Transportation Service
Network Design

Cynthia Barnhart
Massachusetts Institute of Technology

Montreal Spring School on Supply Chain and Transportation Network Design
May 12-14, 2010
Outline

• Transportation Service Network Design
  – Airline scheduling examples
  – Interplay between service network design and schedule reliability

• Impact of service network design and scheduling on aviation system performance
  – Cost of congestion

• Service network design and scheduling responses to reduce congestion and delays
  – The role of schedule slack
  – The role of demand management and competition
Airline Schedule Design

• Schedule design addresses the questions of:
  Ø Where to fly?
  Ø How frequently to fly?
  Ø When to fly?
  Ø How much capacity to provide on each flight leg?

• The output is a set of scheduled flight legs, with assigned capacity, that forms the input to all subsequent planning operations
  - Operating and profitability consequences
Optimization of Airline Service Network Design

• Optimization approaches to schedule design face numerous challenges
  – The ‘tractability’ issue
    • Need to determine where to assign ‘lumpy’ capacity and how to flow individual demands
    • Very large-scale nature of the problems
  – The ‘reliability’ or ‘robustness’ issue
    • Schedules are never executed as planned, and the cost of recovering from unplanned disturbances is huge
  – The competition issue
    • Optimization models ignoring competitive factors result in service network designs that might overestimate revenue capture, and hence, profitability of the service network
Service Network Design and Schedule Reliability: The Fundamental Issue

- Demand for aviation system capacity exceeds amount available
  - At airports in US
    - During operations, the number of operations (flight departures and arrivals) can exceed the capacity at airports
    - Airport capacity is stochastic
      - Bad weather results in reduced airport capacities, allowing fewer departures and arrivals per unit time, to ensure safe operations
    - In US, number of scheduled operations at most airports is NOT limited by airport capacity

- Result of demand-capacity imbalance in aviation system
  - During operations, schedule delays imposed to ensure number of operations does not exceed airport capacity during operations
  - Delays propagate to downstream flights
  - Flights are canceled
  - Passengers misconnect and must be re-accommodated on later flights with available seats- resulting in long passenger delays
Airline Delays

- Nearly 50% of the delays were due to National Aviation System (NAS)

- Majority of National Aviation System (NAS) delays attributed to scheduling more than the realized capacity
  - 90% + of NAS delays

Causes of National Aviation System Delays

- Weather -- 63.45%
- Volume -- 29.05%
- Equipment -- 0.01%
- Closed Runway -- 4.47%
- Other -- 3.02%

### Cost to Airlines

#### Cost to airlines of flight delays ($ Billions):

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<tr>
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Passenger Trip Delays (2007)

- 7.45 million flights
- 487.2 million passengers
- 4437 direct routes between 267 airports.
- Average number of flights between O/D pairs in 2007 was 4.57
- Total Passenger Trip Delay - 28,539 years
- Average Passenger Trip Delay - 31 min/pax
  - Average Passenger Trip Delay for Delayed or Disrupted Passengers
Outline

• Transportation Service Network Design
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• Impact of service network design and scheduling on aviation system performance
  – Cost of congestion

• Service network design and scheduling responses to mitigate delays
  – The role of schedule slack
  – The role of demand management and competition
Service Network Design and Schedule Reliability: Possible Solutions

• The Airline:
  – Build ‘robust’ or ‘reliable’ flight networks that can absorb delays
  – The role of schedule slack

• The Aviation Authority (FAA in US)
  – Limit number of scheduled operations at airports (and in airspace)
  – The role of competition
Slack in Airline Schedules

- Slack is additional time beyond the minimum requirement
  - Aircraft connection, passenger connection, and flight block time

- Slack is desirable in robust schedules
  - Absorb delays in the airline network
  - Reduce the likelihood of downstream propagation of disruptions
  - Obviates need to recover, or provides recovery options

- Slack can be very costly
  - Decreases the utilization of resources in airline operations

- We seek to re-allocate the existing slack such that ..
  - The resulting distribution of slack is more effective in minimizing delays and disruptions
Schedule Aircraft and Passenger Slack
Aircraft Routing/ Scheduling Approach

Jan-Feb 2008 Data

Aircraft routes: slack allocation models

- Each day of operation in Jan-Feb represents one instance of delay scenario $\omega$ ($|\Omega|=60$)
- Assume each delay scenario is equally likely
- Solve 3 different slack allocation models over each day of operation
Robust Aircraft Re-routing (AR)

- Flight schedule and fleet assignments are fixed
- Aircraft assignment of each flight can be changed
  - Affect aircraft connection slack
AR : Optimization Model

• We focus on designing aircraft routes (daily)
  – Assuming that the maintenance feasibility is preserved

• **Goal:** Minimize expected total propagated delay
  – Equivalent to minimizing total delay

• **Constraints:**
  – Every flight is assigned to exactly one aircraft
  – Only use available aircraft

• Use a string-based formulation
  – Each flight string represents a set of flight legs that are operated by a single aircraft on a given day of operation
Minimize \[ \mathbb{E} \left[ \sum_{s \in S} \left( x_s \times \sum_{(i,j) \in S} p_{d_{ij}}^s \right) \right] \]

subject to \[ \sum_{s \in S} a_{is} x_s = 1 \quad \forall i \in F \]
\[ \sum_{s \in S_{m+}} x_s = N_{m^+} \quad \forall m^+ \in M^+ \]
\[ x_s \in \{0, 1\} \quad \forall s \in S \]
AR: Optimization Model

Rewrite the objective function:

\[
\mathbb{E} \left[ \sum_{s \in S} \left( x_s \times \sum_{(i,j) \in S} p_{d_{ij}}^s \right) \right] = \sum_{s \in S} \left( x_s \times \mathbb{E} \left[ \sum_{(i,j) \in S} p_{d_{ij}}^s \right] \right)
\]

\[
= \sum_{\omega \in \Omega} p_\omega \sum_{(i,j) \in S} p_{d_{ij}}^s(\omega)
\]

Can be computed offline!
AR : Alternative Objective Function

- From $PD_{ij} = \max(TAD_i - Slack_{ij}, 0)$
AR : Alternative Objective Function

- Maximizing the total effective slack

\[
\overline{Slack}_{ij} = Slack_{ij} - TAD_i
\]

\[
\overline{Slack}_{ij}(\Gamma_{ij}) = \min(\text{Slack}_{ij} - TAD_i, \Gamma_{ij})
\]
Robust Flight Schedule Re-timing (FR)

• Aircraft routing is fixed
• Departure times are allowed to shifted earlier or later, but block times are fixed
  – Also affect passenger connection slack
FR : Optimization Model

Minimize \[ \sum_{(i,j) \in A} \mathbb{E}[pd_{ij}] = \sum_{(i,j) \in A} \left( \sum_{\omega \in \Omega} p_{\omega}pd_{ij}^{\omega} \right) \]

- \[ aSlack'_{ij} = aSlack_{ij} - x_i + x_j \quad \forall (i,j) \in A \]
- \[ aSlack'_{ij} \geq 0 \quad \forall (i,j) \in A \]
- \[ pSlack'_{ij} = pSlack_{ij} - x_i + x_j \quad \forall (i,j) \in P \]
- \[ pSlack'_{ij} \geq 0 \quad \forall (i,j) \in P \]
- \[ pd_{ij}^{\omega} \geq tad_i^{\omega} - aSlack'_{ij} \quad \forall (i,j) \in A, \forall \omega \in \Omega \]
- \[ pd_{ij}^{\omega} \geq 0 \quad \forall (i,j) \in A, \forall \omega \in \Omega \]
- \[ tad_i^{\omega} \geq IAD_i^{\omega} \quad \forall i \in F_0, \forall \omega \in \Omega \]
- \[ tad_j^{\omega} \geq pd_{ij}^{\omega} + IAD_j^{\omega} \quad \forall (i,j) \in A, \forall \omega \in \Omega \]
- \[ tad_i^{\omega} \geq 0 \quad \forall i \in F, \forall \omega \in \Omega \]
- \[ l_i \leq x_i \leq u_i \quad \forall i \in F \]
- \[ x_i \in \mathbb{Z}^n \quad \forall i \in F \]
FR: Alternative Objective Functions

- Maximizing the total expected effective *aircraft* connection slack:

\[
\text{Maximize } \sum_{(i,j) \in A} \left( \sum_{\omega \in \Omega} p_{\omega} a\text{Slack}_{ij}^\omega \right)
\]

\[
a\text{Slack}_{ij}^\omega \leq a\text{Slack}_{ij}' - tad_i^\omega \quad \forall (i, j) \in A
\]

\[
a\text{Slack}_{ij}^\omega \leq \Gamma_{ij} \quad \forall (i, j) \in A
\]

- Maximizing the total expected effective *passenger* connection slack:

\[
\text{Maximize } \sum_{(i,j) \in P} \left( \sum_{\omega \in \Omega} p_{\omega} p\text{Slack}_{ij}^\omega \right)
\]

\[
p\text{Slack}_{ij}^\omega \leq p\text{Slack}_{ij}' - tad_i^\omega \quad \forall (i, j) \in P
\]

\[
p\text{Slack}_{ij}^\omega \leq \Gamma_{ij} \quad \forall (i, j) \in P
\]
Robust Block Time Adjustment (BA)

- Aircraft routing is fixed
- Departure and arrival times are allowed to change independently
  - Also affect block time slack
BA: Optimization Model

Minimize \( \sum_{i \in F} \mathbb{E}[tad_i] = \sum_{i \in F} \left( \sum_{\omega \in \Omega} p_{\omega} tad_{i,\omega}^\omega \right) \)

- \( aSslack'_{ij} = aSslack_{i,j} - y_i + x_j \quad \forall (i, j) \in A \)
- \( aSslack'_{ij} \geq 0 \quad \forall (i, j) \in A \)
- \( pSslack'_{ij} = pSslack_{i,j} - y_i + x_j \quad \forall (i, j) \in P \)
- \( pSslack'_{ij} \geq 0 \quad \forall (i, j) \in P \)

\[
\begin{align*}
pd_{ij} &\geq tad_{i}^\omega - aSslack_{ij}^\omega \quad \forall (i, j) \in A, \forall \omega \in \Omega \\
xd_{ij} &\geq 0 \\
tad_{i}^\omega &\geq IAD_{i}^\omega + x_i - y_i \quad \forall i \in F_0, \forall \omega \in \Omega \\
tad_{j}^\omega &\geq pd_{ij} + IAD_{j}^\omega + x_j - y_j \quad \forall (i, j) \in A, \forall \omega \in \Omega \\
tad_{i}^\omega &\geq 0 \\
l_i &\leq y_i - x_i \leq u_i \\
l_{x_i} &\leq x_i \leq u_{x_i} \\
l_{y_i} &\leq y_i \leq u_{y_i} \\
x_i, y_i &\in \mathbb{Z}^n
\end{align*}
\]

Aircraft Connection Slack
Passenger Connection Slack
Propagated Delay
Total Arrival Delay
Allowable Changes
BA : Optimization Model

The polyhedron formed by the constraints in the BA formulation is integral, given that all data and parameters in those constraints are integral.

- **Proof:** use Ghouila-Houri’s characterization to show that the coefficient matrix is totally unimodular
- Consequently, we can relax the integrality constraint and solve the problem as an LP
Data and Evaluation Process

Jan-Feb 2008 Data

Routing and scheduling models

Planned Schedule (for March)

March 2008 Data

Simulation

Performance Evaluation Statistics

Passenger Delay is computed based on the Passenger Delay Calculator Algorithm by Bratu and Barnhart (2005)

- Disrupted passengers are re-accommodated on a first-come-first-serve basis
- Maximum passenger delay of 12 hours
AR : Results

Average performance evaluation statistics over 25 days (March 1-25, 2008) for the AR models

<table>
<thead>
<tr>
<th>Flight Delay Statistics</th>
<th>Original</th>
<th>AR_minPD</th>
<th>AR_maxEffACSlack15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Propagated Delay (mins)</td>
<td>1009.60</td>
<td>818.60</td>
<td>-18.92%</td>
</tr>
<tr>
<td>% of Flights with PD &gt; 0</td>
<td>17.74%</td>
<td>14.86%</td>
<td></td>
</tr>
<tr>
<td>Total Arrival Delay (mins)</td>
<td>3141.16</td>
<td>2965.56</td>
<td>-5.59%</td>
</tr>
<tr>
<td>15-min On-Time Performance</td>
<td>76.53%</td>
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<td>96.89%</td>
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<td></td>
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</table>

| Passenger Delay Statistics | | | |
| Total Pax Delay (mins) | 260565 | 250325 | -3.93% | 246903 | -5.24% |
| Total Disrupted Pax (pax) | 47.56 | 45.16 | -5.05% | 44.72 | -5.97% |
AR : Results

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Service Design - Schedule Reliability Interplay -> Role of Slack -> Role of Demand Management and Competition
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<td>+41.21%</td>
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Assume:

1) Each flight is allowed to move at most 15 minutes earlier or later
2) The first and last flights of each string are not allowed to move earlier and later, respectively
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<tr>
<td><strong>Schedule Statistics</strong></td>
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<td></td>
</tr>
<tr>
<td>Total A/C Connection Slack</td>
<td>6676.76</td>
<td>4122.60</td>
</tr>
<tr>
<td>(mins)</td>
<td></td>
<td>-38.25%</td>
</tr>
<tr>
<td>Average Block Time Change</td>
<td>10.55</td>
<td></td>
</tr>
<tr>
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<td></td>
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<td>+30.61%</td>
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1) Each flight is allowed to move at most 15 minutes earlier or later, and the maximum total change in block time is 15 minutes
2) The first and last flights of each string are not allowed to move earlier and later, respectively
## Cost to Airlines

### Cost to airlines of flight delays ($ Billions):

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Service Network Design and Schedule Reliability: Possible Solutions

- **The Airline:**
  - Build ‘robust’ or ‘reliable’ flight networks that can absorb delays
  - *The role of schedule slack*

- **The Aviation Authority (FAA in US):**
  - Limit number of scheduled operations at airports (and in airspace)
  - *The role of competition*
Frequency Competition

- S-curve relationship between market share and frequency share
- Higher frequency shares associated with disproportionately higher market shares
  - And higher delays… but reductions in frequency and hence delays, result in loss of competitive position and profits
## Airline Scheduling under Competition

**LGA-BOS:** 40 direct flights/day

<table>
<thead>
<tr>
<th>Carrier</th>
<th>Flight No.</th>
<th>Dep. Time</th>
<th>Arr. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL</td>
<td>1906</td>
<td>6:00</td>
<td>7:00</td>
</tr>
<tr>
<td>US</td>
<td>2114</td>
<td>6:00</td>
<td>7:00</td>
</tr>
<tr>
<td>DL</td>
<td>1908</td>
<td>6:30</td>
<td>7:34</td>
</tr>
<tr>
<td>MQ</td>
<td>4803</td>
<td>7:00</td>
<td>8:15</td>
</tr>
<tr>
<td>US</td>
<td>2116</td>
<td>7:00</td>
<td>8:12</td>
</tr>
<tr>
<td>DL</td>
<td>1910</td>
<td>7:30</td>
<td>8:37</td>
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<tr>
<td>US</td>
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<td>8:00</td>
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</tr>
<tr>
<td>MQ</td>
<td>4802</td>
<td>8:20</td>
<td>9:30</td>
</tr>
<tr>
<td>DL</td>
<td>1912</td>
<td>8:30</td>
<td>9:40</td>
</tr>
<tr>
<td>US</td>
<td>2120</td>
<td>9:00</td>
<td>10:16</td>
</tr>
<tr>
<td>DL</td>
<td>1914</td>
<td>9:30</td>
<td>10:46</td>
</tr>
<tr>
<td>US</td>
<td>2122</td>
<td>10:00</td>
<td>11:15</td>
</tr>
<tr>
<td>DL</td>
<td>1916</td>
<td>10:30</td>
<td>11:47</td>
</tr>
<tr>
<td>MQ</td>
<td>4805</td>
<td>10:50</td>
<td>12:05</td>
</tr>
<tr>
<td>US</td>
<td>2124</td>
<td>11:00</td>
<td>12:15</td>
</tr>
<tr>
<td>DL</td>
<td>1918</td>
<td>11:30</td>
<td>12:46</td>
</tr>
<tr>
<td>US</td>
<td>2126</td>
<td>12:00</td>
<td>13:10</td>
</tr>
<tr>
<td>DL</td>
<td>1920</td>
<td>12:30</td>
<td>13:39</td>
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<tr>
<td>US</td>
<td>2128</td>
<td>13:00</td>
<td>14:11</td>
</tr>
<tr>
<td>DL</td>
<td>1922</td>
<td>13:30</td>
<td>14:39</td>
</tr>
</tbody>
</table>

[Source: Bonnefoy and Hansman, 2008]
Landing Slot Reduction Schemes

• Number slots available at an airport reduced by x%
  – Total number of allocated slots might equal IFR (bad weather) capacity at the airport

• Mechanisms for allocating the capacity to airlines
  1) Proportionate slot reduction
     - Number of slots at an airport allocated to each airline is \((100-x)\)%
     of the number of flights the airline currently has scheduled at that airport
  2) Reward based slot reduction
     - Slot reduction for each carrier proportional to inverse of passengers/slot
     - Idea is to reward those who are using their slots efficiently
Our Goal

• To understand airline scheduling decisions under competition and to assess the impact of demand-management based congestion mitigation strategies on various concerned stakeholders
Objectives

1) To assess the maximum possible impact of demand management strategies on congestion and delays

2) To model airline competition using game theory and to provide theoretical justification of how competition aggravates the congestion problem

3) To investigate empirically the suitability of the Nash equilibrium solution concept for describing airline decisions

4) To investigate airline response to slot allocation strategies under competition

5) To quantify the benefits and costs of slot allocation strategies to various different stakeholders and overall social welfare
To assess the maximum possible impact of demand management strategies on congestion and delays

OBJECTIVE 1
Computation of a Lower Bound on Airport Congestion

• For the entire US aviation network
  – Design a schedule to minimize airport congestion
  – Assume a single monopolistic airline
  – Carry as many passengers as being carried currently for each market for each time of the day
  – Provide a daily frequency equal to the effective maximum daily frequency provided currently in that market

• Problem solved in 3 stages
  1. Network Design (ND): number of hubs, candidates for non-stop service and allowable airports where passengers can connect
  2. Frequency Planning and Fleet Assignment (FPFA)
  3. Timetable Development (TD)
  – FA is readjusted in the post processing
  – Decisions at a stage may be modified based on feedback from later stages
Integer Programming Formulation for Timetable Development

Minimize $r_{\text{max}}$

Subject to:

$\sum_{i \in I(l)} y_{i} = D_{l}, \forall \ l \in L(m), m \in M$

$x_{f} * C_{f} \geq \sum_{i \in I} \delta_{f}^{i} * y_{i}, \forall \ f \in F$

$\sum_{f \in F(s)} x_{f} \geq f_{s}, \forall \ s \in S$

$\sum_{f \in F} g_{f}^{t} * x_{f} \leq r_{\text{max}} * H C_{t} \forall \ t \in T(a), a \in A$

$x_{f} \in \mathbb{Z}^{+}, \forall \ f \in F$

$y_{i} \in \mathbb{Z}^{+}, \forall \ i \in I$

Minimize maximum utilization ratio

Demand constraint

Seat capacity constraint

Minimum frequency constraint

Utilization ratio cannot exceed maximum utilization
TD Solution Methodology

1. Solve LP Relaxation of Restricted Sub-problem (Top 20% Markets with 69% of Total Demand)

2. Round Fractional Optimal Values Upward

3. Apply Greedy Heuristic to Satisfy the Demand for All Remaining Markets with At Least One Endpoint at a Hub

4. Apply Greedy Heuristic to Satisfy the Demand for All Remaining Markets with Neither Endpoint at a Hub
Results:
Cumulative Distribution of Utilization Ratios

<table>
<thead>
<tr>
<th>Utilization Ratio</th>
<th>Actual Network</th>
<th>Optimized Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;150%</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>&gt; 140%</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>&gt; 130%</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>&gt; 120%</td>
<td>19</td>
<td>0</td>
</tr>
<tr>
<td>&gt; 110%</td>
<td>28</td>
<td>0</td>
</tr>
<tr>
<td>&gt; 100%</td>
<td>55</td>
<td>0</td>
</tr>
<tr>
<td>&gt; 90%</td>
<td>76</td>
<td>35</td>
</tr>
<tr>
<td>&gt; 80%</td>
<td>133</td>
<td>84</td>
</tr>
<tr>
<td>&gt; 70%</td>
<td>196</td>
<td>153</td>
</tr>
<tr>
<td>&gt; 60%</td>
<td>275</td>
<td>212</td>
</tr>
<tr>
<td>&gt; 50%</td>
<td>350</td>
<td>309</td>
</tr>
</tbody>
</table>

- No more than 92% of bad-weather capacity (IFR) is required
- Substantial reduction in airport congestion can be achieved with existing capacity
Delay Calculation

Goal: Given airline networks, calculate expected aircraft delays, capturing effects of delay propagation

• Calculate delays for current airline networks and ‘single airline network’ using actual airport capacity values
  • Airport capacity values available for an entire year
    • Assign each day to one of 5 equal sized buckets
    • 5 realistic scenarios: one corresponding to the median of each bucket
Delay Impact of Airport Congestion: Quantifying the Propagation of Delays in a Network of Airports

- Odoni and Pyrgiotis (2009): Network model to estimate:
  - delays to every flight at individual airports,
  - how these delays propagate through the network of airports, and
  - the effect of real-time mitigation actions, e.g., ground delay programs, airline schedule recovery intervention

Start at T=0 (start of day)

Run QE for every airport:
- Airports treated as M/E/1 queuing systems.
- Calculates the expected delay on landing and takeoff per time of day.

Run DPA:
1. Determine \( t^* \), the time when the first significant delay occurs
2. Process flights operating before \( t^* \)
3. Assign delays and revise arrival and departure times
4. Update airport demand profiles

Input: Updated hourly airport demand profiles

Input aircraft itineraries plus demand and capacity profiles at each airport
## Total Delay Comparison

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Existing Network Delay (aircraft-min)</th>
<th>Single Airline Network Delay (aircraft-min)</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Good</td>
<td>7495.05</td>
<td>3552.97</td>
<td>52.60%</td>
</tr>
<tr>
<td>Good</td>
<td>14682.30</td>
<td>4090.06</td>
<td>72.14%</td>
</tr>
<tr>
<td>Normal</td>
<td>27998.76</td>
<td>5940.40</td>
<td>78.78%</td>
</tr>
<tr>
<td>Bad</td>
<td>35081.44</td>
<td>6289.88</td>
<td>82.07%</td>
</tr>
<tr>
<td>Very Bad</td>
<td>64026.52</td>
<td>7421.76</td>
<td>88.41%</td>
</tr>
<tr>
<td>Average</td>
<td>29856.82</td>
<td>5459.02</td>
<td>81.72%</td>
</tr>
</tbody>
</table>

On the order of an 80% reduction in delays could have been achieved, had there been no competition.
To model airline competition using game theory and to investigate the suitability of the Nash equilibrium solution concept for describing airline decisions

OBJECTIVES 2 AND 3
Optimization Under Slot Constraints

Maximize:
\[ \sum_{s \in S} (P_{a,s} \times Q_{a,s} - C_{a,s} \times f_{a,s}) \]

Maximize total profit = fare revenue − operating cost

Subject to:
\[ Q_{a,s} \leq \frac{f_{a,s}}{\sum_{a' \in A} f_{a',s}} \times M_s \quad \forall s \in S \]

S-curve relationship between market share and frequency share

\[ Q_{a,s} \leq Seats_{a,s} \times f_{a,s} \quad \forall s \in S \]

Seating capacity constraint

\[ \sum_{s \in S} f_{a,s} \leq MAX_{SLOTS_a} \]

Maximum number of available slots

\[ \sum_{s \in S} f_{a,s} \geq MIN_{SLOTS_a} \]

Minimum number of slots that must be utilized (Use-It-Or-Lose-It)

\[ f_{a,s} \in \mathbb{Z}^+ \quad \forall s \in S \]
Multi-Agent Model

• A system of profit maximizing agents

• Optimal frequency decision \((f_{as})\) for an airline \(a\) on segment \(s\) depends on actions by other airlines \((f_{-as})\) and is constrained by number of available slots at airport \(a\)

• **Nash Equilibrium:**
  A frequency profile \(f\) is a Nash Equilibrium if for every airline \(a\),
  \(f_a\) is the best response to \(f_{-a}\)

• **Solution Methodology:** “Myopic Best Response”
  – While there exists a carrier whose current decision is not optimal in relation to others’ decisions, re-optimize for that carrier
Solution Algorithm

- Myopic Best Response algorithm for equilibrium computation
- Successive optimizations using *Dynamic Programming*
  - Slot restrictions are the only coupling constraints across segments
  - Objective function is additive across segments
- No. of stages = No. of segments
- No. of states per stage = Maximum no. of slots

Profit(s, n) = Segment s profit due to exactly n flights per day

\[ R(0,0) = 0, \quad R(0, n) = -\infty \text{ for } n \geq 1 \]

\[ R(s, n) = \max_{0 \leq n' \leq n} \left( R(s - 1, n') + \text{Profit}(s, n - n') \right) \]

Optimal total profit = \( \max_{\text{MIN}_SLOTS \leq n \leq \text{MAX}_SLOTS} R(|S|, n) \)
Model Limitations

• Leg-based demand assumption
  – Ignores passengers connections

• Constant average fares assumption
  – Fare assumed constant for each carrier in each market
  – But in reality there is differential pricing and revenue management
  – Pricing used as a competitive tool and interacts with frequency competition

• Aircraft availability and rotation constraints ignored

• Assumption of constant aircraft sizes for each carrier on each leg
  – This assumption is partially relaxed later
A Typical Best Response Function

Region C

Region B

Region A

$(0, x_0)$

$(y_{cr}, x_i^*(y_{cr}))$

$(y_{th}, x_i^*(y_{th}))$
Central Idea of the Convergence Proof
Convergence Proof

- Best-response algorithm enters the interval I in finite iterations
- Once inside, it cannot exit the interval I
- Once inside I, use a Lyapunov function argument
  \[ L(i+1) = |\gamma_{i+1} - \gamma_{eq}| = |\gamma_{BR}(\chi_i) - \gamma_{BR}(\chi_{eq})| = \left| \int_{\chi_{eq}}^{\chi_i} \frac{\partial \gamma(\chi)}{\partial \chi} \, d\chi \right| \]
  \[ \leq \left| \int_{\chi_{eq}}^{\chi_i} \frac{\partial \gamma(\chi)}{\partial \chi} \, d\chi \right| < \int_{\chi_{eq}}^{\chi_i} 1 \, d\chi = |\chi_i - \chi_{eq}| = L(i) \]
- \( L(i) \) is nonnegative and strictly decreasing function of \( i \) at any non-equilibrium point
- At equilibrium \( L(i) = 0 \)
- Hence convergence!
Accuracy of Frequency Predictions

![Bar chart showing actual vs. model frequency predictions for various airports and airlines. The chart compares actual frequency with model frequency for airports such as ATL, BNA, DFW, MCO, MSP, ORD, and others. The x-axis represents airports and airlines, while the y-axis shows frequency ranging from 0 to 18. The chart indicates the accuracy of frequency predictions using different airlines and airports.]

- **Actual Frequency**
- **Model Frequency**
n-Player Symmetric Case

• n ≥ 2 number of identical players

• Symmetric equilibrium seems to be the only reasonable equilibrium (non-zero frequency values and less than 100% load factor)

• It’s also the ‘worst-case’ equilibrium - equilibrium with maximum ‘social’ cost

• We look at the ratio of the total cost to all competing airlines under the equilibrium with the maximum social costs to the total cost under the system optimal solution

• A measure that is a proxy for:
  – Airport congestion (and delays)
  – Airline profit degradation
  – Total cost of passenger transportation
Result Summary

- Nash equilibrium describes actual decisions reasonably well (within 6.5% error)
- Computational evidence of strong convergence properties:
  - Best response heuristic always found an equilibrium in very few iterations
  - Convergence independent of starting point
- Exact payoff functions are complex, but in the useful range usually strictly concave
Price of Frequency Competition

• Price of airline frequency competition \[
\frac{\alpha pS \ n - 1}{C \ n}
\]
  - The ratio of the total cost to all competing airlines under the equilibrium with the maximum social costs to the total cost under the system optimal solution

• Varies with
  - \(\alpha\): S-curve parameter (intensity of competition); greater the curvature of S-curve, greater is the inefficiency
  - \((pS/C)\): ‘full load profitability’; more profitable the segment, greater is the inefficiency
  - \(n\): number of competitors; greater the number of competitors, greater is the inefficiency

• A typical (median) value = 1.48 (airlines lose about 48% due to decentralization)
To investigate airline response to slot allocation strategies

OBJECTIVE 4
Slot Reduction Schemes

1) Proportionate slot reduction
   - Number of slots available to each carrier reduced by same proportion

2) Reward based slot reduction
   - Slot reduction for each carrier proportional to inverse of passengers/slot
   - Idea is to reward those who are using their slots efficiently

- Airline response evaluated using Nash Equilibrium concept and models presented in previous section
# Overall Impact

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Do Nothing</th>
<th>20% Reduction (Proportionate)</th>
<th>20% Reduction (Reward-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Operating Profit</td>
<td>$1,252,362</td>
<td>$1,568,814 (25.27%)</td>
<td>$1,565,490 (25.00%)</td>
</tr>
<tr>
<td>Passengers Carried</td>
<td>22,260</td>
<td>21,291 (-4.35%)</td>
<td>21,464 (-3.58%)</td>
</tr>
<tr>
<td>NAS Delay per Flight</td>
<td>12.74 min</td>
<td>7.52 min (-40.97%)</td>
<td>7.52 min (-40.97%)</td>
</tr>
</tbody>
</table>
Increase in Profit vs. Slot Reduction Scheme

Proportionate Slot Reduction Scheme

Reward-Based Slot Reduction Scheme
## Impact on Individual Airlines

<table>
<thead>
<tr>
<th>Carrier</th>
<th>100% Profit</th>
<th>Profit Increase</th>
<th>80% Profit</th>
<th>Profit Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>365,582</td>
<td>22.52%</td>
<td>422,943</td>
<td>15.69%</td>
</tr>
<tr>
<td>CO</td>
<td>66,450</td>
<td>10.17%</td>
<td>79,820</td>
<td>20.12%</td>
</tr>
<tr>
<td>DL</td>
<td>188,352</td>
<td>51.59%</td>
<td>274,352</td>
<td>45.66%</td>
</tr>
<tr>
<td>FL</td>
<td>36,908</td>
<td>43.30%</td>
<td>55,406</td>
<td>50.12%</td>
</tr>
<tr>
<td>MQ</td>
<td>33,630</td>
<td>29.58%</td>
<td>35,705</td>
<td>6.17%</td>
</tr>
<tr>
<td>NW</td>
<td>107,006</td>
<td>0.85%</td>
<td>127,265</td>
<td>18.93%</td>
</tr>
<tr>
<td>OH</td>
<td>34,638</td>
<td>56.31%</td>
<td>54,916</td>
<td>58.54%</td>
</tr>
<tr>
<td>UA</td>
<td>200,796</td>
<td>16.13%</td>
<td>241,936</td>
<td>20.49%</td>
</tr>
<tr>
<td>US</td>
<td>170,939</td>
<td>31.75%</td>
<td>227,897</td>
<td>33.32%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1,252,362</strong></td>
<td><strong>25.27%</strong></td>
<td><strong>1,565,490</strong></td>
<td><strong>25.00%</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scheme</th>
<th>-</th>
<th>Proportionate</th>
<th>Reward Based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slots</td>
<td>100%</td>
<td>80%</td>
<td></td>
</tr>
</tbody>
</table>
Impact of Limited Number of Aircraft Upgauges

Decrease in Number of Passengers Vs. Upgauge Percentage
To quantify the benefits and costs to various different stakeholders and the overall social welfare

**OBJECTIVE 5**
Passenger-centric analysis

- **Goal:** measure system performance through passenger delays instead of flight delays
- **Motivation:** non-linear relationship between passenger and flight delays
  - Longer flight delays lead to flight cancellations and missed connections (Bratu, Barnhart 2005)
- **Challenge:** itinerary data is not publicly available
  - Passenger delay estimates vary widely from study to study
    - For 2007 calendar year:
      - $12 Billion (as per US Congress Joint Economic Committee report, 2008)
      - $5 Billion (Air Transport Association, 2008)
      - Both studies ignore passenger delays due to cancellations and missed connections
    - Primary obstacle is the unavailability of disaggregate passenger delay data
      - Publicly available data is aggregated monthly or quarterly
- **Approach:** estimate historical passenger itineraries to calculate passenger delays
Passenger Delay Analysis

• Passenger delays calculated using an extended multi-carrier version of the passenger delay calculator (Bratu and Barnhart, 2005)

• Passenger delays approximately double that of passenger-weighted aircraft delays
  – 50% due to flight delays
  – 33% due to cancellations
  – 17% due to missed connections

• Reducing aircraft delays by about 40%, results in reduction of passenger delay minutes (in 2007) of 5.92 billion minutes of passenger delay, or $3.7 billion savings
  – Assuming $37.6/hr value of passenger time (same as the one used in JEC report), the total cost of passenger delays
Summary

• Airline service network design models that capture competitive effects have increased schedule frequency, with substantial flight and passenger delays, compared to service network designs ignoring competition

• Adding ‘robustness’ to airline schedules, holding all else constant, has (limited) impact, but is expensive

• Demand management strategies such as slot constraints at congested airports can result in reduced flight and passenger delays and increased airline profitability
QUESTIONS?