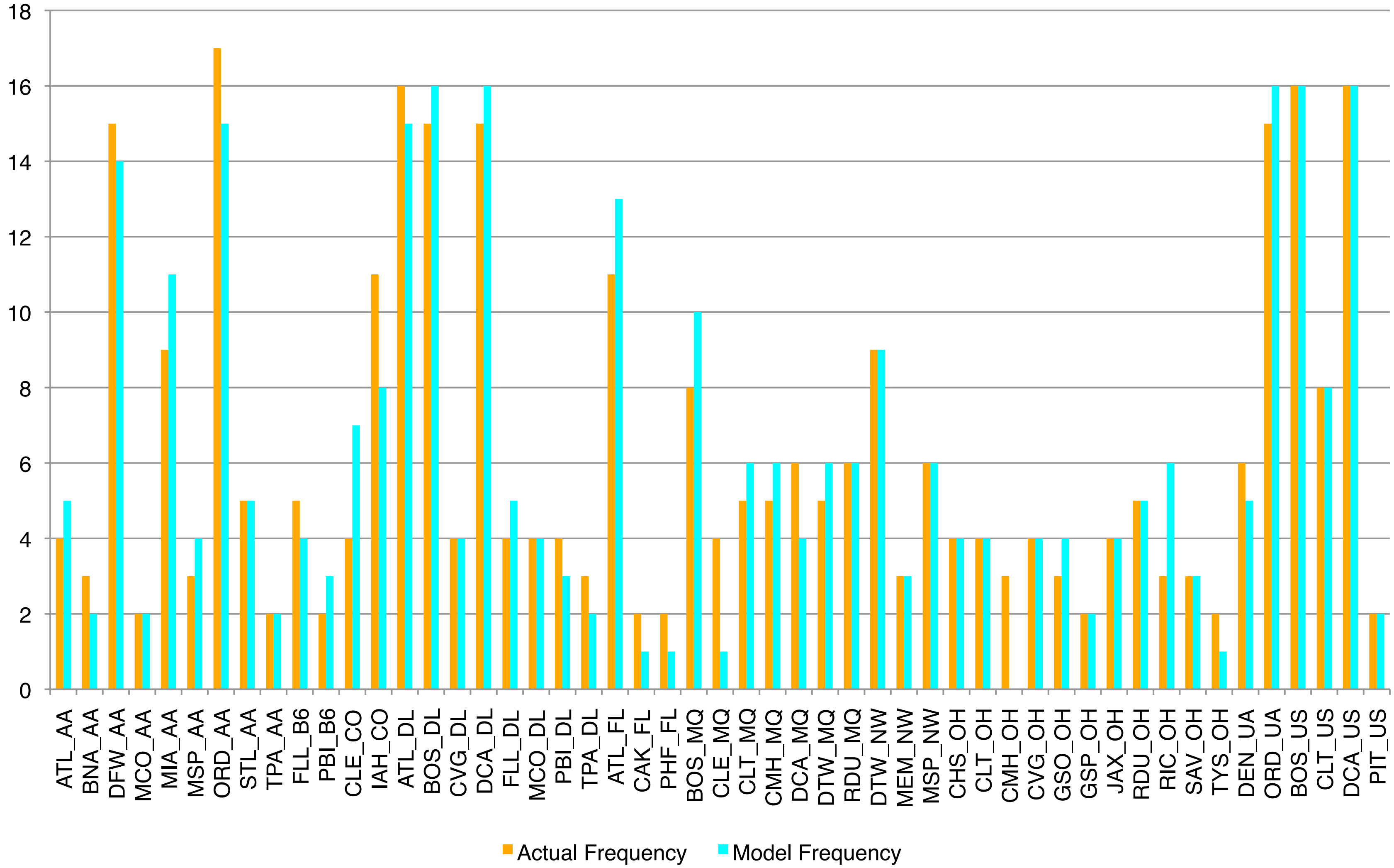
- Best-response algorithm enters the interval I in finite iterations
- Once inside, it cannot exit the interval I
- Once inside I, use a Lyapunov function argument

$$L\left(i+1\right) = \left|\gamma_{i+1} - \gamma_{eq}\right| = \left|\gamma_{BR}\left(\chi_i\right) - \gamma_{BR}\left(\chi_{eq}\right)\right| = \left|\int\limits_{\chi_{eq}}^{\chi_i}\left(\frac{\partial\gamma\left(\chi\right)}{\partial\chi}\right)d\chi\right|$$

$$\leq \left|\int\limits_{\chi_{eq}}^{\chi_i}\left|\frac{\partial\gamma\left(\chi\right)}{\partial\chi}\right|d\chi\right| < \left|\int\limits_{\chi_{eq}}^{\chi_i}1d\chi\right| = \left|\chi_i - \chi_{eq}\right| = L\left(i\right)$$

- L(i) is nonnegative and strictly decreasing function of i at any non-equilibrium point
- At equilibrium L(i) = 0
- Hence convergence!

Accuracy of Frequency Predictions



n-Player Symmetric Case

- $n \geq 2$ number of identical players
- Symmetric equilibrium seems to be the only reasonable equilibrium (non-zero frequency values and less than 100% load factor)
- It's also the 'worst-case' equilibrium- equilibrium with maximum 'social' cost
- We look at the ratio of the total cost to all competing airlines under the equilibrium with the maximum social costs to the total cost under the system optimal solution
- A measure that is a proxy for:
  - Airport congestion (and delays)
  - Airline profit degradation
  - Total cost of passenger transportation

Result Summary

- Nash equilibrium describes actual decisions reasonably well (within 6.5% error)
- Computational evidence of strong convergence properties:
  - Best response heuristic always found an equilibrium in very few iterations
  - Convergence independent of starting point
    - Exact payoff functions are complex, but in the useful range usually strictly concave

Price of Frequency Competition

- Price of airline frequency competition $\frac{\alpha p S}{C} \frac{n-1}{n}$
  - The ratio of the total cost to all competing airlines under the equilibrium with the maximum social costs to the total cost under the system optimal solution
- Varies with
  - α : S-curve parameter (intensity of competition); greater the curvature of S-curve, greater is the inefficiency
  - (pS/C) : 'full load profitability'; more profitable the segment, greater is the inefficiency
  - n: number of competitors; greater the number of competitors, greater is the inefficiency
- A typical (median) value = 1.48 (airlines lose about 48% due to decentralization)

To investigate airline response to slot allocation strategies

OBJECTIVE 4

Slot Reduction Schemes

1) Proportionate slot reduction
   - Number of slots available to each carrier reduced by same proportion
2) Reward based slot reduction
   - Slot reduction for each carrier proportional to inverse of passengers/slot
   - Idea is to reward those who are using their slots efficiently

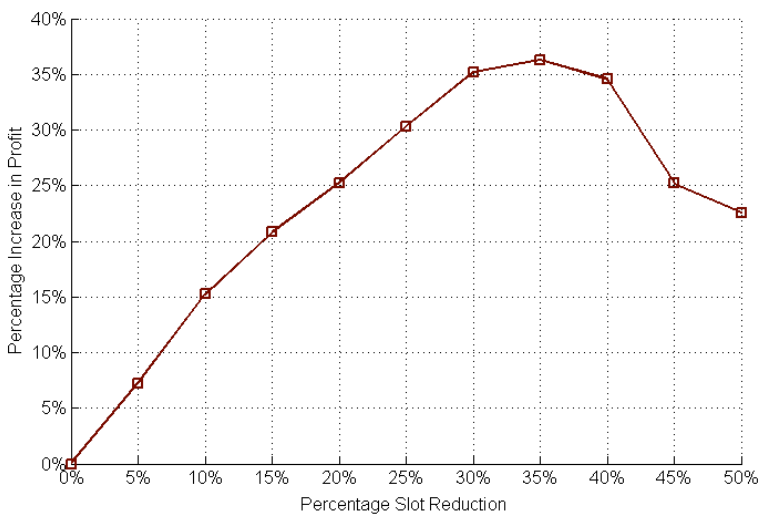
- Airline response evaluated using Nash Equilibrium concept and models presented in previous section

Overall Impact

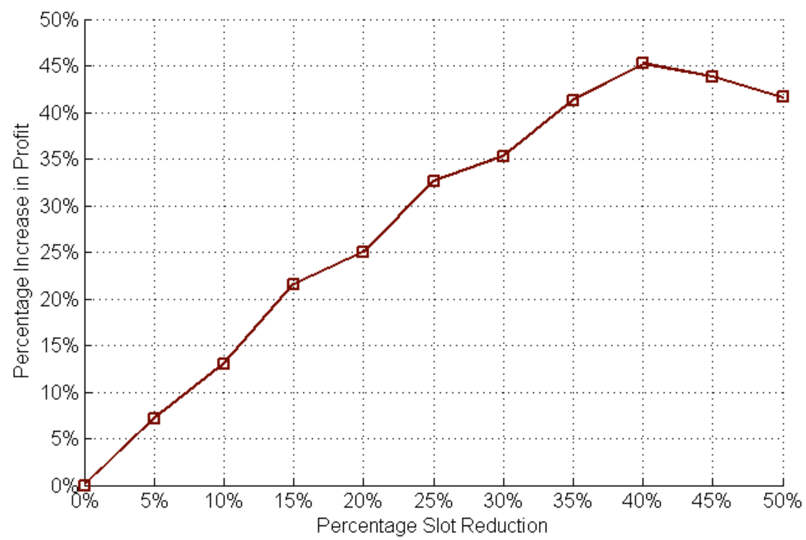
Scheme	Do Nothing	20% Reduction (Proportionate)	20% Reduction (Reward-based)
Total Operating Profit	$ 1,252,362	$ 1,568,814 (25.27%)	$ 1,565,490 (25.00%)
Passengers Carried	22,260	21,291 (-4.35%)	21,464 (-3.58%)
NAS Delay per Flight	12.74 min	7.52 min (-40.97%)	7.52 min (-40.97%)

Increase in Profit vs. Slot Reduction Scheme

Proportionate Slot Reduction Scheme



Reward-Based Slot Reduction Scheme

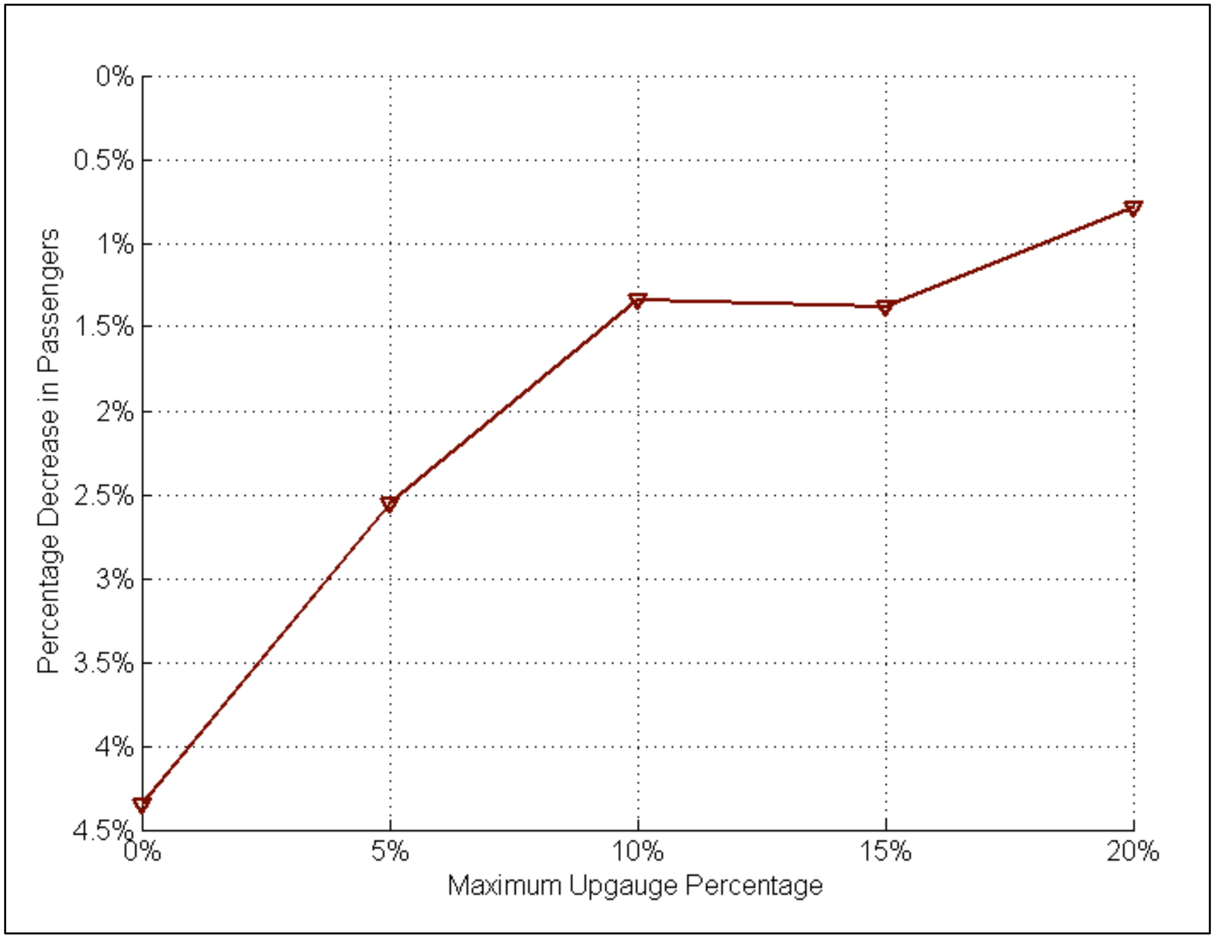


Impact on Individual Airlines

Slots	100%	80%			
Scheme	-	Proportionate		Reward Based	
Carrier	Profit	Profit	Profit Increase	Profit	Profit Increase
AA	365,582	447,897	22.52%	422,943	15.69%
CO	66,450	73,205	10.17%	79,820	20.12%
DL	188,352	285,531	51.59%	274,352	45.66%
FL	36,908	52,891	43.30%	55,406	50.12%
MQ	33,630	43,579	29.58%	35,705	6.17%
NW	107,006	107,920	0.85%	127,265	18.93%
OH	34,638	54,144	56.31%	54,916	58.54%
UA	200,796	233,188	16.13%	241,936	20.49%
US	170,939	225,209	31.75%	227,897	33.32%
Total	1,252,362	1,568,814	25.27%	1,565,490	25.00%

Impact of Limited Number of Aircraft Upgauges

Decrease in Number of Passengers Vs. Upgauge Percentage



To quantify the benefits and costs to various different stakeholders and the overall social welfare

OBJECTIVE 5

Passenger-centric analysis

- **Goal:** measure system performance through passenger delays instead of flight delays
- **Motivation:** non-linear relationship between passenger and flight delays
  - Longer flight delays lead to flight cancellations and missed connections (Bratu, Barnhart 2005)
- **Challenge:** itinerary data is not publicly available
  - Passenger delay estimates vary widely from study to study
    - For 2007 calendar year:
      - $12 Billion (as per US Congress Joint Economic Committee report, 2008)
      - $5 Billion (Air Transport Association, 2008)
      - Both studies ignore passenger delays due to cancellations and missed connections
  - Primary obstacle is the unavailability of disaggregate passenger delay data
    - Publicly available data is aggregated monthly or quarterly
- **Approach:** estimate historical passenger itineraries to calculate passenger delays

Passenger Delay Analysis

- Passenger delays calculated using an extended multi-carrier version of the passenger delay calculator (Bratu and Barnhart, 2005)
- Passenger delays approximately double that of passenger-weighted aircraft delays
  - 50% due to flight delays
  - 33% due to cancellations
  - 17% due to missed connections
- Reducing aircraft delays by about 40%, results in reduction of passenger delay minutes (in 2007) of 5.92 billion minutes of passenger delay, or $3.7 billion savings
  - Assuming $37.6/hr value of passenger time (same as the one used in JEC report), the total cost of passenger delays

Summary

- Airline service network design models that capture competitive effects have increased schedule frequency, with substantial flight and passenger delays, compared to service network designs ignoring competition
- Adding ‘robustness’ to airline schedules, holding all else constant, has (limited) impact, but is expensive
- Demand management strategies such as slot constraints at congested airports can result in reduced flight and passenger delays and increased airline profitability

QUESTIONS?